Practice Exam for Midterm 1

Evaluate the following integrals:

1. \[ \int x \arctan(x) \, dx \]

2. \[ \int_{0}^{3} \frac{x^3 + x}{\sqrt{x^2 + 1}} \, dx \]
3. $\int e^{3t} \cos(t) \, dt$

4. $\int \frac{\sqrt{x^2 - 4}}{x^3} \, dx$ (use the substitution $x = 2 \sec(u)$)
5. \[ \int_{1}^{2} t^2 \ln(t) \, dt \]

6. \[ \int \frac{3x + 2}{x^2 + 5x + 4} \, dx \]
7. Bobby’s farm has the shape of the region enclosed by the parabola $y = x^2$ and the horizontal line $y = 4$. Bobby wants to divide his farm into two parts $O$ and $P$ by a line of the farm $y = h$. Find the value of $h$ such that $O$ and $P$ have the same area.

Figure 1: Bobby’s farm
8. Let $R$ be the region bounded by the $y$-axis, $y = \frac{1}{x}$, $y = 1$ and $y = 2$.

(a) Sketch the shape of this region in the coordinate plane.

(b) Let $S$ be the solid given by rotating the region $R$ about the vertical line $x = -1$. Find the volume of $S$. 
9. Let $S$ be the region obtained by rotating the region enclosed by the $x$-axis and the curves $y = \ln(x^2)$, $x = e$ about the $y$-axis. In order to find the volume of $S$, firstly explain how you slice the region. Then write down the answer as an integral and evaluate the integral.