Reasoning Patterns in Bayesian Games
(Extended Abstract)

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ABSTRACT
Bayesian games have been traditionally employed to describe and analyze situations in which players have private information or are uncertain about the game being played. However, computing Bayes-Nash equilibria can be costly, and becomes even more so if the common prior assumption (CPA) has to be abandoned, which is sometimes necessary for a faithful representation of real-world systems. We propose using the theory of reasoning patterns in Bayesian games to circumvent some of these difficulties. The theory has been used successfully in common knowledge (non-Bayesian) games, both to reduce the computational cost of finding an equilibrium and to aid human decision-makers in complex decisions. In this paper, we first show that reasoning patterns exist for every decision of every Bayesian game, in which the acting agent has a reason to deliberate. This implies that reasoning patterns are a complete characterization of the types of reasons an agent might have for making a decision. Second, we illustrate practical applications of reasoning patterns in Bayesian games, which allow us to answer questions that would otherwise not be easy in traditional analyses, or would be extremely costly. We thus show that the reasoning patterns can be a useful framework in analyzing complex social interactions.

Categories and Subject Descriptors
I.2 [Artificial Intelligence]: Miscellaneous

General Terms
Economics, Human Factors

Keywords
reasoning patterns, Bayesian games, game theory, Bayes-Nash equilibrium, heuristics

1. INTRODUCTION
The real world is a complex place, plagued with uncertainty. Designing agents to reason, make decisions and interact with other agents in such an environment is therefore a challenging problem. The number of states that the agent needs to consider is prohibitively large even in "small" games like poker; moreover, the agent often needs to interact with others who have radically different beliefs about the situation unfolding. Common in real-world situations are private information, inaccurate beliefs about other agents or their strategies, or bounded rationality. In those cases, heuristics or limited reasoning might be employed to reach decisions faster. Furthermore, agents need to be adaptive and perform well even if the situation changes unpredictably, hence they cannot be employed with pre-computed optimal solutions.

Traditional game-theoretic approaches of modeling these systems are often unsatisfactory. If players disagree about the game being played, the situation is usually represented as a Bayesian game, in which the common prior assumption (CPA) is invoked, a requirement that the joint vector of types, describing the private information and beliefs of all the agents, is drawn according to a probability distribution that is common knowledge. The CPA usually serves to simplify the game’s representation and can be justified in some situations. However, the CPA is not always an appropriate modeling choice, especially in diverse populations of agents with different backgrounds in which agreement on a prior through repeated exposure is not warranted (see [10]). In a Bayesian game, agents are usually expected to adopt strategies comprising a Bayes-Nash equilibrium of the game.

This approach overlooks several issues. First, equilibrium solutions are hard to compute. Second, a game usually has a multitude (or even an infinity) of equilibria, and there is no principled way to select one of them. Third, in Bayesian games without a common prior there are technical difficulties (e.g., infinite belief hierarchies) that make optimal solutions very expensive to compute. Also, equilibrium strategies might not be followed by human players, as experiments have demonstrated [8]. And finally, equilibria are mathematical solutions of an optimization problem, and hence leave the actual decision-maker “out of the loop.”

Related Work
Our work aims at extending the ability for analyzing strategic situations beyond traditional game-theoretic analyses. In [7] authors explore “cognitive hierarchies,” a theory that suggests people engage in limited reasoning when analyzing a situation. Their method can be used to circumvent computational issues with equilibrium calculation, although it usually assumes a distribution of the various hierarchy depths (steps of reasoning) people are expected to engage in. Team reasoning (see [12], [13]) seeks to replace individuals as
the simplest reasoning unit with groups. The reasoning pat-
tterns, similarly, relate agents whose decisions influence one
another. Finally, the field of epistemic game theory seeks to
understand the relationship between rationality, players’ be-
lief in rationality, limited reasoning or knowledge, and game-
theoretic outcomes. The reasoning patterns aim at modeling
reasoning at a coarser level than game-theoretic analyses, re-
lexing the assumptions made by traditional game theory, yet
circumventing the complexity or the paradoxes (e.g., see [6])
that rigorous epistemic game theory has revealed.

2. THE REASONING PATTERNS

The original paper [11] defines four reasoning patterns,
which are sets of features that capture the possible effects of
an action on the acting agent’s utility. A proof is provided
that these patterns are “complete,” in the sense that, if a de-
cision of an agent cannot be associated with one of these four
reasoning patterns, then the agent’s choice of action bears
no effect on her utility. This was used to simplify games
for the purpose of computing Nash equilibria in [2]. Rea-
soning patterns (RPs) are shown to correspond to graphical
properties of the Multi-Agent Influence Diagram (MAID)
representation of the game, hence making their detection
computationally easy [1]. Experimentally, when humans are
shown advice generated by looking at the reasoning patterns
in a complex game, they make better decisions [3]. In this
paper we are extending the theory of reasoning patterns to
Bayesian games, with or without a common prior. More-
over, we show that these extended reasoning patterns can be
used to capture interesting social interactions, and help
answer questions that might otherwise be less obvious or
very costly.

To develop the theory of reasoning patterns for Bayesian
games, we rely on the graphical representation developed in
[4], in which a game is represented as a set of blocks. Each
block contains a model of the world and a set of beliefs,
while directed edges represent dependencies among blocks
according to these beliefs. Depending on whether the CPA
holds or not, the graph of blocks may be fully or sparsely
connected. The reasoning patterns developed for Bayesian
games can be explained in detail in the full version of the
paper [5].

3. USING REASONING PATTERNS TO AN-
ALYZE SOCIAL INTERACTIONS

We illustrate the usefulness of reasoning patterns in the
analysis of Bayesian games by means of an example, pre-

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