Rare Macroeconomic Disasters

Robert J. Barro¹ and José F. Ursúa²

¹ Department of Economics, Harvard University, Cambridge, Massachusetts 02138; email: rbarro@harvard.edu

² Department of Economics, Harvard University, Cambridge, Massachusetts 02138; email: jfursua@fas.harvard.edu

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Abstract

The potential for rare macroeconomic disasters may explain an array of asset-pricing puzzles. Our empirical studies of these extreme events rely on long-term data now covering 28 countries for consumption and 40 for GDP. A baseline model calibrated with observed peak-to-trough disaster sizes accords with the average equity premium with a reasonable coefficient of relative risk aversion. High stock-price volatility can be explained by incorporating time-varying long-run growth rates and disaster probabilities. Business-cycle models with shocks to disaster probability have implications for the cyclical behavior of asset returns and corporate leverage, and international versions may explain the uncovered-interest-parity puzzle. Richer models of disaster dynamics allow for transitions between normalcy and disaster, bring in post-crisis recoveries, and use the full time series on consumption. Potential future research includes applications to long-term economic growth and environmental economics and the use of stock-price options and other variables to gauge time-varying disaster probabilities.
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1. INTRODUCTION

One side effect of the macroeconomic, financial, and sovereign-debt crises of 2008-2011 is that economists became more receptive to models of asset pricing and macroeconomic dynamics that emphasize macroeconomic disasters. Theoretical and empirical research suggests that rare-disaster models have explanatory power for an array of asset-pricing puzzles. Moreover, time-varying disaster probabilities may be an important component of closed- and open-economy models of business cycles.

The probability and size distribution of macroeconomic disasters are difficult to quantify empirically because the relevant events are rare and possibly absent in short samples. Thus, the isolation of a substantial number of disasters requires long time series for numerous countries, and the data cannot be missing during many key events, such as wars and major financial crises. Fortunately, the status of long-term national-accounts data for 40 countries has been upgraded by the data effort summarized in Ursúa (2011). The combination of this panel of national-accounts data with expanded long-term information on asset returns facilitates research on rare macroeconomic disasters. We begin with an overview of these data and then review theoretical and empirical advances that use these and other data to study the macro-finance of rare disasters.

Section 2, on measurement issues, emphasizes recent improvements in the long-term national-accounts data, including annual series on real per capita consumer expenditure, C (the main available proxy for consumption), and GDP. The long-term data on returns on stocks, bills, and bonds are also discussed. The macroeconomic and financial data can be used to quantify the frequency and size distribution of rare macroeconomic disasters and to study the interplay of macroeconomic events with asset returns and prices.
Statistical analysis of the time series shows that volatilities of annual growth rates of C and GDP are similar. The data on rates of return exhibit a high equity premium (7%), a substantial term premium between bonds and bills (1-1/2%), and high volatility of stock returns (standard deviation of 32% per year). We stress the fat tails apparent in annual data on growth rates of C and GDP and in the various rates of return.

Section 3 introduces macro-finance models that incorporate rare disasters as a way to explain asset-pricing puzzles. Section 4 presents a baseline model in which disasters have instantaneous and permanent effects on levels of macroeconomic variables. This model yields tractable, closed-form solutions for asset pricing under preference specifications that include the Epstein-Zin-Weil recursive form, which separates risk aversion from the intertemporal elasticity of substitution (IES) for consumption. We assess the baseline model empirically when disaster frequencies and sizes are gauged from peak-to-trough methods using observed histograms (Section 5) or estimated power-law distributions (Section 6). A key issue is whether the coefficient of relative risk aversion required to match the observed average equity premium is “reasonable.”

Section 7 shows that the baseline model does not match the observed volatility of stock prices. This deficiency can be remedied by incorporating shifting long-run growth rates, as in the long-run-risks model considered in Section 8.

An alternative explanation for stock-price volatility, studied in Section 9, allows for shifts in disaster probability. This approach may explain an array of asset-pricing puzzles that extend beyond the high equity premium, low risk-free rate, and high stock-price volatility. A number of recent applications allow for time-varying disaster probability in business-cycle models. In an international context, discussed in Section 10, the framework with time-varying disaster
probability has been applied to the uncovered-interest-parity (UIP) puzzle, which relates to the surprisingly high returns delivered over some periods by carry-trade strategies.

Section 11 considers richer models of the dynamics of disasters, allowing for stochastic transitions between states of normalcy and disaster. In this setting, disasters arise of varying lengths and intensities, and the subsequent recoveries feature abnormally high growth. The allowance for recoveries means that disasters are less permanent than in the baseline model and, hence, have smaller effects on the equity premium. Section 12 discusses the implications of these richer models for the full time series of macroeconomic variables and rates of return. Section 13 concludes by suggesting promising avenues for future research.

2. MEASURING EXTREME MACRO-FINANCIAL EVENTS

From an empirical perspective, the lack of sufficient data on extreme macroeconomic events has been an obstacle. Because these events occur infrequently, assessments of the impact of actual and potential disasters require the pooling of information from many economies and years. Moreover, the estimation of key statistical properties, such as frequency and size distribution, requires the sample to be representative of a broader universe of economies.

Rietz (1988) introduced rare disasters into an asset-pricing model and argued that his extension helped to explain the now famous equity-premium puzzle of Mehra and Prescott (1985). The Rietz idea met skepticism concerning the lack of evidence on the low-probability depressions required by his theory. As Mehra and Prescott (1988, p. 135) argued: “Additional historical evidence in support of Rietz’s hypothesis is needed for it to be taken seriously. … The point is that to determine how useful this theory is, we must identify the possible small-probability events and try to measure the magnitudes of their probability over time.”
Barro (2006) renewed interest in Rietz’s insight by examining long-term data for many countries; thereby including numerous realizations of disaster events. The initial application relied on Maddison’s (2003) data on real per capita GDP. Unfortunately, these data are problematic; partly because of flaws in construction, especially at times of disasters such as wars,\(^1\) and partly because asset-pricing models typically apply to consumption, not GDP. Barro and Ursúa (2008) extended the data set to include estimates of per capita consumption, C (based primarily on personal consumer expenditure) and to improve the measurement of GDP for many countries. The annual data apply over periods extending back before World War I. These data, now available for 40 countries and described in Ursúa (2011), are discussed next, together with a review of historical information on asset returns.

2.1. Recent Improvements in Data on National-Accounts Variables and Asset Returns

The typical variables of interest in the macro-finance literature are growth rates of the main macroeconomic aggregates, real per capita consumption, C, and GDP, and real rates of return on financial assets, notably stocks and government bills. Gaps in the data hinder analyses of rare disasters because of a sample-selection problem, whereby data are most likely missing during the worst crises. Therefore, in a time-series context for a single country, it is important to have estimates for the most difficult periods, often wars. Similarly, in a cross section, it is important not to omit countries with the most difficult macroeconomic histories. Recent extensions of the long-term data to include several challenging cases—China, Russia, and Turkey—represent major improvements in this regard. The basic spirit of our empirical approach to rare macroeconomic disasters is to pool information from the largest possible number of countries and years. Particularly unsatisfactory in this regard is the tendency of

\(^1\)Barro and Ursúa (2008, Table A1) provide a detailed analysis of these measurement problems.
researchers to rely on data for the United States, a practice that, even with the inclusion of the recent Great Recession, puts far too much emphasis on a mostly tranquil macroeconomic history aside from the Great Depression of the early 1930s.

Until recently, the best macroeconomic panel data were the per capita GDP series assembled by Angus Maddison. These series constitute a monumental contribution that has been widely used, notably in works on historical macroeconomic and financial crises, such as Bordo, Eichengreen, Klingebiel and Martinez-Peria (2001) and Reinhart and Rogoff (2009). Shortcomings of Maddison’s data include his tendency to fill in missing observations during crisis periods by interpolating between benchmarks or using information from other countries. Additional problems include lack of data on consumption and omission of major countries. These considerations motivated Barro and Ursúa (2008) to construct the new data set described in Ursúa (2011). The construction of these data was challenging, akin to macroeconomic archaeology. The goal was to improve as much as possible on Maddison’s GDP series and to build a comprehensive new panel for C. The sample-selection criterion was to assemble continuous time series since at least before World War I, while retaining high quality standards. Various methods were implemented to cover periods originally missing or inadequately covered in standard sources.

The macroeconomic data are publicly available\(^2\) and are summarized in Table 1. The information covers 42 countries, falling into five regional groups: Southeast Asia, Latin America, Western Europe, Western Offshoots, and Others. Country starting dates vary, ranging from the early 19th century to 1913. An asterisk indicates that the series has missing data points. The present analysis applies to 40 countries for GDP (21 OECD, 19 non-OECD) and 28 for C

(18 OECD, 10 non-OECD). The corresponding number of annual data points for growth rates from 1870 to 2009 is 5242 for GDP and 3527 for C. Figure 1a shows level series for four major countries (Germany, Japan, United Kingdom, and United States), and Figure 1b show series for four countries with newly available information (China, Egypt, Russia, and Turkey).

Many applications require corresponding information on asset returns. For example, Barro and Ursúa (2009) and Ursúa (2010) used data on real returns on stocks, bills, and bonds. The major source of these data is Global Financial Data (Taylor, 2005), but the coverage was expanded with additional sources, including Morningstar and other sources for Argentina, Brazil, Japan, and Mexico. Table 1 describes the long-term data on these asset returns.

2.2. Statistical Properties of Macroeconomic Growth Rates and Asset Returns

Yearly growth rates and rates of returns are the basic units of measurement for studying extreme macro-financial events over the long run for up to 40 countries. Table 2 shows statistics for growth rates of real per capita GDP and personal consumer expenditure, C, and real rates of return on stocks, bills, and bonds. The real rates of return are computed arithmetically based on total returns and deflation by consumer price indexes. The table shows the mean, standard deviation, and excess kurtosis for each variable for the full sample (starting as early as 1870 and ending in 2009) and the post-WWII period (1948-2009). The statistics apply to countries with data, broken down into the “world” (full set of countries), OECD, and non-OECD.

Average growth rates of per capita C and GDP are around 2% per year for full samples and somewhat higher—between 2.5% and 2.8%—in the post-WWII period. For OECD

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3Our term “OECD” excludes Turkey and recent members. GDP data for Malaysia and Singapore are included in the basic data set but excluded from our analysis because of missing data around WWII. Some of the analysis also omits the Philippines for GDP because of a gap in data around WWII. Greece is included for our GDP analysis despite a missing data point for 1944. We think that 42 countries (21 OECD) come close to those with potentially useable long-term national-accounts data. Possibilities for extension include closing the gaps in GDP data around WWII for Greece, Malaysia, Philippines, and Singapore. A possible 43rd country is Ireland, but we have not yet been successful. Maddison (2003) provides data since 1921, but figures for 1938-1946 come from interpolation.
countries, the standard deviations are around 3% in the post-WWII period but higher, nearly 6%, in full samples. The main reason for this difference is that the full samples contain many realizations of disasters between 1914 and 1946, a period featuring the two world wars, the Great Depression, and the Great Influenza Pandemic. In contrast to observations for the post-WWII United States (such as Campbell and Deaton [1989]), C growth in OECD countries is not smoother than GDP growth—the standard deviations for these two variables are similar.

For non-OECD countries, standard deviations are again higher in the full sample than in the post-WWII period. These standard deviations are also higher in both samples than in the OECD. In the non-OECD, the standard deviation for C growth exceeds that for GDP in both samples. Part of this pattern can reflect poorer measurement for C than for GDP. However, C tends also to decline more than GDP during wartime disasters, in which military spending rises substantially.

Excess kurtosis is positive in both samples for C and GDP growth. These results indicate fat tails; that is, fatter than the normal density. The pattern of high excess kurtosis applies especially to the full sample for OECD countries, likely reflecting the numerous disaster realizations between 1914 and 1946.4

For asset returns, the means for the “world” over the full sample are 8.4% for stocks, 1.3% for bills, and 3.0% for bonds. Thus, these data reveal an average equity premium (stocks versus bills) of 7.1%. However, this value reflects leverage in corporate financial structure; an adjustment assuming a constant debt-equity ratio of 0.5 implies an average unlevered equity premium around 5%. The average term premium (roughly 10-year bonds versus 3-month bills) was 1.7%. The well-known volatility of stock returns shows up as a high standard deviation,

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4Ursúa (2011) shows from bootstrap methods that excess kurtosis is significantly positive for growth rates of C and GDP over full samples. Skewness differs insignificantly from zero.
32%, for the world over full samples (25% for OECD, 44% for non-OECD). There is also substantial volatility of bill returns (11% for the world over full samples) and bond returns (13%). The high values of excess kurtosis signal fat tails for all of the rates of return.\footnote{Ursúa (2011) finds from bootstrap methods that excess kurtosis is significantly positive for the three rates of return over full samples. Skewness is significantly positive for stock returns, significantly negative for bill returns, and insignificantly different from zero for bond returns.}

The new macroeconomic data allowed Ursúa (2011) to use power-law distributions to gauge the fatness of the tails for C and GDP growth. Fat tails were important for negative and positive outcomes, with the former representing disasters and the latter (“bonanzas”) reflecting mainly recoveries from disasters. Tail fatness was stronger for OECD countries than for non-OECD, consistent with the findings on excess kurtosis in Table 2. This pattern likely arises because the biggest disasters associate particularly with WWII in OECD countries.

### 2.3 Macroeconomic Disaster Events

Barro and Ursúa (2008) followed Barro (2006) by using an NBER (National Bureau of Economic Research)-style peak-to-trough measurement of the sizes of macroeconomic contractions. Starting from the annual time series, proportionate decreases in C and GDP were computed peak to trough over one or more years, and declines by 10% or greater were considered. For the four countries in Figure 1a, the events isolated from this method were:

- Germany 4 disasters each for C and GDP,
- Japan 2 disasters each for C and GDP,
- United Kingdom 2 disasters each for C and GDP,
- United States 2 C disasters and 5 GDP disasters.

The largest contractions in this group were the decline in Germany’s GDP by 74% with a trough in 1946 and in its C by 41% with a trough in 1945, the fall in Japan’s C by 64% with a trough in 1945 and in its GDP by 50% with a trough in 1944. The worst U.S. contractions were for GDP by 29% with a trough in 1933, for C by 21% with a trough in 1933, and for C by 16% with a
trough in 1921. The United Kingdom illustrates a pattern where C falls proportionately more than GDP during wartime. The declines in C are 17% with a trough in 1918 and 17% with a trough in 1943, but GDP disasters do not apply to U.K. GDP during the world wars.

Table 3 shows results from the peak-to-trough technique applied to the four countries with newly assembled macroeconomic data: China (long-term data for GDP only), Egypt, Russia (including Soviet Union), and Turkey (including Ottoman Empire). The disasters isolated number 5 for China’s GDP, 5 for Egypt’s GDP and 6 for its C, 7 for Russia’s GDP and 6 for its C, and 8 for Turkey’s GDP and 6 for its C. Some of these events are among the largest depressions ever witnessed: the declines for Russia’s C and GDP by 71% and 62%, respectively, in WWI and the Russian Revolution/Civil War; the fall by 58% in Russian C during WWII; the decline in China’s GDP by 50% from 1936 to 1946 (including WWII); the falls by 49% and 45% in Turkey’s C and GDP, respectively, during WWI; and the decrease in Russia’s GDP by 48% in the transition period 1989-1998.

Figure 2 uses histograms to provide an overview of disaster events applying to 28 countries for C and 40 for GDP. (These samples end in 2006 and include countries with data from before WWI.) The peak-to-trough method isolates 125 disasters for C and 183 for GDP. The average disaster sizes, subject to the threshold of 10%, were similar for the two measures: 0.216 for C and 0.208 for GDP. The mean durations were also similar: 3.7 years for C and 3.6 for GDP.

Table 4 relates the macroeconomic crises to major historical events. World War II is prominent, featuring 21 crises for C (average size of 0.33) and 25 for GDP (average of 0.37). World War I and the Great Depression also stand out. The period 1920-22 may reflect the Great

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6 Border changes were important for Russia and Turkey and also apply to other countries. See Ursúa (2011) on the treatment of border changes—basically, the level series come from smooth pasting of the growth rates from before and after each change.
Influenza Pandemic. The post-WWII period is comparatively tranquil, especially for the OECD, and still appears this way if we extend the end of the sample from 2006 to 2009.\footnote{If we extend to 2009 to include the recent Great Recession, we find many contractions but none in our samples that reach the 10\% threshold. For GDP, Iceland and Japan have declines by 9\%, and Finland and Russia have declines by 8\%. For C, Spain has a decline by 9\% and Mexico by 8\%. Iceland has a decline in C by 25\%, but it is not in our 28-country sample.}

A listing of disaster events—by country, timing, and size—is in Barro and Ursúa (2008, Tables C1 and C2) and can be extended to incorporate Table 3. Barro and Ursúa (2008, Table 9) show that there is no clear pattern on whether C or GDP reaches its trough first during crises; 59\% of the events have the same trough year for C and GDP.

Barro and Jin (2011) show that the frequency distribution for C and GDP disaster sizes (Figure 2) can be characterized by power laws, thereby fitting with Gabaix’s (2009) discussion of the many applications of power-law distributions in finance and other areas. If \( b \) is the proportionate disaster size, the power law applies to the transformed variable, \( z = 1/(1-b) \), which is the ratio of normal to disaster C or GDP. The power law holds for \( b \) above some threshold, taken to be 0.095, which translates into a threshold for \( z \) of 1.105. The single power-law density is then

\[
f(z) = Az^{-(\alpha+1)},
\]

for \( z \geq 1.105 \), where \( A>0, \alpha>0 \). The lower the exponent, \( \alpha \), the fatter the tail of large disasters. Barro and Jin (2011, Table I) show that extending to a double power law substantially improves the fit and that the upper-tail exponent is the key parameter for asset-pricing results. The estimated exponent is 4.2 (s.e.=0.9) for C and 3.5 (s.e.=1.0) for GDP. We use these results later when discussing the link between tail behavior and the coefficient of relative risk aversion, \( \gamma \).
3. RARE DISASTERS AND MACRO-FINANCE MODELS

Models in macro-finance with extreme events have emphasized the interplay between rare disasters and asset-pricing puzzles. The earliest example is Rietz (1988), who introduced a low-probability crash state to explain the equity-premium puzzle of Mehra and Prescott (1985). Another example is Naik and Lee (1990), who developed a continuous-time version of Lucas’s (1978) endowment-economy model to study the pricing of options when random jumps affect the value of the underlying asset. During the 1990s, the literature focused on non-disaster explanations for the equity-premium puzzle, such as the heterogeneous-consumers model of Constantinides and Duffie (1996) and the habit-formation paradigm of Abel (1990) and Campbell and Cochrane (1999). Constantinides (2002) reviews advances in these directions.

In the mid 2000s, the literature returned to the asset-pricing implications of disasters. Longstaff and Piazzesi (2004) stressed the sensitivity of cash flows to large economic shocks and presented a calibrated model with substantially higher equity premia than in standard frameworks. Bansal and Yaron (2004), in an approach now known as the long-run-risks model, emphasized the persistence of changes in expected growth rates and variances of growth rates as determinants of risk premia and the volatility of asset prices. Barro (2006) focused on rare macroeconomic disasters of the short-run type, thereby reviving Rietz’s insight. This model’s tractability and empirical success make it a good platform for understanding how the potential for rare disasters may resolve various asset-pricing puzzles.

4. A BASELINE MODEL OF ASSET PRICING WITH RARE DISASTERS

As developed in Barro (2006, 2009), the baseline model is a variant of Lucas’s (1978) representative-agent, fruit-tree economy, with exogenous and stochastic production. The
The parameter $g \geq 0$ represents exogenous productivity growth. The first random term, $u_{t+1}$, reflects “normal” macroeconomic fluctuations and is assumed to be i.i.d normal with zero mean and constant variance, $\sigma^2$. The second shock, $v_{t+1}$, picks up rare disasters—events in which output contracts over a period by a fraction $b$, where $0 < b < 1$. These events occur with constant probability $p \geq 0$ per period; hence,

probability $1 - p$: $v_{t+1} = 0$,

probability $p$: $v_{t+1} = \log(1 - b)$.

The disaster size, $b$, is subject to some frequency distribution, such as the power-law density discussed before. In this model, the expected growth rate of $C$ and GDP is

$$g^* = g + (1/2)\sigma^2 - p\cdot Eb .$$

(3)

The i.i.d. property implies that shocks have permanent effects on level variables. As shown in Barro (2009), this property yields closed-form solutions for asset pricing under two familiar specifications of preferences for the representative agent: power utility and the recursive preferences developed by Epstein and Zin (1989) and Weil (1990), henceforth called EZW preferences. The advantage of the more general EZW formulation is that it separates the
coefficient of relative risk aversion, $\gamma$, from the reciprocal of the intertemporal elasticity of substitution (IES), $\theta$.\(^8\) As stressed by Bansal and Yaron (2004), EZW preferences avoid counterintuitive predictions from the power-utility case, $\gamma=\theta$, in the context of sufficient risk aversion ($\gamma>1$) for the model possibly to account for the equity premium. Power utility then implies, implausibly, that a rise in the expected growth rate, $g^*$ in Eq. (3), lowers the stock price-dividend ratio, whereas an increase in uncertainty ($\sigma$ or $p$ or an outward shift in the density for $b$) raises this ratio.

As mentioned, EZW preferences allow for a separation between $\gamma$ and $\theta$. Epstein and Zin (1989) and Restoy and Weil (1998, p. 4) show that the first-order optimization condition for the representative agent’s choices of consumption over time is\(^9\)

$$
\beta^{(1-\gamma)/(\gamma-\theta)} \cdot E_t \left[ \frac{C_{t+1}}{C_t} \right]^{-\theta} \cdot R_w^{(\theta-\gamma)/(1-\theta)} \cdot R_{t+1} = 1,
$$

where $\beta$ ($0<\beta<1$) is the one-period discount factor, $R_{w,t+1}$ is the gross return from $t$ to $t+1$ on overall wealth (corresponding to ownership rights on trees in the Lucas-tree model), and $R_{t+1}$ is the gross return from $t$ to $t+1$ on any asset. (The rate of time preference, $\rho$, equals $[1-\beta]/\beta$.) Power utility corresponds to $\gamma=\theta$ and, therefore, to the familiar consumption-based asset-pricing formula:

$$
\beta \cdot E_t \left[ \frac{C_{t+1}}{C_t} \right]^{-\theta} \cdot R_{t+1} = 1. \tag{5}
$$

\(^8\)The formulation of Kreps and Porteus (1978) emphasized attitudes toward early versus late resolution of uncertainty, and Epstein and Zin (1989) began with this perspective. The usual EZW preferences with $\gamma>\theta$ imply that people prefer early resolution. This result is puzzling—why are risk aversion and intertemporal substitution closely linked to preferences about early versus late resolution? The situation is reminiscent of the tight link between risk aversion and intertemporal substitution under power utility. Perhaps an analogous extension can be made to the EZW framework to eliminate the constraint that there are only two degrees of freedom among three apparently distinct dimensions of preferences—risk aversion, intertemporal substitution, and early versus late resolution of uncertainty.

\(^9\)This analysis assumes that the representative agent’s relative risk aversion, $\gamma$, is constant. Empirical support for this familiar specification appears in Brunnermeier and Nagel (2008) and Chiappori and Paiella (2008).
Let $P_t$ be the price of an unlevered equity claim on a Lucas tree. This asset is the only one in positive aggregate net supply in this model. The dividend is the fruit, which equals $C_t$. Therefore, the gross return on wealth is

$$R_{w,t+1} = \left( \frac{C_{t+1} + P_{t+1}}{P_t} \right) = \left( \frac{C_{t+1}}{C_t} \right) \cdot \left( \frac{1 + V_{t+1}}{V_t} \right),$$

where $V_t \equiv P/C_t$ is the price-dividend ratio. Substitution into Eq. (4) yields:

$$\beta^{1(1-\gamma)} \cdot E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \cdot \left( \frac{1+V_{t+1}}{V_t} \right)^{(\theta-\gamma)(1-\theta)} \cdot R_{t+1} \right] = 1. \quad (6)$$

If the shocks to $C_{t+1}/C_t$ (including disasters) are i.i.d., $V_{t+1} = V_t = V$ holds in Eq. (6), and the condition again takes the usual form:

$$\beta^* \cdot E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \cdot R_{t+1} \right] = 1 \ , \quad (7)$$

where $\beta^*$ is a constant that depends on all the underlying parameters. The two differences from Eq. (5) are that the exponent on consumption growth is $\gamma$, not $\theta$, and $\beta^* \neq \beta$. However, since $\beta^*$ is constant, this last difference affects levels of rates of return but not differences between rates, such as the equity premium.

Barro (2009) used Eq. (7), along with the process for $C_t$ in Eq. (2), to generate a formula for the unlevered equity premium that applies when the period length approaches zero:

$$r^e - r^f = \gamma \sigma^2 + p \cdot E\{b \cdot [(1-b)\gamma-1]\} \ , \quad (8)$$

where all terms are measured per unit of time (say per year), $r^e$ is the expected rate of return on unlevered equity, and $r^f$ is the risk-free rate. The first term on the right-hand side is the standard one for normal business fluctuations, as in Mehra and Prescott (1985). As in their analysis, this term is negligible for reasonable values of $\gamma$ and $\sigma$. The second term involves rare disasters and
enters multiplicatively with \( p \), the disaster probability.\(^{10} \) This disaster term ends up doing almost all the work in explaining the equity premium.

The expression in curly brackets in Eq. (8) has a straightforward interpretation under power utility, \( \gamma = \theta \). Then this term is the product of the proportionate decline in equity value during a disaster, \( b \), and the excess of marginal utility of consumption in a disaster state over that in a normal state, \( (1-b)^{\gamma} - 1 \). Note that, in the present (i.i.d.) setting, the proportionate fall in equity value during a disaster, \( b \), equals the proportionate fall in C and GDP during the disaster. Equation (8) can be rewritten as

\[
 r^e - r^f = \gamma \sigma^2 + p \left[ E(1-b)^{\gamma} - E(1-b)^{1-\gamma} - Eb \right]. \tag{9}
\]

Hence, the key properties of the distribution of \( b \) are the expectations of the variable \( 1/(1-b) \) taken to the powers \( \gamma \) and \( \gamma - 1 \). (The \( Eb \) term has a minor impact.)

Equation (9) is best viewed as applying to short periods, approximating continuous time. In the limit, disasters arise as downward jumps at an instant of time, and the disaster size, \( b \), has no time units. In contrast, the underlying data on C and GDP are annual flows. In relating the data to the theory, there is no reason to identify disaster sizes, \( b \), with large contractions in C or GDP observed particularly from one year to the next. In fact, Figure 2 and Table 4 demonstrate that the major disaster events—exemplified by the world wars and the Great Depression—feature cumulative declines over several years, with durations of varying length. In Barro and Ursúa (2008), the disaster jump sizes, \( b \), in the continuous-time model were approximated empirically by the peak-to-trough measures of cumulative, proportionate decline. We first

\(^{10}\) The identification of the risk-free rate, \( r_f \), with the real return on government bills is an approximation, particularly if governments sometimes default formally or through surprise inflation. If defaults occur only during disasters, for which the probability of default is \( q \) and the default fraction is \( b \), then \( p \) in Eq. (8) is replaced by \( p(1-q) \). Therefore, a higher \( q \) reduces the equity premium measured as the difference between stock and bill returns.
consider the explanatory power of this approach and then explore ways to improve on the peak-to-trough measurement.

5. USING HISTOGRAMS FOR PEAK-TO-TROUGH DISASTERS

Barro and Ursúa (2008) gauged the moments involving disaster sizes, \( b \), in Eq. (9) by histograms as in Figure 2, applying to disasters of size 10% or larger and using alternative values of \( \gamma \). The disaster probability, \( p \), was estimated from the empirical frequency of entry into disaster states as 3.6% per year for C and 3.7% for GDP. This methodology assumes that the same process for generating macroeconomic disasters applies within countries over time and across countries. Given this assumption—which provides enough disaster realizations to pin down the relevant parameters with high precision—it is reasonable to use a rational-expectations approach to estimate \( p \) and the size distribution of disasters. That is, agents’ expectations are assumed to correspond to those generated by the true process (as estimated). We discuss later models in which the disaster probability varies over time.

A principal conclusion from Barro and Ursúa (2008, Tables 10 and 11) was that an unlevered equity premium around 5% accorded with the data for C and GDP if \( \gamma \) was around 3.5. Hence, the required risk aversion was substantial but not astronomical. The results were similar for C and GDP, did not change greatly when only OECD data were used, and did not depend much on whether the threshold for declaring a disaster was the assumed 10% value or something higher, such as 15%. However, the results depended on the inclusion of the largest disaster events, many of which associated with wars.

The model’s equity premium in Eq. (9) does not depend on some parameters, such as the expected growth rate \( g^* \) in Eq. (3) and the rate of time preference, \( \rho \) (which equals \( (1-\beta)/\beta \)).
However, levels of rates of return, including the risk-free rate, depend on these parameters. In the calibration of the model, \( \rho \) was chosen—given the other parameter values—to accord with a risk-free rate of 1%.\(^{11}\) Therefore, while the fitted model is consistent with this low risk-free rate, this fit does not constitute a test of the model.

6. USING POWER LAWS FOR DISASTER SIZES

Barro and Jin (2011, Table 1) used the same underlying information on disaster events but estimated the moments involving disaster sizes, \( b \), from the estimated power-law distribution discussed before, rather than histograms. The key parameter for the equity premium in this parametric form is the exponent, \( \alpha \), applicable to the upper tail of the power-law density. The form of Eq. (9) implies that the equity premium involves a race between \( \gamma \)—where a higher value implies a larger equity premium—and the fatness of the upper tail for large disasters—where a higher \( \alpha \) implies a thinner tail and, therefore, a smaller equity premium. The equity premium is finite only if \( \gamma < \alpha \).\(^{12}\) Therefore, with a finite observed unlevered equity premium (about 5%) and an estimated \( \alpha \) around 4, the estimated \( \gamma \) (the value required to match the equity premium) has to be below 4. The results implied an estimated \( \gamma \) of 3.0 (s.e.=0.5) from the C data and 2.8 (s.e.=0.6) from the GDP data.

The likely reason that the parametric approach to gauging disaster sizes (based on power-law distributions) produces smaller estimates of \( \gamma \) than the histogram approach is that the latter method is especially sensitive to a selection bias that screens out the worst disasters from the sample. This selection seems inevitable since economies that are nearly totally destroyed are

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\(^{11}\)In Barro and Ursúa (2008, Tables 10 and 11), the required \( \rho \) was 0.045 with the C data and 0.052 with GDP. The corresponding effective rates of time preference, \( \rho^* \), corresponding to \( \beta^* \) in Eq. (7), were 0.029 and 0.037, respectively.

\(^{12}\)Similarly, Weitzman (2007) shows that the equity premium can be infinite when the underlying shocks are log-normally distributed with an unknown variance. In this context, the frequency distribution for asset pricing is the \( t \)-distribution, for which the tails can be sufficiently fat to generate an infinite equity premium.
unlikely to have data. However, Ursúa’s (2011) recent extension to cases that had been challenging in terms of data and that also feature large disaster events (Russia, Turkey, and China) has lessened the extent of this selection bias.

7. STOCK-PRICE VOLATILITY

One shortcoming of the baseline model is its failure to match the observed high volatility of stock returns. The model’s standard deviation of unlevered stock returns equals that for growth rates of $C$ and GDP, but Table 2 shows for the world over the full sample that the standard deviations were 6.4% for $C$ growth, 6.0% for GDP growth, and 31.7% for stock returns. Allowing for leverage in corporate financial structure explains only part of this discrepancy. The basic problem is that the model implies a constant stock price-dividend ratio, $V$, whereas this ratio is volatile in the data.

In the baseline model, corresponding to Eqs. (2) and (7), the formula for the (constant) dividend-price ratio, $I/V$, is, as in Barro (2009):

$$\frac{1}{V} = \rho + (\theta - 1) \cdot g^\ast - \left(\frac{1}{2}\right) \cdot \gamma \cdot (\theta - 1) \cdot \sigma^2 - p \cdot \left(\frac{\theta - 1}{\gamma - 1}\right) \cdot [E(1 - b)^{1-\gamma} - 1 - (\gamma - 1) \cdot Eb], \quad (10)$$

where $\rho$ is the rate of time preference (equal to $[1-\beta]/\beta$) and $g^\ast = g + (1/2)\sigma^2 - p \cdot Eb$ is the expected growth rate from Eq. (3). Thus, as in Bansal and Yaron (2004), with EZW preferences, the model has “reasonable” properties for effects of one-time changes in the expected growth rate and uncertainty only if $\theta < 1$; that is, if the IES $\geq 1$. In this case, the price-dividend ratio, $V$, rises with an increase in $g^\ast$ and falls with an increase in uncertainty ($\sigma, p$, or an outward shift in the distribution of $b$).

Equation (10) suggests that volatility in $V$—and, hence, in stock returns—can be generated by variations in $g^\ast$ (the long-run risks model of Bansal and Yaron [2004]) or by
variations in parameters, such as the disaster probability, $p$, that govern uncertainty (as in Gabaix [2010], Gourio [2008, 2010], and Wachter [2011]). What is less clear is whether these fixes with regard to volatility have much implication for asset returns, including the equity premium. We consider this issue in the next three sections.

8. SHIFTING LONG-RUN GROWTH RATES

Suppose that shifts to the expected growth rate, $g^*$, occur independently of the other shocks in Eq. (2)—the assumption made by Bansal and Yaron (2004), henceforth BY. With the IES $> 1$, an increase in $g^*$ raises the price-dividend ratio, $V$, as in Eq. (7), and leads, thereby, to a high stock return. Therefore, variability in $g^*$ can generate volatility in stock returns. However, since the realization of $C_{t+1}$ is independent of the shock to $g^*$ that occurs at $t+1$, the movements in $g^*$ do not create non-zero covariance between stock returns and contemporaneous consumption growth. If preferences for consumption were time separable, this lack of covariance implies that the variability of $g^*$ would not influence the equity premium.

However, in an EZW world, preferences over consumption are not time separable. Rather, if $\gamma > \theta$ and $\theta < 1$, as already assumed, EZW preferences imply complementarity between $C_{t+1}$ and anticipated later values of $C$. Because of this complementarity, an increase in $g^*$ at date $t+1$ reduces the value of the pricing kernel (through the term containing $V_{t+1}$ in Eq. [6]) in the way that normally follows from a rise in $C_{t+1}/C_t$. Therefore, the covariance pattern is that marginal utility of consumption is low when stock returns are high. This channel explains why shifting long-run mean growth rates contribute to the equity premium in the BY model.

With endogenous investment, shocks to $g^*$ can generate covariance between stock returns and consumption growth. However, with the IES $> 1$, a rise in $g^*$ tends to reduce contemporaneous consumption, implying that stock returns covary positively with the marginal utility of consumption. Therefore, in this setting, the variability of $g^*$ tends to reduce the equity premium.
The BY model also contains a time-varying variance of the long-run growth rate. An increase in this variance amounts to a rise in uncertainty. With the assumed configuration of preference parameters in the EZW setting, a rise in uncertainty at date \( t+1 \) lowers the price-dividend ratio, \( V_{t+1} \)—this result holds in Eq. (7) for analogous effects from shifts in the uncertainty parameters \( \sigma \) and \( p \). Therefore, shifting uncertainty helps to explain volatility of stock prices. Effects on the equity premium depend again on the lack of time separability of consumption under EZW preferences. Specifically, Eq. (6) implies that the negative effect of a rise in uncertainty on \( V_{t+1} \) raises the value of the pricing kernel, thereby creating a covariance pattern where marginal utility of consumption is high when stock returns are low. Hence, time-varying uncertainty reinforces the effect on the equity premium from a time-varying mean growth rate.

Bansal and Yaron (2004) find by calibrating their model with U.S. data that matching the observed average equity premium depends on high risk aversion; the required \( \gamma \) is around 10. Nakamura, Sergeyev, and Steinsson (2011) get similar results in an extended version of the BY model fit to long-run data on consumer expenditure for 16 OECD countries. The reliance on very high risk aversion is not surprising because the effects of long-run risks on the equity premium depend on complementarity between present and future consumption. Although this channel exists with EZW preferences, the linkage turns out to be too weak to explain much of the equity premium when \( \gamma \) takes on “reasonable” values. Therefore, our conclusion is that variation in long-run mean growth rates and in variances of these growth rates may usefully supplement

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\(^{14}\)Bansal and Yaron (2004, p. 1492) justify a \( \gamma \) as high as 10 by saying: “Mehra and Prescott (1985) argue that a reasonable upper bound for risk aversion is around 10.” However, Mehra and Prescott were actually arguing that, in the context of power utility, a \( \gamma \) of 10 was at the outer bound of what was conceivable. Nakamura, Sergeyev, and Steinsson (2011, p.3) observe that an agent with a \( \gamma \) of 10 would turn down a 50-50 gamble that raised consumption by a factor of 1 million or lowered it by 1%.
analyses that include disaster risk but probably cannot be the main basis for explaining the equity premium and related asset-pricing puzzles explored by Gabaix (2010).

9. SHIFTING DISASTER PROBABILITIES

Gabaix (2010) allows for time-varying probability and severity of disasters; as already noted, these features can generate volatility of price-dividend ratios. Aside from the high equity premium, low risk-free rate, and volatility of stock returns, Gabaix shows that the framework can account for a number of other asset-pricing puzzles. These puzzles include the predictability of stock returns based on price-dividend ratios, the typically upward-sloping nominal yield curve for bonds, the high price of deep out-of-the-money puts on stock-price indexes, and the high corporate-Treasury yield spread (compared with the underlying probability of corporate default). For the last result, the key point is that corporate bonds, especially the highest-rated issues, have their defaults concentrated into the worst of economic times when the marginal utility of consumption is high. The same point explains high prices of deep out-of-the-money puts on stock-price indexes.

Gourio (2010) introduces an exogenous, persistent, time-varying disaster probability into a closed-economy real business-cycle model. Realizations of this shock affect macroeconomic variables, partly through direct influences on productivity and partly through effects on capital accumulation. The shock also influences asset prices. In his preferred calibration, which features an IES>1, a rise in disaster probability leads to a decline of output, investment, stock prices, and the risk-free interest rate and to a rise in the expected rate of return on stocks. Therefore, time-varying disaster probabilities create a counter-cyclical pattern for the equity premium and a procyclical pattern for the risk-free rate.
Gourio (2011) extends the model to include firms’ choices of financial structure, equity versus debt. An expansion of debt has tax advantages but also raises expected bankruptcy costs, and a rise in disaster probability makes the latter consideration more important. Therefore, an increase in disaster probability raises the cost of capital, featuring a rise in the spread between the corporate yield and the risk-free rate. Moreover, as in Gabaix (2010), this spread expands relative to the objective probability of corporate default—because of the larger risk weight associated with disaster-related default. The responses to a higher disaster probability include a reduction in corporate leverage and a greater cutback in investment than in the original model. To put things in reverse, a fall in perceived disaster probability up to 2006 would have raised corporate leverage and led, thereby, to greater vulnerability to a financial shock of the sort experienced in 2007-2009.

10. GAUGING TIME-VARYING DISASTER PROBABILITIES

The models considered in the previous section rely on time-varying disaster probabilities. However, it is a serious empirical challenge to measure these probabilities, as assessed contemporaneously by agents. Even when the disaster probability, \( p \), is constant across countries and over time and is computed based on rational expectations, the empirical estimation of \( p \) is difficult because disaster events are infrequent and may be absent in small samples, such as the post-WWII period for the United States and other OECD countries.\(^\text{15}\) Therefore, reliable estimation required our long-term panel of national-accounts variables, which included several thousand annual data points that generated 125 rare-disaster realizations for C and 183 for GDP (Figure 2). The estimation problem is compounded if \( p \) is allowed to vary across countries or over time within countries, although our estimation procedure would still work if the allowable

\(^{15}\text{See n.7 on the impact of inclusion of observations on the Great Recession up to 2009.} \)
variations in $p$ were limited (for example, to distinguish OECD from non-OECD or to allow for occasional breaks over time for the world).

An alternative approach uses stock-price-index options to infer the disaster probability, $p_t$, that agents perceive. Suppose, to begin, that agents have power utility and there is a fixed size distribution of disasters. In this case, put-option prices on a stock-price index of given maturity depend on $p_t$, the strike price, and the coefficient of relative risk aversion, $\gamma$. At any point in time, the model implies a relationship of option price to strike price (related to “smile” curves), and the conformity of the data with this prediction could be checked. Variations in $p_t$ shift the option-price/strike-price graph, and such shifts could be used to infer changes in $p_t$. However, the analysis is more complicated under EZW preferences.

Bollerslev and Todorov (2011), henceforth BT, use options prices on the S&P 500 from 1996 to 2008 to back out jump (disaster) risk in the underlying stock-price index as priced by investors. The estimation uses close-to-maturity deep out-of-the-money options, thereby relying on claims that are worthless without disaster risk. Their procedure generates “risk-neutral probabilities” for jumps of various sizes (BT, Table 1, column 2). BT then back out equity premia by comparing these measures with objective jump probabilities derived from futures contracts on the S&P 500 (BT, Table 1, column 3). A key conclusion (BT, p.22, n. 33) is that the median of the estimated risk premium due to rare events is 5.6% per year, a large portion of the average premium of around 7%. Hence, BT’s results support our analysis in which the bulk of the explained equity premium came from disaster risk. BT’s estimates (Figure 2, upper panel)

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16Bollerslev and Todorov (2011, Table 1) show that, for objective probabilities, large jumps are rare but reasonably symmetric for positive and negative outcomes. However, for risk-neutral probabilities, the main action reflects negative jumps; bonanzas play a minor role, consistent with our neglect of these episodes in our analysis of asset pricing.
also reveal substantial time variation in the equity risk premium associated with rare events. These results may be interpretable in terms of time-varying disaster probability.

Backus, Chernov, and Martin (2011), henceforth BCM, also use options prices for contracts on the S&P 500, in this case from 1987 to 2003. In contrast to the macroeconomic disasters in Figure 2—which exhibit low probability of occurrence (3.7% per year for C) and large average size (22%)—BCM (Table 3, column 4) find frequent jumps (1.4 per year) of small average size (below 1%). These jumps may reflect changing parameters that the BCM model treats as fixed—notably the perceived disaster probability—rather than realized consumption disasters. Also, the coefficient of relative risk aversion needed to match BCM’s target equity premium of 4% is high, roughly 9. A useful research effort would reconcile the findings of BCM with those of Bollerslev and Todorov (2011).

Instead of looking at asset prices, such as stock-options prices, Berkman, Jacobsen, and Lee (2011), henceforth BJL, gauge time-varying disaster probability by considering the number and severity of international political crises. These political variables directly influence the likelihood of disasters, particularly wars. BJL show that time variations in their political-crisis variable measured at the world level relate to financial variables—stock returns, stock-price volatility, earnings-price ratios, and dividend yields—in ways that would be anticipated for variations in disaster probability. Hence, their world political variable might be a satisfactory proxy for time-varying world disaster probability.

11. DISASTER RISK IN OPEN-ECONOMY MODELS

Another line of research extends the rare-disasters model with time-varying disaster probability to international macroeconomics and finance. Part of this literature adds disaster risk
to the international real-business-cycle modeling that started with Backus, Kehoe, and Kydland (1992). One target of this research is the uncovered-interest-parity (UIP) puzzle, which relates to the carry trade and refers to the disconnect between interest-rate differentials and subsequent changes in exchange rates across countries. Early applications of this idea include Guo (2011); Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011); Jurek (2009); and Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2009). The main ideas are well explained by Farhi and Gabaix (2011).

In the Farhi-Gabaix (2011) model, agents in each country value consumption of tradables and non-tradables in a separable way. Since there is perfect risk sharing in tradables, the pricing kernel for assets depends only on the quantity of world tradables. The realization of a global disaster implies a small quantity of world tradables; hence, high marginal utility; hence, a high value of the pricing kernel.

There are also country-specific shocks, which affect a country’s productivity in converting non-traded into traded goods. These shocks occur at times of world disasters, but countries differ in their sensitivity to the global shock. Comparatively safe countries (Switzerland, Japan, United States?) are relatively immune to world disasters. In the model, the degree of safety is described by a country’s “resilience,” which is subject to world and local shocks but tends to revert over time toward a value that may be country specific. If a disaster occurs, the real exchange rates of low-resilience countries depreciate sharply compared with high-resilience countries.
When a country is risky (resilience is low), its real interest rate is high and its real exchange rate is depreciated.\(^{17}\) If the world disaster probability, \(p_t\), rises, risky countries see an increase in their real interest rates and a depreciation of their real exchange rates when compared to safer countries. At any point in time, the high interest rate in a risky country is compensation for risk—in the form of greater exposure to world disasters and, hence, to sharp currency depreciation should a disaster materialize. Therefore, the carry-trade strategy of borrowing in low interest-rate countries and lending in high interest-rate countries does not “beat the market” once one factors in the frequency of future disasters. The high average return—present even in samples that include the representative number of disasters—is compensation for disaster risk.

To address the UIP puzzle, Farhi and Gabaix (2011) assume that low-resilience countries tend to get more resilient over time and vice versa. This property holds for sure if all countries tend to revert toward a common resilience. In this case, a high-risk country with a high interest rate tends to get less risky over time and tends, therefore, to experience appreciation of its real exchange rate compared to a low-risk country.\(^{18}\) Therefore, in a sample with no realizations of world disasters, Fama (1984)-style regressions of future exchange rate appreciation on current interest-rate differentials for pairs of countries tend to yield coefficients with the “wrong” sign—counter to the standard UIP theory but consistent with much empirical evidence. In contrast, the sign of the coefficient is ambiguous in samples that include the representative number of disasters. If agents are risk neutral, this coefficient equals unity, as in the standard UIP theory.

\(^{17}\)In the model, a country’s real exchange rate and terms of trade depend not just on current productivity but also on the path of expected future productivity. Therefore, a fall in resilience leads immediately to a depreciation of the real exchange rate, even though no disaster has yet occurred.

\(^{18}\)This analysis can be extended to allow for dynamics from the evolution of the world disaster probability, \(p_t\). These shifts generate negative co-movement across countries between exchange-rate appreciation and changes in interest rates (compared to the world average). However, this force may not have much influence on Fama-style regressions.
More realistically, when agents are risk averse and potential disasters are nonzero, the coefficient is less than unity and may be negative.

Jurek (2009) looks empirically at disaster-based explanations of the UIP puzzle; specifically, he examines whether the high average return on carry-trade strategies can be explained by disaster risk. For equal-weighted portfolios involving carry trades between nine OECD currencies and the U.S. dollar, the average rate of return from 1999 to 2007 was positive and statistically significant: 4.7% per year with a t-statistic of 3.2 (Jurek [2009, Table IIb, Panel B]). When the portfolio is hedged for currency-crash risk by using exchange-rate options (at the 0.25δ level19), the average rate of return falls to 3.1% with a t-statistic of 2.5 (Table VII, Panel A). This result suggests that about one-third of the return to the carry trade from 1999 to 2007 represented a premium for exposure to currency-crash risk. However, when the sample is extended to 2008—the crisis year in which carry-trade portfolios experienced losses of around 20%—the average return to the carry trade from 1999 to 2008 when hedged for currency-crash risk was no longer statistically significantly different from zero: 1.7% with a t-statistic of 1.3 (Table IX, Panel A). Thus, in a sample with a disaster realization, the carry trade no longer delivers returns that significantly exceed the premium for currency-crash risk.

12. THE DYNAMICS OF DISASTERS

The baseline rare-disasters model involves unrealistic assumptions that go beyond the assumed constancy of long-run growth rates and disaster probabilities. First, macroeconomic contractions occur over a single period—a jump at an instant of time when the length of the period approaches zero—instead of unfolding over multiple years. Second, disasters have

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19The δ is the sensitivity of a stock-option price to the price of the underlying asset. For calls, δ is positive, near 1 if the option is very far in the money (strike price far below current asset price), close to 0.5 if the option is at the money, and close to 0 if the option is very far out of the money.
permanent level effects, but, as stressed by Gourio (2008), real-world disasters are usually followed by strong recoveries. Third, the model neglects correlation in the timing of disasters across countries. These shortcomings are addressed in the study by Nakamura, Steinsson, Barro, and Ursúa (2011), henceforth NSBU.20

The NSBU model divides consumption, C, into three components—“potential” consumption, a “disaster gap,” and a transitory shock. The disaster gap, which tends to dissipate over time through a first-order Markov process, is the amount by which current C deviates from potential C because of current or past disasters. Potential consumption evolves during disasters and determines the level to which consumption tends to return once a disaster ends. Starting from a “normal” state, entry into a disaster state occurs with a transition probability (equal across time and countries), where the specification allows for correlation across countries in the starts of disasters. As long as the disaster state persists, C growth tends to be substantially negative but with a much larger standard deviation than in normal times. With another probability (equal across time and countries), the economy moves from the disaster state to normalcy. This specification generates disasters of varying cumulative sizes and durations, akin to the histograms shown in Figure 2.

NSBU’s numerical estimation of the model’s parameters uses all the annual observations on real per capita personal consumer expenditure, C, for 24 countries with continuous data from before 1914 to 2006. The estimates imply that the typical disaster reaches its trough after 6 years, with a cumulative drop in C by 30%. However, only about half the decline in C is permanent; that is, post-crisis recoveries make up about half the level of C lost during a disaster.

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20These extensions address criticisms of the baseline model offered by Constantinides (2008) and Julliard and Ghosh (2008).
The main implications of the estimated model for the equity premium concern the recoveries. Since, on average, only half the decline in consumption during a disaster is permanent, the model’s predicted equity premium falls short of the value in Eq. (9). The other extensions have less influence on the equity premium, although the stochastic duration of disasters matters because of effects on the correlation between consumption growth and stock returns during disasters. The use of EZW preferences is important here because C growth deviates substantially from an i.i.d. process, especially during disasters but also in recoveries. For example, in the early stages of a disaster, such as a war, stock prices typically have already fallen a lot, whereas C has fallen only modestly. However, there is a strong belief that future C will decline much more but by a highly uncertain amount before the eventual end of the disaster tends to generate a recovery. These expectations bring in effects involving the lack of time separability in consumption under EZW preferences.

Overall, the estimated model generates a sizable equity premium from disaster risk but one that is smaller than in the baseline model, in which disasters are permanent and instantaneous. To accord with an average unlevered equity premium of 5%, the NSBU model requires a coefficient of relative risk aversion, \( \gamma \), of 6.4, compared with the value around 3.5 in Barro and Ursúa (2008, Tables 10 and 11) and around 3.0 in Barro and Jin (2011, Table 1).\(^{21}\)

To put the results another way, since a \( \gamma \) of 6 seems implausibly high, the NSBU model requires the addition of other features that raise the equity premium in order to get the estimated \( \gamma \) down to a plausible range. One promising extension is to allow for shocks to the mean and variance of the long-run growth rate; that is, to bring together the long-run risks model of Bansal and Yaron (2004), considered in part 8, with the dynamics of disasters considered in NSBU.

\(^{21}\)Nakamura, Steinsson, Barro and Ursúa (2011, Table VII) show that the required \( \gamma \) falls to 4.4 if disasters are permanent; that is, when there are no recoveries. The required \( \gamma \) declines further, to 3.0, if the disaster shocks are also instantaneous, rather than applying over periods of finite and random length.
Another idea is to incorporate time-varying disaster probabilities, considered in Section 9, into the NSBU setting.

13. INCORPORATING THE FULL TIME SERIES OF RATES OF RETURN

The empirical exercises described thus far seek to fit particular properties of asset returns, such as a high average equity premium, low average risk-free rate, and high volatility of stock returns. However, additional tests of the model can be carried out with the full time series on rates of return.

In the studies described before, Barro and Ursúa (2008) and Barro and Jin (2011) followed the spirit of Mehra and Prescott (1985) and Rietz (1988) by examining whether the unlevered equity premium generated from Eq. (8) matched the target of around 5%, given that stock returns behave in ways hypothesized by the model particularly during disasters. Specifically, in the term \( b \cdot [(1 - b) - 1] \) in Eq. (8), the first \( b \) represents the proportionate fall in stock prices during a disaster, whereas the second represents the proportionate fall in consumption, \( C \). However, in the data, stock returns during disasters are not as correlated with \( C \) growth as the model assumes.

Barro and Ursúa (2009) looked directly at the first-order condition that applies, as in Eq. (8), when the underlying shocks are i.i.d. The key term is the cross-product,

\[
E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \cdot R_{t+1} \right],
\]

where \( R \) is the gross real return on stocks. For a panel of countries with data on \( C \) and \( R \), the sample average for this term depends particularly on how stock returns behave during disasters. One finding in Barro and Ursúa (2009) is that the correlation between stock returns and \( C \) growth (measured over a fixed period length, which could be one or more years) is not nearly high enough during disasters for this approach to accord with the average real
rate of return on stocks observed over the full sample. Another finding is that, when the timing of stock-market crashes and depressions is interpreted “flexibly,” the co-movement between C growth and stock returns during disasters is strong enough to be consistent with the observed average return on stocks. However, to be convincing, this approach requires a disciplined model in which the timing between low C growth and low stock returns is not coincident during disasters.

As noted before, when preferences are of the EZW type and shocks are not i.i.d., the model does not require stock returns to co-move precisely with C growth during disasters. The key term from Eq. (6) is the cross product, \( E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{\gamma} \cdot \frac{1+V_{t+1}}{V_t} \cdot \left( \frac{1}{1+\theta} \right) \cdot R_{t+1} \right] \), where \( V \) is the ratio of wealth to consumption, corresponding in a simple model to the ratio of stock prices to dividends.\(^{22}\) It is an open question whether an analysis extended to allow for this influence from stock-price changes will perform satisfactorily when implemented with the full panel of data on C growth, price-dividend ratios, and stock returns.\(^{23}\) A serious empirical challenge for this implementation is that data on real stock returns tend systematically to be missing during the worst disasters, particularly due to closed markets during wartime.\(^{24}\) Moreover, even when

\(^{22}\)Note that EZW preferences imply a particular form of time non-separability for consumption. It would be possible to modify the form of this non-separability by including, for example, the habit-formation idea of Abel (1990) and Campbell and Cochrane (1999) into a model with EZW preferences.

\(^{23}\)Nakamura, Steinsson, Barro and Ursúa (2011, p. 20) incorporate these effects from stock-price changes during disasters without using the full time series of stock prices and returns: “When the news arrives that a disaster has struck, the stock market crashes. This crash … coincides with a sizable drop in consumption. The fact that stocks pay off poorly at the onset of disasters, when consumption is low and the marginal utility of consumption is high, implies that stocks must yield a considerable return-premium over bills in normal times. … [However,] the consumption decline in any given year of a disaster is substantially smaller than the peak-to-trough declines used to calibrate simpler disaster models … How, then do our estimates generate a sizable disaster premium? The key point is that the current short-run decline in consumption is paired with news about future declines in consumption and a large increase in uncertainty about future consumption, … [which] contribute … to the premium households are willing to pay for assets that insure against disaster events.”

\(^{24}\)Examples are Belgium 1914-1918 and 1944-1946, France 1940-1941, Greece 1941-1952, Mexico 1915-1918 (revolution and civil war), Netherlands 1944-1946, Portugal 1974-1977 (“Carnation Revolution”), Spain 1936-1940 (Spanish Civil War), and Switzerland 1914-1916. The data in these cases do allow for reasonable measurement of
financial markets remained open, the data on real stock prices and real stock returns were often substantially mis-measured during wartime because of controls on prices, sometimes extended to prices of financial assets.\(^{25}\)

### 14. CONCLUDING THOUGHTS AND FUTURE RESEARCH

The rare-disasters perspective provides an important bridge between macroeconomics and finance and helps to explain an array of asset-pricing puzzles, including the high equity premium. From the perspective of macroeconomic fluctuations, shocks to the perceived probability of disaster provide a potentially important supplement to existing closed- and open-economy real business-cycle models.

We conclude with a list of promising extensions of research on rare disasters.

1. Rare disasters of uncertain timing and magnitude can be incorporated into models of long-run economic growth.

2. In the environmental-economics literature related to climate change (Nordhaus [2007], Weitzman [2009]), a key issue is the appropriate discount rate for highly uncertain flows that arise in the distant future. This uncertainty applies to effects of climate change on the economy, effects of policies on climate change, and future real GDP. The standard, deterministic neoclassical growth model used in parts of the existing literature is not helpful for this kind of analysis. An appropriate treatment requires an explicit modeling of uncertainty, along the lines of the disaster models that we discussed.

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\(^{25}\)An extreme example is Germany during WWII, where controls—which held down reported consumer prices starting in 1936 and propped up reported stock prices starting in 1943—lapsed only in 1948. During the war and through 1947, the underestimation of true inflation and the propping up of reported stock prices led to a substantial overstatement of real stock returns. Then the measured real stock return of −0.89 when controls were lifted in 1948 is misleadingly low. However, the data in this and other cases allow for reasonable measurement of real stock returns over periods long enough to bridge the interval of controls; for example, from 1935 to 1948 in Germany.
3. The framework developed by Nakamura, Steinsson, Barro and Ursúa (2011) can be extended to incorporate the long-run risk ideas of Bansal and Yaron (2004).

4. Stock-price index options, available for the United States and possibly other countries, can be used to gauge shifting disaster probabilities, \( p_t \). These results would be important for asset-pricing research and for assessing time-varying disaster probabilities as an input into business-cycle models. Exchange-rate options can be used in a similar way to measure changes in country “resiliences,” which appear in international business-cycle models and in studies of the uncovered interest-parity condition.
ACKNOWLEDGEMENTS

This research is supported by grant SES-0949496 from the National Science Foundation. We appreciate helpful comments from David Backus, Xavier Gabaix, Tao Jin, Ian Martin, Emi Nakamura, and Jón Steinsson.
LITERATURE CITED


### Table 1: Available Macroeconomic Growth and Financial-Returns Data by Regional Group

<table>
<thead>
<tr>
<th>Southeast Asia</th>
<th>Latin America</th>
<th>Western Europe</th>
<th>Western Offshoots</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GDP:</strong></td>
<td><strong>C:</strong></td>
<td><strong>Country</strong></td>
<td><strong>Country</strong></td>
<td><strong>Country</strong></td>
</tr>
<tr>
<td>China</td>
<td></td>
<td>Argentina</td>
<td>Austria</td>
<td>Australia</td>
</tr>
<tr>
<td>1890</td>
<td>1952</td>
<td>1875, 1875</td>
<td>1870, 1870</td>
<td>1820, 1861</td>
</tr>
<tr>
<td><strong>Japan</strong>:</td>
<td><strong>C:</strong></td>
<td>Brazil</td>
<td>Belgium</td>
<td>Canada</td>
</tr>
<tr>
<td>1870</td>
<td>1974</td>
<td>1850, 1901</td>
<td>1846, 1913</td>
<td>1870, 1871</td>
</tr>
<tr>
<td><strong>Indonesia</strong>:</td>
<td><strong>C:</strong></td>
<td>Chile</td>
<td>Denmark</td>
<td>New Zealand</td>
</tr>
<tr>
<td>1880</td>
<td>1960</td>
<td>1860, 1900</td>
<td>1818, 1844</td>
<td>1860, 1878</td>
</tr>
<tr>
<td><strong>Malaysia</strong>:</td>
<td><strong>C:</strong></td>
<td>Colombia</td>
<td>Finland</td>
<td>United States</td>
</tr>
<tr>
<td>1900*</td>
<td>1900*</td>
<td>1905, 1925</td>
<td>1860, 1860</td>
<td>1790, 1834</td>
</tr>
<tr>
<td><strong>Philippines</strong>:</td>
<td><strong>C:</strong></td>
<td>Mexico</td>
<td>France</td>
<td>Sri Lanka</td>
</tr>
<tr>
<td>1902*</td>
<td>1946</td>
<td>1895, 1900</td>
<td>1820, 1824</td>
<td>1870, 1960</td>
</tr>
<tr>
<td><strong>Singapore</strong>:</td>
<td><strong>C:</strong></td>
<td>Peru</td>
<td>Germany</td>
<td>Turkey</td>
</tr>
<tr>
<td>1900*</td>
<td>1900*</td>
<td>1896, 1896</td>
<td>1851, 1851</td>
<td>1875</td>
</tr>
<tr>
<td><strong>South Korea</strong>:</td>
<td><strong>C:</strong></td>
<td>Uruguay</td>
<td>Greece</td>
<td></td>
</tr>
<tr>
<td>1911</td>
<td>1911</td>
<td>1870, 1960</td>
<td>1833*, 1938</td>
<td></td>
</tr>
<tr>
<td><strong>Taiwan</strong>:</td>
<td><strong>C:</strong></td>
<td>Venezuela</td>
<td>Iceland</td>
<td></td>
</tr>
<tr>
<td>1901</td>
<td>1901</td>
<td>1883, 1923</td>
<td>1870, 1945</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Starting dates are in first rows for GDP and second rows for C. Symbol * means that series has missing years: for GDP, these are Greece (1944), Malaysia (1943-46), Singapore (1940-49), and Philippines (1941-45); for C, these are Austria (1919-23), Malaysia (1940-46), and Singapore (1940-47). Italics for starting year means that series lacks continuous data since at least 1913. End year in all cases is 2009. Superscript ° indicates inclusion in "OECD" sample. Subscripts [r t b p] indicate that country has long-term information on stock returns, bill returns (usually 3-month government bills), bond returns (usually 10-year government bonds), and consumer price inflation, respectively.
Table 2
Sample Moments for Growth Rates and Rates of Return

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>full sample</td>
<td>post-WWII</td>
<td>full sample</td>
</tr>
<tr>
<td>World:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP growth</td>
<td>0.020</td>
<td>0.028</td>
<td>0.060</td>
</tr>
<tr>
<td>C growth</td>
<td>0.018</td>
<td>0.026</td>
<td>0.064</td>
</tr>
<tr>
<td>Stock return</td>
<td>0.084</td>
<td>0.108</td>
<td>0.317</td>
</tr>
<tr>
<td>Bill return</td>
<td>0.013</td>
<td>0.011</td>
<td>0.106</td>
</tr>
<tr>
<td>Bond return</td>
<td>0.030</td>
<td>0.025</td>
<td>0.126</td>
</tr>
<tr>
<td>OECD:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP growth</td>
<td>0.019</td>
<td>0.027</td>
<td>0.057</td>
</tr>
<tr>
<td>C growth</td>
<td>0.017</td>
<td>0.025</td>
<td>0.057</td>
</tr>
<tr>
<td>Stock return</td>
<td>0.077</td>
<td>0.094</td>
<td>0.254</td>
</tr>
<tr>
<td>Bill return</td>
<td>0.013</td>
<td>0.013</td>
<td>0.103</td>
</tr>
<tr>
<td>Bond return</td>
<td>0.031</td>
<td>0.028</td>
<td>0.127</td>
</tr>
<tr>
<td>non-OECD:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP growth</td>
<td>0.021</td>
<td>0.028</td>
<td>0.063</td>
</tr>
<tr>
<td>C growth</td>
<td>0.022</td>
<td>0.028</td>
<td>0.077</td>
</tr>
<tr>
<td>Stock return</td>
<td>0.102</td>
<td>0.139</td>
<td>0.438</td>
</tr>
<tr>
<td>Bill return</td>
<td>0.014</td>
<td>-0.010</td>
<td>0.129</td>
</tr>
<tr>
<td>Bond return</td>
<td>0.022</td>
<td>0.007</td>
<td>0.114</td>
</tr>
</tbody>
</table>

Note: Data run as far back as 1870 to 2009 (see Table 1). World samples have 39 countries (21 OECD) for GDP (not including Malaysia, Philippines, and Singapore because of breaks in data during WWII), 28 countries (18 OECD) for C, 30 countries (20 OECD) for stock returns, 19 countries (17 OECD) for bill returns, and 17 countries (15 OECD) for bond returns.
Table 3 Macroeconomic Disasters for Four Countries Added to Data Set  
(China, Egypt, Russia, Turkey)

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP Disaster</th>
<th>C Disaster</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>C data start 1952</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1892-1895: -0.25</td>
<td></td>
<td></td>
<td>Sino-Japanese War I, 1894-1895</td>
</tr>
<tr>
<td>1929-1934: -0.11</td>
<td></td>
<td></td>
<td>Japanese invasion, 1931-1932</td>
</tr>
<tr>
<td>1936-1946: -0.50</td>
<td></td>
<td></td>
<td>Sino-Japanese War II, WWII, 1937-1945</td>
</tr>
<tr>
<td>Egypt</td>
<td>Data start 1894</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1899-1908: -0.12</td>
<td>1899-1908: -0.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1912-1918: -0.13</td>
<td>1912-1918: -0.10</td>
<td></td>
<td>WWI, Egypt separated from Ottoman Empire</td>
</tr>
<tr>
<td>1919-1921: -0.17</td>
<td>1919-1921: -0.23</td>
<td></td>
<td>1919 revolution, 1922 independent kingdom</td>
</tr>
<tr>
<td>1922-1926: -0.11</td>
<td>1922-1926: -0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1938-1943: -0.18</td>
<td>1939-1941: -0.20</td>
<td></td>
<td>WWII, U.K. used as base</td>
</tr>
<tr>
<td></td>
<td>1951-1953: -0.17</td>
<td></td>
<td>Revolution, 1952-1953</td>
</tr>
<tr>
<td>Russia</td>
<td>C data start 1885</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1878-1880: -0.13</td>
<td></td>
<td></td>
<td>Russo-Turkish War, 1877-1878</td>
</tr>
<tr>
<td>1882-1886: -0.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1887-1891: -0.18</td>
<td>1887-1891: -0.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1904-1907: -0.19</td>
<td>1904-1906: -0.13</td>
<td></td>
<td>Russo-Japanese War, 1904-1905</td>
</tr>
<tr>
<td>1913-1921: -0.62</td>
<td>1913-1921: -0.71</td>
<td></td>
<td>WWII, Revolution/Civil War, 1913-1921</td>
</tr>
<tr>
<td></td>
<td>1929-1932: -0.16</td>
<td></td>
<td>Great Depression</td>
</tr>
<tr>
<td>1939-1942: -0.30</td>
<td>1937-1943: -0.58</td>
<td></td>
<td>WWII</td>
</tr>
<tr>
<td>1989-1998: -0.48</td>
<td>1990-1996: -0.16</td>
<td></td>
<td>Transition economy</td>
</tr>
<tr>
<td>Turkey</td>
<td>Data start 1875</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1876-1880: -0.40</td>
<td>1876-1880: -0.38</td>
<td></td>
<td>Russo-Turkish War, 1877-1878</td>
</tr>
<tr>
<td>1885-1888: -0.20</td>
<td>1885-1888: -0.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1892-1895: -0.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1913-1920: -0.45</td>
<td>1913-1919: -0.49</td>
<td></td>
<td>WWII, breakup of Ottoman Empire</td>
</tr>
<tr>
<td>1926-1927: -0.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1931-1932: -0.12</td>
<td>1929-1932: -0.12</td>
<td></td>
<td>Great Depression</td>
</tr>
<tr>
<td>1939-1945: -0.40</td>
<td>1938-1946: -0.30</td>
<td></td>
<td>WWII neutrality</td>
</tr>
<tr>
<td>1999-2001: -0.10</td>
<td>1997-2001: -0.12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Underlying data on real per capita personal consumer expenditure, C, and GDP for these countries are graphed in Figure 1. The proportionate declines in GDP or C are those of 0.095 or greater when computed from a peak-to-trough approach over multiple years.
Table 4  Breakdown of Macroeconomic Crises, 1870-2006

<table>
<thead>
<tr>
<th>Episode/Period</th>
<th>Number of Events</th>
<th>Mean Fall</th>
<th>Number of Events</th>
<th>Mean Fall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1914</td>
<td>31</td>
<td>0.16</td>
<td>51</td>
<td>0.17</td>
</tr>
<tr>
<td>World War I</td>
<td>20</td>
<td>0.24</td>
<td>31</td>
<td>0.21</td>
</tr>
<tr>
<td>Early 1920s (Flu?)</td>
<td>10</td>
<td>0.24</td>
<td>8</td>
<td>0.22</td>
</tr>
<tr>
<td>Great Depression</td>
<td>14</td>
<td>0.20</td>
<td>23</td>
<td>0.20</td>
</tr>
<tr>
<td>World War II</td>
<td>21</td>
<td>0.33</td>
<td>25</td>
<td>0.37</td>
</tr>
<tr>
<td>Post-World War II</td>
<td>24</td>
<td>0.18</td>
<td>35</td>
<td>0.17</td>
</tr>
<tr>
<td>OECD</td>
<td>6</td>
<td>0.12</td>
<td>6</td>
<td>0.13</td>
</tr>
<tr>
<td>Non-OECD</td>
<td>18</td>
<td>0.19</td>
<td>29</td>
<td>0.17</td>
</tr>
<tr>
<td>Other</td>
<td>5</td>
<td>0.19</td>
<td>10</td>
<td>0.15</td>
</tr>
<tr>
<td>Overall</td>
<td>125</td>
<td>0.22</td>
<td>183</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Note: These results update Barro and Ursúa (2008, Table 7) to include the four countries with newly constructed data as shown in Table 3. (China is not included for C. New Zealand is included for C but was not in Barro and Ursúa [2008].) Declines in real per capita personal consumer expenditure, C, and GDP are those of 0.095 or greater when computed from a peak-to-trough approach over multiple years.
Figure 1a: C and GDP for Four Major Countries

<table>
<thead>
<tr>
<th>Germany</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>GDP</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
</tr>
</tbody>
</table>

Note: The data, ending in 2009, are for real per capita personal consumer expenditure, C, and real per capita GDP. Levels of series were anchored at 2005 values of series on PPP-adjusted, constant-dollar aggregates from *World Development Indicators*. Scales are in natural logs.
Figure 1b: C and GDP for Four Countries with Newly Available Macroeconomic Data

China

Egypt

Russia

Turkey

Note: See note to Figure 1a.
Figure 2
Disaster Sizes and Durations

Note: This sample of disaster events extends Barro and Ursúa (2008, Figures 1 and 2) to include the four countries with newly constructed data as shown in Table 3. (China is not included for C. New Zealand is included for C but was not in Barro and Ursúa [2008].) Proportionate declines in real per capita personal consumer expenditure, C, and GDP are those of 0.095 or greater when computed from a peak-to-trough approach over multiple years. Duration is the number of years from peak to trough. Samples have 28 countries for C and 40 for GDP.