Control of Beam’s Snapback Responses
Using Multiple Flexoelectric Actuators

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Abstract

Flexoelectric material under certain inhomogeneous electric field can be utilized to control the vibration of flexible structure based on the converse flexoelectric effect. The atomic force microscope (AFM) probe placed on the top of flexoelectric patch and the electrode under it with certain voltage applied between them are used to generate the inhomogeneous electric field. The actuation membrane force and bending moment induced by flexoelectric patch in turn influence the vibration of shells. In this study, an elastic cantilever beam excited by an initial static mechanical force locating at the tip end is considered. As the mechanical force withdraws the snapback response appears where the cantilever beam vibrates with all mode participating and their amplitudes decaying exponentially. Multiple actuators, consist of AFM probes and flexoelectric patches, are used here to accelerate the decrease of snapback response. On each probe voltages of multiple frequencies are applied to obtain the control effects on multiple modes. The actuator’s positions minimizing the tip displacement of the beam is selected as the optimal ones. Two way of determining the control voltage, i.e., open loop and close loop, is introduced and the information of positions of the actuators are emphasized in both methods. In case studies, the free damping vibration of the beam without control is evaluated and the first mode dominates the vibration. The open loop method, with constant amplitude of control voltage, doesn’t work well in a relatively long time scale. While the close loop method, with its control voltage related to the tip displacement detected by a sensor, turns out to be more stable and effective. The optimal positions for the actuations of both methods are clustered near the fixed end, where the actuation factor reaches its maximal value. At last, limitation on voltage amplitude is implemented to avoid the stress concentration, under which the control effect does not weaken much. This study serves as an introduction to flexoelectric material & multiple AFM probe based vibration control and a theoretical foundation for further researches.

Keywords: AFM probe, open loop, close loop, actuation factor

1. INTRODUCTION

Cantilever beam model is among the most frequently used structures in engineering applications, i.e., wings, antennas, robot arms, etc. Studies on dynamics of cantilever beams have been thoroughly conducted using modal expansion method [1]. The snap-back response has been considered as an important characteristic [2-4] of the cantilever beam structure and served as a basic testing method [5].
Piezoelectric materials, developing rapidly in the last decades, have been used on sensors, actuators and active control of flexible beam models [6-11]. Flexoelectric materials, based on the converse flexoelectric effect [12], are newly developed materials which can be utilized, as a substitution to piezoelectric materials, to achieve precision actuation and active vibration control. This effect that the stress inside the flexoelectric material can be induced by certain inhomogeneous electric field has been studied both theoretically [13] and experimentally [14,15]. Many researches using the direct flexoelectric effect to build flexoelectric sensors has been carried out on various structures, i.e., cantilever beams [16,17], rings [18,19], cylindrical shells [20] as well as the effect comparison with piezoelectric ones [21]. Actuation studies have been done based on the inhomogeneous field generated by atomic force microscope (AFM) probes [22] and asymmetric electrodes [23]. Comparing with single-actuator (consist of AFM probe and flexoelectric patch in this study) control method, multiple-actuator control gives more arrangement possibility and thus more efficient. The efficiency of multiple-actuator control method has been verified by previous piezoelectricity-based multiple-actuator control method [24,25] and distributed actuating systems [26,27]. However similar researches on flexoelectricity-based method have not been reported.

In this study, snap-back is applied to test the validity of the multiple flexoelectric actuators’ control effect. The dynamic response of snap-back cantilever beam is analyzed first, followed by the control theories on flexoelectric control using multiple actuators. Two kinds of control method, open loop and close loop, are presented respectively and their control effect are evaluated. Limitation on voltage amplitude is applied in close loop case to avoid stress concentration near the contact point (with AFM probe). This study introduces an application for AFM probe & flexoelectric material based actuators.

2. THE BEAM MODEL OF FLEXOELECTRIC CONTROL

A cantilever beam model, used to demonstrate the control effect of the snap-back vibration, is shown in Fig.1. Multiple probes (generally n probes) are applied here, with corresponding flexoelectric patches, to generate the control forces. According to the converse flexoelectric effect, the flexoelectric patches need to be actuated by an inhomogeneous electric field and such electric field is induced by AFM probes. In another word, the actuators used here consist of AFM probes and flexoelectric patches. The coordinates x and z denotes the longitudinal and transverse directions; L is the length, b is the width and h is the thickness of the elastic cantilever beam. Generally, the control effect of n actuators, as demonstrated in Fig.1, is considered. For each actuator, the geometrical size of AFM probe and flexoelectric patch is unified as probe radius R, patches thickness ha and length La. The position of each actuator, which plays an important role in evaluating the control effect, is denoted by xi, where the subscript “i” indicates the i-th actuator.

![Figure 1. The cantilever beam model of flexoelectric control.](image-url)
Note that it is assumed that the thickness of the flexoelectric patch $h^a$ is much greater than the AFM probe radius $R$ while much smaller than the thickness of cantilever beam $h$, i.e., $R \ll ha \ll h$.

3. THE FREE VIBRATION OF SNAP-BACK CANTILEVER BEAM

The free damping vibration of the snap-back cantilever beam without the control of actuators is analyzed first. A mechanical force $F$ is placed on the tip end of the cantilever beam and certain static deformation is formed. It is assumed that the mechanical force is removed instantly at the time $t=0$. The cantilever beam will oscillate due to the initial condition of static deformation.

According to modal expansion method, the transverse displacement can be expressed as the summation of all the modes that participates the vibration multiplied the corresponding modal participation factors, as [1]

$$u_1(x,t) = \sum_{k=1}^{\infty} \eta_k(t) U_{3k}(x),$$

where $k$ is mode number, $\eta_k$ is the $k$-th modal participation factor and $U_{3k}$ is the cantilever beam’s $k$-th mode shape function which can be written as

$$U_{3k}(x) = C_k \left[ \cosh \lambda_k x - \cos \lambda_k x - \frac{\cosh \lambda_k L + \cos \lambda_k L}{\sinh \lambda_k L + \sin \lambda_k L} \left( \sinh \lambda_k x - \sin \lambda_k x \right) \right],$$

where $\lambda_k L$ is the root of the equation $1 + \cosh (\lambda L) \cos (\lambda L) = 0$ and the first four roots are 1.875, 4.694, 7.855, 10.996 respectively and $C_k$ is an arbitrary constant. Utilizing the orthogonality of the mode shape functions of cantilever beam, the modal participation factor equation can be written as [1]

$$\dot{\eta}_k + 2\zeta_k \omega_k \eta_k + \omega_k^2 \eta_k = \hat{F}_k(t),$$

where $\omega_k$ is the $k$-th natural frequency; $\zeta_k$ represents the modal damping ratio of the ring, which depends on the equivalent damping constant $c$ and the natural frequency $\omega_k$, i.e., $\zeta_k = c/(2\rho \pi \omega_k)$; $\hat{F}_k(t)$ is the modal force for the $k$-th mode which is absent from the analysis of the free damping vibration of the cantilever beam when $t > 0$. Using Laplace transformation, the solution of the modal participation factor equation of subcritical case can be expressed as

$$\eta_k(t) = e^{-\zeta_k \omega_k t} \left\{ \eta_k(0) \cos(\omega_k t) + \left[ \eta_k(0) \zeta_k \omega_k + \dot{\eta}_k(0) \right] \frac{\sin(\omega_k t)}{\omega_k} \right\} U_{3k}(x)$$

$$+ \frac{1}{\omega_{kl}} \int_0^t \hat{F}_k(t-\tau) e^{-\zeta_k \omega_k (t-\tau)} \sin[\omega_{kl} (t-\tau)] d\tau,$$

where $\omega_{kl}$ is the damped frequency, which is defined as $\omega_{kl} = \omega_k \sqrt{1-\zeta_k^2}$; $\eta_k(0)$ and $\dot{\eta}_k(0)$ are the initial conditions of the modal participant factors. Note that the influence of the modal force does not exist at $t > 0$ for the value of integration in the second term is zero. Such result is reasonable since the vibration is free damping without any mechanical force. $\eta_k(0)$ can be solved from the static modal participation factor function which can be expressed as $\omega_{kl}^2 \eta_k = \hat{F}_k(0)$, where $\hat{F}_k(0)$ is the modal force induced by the mechanical force $F$ at $t=0$, which can be expressed, using the orthogonality of the mode shape functions, as
\[
\hat{F}_k(0) = \frac{1}{\rho bh N_k} \int_0^L F \delta(x-L) U_{3k}(x) \, dx = \frac{F U_{3k}(L)}{\rho bh N_k}.
\]

Substituting the expression of mode force at t=0 (Eq. (6)) into the static modal participation factor function (Eq. (5)), the initial condition of modal participant factors can be expressed as

\[
\eta_k(0) = \frac{\hat{F}_k(0)}{\omega_k^2 \rho bh N_k} = \frac{F U_{3k}(L)}{\omega_k^2 \rho bh N_k}.
\]

Substituting Eq. (7) into the solution of modal participation factor equation, i.e., Eq. (4) and noting that \( \dot{\eta}_k(0) = 0 \) and \( \ddot{F}_k(t) = 0 \) when \( t > 0 \), the displacement of the beam can be written, by the modal expansion expression (Eq. (1)), as

\[
\begin{aligned}
    u_z(x,t) &= \sum_{k=1}^n e^{-\xi_k \omega_k t} \frac{F U_{3k}(L) U_{3k}(x)}{\omega_k^2 \rho bh N_k} \left[ \cos(\omega_k t) - \frac{\zeta_k}{\sqrt{1 - \zeta_k^2}} \sin(\omega_k t) \right], \\
    &= \sum_{k=1}^n e^{-\xi_k \omega_k t} \frac{F U_{3k}(L) U_{3k}(x) e^{j(\omega_k t - \phi)}}{\omega_k^2 \rho bh N_k} \sqrt{1 - \zeta_k^2}.
\end{aligned}
\]

As shown in the displacement expression of the cantilever beam, i.e., Eq. (7), the initial condition of static force triggers every mode participating the vibration and each mode decays exponentially with different speed (generally higher mode decays faster).

4. VIBRATION CONTROL BY MULTIPLE ACTUATORS

As the participation factor of higher mode can be small enough to neglect, only initial \( m \) modes are considered. To obtain a better control effect on all \( m \) modes, for every probe \( m \) harmonic voltages with different frequencies are applied. The expression of the voltage on probe \( i \) can be written as

\[
\phi_i^a = \sum_{p=1}^m \phi_{pz}^i e^{j(\omega_p t - \phi_i)}.
\]

Applying the actuation voltage \( \phi_i^a \) between the electrode of the \( i \)-th flexoelectric patch and \( i \)-th AFM probe, the transverse electric field between them can be expressed according to Abplanalp’s approximate [28,29] as

\[
E_{zi}(x-x_i, z) = -\frac{\phi_i^a R\left(R + h / 2 + h^a - z\right)}{\left[(x-x_i)^2 + (R + h / 2 + h^a - z)^2\right]^{3/2}},
\]

where \( E_{zi} \) denotes the transverse electric field induced by the AFM probe. Note that the positions of the actuators are emphasized by regarding them as independent variables. The gradient of the electric field, as predicted by converse flexoelectric effect, can induces the stress on the flexoelectric material, particularly the normal stress in the longitudinal direction can be expressed as [22]

\[
T_{zi}^a(x-x_i, z) = \pi_{12} \frac{\partial E_{zi}}{\partial z},
\]

where \( T_{zi}^a \) denotes the longitudinal normal stress induced by the \( i \)-th actuator, where the superscript
“a” indicates the actuator induced component. The membrane force and control moment can be calculated according to the expression of the induced stress as

\[ N_{xx}^a(x-x') = \frac{h+h'}{2} T_{xx}^a(x-x', z)dz, \]

\[ M_{xx}^a(x-x') = \frac{h+h'}{2} \times N_{xx}^a(x-x'), \]

(10)

where \( N_{xx}^a \) denotes the membrane force and \( M_{xx}^a \) denotes the membrane force induced by the \( i \)-th actuator. The displacement of the cantilever beam induced by \( n \) actuators can be obtained by adding the effect of each actuator as [27]

\[ u_k(x) = \sum_{k=1}^{n} \sum_{i=1}^{n} \frac{U_{3k}(x) \tilde{F}_{ik}^a(x') e^{i(\omega - \phi^i)}}{\omega_k^2 \sqrt{1 - \frac{\omega^2}{\omega_k^2}} + 4 \zeta_k^2 \left( \frac{\omega}{\omega_k} \right)^2}, \]

(11)

where \( \tilde{F}_{ik}^a \) is the \( k \)-th modal force induced by the \( i \)-th actuator. The expression of the actuator induced modal force can be written as [27]

\[ \tilde{F}_{ik}^a = \sum_{i=1}^{n} \tilde{F}_{ik}^a = \sum_{i=1}^{n} \frac{1}{\rho h N_k} \int_{z_1}^{z_2} \left( \frac{\partial^2 M_{xx}^a(x-x', z)}{\partial x^2} \right) U_{3k}(x) dx, \]

(12)

where \( N_k = \int_{0}^{L} U_{3k}^2 dx \); The modal actuation factor of \( i \)-th probe and \( k \)-th mode is defined as \( A_k^a(x') = \tilde{F}_{ik}^a / \phi^a \), which indicates the modal force generated by the actuator with unit actuation voltage. According to the definition of the modal actuation factor and the expression of the applied control voltage, the \( k \)-th modal force induced by the \( i \)-th actuator can be written as

\[ \tilde{F}_{ik}^a(x') = A_k^a(x') \phi^a = A_k^a(x') \sum_{p=1}^{\infty} \tilde{F}_{ik}^a e^{i(\omega_p - \phi^a)}. \]

(13)

Note that the modal actuation factor \( A_k^a(x') \) demonstrates the information of the \( i \)-th actuator’s position and the applied voltage \( \phi^a \) contains the magnitude, frequency and phase difference information. By introducing the concept of modal actuation factor, the influence of the position of the actuator and the actuation characteristics of the actuator itself are separated, which gives a clear picture of how the design parameters influence the control effect of the vibration of cantilever beam.

Substituting Eq.(13) into the actuators induced displacement expression, i.e., Eq.(11) yields

\[ u_k(x) = \sum_{k=1}^{n} \sum_{i=1}^{n} \sum_{p=1}^{\infty} \frac{U_{3k}(x) A_k^a(x') \tilde{F}_{ik}^a e^{i(\omega_p - \phi^a)}}{\omega_k^2 \sqrt{1 - \frac{\omega^2}{\omega_k^2}} + 4 \zeta_k^2 \left( \frac{\omega}{\omega_k} \right)^2}. \]

(14)

Note that in Eq.(14), \( k \) denotes the mode number; \( i \) denotes the \( i \)-th actuator and \( p \) denotes the \( p \)-th harmonic voltage applied on the \( i \)-th actuator. To control the snap-back free damping vibration of the
cantilever beam, a basic method is to cancel out the displacement of the vibration by the displacement induced by the actuators and thus accelerate the damping process. To make the method practicable, the frequencies of the induced vibration must be coincidence with the original free damping vibration. The $p$-th frequency on the $i$-th probe is setting to be the same as the corresponding free damping frequency for each actuator as, $\omega_p = \omega_{p_i}$. The actuators induced displacement expression, considering only first $m$ modes, can be expressed as

$$u_{ij}(x) = \sum_{i=1}^{m} \sum_{i=1}^{m} \sum_{p=1}^{m} \frac{U_{ij}(x) A_{ik}^p(x_i) \phi_k}{\omega_k^2} e^{i(\omega_{p_i} t - \phi_{p_i} - \phi_k)}$$

(15)

where $\phi_{p_k}$ is the phase angle lagging behind the $p$-th harmonic excitation (voltage) of the $i$-th actuator on $k$-th mode. Note that this phase angle deducted from $\phi_{p_i}$ to $\phi_{p_k}$ because it is irrelevant to the actuator number for the actuation frequencies on each actuator are set to the same.

More than setting the actuator frequencies harmonic with the free damping ones, the phase difference between them, if possible, should be zero to enable the maximal control effect. That is $\phi_k = \phi_{p_i} + \phi_{p_k}$ for each $k$, $i$ and $p$ while only the phase angle of actuation voltage $\phi_{p_i}$ is adjustable. However, satisfying the zero phase difference condition for each $k$, $i$ and $p$ is impossible, and also, unnecessary. The $p \neq k$ circumstance stands for the vibration of $k$-th mode induced by the $p$-th damped frequency. Comparing with the vibration of $p = k$ condition, which is nearly resonance, the $p \neq k$ components are negligible. Thus, only the $p = k$ terms is ensured to be of no phase difference. The phase angle for the $i$-th actuator can be determined as $\phi_{p_i} = \phi_p - \phi_{p_k}$.

The displacement with the control of multiple AFM probes can be obtained by the superposition of free damping expression (Eq.(7)) and actuator induced expression (Eq.(15)), and the phase angle of each actuation voltage on each actuator. When only first $m$ modes are considered, the displacement after control by multiple AFM probes can be expressed as

$$u_{ij}(x,t) = \sum_{i=1}^{m} \sum_{i=1}^{m} \sum_{p=1}^{m} \frac{U_{ij}(x) A_{ik}^p(x_i) \phi_k}{\omega_k^2} e^{i(\omega_{p_i} t - \phi_{p_i} - \phi_k)}$$

(16)

Note that the first term in Eq.(16) is the $p = k$ components where the free damping terms and control terms are harmonic without phase difference. As discussed before, this term almost dominates the vibration; the second term is the $p \neq k$ component where the terms of different phase angle coupling together, which is very difficult to analyze. Fortunately, the contribution of the second term can be neglected as the vibration induced by it is much smaller than the first term. So the first term can be picking as the expression of response under control. If a constant amplitude of control voltage is
selected for the open loop control, by carefully choose the positions of actuators, one can suppress the incipient vibration efficiently. However notice that the first term in the bracket is of transient state, as the time marches when the initial static force induced vibration dies down the vibration induced by the actuator still exist and such phenomenon will be carefully analyzed in case studies.

5. THE FEEDBACK CONTROL

As proposed previously, if constant control voltage amplitude is used, the control effect is bad in a relatively long time scale. Thus the amplitude of the voltage should also be of transient state and this can be realized by introducing close loop control, i.e., adding feedbacks to the control system. The control voltages can be determined by frequency-domain analysis on the displacement signal detected by the displacement sensor, the sensed modal participation factor for $k$-th mode can be obtained as $\eta^*_k$, where the superscript “s” denotes the sensing signal. Then the $p$-th actuation voltage on $i$-th actuator, noting that only $p = k$ conditions is considered here, can be determined according to the sensing signal as $\hat{u}_{ik} = G_{ik} \eta^*_k$, where $G_{ik}$ is the feedback gain factor for $p$-th ($k$-th) actuation factor on $i$-th actuator.

Taking the expression of actuation voltage and considering only the first term, Eq.(16) can be written as

$$u_s(L,t) = \sum_{i=1}^{n} \frac{U_{3i}(L)}{a_i^2} \left[ \frac{FU_{3i}(L)}{\rho \beta h N_i} e^{-\xi_i \omega_i t} \sqrt{1-\xi_i^2} + \sum_{i=1}^{n} \frac{A^u_a(x_i^*) G_{ik} \eta^*_k}{2 \omega_i^2 \xi_i \sqrt{1-\xi_i^2}} \right] e^{i(\omega_i t - \phi_i)} \tag{17}$$

Ideally, the true time response is equal to the displacement outputted by sensor. Using modal expansion method to both true time response and sensor output displacement yields $\hat{\eta}_k = \hat{\eta}_k$. With the flexoelectric control, the sensor output is multiplied by a gain factor $G_{ik}$ in order to enhance the control effect. Thus one needs to determine the modal participation factor by solving the following equation

$$\hat{\eta}_k(t) = \frac{FU_{3i}(L)}{\rho \beta h N_i a_i^2} \sqrt{1-\xi_i^2} + \sum_{i=1}^{n} \frac{A^u_a(x_i^*) G_{ik} \hat{\eta}_k}{2 \omega_i^2 \xi_i \sqrt{1-\xi_i^2}} \tag{18}$$

The solution of Eq.(18), i.e., the true time modal response of the snap-back beam under the control of multiple actuators can be written as

$$\hat{\eta}_k(t) = \frac{FU_{3i}(L)}{\rho \beta h N_i a_i^2} \sqrt{1-\xi_i^2} \left[ 1 - \sum_{i=1}^{n} \frac{A^u_a(x_i^*) G_{ik}}{2 \omega_i^2 \xi_i \sqrt{1-\xi_i^2}} \right]$$

The modal snap back response of the cantilever beam without control can be reduced from Eq.(19) by setting the gain factor $G_{ik}$ equal to zero and the result modal response is identical with that of the free damping snap-back beam demonstrated in Eq.(7). The control effect for each mode of cantilever beam depends on the value of the denominator of the expression of true time modal response of the beam. The denominator is determined by both the positions of actuators $x_i^*$ and the gain factors $G_{ik}$. Due to the stress concentration problems induced by AFM probe, the control voltage must be limited, which in turn limits the gain factor $G_{ik}$. Thus for the limited gain factor a set of optimal positions must be found first to achieve better control effect by maximize the absolute value of the denominator for
each mode. While the optimal positions for actuators are different from one mode to another, one should first choose the principle mode and then set the positions of actuators according to the optimal positions for this mode. With the parameters for both elastic beam and actuator listed in Table 1, some detailed studies are conducted in case studies.

**Table 1. Parameters and properties of the model.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Other properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam length, $L$, (m)</td>
<td>0.100</td>
<td>Flexoelectric patches thickness, $h_c$ ($\mu$m)</td>
</tr>
<tr>
<td>Beam width, $b$, (m)</td>
<td>0.010</td>
<td>Flexoelectric patches length, $L_a$ (m)</td>
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<tr>
<td>Beam thickness, $h$ (m)</td>
<td>0.001</td>
<td>AFM probe tip radius, $R$ (nm)</td>
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<td>Young’s modulus of beam, $Y$ (N/m²)</td>
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<td>Actuation voltage, $\phi$ (V)</td>
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<tr>
<td>Beam mass density, $\rho$ (kg/m³)</td>
<td>1100</td>
<td>Flexoelectric constant, $\pi_2$ ($\mu$C/m)</td>
</tr>
<tr>
<td>Initial force, $F$ (N)</td>
<td>0.05</td>
<td>Gain factor, $G$ (V/m)</td>
</tr>
</tbody>
</table>

### 6. CASE STUDIES

#### 6.1 Open loop control

In the case study, only three actuators are used and on each actuator only three harmonic voltages are applied with the same magnitude $\hat{\phi}_{ip}=33.33$V. Note that the voltage actually applied on the AFM probes are the superposition of voltages of different modal frequencies, thus actual voltage applied on the actuators can as large as 100V. To evaluate the control effect of open loop method, initial static forces of different magnitudes are applied on the tip end of the beam. The optimal positions of the actuators are selected to minimize the vibration displacement. Response of the cantilever beam, with and without control, are shown in Fig 5 with the initial force of 0.1N, 0.05N, 0.03N and 0.02N.

![Figure 5](image_url)

**Figure 4.** Tip displacement of the beam with initial static force of (a) 0.1N; (b) 0.05N; (c) 0.03N and (d) 0.02N.
The control effect from 0s to 1s, as shown in Fig. 4 (a) and (b), enhances as the magnitude of initial static force decreases, which is reasonable because the control ability, i.e., actuation voltage, keeps unchanged during the original vibration become smaller. Most important, the optimal positions for first two cases when initial force is given by 0.1N and 0.05N are all 0.005m, 0.015m and 0.025m. For the relatively big initial deflection, the optimal arrangement is to place the actuators as close to $x=0$ as possible, where the actuation effect of first mode achieves greater value. However, when initial deflection is small the vibration becomes out of control in this time scale, as shown by Fig. 4 (c) and (d). The optimal positions here are no longer clustered near the fixed end. Because as the initial static force decreases, the displacement induced by the initial condition are outweighed by the vibration induced by the actuators, which makes the actuators change their favorite positions to avoid over-generating the displacement and substitute with one less effective.

Thus open loop control method is not good enough for a relatively long time scale, one may solve this problem by turning off the actuators when the tip displacement is small enough, i.e., below a certain threshold, or, as discussed next, adding a feedback to the actuators.

6.2 Feedback control

To avoid the emergence of chaos demonstrated in open loop control, feedback control is analyzed here. The true time response of the cantilever beam can be obtained by the displacement expression. Both the positions of the actuators and the value of the gain factor will influence the effect of control. The aim is to obtain optimal control effect under the same gain factor by adjusting the positions of the actuators. Recall that the first mode dominant the others in snap-back vibration, the positions will be selected here according the optimal positions for first mode, i.e., 0.005m, 0.010m and 0.015m. The tip displacements of both the free and controlled vibrations, when the initial static force is set to be 0.05N, are plotted in Fig. 5(a).

Figure 5. Control effect of beam in (a) real time, (b) first (c) second and (d) third mode.
The first mode response almost represents the true time response of the cantilever beam as shown in Fig.5(b) as this mode is the dominant one in snap-back vibration. The first mode also possess the best control effect in first three modes (comparing to Fig.5(b) and (c)), because that the positions of the actuators is chosen in favor of the first mode rather than the others.

To further analyze the feedback control of the cantilever beam, the feedback voltages of three respective modes are plotted in Fig.6(a). Under this gain factor the maximal feedback voltage is about 55V, which is much lower than the open loop control as discussed before. Note that control voltages are relatively large at incipient stage, thus certain limitation on maximal control voltage can be introduced to avoid the stress concentration problem. A limitation of 33.33V on the amplitude of control voltage is posed here, and the feedback control voltage under this limitation is plotted in Fig.6(b).

![Figure 6](image)

**Figure 6.** The control voltages of the actuators (a) without limitation; (b) with limitation and (c) real time responses of both cases.

The limitation forced the feedback voltage to be smaller than the maximal value, i.e., 33.33V. The control effect after introducing the limitation, before limitation and free damping are shown in Fig.6(c). Comparing with the one without voltage limitation, the incipient control effect is weakened because the control voltage is relatively small. However, the overall control effect in a sufficiently long time scale is almost the same as the one without voltage limitation. Recall that voltage limitation is introduced to avoid the stress concentration problem which is of great significance, thus the feedback control under voltage limitation is considered superior than the one without.

7. CONCLUSION

This study concentrated on the optimal positions of the actuators and control strategies of flexoelectric-based vibration control on cantilever beam structure. The snap-back vibration of the beam, excited by withdrawing the initial static force at \( t=0 \) moment, was considered and the goal is to accelerate its damping process. Multiple actuators consist of flexoelectric patches and AFM probes...
were used and on each AFM probe voltages of different frequencies are applied. To obtain maximal control effect, the frequencies of control voltages are set equal to the natural frequencies of the cantilever beam model and the phase angle of them is selected to diminish the phase difference in dominant components while neglect the nonsignificant ones. Two kinds of control method, open loop and close loop, was introduced based on the former theoretical results. A problem of open loop control is that the actuators will continue to actuate the beam after the initial transient snap-back vibration diminishes. One possible solution is to turn down the actuators when the amplitude of the vibration is below a certain threshold (small enough). The close loop control, detecting the displacement information from the beam’s tip end, feedback the vibration conditions to the actuators. This method obtains similar control effect without induces extra vibration, and furthermore, less voltage magnitude is required. The voltage can be further limited in close loop to avoid the stress concentration. The voltage limited one, by giving a limitation on the voltage magnitude applied, works almost as well as the unlimited one but with much less stress concentration.

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