We study cross-country risk sharing as a second-best problem for members of a currency union using an open economy model with nominal rigidities and provide two key results. First, we show that, if financial markets are incomplete, the value of gaining access to any given level of aggregate risk sharing is greater for countries that are members of a currency union. Second, we show that, even if financial markets are complete, privately optimal risk sharing is constrained inefficient. A role emerges for government intervention in risk sharing to both guarantee its existence and to influence its operation. The constrained efficient risk sharing arrangement can be implemented by contingent transfers within a fiscal union. The benefits of such a fiscal union are larger, the bigger the asymmetric shocks affecting the members of the currency union, the more persistent these shocks, and the less open the member economies.

1 Introduction

The benefits of flexible exchange rates were famously argued by Friedman (1953) and are widely accepted by economists. Countries in a currency union forego the possibility of adjustments to their exchange rates in response to asymmetric shocks. How costly is this loss in flexibility and what can be done to compensate it? These questions are precisely those tackled by the Optimal Currency Area (OCA) literature (for the pioneering articles, see Mundell, 1961; McKinnon, 1963; Kenen, 1969).

In a seminal contribution, Kenen (1969) argued that fiscal integration was critical to a well-functioning currency union:

“It is a chief function of fiscal policy, using both sides of the budget, to offset or compensate for regional differences, whether in earned income or in unemployment rates. The large-scale transfer payments built into fiscal systems are interregional, not just interpersonal [...]” (pg. 47)
Countries such as the United States, which can be thought as a currency and fiscal union of regions, share federal revenue and transfers—through the unemployment insurance program, federal income and social security taxes and, in extreme cases, direct federal assistance—in a manner that provides automatic stabilizers across regions. The ongoing crisis in the Eurozone, where such mechanisms are lacking, is seen by many as a vindication of Kenen’s fiscal integration criterion. Going forward, many policy discussions center around the construction of a fiscal union. How should a fiscal union be designed and how effective can we expect it to be?

Unfortunately, the OCA literature is couched in terms of Keynesian models that lack proper micro-foundations. As a result, the treatment of welfare is cursory. Recently, the New Open Economy Macro literature has developed open economy New Keynesian models with explicit micro-foundations and applied them to currency areas (see e.g. Benigno 2004; Beetsma and Jensen 2005; Gali and Monacelli 2008; Ferrero 2009). Our goal is to revisit Kenen’s idea using such a model. This allows for a rigorous treatment of optimal policy design. Indeed, we are able to deliver a complete characterization of the required transfers and of their effectiveness as a function of a small number of key characteristics of the economy.

We tackle the design of a fiscal union within a currency union using a simple model. We begin our analysis with the simplest possible model: a static setting with a traded good, a non-traded good and labor as in Obstfeld and Rogoff (2000). We then extend the analysis to a standard dynamic model featuring non-trivial intra-temporal trade and price adjustment dynamics that builds on Gali and Monacelli (2005, 2008). The key features in both settings are price or wage stickiness and limited openness, in the form of non-traded goods or home bias. In this context, we set up and study the second-best planning problem for constrained efficient risk sharing transfers among countries in a currency union.¹

Transfers have a dual role. First, they help smooth consumption—the usual direct role of risk sharing. Second, under a fixed exchange rate, in the presence of nominal price or wage rigidities, and with non-traded goods or home bias, transfers also have an indirect effect by affecting the pattern of spending, which in turn affects output and hence income or wealth—a mechanism first discussed in the famous Transfer Problem debate involving Keynes (1929) and Ohlin (1929)—and this helps mitigate recessions (or, in the other direction, curb booms). We show that this gives rise to an aggregate demand externality: the social benefits from risk sharing are greater than what is appreciated by private economic agents, since they do not internalize these indirect macro stabilizing effects and only value the direct private consumption smoothing role. Indeed, our main result is that, even under ideal, complete-market conditions the equilibrium without intervention does not provide the

¹We follow the approach of the OCA literature by taking the existence of a currency union as an exogenous constraint and not attempting to model the reasons for its formation in the first place. In other words, we abstract from the potential benefits and focus on the costs of currency unions. We characterize to what extent these costs can be mitigated by the establishment of a fiscal union. Of course, one potential concern is that the factors leading to the formation of currency unions could influence the optimal design of fiscal unions. Unfortunately, there is no consensus among economists on the benefits of currency unions. In addition, at least in the case of the Eurozone, the adoption of the euro was part of a larger political unification project. For all these reasons we believe that treating the existence of a currency union as an exogenous constraint is a useful starting point.
The constrained inefficiency of private risk sharing can be addressed by government intervention. Indeed constrained efficient outcomes can be implemented in a number of ways. If individuals do have access to private asset markets that are complete, then constrained efficiency can be ensured by providing quantity restrictions or tax incentives that distort their individual portfolios choices. We provide a simple formula for the required implicit tax: the subsidy on the portfolio return in a particular state of the world equals the product of the labor wedge (a measure of the state of the business cycle) and the relative expenditure share of non-traded goods. A second possibility is for the government to take over risk sharing by assuming the necessary insurance positions in financial markets itself. Equivalently, instead of using financial markets, it can arrange ex ante for state contingent transfers or “bailouts” with other union members. In either case, it must then also take steps to ensure that the private sector does not undo these arrangements, by setting up the aforementioned tax incentive system or employing more extreme measures, such as banning financial markets.

We view the complete financial markets paradigm as a useful assumption to highlight that the constrained inefficiency of private risk sharing that we derive does not arise from inefficiencies in financial markets. However, our preferred interpretation is that financial markets are incomplete, so that markets for sharing aggregate (macro) risk across countries are imperfect or nonexistent. This only strengthens the argument for building a fiscal union that creates risk sharing arrangements across members within a currency union. Indeed, the constrained efficient risk sharing arrangement can then be implemented through ex-post transfers or “bailouts” that are contingent on the shocks experienced by each country. Since agents have no access to financial markets, neither restrictions nor taxes on private insurance are needed. Under this interpretation, our paper can be seen as offering a precise characterization of these ex-post transfers and clarifying that for members of a currency union: (i) the value of gaining access to risk sharing, for any given level of insurance, is greater; and (ii) transfers should go beyond emulating the outcome that private risk sharing would reach if only asset markets were complete. These two points are distinct but complement each other to motivate the formation of fiscal unions within currency unions.

Importantly, we do not reach the same conclusion for countries outside a currency union, with flexible exchange rates. As long as they exercise their independent monetary policy optimally, it is optimal to let agents trade freely in a complete set of financial markets, or to replicate that outcome through fiscal transfers. Our argument for government involvement in risk sharing relies on membership in a currency union precisely because this constrains monetary policy and prevents stabilization of asymmetric shocks. Fiscal and monetary unions go hand in hand.

Our results qualify a view often presented in the OCA literature that transfers and risk sharing through private financial markets are substitutes—both providing adequate buffers against asymmetric macroeconomic shocks in a currency union. For example, Mundell (1973) argues that a common currency could help improve risk sharing, by increasing cross holdings of assets or deepening

\[\text{Atkeson and Bayoumi (1993) examine cross-regional insurance in the United States and conclude that “integrated capital markets are [...] unlikely to provide a substantial degree of insurance against regional economic fluctuations [...] This task will continue to be primarily the business of government.”}\]
financial markets. While our model is silent on whether a currency union may facilitate the development of financial markets, it shows that the benefits of risk sharing are larger in a currency union and that government intervention is needed to reap the full benefits. Indeed, we establish that private risk sharing is not constrained Pareto efficient in a currency union, so that financial integration alone is not sufficient.

We emphasize three key determinants of the effectiveness of transfers as a stabilization tool in a currency union: the asymmetry of the shocks hitting the members of the currency union, the persistence of these shocks (in the dynamic version of the model) and the openness of the member economies. Indeed, symmetric shocks can be accommodated with union wide monetary policy so that transfers should be used only in the face asymmetric shocks. Optimal transfers are increasing in the persistence of the shocks, but hump-shaped as a function of openness. However a given transfer is more effective at stabilizing the economy when the economy is more closed. Hence more stabilization is achieved at the optimum both when the economy is more closed, and when shocks are more persistent. Indeed, we show that full stabilization is achieved in the limit as shocks become permanent and the economy becomes closed. This contrasts with the ideas in McKinnon (1963), who discusses reasons why openness may mitigate the costs of currency unions. However, our results are fully compatible with the notion that openness is beneficial in a currency union lacking a fiscal union because our results only apply when an optimal fiscal union is in place.

It is interesting to compare fiscal unions with other macroeconomic stabilization instruments in currency unions, such as government spending (as analyzed in Beetsma and Jensen 2005; Gali and Monacelli 2008; Ferrero 2009) and capital controls (Farhi and Werning 2012; Schmitt-Grohe and Uribe 2012). It is best to jointly use these different instruments to the extent that they are available, and our main results about the optimal design of fiscal unions are robust to the inclusion of these other instruments. But one might also want to choose among these different instruments instead of using them jointly, which requires an assessment of their relative performance. To that effect, we also discuss theoretically and illustrate numerically the relative performance of these different instruments depending on a number of important parameters of the economy such as openness and the persistence of shocks.

Finally, we explore the robustness of our results in the presence of agency problems at the national level, such as limited commitment or moral hazard. We show how these incentive issues together with the forces that we have identified above jointly influence the optimal design of fiscal unions, and that our main insights carry over.

The rest of the paper is organized as follows. The static model is covered in Sections 2 and 3. The dynamic model is contained in Sections 4 and 5. Section 6 contains our conclusions. All the proofs are contained in the appendix.

Interestingly, we should expect more stabilization to be achieved if countries in a bust also faced credit constraints (a possibility that we abstract from). Indeed, this would raise their marginal propensity to consume and hence increase the stabilization benefits of transfers.
Related literature. First and foremost, our paper is related to the Optimal Currency Area (OCA) literature. This literature has emphasized a number of important factors for successful currency unions: factor mobility (Mundell, 1961), openness (McKinnon, 1963), fiscal integration (Kenen, 1969), and financial integration (Mundell, 1973). Our paper formalizes and refines the arguments of Kenen (1969), by seeing fiscal unions as the implementation of an optimal risk sharing arrangement within in a currency union, in a model with explicit micro-foundations. We offer a precise characterization of the size, direction, and effectiveness of fiscal transfers. Our results qualify the view implicit in Mundell (1973) that financial integration is a substitute to fiscal integration. Finally, our work contrasts with the ideas in McKinnon (1963), who discusses reasons why openness may mitigate the costs of currency unions. In our paper, fiscal unions are more effective when member countries are more closed. However, our results are fully compatible with the notion that openness is beneficial in a currency union lacking a fiscal union.

Our modeling approach follows the New Keynesian tradition embraced by the New Open Economy Macro literature. In particular, our static analysis builds on the model of Obstfeld and Rogoff (2000), and our dynamic analysis builds on the model of Gali and Monacelli (2005, 2008). A flexible exchange rate allows the implementation of the flexible price allocation (see e.g. Benigno, 2000; Clarida et al., 2002; Gali and Monacelli, 2005). A fixed exchange rate represents a constraint on macroeconomic stabilization, and raises the question of the optimal use of monetary policy in a currency union. Benigno (2004) analyzes the case of a currency union with complete markets, shows that monetary policy at the union level cannot achieve perfect stabilization in the face of asymmetric shocks and characterizes optimal monetary policy at the union level.

Our paper explores the optimal use of macroeconomic instruments beyond monetary policy, focusing, in particular, on cross-country transfers or interventions in financial markets. Previous literature has studied other policy tools. Beetsma and Jensen (2005) and Gali and Monacelli (2008) analyze optimal fiscal policy in a currency union, by characterizing how government purchases of home goods can help stabilize the economy in response to asymmetric shocks. Adao et al. (2009) and Farhi et al. (2011) show that with a rich enough set of distortionary taxes, the flexible price allocation can be achieved. However, in our view, there are practical limitations that limit the extent to which these tax incentives can be used, leaving considerable room for other instruments. Ferrero (2009) analyzes another dimension of fiscal policy, focusing on distortionary taxes and government debt. Farhi and Werning (2012) and Schmitt-Grohe and Uribe (2012) analyze capital controls. None of these papers considers fiscal transfers across union members and most assume complete private financial markets. Our work complements these contributions by analyzing fiscal transfers as another macroeconomic tool.

Few papers consider optimal policy with incomplete financial markets. An exception is Benigno (2009) who analyzes optimal monetary policy in the case of incomplete markets and flexible exchange rates. Nominal rigidities create a tradeoff between completing markets and stabilizing the economy. On the one hand, if prices were flexible, the optimum would imitate complete markets, by tailoring the real returns of international bonds. On the other hand, if markets could be com-
pleted, or if transfers imitate complete markets, the optimum would be fully efficient. Our modeling assumptions and results are essentially the polar opposite. Our analysis assumes that the exchange rate is fixed, so that the aforementioned tradeoff is not considered. Moreover, in the presence of non-traded goods or home bias, our main result is that complete markets, or transfers that imitate complete markets, lead to a suboptimal outcome.

The key ingredient of the New Open Economy Macro literature is the presence of nominal rigidities. Another important ingredient, present in some but not all papers in that literature is the assumption of home bias or non-traded goods, allowing for movements in the real exchange rate. This ingredient is absolutely central for our theory—as it is in any serious analysis of the Transfer Problem. Finally, we study an instrument (intervention in financial markets with complete markets, or international transfers with incomplete markets) that had not been considered in the literature.

2 A Static Model of a Currency Union

We start with a simple static model that illustrates our main idea most transparently. Later we show that the same effects are present in standard dynamic open economy models. The model’s environment builds on the model with traded and non-traded goods presented in Obstfeld and Rogoff (2000). It features a traded good, a non-traded good and labor. The traded good is supplied inelastically and traded competitively. The non-traded good is supplied from labor by monopolistic firms. The prices set by these monopolistic firms are sticky.\(^4\)

We offer two market settings and associated policy interventions for the same model environment. The first assumes complete markets and features portfolio taxes as the policy instrument to influence equilibrium risk sharing across countries. The second assumes incomplete markets, so that private agents have no opportunities to share risk. In this case we focus on government arranged fiscal transfers across countries to provide international risk sharing. Importantly, we show that both settings lead to the same set of implementable allocations. This allows us to characterize efficient allocations using the same second-best Ramsey planning problems for both settings in Section 3.

In our view, the first setting, while less realistic offers several conceptual advantages. First, it allows us to make the point that constrained efficient allocations require government intervention even if financial markets are complete. By implication, if markets are incomplete, government intervention should not simply mimic the complete-markets outcome. Second, we can provide simple formulas for the interventions in the form of portfolio taxes. The incomplete markets setting, on the other hand, seems more realistic and the implementation of constrained efficient allocations involves cross country risk sharing through fiscal transfers, providing a foundation for fiscal unions. In any case, although we favor the incomplete-market setting and its implementation in practical terms, the characterization using complete markets sheds light on both.

\(^4\)In the online appendix A.5, we show that all our results go through if wages are nominally rigid instead of prices. In particular, Propositions 1–12 are still valid.
2.1 Households

There is a single period and a continuum of countries indexed by $i \in [0, 1]$. We start by assuming that all countries belong to a currency union, but will relax this later. Uncertainty affects preferences and technology: the state of the world $s \in S$ has density $\pi(s)$ and determines preferences and technology, possibly asymmetrically, in all countries.

In each country $i \in I$, there is a representative agent with preferences over non-traded goods, traded goods and labor given by the expected utility

$$\int U^i(C^i_{NT}(s), C^i_T(s), N^i(s); s)\pi(s)ds.$$ 

Below we make some further assumptions on preferences.

In the complete-market setting, agents can trade in a complete set of financial markets before the realization of the state of the world $s \in S$ (we discuss the incomplete market setting in subsection 2.5). Households are subject to the following budget constraints

$$\int D^i(s)Q(s)\pi(s)ds \leq 0, \tag{1}$$

$$P^i_{NT}C^i_{NT}(s) + P^i_T C^i_T(s) \leq W^i(s)N^i(s) + P^i_T E^i_T(s) + \Pi^i(s) + T^i(s) + (1 + \tau^i_D(s))D^i(s), \tag{2}$$

where $P^i_{NT}$ is the price of non-traded goods which as we will see shortly, does not depend on $s$ due to the assumed price stickiness; $P^i_T(s)$ is the price of traded goods in state $s$; $W^i(s)$ is the nominal wage in state $s$; $E^i_T(s)$ is country $i$’s endowment of traded goods in state $s$; $\Pi^i(s)$ represents aggregate profits in state $s$; $T^i(s)$ is a lump sum rebate; $D^i(s)$ is the nominal payoff of the household portfolio in state $s$; $Q(s)$ is the price of one unit of currency in state $s$ in world markets, normalized by the probability of state $s$; and $\tau^i_D(s)$ is a state contingent portfolio return subsidy.\(^5\) The lump sum rebate $T^i(s)$ is used to rebate the proceeds from the tax on financial transactions to households. We sometimes also consider lump-sum transfers over and above such rebates to redistribute wealth across countries. Note that the nominal price of traded goods is assumed to be the same across countries, reflecting the law of one price and the fact that all countries in the union share the same currency.

The households’ first order conditions can be written as

$$\frac{U^i_{C_T}(s)(1 + \tau^i_D(s))}{Q(s)P^i_T(s)} = \frac{U^i_{C_T}(s')(1 + \tau^i_D(s'))}{Q(s')P^i_T(s')}, \tag{3}$$

$$\frac{U^i_{C_T}(s)}{P^i_T(s)} = \frac{U^i_{C_{NT}}(s)}{P^i_{NT}}, \tag{4}$$

$$\frac{U^i_N(s)}{W^i(s)} = \frac{U^i_{C_{NT}}(s)}{P^i_{NT}}. \tag{5}$$

\(^5\)Above we assumed that the returns from firms are not subsidized. Another possibility is to subsidize profits $\Pi^i(s)$ at the same rate $\tau^i_D(s)$ as financial returns. None of our analysis or conclusions are affected by this modeling choice.
2.2 Firms

We assume that the traded good is in inelastic supply: each country is endowed with a quantity $E_T(s)$ of traded goods. These goods are traded competitively in international markets.

Non-traded goods are produced in each country by competitive firms that combine a continuum of non-traded varieties indexed by $j \in [0, 1]$ using the constant returns to scale CES technology

$$Y_{NT}^i(s) = \left( \int_0^1 Y_{NT}^{ij}(s)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}},$$

with elasticity $\varepsilon > 1$.

Each variety is produced by a monopolist using a linear technology:

$$Y_{NT}^{ij}(s) = A^i(s) N^{ij}(s).$$

Each monopolist hires labor in a competitive market with wage $W^i(s)$, but pays $(1 + \tau^i_L)W^i(s)$ net of a country specific tax on labor. Monopolists must set prices in advance, at the beginning of the period, before the realization of uncertainty. The demand for each variety is given by

$$C_{NT}^i(s)\left( \frac{P_{NT}^i}{P_{NT}^i} \right)^{-\varepsilon}$$

where $P_{NT}^i = (\int (P_{NT}^{ij})^{1-\varepsilon} dj)^{1/(1-\varepsilon)}$ is the price of non traded goods.

With complete markets (we discuss the incomplete markets case further below) they solve

$$\max_{P_{NT}^{ij}} \int Q(s) \left( 1 + \tau^i_D(s) \right) \Pi^{ij}(s) \pi(s) ds,$$

where

$$\Pi^{ij}(s) = \left( P_{NT}^{ij} - \frac{1 + \tau^i_L}{A^i(s)} W^i(s) \right) C_{NT}^i(s) \left( \frac{P_{NT}^{ij}}{P_{NT}^i} \right)^{-\varepsilon}.$$

Aggregate profits are given by $\Pi^i(s) = \int \Pi^{ij}(s) dj$. In a symmetric equilibrium, all monopolists in country $i$ set the same profit maximizing price. Rearranging the first-order condition yields the familiar expression for the price as a markup over a weighted average across states of the marginal cost

$$P_{NT}^i = (1 + \tau^i_L) \frac{\varepsilon}{\varepsilon - 1} \left( \frac{\int Q(s) W^i(s) C_{NT}^i(s) \pi(s) ds}{\int Q(s) C_{NT}^i(s) \pi(s) ds} \right)^{\frac{1}{\varepsilon}}.$$

(6)

2.3 Government

The government is subject to the budget constraint

$$T^i(s) = \tau^i_L W^i(s) N^i(s) - \tau^i_D(s) D^i(s) + \hat{T}^i(s).$$

(7)
Here \( \hat{T}_i(s) \) are net international fiscal transfers, satisfying
\[
\int \hat{T}_i(s) di = 0,
\]
for all \( s \in S \), that redistributes resources across countries via the governments’ budgets.

### 2.4 Equilibrium with Complete Markets

An equilibrium is such that households and firms maximize, the government’s budget constraint is satisfied, and markets clear:
\[
C^i_{NT}(s) = A^i(s)N^i(s),
\]
\[
\int C^i_T(s) di = \int E^i_T(s) di.
\]

These conditions imply that the bond market is cleared, i.e. \( \int D^i(s) di = 0 \) for all \( s \in S \).

The conditions for an equilibrium (1)–(10) act as constraints on the planning problem we study next in Section 3. However, in a spirit similar to Lucas and Stokey (1983), we seek to drop variables and constraints as follows. Given quantities, equations (3), (5) and (6) can be used to back out certain prices, wages and taxes. Since these variables do not enter the welfare function they can be dispensed with from our planning problem, along with equations (1), (2), (3), (5), (6), (7), and (8). We summarize these arguments in the following proposition.

**Proposition 1** (Implementability, Complete Markets). An allocation \( \{C^i_T(s), C^i_{NT}(s), N^i(s)\} \) together with prices \( \{P^i_T(s), P^i_{NT}\} \) form part of an equilibrium with complete markets if and only if equations (4) and (9) hold for all \( i \in I, s \in S \) and equation (10) holds for all \( s \in S \).

Importantly, we cannot dispense with equation (4). This equation summarizes the restriction imposed by a currency union, that the price of traded goods cannot vary across countries, and price stickiness, that the price of non-traded goods cannot vary across states of the world. Consider attempting to use equation (4) as a residual to back out prices that support an allocation, as we did with equations (3), (5) and (6). Equation (4) requires that the relative price of traded to non-traded goods equal \( U^i_{CT}(s)/U^i_{C_{NT}}(s) \). For any arbitrary allocation, this required relative price can be computed, but the problem is that it may not be possible to express it as a ratio of a price that is independent of \( i \) and a price that is independent of \( s \), i.e. as a ratio \( P^i_T(s)/P^i_{NT} \). This is why we must keep equation (4) as a constraint.

Our constructive proof shows that an allocation \( \{C^i_T(s), C^i_{NT}(s), N^i(s)\} \) and prices \( \{P^i_T(s), P^i_{NT}\} \) that satisfy the conditions in the propositions are actually part of several equilibria. We have emphasized two dimensions of indeterminacy. First, we can choose any set of state prices \( Q(s) \). Second, we

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6 Our notation already takes into account the symmetry of prices, output and labor across varieties \( j \) within each country \( i \).

7 In addition, the budget constraints (1) and (2) must hold as an equality.
can choose different ex-post fiscal transfers \( \hat{T}_i(s) \). These two dimensions are actually related in the sense that different state prices require different ex-post fiscal transfers.

The first dimension of indeterminacy can be intuitively understood as follows. The relevant state prices for households are adjusted for portfolio taxes \( \frac{Q(s)}{1+\tau_D(s)} \). Scaling up state prices \( Q(s) \) and the corresponding portfolio taxes \( 1+\tau_D(s) \) by a function \( \lambda(s) \) leaves these tax-adjusted state prices unchanged. However this change indirectly transferring resources across countries and states. These indirect transfers need to be compensated by adjusting ex-post fiscal transfers \( \hat{T}_i(s) \).

The second dimension of indeterminacy can be intuitively understood as follows. How much transfers across countries actually operate through financial markets \( D^i(s) \) or ex-post fiscal transfers \( \hat{T}_i(s) \) is not pinned down. For example, one possibility is to constrain ex-post fiscal transfers to be non-state contingent \( \hat{T}^i(s) = \hat{T}^i \). All risk sharing is then being delivered through financial markets, and portfolio taxes are required to make sure that private agents secure the right amount of insurance \( D^i(s) \). Another possibility is to set \( \hat{T}^i(s) = P_T(s)(C_T^i(s) - E^i_T(s)) \). In that case, all risk sharing is being delivered through ex-post fiscal transfers. Portfolio taxes are then required to ensure that private agents do not “undo” these transfers and indeed choose \( D^i(s) = 0 \).

### 2.5 Equilibrium with Incomplete Markets

We also consider an alternative setup where markets are incomplete, in the sense that there are no financial markets before the realization of the state of the world \( s \in S \). We split the representative agent in country \( i \) into a continuum of households \( j \in [0,1] \). Household \( j \) is assumed to own the firm of variety \( j \). Households \( j \) maximizes utility

\[
\int U^i(C^i_{NT}(s), C^i_T(s), N^i(s); s) \tau(s) ds,
\]

by choosing \( \{C^i_T(s), C^i_{NT}(s), N^i(s)\} \) and the prices set by its own firm \( P^i_{NT} \), taking aggregate prices and wages \( \{P_T(s), P^i_{NT}, W^i(s)\} \) and aggregate demand \( \{C^i_{NT}(s)\} \) as given, subject to

\[
P^i_{NT}C^i_{NT}(s) + P_T(s)C^i_T(s) \leq W^i(s)N^i(s) + P_T(s)E^i_T(s) + \Pi^{ij}(s) + T^i(s),
\]

where

\[
\Pi^{ij}(s) = \left( P^i_{NT} - 1 + \frac{\tau^i}{A^i(s)} W^i(s) \right) C^i_{NT}(s) \left( \frac{P^i_{NT}}{P^i_{NT}} \right)^{-\varepsilon}.
\]

The corresponding first-order conditions are symmetric across \( j \) and given by (4) and (5) and the price setting condition

\[
P^i_{NT} = (1 + \tau^i) \varepsilon \frac{\int U^i_{NT}(s) W^i(s) C^i_{NT}(s) \tau(s) ds}{\int U^i_{NT}(s) P^i_T(s) C^i_{NT}(s) \tau(s) ds}.
\]
Of course, in equilibrium we impose the consistency condition that $C^i_{NT}(s) = C^i_{NT}(s)$ for all $i$ and $s$.

The government budget constraint simplifies to

$$T^i(s) = \tau^i_L W^i(s) N^i(s) + \hat{T}^i(s).$$

(13)

We can now define an equilibrium with incomplete markets. An equilibrium specifies quantities $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$, prices and wages $\{P_T(s), P^i_{NT}, w^i(s)\}$, taxes $\{\tau^i_L, T^i(s)\}$ and international fiscal transfers $\{\hat{T}^i(s)\}$ such that households and firms maximize, the government’s budget constraint is satisfied, and markets clear. More formally, the conditions for an equilibrium are given by (4), (5), (8), (11) holding with equality, (12) with $\hat{C}^i(s) = C^i(s)$, and (13).

As in the complete markets implementation, we can drop variables and constraints as follows. Given quantities, equations (5) and (12) can be used to back out certain prices, wages and taxes. Since these variables do not enter the welfare function they can be dispensed with from our planning problem, along with equations (5), (8), (11), (12), and (13). We summarize these arguments in the following proposition.

**Proposition 2** (Implementability, Incomplete Markets). An allocation $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$ together with prices $\{P_T(s), P^i_{NT}\}$ form part of an equilibrium with incomplete markets if and only if equations (4) and (9) hold for all $i \in I, s \in S$ and equation (10) holds for all $s \in S$.

Propositions 1 and 2 reach the same implementability conditions for the complete- and incomplete-market settings. Of course, although the set of implementable quantities $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$ and prices $\{P_T(s), P^i_{NT}\}$ is the same, the required policy instruments are different.

Under complete markets, portfolio taxes $\{\tau^i_D(s)\}$ are needed, and international transfers $\{\hat{T}^i(s)\}$ are largely indeterminate. This can easily be seen by starting with the household’s budget constraint, holding with equality, and substituting out profits $\Pi^i(s)$ and transfers $T^i(s)$ to arrive at the following country budget constraint

$$\int Q(s) \left[ P_T(s)(C^i_T(s) - E^i_T(s)) \right] \pi(s) = \int Q(s) \hat{T}^i(s) \pi(s) ds,$$

which states that the value of the trade balance must be covered by the value of international fiscal transfers. Indeed, this is the only restriction on fiscal transfers, any $\{\hat{T}^i(s)\}$ satisfying this equation helps implements an equilibrium. One simple case is to assume that transfers that are not state contingent, making $\hat{T}^i(s)$ independent of $s$ for all $i$.

In contrast, in the incomplete market setting no restriction on private portfolios are introduced since no assets are available to private agents. In this case, the international transfers $\{\hat{T}^i(s)\}$ are uniquely determined and are typically state contingent.

### 2.6 Homothetic Preferences

Next, we characterize this key condition (4) further by making some weak assumptions on preferences. We make two assumptions on preferences: (i) preferences over consumption goods are weakly
separable from labor; and (ii) the preference over consumption goods are homothetic. Denoting by
\[ p^i(s) = \frac{P^i(s)}{P^i_{NT}} \]
the relative price of traded goods in state \( s \) in country \( i \), these assumptions imply that
\[ C^i_{NT}(s) = \alpha^i(p^i(s);s)C^i_T(s), \]
for some function \( \alpha^i(p; s) \) that is increasing and differentiable in its first argument. This conveniently
encapsulates the restriction implied by the first order condition (4). This condition is crucial because
the stickiness of non-traded prices, together with the lack of monetary independence, places restric-
tions on the possible variability across \( i \in I \), for any state of the world \( s \), in the relative price \( p^i(s) \).

3 Constrained Efficient Risk Sharing in the Static Model

Define the indirect utility function
\[ V^i(C_T, p; s) \equiv U^i\left(\alpha^i(p; s)C_T, C_T, \frac{\alpha^i(p; s)}{A^i(s)}C_T; s\right). \]

In an equilibrium with \( C^i_T(s) \) and \( p^i(s) \), ex post welfare in state \( s \) in country \( i \) is then given by
\[ V^i(C^i_T(s), p^i(s); s). \] The derivatives of the indirect utility function will prove useful for our analysis.

To describe these derivatives, it is useful to first introduce the labor wedge\(^8\)
\[ \tau^i(s) \equiv 1 + \frac{1}{A^i(s)} \frac{U^i_{C_T}(s)}{U^i_{C^i_{NT}}(s)}. \]

The labor wedge is zero at a first-best allocation.

**Proposition 3.** The derivatives of the value function are
\[ V^i_p(s) = \frac{\alpha^i_p(s)}{p^i(s)}C^i_T(s)U^i_{C^i_T}(s)\tau^i(s) \quad \text{and} \quad V^i_{C^i_T}(s) = U^i_{C^i_T}(s)\left(1 + \frac{\alpha^i(s)}{p^i(s)}\tau^i(s)\right). \]

These observations about the derivatives and their connection to the labor wedge will be key
to our results. A private agent values a transfer in traded goods according to its marginal utility
\[ U^i_{C^i_T}(s), \] but the actual marginal value in equilibrium is \( V^i_{C^i_T}(s) \). The wedge between the two equals
\[ \frac{\alpha^i(s)}{p^i(s)}\tau^i(s) = \frac{P^i_{NT}C^i_{NT}(s)}{P^i_{NT}(s)C^i_T(s)}\tau^i(s), \] the labor wedge weighted by the relative expenditure share of non-traded
goods relative to traded goods. We will sometimes refer to it as the *weighted labor wedge* for short.

In particular, a private agent undervalues transfers \( V^i_{C^i_T}(s) > U^i_{C^i_T}(s) \) whenever the economy is
experiencing a recession, in the sense of having a positive labor wedge \( \tau^i(s) > 0 \). Conversely, private
agents overvalue the costs of making transfers \( V^i_{C_T}(s) < U^i_{C_T}(s) \) whenever the economy is booming,

\(^8\)In this and other expressions and functions we streamline the notation by leaving the dependence on some of the
arguments implicit.
in the sense of having a negative labor wedge $\tau^i(s) < 0$. These effects are magnified when the economy is relatively closed, so that the relative expenditure share of non-traded goods is large.

When country $i$ receives a transfer, its consumers feel richer and increase their spending on both traded and non-traded goods in equal proportions. Since prices are fixed, the resulting increased demand for non-traded goods translates one-for-one into an increase in output. This in turn generates more income, further raising spending etc. This mechanism is at the core of the famous Transfer Problem controversy between Keynes (1929) and Ohlin (1929). These equilibrium effects, which are not internalized by private agents, open up a wedge between the social and private marginal values of transfers.

Since the increase in demand for both goods is proportional, the “dollar-for-dollar” output multiplier of transfers is precisely given by the relative expenditure share of non-traded to traded goods $\frac{P_{NT}C_{NT}(s)}{P_T(s)C_T(s)}$. The labor wedge $\tau^i(s)$ summarizes the net calculation for utility of the increase in non-traded consumption and the increase in labor that accompany the increase in output. This explains why the wedge between the social and private marginal valuations is precisely $\frac{P_{NT}C_{NT}(s)}{P_T(s)C_T(s)} \tau^i(s)$.

It is theoretically possible for the marginal value of a transfer to be negative $V^i_{C_T}(s) < 0$ if the labor wedge is sufficiently negative, especially if the share of non traded goods, relative to traded goods, is large enough. In this extreme case a country can improve welfare by making gift transfers, without any counterpart transfer in the opposite direction. If $\tau^i$ is sufficiently negative then unilateral gift transfers to other countries are welfare enhancing for country $i$. This extreme case will not be our focus and is not employed in any of our results below. However, it is a stark example of just how divergent public and private valuations of transfers can become.

### 3.1 Second-Best Ramsey Planning Problem

We consider a planning problem that allows us to characterize constrained Pareto efficient allocations. The planning problem is indexed by a set of nonnegative Pareto weights $\lambda^i$. By varying these Pareto weights, we can trace out the entire constrained Pareto frontier. The second-best planning problem is

$$\max_{P_T(s), P_{NT}C_T(s)} \int \int \lambda^i V^i \left( C^i_T(s), \frac{P_T(s)}{P_{NT}} ; s \right) \pi(s) \, ds \, ds$$

subject to

$$\int C^i_T(s) \, ds = \int E^i_T(s) \, ds.$$ 

Let $\mu(s) \pi(s)$ be the multiplier on the resource constraint in state $s \in S$. The first order conditions for $C^i_T(s), P_T(s)$ and $P_{NT}^i$ are, respectively,

$$\lambda^i V^i_{C_T}(s) = \mu(s),$$

$$\int V^i_{p_i}(s) \frac{1}{P_{NT}} \lambda^i \, ds = 0,$$
\[ \int V_p^i(s)p^i(s)\pi(s)ds = 0. \]

These first-order conditions tightly characterize the solution. The first order condition for \( P_{NT}^i \) implies our first proposition.

**Proposition 4 (Optimal Price Setting).** At a constrained Pareto efficient equilibrium, for every country \( i \), a weighted average of labor wedges across states is zero:

\[ \int \alpha_p^i(s)C_T^i(s)U_{CT}^i(s)\tau^i(s)\pi(s)ds = 0. \]

In the absence of uncertainty this proposition implies a zero labor wedge \( \tau^i(s) = 0 \), obtained by setting the labor tax to cancel the monopolistic markup: \( \tau_L^i = -1/\epsilon \). With uncertainty, in general \( \tau_L^i \neq -1/\epsilon \) and the labor wedge takes on both signs with a weighted average of zero.\(^9\)

The first-order condition for \( P_T(s) \) implies the following proposition.

**Proposition 5 (Optimal Monetary Policy).** At a constrained Pareto efficient equilibrium, in every state \( s \), a weighted average of labor wedges across countries is zero:

\[ \int \alpha_p^i(s)C_T^i(s)U_{CT}^i(s)\tau^i(s)\lambda^i di = 0. \]

This proposition establishes that optimal monetary policy targets a weighted average across countries for the labor wedge. It sets this target to zero in each state of the world. The intuition for the result is that monetary policy can be chosen at the union level, and can adapt across states to the average condition. If all countries are identical and the shock is symmetric, then we obtain perfect stabilization in each country: \( \tau^i(s) = 0 \) for all \( i \in I, s \in S \). By contrast, when shocks across countries are not symmetric then perfect stabilization is impossible. However, at the union level the economy is stabilized in the sense that the weighted average for the labor wedge across countries is set to zero for all states of the world \( s \in S \).\(^10\)

Finally, the first order condition for \( C_T(s) \) says that the marginal utility of transfers in traded goods adjusted for the Pareto weight \( \lambda^iV_{CT}^i(s) \) should be equalized across countries for every state \( s \). It is more revealing to rewrite this condition using our expressions for the derivative of \( V_{CT}^i(s) \).

**Proposition 6 (Optimal Risk Sharing).** For every pair of states \( (s, s') \), and pair of countries \( (i, i') \), optimal

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\(^9\)When the sub-utility function between \( C_{NT} \) and \( C_T \) is a CES so that \( \alpha(\cdot; s) \) has constant elasticity, independent of \( s \), then \( \tau_L^i = -1/\epsilon \) is optimal even with uncertainty. The proof is contained in the online appendix A.3.

\(^10\)The result is related to the result in Benigno (2004) and Gali and Monacelli (2008) that optimal monetary policy in a currency union ensures that the union average output gap, in a linearized version of the model, is zero in every period. Here the result is obtained without linearizing the model and it is expressed in terms of the labor wedge, instead of the output gap.
risk sharing takes the following form:

\[
\frac{U^i_C(s)}{U^i_C(s')} \left( 1 + \frac{\alpha^i(s)}{p^i(s)} \tau^i(s) \right) = \frac{U'^i_C(s)}{U'^i_C(s')} \left( 1 + \frac{\alpha'^i(s)}{p'^i(s)} \tau'^i(s) \right).
\] (15)

If portfolio taxes are not employed, then the risk sharing condition (3) imposes the additional constraint that for every pair of states \((s, s')\), and pair of countries \((i, i')\),

\[
\frac{U^i_C(s)}{U^i_C(s')} = \frac{U'^i_C(s)}{U'^i_C(s')}.
\] (16)

Comparing these conditions, one may expect the private risk sharing condition (16) to be incompatible with the efficiency condition (15) except in special cases. Indeed, we next show that because labor wedges must average to zero across states and countries according to Propositions 4 and 5, they are indeed incompatible unless the first best is attainable. This implies that equilibria with privately optimal risk sharing (without portfolio taxes) are constrained Pareto inefficient.

**Proposition 7** (Inefficiency of Private Risk Sharing). An equilibrium with complete markets and no portfolio taxes \((\tau^i_D(s) = 0\) for all \(i \in I, s \in S)\) is constrained Pareto inefficient unless \(\tau^i(s) = 0\) for all \(i \in I, s \in S\), in which case it is first best.

Without interventions in financial markets, private agents secure the constrained efficient amount of risk sharing in financial markets. They do not fully internalize the macroeconomic stability consequences of their portfolio decisions, opening a role for government intervention in financial markets.\(^\text{11}\) Government intervention secures additional transfers from low weighted labor wedge countries (“boom” countries) to high weighted labor wedge countries (“bust” countries). This reduces the demand for non-traded goods in the boom countries and increases it in the bust countries, stabilizing output and income. These stabilization benefits are not internalized by private agents, hence the need for government intervention.

### 3.2 Implementation

We now turn to the implementation of constrained Pareto efficient allocations. With complete markets, constrained Pareto efficient equilibria can be decentralized with appropriate labor taxes \(\tau^i_L\) and corrective portfolio taxes \(\tau^i_D(s)\). Proposition 6 leads to a neat characterization of the required taxes.

**Proposition 8** (Complete Markets and Portfolio Taxes). If private asset markets are complete, constrained Pareto efficient allocations can be implemented by the following portfolio return subsidy/taxes

\[
\tau^i_D(s) = \frac{\alpha^i(s)}{p^i(s)} \tau^i(s).
\]

\(^\text{11}\) We should also point out that the Propositions 5 and 6 go through if non-traded goods prices are entirely predetermined (i.e. are exogenously fixed).
Insurance for bad states of the world, where the weighted labor wedge is high, should be relatively subsidized. It is interesting to note that the taxes do not depend directly on the Pareto weights \(\{\lambda^i\}\), but only indirectly through the relative expenditure share of non-traded goods and the labor wedge. This underscores the fact that they are imposed to correct a macroeconomic aggregate demand externality and not to redistribute. As we move along the constrained Pareto efficient frontier by varying Pareto weights \(\{\lambda^i\}\), the net present value of transfers to each country varies according to

\[
\int U^i_{C_T}(s)(1 + \tau^i_D(s)) \frac{\hat{T}^i(s)}{P_T(s)} \pi(s)ds = \int U^i_{C_T}(s)(1 + \tau^i_D(s))(C^i_T(s) - E^i_T(s)) \pi(s)ds.
\]

When markets are complete, how much transfers across countries actually operate through financial markets or ex-post fiscal transfers is indeterminate. For example, one possibility is to constrain ex-post fiscal transfers to be non-state contingent \(\hat{T}^i(s) = \hat{T}^i\). In this case all risk sharing is being delivered through financial markets, and portfolio taxes are required to make sure that private agents secure the right amount of risk sharing. Another possibility is to set \(\hat{T}^i(s) = P_T(s)(C^i_T(s) - E^i_T(s))\). In this case, all risk sharing is being delivered through ex-post fiscal transfers, and portfolio taxes are required to ensure that agents do not “undo” this risk sharing.

The implementation of the socially optimum with corrective portfolio taxes is only one interesting possibility. Another equally interesting interpretation of our results assumes that private asset markets are nonexistent, so that private opportunities for risk sharing are unavailable. The optimum can then be implemented through ex-post transfers contingent on the shocks experienced by each country.

**Proposition 9 (Incomplete Markets and Ex-Post Transfers).** If private asset markets are incomplete so that state contingent-assets are unavailable, constrained Pareto efficient allocations can also be implemented through ex-post transfers contingent on the shock experienced by each country

\[
\hat{T}^i(s) = P_T(s)(C^i_T(s) - E^i_T(s)).
\]

Under this alternative implementation, no restriction on private portfolios are needed since no assets are available to private agents. Our results can then be seen as offering a precise characterization of the required ex-post transfers. A key conclusion of our analysis is that these transfers would go beyond replicating the outcome that private risk sharing decisions would achieve if markets were complete.

It is also possible to imagine implementations that are in between the two polar cases of corrective portfolio taxes with complete markets and ex-post transfers with incomplete markets. In general, government positions in asset markets, or ex-post transfers contingent on the shocks experienced by each country, combined with some restrictions or tax incentives on agents private portfolios are required.

\[12\]The exact value of the transfer is \(\hat{T}^i = \int \frac{U^i_{C_T}(s)(1 + \tau^i_D(s))(C^i_T(s) - E^i_T(s)) \pi(s)ds}{\int U^i_{C_T}(s)(1 + \tau^i_D(s)) \pi(s)ds} \).
3.3 Countries Outside the Currency Union

Up to this point we have assumed that all countries belong to the currency union. Now, imagine that only a subset of countries $I \subseteq [0, 1]$ are members. The rest manage monetary policy independently as follows. Country $i \notin I$ sets its own local nominal price for the traded good $P^i_T(s) = E^i(s)P^i_T(s)$ in its home currency by manipulating the level of its exchange rate $E^i(s)$ against the union’s currency.\(^\text{13}\)

The planning problem becomes

$$\max \int_{i \in I} \lambda^i V^i \left( C^i_T(s), \frac{P^i_T(s)}{P^i_{NT}} ; s \right) di + \int_{i \notin I} \lambda^i V^i \left( C^i_T(s), \frac{P^i_T(s)}{P^i_{NT}} ; s \right) di$$

subject to

$$\int C^i_T(s) di = \int E^i_T(s) di.$$  \hspace{1cm} (17)

For a country $i \notin I$ outside the union, the first order condition for $P^i_T(s)$ is

$$V^i_p(C^i_T(s), p^i(s); s) = \alpha^i_p(s) \frac{C^i_T(s)}{p^i(s)} \tau^i_C^i(s) \tau^i(s) = 0.$$  

By implication

$$\tau^i(s) = 0 \quad \text{for all } s \in S, i \notin I.$$

A flexible exchange rate leads to perfect stabilization, in the sense that the labor wedge is set to zero for all states of the world. This result is reminiscent of the arguments set forth by Friedman (1953) and Mundell (1961) in favor of flexible exchange rates. For countries in the currency union optimal monetary policy is still imperfect and characterized by the average condition for the labor wedge in Proposition 5.

The optimal risk sharing condition in Proposition 6 still applies to all countries, inside or outside the currency union. However, since $\tau^i(s) = 0$ for $s \in S, i \notin I$, this condition coincides with the privately optimal risk sharing condition for countries outside the currency union. As a result, there is no need to upset private risk sharing.

**Proposition 10 (Countries Outside the Currency Union).** None of the results are affected by considering countries outside the union. Countries that have independent monetary policy manage to obtain a zero labor wedge $\tau^i(s) = 0$. If markets are incomplete, they should not intervene in financial markets $\tau^i_D(s) = 0$. If markets are incomplete, they should seek to secure ex-post transfers $\hat{T}^i(s)$ that replicate private risk sharing outcomes.

If markets are incomplete, then ex-post fiscal transfers might be required even outside a currency union. Interestingly, we will show in the dynamic version of the model with only traded goods and

\(^{13}\text{Since the price of traded goods is modeled as flexible here, we do not require assumptions about producer currency pricing (PCP) versus local currency pricing (LCP); these are alternative assumptions regarding the form price stickiness takes.}\)
home bias in preferences, there are cases (the Cole-Obstfeld case) where ex-post fiscal transfers are not be required for countries outside a currency union, whereas they are required for countries inside a currency union. Crucially, our results establish that that inside a currency union, ex-post fiscal transfers should go beyond replicating the outcome that would arise if markets were complete. In this sense, our results yield two important insights. First currency unions and fiscal unions go hand in hand. Second, fiscal integration and financial integration are not perfect substitutes.

How are attitudes towards risk affected by membership in a union? We show that members are more risk averse in the following sense. Suppose country $i$ belongs to the currency union with equilibrium relative price $p^i(s)$. The advantage of leaving the union is that the relative price $p^i(s)$ is not constrained and welfare attains the first best level conditional on $C^i_T$. It follows that

$$v^i(C^i_T; s) \equiv V^i(C^i_T, p^i(s); s) \leq \max_p V^i(C^i_T, p; s) \equiv V^i_s(C^i_T; s), \quad (18)$$

with equality if and only if $p^i(s) \in \arg \max_p V^i(C^i_T, p; s)$, in which case the labor wedge is zero, $\tau(s) = 0$. Thus, for every state $s$, the function $V^i_s$ is the upper envelope over $v^i$ and is tangent to it precisely at a level of $C^i_T$ that implies $\tau(s) = 0$. In this sense, $v^i$ is more concave than $V^i_s$ and member countries are more risk averse. We shall put this inequality to use in the next section.

### 3.4 Value of Risk Sharing

Our simple model allows for three random disturbances: (i) shocks to productivity of labor in the production of non-traded goods; (ii) shocks to preferences (demand); and (iii) shocks to the endowment of traded goods. Proposition 7 shows that if the equilibrium without portfolio taxes does not attain the first best, then it is constrained inefficient. As we show next, this is true except in a knife-edge cases. Examining these knife-edge cases turns out to be interesting, because even when the equilibria coincides with the first best we find that the planner values the availability of insurance strictly more than private agents do. Risk sharing is of greater social than private value. Extrapolating beyond our model, this could help explain why macro insurance markets may be missing, even if their social value is significant.

To concoct an example where the first best is attainable it is useful to specialized our model to the utility function

$$U^i(C_T, C_{NT}, N; s) = \log(C_T) + \alpha^i(s) \log(C_{NT}) - \frac{1}{1 + \varphi} N^{1+\varphi}, \quad (19)$$

with $\varphi \geq 0$.

**Proposition 11.** Suppose the utility function is given by (19), then the equilibrium without portfolio taxes is constrained efficient if and only if productivity shocks and preference shocks are such for all pairs of countries $(i, i')$,

$$\frac{A^i(s)}{A^{i'}(s)} \left( \frac{\alpha^i(s)}{\alpha^{i'}(s)} \right)^{-\frac{\varphi}{1+\varphi}}$$
is constant for all \( s \in S \); the shocks to the endowment of traded goods \( E^i(s) \) can be arbitrary.

This proposition defines a precise notion of symmetric shocks to productivity and preferences for which the first best allocation is attainable without portfolio taxes. For example, if the only shocks are to productivity, then this condition requires that productivity vary proportionally across countries. A currency union can handle such a shock using union-wide monetary policy. A similar point applies to taste shocks. More generally, the key constraint imposed by nominal rigidities and a single monetary policy is condition (4), rewritten here for convenience as

\[
\frac{U_{CT}^i(s)}{U_{CT}^i(s)} = \frac{P_{NT}^i}{P_T(s)}
\]

where \( P_T(s) \) is only allowed to vary with \( s \) not \( i \), while \( P_{NT}^i \) is allowed to vary with \( i \) but not \( s \). In other words, one can handle fixed differences across countries and union-wide shocks to this marginal rate of substitution, but not individual variations. This refines the notion of symmetric shocks that is required for the first best. Monetary policy in a currency union is constrained, affecting the adjustment in prices, but in some special circumstances no adjustment is needed.

This discussion highlights just how special these circumstances are. Note, however, that the proposition implies that endowment shocks can be properly insured without portfolio taxes. To understand this result, suppose we only have shocks to endowments. Then the first best features perfect risk sharing in the consumption of traded goods: only aggregate fluctuations in traded goods affect the consumption of traded goods. Due to separability of preferences, the first best allocation for non traded goods and labor is not affected by these shocks. It follows that the marginal rate of substitution only varies with union-wide shocks and the first best is implementable as an equilibrium. The marginal rate of substitution only varies with union-wide shocks—and does so symmetrically—implying that the first best is implementable as an equilibrium.\(^{14}\)

Of course, the case of endowment shocks is somewhat artificial, relying on the modeling asymmetry that non traded goods are produced but traded goods are not. If instead traded goods were produced from labor and another fixed input (capital or land) subject to (industry specific) productivity shocks, then these shocks would also have to satisfy the restriction of being symmetric to attain the first best—just as in the case of productivity shocks in the non traded goods.

It is useful to have a case, however artificial, where private risk sharing is constrained efficient so that we can isolate a separate result. We show that members of a currency union value this risk sharing more than non members. Moreover, this is is not the true of the value placed on risk sharing by private individuals. This highlights the role of the aggregate demand externality from risk sharing decisions, which is not internalized by private agents.

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\(^{14}\)In more detail, suppose \( A^i(s) = A^i \) and \( \alpha^i(s) = \alpha^i \). The first best allocation features \( C_T^i(s) = \frac{1}{\pi} \int_0^1 E^i(s)di, N^i(s) = (\alpha^i)^{\frac{1}{\lambda^i}}, \) and \( C_{NT}^i(s) = A^i (\alpha^i)^{\frac{1}{\lambda^i}} \). This allocation is supported as an equilibrium without portfolio taxes by \( P_{NT}^i = (\alpha^i)^{\frac{1}{\pi^i}} / (\lambda^i A^i), P_T(s) = (\int_0^1 E^i(s)di)^{-1}, W^i(s) = (\alpha^i)^{\frac{1}{\pi^i}} / \lambda^i, Q(s) = 1 \) and \( 1 + \tau_L^i = \frac{\pi - 1}{\tau} \).
Proposition 12. Suppose there are only endowment shocks and that all risk is idiosyncratic, so that the aggregate endowment is constant across states. If we exclude a country from risk sharing, then its utility loss is greater if it belongs to a currency union. And if we exclude a single individual within a country from risk sharing, then his utility loss is the same whether or not his country belongs to a currency union.

Figure 1 illustrates the basic logic behind the first part this proposition for an example with two the equiprobable endowment values. Since the aggregate endowment is constant, the price of traded goods is constant and perfect financial markets offer fair insurance. The resulting equilibrium features constant consumption of the traded good at the average value of the endowment and constant prices and wages. This is true for both members and non members. When the country is excluded from risk sharing its consumption of the traded good must now fluctuate with its endowment, creating a mean-preserving spread in consumption of traded goods and a loss in expected utility. The crucial point is that the loss is greater for union members because they are more risk averse, according to inequality (18). Indeed, given that prices are constant and the utility function is independent of the state $s$ this inequality simplifies to $V^i(C_T, \bar{p}) \leq \max_p V^i(C_T, p) \equiv V^{i*}(C_T)$. These two value functions are depicted in the figure. They are tangent at the average value of the endowment $\bar{E}$ because this represents the equilibrium consumption level with risk sharing.

As to the second part of the proposition, it follows easily from the observation that in the context of this specific example, the equilibrium with risk sharing is the same whether or not the country belongs to the currency union. In both cases the first best allocation is attained. Therefore, if an individual is excluded from risk sharing, he faces the same prices whether the country is a member or not. Thus, the drop in utility is the same.

3.5 Limited Commitment

Explicit or implicit insurance (risk sharing) arrangements inevitably raise concerns of incentives. We have abstracted from these considerations, not because we believe them to be unimportant, but in order to isolate the effects that our aggregate demand externality has on optimal risk sharing.
Modeling limits to insurance due to incentive problems requires making specific choices about the underlying shocks, the asymmetry of information, the available monitoring technologies, or the type of commitment problem, etc. Although the possibilities are vast and exploring them all is beyond the scope of this paper, we believe the main insights of our analysis would carry over.\footnote{In practice of course, institutional mechanisms exist to mitigate these agency problems. For example, most fiscal unions such as the US channel a large part of their transfers through more or less ex-ante-rules-based automatic stabilizers (through the unemployment insurance program, federal income and social security taxes, bailout funds), probably for reasons of political acceptability and transparency, but also to mitigate the difficulties associated with collective and distributing discretionary ex-post transfers in a world with limited commitment. Another example is state debt-limit in the US, or collective budget procedures and enforcement mechanisms that already exist in Europe.}

In the online appendix A.6, we analyze an example with moral hazard. Here instead, we develop an example with limited commitment. Consider the implementation with incomplete markets and international transfers, where all international risk sharing occurs through international transfers. Ex post, in every state of the world \(s\), some countries \(i\) are net contributors to the union with \(\hat{T}_i(s)\leq 0\), and some countries \(i\) are net beneficiaries with \(\hat{T}_i(s)\geq 0\). This poses no particular problem to the extent that there exists a strong enough union-wide institutional enforcement mechanism. But with imperfect enforcement and limited commitment, the concern arises that governments of ex-post net contributor countries do no in fact contribute the transfers that were agreed upon ex ante behind the veil of ignorance before the realization of the shock.

To make things stark, consider the extreme case where there is no institutional enforcement mechanism. Governments can default on their promised transfers \(\hat{T}_i(s)\), and have no ability to commit. We assume that default leads to a utility loss for which we adopt a flexible parametrization \(K_i(s)\).

The planning problem can now be written as

\[
\max_{P_T, P_{NT}, C_T(s)} \int \int \lambda^i V^i \left( C_T^i(s), \frac{P_T}{P_{NT}} ; s \right) \pi(s) dids \tag{20}
\]

subject to

\[
\int C_T^i(s) di = \int E_T^i(s) di \tag{21}
\]

and

\[
V^i \left( C_T^i(s), \frac{P_T}{P_{NT}} ; s \right) \geq V^i \left( E_T^i(s), \frac{P_T}{P_{NT}} ; s \right) - K^i(s). \tag{22}
\]

The only difference introduced by the limited commitment problem is the presence of the incentive compatibility constraint (22), requiring each country to be better off sticking to the fiscal union arrangement than defaulting and reverting to autarky while experiencing the endowment loss associated with default.

Let \(\mu > 0\) be the multiplier on (21) and \(\nu^i(s) \geq 0\) be the multiplier on (22). The condition for socially optimal risk sharing becomes

\[
U^i_{C_T}(s) \left( 1 + \frac{\alpha^i(s)}{p^i(s)} \tau^i(s) \right) \left( 1 + \nu^i(s) \right) = \mu. \tag{23}
\]
By contrast, the corresponding condition for a country outside the currency union is

\[ U^i_{CT}(s)(1 + \nu^i(s)) = \mu. \]  

(24)

Condition (24) shows that even with flexible exchange rates, limited commitment endogenously limits insurance (risk sharing) possibilities. A high value of the multiplier \( \nu^i(s) \) indicates that it is relatively tempting for country \( i \) to default in a state \( s \). The optimal contract then adjusts the transfer \( \hat{T}^i(s) \) and the traded goods consumption \( C^i_T(s) \) so that default is prevented. Condition (23) shows how the optimal provision of insurance (risk sharing) and incentives must be modified when the country is in a currency union. The provision of incentives requires the private consumption of traded goods to vary with the realization of government consumption. Because prices are sticky, this generates a non-zero pattern of labor wedges \( \tau^i(s) \). This in turn opens up a wedge between the social and private marginal utility of income, which creates another force against the perfect equalization of consumption of traded goods across states for each country.

This example shows that the optimal risk sharing arrangements are different for countries that belong to a currency union than that for countries who have a flexible exchange rate. This is true with or without enforcement frictions. In both cases, the optimal arrangement involves a key statistic, the social marginal utility of transfers \( (U^i_{CT}(s) \left( 1 + \frac{\hat{T}^i(s)}{p^i(s)} \tau^i(s) \right) \) vs. \( U^i_{CT}(s) \).

3.6 Government Spending

We introduce government spending in the model. We characterize the joint optimal use of international transfers and government spending. Our analysis underscores that both instruments should be used in conjunction. Moreover, we show that our characterization of fiscal unions is robust to the availability of government spending as an additional instrument. We also compare their relative performance depending on a number of deep economic parameters by studying a few limit cases.

Introducing government spending. Following the literature, we focus on the case where government spending is concentrated on non-traded goods, which we view as the most practically relevant case.\(^\text{16}\) In each state \( s \) and country \( i \), the government spends \( p^i_{NT} G^i_{NT}(s) \) to finance government consumption of \( G^i_{NT}(s) \) of non-traded goods. As is standard, we capture agents’ preferences of government consumption by including it in the utility function and write

\[ U^i(G^i_{NT}(s), C^i_{NT}(s), C^i_T(s), N^i(s); s) \]

for the state-\( s \) utility function of country \( i \) agents. We assume that that preferences are weakly separable over government consumption on the one hand, and private consumption and labor on the

\(^{16}\)For example Beetsma and Jensen (2005) and Gali and Monacelli (2008) introduce government spending on domestic goods in models where all goods are traded, with or without home bias in consumption. The natural equivalent in our setup is to study government spending on non-traded goods. We have also analyzed government spending on traded goods. The analysis is available upon request.
other hand. In addition, we continue to assume that preferences over consumption goods are weakly separable from labor, and that the preference over consumption goods are homothetic.

Apart from that, there are only minor differences with the setup of the main model. These differences involve the government budget constraint, the resource constraint for non-traded goods, and the price setting conditions. Our implementability results in Propositions 1 and 2 can be extended in a straightforward way.

**Second-Best Planning problem.** In order to write down the second-best Ramsey planning problem jointly characterizing international transfers and government spending, we modify the indirect utility function. We define

\[ V^i(G^i_{NT}(s), C^i_T, p^i; s) \equiv U^i \left( G^i_{NT}(s), \alpha^i(p^i; s)C^i_T, \frac{\alpha^i(p^i; s)C^i_T + G^i_{NT}(s)}{A^i(s)}; s \right). \]

In an equilibrium with \( G^i_{NT}(s), C^i_T(s) \) and \( p^i(s) \), ex post welfare in state \( s \) in country \( i \) is then given by

\[ \hat{V}^i(G^i_{NT}(s), C^i_T(s), p^i(s); s). \]

The second-best planning problem is

\[ \max_{G^i_{NT}(s), P^i_T(s), C^i_T(s)} \int \int \hat{V}^i \left( G^i_{NT}(s), C^i_T(s), \frac{P^i_T(s)}{P^i_{NT}}; s \right) \lambda^i \pi(s) \, di \, ds \]

subject to

\[ \int C^i_T(s) \, di = \int E^i_T(s) \, di. \]

We can solve this planning problem recursively by defining

\[ V^i(C^i_T, p^i; s) \equiv \max_{G^i_{NT}(s)} U^i \left( G^i_{NT}(s), \alpha^i(p^i; s)C^i_T, \frac{\alpha^i(p^i; s)C^i_T + G^i_{NT}(s)}{A^i(s)}; s \right), \tag{25} \]

and then solving

\[ \max_{P^i_T(s), C^i_T(s)} \int \int V^i \left( C^i_T(s), \frac{P^i_T(s)}{P^i_{NT}}; s \right) \lambda^i \pi(s) \, di \, ds \]

subject to

\[ \int C^i_T(s) \, di = \int E^i_T(s) \, di. \]

\[ \text{The government budget constraint is now } T^i(s) + P^i_{NT}G^i_{NT}(s) = \tau^i L^i W^i(s)N^i(s) - \tau^i D^i(s)D^i(s) + \hat{T}^i(s). \]

The resource constraint for non-traded goods is now \( C^i_{NT}(s) + G^i_{NT}(s) = A^i(s)N^i(s) \). The price setting constraint is now \( P^i_{NT} = (1 + \tau^i L^i)^{-1} \frac{\int \frac{\pi(s)}{1 + \tau^i D^i(s)} \left[ C^i_{NT}(s) + G^i_{NT}(s) \right] \pi(s) \, ds}{\int \frac{\pi(s)}{1 + \tau^i D^i(s)} \left[ C^i_{NT}(s) + G^i_{NT}(s) \right] \pi(s) \, ds}. \]
**Constrained Pareto efficient allocations.** With these notations, the analysis is identical to that of the model without government spending. Indeed, the derivatives of the indirect utility function $V^i(C_T, p; s)$ are given by exactly the same formula as in Proposition 3, and as a result, Propositions 4-10 as well as Proposition 12 carry through without any modification.\(^{18}\) Hence our analysis of fiscal unions is robust to the availability of government spending as an additional instrument.

Of course, this does not mean that the resulting allocation is unchanged. Away from this case, optimal government spending can reduce the deviations of the labor wedge $\tau^i(s)$ from zero, but it does eliminate them.\(^ {19}\) There are two informative ways to write the optimality condition for government spending, both of which follow directly from the definition of $V^i(C_T, p; s)$ in equation (25):

$$U^i_{GNT}(s) = -\frac{1}{A^i(s)} U^i_N(s), \quad (26)$$

or

$$U^i_{GNT}(s) = (1 - \tau^i(s)) U^i_{CNT}(s). \quad (27)$$

To understand these formulas, it is best to analyze first the case when prices or exchange rates are flexible. Optimal government spending is then characterized by equation (26) or equation (27) with $\tau^i(s) = 0$. Both equations equalize the marginal benefit $U^i_{GNT}(s)$ of government consumption with its marginal cost, but express the marginal cost in two different (but equivalent) ways. Equation (26) expresses the marginal cost $-\frac{1}{A^i(s)} U^i_N(s)$ in terms of the marginal increase labor that would be required to service the marginal increase in government consumption, while equation (27) with $\tau^i(s) = 0$ expresses the marginal cost $U^i_{CNT}(s)$ in terms of the marginal reduction in private consumption that would be required to service the marginal increase in government consumption. These are two equivalent ways of stating the Samuelson rule (see Samuelson 1954) for the optimal provision of public goods.

Depending on which of these formulations one prefers to focus on, rigid prices and fixed exchange rates either require no deviation from the Samuelson rule (equation (26)) or a deviation from the Samuelson rule (equation (27)). The reason is that the social marginal cost of government spending is still given by $-\frac{1}{A^i(s)} U^i_N(s)$ but not by $U^i_{CNT}(s)$ and instead by $(1 - \tau^i(s)) U^i_{CNT}(s)$. This is because the price of non-traded goods does not reflect the marginal cost of producing them. The discrepancy is precisely given by the labor wedge. The government internalizes this wedge when it decides its consumption of non-traded goods, but private agents do not. As a result, in recessions when $\tau^i(s) > 0$, it is optimal to tilt the mix of government and private consumption of non-traded goods in the direction of the former, and the opposite holds true in booms when $\tau^i(s) < 0$.\(^ {20}\)

\(^{18}\)The exact conditions in Proposition 11 for the constrained efficiency of the complete markets equilibrium without portfolio taxes are different in the presence of government spending.

\(^{19}\)Formally, this is true except in the knife-edge cases where the optimal allocation with flexible prices can be implemented with a fixed exchange rate.

\(^{20}\)This analysis assumes that prices are entirely rigid. If there is some adjustment in prices, then increases in government spending stimulate inflation. Given a fixed exchange rate, and other things equal, this leads to an appreciation of the real exchange rate which depresses private spending on non-traded goods and counteracts the direct effect of government spending on total spending on non-traded goods (see e.g. Farhi-Werning 2012). This lessens the macroeco-
Having characterized the jointly optimal use of international transfers and government spending and shown the robustness of our characterization of optimal international transfers to the availability of government spending as an additional instrument, we now compare the relative performance of government spending and international transfers in a few enlightening limit cases. We first treat the case of the closed economy limit. We show that international transfers achieve perfect macroeconomic stabilization, with no residual role for government spending. By contrast, in the perfectly open economy limit, international transfers are not used for macroeconomic stabilization, but government spending is. We then treat the cases where the disutility of labor is linear or government spending is purely wasteful. In both cases, we show that even though the optimum is away from the first best, government spending is not useful for macroeconomic stabilization.

Closed economy limit. Consider first the closed economy limit. This limit can be understood as follows. Suppose for simplicity that preferences are given by
\[ v(G^i(s)) + \frac{C^i(s)^{1-\gamma}}{1-\gamma} - \phi(N^i(s)) \]
where
\[ C^i(s) = \left[ (1 - \alpha) \frac{1}{\eta} C^i_{NT}(s) \frac{\eta - 1}{\eta} + \frac{1}{\eta} C^i_{T}(s) \frac{\eta - 1}{\eta} \right] \frac{\eta}{\eta - 1}. \]
Then for any \( p_i \), \( \alpha^i(p; s) \) is decreasing in \( \alpha \). The closed economy limit is obtained in the limit where \( \alpha \) goes to zero and \( \alpha^i(p; s) \) goes to infinity, so that the relative expenditure share on non-traded to traded goods goes to infinity. In this limit, the first-best level of welfare is achieved. This is because international transfers are extremely powerful in relatively closed economies. Indeed, we have already emphasized that the “dollar-for-dollar” output multiplier of transfers is precisely given by the relative expenditure share of non-traded to traded goods. And this multiplier goes to infinity in the closed-economy limit. As we approach the closed economy limit, vanishingly small departures (as a fraction of each country’s nominal income) from the international transfers that support the first-best allocation are enough to perfectly stabilize the economy and deliver \( \tau^i(s) = 0 \) for all \( i \) and \( s \). There is no residual macroeconomic stabilization role for government spending, which then simply follows the first-best Samuelson rule.

Perfectly open economy limit. The relative usefulness of international transfers and government spending is reversed in the limit where countries are perfectly open, which we capture by letting \( \alpha \) go to one. In this limit, international transfers are not used for macroeconomic stabilization, in the sense that social and private optimal risk sharing coincide, so that optimal international transfers are only needed when markets are incomplete, in order to replicate the complete markets allocation with privately optimal risk sharing. By contrast, government spending is used for macroeconomic stabilization as characterized by the same optimality conditions (26) and (27).

Linear disutility of labor. Suppose now that the disutility from labor is linear. We maintain the same parametrization of preferences and assume in addition that \( \phi(N^i(s)) = \phi N^i(s) \) for some constant \( \phi > 0 \). In this case, the first order condition for optimal government spending (26) becomes
\( v'(G^i_{NT}(s)) = \frac{\phi}{A^i(s)} \). This formula, which pins down \( G^i_{NT}(s) \) as a function of \( A^i(s) \), holds both under rigid prices and fixed exchange rates, and under flexible prices or flexible exchange rates. Hence there is a sense in which government spending is not used for macroeconomic stabilization, despite the fact that macroeconomic stabilization is imperfect. The same is not true of international transfers.

**Purely wasteful government spending.** Another enlightening case is the case in which government spending is purely wasteful, so that it does not enter preferences.\(^{21}\) In that case, formulas (26) and (27) indicate that it is optimal not to use government spending, both with rigid prices and fixed exchange rates and with flexible prices or flexible exchange rates. Hence, once again, there is a sense in which government spending is not used for macroeconomic stabilization, despite the fact that macroeconomic stabilization is imperfect. The same is not true of international transfers.

## 4 A Dynamic Model

The static model reveals some key results in a simple and transparent manner. However, it is perhaps too simple to explore the issues in greater depth, and in particular to think about two key determinants of fiscal unions: price adjustment dynamics and the persistence of shocks. We now build a richer, dynamic model similar to Farhi and Werning (2012) which in turn builds on Gali and Monacelli (2005, 2008). We present the model with incomplete markets where agents can only trade short-term risk free bonds as in Farhi and Werning (2012), although we will also compare it to the complete financial market case when we turn to the log-linearized version of the model in Section 5.

In Farhi and Werning (2012), we focused on capital controls. Here instead we do no consider capital controls. Instead, our focus, just as in the static model, is on the design of ex-post transfers between countries that are contingent on the shocks experienced by all countries.

We focus on one-time shocks, starting in a symmetric steady state. At \( t = 0 \), the path for productivity in each country is realized. There is no further uncertainty. In the log-linearized version of the model, which we focus our analysis on, it is well known that a certainty equivalence principle holds so that this assumption is irrelevant. In other words, our analysis can simply be understood as an impulse response characterization in a setup where shocks might keep occurring in every period.

### 4.1 Households

There is a continuum measure one of countries \( i \in [0, 1] \). We focus attention on a single country, which we call Home, and can be thought of as a particular value \( H \in [0, 1] \). In every country, there is a representative household with preferences represented by the utility function

\[
\sum_{t=0}^{\infty} \beta^t \left[ \frac{C^i_t^{1-\sigma}}{1-\sigma} - \frac{N^i_t^{1+\phi}}{1+\phi} \right],
\]

\(^{21}\) Formally, this means that \( U^i(G^i_{NT}(s), C^i_{NT}(s), C^i_T(s), N^i(s); s) \) is independent of \( G^i_{NT}(s) \).
where \( N_t \) is labor, and \( C_t \) is a consumption index defined by
\[
C_t = \left[ (1 - \alpha)^{\frac{\gamma - 1}{\gamma}} C_{H,t}^{\gamma} + \alpha^{\frac{\gamma - 1}{\gamma}} C_{F,t}^{\gamma} \right]^{\frac{\gamma}{\gamma - 1}},
\]
where \( C_{H,t} \) is an index of consumption of domestic goods given by
\[
C_{H,t} = \left( \int_0^1 C_{H,t}(j)^{\frac{\gamma - 1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma - 1}},
\]
where \( j \in [0, 1] \) denotes an individual good variety. Similarly, \( C_{F,t} \) is a consumption index of imported goods given by
\[
C_{F,t} = \left( \int_0^1 C_{i,t}(j)^{\frac{\gamma - 1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma - 1}},
\]
where \( C_{i,t} \) is, in turn, an index of the consumption of varieties of goods imported from country \( i \), given by
\[
C_{i,t} = \left( \int_0^1 C_{i,t}(j)^{\frac{\gamma - 1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma - 1}}.
\]

Thus, \( \epsilon \) is the elasticity between varieties produced within a given country, \( \eta \) the elasticity between domestic and foreign goods, and \( \gamma \) the elasticity between goods produced in different foreign countries. An important special case obtains when \( \sigma = \eta = \gamma = 1 \). We call this the Cole-Obstfeld case, in reference to Cole and Obstfeld (1991). This case is more tractable and has some special implications that are worth highlighting. Thus, we devote special attention to it, although we will also derive results away from it.

The parameter \( \alpha \) indexes the degree of home bias, and can be interpreted as a measure of openness. Consider both extremes: as \( \alpha \to 0 \) the share of foreign goods vanishes; as \( \alpha \to 1 \) the share of home goods vanishes. Since the country is infinitesimal, the latter captures a very open economy without home bias; the former a closed economy barely trading with the outside world.

Households seek to maximize their utility subject to the sequence of budget constraints
\[
\int_0^1 P_{H,t}(j)C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j) dj di + D_{t+1} + \int_0^1 E_{i,t}D_{i,t+1} di 
\leq W_t N_t + \Pi_t + T_t + (1 + i_{t-1})D_t + \int_0^1 E_{i,t}(1 + i_{t-1})D_{i,t} di
\]
for \( t = 0, 1, 2, \ldots \) In this inequality, \( P_{H,t}(j) \) is the price of domestic variety \( j \), \( P_{i,t}(j) \) is the price of variety \( j \) imported from country \( i \), \( W_t \) is the nominal wage, \( \Pi_t \) represents nominal profits and \( T_t \) is a nominal lump sum transfer. All these variables are expressed in domestic currency. The portfolio of home agents is composed of home and foreign bond holding: \( D_t \) is home bond holdings of home agents, \( D_{i,t} \) is bond holdings of country \( i \) of home agents. The returns on these bonds are determined by the nominal interest rate in the home country \( i_t \), the nominal interest rate \( i_{i,t} \) in country \( i \), and the evolution of the nominal exchange rate \( E_{i,t} \) between home and country \( i \).

The nominal lump sum transfer is the focus of our analysis. More precisely, we allow for ex-post transfers across countries, contingent on the shocks experienced by these countries. We will provide a sharp characterization of these optimal transfers in the log-linearized version of the model. We will also compare these transfers to the implicit transfers that would occur through financial markets if asset markets were complete and private agents freely chose their portfolios.
4.2 Firms

Technology. A typical firm in the home economy produces a differentiated good with a linear technology given by

\[ Y_i(j) = A_{H,t} N_i(j) \] (29)

where \( A_{H,t} \) is productivity in the home country. We denote productivity in country \( i \) by \( A_{i,t} \).

We allow for a constant employment tax \( 1 + \tau_L \), so that real marginal cost deflated by Home PPI is given by

\[ MC_t = \frac{1 + \tau_L}{A_{H,t}} \frac{W_i}{P_{H,t}}. \]

We take this employment tax to be constant in our model. We pin this tax rate down by assuming that it is optimally set cooperatively at a symmetric steady state with flexible prices. The tax rate is simply set to offset the monopoly distortion so that \( \tau_L = -\frac{1}{\varepsilon} \).

Price-setting assumptions. As in Gali and Monacelli (2005), we maintain the assumption that the Law of One Price (LOP) holds so that at all times, the price of a given variety in different countries is identical once expressed in the same currency. This assumption is known as Producer Currency Pricing (PCP) and is sometimes contrasted with the assumption of Local Currency Pricing (LCP), where each variety’s price is set separately for each country and quoted (and potentially sticky) in that country’s local currency. Thus, LOP does not necessarily hold. It has been shown by Devereux and Engel (2003) that LCP and PCP may have different implications for monetary policy. However, for our purposes, these two polar cases are equivalent since, for the most part, we will study the model assuming fixed exchange rates.

We consider Calvo price setting, where in every period, a randomly selected fraction \( 1 - \delta \) of firms can reset their prices. Those firms that get to reset their price choose a reset price \( P^r_i \) to solve

\[ \max_{P^r_i} \sum_{k=0}^{\infty} \delta^k \left( \prod_{h=1}^{k} \frac{1}{1 + i_{t+h}} \right) (P^r_i Y_{t+k|t} - P_{H,t} MC_t Y_{t+k|t}) \]

where \( Y_{t+k|t} = \left( \frac{P^r_i}{P_{H,t+k}} \right)^{-\frac{1}{\varepsilon}} C_{t+k} \), taking the sequences for \( MC_t, Y_t \) and \( P_{H,t} \) as given.

4.3 Terms of Trade, Exchange Rates and UIP

It is useful to define the following price indices: home’s Consumer Price Index (CPI) \( P_t = [(1 - \alpha) P^1_{H,t} + \alpha P^1_{F,t}]^{\frac{1}{\eta}} \), home’s Producer Price Index (PPI) \( P_{H,t} = [\int_{0}^{1} P_{H,t}(j)^{1-\varepsilon} dj]^{\frac{1}{1-\varepsilon}} \), and the index for imported goods \( P_{F,t} = [\int_{0}^{1} P_{i,t}(j)^{1-\varepsilon} dj]^{\frac{1}{1-\varepsilon}} \), where \( P_{i,t} = [\int_{0}^{1} P_{i,t}(j)^{1-\varepsilon} dj]^{\frac{1}{1-\varepsilon}} \) is country \( i \)’s PPI.

Let \( E_{i,t} \) be nominal exchange rate between home and \( i \) (an increase in \( E_{i,t} \) is a depreciation of the home currency). Because the Law of One Price holds, we can write \( P_{i,t}(j) = E_{i,t} P^i_{i,t}(j) \) where \( P^i_{i,t}(j) \) is country \( i \)’s price of variety \( j \) expressed in its own currency. Similarly, \( P_{i,t} = E_{i,t} P^i_{i,t} \) where \( P^i_{i,t} = [\int_{0}^{1} P^i_{i,t}(j)^{1-\varepsilon} ]^{\frac{1}{1-\varepsilon}} \) is country \( i \)’s domestic PPI in terms of country \( i \)’s own currency. We therefore
have \( P_{F,t} = E_t P_t^* \), where \( P_t^* = [\int_0^1 P_{i,t}^{1-\gamma} di]^{1/\gamma} \) is the world price index and \( E_t \) is the effective nominal exchange rate.\(^{22}\)

The effective terms of trade are defined by \( S_t = \frac{P_{F,t}}{P_{H,t}} = \left( \int_0^1 S_{i,t}^{1-\gamma} di \right)^{1/\gamma} \), where \( S_{i,t} = P_{i,t}/P_{H,t} \) is the terms of trade of home versus \( i \). The terms of trade can be used to rewrite the home CPI as \( P_t = P_{H,t}[1 - \alpha + \alpha S_{i,t}^{1-\eta}]^{1/\eta} \).

Finally we can define the real exchange rate between home and \( i \) as \( Q_{i,t} = E_{i,t} P_t^i / P_t \) where \( P_t^i \) is country \( i \)'s CPI. We define the effective real exchange rate be \( Q_t = \frac{E_t P_t^*}{P_t} \).

### 4.4 Equilibrium Conditions

We now summarize the equilibrium conditions. Equilibrium in the home country can be described by the following equations. We find it convenient to group these equations into two blocks, which we refer to as the demand block and the supply block.

The demand block is independent of the nature of price setting. It is composed of the Backus-Smith condition

\[
C_t = \Theta^i C^i_t Q^{1}_{i,t}, \tag{30}
\]

where \( \Theta^i \) is a relative Pareto weight which depends on the realization of the shocks, the goods market clearing condition

\[
Y_t = \left( \frac{P_{H,t}}{P_t} \right)^{\eta} \left[ (1 - \alpha) C_t + \alpha \int_0^1 C^i_t (S^i_t S_{i,t})^{\gamma-\eta} Q^{\eta}_{i,t} di \right], \tag{31}
\]

where \( S^i_t \) denotes the effective terms of trade of country \( i \), the labor market clearing condition

\[
N_t = \frac{Y_t}{A_{H,t}} \Delta_t \tag{32}
\]

where \( \Delta_t \) is an index of price dispersion \( \Delta_t = \int_0^1 \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-e} \), the Euler equation

\[
1 + i_t = \frac{\beta^{-1} C_t^{\gamma+1} \Pi_{t+1}}{C_t^\sigma \Pi_{t+1}} \tag{33}
\]

where \( \Pi_t = \frac{P_{t+1}}{P_t} \) is CPI inflation, the arbitrage condition between home and foreign bonds

\[
1 + i_t = (1 + i^*_t) \frac{E_{i,t+1}}{E_{i,t}}, \tag{33}
\]

\(^{22}\)The effective nominal exchange rate is defined as \( E_t = [\int_0^1 E_{i,t}^{1-\gamma} P_{i,t}^{1-\gamma} di]^{1/\gamma} / [\int_0^1 P_{i,t}^{1-\gamma} di]^{1/\gamma} \).
for all \( i \in [0, 1] \), and the country budget constraint

\[
NFA_t = -(P_{H,t}Y_t - P_tC_t) + \frac{1}{1 + i_t} NFA_{t+1}
\]

(34)

where \( NFA_t \) is the country’s net foreign assets at \( t \), which for convenience, we measure in home numeraire. We also impose a No-Ponzi condition so that we can write the budget constraint in present-value form

\[
NFA_0 = -\sum_{t=0}^{\infty} \left( \prod_{s=0}^{t-1} \frac{1}{1 + i_s} \right) (P_{H,t}Y_t - P_tC_t).
\]

(35)

The value of \( NFA_0 \), which depends on the realization of shocks, is a measure of the (net present value) transfer to the home country. Characterizing the optimal value of \( NFA_0 \) depending on the shocks is of the main focuses of our analysis below. Absent ex-post transfers across countries, we would have \( NFA_0 = 0 \) since countries are ex-ante identical and only risk-free bonds can be traded. We will also compare the optimal value of \( NFA_0 \) to the value that would obtain if private agents could engage in risk-sharing through a complete set of financial markets. One of our main results will establish that these values differ, and to characterize how they differ.

Finally with Calvo price setting, the supply block is composed of the equations summarizing the first-order condition for optimal price setting. These conditions are provided in Appendix A.7. We will only analyze a log-linearized version of the model with Calvo price setting (see Section 5).

For most of the paper, we will be concerned with fixed exchange rate regimes (either pegs or currency unions) in which case we have the additional restriction that \( E_t = E_0 \) for all \( t \geq 0 \) where \( E_0 \) is predetermined.

5 Optimal Transfers in the Dynamic Model

As is standard in the literature, we work with a log-linearized approximation of the model. As before, at \( t = 0 \), the economy is hit with an unanticipated shock. It is convenient to work with a continuous time version of the model. This does not affect our results, but it is useful because it implies that no price index can jump at \( t = 0 \) and this simplifies the derivation of initial conditions characterizing the equilibrium. We denote the instantaneous discount rate by \( \rho \), and the instantaneous arrival rate for price changes by \( \rho_\delta \).

From now on we focus on the Cole-Obstfeld case \( \sigma = \eta = \gamma = 1 \). This case is attractive for the following reason. Even when prices are sticky, the with incomplete markets and no transfers coincides with the equilibrium with complete markets and no interventions in financial markets. Once again, risk sharing is delivered with balanced trade. This means that we can interpret any deviation from balanced trade at the optimum with transfers as an indication that private risk sharing through complete financial markets (if those were available) would be constrained inefficient. Third, it is possible to derive a simple second-order approximation of the welfare function around the symmetric deterministic steady state. Away from the Cole-Obstfeld case the welfare function is more involved.
We start by considering the case where all countries are members of the same currency union. Later, we consider the case where some countries are in a currency union, while others remain outside, with a flexible exchange rate and independent monetary policy.

5.1 The Log-Linearized Economy

We denote with lowercase variables the log deviations from steady state of the corresponding uppercase variable introduced in Section 4.

The natural allocation. We define a reference allocation which corresponds to the flexible price counterparts. We denote by \( \tilde{y}_t \) the log deviations from steady state of the corresponding variable introduced in Section 4.

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We denote with a star the union average of a given variable. For example, \( \tilde{y}_t = \int_0^1 \tilde{y}_t^i di \), \( \tilde{c}_t^i = \int_0^1 \tilde{c}_t^i di \) and \( a_t^* = \int_0^1 a_{i,t}^i di \). At the natural allocation, output in country \( i \) is given by \( \tilde{y}_t^i = a_{i,t}^i \), consumption is given by \( \tilde{c}_t^i = a_{i,t}^i + (1 - \alpha) a_{i,t}^i \), labor is given by \( \tilde{\lambda}_t^i = 0 \), and the terms of trade are given by \( \tilde{s}_t^i = a_{i,t}^i - a_t^* \).

In addition, trade is balanced. Finally, aggregate output is equal to aggregate consumption and is given by \( \tilde{y}_t = \tilde{c}_t = a_t^* \). Note that by construction \( \int_0^1 \tilde{s}_t^i di = 0 \),

Summarizing the system in gaps. We denote by \( \hat{y}_t^i \) and \( \hat{\theta}_t^i \) the deviations of \( y_t^i \) and \( \theta_t^i \) from their flexible price counterparts. We denote by \( \tilde{y}_t = \hat{y}_t^i - \hat{y}_t^i \) and \( \hat{\theta}_t = \hat{\theta}_t^i - \hat{\theta}_t^i \) where \( \hat{y}_t^i = \int_0^1 \hat{y}_t^i di \) and \( \hat{\theta}_t^i = \int_0^1 \hat{\theta}_t^i di \) the deviations of these variables from their corresponding aggregates; also let \( \pi_{H,t}^i = \pi_{H,t}^i - \pi_t^* \) where \( \pi_t^* = \int_0^1 \pi_{H,t}^i di \). Note that \( \hat{\theta}_t^i \) is already a normalized variable so that \( \hat{\theta}_t = \hat{\theta}_t^i \).

The trade balance is constant and equals \(-a\hat{\theta}_t^i\). The net foreign asset position must pay for the present value of the trade deficits, so that starting from a position of zero net foreign assets, transfers must bring the net foreign asset position to \( NFA_0 = \frac{\hat{\rho}_t}{\hat{\rho}_t} \hat{\theta}_t^i \).

The disaggregated variables solve the ordinary differential equations, corresponding to the Phillips curve \( \pi_{H,t}^i = \rho \pi_{H,t}^i - \kappa \hat{y}_t^i - \lambda \hat{\theta}_t^i \) and the Euler equation \( \hat{y}_t^i = -\pi_{H,t}^i - \hat{s}_t^i \), with initial condition \( \tilde{y}_0^i = (1 - \alpha)\hat{\theta}_t^i - \tilde{s}_t^i \), where \( \lambda = \rho \delta (\rho + \rho \delta) \) and \( \kappa = \lambda (1 + \phi) \) index price flexibility.

Since \( \int_0^1 \tilde{s}_t^i di = 0 \), as long as \( \int_0^1 \hat{\theta}_t^i di = 0 \) the following aggregation constraints are verified for any bounded solution of the system above: \( \int_0^1 \tilde{y}_t^i di = 0 \), and \( \int_0^1 \tilde{\pi}_{H,t}^i di = 0 \). We will assume that the zero lower bound on the nominal interest is not binding. Then the only constraint on the aggregates is that they must satisfy the aggregate New Keynesian Philips Curve \( \pi_t^* = \rho \pi_t^* - \kappa \hat{y}_t^i \). Thus, there are many possible paths for the aggregate variables, depending on the stance of monetary policy at the union level.

From these equations we can infer aggregate consumption \( \hat{c}_t^* = \hat{y}_t^* \). We can also infer the disaggregated variables for country \( i \) as follows. The terms of trade gap \( \tilde{s}_t^i \) can be backed out from \( \hat{y}_t^i = (1 - \alpha)\hat{\theta}_t^i + \tilde{s}_t^i \), which combines the market clearing condition with the Backus-Smith condition.

\(^{23}\) Although we do not need it for our analysis, note that the natural interest rate is given by \( \bar{r}_t^i = \hat{a}_{i,t}^i \).
Similarly, we can back out the employment gap $\bar{n}_i^t$ and the consumption gap $\bar{c}_i^t$ from technology $\bar{y}_i^t = \bar{n}_i^t$ and market clearing $\bar{y}_i^t = \bar{c}_i^t + \alpha \bar{s}_i^t - \alpha \bar{t}_i^t$.

**Loss function.** We are interested in the symmetric constrained Pareto efficient allocation that provides optimal ex-ante risk sharing behind the veil of ignorance, before shocks are realized. To solve for this we maximize an unweighted Utilitarian welfare function. A simple representation of this welfare function associated with this welfare criterion is as follows (see Farhi and Werning, 2012):

\[
\frac{(1-\alpha)(1+\phi)}{2} \int_0^\infty \int_0^1 e^{-\rho t} \left[ \alpha_\pi (\pi_i^t + \pi_i^* t)^2 + (\hat{y}_i^t + \hat{y}_i^*)^2 + \alpha_\theta (\theta^i t)^2 \right] dt
di
\]

where $\alpha_\pi = \frac{\epsilon}{\lambda(1+\phi)}$ and $\alpha_\theta = \frac{\alpha(2-\alpha)}{1+\phi}$.

The first two terms in the loss function are familiar in New-Keynesian models and are identical to those obtained by Gali and Monacelli (2005, 2008). The third term captures the direct welfare effects of transfers—it penalizes deviations from privately optimal risk sharing. In the closed economy limit, as $\alpha \to 0$, this term goes to zero since $\alpha_\theta \to 0$.

### 5.2 Optimal Transfers in a Currency Union

In the online appendix A.10, we solve for the positive effects of transfers. We now explore the associated normative questions: What is the optimal use of transfers in a currency union? How do they differ from the transfers implicit in the solution with complete markets and no interventions in financial normative questions: What is the optimal use of transfers in a currency union? How do they

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Using the fact that $\int_0^1 \bar{y}_i^t dt = \int_0^1 \pi_i^t dt = 0$, we are led to the following second-best planning problem which characterizes optimal transfers:

\[
\min \frac{1}{2} \int_0^\infty \int_0^1 e^{-\rho t} \left[ \alpha_\pi (\pi_i^t + \pi_i^* t)^2 + (\hat{y}_i^t + \hat{y}_i^*)^2 + \alpha_\theta (\theta^i t)^2 \right] dt
di
\]

subject to

\[
\begin{align*}
\dot{\pi}_i^t &= \rho \pi_i^t - \dot{\pi}_i^t - \lambda \dot{\theta}_i^t, \\
\dot{y}_i^t &= -\pi_i^t - \dot{s}_i^t, \\
\dot{y}_i^0 &= (1-\alpha) \dot{\theta}_i^t - \dot{s}_i^t
\end{align*}
\]

\[\text{Note that from the perspective of an individual country } i, \text{ transfers also have a first order effect on welfare—the loss function of an individual country inherits a term } -\frac{1}{2} \int_0^\infty e^{-\rho t} \frac{2\alpha(2-\alpha)}{1+\phi} \dot{\theta}_i^t dt. \text{ This term represents the pure distributional aspect of transfers. These distributional concerns are zero sum and wash out in the aggregate since } \int_0^1 \dot{\theta}_i^t = 0. \]

The best equilibrium with complete markets and no intervention in financial markets has $\dot{\theta}_i^t = 0$. Under the Cole-Obstfeld specification considered here, it coincides with the best equilibrium with incomplete markets and no government transfers. In both cases, optimal monetary policy ensures that the aggregate output gap and inflation are zero $\bar{y}_i^* = \pi_i^* = 0$. See the online appendix A.9 for details.
\[ \int_0^1 \tilde{\theta}^i di = 0, \quad (40) \]
\[ \tilde{\pi}^*_i = \rho \tilde{\pi}^*_i - \hat{k} \hat{\tilde{y}}^*_i, \quad (41) \]

where the minimization is over the variables \( \hat{\tilde{\pi}}^i, \tilde{\pi}^*_i, \tilde{\theta}^i, \hat{\tilde{y}}^*_i, \hat{\tilde{y}}^i \). In the online appendix A.8, we show how to decompose this planning problem into an aggregate planning problem that determines \( \tilde{\pi}^*_i \) and \( \hat{\tilde{y}}^*_i \), and a disaggregated planning problem that determines \( \hat{\tilde{\pi}}^i, \tilde{\pi}^*_i, \tilde{\theta}^i \) for all \( i \). Furthermore, we show that the disaggregated planning problem can be relaxed and broken down into separate component planning problems indexed by \( i \).

Monetary policy can be chosen at the union level so that monetary conditions are adapted to the average country, a result which echoes Proposition 5 from the static model, are reminiscent of the results in Benigno (2004) and Gali and Monacelli (2008).

**Proposition 13 (Optimal Monetary Policy).** At the second-best optimum, union-wide aggregates are zero \( \hat{\tilde{y}}^*_i = \pi^*_i = 0 \).

We now characterize disaggregated variables at the second-best optimum, focusing on transfers. We provide closed-form solutions for two enlightening special cases, rigid prices and the closed economy limit. We then explore the general case using numerical simulations.

**The case of rigid prices.** We first treat the case of rigid prices where \( \hat{k} = 0 \).

**Proposition 14 (Rigid Prices).** Suppose prices are rigid, then the second-best optimum has

\[ N\tilde{FA}^i_0 = \frac{\alpha (1 - \alpha)}{(1 - \alpha)^2 + \alpha \theta} \int_0^\infty e^{-\rho t} \tilde{z}_i dt \quad \text{and} \quad \hat{\tilde{\theta}}^i = \frac{\rho (1 - \alpha)}{(1 - \alpha)^2 + \alpha \theta} \int_0^\infty e^{-\rho t} \tilde{z}_i dt. \]

Importantly, we find that \( N\tilde{FA}^i_0 \neq 0 \) and \( \hat{\tilde{\theta}}^i \neq 0 \), so that the optimal solution does not coincide with the solution with complete markets and no interventions in financial markets. Government intervention in risk sharing, either through ex-post transfers or through assets markets, is a necessary feature of the optimum.

Countries experiencing shocks that depreciate their natural terms of trade \( \tilde{z}_i \) should receive positive transfers. The optimal transfers are increasing in the size and persistence of shocks. This helps alleviate the recession resulting from the inability of the terms of trade to adjust to that level in the short-run. With positive home bias \( \alpha < 1 \), transfer increases the demand for home goods and reduces that for foreign goods—once again, a manifestation of the Transfer Problem.

Optimal transfers are increasing the persistence of the shocks. This is intuitive. Transfers affect the economy permanently and are therefore better suited to deal with persistent shocks.

Optimal transfers \( N\tilde{FA}^i_0 \) depend crucially on the openness of the economy, as captured by the degree of home bias \( \alpha \). They are non-monotonic in the degree of openness. Indeed, \( N\tilde{FA}^i_0 \) is zero for both \( \alpha = 0 \) (closed economy) and \( \alpha = 1 \) (fully open economy). In contrast, the coefficient \( \hat{\tilde{\theta}}^i \) equals \( \rho \) for \( \alpha = 0 \) and zero for \( \alpha = 1 \).
This shows that the reason for zero transfers for $\alpha = 0$ and for $\alpha = 1$ are very different. Basically for $\alpha$ close to 0 (extreme home bias), small transfers have large expenditure switching effect across different goods. Small transfers therefore have large effects on output. For $\alpha$ close to 1, transfers have no expenditure switching effects, and therefore have no effects on output. So for $\alpha$ close to 0, we get small transfers because small transfers are very effective (they have very large effects on output). By contrast, for $\alpha$ close to 1, we get small transfers because transfers are very ineffective (they have small effects on output).

The effectiveness of small transfers when $\alpha$ is small can be further illustrated in the case $\alpha \to 0$ and permanent shocks $\bar{s}_i = \bar{s}_i$ in which case we get perfect stabilization $\bar{y}_i = 0$ at the optimum (we achieve the natural allocation). We show this conclusion holds more generally, even when prices are not perfectly rigid, in Corollary 1.

The case of the closed economy limit $\alpha \to 0$. We now return to the case where prices are not entirely rigid, $\hat{k} > 0$, so that the costs of inflation must also be weighed against the stabilization of output gaps. Things simplify in the closed economy limit $\alpha \to 0$.

**Proposition 15 (Closed Economy Limit).** In the closed economy limit, when $\bar{s}_i = \bar{s}_i e^{-\psi t}$, at the second-best optimum, we have

$$N\bar{F}A_0^i = 0 \quad \text{and} \quad \bar{\theta}_i = \bar{s}_i^i \left[ 1 - \frac{\psi^2}{(\psi + \nu)(\psi + \rho - \nu)} + \frac{\psi (\nu \pi \hat{k} + \psi)}{(\psi + \nu)(\psi + \rho - \nu)^2 \alpha \pi \nu^2 + 1} \right].$$

For $\alpha$ close to 0 (extreme home bias), small transfers have large expenditure switching effect across different goods. Small transfers therefore have large effects on output. Indeed, in the limit, we get $\bar{\theta}_i \neq 0$ despite the fact that $N\bar{F}A_0^i = 0$. Transfers are particularly useful in the case where shocks are permanent: if $\psi = 0$ then $\bar{\theta}_i = \bar{s}_i^i$ and we get perfect stabilization of output and inflation.

**Corollary 1 (Closed Economy Limit, Permanent Shocks).** In the closed economy limit, in response to a permanent shock $\bar{s}_i = \bar{s}_0^i$, the second-best optimum achieves the first best $\bar{y}_i = \bar{\pi}_H^i, t = 0$ with

$$N\bar{F}A_0^i = 0 \quad \text{and} \quad \bar{\theta}_i = \bar{s}_0^i.$$

This result is striking. For rather closed economies in a currency union, modest transfers achieve large stabilization benefits. This result is interesting as a contrast to the arguments presented by McKinnon (1963) that common currencies are more costly for economies that are more closed. McKinnon did not consider transfers, however. Our result shows that this matters: closed economies make transfers more potent.

**Numerical exploration of the general case.** In the general case, we resort to numerical simulations. We show in the appendix that $\bar{\theta}_i$ solves a simple static quadratic minimization problem that is very tractable. For our simulations, we set the benchmark parameters at: $\phi = 3$, $\rho = 0.06$, $\epsilon = 11$ and $\rho_\delta = 0.6$. We explore different values of the remaining parameters.
Figure 2 displays the behavior of the economy with optimal transfers and with no transfers in response to a permanent shock with $\bar{s}_t = 0.05$. The top panel corresponds to $\alpha = 0.01$, the middle panel to $\alpha = 0.1$ and the bottom panel to $\alpha = 0.4$. In this figure, time is measured in years and inflation is annualized. The allocation without transfers features deflation and a recession (in gaps) in the short run which vanishes in the long run as prices adjust: the output gap increases from $-5\%$ to 0 and the inflation rate from $-6\%$ to 0. The allocation with transfers features less deflation and smaller recession in the short run, but lower output in the long run (in gaps). For example, with $\alpha = 0.1$, the output gap at impact is only $-1.2\%$ and the inflation rate $-1.4\%$. The allocation without transfer is independent of openness $\alpha$. By contrast, the solution with optimal transfers is more stable, the more closed the economy (the lower $\alpha$). Optimal transfers stabilize the economy more effectively when the economy is more closed.

Figure 3 displays a measure of stabilization due to transfers. We compare the impact on the output gap of a shock with and without optimal transfers and report the mitigation factor—the difference between the two as a fraction of the latter. We feed in exponentially decaying shocks $\bar{s}_t = e^{-\psi t}$ and normalize the initial shock $\bar{s}_0$ to 0.05. We then plot our stabilization measure as a function of openness $\alpha$ and the persistence of the shock as measured by its half life ($-\log(0.5)/\psi$). Using the same shock, Figure 4 displays transfers $\tilde{NFA}_0$ as a function of the same two parameters; these numbers can be interpreted as transfers as a fraction of GDP.

Stabilization is increasing in the persistence of the shock and decreasing in openness. The optimal transfer is increasing in the persistence of the shock starting at zero for fully transitory shocks, but hump-shaped as a function of openness, starting at zero at $\alpha = 0$. Significant stabilization is achieved with relatively modest transfers when the economy is relatively closed and shocks are relatively permanent.

The role of fixed exchange rates. In the online appendix A.15, we clarify the role of fixed exchange rates. We assume that only a subset of countries $I \subseteq [0, 1]$ are in the currency union. These countries have flexible exchange rates. We show that, for countries outside the currency union, it is optimal not to make or receive any transfers to other countries $\tilde{\theta}_i = 0$, which implements the same allocation as the best equilibrium with complete markets and no interventions in financial markets. They achieve perfect stabilization $\pi^i_{H,t} = \bar{y}_i = 0$. It follows that any role for transfers can be solely attributed to the fixed exchange rates prevailing in a currency union, which echoes Proposition 10.

5.3 Capital Controls and Government Spending

In Farhi and Werning (2012), we used the same model presented in this section to study two other policy instruments. The first instrument, which is the main focus of the paper, is capital controls. The second instrument, for which our analysis simply replicates Gali and Monacelli (2008), is government spending on home goods.27

27See Section 4.6 of the NBER working paper version of Farhi and Werning (2012).
It is important to clarify the difference between international transfers and capital controls. While international transfers affect the allocation of consumption across states, capital controls affect the allocation of consumption over time in each state. The optimal use of capital controls can be derived by studying a planning problem related to (36). The differences are as follows. First, in the objective function (36) and in the New Keynesian Philips curve (37), the constant variable \( \tilde{\theta} \) is now replaced by time-dependent variable \( \tilde{\theta}_t \). Second, the initial condition (39) now features \( \tilde{\theta}_0 \) instead of \( \tilde{\theta} \). Third, there is an additional set of constraints \( \int e^{-\rho_t} \tilde{\theta}_t dt = 0 \) if international transfers are not available but only capital controls are available.

It is beyond the scope of this paper to provide a full quantitative evaluation of the joint optimal use of these different instruments, or of the relative performance of each individual instrument. Here, we only wish to highlight a few important points.

First, international transfers, capital controls, and government spending are different instruments. In general no single instrument can achieve perfect macroeconomic stabilization, and it is best to use all three in conjunction.

Second, the relative performance of these instruments depends on the parameters of the economy, and in particular on two key parameters: the degree of openness, and the persistence of the shocks. In general, international transfers and capital controls instruments are more potent when the economies are relatively closed.\(^{28}\) By contrast the effectiveness of government spending is largely independent of the degree of openness.\(^{29}\) International transfers are better suited to deal with persistent shocks, capital controls are better suited to deal with transitory shocks. Loosely speaking, international transfers are a permanent solution, and are better to deal with permanent shocks, while capital controls are a temporary solution, and are better suited to deal with temporary shocks. The effectiveness of government spending is less dependent on the persistence of shocks.

More formally, in the closed economy limit, when shocks \( \tilde{s}_i = a_{i,t} - a_{i,t}^* \) are permanent, international transfers achieve the first best, while capital controls and government spending do not. In this case, there is no residual macroeconomic stabilization role for capital controls (which are optimally set to zero) or government spending (government spending is equal to its efficient value at the natural allocation) once international transfers are available. Conversely, in the limit where productivity shocks \( \tilde{s}_i = a_{i,t} - a_{i,t}^* \) are purely transitory, optimal international transfers are zero and do not perform any macroeconomic stabilization role, while optimal capital controls and government spending do help stabilize the economy.

One scale-free way (independent of the distribution of shocks \( \tilde{s}_i \)) to illustrate these properties in our benchmark numerical example is to compare the welfare loss with only international transfers \( L_1 \), with only capital controls \( L_2 \), and with only government spending \( L_3 \).\(^{30}\) We start with a permanent shock. We find that welfare losses are in increasing order \( L_1, L_2, \text{and } L_3 \). When \( \alpha = 0.4, L_1 \) is

\(^{28}\)We have made this point in formal propositions and simulations in this paper and in Farhi and Werning (2012).

\(^{29}\)In fact, in our numerical example, we show in Farhi and Werning (2012) that optimal government spending, the output gap, and inflation are literally independent of openness.

\(^{30}\)Following Gali and Monacelli (2008), we take the utility function to be \( (1 - \lambda) \frac{c^{1-\sigma}}{1-\sigma} + \lambda \frac{c^{1-\sigma}}{1-\sigma} - N_{1+i}^{1+i} \), and we calibrate \( \lambda \) to match a steady state government spending share of 0.25.
10% lower than $L_2$ and 25% lower than $L_3$. When $\alpha = 0.1$, $L_1$ is 52% lower than $L_2$ and 80% lower than $L_3$. We then perform the same exercise for a shock with a half-life of one year. We find that welfare losses are in increasing order $L_2$, $L_3$ and $L_1$. When $\alpha = 0.4$, $L_2$ is 58% lower than $L_1$ and 16% lower than $L_3$. When $\alpha = 0.1$, $L_2$ is 91% lower than $L_1$ and 84% lower than $L_3$.

6 Conclusion

Even if private asset markets are perfect, we find that private risk sharing is constrained inefficient within a currency union. A role emerges for governments intervention in risk sharing, providing a rationale for a fiscal union within a currency union. We give a precise characterization the effectiveness of such a fiscal union and the size of the underlying transfers as a function of a small numbers of key characteristics of the currency union, such as the asymmetry of shocks, the openness of member countries, the persistence of shocks, and the rigidity of prices.

References


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Figure 2: Allocations with optimal transfers (blue) and no transfers (green). The top panel corresponds to $\alpha = 0.01$, the middle panel to $\alpha = 0.1$ and the bottom panel to $\alpha = 0.4$. Inflation is annualized and time is measured in years.
Figure 3: Optimal initial output gap mitigation at impact as a function of openness $\alpha \in (0, 1)$ and persistence (half-life of the shock) $-\frac{\log(0.5)}{\psi} \in (0, 10)$.

Figure 4: Transfers (as fraction of GDP) for a 1% shock to the terms of trade as a function of openness $\alpha \in (0, 1)$ and persistence (half-life of the shock) $-\frac{\log(0.5)}{\psi} \in (0, 10)$. 
A Online Appendix

A.1 Proof of Proposition 1

We have already proved that the conditions in the proposition are necessary for an allocation together with prices to form part of an equilibrium with complete markets. We now need to establish these conditions are sufficient. The proof is constructive. Start with an allocation together with prices that satisfy these conditions. We choose wages \( W^i(s) \) to satisfy the labor-leisure condition (5) for each \( i \in I \) and \( s \in S \). Given some set of state prices \( Q(s) \), we pick portfolio taxes \( \tau_D^i(s) \) to satisfy the risk sharing condition (3) for each \( i \in I \) and \( s \in S \). Note a first dimension of indeterminacy here: we can always multiply state prices \( Q(s) \) and portfolio taxes \( 1 + \tau_D^i(s) \) by some arbitrary common function \( \lambda(s) \) of \( s \). We then pick labor taxes \( \tau_L^i \) to satisfy the price setting equation (6). Finally, for a given set of ex-post fiscal transfers \( \hat{T}^i(s) \) that satisfy the country budget constraint (8), we compute transfers to households \( T^i(s) \) using the government budget constraint (7). We can then compute the required portfolio positions \( D^i(s) \) using the ex-post household budget constraint (2). These choices guarantee that the ex-ante household budget constraint (1) is verified. Note a second dimension of indeterminacy, as we have some degree of freedom in choosing ex-post fiscal transfers \( \hat{T}^i(s) \).

A.2 Proof of Proposition 2

We have already proved that the conditions in the proposition are necessary for an allocation together with prices to form part of an equilibrium with complete markets. We now need to establish these conditions are sufficient. The proof is constructive. Start with an allocation together with prices that satisfy these conditions. We choose wages \( W^i(s) \) to satisfy the labor-leisure condition (5) for each \( i \in I \) and \( s \in S \). We then pick labor taxes \( \tau_L^i \) to satisfy the price setting equation (12). We choose transfers \( T^i(s) \) to satisfy the household budget constraint (11). We then choose ex-post fiscal transfers \( \hat{T}^i(s) \) to satisfy the government budget constraint (13). We can verify that these choices satisfy (8).

A.3 Price Setting with Constant Elasticity of Substitution

We have

\[
1 - \frac{\int \tau_L^i(s) U_{CNT}^i(s) C_{NT}^i(s) \pi(s) ds}{\int U_{CNT}^i(s) C_{NT}^i(s) \pi(s) ds} = \frac{1}{1 + \tau_L^i} \frac{\epsilon - 1}{\epsilon}.
\]

We can rewrite the first order condition for \( p_{NT}^i \) as

\[
\int \frac{\alpha_p^i(s)}{\alpha^i(s)} p^i(s) \frac{1}{p^i(s)} U_{CNT}^i(s) \tau^i(s) \pi(s) ds = 0.
\]
If \( \frac{\alpha_i^s(s)}{\alpha^i(s)} p^i(s) \) is constant then this implies that

\[
\int C_{NT}(s) U_{CNT}^i(s) \tau^i(s) \pi(s) ds = 0.
\]

Thus in this case \( \frac{1}{1 + \tau^i_L} \approx 1 \) or \( \tau^i_L = -1/\varepsilon \).

### A.4 Proof of Proposition 7

Consider an equilibrium such that \( \tau^i(s) \neq 0 \) for some \( i \in I, s \in S \). Assume, towards a contradiction, that the allocation is constrained Pareto efficient.

We consider two cases in turn. First, suppose that \( V^i_{CT}(s) = U^i_{CT}(s) (1 + \frac{\alpha^i(s)}{p^i(s)} \tau^i(s)) < 0 \) for some set \( \Omega \subset I \times S \) of positive measure of countries and states. Define the sections \( \Omega(s) = \{i : (i, s) \in \Omega\} \).

Then there exists a perturbation that for each \( s \in S \): (a) lowers \( C^i_T(s) \) for \( i \in \Omega(s) \) and improves welfare \( V^i(s) \); (b) increases \( C^i_T(s) \) for \( i \notin \Omega(s) \) and improves welfare \( V^i(s) \); and (c) satisfies the resource constraint \( \int C^i_T(s) di = \int E^i_T(s) di \). This perturbation is feasible and creates a Pareto improvement, a contradiction.

Next, consider the case where \( 1 + \frac{\alpha^i(s)}{p^i(s)} \tau^i(s) \geq 0 \) for all \( i \in I, s \in S \). For each state \( s \) consider ranking countries by their weighted labor wedge \( \frac{\alpha^i(s)}{p^i(s)} \tau^i(s) \). By Proposition 6 it must be that

\[
\frac{1 + \frac{\alpha^i(s)}{p^i(s)} \tau^i(s)}{1 + \frac{\alpha^{i'}(s)}{p^{i'}(s)} \tau^{i'}(s)} = \frac{1 + \frac{\alpha^i(s)}{p^i(s)} \tau^i(s')}{1 + \frac{\alpha^{i'}(s)}{p^{i'}(s)} \tau^{i'}(s')}
\]

for all \( i, i', s \) and \( s' \). This implies that the ranking must be the same in all states \( s \). It follows that there is a country \( i^* \) that is at top of the ranking for all states \( s \), i.e. \( i^* \in \cap_{s \in S} \arg \max_{i \in I} \frac{\alpha^i(s)}{p^i(s)} \tau^i(s) \). Proposition 5 then implies that this country has a positive labor wedge: \( \tau^{i^*}(s) \geq 0 \) for all \( s \). Proposition 4 then implies that \( \tau^{i^*}(s) = 0 \) for all \( s \). Therefore we have that \( \tau^i(s) \leq 0 \) for all \( i \in I, s \in S \). Proposition 5 then implies that actually \( \tau^i(s) = 0 \) for all \( i \in I, s \in S \).

### A.5 Sticky Wages

In order to have a well defined wage setting problem we assume that labor services are produced by combining a variety of differentiated labor inputs according to the constant returns CES technology

\[
N^i(s) = \left( \int_0^1 N^{i,h}(s)^{1-\frac{1}{\tau_w}} dh \right)^{\frac{1}{1-\frac{1}{\tau_w}}}
\].
The rest of the technology is as before. We assume that in each country there is a continuum of workers \( h \in [0,1] \), each supplying a particular variety \( h \in [0,1] \) with preferences

\[
\int U^i(C^i_{NT}(s), C^i_T(s), N^i_{NT}(s); s) \pi(s) ds.
\]

The budget constraints are the same as before

\[
\int D^i(h)(s)Q(s)\pi(s)ds \leq 0,
\]

\[
P^i_{NT}(s)C^i_{NT}(s) + P^i_T(s)C^i_T(s) \leq (1 - \tau^i_l)W^i,h N^i_{NT}(s)
\]

\[
+ P^i_T(s)E^i_T(s) + \Pi^i(s) + T^i(s) + (1 + \tau^i_D(s))D^i(h),
\]

except that the wage \( W^i,h \) is now specific to each worker \( h \) but independent of \( s \) because wages are set in advance of the realization of the state \( s \). Note that prices of non-traded goods are now state contingent. For convenience, we now assume that the worker pays for the labor tax; firms are untaxed.

Workers set their own wages \( W^i,h \) taking into account that in each state of the world \( s \) labor demand is given by \( N^i(s)(W^i,h/W^i)^{-\epsilon_w} \) where \( W^i = (\int (W^i,h)^{1-\epsilon_w} dh)^{1/(1-\epsilon_w)} \) is the wage index for labor services. In a symmetric equilibrium, all workers set the same wage \( W^i,h = W^i \), and consume and work the same so that \( C^i_{NT}(s) = C^i_{NT}(s), C^i_T(s) = C^i_T(s) \) and \( N^i_{NT}(s) = N^i(s) \). The wage \( W^i \) is given by

\[
W^i = \frac{1}{1 - \tau^i_l \epsilon_w - 1} \frac{\int -N^i(s)U^i_{NT}(s)\pi(s)ds}{\int U^i_{NT}(s) N^i(s)\pi(s)ds}.
\]

All varieties sell at the same price so that \( P^i_{NT}(s) = P^i_{NT}(s) \). This price is given by

\[
P^i_{NT}(s) = \frac{e}{e - 1} \frac{W^i}{A^i(s)}.
\]

All the results that we derived in the version of the model with sticky prices carry through with no modification to this specification with sticky wages. In particular, Propositions 1–12 are still valid.

### A.6 Moral Hazard

Suppose that the government can exert effort \( e \) ex ante to affect the distribution of the endowment of the traded good ex post, but that effort \( e \) is not observable, creating a moral hazard problem. We focus on a single country \( i \in [0,1] \). We assume that the shock \( s \) is purely idiosyncratic and only affects the value of the endowment \( E^i_T(s) \) in country \( i \). Naturally, monetary policy at the union level should not react to to the idiosyncratic shocks \( s \) of an infinitesimal country, so that \( P^i_T(s) = P_T \) is
constant. These assumptions simplify the exposition. The principal-agent problem is then

\[
\max_{P_{NT}, C_T(s), e} \int V^i \left( C_T(s), \frac{p_T}{p_i} \right) \pi(s|e) ds - h(e) \quad (42)
\]

subject to

\[
\int (C_T(s) - E_T(s)) \pi(s|e) ds \leq 0 \quad (43)
\]

and

\[
\int V^i \left( C_T(s), \frac{p_T}{p_i} \right) \pi(s|e) ds - h(e) \geq \int V^i \left( C^*_T(s), \frac{p_T}{p_N} \right) \pi(s|e') ds - h(e') \quad \text{for all } e'. \quad (44)
\]

The first constraint (43) simply conditions the average level of expected transfers; this reflects the fact that insurance is priced fairly i.e. \((Q(s) = 1)\), since the shock is experienced by a single country and does not affect aggregate resources at the union level. The last constraint (44) is the incentive compatibility condition, requiring the country’s effort to be optimal, taking the schedule \(C_T^*(s)\) as given.

In the absence of nominal rigidities or for a country with flexible exchange rates and independent monetary policy, we would solve the same problem but using \(V^i*(C_T^i(s)) = \max_{p^i} V^i (C_T^i(s), p^i)\) in place of \(V^i \left( C_T(s), \frac{p_T}{p_i} \right)\). Note that \(V^i (C_T, p^i) \leq V^i*(C_T^i)\) with equality at a single value of \(C_T^i\), so that \(V^i*\) is an upper envelope of \(V^i\). When prices are rigid it is as if the country were more risk averse, in the sense described earlier. In the presence of moral hazard, higher risk aversion affects the optimal insurance (risk sharing) contract \(C^*_T(\cdot)\).

Consider the planning problem (42). Let \(\mu\) be the multiplier on (43) and \(dv(e')\) be the measure multiplier on (44). The corresponding for socially optimal risk sharing becomes

\[
U^i_{C_T} \left( 1 + \frac{\alpha^i(s)}{p^i(s)} \tau^i(s) \right) \left( 1 + \int \frac{\pi(s|e) - \pi(s|e')}{\pi(s|e)} dv(e') \right) = \mu. \quad (45)
\]

By contrast, the corresponding corresponding condition for a country outside the currency union is

\[
U^i_{C_T} \left( 1 + \int \frac{\pi(s|e) - \pi(s|e')}{\pi(s|e)} dv(e') \right) = \mu. \quad (46)
\]

Condition (46) shows that even with flexible exchange rates, moral hazard endogenously limits insurance (risk sharing) possibilities. There is a meaningful tradeoff between insurance (risk sharing) and incentives and providing incentives for the country’s government to exert the adequate level effort requires the government to have “skin in the game”. The private consumption of traded goods must vary with the realization of government spending on traded goods. It must be high whenever the particular realization of government spending is more likely (as measured by the likelihood ratio \(\frac{\pi(s|e) - \pi(s|e')}{\pi(s|e)}\)) under the desired effort level than under alternative effort levels that the gov-
ernment is tempted to exert (as measured by the measure multiplier $dv(e')$ on the corresponding incentive compatibility constraint). This is accomplished by reducing the level of transfers to the country when government spending on traded goods is high.

Condition (45) shows how the optimal provision of insurance (risk sharing) and incentives must be modified when the country is in a currency union. The provision of incentives requires the private consumption of traded goods to vary with the realization of government consumption. Because prices are sticky, this generates a non-zero pattern of labor wedges $\tau_i(s)$. This in turn opens up a wedge between the social and private marginal utility of income, which creates another force agains the perfect equalization of consumption of traded goods across states. As a result, the optimal insurance insurance (risk sharing) arrangements are different for countries that belong to a currency union than that for countries who have a flexible exchange rate.

A.7 Nonlinear Calvo Price Setting Equations

The equilibrium conditions for the Calvo price setting model can be expressed as follows

$$\frac{1 - \delta \Pi_{H,t}^{-1}}{1 - \delta} = \left( \frac{F_t}{K_t} \right)^{\epsilon - 1},$$

$$K_t = \frac{\epsilon}{\epsilon - 1} \frac{1 + \tau^L_i}{A_{H,t}} Y_i N_i^\phi + \delta \beta \Pi_{H,t+1}^\epsilon K_{t+1},$$

$$F_t = Y_t C_t^{-\sigma} S_t^{-1} Q_t + \delta \beta \Pi_{H,t+1}^\epsilon F_{t+1},$$

together with an equation determining the evolution of price dispersion

$$\Delta_t = h(\Delta_{t-1}, \Pi_{H,t}),$$

where $h(\Delta, \Pi) = \delta \Pi^\epsilon + (1 - \delta) \left( \frac{1 - \delta \Pi^{-1}}{1 - \delta} \right)^{\epsilon - 1}.$

A.8 Decomposing the Planning Problem (36)

We can break down the planning problem into two parts. First, there is an aggregate planning problem determining the average output gap and inflation $\hat{y}_t^*$ and $\pi_t^*$

$$\min \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ a_\pi (\pi_t^*)^2 + (\hat{y}_t^*)^2 \right] dt$$

subject to (41).

Second, there is a disaggregated planning problem determining deviations from the aggregates
for output gap, home inflation and consumption smoothing, $\tilde{y}_i^t$, $\tilde{\pi}^{i}_{H,t}$ and $\tilde{\theta}_i^t$

$$\min \frac{1}{2} \int_{0}^{\infty} \int_{0}^{1} e^{-\rho t} \left[ \alpha_\pi (\tilde{\pi}^{i}_{H,t})^2 + (\tilde{y}_i^t)^2 + \alpha_\theta (\tilde{\theta}_i^t)^2 \right] dt$$

subject to (37), (38), (39), (40). Note that because the forcing variables in this linear quadratic problem satisfy $\int_{0}^{1} \tilde{s}_i^t dt = 0$, the aggregation constraint (40) is not binding. We can therefore drop it from the planning problem. The resulting relaxed planning problem can be broken down into separate component planning problems for each country $i \in [0, 1]$

$$\min \frac{1}{2} \int_{0}^{\infty} e^{-\rho t} \left[ \alpha_\pi (\tilde{\pi}^{i}_{H,t})^2 + (\tilde{y}_i^t)^2 + \alpha_\pi (\pi^*_t)^2 + (\hat{y}_i^t)^2 \right] dt$$

subject to (37), (38) and (39).

**A.9 Incomplete Markets and No Transfers in a Currency Union**

Here we analyze the solution with incomplete markets and no transfers. This solution imposes $\tilde{\theta}_i^t = 0$ and coincides with the solution with complete markets and no interventions in financial markets, a well-known property of the Cole-Obstfeld case, where the lack of complete markets is not a constraint on private risk sharing.

Using the fact that $\int_{0}^{1} \tilde{y}_i^t dt = \int_{0}^{1} \tilde{\pi}^{i}_{H,t} dt = 0$, we are led to the following planning problem:

$$\min \frac{1}{2} \int_{0}^{\infty} \int_{0}^{1} e^{-\rho t} \left[ \alpha_\pi (\tilde{\pi}^{i}_{H,t})^2 + (\tilde{y}_i^t)^2 + \alpha_\pi (\pi^*_t)^2 + (\hat{y}_i^t)^2 \right] dt$$

subject to

$$\hat{\pi}_i^t = \rho \hat{\pi}_i^t - \kappa \hat{y}_i^t,$$
$$\hat{y}_i^t = -\hat{\pi}_i^t - \hat{s}_i^t,$$
$$\hat{y}_0^t = -\hat{s}_0^t,$$
$$\hat{\pi}_i^t = \rho \pi^*_t - \kappa \hat{y}_i^t,$$

where the minimization is over the variables $\pi^*_t$, $\tilde{\pi}^{i}_{H,t}$, $\tilde{y}_i^t$, $\hat{y}_i^t$. Note that since $\tilde{\theta}_i^t = 0$, the two aggregation constraints $\int_{0}^{1} \tilde{y}_i^t dt = 0$ and $\int_{0}^{1} \tilde{\pi}^{i}_{H,t} dt = 0$ are automatically verified.

The solution of the planning problem is then simply $\hat{y}_i^t = \pi^*_t = 0$ for the aggregates. This result is a restatement of the result in Benigno (2004) and Gali and Monacelli (2008) that optimal monetary policy in a currency union ensures that the union average output gap and inflation are zero in every period. Monetary policy can be chosen at the union level so that monetary conditions are adapted to the average country. The disaggregated variables $\tilde{\pi}^{i}_{H,t}$ and $\tilde{y}_i^t$ solve the following
system of differential equations,

\[ \hat{\pi}_i^{H,t} = \rho \hat{\pi}_i^{H,t} - \hat{\kappa} \hat{y}_i, \]
\[ \dot{\hat{y}}_i = -\hat{\pi}_i^{H,t} - \hat{s}_i, \]

with initial condition

\[ \hat{y}_0 = -\hat{s}_0. \]

**Proposition 16.** The solution with incomplete markets and no interventions in financial markets \((NFA_0^i = \hat{\theta}^i = 0)\) coincides with the solution with complete markets and no interventions in financial markets. In both cases, union-wide aggregates are zero

\[ \hat{y}^*_t = \pi^*_t = 0. \]

### A.10 Transfer Multipliers in a Currency Union

Before solving the normative problem it is useful to review the positive effects of transfers. The next proposition characterizes the response of the economy to a marginal increase in transfers.

**Proposition 17 (Transfer Multipliers).** Let \( v = \frac{\rho - \sqrt{\rho^2 + 4 \hat{\kappa}}}{2}. \) Transfer multipliers are given by

\[ \frac{\partial \hat{y}_i^t}{\partial NFA_0^i} = e^{vt} \rho \frac{1 - \alpha}{\alpha} - (1 - e^{vt}) \rho \frac{1}{1 + \phi}, \]
\[ \frac{\partial \hat{\pi}_i^{H,t}}{\partial NFA_0^i} = -ve^{vt} \left[ \frac{1 - \alpha}{\rho} + \rho \frac{1}{1 + \phi} \right], \]
\[ \frac{\partial \hat{s}_i^t}{\partial NFA_0^i} = -[1 - e^{vt}] \left[ \frac{1 - \alpha}{\rho} + \rho \frac{1}{1 + \phi} \right]. \]

The presence of the discount factor \( \rho \) in all these expressions is natural because what matters is the annuity value \( \rho NFA_0^i \) of the transfer. Note that the terms of trade gap equals accumulated inflation: \( \hat{s}_i = -\int_0^t \hat{\pi}_i^{H,t} ds. \)

Transfers have opposite effects on output in the short and long run. In the short run, when prices are rigid, there is a Keynesian effect due to the fact that transfers stimulate the demand for home goods: \( \frac{\partial \hat{y}_i^0}{\partial NFA_0^i} = \rho \frac{1 - \alpha}{\alpha}. \) In the long run, when prices adjust, the neoclassical wealth effect on labor supply lowers output: \( \lim_{t \to \infty} \frac{\partial \hat{y}_i^t}{\partial NFA_0^i} = -\rho \frac{1}{1 + \phi}. \) In the medium run, the speed of adjustment, from the Keynesian short-run response to the neoclassical long-run response, is controlled by the degree of price flexibility \( \hat{\kappa} \), which affects \( v. \)

Note that the determinants of the Keynesian and neoclassical wealth effects are very different. The strength of the Keynesian effect hinges on the relative expenditure share of home goods \( \frac{1 - \alpha}{\alpha} \); the more closed the economy, the larger the Keynesian effect. The strength of the neoclassical wealth effect

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31 Note that \( v \) is decreasing in \( \hat{\kappa} \), with \( v = 0 \) when prices are rigid \( (\hat{\kappa} = 0) \), and \( v = -\infty \) when prices are flexible \( (\hat{\kappa} = \infty) \).
effect depends on the elasticity of labor supply $\phi$: the more elastic labor supply, the larger the neoclassical wealth effect.

Positive transfers also increase home inflation. The long-run cumulated response in the price of home produced goods equals $\rho^{1-\alpha} + \rho^{1+\phi}$. The first term $\rho^{1-\alpha}$ comes from the fact that transfers increase the demand for home goods, due to home bias. The second term $\rho^{1+\phi}$ is due to a neoclassical wealth effect that reduces labor supply, raising the wage. How fast this increase in the price of home goods occurs depends positively on the flexibility of prices through its effect on $\nu$.$^{32}$

The effects echo the celebrated Transfer Problem controversy of Keynes (1929) and Ohlin (1929). With home bias, a transfer generates a boom when prices are sticky, and a real appreciation of the terms of trade when prices are flexible. The neoclassical wealth effect associated with a transfer comes into play when prices are flexible, and generates an output contraction and a further real appreciation.

**A.11 Proof of Proposition 14**

In this case, $\kappa = 0$ and the constraint set boils down to $\dot{y}_i^0 = (1-\alpha)\theta_i^0 - \bar{s}_i^0$, and we are therefore left with the following component planning problem

$$\min \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ ((1-\alpha)\theta_i^0 - \bar{s}_i^0)^2 + \alpha \theta_i^0 (\bar{s}_i^0)^2 \right] dt.$$ 

The result follows.

**A.12 Proof of Proposition 17**

We use the decomposition of the planning problem given in Appendix A.8. We focus on the component planning problem for a country $i$. We first solve the behavior of an economy for a given transfer $\bar{\theta}_i$. Then in Appendix A.13, we solve for the optimal $\bar{\theta}_i$.

$$\dot{\bar{\theta}}_i = \rho \bar{\theta}_i - \kappa \bar{\theta}_i - \lambda \alpha \bar{\theta}_i,$$

$$\dot{\bar{s}}_i = -\bar{\theta}_i.$$

Define $E_1 = [1,0]'$ and $E_2 = [0,1]'$. Let $X_i^0 = [\bar{\theta}_i^0, \bar{s}_i^0]'$, $B_i = [-\lambda \alpha \bar{\theta}_i, -\bar{s}_i]' = -\lambda \alpha \bar{\theta}_i E_1 - \bar{s}_i E_2$.

Define $A = \begin{bmatrix} \rho & -\kappa \\ -1 & 0 \end{bmatrix}$. Let $\nu = \frac{\rho - \sqrt{\rho^2 + 4\kappa}}{2} < 0$ be the (only) negative eigenvalue of $A$, and $X_{\nu} = [-\nu,1]'$ and be an eigenvector associated with the negative eigenvalue of $A$. The solution is given by

$$X_i = e^{\nu t} \bar{\theta}_i X_{\nu} - \int_t^\infty e^{\nu (t-s)} B_i \dot{s}_i ds = e^{\nu t} \bar{\theta}_i X_{\nu} + \lambda \alpha \bar{\theta}_i A^{-1} E_1 + \int_t^\infty \bar{s}_i e^{\nu (t-u)} E_2 du,$$ 

$^{32}$Recall that $\nu$ is decreasing in the degree of price flexibility $\kappa$. 

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where
\[ X_0^i + \int_0^\infty e^{-As}B_2^i ds = \alpha_v^i X_v, \]
\[ E_2'X_0^i = (1 - \alpha)\tilde{\theta}^i - \bar{g}_0^i. \]

We find
\[ \alpha_v^i = \left[ (1 - \alpha) - \lambda \alpha E_2'A^{-1}E_1 \right] \tilde{\theta}^i - \bar{g}_0^i - \int_0^\infty \bar{g}_0^i E_2' e^{-At}E_2 dt. \]

from which we can infer the path for output
\[ \bar{y}_t^i = e^{\nu t} \alpha_v^i + \lambda \alpha \tilde{\theta}^i E_2'A^{-1}E_1 + \int_t^\infty \bar{g}_0^i E_2' e^{A(t-u)}E_2 du, \]

and inflation
\[ \bar{\pi}_{H,t}^i = -\nu e^{\nu t} \alpha_v^i + \lambda \alpha \tilde{\theta}^i E_1'A^{-1}E_1 + \int_t^\infty \bar{g}_0^i E_1' e^{A(t-u)}E_2 du, \]

Using \( E_2'A^{-1}E_1 = -\hat{\kappa}^{-1} \), and \( E_1'A^{-1}E_1 = 0 \), we can then compute the transfer multipliers.

### A.13 Derivation of the Optimum in Section 5.2

In Appendix A.12, we solved for the behavior of the disaggregated variables \( X_j^i = [\bar{\pi}_{H,t}^i, \bar{y}_t^i]' \) for a given \( \tilde{\theta}^i \). We now solve for the optimal \( \tilde{\theta}^i \). We apply the results of Appendix A.12 in the particular case where \( \bar{g}_0^i = \bar{g}_0^i e^{-\psi t} \). We get
\[ X_j^i = e^{\nu t} \alpha_v^i X_v + \lambda \alpha \tilde{\theta}^i A^{-1}E_1 - \psi e^{-\psi t} \bar{g}_0^i (A + \psi I)^{-1}E_2, \quad (50) \]

where
\[ \alpha_v^i = \left[ (1 - \alpha) - \lambda \alpha E_2'A^{-1}E_1 \right] \tilde{\theta}^i - \bar{g}_0^i + \psi \bar{g}_0^i E_2'(A + \psi I)^{-1}E_2, \]
\[ E_1 = [1, 0]', E_2 = [0, 1]', A = \begin{bmatrix} \rho & -\hat{\kappa} \\ -1 & 0 \end{bmatrix}, \nu = \frac{\rho - \sqrt{\rho^2 + 4\hat{\kappa}}}{2} < 0 \text{ is the negative eigenvalue of } A, \text{ and} \]
\[ X_v = [-\nu, 1]' \text{ is an eigenvector associated with the negative eigenvalue of } A. \]

We need to solve
\[ \min_{\tilde{\theta}} \frac{1}{2} \int_0^\infty e^{-pt} \left[ (X_j^i)' \Omega (X_j^i) + (1 - \alpha)\alpha_v(\tilde{\theta}^i)^2 \right] dt, \]

where
\[ \Omega \equiv \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix}. \]

Replacing the \( X_j^i \) by its expression as a function of \( \tilde{\theta}^i \) given in (50), we find that \( \tilde{\theta}^i \) minimizes the following quadratic form:
\[ \frac{1}{2} (1 - \alpha)\alpha_v(\tilde{\theta}^i)^2 + \frac{1}{2} (\alpha_v^i)^2 \frac{1}{\rho - 2\nu} (X_v' \Omega X_v) + \frac{1}{2} (\tilde{\theta}^i)^2 (\lambda \alpha)^2 \frac{1}{\rho} (E_1' (A')^{-1} \Omega A^{-1} E_1) \]
\[ + \frac{1}{2} (\bar{s}_0^i)^2 (\psi)^2 \frac{1}{\rho + 2\psi} (E_2'(A' + \psi I)^{-1}(A + \psi I)E_2) + \alpha_i \hat{\delta} \lambda \Delta \sqrt{\frac{1}{\rho - \nu}} (X'_i \Omega A^{-1} E_1) - \alpha_i \bar{s}_0 \psi \frac{1}{\rho + \psi - \nu} (X'_i \Omega (A + \psi I)^{-1} E_2) - \hat{\delta} \bar{s}_0 \psi \lambda \Delta \sqrt{\frac{1}{\rho + \psi}} (E'_1 (A')^{-1}(A + \psi I)^{-1} E_2), \]

where \( \alpha_i \) is the linear function of \( \hat{\delta} \) and \( \bar{s}_0 \) derived above. Solving the corresponding FOC gives us the solution.

### A.14 Proof of Proposition 15

The solution for the closed economy limit can be obtained as a particular case of the analysis in Appendix A.13. When \( \bar{s}_i = \bar{s}_0^i e^{-\psi t} \), for a given \( \bar{\theta}, \) we have that \( X_i(t) = [\hat{\pi}_t, \hat{\psi}_t]' \) is given by

\[ X_i(t) = e^{\psi t} \bar{s}_i \alpha X_v - \psi e^{-\psi t} \bar{\pi}_t (A + \psi I)^{-1} E_2, \]

where

\[ \bar{s}_i = \bar{s}_0^i - \hat{\delta} \bar{s}_0 E_2 (A + \psi I)^{-1} E_2. \]

We find that \( \bar{\theta} \) minimizes the following quadratic form:

\[ \frac{1}{2} (\bar{s}_i)^2 (\psi)^2 \frac{1}{\rho + \psi - \nu} (X'_i \Omega (A + \psi I)^{-1} E_2) + \frac{1}{2} (\bar{s}_0^i)^2 (\psi)^2 \frac{1}{\rho + 2\psi} (E_2'(A' + \psi I)^{-1}(A + \psi I)E_2). \]

The solution is

\[ \bar{\theta} = \bar{s}_0^i \left[ 1 - \psi E_2'(A + \psi I)^{-1} E_2 + \psi \frac{\rho - 2\psi}{\rho + \psi - \nu} X'_i \Omega (A + \psi I)^{-1} E_2 \right]. \]

Using \( E_2'(A + \psi I)^{-1} E_2 = \frac{\psi}{(\psi + \nu)(\psi + \rho - \nu)} X'_i \Omega (A + \psi I)^{-1} E_2 = \frac{\nu \alpha \sigma_\psi + \psi}{(\psi + \nu)(\psi + \rho - \nu)} \) and \( X'_i \Omega X_v = \alpha \nu^2 + 1 \), we get the proposition.

### A.15 The Role of Fixed Exchange Rates: Countries Outside a Currency Union

In this section, we seek to clarify the role of fixed exchange rates. We now assume that only a subset of countries \( I \subseteq [0, 1] \) are in the currency union. These countries have flexible exchange rates. We can write down the corresponding planning problem as follows:

\[ \min \frac{1}{2} \int_0^\infty \int_0^{\infty} e^{-rt} \left[ \alpha_\nu (\hat{\pi}_t)^2 + (\hat{\psi}_t)^2 + \alpha_\theta (\bar{\theta})^2 + \alpha_\nu (\hat{\pi}_t)^2 + (\hat{\psi}_t)^2 \right] dt \]

subject to

\[ \hat{\pi}_t = \rho \hat{\pi}_t - \hat{\kappa} \hat{\psi}_t. \]
\[
\int_0^1 \tilde{\theta}^i di = 0,
\]

for \( i \in I, \)

\[
\begin{align*}
\dot{\bar{\pi}}^i_{H,t} &= \rho \bar{\pi}^i_{H,t} - \hat{\kappa} \bar{y}^i_t - \lambda \alpha \hat{\theta}^i, \\
\bar{y}^i_t &= -\bar{\pi}^i_{H,t} - \bar{s}^i_t, \\
\bar{y}^i_0 &= (1 - \alpha) \tilde{\theta}^i - \bar{s}^i_0,
\end{align*}
\]

and for \( i \not\in I, \)

\[
\dot{\bar{\pi}}^i_{H,t} = \rho \bar{\pi}^i_{H,t} - \hat{\kappa} \bar{y}^i_t - \lambda \alpha \hat{\theta}^i.
\]

For countries outside the currency union the only constraint is the Phillips curve. The Euler equation and the initial condition do not appear as constraints because with a flexible exchange rate \( \tilde{e}^i_t \) these become

\[
\begin{align*}
\dot{\bar{y}}^i_t &= \tilde{e}^i_t - \bar{\pi}^i_{H,t} - \bar{s}^i_t, \\
\bar{y}^i_0 &= \tilde{e}^i_0 + (1 - \alpha) \tilde{\theta}^i - \bar{s}^i_0.
\end{align*}
\]

Thus, these equations simply define the required value for the exchange rate \( \tilde{e}^i_t \). As a result, the solution entails \( \bar{\pi}^i_{H,t} = \bar{y}^i_t = \tilde{\theta}^i = 0 \) for \( i \not\in I \). These countries do not send or receive transfers, and reach the same allocation as under complete markets with no interventions in financial markets.

**Proposition 18** (Countries Outside the Currency Union). *For countries outside the currency union, second-best optimal transfers are zero and \( \tilde{\theta}^i = 0 \). They achieve perfect stabilization \( \bar{\pi}^i_{H,t} = \bar{y}^i_t = 0 \).*