The Safety Trap

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Abstract

In this paper we provide a model of the macroeconomic consequences of a shortage of safe assets. In particular, we discuss the emergence of a deflationary *safety trap* equilibrium which is an acute form of a liquidity trap. In this context, issuing public debt, swapping private risky assets for public debt, or increasing the inflation target, stimulate aggregate demand and output, while forward guidance is ineffective. The safety trap can be arbitrarily persistent, as in the secular stagnation hypothesis, despite the existence of infinitely lived assets. When we endogenize the private securitization capacity, we show that in a safety trap there is a securitization externality that leads to underprovision of safe assets.

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1 Introduction

One of the main structural features of the global economy in recent years is the apparent shortage of safe assets. The signature of the growing shortage of safe assets is the secular downward trend in equilibrium real interest rates for more than two decades.[1]

In this paper we provide a simple model of the macroeconomic implications of such a shortage. In particular, we discuss the emergence of a deflationary *safety trap* equilibrium. It is an acute form of liquidity trap, in which the shortage of a specific form of assets (safe assets), as opposed to a general shortage of assets, is the fundamental driving force. The safety trap can be arbitrarily persistent, or even permanent, consistent with the secular stagnation hypothesis (Hansen 1939, Summers 2013), and despite the existence of infinitely lived assets.

In this context, policies that increase the stock of safe assets, such as public debt issuances and some versions of QE, stimulate aggregate demand and output in a safety trap.[2] Instead, policies that seek (and fail) to stimulate aggregate demand by directly increasing the value of risky assets, such as forward guidance, are largely ineffective because they are mostly dissipated in higher risk premia. The relative ineffectiveness of forward guidance in safety traps contrasts with its effectiveness in standard liquidity trap models. In this sense, the safety trap offers a possible rationalization of the "forward guidance puzzle", a term that refers to the limited effect of forward guidance on economic activity observed in the data.[3] Finally, policies that seek to directly reduce the safe real interest rate, such as a large enough increase in the inflation target, lead to the emergence of a good equilibrium with no recession, positive inflation, and negative safe real interest rates.

The model is a perpetual youth OLG model with heterogeneous agents: Neutrals (risk

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[1] We take the term shortage to be an indicator of the excess demand for safe assets at any given safe real interest rate.

[2] In a previous version of this paper, we argued that the benefit of QE1 type policies are unlikely to extend to the swapping of short-run public debt for long-run public debt (which we refer to as Operation Twist (OT), and which encompass the recent QE2 and QE3 in the U.S). In fact, OT can be counterproductive since long term public debt, by being a "bearish" asset that can be used to hedge risky private assets, has a *safe asset multiplier effect* that short term public debt lacks. That is, long term public debt is not only a safe asset in itself, but also makes risky private assets safer through portfolio effects.

Of course, part of the benefit of OT policies is to support the bearish nature of long term public debt, and in this sense it is the *commitment to future* support of these assets, should conditions deteriorate, that generates the benefit, for reasons similar to those we highlight in QE type policies.

We refer the reader to a previous version of this paper (Caballero and Farhi 2013) for a detailed exposition.

[3] The term "forward guidance puzzle" was coined by Del Negro et al. (2013), who also documented this fact.

neutral) and Knightians (infinitely risk averse). Neutrals own risky Lucas trees (aggregate risk) and issue safe assets to Knightians. This securitization process is hampered by a financial friction. As the supply of safe assets shrinks relative to demand (at a given safe interest rate), the safe interest rate drops and the risk premium rises. This mechanism transfers resources from Knightians to Neutrals, reduces the demand for safe assets, and restores equilibrium in the safe asset market.

Once the safe rate hits the zero lower bound, the transfer mechanism breaks down, and instead equilibrium in the safe asset market is restored through a drop in output, which reduces the demand for safe assets. If deflationary forces emerge, then the transfer flow is actually reversed as safe real interest rates rise, resulting in an even larger drop in output. This leads to a dual view of safe asset shortages. As long as safe interest rates are positive, safe asset shortages are essentially benign. But when safe interest rates reach the zero lower bound, they become malign. This is because at the zero lower bound tipping point, the virtuous equilibration mechanism through a reduction in safe interest rates is replaced by a perverse equilibrating mechanism through a reduction in output.

The equilibrium of our model admits an aggregate supply-aggregate demand representation, where the stock of safe assets plays the role of an aggregate demand shifter. The equilibrium impact on output is magnified by a Keynesian multiplier. The multiplier is higher, the more responsive is inflation to output (the steeper is the Phillips curve), or equivalently, the more flexible prices are.

A key aspect of the policy section is the government's capacity to increase the supply of safe assets. This capacity depends on two factors: fiscal capacity and crowding out of private safe assets by public safe assets. In a safe asset shortage situation, the relevant form of fiscal capacity is the government's ability to raise taxes in the bad events feared by Knightians. Crowding out, on the other hand, depends on how much these taxes reduce the private sector's capacity to issue safe claims backed by the risky dividends of Lucas trees. In our model, there is less crowding out when the securitization capacity of the economy is impaired (when the financial friction is severe). In a safety trap, issuing public debt, and possibly purchasing private risky assets, increases the supply of safe assets and stimulates the economy.

The low rates of a safety trap environment create a fertile ground for the emergence of bubbles. However, we show that risky bubbles do not alleviate the safety trap situation, as they do not expand the stock of safe assets. This formalizes some observations in Summers

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(2013) that in secular stagnation environments, even large financial bubbles only seem to create moderate economic expansions. Conversely, safe bubbles do alleviate the problem. We associate the latter concept to that of public debt, and show that the existence of a bubbly region expands the fiscal capacity of the government and reduces the crowding out effect, as bubble-debt does not require future taxation if real rates remain secularly low.

Finally, we show that when the securitization capacity of the economy is endogenous, private securitization decisions are efficient outside of a safety trap, but inefficient inside of it. This is because in a safety trap, private agents do not internalize the stimulative effects of safe asset creation.

Related literature. Our paper is related to several strands of literature. First and most closely related is the literature that identifies the shortage of safe assets as key macroeconomic fact (see e.g. Caballero 2006, Caballero et al. 2008a and 2008b, Caballero and Krishnamurthy 2009, Caballero 2010, Bernanke et al. 2011, and Barclay's 2012).[4] Our paper provides a model that captures many of the key insights in that literature and that allows us to study the main macroeconomic policy implications of this environment more precisely. Like us, Barro and Mollerus (2014) considers an environment with heterogenous risk aversion. They show that such a model can quantitatively match the value of safe assets to GDP as well as a number of asset pricing facts, and also study the crowding out of private safe assets by public safe assets. He et al. (2015) emphasize that the public supply of safe assets is determined not only by fundamentals such as fiscal capacity (as in our paper), but also by self-fulfilling expectations supported by strategic complementarities among investors arising in the presence of default decisions.

Second, there is the literature on liquidity traps (see e.g. Keynes 1936, Krugman 1998, Eggertsson and Woodford 2003, Christiano, Eichenbaum and Rebelo 2011, Correia et al. 2012, and Werning 2012). This literature emphasizes that the binding zero lower bound on nominal interest rates presents a challenge for macroeconomic stabilization. In most models of the liquidity trap, the corresponding asset shortage arises from an exogenous increase in the propensity to save (a discount factor shock). Some recent models (see e.g. Guerrieri and Lorenzoni 2011, and Eggertsson and Krugman 2012) provide deeper microfoundations

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[4]Caballero et al. (2008a,b) developed the idea that global imbalances originated in the superior development of financial markets in developed economies, and in particular the U.S. Global imbalances resulted from an asset imbalance. Although we do not develop the open economy version of our model here, our model could capture a specific channel that lies behind global imbalances: The latter were caused by the funding countries' demand for financial assets in excess of their ability to produce them, but this gap is particularly acute for safe assets since emerging markets have very limited institutional capability to produce them.

and emphasize the role of tightened borrowing constraints in economies with heterogeneous agents (borrowers and savers). Similarly, our model can be seen as providing deeper micro-foundations, introducing a distinction between safe and risky assets (a distinction which is irrelevant in most liquidity trap analyses which are either deterministic or linearized), and exploring the specific role of safe assets shortages. This distinction has important policy implications.

Third, there is an emerging literature on secular stagnation: the possibility of a permanent zero lower bound situation (see e.g. Kocherlakota 2013 and especially Eggertsson and Mehrota 2014). Like us, they use an OLG structure with a zero lower bound. Unlike us, they do not consider risk and risk premia. As we show, this difference has important consequences for the relative effectiveness of different policy options. An additional difference has to do with the theoretical possibility of permanent zero lower bound equilibrium in the presence of infinitely-lived assets, such as land. Indeed, infinitely-lived assets would rule out a permanent zero lower bound in Kocherlakota (2013) and Eggertsson and Mehrota (2014). In our model, trees are indeed infinitely lived. But their value remains finite even when the safe interest rate is permanently at zero. This is only possible because our model features risk and risk premia, which are ignored in most liquidity trap analyzes. Finally, our modelling of inflation in Section 5 borrows heavily from Eggertsson and Mehrota (2014).

Fourth, our paper is related to the literature on aggregate liquidity (see e.g. Woodford 1990 and Holmström and Tirole 1998), which analyzes the role of governments in providing (possibly contingent) stores of value that cannot be created by the private sector. Our paper shares the idea that liquidity shortages are important macroeconomic phenomena, and that the government has a special role in alleviating them. However, it shifts the focus to a very specific form of liquidity—safe assets—and works out its distinct consequences.

Fifth, there is a literature that documents significant deviations from the predictions of standard asset-pricing models—patterns which can be thought of as reflecting money-like convenience services—in the pricing of Treasury securities generally, and in the pricing of short-term T-bills more specifically (Krishnamurthy and Vissing-Jorgensen 2011, 2012, Greenwood and Vayanos 2010, Duffee 1996, Gurkaynak et al. 2006). Our model offers an interpretation of these stylized facts, where the "specialness" of public debt is its safety during bad aggregate states.

Sixth, there is a literature which emphasizes how the aforementioned premium creates incentives for private agents to rely heavily on short-term debt, even when this creates

systemic instabilities (Gorton 2010, Stein 2012, Woodford 2012, Gennaioli et al. 2012). Greenwood et al. (2012) consider the role of the government in increasing the supply of short-term debt and affecting the premium. Gorton and Ordonez (2013) also consider this question but in the context of a model with (asymmetric) information acquisition about collateral where the key characteristic of public debt that drives its premium is its information insensitivity. The inefficiency takes the form of too much securitization. It occurs ex ante (before the crisis), because of a pecuniary externality (fire sales). Instead, in our model, the inefficiency takes the form of too little securitization. The inefficiency occurs ex post (during the crisis) if there is a safety trap, and it does not originate in a pecuniary externality, but rather in a Keynesian externality operating through the level of aggregate demand.

Finally, our paper relates to an extensive literature, both policy and academic, on fiscal sustainability and the consequences of current and future fiscal adjustments (see, e.g., Giavazzi and Pagano 1990, 1996, Alesina and Ardagna 1998, IMF 1996, and Guihard et al. 2007). Our paper revisits some of the policy questions in this literature but highlights the government's capacity to create safe assets at the margin, as the key concept to determine the potential effectiveness of further fiscal expansions as well as the benefits of future fiscal consolidations.

The paper is organized as follows. Section 2 describes our basic model and introduces the key mechanism of a safety trap. Section 3 introduces public debt and considers the effects of QE policies. Section 4 analyzes the role of forward guidance. Section 5 introduces inflation. Section 6 endogenizes securitization and establishes the existence of a securitization externality in a safety trap. Section 7 explores the possibility and consequences of rational bubbles. Section 8 develops a version of a liquidity trap in the context of our model but with no risk premia, where the distinction between safe and risky assets is irrelevant, and explains the similarities and differences with a safety trap. Section 9 concludes.

2 A Model

In this section we introduce our basic model. We use a simple stochastic overlapping generations model. We start by developing a real model of an endowment economy. We then extend it to a production economy with nominal rigidities and money. In the model, as long as safe nominal interest rates are positive, safe assets shortages can be accomodated with reductions in safe nominal interest rates. The associated reduction in the return of safe

assets reduces their demand and restores equilibrium in the market for safe assets. But if safe nominal interest rates are at the zero lower bound, then a safety trap emerges and asset markets are cleared through a recession in goods markets instead.

2.1 Real Model

The horizon is infinite and time is continuous.

Demographics. Population is constant and normalized to one. Agents are born and die at hazard rate θ, independent across agents. Each dying agent is instantaneously replaced by a newborn. Therefore, in an interval dt, θdt agents die and θdt agents are born.

Output and aggregate risk. We model aggregate risk as follows. There are two *aggregate* Poisson processes: a good Poisson process with intensity λ^+ and a bad Poisson process with intensity λ^-. We denote by σ^+ and σ^- the stopping times for the realizations of the good and bad Poisson processes.

We define a Poisson event to be the first realization of either the good or the bad Poisson process, with corresponding stopping time $\sigma = \min\{\sigma^+, \sigma^-\}$. We say that the Poisson event is good if $\sigma = \sigma^+$ and bad if $\sigma = \sigma^-$.

Before the Poisson event, for $t < \sigma$, output X_t per unit of time (output for short) is equal to X. After the Poisson event, for $t \geq \sigma$, output X_t is equal to $\mu^+ X > X$ if the Poisson event is good, and to $\mu^- X < X$ if the Poisson event is bad.

Preferences. We assume that agents only have an opportunity to consume when they die, in which case we denote their consumption by c_t. We denote by σ_θ the stopping time for the idiosyncratic Poisson process controlling death for the agent under consideration.

There are two types of agents in constant fractions in the population: a fraction α of Knightians and $1 - \alpha$ of Neutrals, with identical demographics. These agents have different preferences over risk: Knightians are infinitely risk averse over short time intervals, while Neutrals are risk neutrals over short time intervals. More precisely, for a given stochastic consumption process $\{c_t\}$ which is measurable with respect to the information available at date t, we define the utility U_t^N of a Neutral alive at date t, and U_t^K of a Knightian alive at date t, with the following stochastic differential equations

$$U_t^N = 1_{\{t-dt \leq \sigma_\theta < t\}} c_t + 1_{\{t \leq \sigma_\theta\}} \mathbb{E}_t[U_{t+dt}^N],$$

and
$$U_t^K = 1_{\{t-dt \le \sigma_\theta < t\}} c_t + 1_{\{t \le \sigma_\theta\}} \min{}_t\{U_{t+dt}^K\},$$

where we use the notation $\mathbb{E}_t[U_{t+dt}^N]$ to denote the expectation of U_{t+dt}^N conditional on the information available at date t and $\min_t\{U_{t+dt}^K\}$ to denote the minimum possible realization of U_{t+dt}^K given the information available at date t.

Note that the information at date t contains the information about the realization of the idiosyncratic and aggregate Poisson shocks up to t, implying that $1_{\{t-dt \le \sigma_\theta < t\}}$ and c_t are known at date t. Similarly the conditional expectation $\mathbb{E}_t$ is an expectation over both aggregate shocks and idiosyncratic Poisson death shocks.

Basically, Neutrals are risk neutral with no discounting, and Knightians have Epstein-Zin preferences with infinite relative risk aversion and infinite intertemporal elasticity of substitution, with no discounting.[5] When there is no aggregate risk (as happens after a Poisson event), then the preferences of Knightians and Neutrals coincide.

Endowments, assets, and limited pledgeability. Between t and $t+dt$, output $X_t dt$ is divided into an endowment $(1-\delta)X_t dt$ distributed equally to agents who are born during that interval of time, and the dividend $\delta X_t dt$ of a unit measure of identical infinitely lived Lucas trees.

Only Neutrals can own and operate Lucas trees (a Lucas tree owned and operated by a Knightian yields no dividends). A Neutral can then securitize (borrow against) a tree that he owns by issuing arbitrary state-contingent securities to outside investors (other Neutrals or Knightians) against the cash flows of that tree.

Assumption 1 *(Financial friction): The securitization process is hampered by an agency problem: only a fraction ρ of the cash flows of each tree can be pledged to outside investors, where $\rho > \alpha$.*

This assumption could be motivated in various ways. One popular microfoundation in the financial constraints literature (see e.g. Holmstrom and Tirole 1998, Kiyotaki and Moore 1997, and a vast literature since then) is the existence of a moral hazard problem whereby the owner of a tree can abscond with a fraction $1 - \rho$ of the cash flows.

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[5]Because agents cannot substitute intertemporally (they can only consume when they die), the value of the intertemporal elasticity is irrelevant for our equilibrium, and could be taken to be any number without any modification to our subsequent analysis.

We require that $\rho > \alpha$ in order to ensure that the financial friction would have no bite if there were no aggregate risk (i.e., if we had $\mu^- = 1$). This allows us to isolate the limits to the securitization of safe assets, from the more standard financial friction that limits the securitization of assets in general.[6] In particular, this implies that the financial friction is slack after the good and bad Poisson shocks. This matters when we derive the value of safe assets $V^S = \rho \mu^- \frac{X}{\theta}$ below.

Equilibrium. Newborns trade their endowments for assets. They keep reinvesting and rebalancing their portfolio until they die, at which point they sell their assets for goods and consume them. We focus on the period *before* the (aggregate) Poisson event and, for simplicity, we study the limit $\lambda^- \to 0$ and $\lambda^+ \to 0$.[7] Agents choose their portfolios of assets to maximize their utility. Crucially, Knightians and Neutrals choose different portfolios.

The main features of the portfolios can be understood intuitively. The formal derivations are in Appendix A.1. In order to maximize his utility at date t, a Knightian agent chooses to invest his wealth between t and $t+dt$ in safe assets; that is assets whose value is independent of the realization of aggregate shocks between t and $t + dt$. Similarly, in order to maximize his utility at date t, a Neutral agent chooses to hold some Lucas trees and to issue some safe assets to Knightians against their pledgeable dividends. This is all we need to know about optimal portfolios in order to derive the equilibrium.

Because of the linearity of preferences and the i.i.d. (across agents and time) nature of death, the model aggregates cleanly. We denote by W_t^K the total wealth of Knightians and W_t^N the total wealth of Neutrals. We denote by V_t^S the total value of safe assets that can be issued against the Lucas trees, and by V_t^R the total value of risky assets, by which we mean the value of the Lucas trees net of the value of the safe assets that can be issued against them. Note that we have defined V_t^S as the total value of safe assets that *can* be issued, not the value of safe assets that *are* actually issued by Neutrals to Knightians. It is therefore possible that some safe assets are held by Neutrals. We denote total wealth by W_t with $W_t = W_t^K + W_t^N$, and the total value of assets by $V_t = V_t^R + V_t^S$. Note that V_t is the total value of Lucas trees, the only assets in positive net supply.

Between t and $t + dt$ a fraction θ of agents die and consume. Because dying agents are a representative sample of the population, consumption between t and $t + dt$ is $\theta W_t dt$. Given

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[6]If $\rho < \alpha$, then in steady state, even if there were no risk, so that Knightians and Neutrals had identical preferences, the assets held by Knightians would feature lower interest rates than the rates of return obtained by Neutrals. This would be a pure liquidity premium arising because of the financial friction.

[7]Note that our focus is on the period before the Poisson recession event, and we analyze the consequences of the possibility of a shock rather than the realization of a shock in itself.

that output between t and $t + dt$ is $X\,dt$, market clearing in the goods market pins down the equilibrium level of wealth:

$$W_t = W = \frac{X}{\theta}.$$

Asset market clearing then determines the value of existing assets:

$$V_t = V = W = \frac{X}{\theta}.$$

We can find V_t^S by solving backwards. After the bad Poisson shock, the total value of Lucas trees can be found by applying a similar logic to that prior the shock, so that:

$$V^{\mu^-} = \mu^- \frac{X}{\theta}.$$

Given that only a fraction ρ of the cash flows after the bad Poisson shock can be pledged, and that the financial constraint is slack after the bad Poisson shock (because $\rho > \alpha$), the maximal value of safe assets is ρV^{μ^-}. We will verify below that the equilibrium value of safe assets before (and after) the Poisson shock is indeed

$$V_t^S = V^S = \rho \mu^- \frac{X}{\theta}.$$

For now, we proceed as if it were the case. Risky assets (before the Poisson event) are worth the residual

$$V_t^R = V^R = (1 - \rho\mu^-)\frac{X}{\theta}.$$

Let r_t, r_t^K, and δ_t^S denote the rate of return on risky assets, the rate of return on safe assets, and the dividend paid by safe assets, respectively. Then equilibrium is characterized by the following equations:[8]

$$r_t^K V^S = \delta_t^S X,$$

$$r_t V^R = (\delta - \delta_t^S) X,$$

$$\dot{W}_t^K = -\theta W_t^K + \alpha(1 - \delta)X + r_t^K W_t^K,$$

$$\dot{W}_t^N = -\theta W_t^N + (1 - \alpha)(1 - \delta)X + r_t W_t^N,$$

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[8]Note that in the limit that we consider ($\lambda^+ \to 0$ and $\lambda^- \to 0$), μ^+ does not appear in the equilibrium equations before the Poisson event. Only μ^- does, because it determines the supply of safe assets.

$$W_t^K + W_t^N = V^S + V^R.$$

The first two equations are the standard asset pricing equation for safe and risky assets. The third and fourth equations are the wealth evolution equations for Knightians and Neutrals. The fifth equation is just the asset market clearing equation.

The asset pricing equations equate the rates of returns to dividend yields and capital gains for both safe and risky assets, taking into account that capital gains are zero because the value of safe and risky assets are constant over time. Focusing on the evolution equation for Knightian wealth (the intuition for the evolution equation of Neutral wealth is similar), the wealth equations can be understood as follows: First, between t and $t + dt$, a fraction θdt of Knightians die, sell their assets, and consume. Because the dying Knightians are a representative sample of Knightians, this depletes the stock of Knightian wealth by $\theta W_t^K dt$. Second, between t and $t + dt$, $\theta \alpha dt$ new Knightians are born with a total endowment $\alpha(1 - \delta)X dt$, which they sell to acquire assets. This increases Knightian wealth by $\alpha(1 - \delta)X dt$. Third, between t and $t + dt$, Knightians collect interest rates $r_t^K W_t^K dt$. Overall, the increase in Knightian wealth is therefore $W_{t+dt}^K - W_t^K = -\theta W_t^K dt + \alpha(1 - \delta)X dt + r_t^K W_t^K dt$. Taking the limit $dt \to 0$ yields the stated equation.

Taken together, these equations constitute an equilibrium if and only if two conditions are satisfied: $W_t^K \leq V^S$ and $\delta_t^S \leq \delta\rho$. These conditions are necessary and sufficient for the pledgeability constraints to be verified; i.e. that no Neutral pledges more than a fraction ρ of the cash flows of his Lucas trees to Knightians in the form of safe assets. We shall see below that in equilibrium we can either have $W_t^K < V^S$ or $W_t^K = V^S$, but that we always have $\delta_t^S < \delta\rho$. This validates our earlier claim that the value of safe assets is given by $V^S = \rho V^{\mu^-}$.

Two regimes. We focus on steady states and drop t subscripts. There are two regimes, depending on whether the constraint $W_t^K \leq V^S$ is slack (unconstrained regime) or binding (constrained regime).

In the unconstrained regime, Neutrals are the marginal holders of safe assets so that safe and risky rates are equalized. A couple of steps of algebra show that in this case:

$$\delta^S = \delta\rho\mu^- < \delta\rho,$$

$$r = r^K = \delta\theta.$$

The interesting case for us is the *constrained* regime, where Knightians are the marginal holders of safe assets, and which captures the safe asset shortage environment. In it, Knigh-

tians gobble up all safe assets:

$$W^K = V^S = \rho\mu^- \frac{X}{\theta}.$$

It is easy to verify that this regime holds (after possibly a transitional period) as long as the following *safe asset shortage condition* holds (which we shall assume holds henceforth).

Assumption 2 *(Safe asset shortage):* $\alpha > \rho\mu^-$.

With this assumption we have:

$$\delta^S = \delta\rho\mu^- - (\alpha - \rho\mu^-)(1-\delta) < \delta\rho\mu^- < \delta\rho,$$

$$r^K = \delta\theta - (1-\delta)\theta\frac{\alpha - \rho\mu^-}{\rho\mu^-} < \delta\theta,$$

$$r = \delta\theta + (1-\delta)\theta\frac{\alpha - \rho\mu^-}{1 - \rho\mu^-} > \delta\theta.$$

It follows that in this region there is a safety premium

$$r - r^K = (1-\delta)\theta\frac{\alpha - \rho\mu^-}{\rho\mu^-(1 - \rho\mu^-)} > 0.$$

The supply of safe assets is determined by the severity of the potential bad shock (μ^-) and the ability of the economy to create safe assets (ρ). In fact ρ and μ^- enter the equilibrium equations only through the sufficient statistic $\rho\mu^-$. Similarly, the demand for safe assets is summarized by the fraction of Knightians (α). Together, these sufficient statistics determine whether we are in the unconstrained regime ($\alpha \leq \rho\mu^-$) or in the constrained regime ($\alpha > \rho\mu^-$).

Remark 1 *Our demographics and preferences allow us to capture a stylized version of a life-cycle model with a portfolio choice, abstracting from intertemporal substitution in consumption which is not central to the questions we want to analyze. The only decision that agents are making is how to invest their wealth at every point in time, and because Knightians and Neutrals have different attitudes towards risk, they choose different portfolios.*

Remark 2 *Our model features two forms of market incompleteness. The first one is tied to our overlapping generations structure, which makes future endowment ("wages") non-pledgeable. The second market incompleteness is the pledegeability of dividends constraint*

(ρ) *which limits the ability to securitize (tranche) Lucas trees. Tranching is desirable because it decomposes an asset into a safe tranche which can be sold to Knightian agents and a risky tranche. In the policy discussion we will not exploit the first form of incompleteness (we use the OLG structure mostly because it yields a stationary distribution of wealth between Neutrals and Knightians) and focus instead on the second form.*[9] *The latter implies that in equilibrium risky assets held by Neutrals contain an unpledgable safe claim of size* $(1-\rho)\mu - \frac{X}{\theta}$ *which is the starting point of our macroeconomic policy analysis later on. See Section 3 for a more detailed discussion.*

2.2 Aggregate Demand and the Safety Trap

In this section, we extend the real model to a production economy with nominal rigidities and money. The flexible price equilibrium of this extended model is the same as that of the real model. We call the resulting safe interest rate and allocation the Wicksellian *safe natural interest rate* and the natural allocation. We write $r^{K,n}$ for the safe natural interest rate. For natural output X, we also sometimes use the term potential output.

With nominal rigidities, appropriate monetary policy can ensure that actual output is at potential as long as the safe natural interest rate is positive. But when the safe natural interest rate is negative, the economy reaches the zero lower bound and actual output drops below potential, a situation which we call a *safety trap*. The zero lower bound on safe nominal interest rates arises endogenously because money can be held as a safe store of value.

2.2.1 A New Keynesian Cash-In-Advance Economy

We extend the model in two steps. The first step consists of making output demand determined and to associate real to nominal safe rates by adding standard New Keynesian features. The second step adds money and captures its transaction role with a Cash-In-Advance constraint, which introduces a zero lower bound for safe nominal rates.

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[9]An alternative would have been to use a model where are all agents are alive from the initial period onwards. The problem is that differences in risk aversion lead to explosive wealth distribution dynamics to induce stationarity. Barro and Mollerus (2014), who follow this route, must introduce mean reversion in risk aversion in the form of an idiosyncratic Poisson shock that transforms a Neutral into a Knightian and vice versa.

Demand determined output. Let us incorporate the traditional ingredients of New Keynesian economics: imperfect competition, sticky prices, and a monetary authority.

In this setting, in every period, non-traded inputs are used to produce differentiated varieties of goods x_k indexed by $k \in [0,1]$ where each variety is produced using a different variety of non-traded good also indexed by $k \in [0,1]$. We index trees by $i \in [0,\delta]$, where each tree i yields a dividend of X non-traded goods. Similarly, we index newborns by $j \in [\delta,1]$ where each newborn j is endowed with X non-traded goods. Goods with indices $k \in [0,\delta]$ are produced with the non-traded inputs from the dividends of trees indexed by k, and goods with indices $k \in [\delta,1]$ are produced with the non-traded inputs from the endowments of the newborns indexed by k. Each variety is sold by a monopolistic firm. Firms post prices p_k in units of the numeraire. These differentiated varieties of goods are valued by consumers according to a standard Dixit-Stiglitz aggregator $C = \left(\int_0^1 x_k^{\frac{\sigma-1}{\sigma}} dk \right)^{\frac{\sigma}{\sigma-1}}$, and consumption expenditure is $PC = \int_0^1 p_k x_k dk$, where the price index is defined as $P = \left(\int_0^1 p_k^{1-\sigma} dk \right)^{\frac{1}{1-\sigma}}$. The resulting demand for each variety is given by $x_k = \left(\frac{p_k}{P} \right)^{-\sigma} C$.

The prices of different varieties are entirely fixed (an extreme form of sticky prices, which we shall relax later in Section 5) and equal to each other, $p_k = P$. Firms accommodate demand at the posted price, and their profits accrue to the agent owning and supplying the corresponding non-traded input. Without loss of generality, we use the normalization $P = 1$. Note that because the prices of all varieties are identical, the demand for all varieties is the same. Output is demand-determined, and as a result, $x_k = C = \xi X$ for all k where the capacity utilization rate $\xi \leq 1$ is the same for all firms. Capacity utilization ξ represents the wedge between actual output ξX and potential output X.

Finally, a monetary authority sets a safe nominal interest rate i. Because prices are rigid, this determines the safe real interest rate $r^K = i$.

Money, the zero lower bound and the cashless limit. To justify a zero lower bound, $r^K \geq 0$, we introduce money into the model. We then define and focus on the cashless limit (see e.g. Woodford 2003).

We represent the demand for real money balances for transactional services using a Cash-In-Advance constraint that stipulates that individuals with wealth w_t and money holdings m_t can only consume $\min(w_t, \frac{m_t}{\varepsilon})$. When $i > 0$, money is held only for transaction services. When $i = 0$ money is also held as a safe store of value, which competes with its transaction services. This model has no equilibrium with $i < 0$, because then money would dominate

other safe assets. Hence there is a zero lower bound $i \geq 0$ or equivalently $r^K \geq 0$.

The demand for real money balances for transactional services is εW_t^K and εW_t^N for Knightians and Neutrals respectively. We assume that the money supply is $\varepsilon M^\varepsilon$ with $M^\varepsilon = \frac{X}{\theta}$ before the Poisson event and that it adjusts to accommodate one for one the change in potential output after the Poisson shock ($M^{\varepsilon-} = \mu^- \frac{X}{\theta}$ and $M^{\varepsilon+} = \mu^+ \frac{X}{\theta}$ for bad and good shocks, respectively).

After the (bad) Poisson shock, the value of the safe tranches of trees is a fraction ρ of the total value of assets excluding money (government liability), and the total value of safe assets is

$$
\begin{aligned}
V^S &= \rho(\mu^- \frac{X}{\theta} - \varepsilon M^{\varepsilon-}) + \varepsilon M^{\varepsilon-}, \\
&= \rho \mu^- (1 - \varepsilon) \frac{X}{\theta} + \varepsilon \mu^- \frac{X}{\theta}.
\end{aligned}
$$

and the equilibrium equations are now,

$$
r^K V^S = \delta^S \xi X,
$$

$$
r V^R = (\delta - \delta^S)\xi X,
$$

$$
\dot{W}_t^K = -\theta W_t^K + \alpha (1 - \delta) \xi X + r^K (1 - \varepsilon) W_t^K,
$$

$$
\dot{W}_t^N = -\theta W_t^N + (1 - \alpha) (1 - \delta) \xi X + r (1 - \varepsilon) W_t^N,
$$

$$
\varepsilon(W_t^K + W_t^N) \leq \varepsilon M^\varepsilon \quad \text{with equality if} \quad r^K > 0
$$

$$
W_t^K + \varepsilon W_t^N \leq V^S,
$$

$$
W_t^K + W_t^N = V^S + V^R,
$$

and the requirement that

$$
r^K = i \geq 0.
$$

In the rest of the paper, we focus on the cashless limit $\varepsilon \to 0$.[10] We also focus on steady states and drop t-subscripts.

---

[10]We refer the reader to Appendix A.2 for an analysis of some interesting issues that arise away from the cashless limit.

2.2.2 The Zero Lower Bound and the Safety Trap

If the safe natural interest rate is positive, then by setting the nominal interest rate equal to the natural safe interest rate $i = r^{K,n} > 0$, the monetary authority can replicate the natural allocation, ensuring that output is at potential $\xi = 1$.

The safety trap. If the safe natural interest rate is negative, then there is a recession ($\xi < 1$) even with $i = r^K = 0$. To determine the severity of the recession, let us work backwards and recall that we have assumed that actual and potential (now lower) output coincide after the bad Poisson shock and therefore the value of safe assets (before the shock) is still given by

$$V^S = \rho \mu^- \frac{X}{\theta}.$$

Mechanically, the expanded (to incorporate a zero lower bound) model is identical to the basic real model but with $\rho \mu^-$ replaced by $\frac{\rho \mu^-}{\xi}$ and X replaced by ξX. The requirement that $r^K = 0$ determines the severity of the recession ξ:

$$0 = \delta \theta - (1 - \delta) \theta \frac{\alpha - \frac{\rho \mu^-}{\xi}}{\frac{\rho \mu^-}{\xi}},$$

yielding

$$\xi = \frac{\theta}{\theta - r^{K,n}} = \frac{\rho \mu^-}{\underline{\rho \mu^-}} < 1,$$

where $\underline{\rho \mu^-} = \alpha (1 - \delta)$ corresponds to the value of these combined parameters for which zero is the natural safe interest rate.[11]

The mechanism is a form of liquidity trap which we call a *"safety trap"*. At full employment, there is an excess demand for safe assets. A recession lowers the absolute demand for safe assets while keeping the absolute supply of safe assets fixed and restores equilibrium. Figure 1 illustrates this mechanism, which we describe next.

The supply of safe assets is given by $V^S = \rho \mu^- \frac{X}{\theta}$ and the demand is given by $W^K = \frac{\alpha(1-\delta)\xi X}{\theta - r^K}$. Equilibrium in the safe asset market requires that $W^K = V^S$, i.e.

---

[11]Note that the risky interest rate r is increasing in ξ, so that the deeper the recession, the lower is r:

$$r = \delta \theta + (1 - \delta) \theta \frac{\alpha - \frac{\rho \mu^-}{\xi}}{1 - \frac{\rho \mu^-}{\xi}}.$$
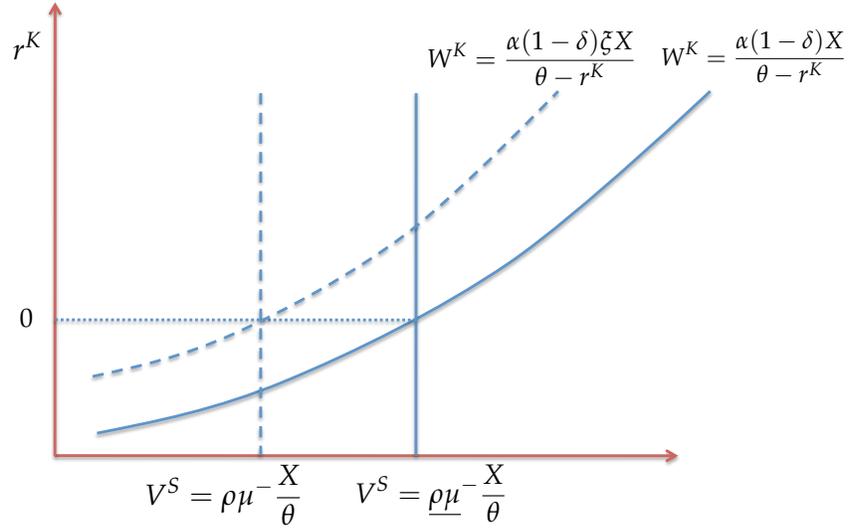
The figure shows curves labeled:

$$W^K = \frac{\alpha(1-\delta)\xi X}{\theta - r^K} \qquad W^K = \frac{\alpha(1-\delta) X}{\theta - r^K}$$

$$V^S = \rho\mu^- \frac{X}{\theta} \qquad V^S = \underline{\rho\mu}^- \frac{X}{\theta}$$

Figure 1: Safety trap. Recession caused by a decrease in the supply of safe assets. The safe asset supply curve shifts left $(\underline{\rho\mu}^- < \rho\mu^-)$, the endogenous recession shifts the safe asset demand curve left $(\xi < 1)$, and the safe interest rate remains unchanged at 0.

$$\frac{\alpha(1-\delta)\xi X}{\theta - r^K} = \rho\mu^- \frac{X}{\theta}.$$

Consider an unexpected (zero ex-ante probability) shock that lowers the supply of safe assets (a reduction in $\rho\mu^-$). The mechanism by which equilibrium in the safe asset market is restored has two parts. The first part immediately reduces Knightian wealth W^K to a lower level, consistent with the lower supply of safe assets $\rho\mu^- \frac{X}{\theta}$. The second part *maintains* Knightian wealth W^K at this lower level.

The first part of the mechanism is as follows. The economy undergoes an immediate wealth adjustment (the wealth of Knightians drops) through a round of trading between Knightians and Neutrals born in previous periods. At impact, Knightians hold assets that now carry some risk. They react by selling the risky part of their portfolio to Neutrals. This shedding of risky assets catalyzes an instantaneous fire sale whereby the price of risky assets collapses before immediately recovering once risky assets have changed hands. Needless to say, in reality this phase takes time, which we have removed to focus on the phase following the initial turmoil.

The second part of the equilibrating mechanism differs depending on whether the safe interest rate $r^K = i$ is above or at the zero lower bound. If $r^K = i > 0$, then a reduction in
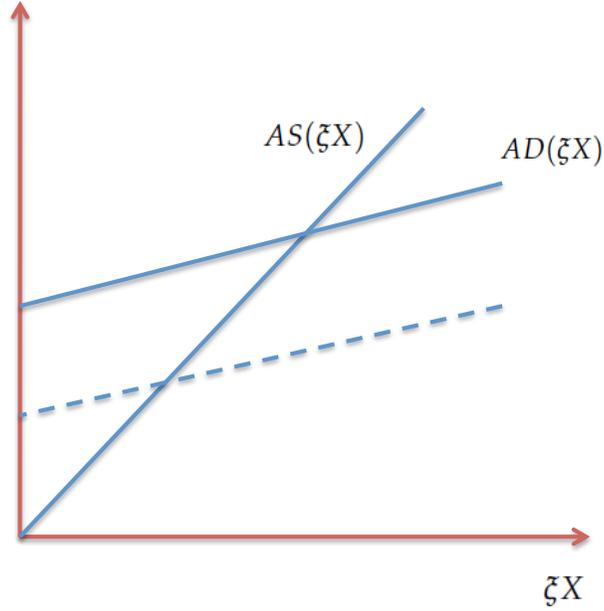
Figure 2: AS-AD representation and the Keynesian cross. Recession caused by a decrease in the supply of safe assets. Aggregate demand shifts down.

the safe interest rate $r^K = i$ takes place. This reduction in the safe interest rate effectively operates a transfer (in every period) from Knightians to Neutrals, which limits the growth of Knightian wealth. As a result, the safe asset market remains in equilibrium, and so does the goods market. If the safe interest rate is against the zero lower bound $r^K = i = 0$, then this reduction in the safe interest rate cannot take place and the associated transfer cannot occur. The only adjustment mechanism is with a decline in output (income), which also drags down Neutral's wealth.

AS-AD representation and the Keynesian cross. With safe interest rates fixed at $r^K = i = 0$, output is determined as follows. Recall that $rW^N = rV^R = (\delta - \delta^S)X = \delta X - r^K V^S$, which replaced into the wealth accumulation equation of Neutrals yields the total value of Neutral wealth:

$$W^N = (1 - \alpha)(1 - \delta)\xi\frac{X}{\theta} + \delta\xi\frac{X}{\theta} - \frac{r^K}{\theta}V^S$$

while the total value of Knightian wealth is $W^K = V^S$. Aggregate demand for goods is

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therefore given by $\theta \left(W^N + W^K \right) = AD \left(\xi X \right)$ where

$$AD \left(\xi X \right) = (1 - \alpha)(1 - \delta)\xi X + \delta \xi X + \left(\theta - r^K \right) V^S,$$

with $r^K = 0$. Aggregate supply is simply the 45 degree line

$$AS \left(\xi X \right) = \xi X.$$

Aggregate demand and aggregate supply are two increasing functions of income ξX. Aggregate demand is flatter than aggregate supply. Equilibrium is determined by a *Keynesian cross* at the intersection of the aggregate demand and aggregate supply curves. Crucially, *a reduction $dV^S < 0$ in the value of safe assets V^S represents an adverse (downward) shift to aggregate demand.* This reduction in aggregate demand lowers output, which further reduces aggregate demand, etc. ad infinitum. This is a familiar Keynesian multiplier

$$\frac{\xi X}{\theta V^S} > 1,$$

which amplifies the effect of the initial reduction θdV^S in aggregate demand to a final effect of $d \left(\xi X \right) = \frac{\xi X}{\theta V^S} \theta dV^S$—a proportional increase in output.[12,13] Figure 2 provides a graphical illustration of this adjustment.

Secular stagnation. Because we are studying the limit $\lambda^- \to 0$ and $\lambda^+ \to 0$, the safety trap is essentially permanent. The safety trap also can be seen as a model of secular stagnation, understood by most recent observers to be a situation where the economy is at

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[12]The fact that the final effect of a decrease in V^S is a proportional decrease in output ξX is an immediate consequence of the fact that the Keynesian cross is a linear system of two equations in ξX and V^S.

[13]The analysis above goes through as well if we raise the share of Knightian agents α instead of reducing $\rho \mu^-$, in which case the recession factor is

$$\xi = \frac{\theta}{\theta - r^{K,n}} = \frac{\underline{\alpha}}{\alpha} < 1,$$

where $\underline{\alpha} = \frac{\rho \mu^-}{1 - \delta}$ corresponds to the value of this parameter for which zero is the natural safe interest rate. The increase in α acts as an adverse shift in aggregate demand. This interpretation resembles the Keynesian paradox of thrift. Combining both, asset supply and demand factors, we have that the severity of the recession is determined by the sufficient statistic $\frac{\rho \mu^-}{\alpha}$ according to the simple equation:

$$\xi = \frac{\alpha}{\underline{\rho \mu^-}} \frac{\rho \mu^-}{\alpha},$$

where $\frac{\underline{\rho \mu^-}}{\alpha} = 1 - \delta$ corresponds to the value of these combined parameters for which zero is the natural safe interest rate.

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the zero lower bound forever.

The theoretical possibility of permanent zero lower bound equilibrium is sometimes disputed on the grounds that this would imply an infinite value for infinitely-lived assets, such as land. In our model, trees are indeed infinitely lived. But their value remains finite even when the safe interest rate is $r^K = i = 0$ essentially permanently. This is only possible because our model features risk and endogenous risk premia, which are ignored in most liquidity trap analyses.

Policy options: a roadmap. In the next sections we analyze policy options in a safety trap. The only determinants of aggregate demand that these policies will affect are the value of safe assets V^S (public debt and QE in Section 3) and r^K (increasing the inflation target in Section 5). In particular, it is important to note that the risky rate r does not appear either in aggregate demand or in aggregate supply. The equilibrium is block-recursive in ξ and (r, V^R). Even when $\lambda^+ > 0$, the future value of risky assets after the good Poisson shock does not affect the determination of output ξX. This point will prove crucial later on when we discuss the ineffectiveness of forward guidance in a safety trap (Section 4).

3 Public Debt and Quantitative Easing

In this section we introduce public debt to our analysis and discuss situations where the government can use its debt capacity to alleviate the safety trap. We make three points: First, there is a region in which public debt crowds out one for one the private sector's capacity to produce its own safe debt. Second, there is a region in which this crowding out does not occur, and instead public debt increases the total supply of safe assets one for one. And third, we show that in such contexts, QE type policies can be effective.

3.1 Public Debt and Fiscal Capacity

Public debt. We start by introducing public debt and discussing the role of public purchases and sales of such debt. The government taxes dividends, δX. The tax rate is τ^+ after the good Poisson shock occurs, τ^- after the bad Poisson shock occurs, while the tax rate before the Poisson event is set to a value τ_t that satisfies the government flow budget constraint. The government issues a fixed amount of risk-free bonds that capitalize future tax revenues

and pays a variable rate r_t^K.[14] The proceeds of the sales of these bonds are rebated lump-sum to agents at date 0.

Let the value of public debt be given by D. We have

$$D = \tau^- \mu^- \frac{X}{\theta}.$$

Assumption 3 *(Regalian taxation power): Taxes backing government safe debt can be levied on the claim $(1-\rho)\mu^- \frac{X}{\theta}$ to the privately unpledgable part of future dividends.*

That is, the government is essentially better than private investors at collecting dividend revenues from Neutrals once borrowers' incentives are weak. This confers the government a comparative advantage in the production of safe assets.[15] In Section 6, we develop a different rationale for government intervention in the securitization market. There, we endogenize the securitization capactiy ρ and show that in a safety trap (but not outside of it) there is a securitization externality that justifies government intervention; we postpone a full discussion of this justification for intervention until then.

Note, however, that the scope for policy depends crucially on the degree of financial development of the economy (indexed by ρ). In a very developed market, ρ is high, and soon public debt starts to crowd out privately produced safe assets. To see this, note that because the consumption of a Neutral cannot be negative (a form of limited liability), the fraction of dividends that it can pledge is now $\rho(\tau^-) = \min\{\rho, 1-\tau^-\}$. Thus, as long as $\tau^- \leq 1-\rho$ there is no crowding out, but above this threshold public safe assets crowd out private safe assets one for one (we return to this issue below).

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[14]It is the latter feature that makes this debt "short-term," since its value remains constant over time as its coupons vary with the riskless rate. In the previous version of this paper (Caballero and Farhi 2012), we introduce long-term public debt and study Operation Twist (OT) policies that swap long-term debt for short-term debt.

[15]This mechanism has some commonality with the idea in Holmström and Tirole (1998) that the government has a comparative advantage in providing liquidity. In their model like in ours, this result arises from the assumption that some agents (consumers in their model) lack commitment and hence cannot borrow because they cannot issue securities that pledge their future endowments. This can result in a scarcity of stores of value. The government can alleviate this scarcity by issuing public debt and repaying this debt by taxing consumers. The proceeds of the debt issuance can actually be rebated to consumers. At the aggregate level, this essentially relaxes the borrowing constraint of consumers: They borrow indirectly through the government. In their model like in ours, the comparative advantage of the government in providing liquidity arises because it is better than private lenders at collecting revenues from consumers.

The total (private and public) value of safe assets is then given by

$$V^S = \rho(\tau^-)\mu^- \frac{X}{\theta} + D = [\rho(\tau^-) + \tau^-]\mu^- \frac{X}{\theta}.$$

The model is isomorphic to the one described in Section 2, with ρ replaced by $[\rho(\tau^-) + \tau^-]$ (and note that $\rho \leq \rho(\tau^-) + \tau^- \leq 1$). Thus we have

$$r^K = \delta\theta - (1-\delta)\theta \frac{\alpha - [\rho(\tau^-) + \tau^-]\mu^-}{[\rho(\tau^-) + \tau^-]\mu^-},$$

$$r = \delta\theta + (1-\delta)\theta \frac{\alpha - [\rho(\tau^-) + \tau^-]\mu^-}{1 - [\rho(\tau^-) + \tau^-]\mu^-},$$

and we can also use $r^K D = \tau\delta X$ to compute

$$\tau = \tau^- \mu^- \frac{r^K}{\delta\theta}.$$

The economy is in the constrained regime if and only if $\alpha > [\rho(\tau^-) + \tau^-]\mu^-$, which we assume. The safety premium is then given by

$$r - r^K = \theta(1-\delta) \frac{\alpha - [\rho(\tau^-) + \tau^-]\mu^-}{[\rho(\tau^-) + \tau^-]\mu^-[1 - [\rho(\tau^-) + \tau^-]\mu^-]} \geq 0.$$

Crowding out. As we mentioned earlier, in this model *government debt acts exactly like tranching, with τ^- playing the same role as ρ,* as long as public debt is low enough ($\tau^- < 1 - \rho$). In this non-Ricardian region, issuing public debt does not crowd out private safe assets, resulting in a one for one expansion of the supply of safe assets V^S, an increase in the safe interest rate r^K, a reduction in the risky interest rate r, and a reduction in the safety premium $r - r^K$. There is also a Ricardian region where public debt is high enough ($\tau^- \geq 1 - \rho$) so that issuing public debt crowds out private safe assets one for one, leaving unchanged the supply of safe assets V^S, the safe and risky interest rates r^K and r as well as the safety premium $r - r^K$. The economy is more likely to be in the Ricardian region than in the non-Ricardian region, the higher is the securitization capacity ρ.

It will sometimes prove convenient to write

$$V^S = \frac{\mu^- X}{\theta} v^S(\frac{D}{X})$$

where the function $v^S(\frac{D}{X})$ is defined together with the function $\tau^-(\frac{D}{X})$ by the following

equations

$$v^S(\frac{D}{X}) = \rho(\tau^-(\frac{D}{X})) + \frac{\theta}{\mu^-}\frac{D}{X} = \rho(\tau^-(\frac{D}{X})) + \tau^-(\frac{D}{X}),$$

and

$$\tau^-(\frac{D}{X}) = \frac{\theta}{\mu^-}\frac{D}{X}.$$

Crowding out of private safe assets $\frac{\mu^- X}{\theta}v^S(\frac{D}{X}) - D$ by public debt D is

$$1 - \frac{\mu^-}{\theta}\frac{dv^S}{d(\frac{D}{X})} = -\frac{d\rho}{d\tau^-}$$

is either 0 (in the non-Ricardian region) or 1 (in the Ricardian region).

Intermediate crowding out. We can easily extend the model to feature intermediate crowd out by assuming that the trees differ in their pledgeability $\tilde{\rho}$ with a distribution $dF(\tilde{\rho})$. This extension can be exactly mapped into the model above. The only difference is that we now have

$$\rho(\tau^-) = \int \min\{\tilde{\rho}, 1 - \tau^-\} dF(\tilde{\rho}),$$

so that crowding out is now given by

$$-\frac{d\rho}{d\tau^-} = [1 - F(1 - \tau^-)] \in [0, 1].$$

Crowding out and Ricardian equivalence. The tight link between imperfect crowding out and the failure of Ricardian equivalence that exists in our model relies on the assumption that taxes are capitalized. It would not hold if, for example, taxes were levied on the endowment of newborns.[16] We view this feature as desirable since it allows us to focus on the market failure that is central to our analysis—the financial friction that hampers the securitization process—rather than on more conventional and perhaps debatable features of OLG models.

Public debt and the safety trap. Now imagine that the economy is in a safety trap

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[16]More generally, the distribution of taxes matters for this result. We refer the reader to an earlier version of this paper, Caballero and Farhi (2013), for an exploration of this idea. In a recent paper, Barro and Mollerus (2014) consider a model with heterogeneous risk aversion and assume that taxes are distributed independently of risk aversion. They generate a crowding out of 0.5 despite the fact that Ricardian equivalence holds in their model. See Abel (2015) for a detailed exploration of the determinants of crowding out in Ricardian economies.

where the safe interest rate is fixed at zero and output is below potential with $\xi < 1$. Then increasing public debt from D to $\hat{D} > D$ increases the supply of safe assets $\hat{V}^S > V^S$ where $\hat{V}^S = \frac{\mu - X}{\theta} v^S(\frac{\hat{D}}{X})$ and $V^S = \frac{\mu - X}{\theta} v^S\left(\frac{D}{X}\right)$ as long as there is less than full crowding out. This then stimulates output, increasing ξ to $\hat{\xi}$ where

$$\hat{\xi} = \frac{\hat{V}^S}{V^S}\xi > \xi.$$

The stimulative effects of public debt can be understood most clearly by going back to our AS-AD equilibrium representation. Because issuing public debt increases the supply of safe assets V^S, it produces and upward shift in aggregate demand, which in turn results in a proportional increase in output through the Keynesian multiplier.

In the model, we necessarily have $\tau^+ \mu^+ = \tau^- \mu^-$, which implies that $\tau^+ < \tau^-$. For this reason, it is natural to expect fiscal constraints to be more binding after the bad Poisson shock than after the good Poisson shock. This is why we adopt τ^-, a measure of the ability of the government to raise tax revenues after the bad Poisson shock, as our measure of future fiscal capacity. In particular, in a safety trap, increasing the supply of public debt to $\hat{D}$ requires the government to have spare future fiscal capacity, that is to have the ability to raise more taxes after the bad Poisson shock

$$\hat{\tau}^- = \frac{\hat{D}}{D}\tau^- > \tau^-.$$

Note however that taxes do not have to be raised while the economy is in the safety trap before the Poisson event. Indeed $\hat{\tau} = \tau$ (and both are equal to 0) since safe interest rates on debt are $r^K = 0$.

Remark 3 *Issuing money while at the zero bound is equivalent to issuing short-term bonds, and both are constrained by the long-term fiscal capacity of the government. Indeed, after the bad Poisson shock, the government must raise taxes to retire the additional money that it has issued before the Poisson event. See Appendix A.2 for a detailed exposition of these arguments.*

3.2   Quantitative Easing

Here we use the term QE loosely to encompass policies that swap risky assets for safe assets such as QE1, LTRO, and many other lender of last resort central bank interventions. We

model QE as follows. The government issues additional short term debt and purchases private risky assets (the value of which drops to zero after the bad Poisson shock). Let $\hat{\beta}^g$ be the fraction of the value of risky assets purchased by the government where[17]

$$\hat{\beta}^g[1 - [\rho(\hat{\tau}^-) + \hat{\tau}^-]\mu^-]\frac{X}{\theta} = \hat{D} - D.$$

The key difference between QE and simply issuing more public debt is what the government does with the proceeds from the debt issuance. In QE, the government uses the proceeds to purchase private risky assets instead of simply rebating them lump sum to private agents. The revenues from the debt issuance taxes $\hat{\tau}$ before the bad Poisson shock can be lowered to

$$\hat{\tau} = \tau - \frac{r}{\delta\theta}\hat{\beta}^g[1 - [\rho(\hat{\tau}^-) + \hat{\tau}^-]\mu^-]$$

because the government can now avail itself of additional investment revenues from its holdings of private risky assets. If $\tau = 0$, as would be the case in a safety trap, then we can get $\hat{\tau} < 0$. This should then be interpreted as a possibility to reduce taxes if there were some other reasons for which taxes had to be raised.

As long as there is less than full crowding out, the safe asset shortage is alleviated by this policy: r^K increases, r decreases, and the safety premium shrinks. Here QE works not so much by removing risky private assets from private balance sheets, but rather by injecting public assets into private balance sheets. In other words, QE works by increasing the supply of safe assets. The proceeds of the extra debt issuance can either be rebated lump sum or reinvested in a portfolio of private assets as long as these assets are risky and not safe.

If the economy is in a safety trap where the safe interest rate is fixed at zero and output is below potential with $\xi < 1$, QE acts by stimulating output, increasing the value of ξ to $\hat{\xi}$ where

$$\hat{\xi} = \frac{\hat{V}^S}{V^S}\xi > \xi,$$

with as before $\hat{V}^S > V^S$ where $\hat{V}^S = \frac{\mu^- X}{\theta}v^S(\frac{\hat{D}}{X})$ and $V^S = \frac{\mu^- X}{\theta}v^S\left(\frac{D}{X}\right)$ as long as there is less than full crowding out.

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[17]As long as $\hat{\beta}^g$ is low enough, this can be done without violating the pledgeability condition. The precise condition is

$$\hat{\beta}^g < \frac{\delta\rho - \{\delta[\hat{\tau}^- + \rho(\hat{\tau}^-)]\mu^- - [\alpha - [\hat{\tau}^- + \rho(\hat{\tau}^-)]\mu^-](1-\delta)\}}{\delta - \{\delta[\hat{\tau}^- + \rho(\hat{\tau}^-)]\mu^- - [\alpha - [\hat{\tau}^- + \rho(\hat{\tau}^-)]\mu^-](1-\delta)\}}.$$

4   Forward Guidance

Another major policy tool advocated in the context of zero lower bound of interest rates is forward guidance (the commitment to low future interest rates once the economy recovers). However, in this section we show that when the reason for this low interest rate is a shortage of safe assets, the policy is ineffective.

The reason is that only policy commitments that support future *bad* states work in safety traps. This is a higher level of requirement than in the standard New-Keynesian liquidity trap mechanism where *any* future wealth increase has the potential to stimulate the economy, including wealth created after the recovery is completed.

We illustrate this point with an example of forward guidance policy that would work in a standard liquidity trap environment but not in a safety trap. We refer the reader to Section 8 for a version of the standard liquidity trap in the context of our model and the demonstration of forward guidance effectiveness in that context. In this section we focus on the safety trap case.

Since public debt is not key to our main concern here, we temporarily revert to our model in Section 2 where there are only private assets. We introduce two modifications to that model. First, we temporarily (only for this section) assume a non-zero intensity of the good Poisson shock $\lambda^+ > 0$. Second, we allow agents to produce $\zeta > 1$ units of output per unit of input. However, we imagine that there is a large utility loss from doing so.

This model functions as in Section 2 with $\lambda^+ = 0$. Indeed all the variables V^S, V^R, W^K, W^N, r^K, δ^S have exactly the same equilibrium values. The only difference is in the (risky) interest rate r. The interest rate r is now determined by the following set of equations (and λ^+ only enters the last of these equations):

$$r = \delta\theta + (1 - \delta)\,\theta\frac{\alpha - \rho\mu^-}{1 - \rho\mu^-} + \lambda^+\frac{\mu^+ - \rho\mu^-}{1 - \rho\mu^-}.$$

We can also look at safety traps where $r^K = i = 0$. Output ξX is determined by the exact same equation as in the model with $\lambda^+ = 0$. To see this more transparently, it is useful to go back to our AS-AD representation. The key is to note that both the aggregate supply $AS\,(\xi X) = \xi X$ and aggregate demand $AD\,(\xi X) = (1-\alpha)(1-\delta)\xi X + \delta\xi X + (\theta - r^K)V^S$ (with $r^K = 0$) relations are unchanged.[18] The risky interest rate r is then determined by the

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[18]This occurs because the value of safe assets $V^S = \rho\mu^- \frac{X}{\theta}$ is unchanged. As a result, the dividend

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same equation as above but with X replaced by ξX, μ^+ replaced by $\frac{\mu^+}{\xi}$ and μ^- replaced by $\frac{\mu^-}{\xi}$:

$$r = \delta\theta + (1-\delta)\,\theta\frac{\alpha - \frac{\rho\mu^-}{\xi}}{1 - \frac{\rho\mu^-}{\xi}} + \lambda^+\frac{\frac{\mu^+}{\xi} - \frac{\rho\mu^-}{\xi}}{1 - \frac{\rho\mu^-}{\xi}}.$$

In New-Keynesian models of the liquidity trap (see e.g. Krugman 1998, Eggertsson and Woodford 2003, and Werning 2012), committing to keep the interest rate low in the future once the economy recovers (after the good Poisson shock) stimulates the economy—a policy often referred to as forward guidance. The latter works by creating a boom in the future, which raises current demand through a combination of a wealth effect (higher income in the future) and substitution effect (lower real interest rates because of inflation). In Section 8, we show how to model a standard liquidity trap without a specific safe asset shortage (and instead with a general asset shortage) in the context of our framework. There, we show that forward guidance does stimulate output in our version of the liquidity trap.[19]

In contrast, in a safety trap the commitment to low interest rates *after the good Poisson shock* translates into an increase in r but fails to stimulate the economy, as it does not affect value of assets V^S, which is the key shifter of aggregate demand. The reconciliation of this ineffectiveness result with the obvious ex-post (after the good Poisson shock) increase in risky asset values and Neutral wealth of the policy is that the risky rate r rises just enough to offset the present value effect of the policy. We show this result formally next.

Consider the following policy: Suppose that the good Poisson shock occurs at σ^+. After the good Poisson shock, the central bank stimulates the economy by setting the interest rate i_t below the natural interest rate $\delta\theta$ until $\sigma^+ + T$, at which point it reverts to setting the nominal interest rate equal to the natural interest rate $i = \delta\theta$. For $t > \sigma^+ + T$, output is equal to potential so that $\zeta_t = 1$. For $\sigma^+ \leq t \leq \sigma^+ + T$, output is above potential, and capacity utilization satisfies a simple differential equation

$$\frac{\dot{\zeta}_t}{\zeta_t} = i_t - \delta\theta \leq 0,$$

accruing to Neutrals $(\delta - \delta^S)X = \delta\xi X - r^K V^S$ is itself unchanged. Plugging it back into the unchanged wealth accumulation equation for Neutrals $\dot{W}^N = (1-\alpha)(1-\delta)\xi\frac{X}{\theta} + \delta\xi\frac{X}{\theta} - \frac{r^K}{\theta}V^S$ yields the result.

[19]The model with constant prices de facto shuts down the effect through inflation and lower real interest rates. The expectation of a boom still produces a positive wealth effect which stimulates output. We can also introduce inflation as in Section 5. Then forward guidance gains extra kick by increasing inflation and reducing real interest rates.

with terminal condition

$$\zeta_{\sigma^+ + T} = 1.$$

The solution is

$$\zeta_t = e^{\int_t^{\sigma^+ + T}(\delta\theta - i_s)ds}.$$

By lowering interest rates, the central bank creates a temporary boom after the Poisson shock. This boom boosts the total value of assets immediately after the good Poisson shock from

$$\mu^+ \frac{X}{\theta}$$

to

$$\mu^+ \zeta_{\sigma^+} \frac{X}{\theta} > \mu^+ \frac{X}{\theta}.$$

Working backwards from expression into the equilibrium equations pre-Poisson shows that the only effect of this policy is to *increase* the interest rate r during the safety trap to

$$r = \delta\theta + (1-\delta)\,\theta \frac{\alpha - \frac{\rho\mu^-}{\xi}}{1 - \frac{\rho\mu^-}{\xi}} + \lambda^+ \frac{\frac{\zeta_{\sigma^+}\mu^+}{\xi} - \frac{\rho\mu^-}{\xi}}{1 - \frac{\rho\mu^-}{\xi}}.$$

This increase in the interest rate is such that the contemporaneous value of risky assets V^R (and hence the wealth of Neutrals W^N) is unchanged, despite the fact that its future value after a good Poisson shock has increased to $\frac{\zeta_{\sigma^+}\mu^+ X}{\theta} - V^S$. But there is no effect on output ξX. This can be seen most clearly by going back to the AS-AD equilibrium representation. Neither ζ_{σ^+} nor r affects aggregate demand. The future increase in risky asset values and the contemporaneous increase in risky interest rates are *orthogonal* to the safe-asset shortage problem. Since the policy leaves the supply of safe assets unchanged, it does not expand aggregate demand or output.[20]

This difference between the safety trap and standard liquidity trap models offers a possible rationalization of the "forward guidance puzzle", an expression that refers to the gap between

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[20]There is one caveat to this conclusion. We have assumed that prices are entirely rigid. If prices could adjust gradually over time in a forward looking manner, then forward guidance could regain some kick: A commitment to lower interest rates after the good Poisson shock could increase inflation while the economy is in a safety trap. This would lower the safe interest rate r^K and mitigate the recession. Note than when we model inflation in Section 5, we assume, motivated by a desire to capture downward wage rigidity, that inflation is determined by a myopic Philipps curve rather than an expectations-augmented Philipps curve, so that this effect does not arise.

A similar comment applies to the unconventional tax policies considered by Correia et al. (2012), which here could simply take the form of an increasing path of sales taxes—say through a sales tax holiday—which would create inflation in consumer prices and hence reduce r^K.

the limited effect of forward guidance on economic activity observed in the data, and the remarkable effectiveness of forward guidance in traditional liquidity trap models.[21]

A safety trap is addressed more directly by committing to provide support during bad rather than good times, as would be the case of a commitment to lower interest i_t rates after the bad Poisson shock.[22] By setting the nominal interest rate i_t below the natural interest rate $\delta\theta$ after the bad Poisson shock, monetary authorities stimulate the economy and inflate the value of safe assets to

$$\hat{V}^S = \rho\mu^{-\frac{\zeta_{\sigma^+}}{\theta}}X,$$

where

$$\zeta_{\sigma^+} = e^{\int_{\sigma^+}^{\sigma^++T}(\delta\theta - i_s)ds}.$$

This mitigates the recession in the safety trap by raising ξ to $\xi\zeta_{\sigma^+} > \xi$ (the analysis is almost identical to that of a monetary stimulus after the good Poisson shock explained above).[23,24]

However, it is natural to question whether monetary authorities would have the ability to lower interest rates in that state. If indeed the bad state happens to coincide with yet another safety or liquidity trap, monetary authorities could find themselves unable to deliver a lower interest rate. Perhaps a more realistic policy option would be a commitment by the authorities to buy up safe assets at an inflated price after the Poisson shocks—a form of government (central bank?) put. A commitment to buy up safe private assets at an inflated value $\omega\rho\mu^{-\frac{X}{\theta}} > \rho\mu^{-\frac{X}{\theta}}$ would mitigate the recession and increase the value of ξ to $\hat{\xi}$ where

$$\hat{\xi} = \omega\xi > \xi.$$

It could be carried out by monetary authorities but it does require spare fiscal capacity (in the form of taxes or seigniorage). This kind of public insurance policy can potentially play a crucial role in a safety trap.[25]

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[21] See e.g. Carlstrom et al. (2012) and Del Negro et al. (2013) for an exposition of this puzzle. McKay et al. (2015) offer a different but related rationalization in a model with precautionary savings where agents respond little to changes in future interest rates because their time horizons are endogenously limited.

[22] Another example is the OMT (outright monetary transactions) program established by the ECB in late 2012, which had an immediate impact on the Eurozone risk perception.

[23] Note we could just as well have used the model with public debt. The central banker's put works by increasing both the public and private sectors' ability to provide safe assets.

[24] Just like standard models of the liquidity trap, and to the extent that they are possible at all, these forms of policy commitments raise time-consistency issues: Their efficacy hinges on the ability of monetary authorities to carry out credible commitments.

[25] See, e.g., Caballero and Kurlat (2010) for a proposal to increase the resilience of the financial system in a shortage of safe assets environment. Also, see Brunnermeir et al (2012) for a related proposal in the

5 Inflation

In this section we relax the extreme sticky price rigidity assumption and show how safety traps trigger deflationary forces, which exacerbate the output drop. We also show that the policy multiplier (of increasing safe assets) is enhanced by its positive effect on inflation. Finally, as always, inflation also opens the door for expectations and targets to alleviate the trap by lowering real rates.

5.1 Safety Trap, Deflation, and Inflation Targets

We assume that prices cannot fall faster than at a certain pace:[26]

$$\pi_t \geq -(\kappa_0 + \kappa_1(1 - \xi_t)).$$

The more slack there is in the economy, the more prices can fall.

We impose that if there is slack in the economy, prices fall as fast as they can: $\xi_t < 1$ implies $\pi_t = -(\kappa_0 + \kappa_1(1 - \xi_t))$. We capture this requirement with the complementary slackness condition

$$[\pi_t + (\kappa_0 + \kappa_1(1 - \xi_t))](1 - \xi_t) = 0.$$

This is a Phillips curve which we plug in our model in conjunction with a (truncated) Taylor rule

$$i_t = \max\{0, r_t^{K,n} + \pi^* + \phi(\pi_t - \pi^*)\},$$

with $\phi > 1$, $\pi^* \geq 0$ where $r_t^{K,n}$ is the safe natural interest rate at t.

Now imagine that the real model is such that the safe interest rate is negative $r^{K,n} < 0$. The equilibrium equations are then

$$\max\{0, r^{K,n} + \pi^* + \phi(\pi - \pi^*)\} - \pi = \delta\theta - (1 - \delta)\theta \frac{\alpha - \frac{\rho\mu^-}{\xi}}{\frac{\rho\mu^-}{\xi}},$$

$$[\pi + (\kappa_0 + \kappa_1(1 - \xi))](1 - \xi) = 0,$$

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context of the current Euro crisis.

[26]Our motivation is to capture downward wage rigidities. This requires reinterpreting our varieties as varieties of labor which can then be transformed into final goods by competitive firms with a one-to-one technology. Our modeling strategy in this section borrows heavily from Eggertsson and Mehrota (2014).
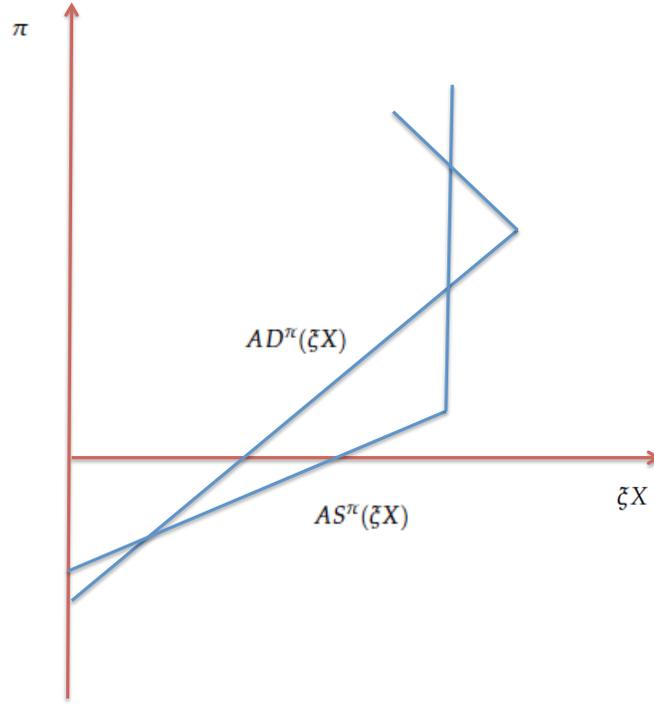
Figure 3: Aggregate supply and aggregate demand with inflation.

where

$$r^{K,n} = \delta\theta - (1-\delta)\,\theta\frac{\alpha - \rho\mu^-}{\rho\mu^-} < 0.$$

The first equation is simply the requirement that $i - \pi = r^K$, where we have used the Taylor rule to replace i and the equilibrium equations to replace r^K. The second equation is the Phillips curve. These two correspondences link inflation π and output ξX and can be interpreted as aggregate demand and aggregate supply respectively. We denote them by $\pi = AD^\pi(\xi X)$ and $\pi = AS^\pi(\xi X)$ to distinguish them from the aggregate supply and demand functions $AS(\xi X)$ and $AD(\xi X)$ that we introduced in the Keynesian cross equilibrium representation in Section 2. We discuss the link between these two representations in detail below.

We make the further assumptions that there is "enough" price rigidity so that the deflationary spiral (the feedback loop between inflation and output) remains bounded:

$$\kappa_1 < (1-\delta)\,\theta\frac{\alpha}{\rho\mu^-},$$

$$\kappa_0 + \kappa_1 < \theta.$$

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In this context, there is always a deflationary safety trap equilibrium (with $\pi \leq 0$ and $\xi < 1$). It is determined by the following equations

$$\pi = (1 - \delta)\,\theta\,\frac{\alpha - \frac{\rho\mu^-}{\xi}}{\frac{\rho\mu^-}{\xi}} - \delta\theta,$$

$$\pi = -(\kappa_0 + \kappa_1(1 - \xi)).$$

Both equations expresses π as increasing linear functions of ξ. The first one is aggregate demand $\pi = AD^\pi(\xi X)$, while the second is aggregate supply $\pi = AS^\pi(\xi X)$. With our assumptions above, aggregate demand is steeper than aggregate supply and both curves intersect exactly once on $[0, 1]$ at some value ξ. This is the "bad" safety trap equilibrium, analog to the one discussed in the previous sections.

But this model may also feature inflationary full employment equilibria (i.e., $\pi > 0$ and $\xi = 1$). To see this, note that the supply curve becomes vertical at $\xi = 1$ and the demand curve has a kink at the value of $\pi = \tilde{\pi}$ that solves $r^{K,n} + \pi^* + \phi(\pi - \pi^*) = 0$: the demand curve is an upward sloping function $\xi(\pi)$ for $\pi \leq \tilde{\pi}$ and downward sloping for $\pi > \tilde{\pi}$. As a result, there are either one or three intersections between the supply and demand curves. We have already seen the bad equilibrium; the other equilibria (if they exist) feature $\pi > 0$ and $\xi = 1$.

In an equilibrium with $\pi > 0$ and $\xi = 1$, inflation is determined by

$$\pi = \frac{1}{\phi - 1}[\phi\pi^* + r^{K,n}],$$

if $r^{K,n} + \pi^* + \phi(\pi - \pi^*) > 0$ (in which case $i = r^{K,n} + \pi^* + \phi(\pi - \pi^*)$) or by

$$\pi = -r^{K,n},$$

if $r^{K,n} + \pi^* + \phi(\pi - \pi^*) \leq 0$ (in which case $i = 0$). The condition for existence of both equilibria is the same and is given by

$$0 \leq r^{K,n} + \pi^*.$$

Hence the requirement is (roughly speaking) that the inflation target π^* be high enough.

To summarize, when $r^{K,n} < 0$, there is always a safety trap equilibrium with $\xi < 1$ and $\pi < 0$. If the inflation target is high enough ($0 \leq r^{K,n} + \pi^*$), then there are also two other

equilibria with $\xi = 1$ (and two different nominal interest rates $i = 0$ and $i > 0$ and two different inflation rates $\pi = -r^{K,n}$ and $\pi = \frac{1}{\phi-1}[\phi\pi^* + r^{K,n}] > -r^{K,n}$). In other words, the increase in the inflation target needs to be large enough to even have a chance to work. Note however that even when there is a "good" equilibrium with $\xi = 1$ and $i > 0$, the "bad" equilibrium with $\xi < 1$ and $i = 0$ still exists. The best a high inflation target can achieve is the possibility of a good equilibrium, not the elimination of the possibility of the bad equilibrium. Figure 3 provides a graphical illustration.

Let us now connect our discussion with the Keynesian cross equilibrium representation developed in Section 2. The difference here is that output ξX and inflation π are jointly determined by the new aggregate demand and aggregate supply relations $\pi = AD^\pi (\xi X)$ and $\pi = AS^\pi (\xi X)$. We can still use the Keynesian cross equilibrium representation to determine output once inflation has been solved out. Indeed we now have

$$AD (\xi X) = (1 - \alpha)(1 - \delta)\xi X + \delta \xi X + \left(\theta - r^K\right) V^S,$$

$$AS (\xi X) = \xi X,$$

with $r^K = i - \pi$ and $i = 0$. Higher inflation therefore helps to reduce safe interest rates, causing an upward shift in aggregate demand and resulting in a increase in output. In the bad safety trap equilibrium, this logic works in reverse, with the recession causing deflation, increasing safe real interest rates, reducing aggregate demand, further reducing output etc. In fact, adding an inflation channel to the model increases the value of the Keynesian multiplier to

$$\frac{1}{1 - \frac{\kappa_0 + \kappa_1}{\theta}} \frac{\xi X}{\theta V^S}$$

so that $d(\xi X) = \frac{1}{1 - \frac{\kappa_0 + \kappa_1}{\theta}} \frac{\xi X}{\theta V^S}(\theta - r^K)dV^S$. This is because increases in the value of safe assets increase aggregate demand, which increases output, increasing inflation, reducing the safe real interest rate, further increasing aggregate demand and output etc. ad infinitum. The more responsive is inflation to capacity utilization ξ (the larger is κ_1), the larger is the Keynesian multiplier. In other words, increased price flexibility is destabilizing as in DeLong and Summers (1986).

5.2 Public Debt, QE and Forward Guidance with Inflation

We now turn to the effects of the policies considered in Sections 3 and 4 in this extended model with inflation. We focus on the safety trap equilibrium and show that the main conclusions remain qualitatively unchanged but the power of QE is enhanced.

We start with public debt and QE. In a safety trap, $\xi < 1$ and $\pi < 0$ are determined by the intersection of the demand and supply curves:

$$\pi = (1-\delta)\,\theta\,\frac{\alpha - \frac{v^S\left(\frac{D}{X}\right)\mu^-}{\xi}}{\frac{v^S\left(\frac{D}{X}\right)\mu^-}{\xi}} - \delta\theta,$$

$$\pi = -(\kappa_0 + \kappa_1(1-\xi)).$$

Clearly, increasing D to $\hat{D} > D$ (and either rebating the proceeds to consumers or purchasing private risky assets) shifts the demand curve down which, given that it is steeper than the supply curve, results in an increase in ξ to $\hat{\xi} > \xi$. Note that the stimulus is stronger in this extended setup with endogenous inflation

$$\hat{\xi} > \frac{\hat{V}^S}{V^S}\xi,$$

with $V^S = v^S\left(\frac{D}{X}\right)$ and $\hat{V}^S = v^S\left(\frac{\hat{D}}{X}\right)$. This is because of a virtuous circle whereby additional safe assets increase output, which increases inflation (reduces deflation), which lowers the real interest, further stimulating output etc. ad infinitum, resulting in a larger value of the Keynesian multiplier as explained above.

We now turn to forward guidance. In order for monetary policy to be able to generate a boom after the good Poisson shock, we introduce the following modification of our setup. We assume that the Phillips curve only becomes vertical at a value $\bar{\xi} > 1$ so that we now have

$$[\pi_t + (\kappa_0 + \kappa_1(1-\xi_t))](\bar{\xi} - \xi_t) = 0.$$

Furthermore, we assume that $\kappa_0 = -\pi^*$ so that inflation is π^* if the economy is at capacity.

To capture forward guidance, we continue to assume that monetary policy follows the truncated Taylor rule

$$i_t = \max\{0, r_t^{K,n} + \pi^* + \phi(\pi_t - \pi^*)\}$$

before and after the bad Poisson shock, but we allow monetary policy to depart from this

rule after the good Poisson shock and follow instead (recall that $r_t^{K,n} = \delta\theta$ after the good Poisson shock)

$$i_t = \max\{0, \hat{\imath}_t + \pi^* + \phi(\pi_t - \pi^*)\}$$

where $\hat{\imath}_t < \delta\theta$ for $\sigma^+ \leq t \leq \sigma^+ + T$ and $\hat{\imath}_t = \delta\theta$ for $t > \sigma^+ + T$. Then output is above potential after the good Poisson shock. Capacity utilization satisfies the differential equation

$$\frac{\dot{\zeta}_t}{\zeta_t} = (\hat{\imath}_t - \delta\theta) + (\phi - 1)(\pi_t - \pi^*) \leq 0,$$

$$\pi_t - \pi^* + \kappa_1(1 - \zeta_t) = 0$$

with terminal condition

$$\zeta_{\sigma^+ + T} = 1.$$

Just as in Section 4, the solution features $\zeta_t > 1$ and $\pi_t \geq \pi^*$ for $\sigma^+ \leq t < \sigma^+ + T$. The rest of the analysis is identical and the conclusion is identical. Forward guidance stimulates the economy after the good Poisson shock, resulting in a boom and inflation above target. But this fails to stimulate the economy before the Poisson even when the economy is in a safety trap.

6 Securitization Externality

In this section, we endogenize the securitization capacity of the economy. We assume that by investing resources $j_t X dt$, a Neutral agent can increase $\rho(j_t)$ and, with it, increase the supply of safe assets (i.e., the share of the tree's revenue in the bad state of the world that is pledgable today).

We show in this extension that outside of a safety trap, the competitive equilibrium is constrained Pareto efficient, but that in a safety trap, it is constrained inefficient (there is underprovision of safe assets).

We trace back this inefficiency to a *securitization externality*. In a safety trap, private agents do not internalize the full social benefit of creating safe assets. More specifically, they do not take into account the stimulative effects of these assets, which creates a role for government intervention in the securitization market. This argument is distinct from the comparative advantage of the government in safe asset creation that we analyzed in Section 3, but it can also be used to support it if we associate the government extra-taxation power

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to the private sector's technology to increase safe assets (at a cost).

The equilibrium equations are now

$$r_t^K V_t^S = \delta_t^S X + \dot{V}_t^S,$$

$$r_t V_t^R = (\delta - \delta_t^S - j_t)X + \dot{V}_t^R,$$

$$\dot{W}_t^K = -\theta W_t^K + \alpha(1-\delta)X + r_t^K W_t^K,$$

$$\dot{W}_t^N = -\theta W_t^N + (1-\alpha)(1-\delta)X + r_t W_t^N,$$

$$V_t^S = \frac{\rho(j_t)\mu^- X}{\theta},$$

$$(r_t - r_t^K)\frac{\rho'(j_t)\mu^-}{\theta} = 1.$$

There are two differences with the baseline model of Section 2. First, the asset pricing equation for risky assets reflects the fact that dividends are reduced by securitization investment $j_t X$. Second, there is a new equation (the last one) which is simply the first order condition for securitization. This condition is intuitive: a Neutral managing a tree equates the marginal cost of increasing securititzation investment by $X dj_t$ between t and $t + dt$ to the marginal benefit of issuing $\frac{\rho'(j_t)\mu^-}{\theta}X dj_t$ additional safe assets on which he earns a spread $(r_t - r_t^K)$ between t and $t + dt$.

Apart from that, the analysis of the equilibrium is almost identical to that of the baseline model. In particular, and focusing on steady states, there is an unconstrained regime with $r = r^K$ and a constrained regime with $r > r^K$. In the constrained regime, we can enter a safety trap where $r^K = 0$ and output ξX is below potential with $\xi < 1$. We denote by j the associated level of investment. The details are omitted in the interest of space.

We now investigate the efficiency properties of the competitive equilibrium. To do so, it is convenient to introduce lump sum taxes that allow arbitrary redistribution within Knightians and within Neutrals but not across these two groups. Because they only redistribute within groups, these taxes do not change the characterization of the equilibrium.

Consider the steady state of the competitive equilibrium with Neutral wealth W^N, Knightian wealth W^K, interest rates r and r^K, and securitization j. We focus on the constrained regime throughout. We analyze first the case where prices are flexible. We then analyze the case with rigid prices. The first case corresponds to the real model where output is always at capacity. The corresponding planning problem is easier to analyze. Moreover, as long

as the natural safe real interest rate is positive, this flexible prices planning problem is a constrained version of the sticky prices planning problem, but the corresponding constraints are satisfied at the optimum of the latter, so that their solutions coincide. This is no longer the case when the natural safe real interest rate is negative.

We are interested in whether it is possible to generate a Pareto improvement by controlling securitization j_t (either through taxes or quantity restrictions) and setting it at a different level than would occur in a competitive equilibrium without government interventions in securitization. With the lump sum taxes mentioned above, this happens if and only if we can find processes $\hat{W}_t^K$, $\hat{W}_t^N$, $\hat{W}_t$, $\hat{V}_t^S$, $\hat{V}_t^R$, $\hat{V}_t$, $\hat{r}_t$, $\hat{r}_t^K$, $\hat{\delta}_t^S$ and $\hat{j}_t$ that verify all the equilibrium equations except the first order condition for $\hat{j}_t$, and such that $\hat{W}_t^N \geq W_t^N$ and $\hat{W}_t^K \geq W_t^K$ for all t and a strict inequality for some positive measure of t.

Therefore, the steady state of the competitive equilibrium is constrained Pareto efficient if and only if we can find Pareto weights $\lambda_t^N > 0$ and $\lambda_t^K > 0$ such that the solution of the following planning problem is such that $\hat{W}_t^N = W^N$, $\hat{W}^K = W^K$, and $\hat{j}_t = j$:

$$\max_{\hat{W}_t^K, \, \hat{W}_t^N, \, \hat{j}_t} \int_0^\infty \lambda_t^N \theta \hat{W}_t^N + \lambda_t^K \theta \hat{W}_t^K dt$$

subject to $\hat{W}_t^N = \frac{1}{\theta} - \frac{[\hat{j}_t + \rho(\hat{j}_t)\mu^-]}{\theta}$, $\hat{W}_t^K = \frac{\rho(\hat{j}_t)\mu^-}{\theta}$ and $\hat{W}_0^K = W^K$.

The solutionto this problem is

$$(\lambda_t^K - \lambda_t^N)\frac{\rho'(\hat{j}_t)\mu^-}{\theta} = \frac{\lambda_t^N}{\theta}.$$

Taking $\lambda_t^K = \lambda_t^N(1 + \frac{r - r^K}{\theta})$ and $\lambda_t^N > 0$ arbitrary such that $\int \lambda_t^N dt < \infty$, we can rewrite the solution as $(r_t - r_t^K)\frac{\rho'(\hat{j}_t)\mu^-}{\theta} = 1$ or equivalently $\hat{j}_t = j$. This shows that with flexible prices, the competitive equilibrium is constrained Pareto efficient.

Now suppose that prices are entirely rigid. The steady state of the competitive equilibrium (which may or may not feature a safety trap) is constrained Pareto efficient if and only if we can find Pareto weights $\lambda_t^N > 0$ and $\lambda_t^K > 0$ such that the solution of the following planning problem is such that $\hat{W}_t^N = W^N$, $\hat{W}^K = W^K$, $\hat{j}_t = j$ and $\hat{\xi}_t = \xi$:

$$\max_{\hat{W}_t^K, \, \hat{W}_t^N, \, \hat{j}_t, \, \hat{r}_t^K, \, \hat{\xi}_t} \int_0^\infty \lambda_t^N \theta \hat{W}_t^N + \lambda_t^K \theta \hat{W}_t^K dt$$

subject to $\hat{W}_t^N = \frac{\hat{\xi}_t}{\theta} - \frac{[\hat{j}_t + \rho(\hat{j}_t)\mu^-]}{\theta}$, $\hat{W}_t^K = \frac{\rho(\hat{j}_t)\mu^-}{\theta}$, $\hat{W}_0^K = W^K$, $\frac{d\hat{j}_t}{dt} = \frac{\theta}{\rho'(\hat{j}_t)\mu^-}[\alpha(1-\delta) - $

$\frac{\rho(\hat{\jmath}_t)\mu^-}{\hat{\xi}_t}]\hat{\xi}_t + \hat{r}_t^K \frac{\rho(\hat{\jmath}_t)}{\rho'(\hat{\jmath}_t)}$, $\hat{r}_t^K \geq 0$ and $\hat{\xi}_t \leq 1.$[27]

This is an optimal control problem with a state variable $\hat{\jmath}_t$. However as long as $\hat{r}_t^K > 0$, the corresponding costate variable $\hat{\nu}_t$ is equal to zero so that $\hat{\xi}_t = 1$ and the solution coincides with that of the flexible prices planning problem. Below we state the main results and omit some derivations. We refer the reader to the appendix for the details.

If the steady state of the competitive equilibrium does not feature a safety trap ($r^K > 0$ and $\xi = 1$), then taking $\lambda_t^K = \lambda_t^N(1 + \frac{r - r^K}{\theta})$ and $\lambda_t^N > 0$ arbitrary such that $\int \lambda_t^N dt < \infty$, the solution of the planning problem coincides with the the competitive equilibrium, showing that the competitive equilibrium is constrained Pareto efficient.

But if the steady state of the competitive equilibrium does feature a safety trap ($r^K = 0$ and $\xi < 1$), then it is not possible to find weights $\lambda_t^N > 0$ and $\lambda_t^K > 0$, such that the solution of the planning problem coincides with the competitive equilibrium. This shows that the competitive equilibrium is not constrained Pareto efficient.

We can take $\lambda_t^K = \lambda_t^N(1 + \frac{r - r^K}{\theta})$ with $\lambda_t^N = e^{-\phi t}$ with $\phi > 0$. As is usual in such problems, we renormalize the costate variable by multiplying it by $e^{\phi t}$. Then, as long as the allocation (including the costate renormalized costate) converge to a non-degenerate steady state when t goes to ∞, we have that $(r - r^K)\frac{\rho'(\hat{\jmath}_\infty)\mu^-}{\theta} = 1 + \theta(\sigma + \theta)\hat{\nu}_\infty < 1$. This implies that $\hat{\jmath}_\infty > j$, and by implication $\hat{\xi}_\infty = \frac{\rho(\hat{\jmath}_\infty)\mu^-}{\alpha(1-\delta)} > \xi$.

Moreover, one can show that $\hat{r}_\infty < r_\infty$ and $r_\infty^K = 0$ so that $\hat{r}_\infty - \hat{r}_\infty^K < r - r^K$. It follows that:

$$(\hat{r}_\infty - \hat{r}_\infty^K)\frac{\rho'(\hat{\jmath}_\infty)\mu^-}{\theta} < 1.$$

These results show that there is a positive externality from securitization. This is because securitization stimulates economic activity, which is not internalized by private agents, creating a role for government intervention in the securitization market.

The government could use taxes or quantity restrictions to encourage securitization. This should not be interpreted too narrowly. Indeed, in practice, one possible interpretation of j_t is as a proxy for the net worth of financial intermediaries. Forcing financial intermediaries to increase j_t could then be interpreted as forcing them to raise more fresh capital than they would do otherwise. This would improve welfare by stimulating the economy. Another

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[27]To undestand where the equation for $\frac{d\hat{\imath}_t}{dt}$ is coming from, combine the asset pricing equation for safe assets with the wealth accumulation equation for Neutrals to get $\frac{\rho(\hat{\jmath}_t)\mu^-}{\hat{\xi}_t} = \alpha(1 - \delta) + \hat{\delta}_t^S$. Solve out this equation for $\hat{\delta}_t^S$ and replace in the asset pricing equation for safe assets $\hat{r}_t^K \frac{\rho(\hat{\jmath}_t)\mu^-}{\theta} = \delta_t^S \hat{\xi}_t + \frac{\rho'(\hat{\jmath}_t)\mu^-}{\theta} \frac{d\hat{\jmath}_t}{dt}$. The equation in the text follows.

interpetation is that ρ is increased by monitoring, either private or public. Assume, for example, that public and private monitoring are perfect substitutes with associated costs $j_t^G \geq 0$ and $j_t^P \geq 0$ and that $\rho(j_t)$ is increasing in total monintoring $j_t = j_t^G + j_t^P$. With the additional assumption that public monitoring is financed by taxes on dividends, this model can be mapped exactly to the model in this section. Then a possible implementation of constrained Pareto efficient allocations in safety traps is an increase in public monitoring $j_t^G = \hat{j}_t$ and $j_t^P = 0$ (assuming that monitoring cannot be negative).[28]

7   Bubbles and Fiscal Capacity

The very low interest rates that characterize a safety trap raises the issue of whether speculative bubbles may emerge, and whether these can play a useful role through their wealth effect. We show that bubbles can indeed arise in safety traps, but that only the emergence of safe bubbles (as opposed to risky bubbles) can stimulate economic activity. This is because only safe bubbles alleviate the shortage of safe assets. We associate the latter to public debt, and in fact the existence of a bubbly-region is equivalent to an expansion of the fiscal capacity of the government.

7.1   Growth

We extend the model to allow for bubbles. It is well understood in the rational bubbles literature that the growth rate of the economy is a key determinant of the possibility and size of bubbles.

We generalize our model by allowing for an arbitrary growth rate $g > 0$. At every point in time, there is a mass X_t of trees. A mass $\dot{X}_t = gX_t$ of new trees are created, which are claims to a dividend of δ units of goods at every future date until a Poisson event occurs, at which point the dividend jumps permanently to $\delta\mu^+$ if the good Poisson shock takes place and to $\delta\mu^-$ if the bad Poisson shock takes place. For reasons that will appear clear below, we assume that new trees are initially endowed to Neutral newborns.[29] Endowments also

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[28]This latter interpretation can be seen as another rationalization for the sort of government interventions in securitization markets (public debt and QE) considered in Section 3, with public monitoring j_t^G playing a role similar to the role of taxes τ_t in Section 3. The difference is that public monitoring is costly in terms of resources while taxes are not because they are not distortive.

[29]If new trees are endowed in equal proportions to Knightians and Newborns, then bubbles do stimulate the economy in a safety trap because they reduce the value of the new trees endowed to Knigithian newborns

grow at the rate g.

We some abuse of notation, we suppress time indices throughout. Hence we write X, V^S, V^R, V, W^K, W^N, W for X_t, V_t^S, V_t^R, V_t, W_t^K, W_t^N, W_t. All these variables grow at rate g in equilibrium. We also write r^K, r, δ^S for r_t^K, r_t, δ_t^S. All these variables are constant in equilibrium.

We focus on the constrained regime where $W^K = V^S = \frac{\rho\mu^- X}{\theta}$ and $r > r^K$. This occurs as long as

$$\frac{\alpha - \rho\mu^-}{\rho\mu^-} > \frac{g}{(1-\delta)\,\theta}.$$

The equilibrium equations in the constrained regime are

$$r^K V^S = \delta^S X,$$

$$r V^R = (\delta - \delta^S) X,$$

$$g W^K = -\theta W^K + \alpha\,(1-\delta)\,X + r^K W^K,$$

$$g W^N = -\theta W^N + (1-\alpha)\,(1-\delta)\,X + g(V^S + V^R) + r W^N,$$

$$W^K + W^N = V^S + V^R,$$

$$W^K = V^S = \frac{\rho\mu^- X}{\theta}.$$

We then have

$$\delta^S = g\frac{\rho\mu^-}{\theta} + \delta\rho\mu^- - (1-\delta)\left(\alpha - \rho\mu^-\right),$$

$$r^K = g + \delta\theta - \theta\,(1-\delta)\,\frac{\alpha - \rho\mu^-}{\rho\mu^-},$$

$$r = g + \delta\theta - \frac{g}{1 - \rho\mu^-} + (1-\delta)\,\theta\frac{\alpha - \rho\mu^-}{1 - \rho\mu^-}.$$

Now suppose that we are in a safety trap where

$$g + \delta\theta - \theta\,(1-\delta)\,\frac{\alpha - \rho\mu^-}{\rho\mu^-} < 0,$$

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and hence reduce the growth rate of Knightian wealth. Endowing the new trees exclusively to Neutrals shuts down this somewhat artificial effect of bubbles on safe asset demand.

then as in Section 2, we have a recession determined by

$$0 = g + \delta\theta - (1 - \delta)\,\theta\,\frac{\alpha - \frac{\rho\mu^-}{\xi}}{\frac{\rho\mu^-}{\xi}}.$$

This formula shows that in the constrained regime with a scarcity of safe assets, the lower is the structural growth rate g of the economy, the lower is the natural safe interest rate, and if the latter is negative, the deeper is the recession (the lower is utilization capacity ξ). In this sense supply side secular stagnation can lead to or reinforce demand side secular stagnation—a powerful and perverse complementarity.

7.2   Bubbles

To ensure that bubbles are stationary, we allow for new bubbles to be created. Just like new trees, new bubbles are endowed to Neutral newborns. The total value of the bubble B_t grows at rate $\dot{B}_t = gB_t$ until the bad Poisson shock occurs, at which point the bubble drops to $B_t^- < B_t$ and then keeps growing at rate g, or until the good Poisson shock occurs, at which point the bubble jumps to $B_t^+ \geq B_t$. The value of the bubble B_t^+ after the good Poisson shock is irrelevant for our analysis. The bubble can be separated into a safe bubble $B_t^S = B_t^-$ with rate of return r_t^K and a risky bubble $B_t^R = B_t - B_t^-$ with rate of return r_t. New safe bubbles per unit of time are then given by $(g - r_t^K)B_t^S$, and new risky bubbles per unit of time by $(g - r_t)\,B_t^R$, which must both be positive for the equilibrium to exist. Again, we focus on balanced growth paths and suppress the dependence on time and write B, B^-, B^+, B^S and B^R for B_t, B_t^-, B_t^+, B_t^S and B_t^R.

After the bad Poisson shock, goods market clearing requires that

$$\theta(B^- + \frac{\delta\mu^- X}{r^-}) = \mu^- X.$$

This implies that the interest rate r^- is given by

$$r^- = \frac{\delta\mu^- X}{\frac{\mu^- X}{\theta} - B^-} = \frac{\delta\theta}{1 - \frac{\theta B^-}{\mu^- X^-}}.$$

The value of safe assets before the bad Poisson shock is therefore

$$V^S = B^- + \frac{\delta\rho\mu^- X}{r^-},$$

which can be rewritten as a function

$$V^S = \frac{\mu^- X^-}{\theta} v^{S,B}(\frac{B^-}{X})$$

with

$$v^{S,B}(\frac{B^-}{X^-}) = \rho + \frac{\theta}{\mu^-}\frac{B^-}{X^-}(1-\rho).$$

This expression makes clear that safe bubbles increase the value of safe assets because of the implicit assumption that there is no agency problem involved in the tranching of bubbles into a safe and a risky part.

As above, we focus on the constrained regime where $W^K = V^S$ and $r > \delta\theta > r^K$. The equilibrium equations in the constrained regime in the presence of bubbles are then

$$r^K(V^S - B^S) = \delta^S X,$$

$$r(V^R - B^R) = (\delta - \delta^S)X,$$

$$gW^K = -\theta W^K + \alpha(1-\delta)X + r^K W^K,$$

$$gW^N = -\theta W^N + (1-\alpha)(1-\delta)X + [g(V^S + V^R) - r^K B^S - rB^R] + rW^N,$$

$$W^K + W^N = V^S + V^R,$$

$$W^K = V^S = \frac{\mu^- X}{\theta} v^{S,B}(\frac{B^-}{X}).$$

We find the following expressions for the safe and risky interest rates:

$$r^K = g + \delta\theta - \theta(1-\delta)\frac{\alpha - \mu^- v^{S,B}(\frac{B^-}{X})}{\mu^- v^{S,B}(\frac{B^-}{X})},$$

$$r = \frac{(g+\theta)[1 - \mu^- v^{S,B}(\frac{B^-}{X})] - (1-\alpha)(1-\delta)\theta - g + r^K \frac{\theta B^-}{X}}{1 - \mu^- v^{S,B}(\frac{B^-}{X}) - \frac{\theta B^R}{X}}.$$

The condition for the constrained regime is that $r > r^K$ and the condition for bubbles to exist is that $\max\{r, r^-, r^+\} \le g$, where r^+ is the interest rate after the good Poisson shock $r^+ = \frac{\delta\mu^+ X}{\frac{\mu^+ X}{\theta} - B^+}$.

Now suppose that we are in a safety trap. Then we have a recession with $\xi < 1$ determined

exactly as in the bubbleless equilibrium analyzed in Section 7.1 above

$$0 = g + \delta\theta - \theta\left(1 - \delta\right) \frac{\alpha - \frac{v^{S,B}\left(\frac{B^-}{X}\right)\mu^-}{\xi}}{\frac{v^{S,B}\left(\frac{B^-}{X}\right)\mu^-}{\xi}}.$$

Clearly, output ξX is increasing in the size of the safe bubble $\frac{B^-}{X}$. A larger safe bubble $\frac{\hat{B}^-}{X} > \frac{B^-}{X}$ stimulates output to $\hat{\xi}X$ with

$$\hat{\xi} = \frac{\hat{V}^S}{V^S} > \xi,$$

where $\hat{V}^S = v^{S,B}(\frac{\hat{B}^-}{X})$ and $V^S = v^{S,B}(\frac{B^-}{X})$. However output is invariant to the size of the risky bubble $\frac{B^R}{X}$. This is because the safe bubbles increase the supply of safe assets V^S while the risky bubbles only increase the risky interest rate r (they fail to increase the supply of risky assets V^R since they perfectly crowd out other risky assets through an increase in the risky interest rate r). In other words, in terms of the AS-AD equilibrium representation, only safe bubbles increase aggregate demand, not risky bubbles.

The fact that risky bubbles have no effect on output in a safety trap formalizes some interesting observations in Summers (2013) that in secular stagnation environments, even large financial bubbles only seem to create moderate economic expansions.

7.3   Fiscal Capacity: Debt as a Safe Bubble

A natural interpretation of safe bubbles is that they are a form of government debt. To develop this idea, assume that the risky bubble is equal to zero, and interpret the safe bubble B^- as government debt $B^- = D$. When $r < g$, and by implication $\delta\theta < g$, the government can sustain a stable debt to output ratio without ever having to levy any taxes ($\tau = \tau^+ = \tau^- = 0$). The government can then increase the supply of safe assets by levering on the bubble in $V^S = \frac{\mu^- X}{\theta} v^{S,B}(\frac{B^-}{X})$, with a crowding out (of private safe assets) by public debt of:

$$\frac{\mu^-}{\theta}\left[1 - \frac{dv^{S,B}\left(\frac{B^-}{X}\right)}{d\left(\frac{B^-}{X}\right)}\right] = \rho < 1.$$

Finally, note that as the government issues more public debt, r^- and r^+ increase, which limits how much public debt can be issued without ever having to tax, namely $\frac{B^-}{X} \leq \frac{\mu^-}{\theta}(1 - \frac{\delta\theta}{g})$. In a safety trap, this gives the government some fiscal space to increase debt and stimulate

the economy.

8   A Liquidity Trap without Safe Asset Shortage

We conclude by highlighting the role of the safety component of our liquidity trap model. To do so, we develop a version with no safety premia, where the economy can be in a liquidity trap but where there is no specific shortage of safe assets. Rather, there is a shortage of assets in general which all carry the same zero interest rate.

We assume either that there are no Knightians ($\alpha = 0$) or that there are Knightians ($\alpha > 0$) but that the economy is in the unconstrained regime so that $r = r^K$. In both cases, the distinction between risky and safe assets is irrelevant, and the analysis is similar. We make one modification: The possibility of the bad shock is $\lambda^- > 0$ rather than studying the limit $\lambda^- \to 0$. This is necessary for the natural interest rate r to reach zero (and it cannot go below zero because of the zero lower bound on nominal interest rates). We maintain our focus on the limit $\lambda^+ \to 0$ for now.

In this context the equilibrium equations are

$$rV = \delta X + \lambda^- (\frac{\mu^- X}{\theta} - V),$$

$$0 = -\theta W + (1 - \delta)X + rW - \lambda^- (\frac{\mu^- X}{\theta} - V) + gV,$$

$$V = W.$$

As long as the zero bound is not binding, we have $V = W = \frac{X}{\theta}$ and

$$r = \delta\theta - \lambda^- \left(1 - \mu^-\right) > 0.$$

When the zero bound $r = 0$ binds, the economy enters a recession ($\xi < 1$) where ξ is determined by the requirement that $r = 0$:

$$0 = \delta\theta - \lambda^- \left(1 - \frac{\mu^-}{\xi}\right),$$

i.e.

$$\xi = \frac{\mu^-}{1 - \frac{\delta\theta}{\lambda^-}}.$$

The recession originates from a scarcity of assets (stores of value). It is more severe, the worse the expected bad shock (the lower is μ^-), the more likely is the bad shock (the higher λ^-), the higher the propensity to save (the lower θ), and the lower is the ability of the economy to create assets that capitalize future income (the lower is δ).

We can use this model to examine the effects of the same policies that we have considered in the context of the safety trap: balance sheet policies (QE) and monetary policy commitments (forward guidance). We can also examine the possibility and the consequences of bubbles.

Public Debt and QE. We start with public debt and QE. We introduce public debt in the model exactly as in Section 3. The key point is that public debt issuances and QE have *no* effect at all on the recession ξ. This irrelevance result relies on our assumption (made throughout the paper) that dividends are taxed while the endowment of newborns (wages) is not. As a result, public debt issuances and QE simply reshuffle the fraction of dividends that accrues to private asset holders and the fraction of dividends that is absorbed by taxes to pay interest on debt of various maturities. This assumption essentially renders our framework Ricardian, despite the fact that we have overlapping generations of agents.[30]

These conclusions about the irrelevance of public debt issuances and QE in this standard liquidity trap environment must be contrasted with those reached in Section 3 for safety traps. The effects of public debt issuances and QE in safety traps rely entirely on the (assumed) superior ability of the government to address a form of market incompleteness— the difficulty to isolate safe from risky assets. (Note that ρ does not show up anywhere in the equilibrium equations of this section. Thus, the power of policy comes not from the existence of a financial friction per se, but from the implication of the latter for private agents ability to tranch the full asset).

Forward Guidance. We now turn to monetary policy commitments. To do so, we introduce the possibility of a good shock as in Section 4. We temporarily (only for this section) assume that $\lambda^+ > 0$. In a liquidity trap we have that $r = 0$ and the recession is now determined by

$$0 = \delta\theta - \lambda^-(1 - \frac{\mu^-}{\xi}) - \lambda^+(1 - \frac{\mu^+}{\xi}),$$

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[30]If we allowed the endowments of newborns to be taxed, then public debt issuances and QE could have some non-Ricardian effects, depending on exactly how these taxes are levied, and hence affect economic activity in a liquidity trap. For example, Kocherlakota (2013) studies a non-Ricardian environment where issuing public debt can stimulate the economy in a liquidity trap.

i.e.

$$\xi = \frac{\frac{\lambda^-}{\lambda^- + \lambda^+}\mu^- + \frac{\lambda^+}{\lambda^- + \lambda^+}\mu^+}{1 - \frac{\delta\theta}{\lambda^- + \lambda^+}}.$$

Consider the following policy: After the good Poisson shock, which occurs at σ^+, the central bank stimulates the economy by setting the interest rate i_t below the natural interest rate $\delta\theta$ until $\sigma^+ + T$, at which point it reverts to setting the nominal interest rate equal to the natural interest rate $i = \delta\theta$. For $t > \sigma^+ + T$, output is equal to potential so that $\zeta_t = 1$. For $\sigma^+ \leq t \leq \sigma^+ + T$, output is above potential, and capacity utilization satisfies a simple differential equation

$$\frac{\dot{\zeta}_t}{\zeta_t} = i_t - \delta\theta \leq 0,$$

with terminal condition

$$\zeta_{\sigma^+ + T} = 1.$$

The solution is

$$\zeta_t = e^{\int_t^{\tau+T}(\delta\theta - i_s)ds}.$$

By lowering interest rates, the central bank creates a temporary boom after the good Poisson shock. This boom boosts the value of risky assets immediately after the good Poisson shock from

$$\mu^+ \frac{X}{\theta}$$

to

$$\mu^+ \zeta_{\sigma^+} \frac{X}{\theta} > \mu^+ \frac{X}{\theta}.$$

This policy alleviates the recession while the economy is in a liquidity trap, pushing ξ to $\hat{\xi}$ where

$$\hat{\xi} = \xi \frac{\frac{\lambda^-}{\lambda^- + \lambda^+}\mu^- + \frac{\lambda^+}{\lambda^- + \lambda^+}\zeta_\tau\mu^+}{\frac{\lambda^-}{\lambda^- + \lambda^+}\mu^- + \frac{\lambda^+}{\lambda^- + \lambda^+}\mu^+} > \xi.$$

Basically, committing to low interest rates after the good Poisson shock increases the value of assets while the economy is in the liquidity trap. This wealth effect increases demand and mitigates the recession. Forward guidance works by alleviating the asset shortage that is at the root of the recession.

Forward guidance trades off a future boom against a mitigation of the current recession and hence raises time-consistency issues. Because of the utility loss that comes with the boom, monetary authorities might be tempted to renege on their commitment to keep interest

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rates low when the time comes to deliver on this promise. Nevertheless, our main point here is that the effectiveness of forward guidance in this standard liquidity trap is to be contrasted with its relative ineffectiveness in the safety trap model developed in this paper.

Inflation. We could also introduce inflation just like in Section 5, with a Philipps curve

$$[\pi_t + (\kappa_0 + \kappa_1(1 - \xi_t))](\bar{\xi} - \xi_t) = 0,$$

and a truncated Taylor rule

$$i_t = \max\{0, r_t^n + \pi^* + \phi(\pi_t - \pi^*)\}.$$

This does not change our results for public debt and QE in a safety trap. Forward guidance gains an extra kick by increasing inflation, reducing real interest rates, further stimulating output and inflation, and so on.

Bubbles. To consider the possibility and consequences of bubbles, we could generalize the environment to allow for growth as in Section 7 and introduce safe and risky bubbles $B_t = B_t^R + B_t^S$. Such bubbles are possible as long as

$$\delta\theta < g.$$

Now suppose that we are in a liquidity trap. We have

$$r(V - B) = \delta\xi X + \lambda^-[\frac{\mu^- X}{\theta} - B^- - (V - B)],$$

$$\theta V = \xi X,$$

$$r = 0,$$

which yields

$$0 = \delta\xi X + \lambda^-[\frac{\mu^- X}{\theta} - B^- - (\frac{\xi X}{\theta} - B)],$$

or

$$\xi = \frac{\mu^- + \frac{\theta}{X}(B - B^-)}{1 - \frac{\delta\theta}{\lambda^-}} = \frac{\mu^- + \frac{\theta}{X}B^R}{1 - \frac{\delta\theta}{\lambda^-}}.$$

Hence, in contrast to a safety trap environment, in a standard liquidity trap environment it is only risky bubbles (and not safe bubbles) that stimulate output. This is because safe bubbles, in contrast with risky bubbles, entirely crowd out the future value of other assets,

and hence do not result in an increase of the present value of assets at a given interest rate $r = 0$.

9   Final Remarks

In this paper we provided a model that captures some of the most salient macroeconomic consequences and policy implications of a safety trap. Given the faster growth of safe-asset-consumer economies than that of safe-asset-producer economies as well as the aging of wealth-rich economies, absent major financial innovations, the shortage of safe assets is only likely to worsen over time, perhaps as a latent factor during booms but reemerging in full force during contractions. It is our conjecture that the shortage of safe assets will remain as a structural drag, lowering safe rates, increasing safety spreads, straining the financial system, and weakening the effectiveness of conventional monetary policy during contractions.

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A Appendix

A.1 Derivation of the optimal portfolios of Knightians and Neutrals

Recall that we focus on the period *before* the Poisson event and that we study the limit $\lambda^- \to 0$ and $\lambda^+ \to 0$. Between t and $t + dt$, three events can occur. First there can be no Poisson event. Second, there can be a bad Poisson event. Third, there can be a good Poisson event.

We denote by $q_{t,t+dt}$ the price at date t of an asset that pays one at date $t + dt$ if and only if the first event occurs. We denote by $q^-_{t,t+dt}$ the price at date t of an asset that pays one at date $t + dt$ if the second event occurs. And we denote by $q^+_{t,t+dt}$ the price at date t of an asset that pays one at date $t + dt$ if the third event occurs. Finally we denote by v_t the price of a tree, and we denote the price of the tree at date $t + dt$ by v_{t+dt} if the first event occurs, v^-_{t+dt} if the second event occurs, and v^+_{t+dt} if the third event occurs. In equilibrium $v_t = v_{t+dt} = \frac{X}{\theta}$, $v^-_{t+dt} = \mu^- \frac{X}{\theta}$ and $v^+_{t+dt} = \mu^+ \frac{X}{\theta}$. The equilibrium has $q_{t,t+dt} > 0$, $q^-_{t,t+dt} \geq 0$, and $q^+_{t,t+dt} = 0$. We define

$$1 + r^K_t dt = \frac{1}{q_{t,t+dt} + q^-_{t,t+dt}}$$

and

$$1 + r_t dt = \frac{1}{q_{t,t+dt}}$$

so that to a first order in dt, $q^-_{t,t+dt} = (r_t - r^K_t)dt$. The case $q^-_{t,t+dt} = 0$ corresponds to the unconstrained regime where $r_t = r^K_t$. The case $q^-_{t,t+dt} > 0$ corresponds to the constrained regime where $r_t > r^K_t$.

Consider a Knightian agent with wealth w^K_t with $\sigma_\theta \geq t$. Let β^K_t, β^{K-}_t, and β^{K+}_t be the numbers of each assets in the agent's portfolio, with $\beta^K_t q_{t,t+dt} + \beta^{K-}_t q^-_{t,t+dt} = w^K_t$ and $\beta^K_t \geq 0$, $\beta^{K-}_t \geq 0$ and $\beta^{K+}_t \geq 0$. His utility is given by

$$w^K_t \min\{\beta^K_t, \beta^{K-}_t, \beta^{K+}_t\}.$$

Clearly the solution is $\beta^K_t = \beta^{K-}_t = \beta^{K+}_t = \frac{w^K_t}{q_{t,t+dt} + q^-_{t,t+dt}}$, so that the Knightian agent chooses to invest his wealth between t and $t + dt$ in a safe asset.

Consider now a Neutral agent with wealth w^N_t with $\sigma_\theta \geq t$. Let β^N_t, β^{N-}_t, and β^{N+}_t

be the numbers of each asset in the agent's portfolio, and let β_t^{Nv} be the number of trees owned by the agent, with $\beta_t^N q_{t,t+dt} + \beta_t^{N-} q_{t,t+dt}^- + \beta_t^{Nv} v_t = w_t^N$ and $\beta_t^N \geq -\rho \beta_t^{Nv} v_{t+dt}$, $\beta_t^{N-} \geq -\rho \beta_t^{Nv} v_{t+dt}^-$, $\beta_t^{N+} \geq -\rho \beta_t^{Nv} v_{t+dt}^+$, and $\beta_t^{Nv} \geq 0$. His utility is given by

$$w_t^N[\beta_t^N + \beta_t^{Nv}(\delta X dt + v_{t+dt}]).$$

If we are in the unconstrained regime with $q_{t,t+dt}^- = 0$, a necessary condition for an interior solution in β_t^N and β_t^{Nv} (which must be the case in equilibrium) is $\frac{1}{q_{t,t+dt}} = \frac{\delta X dt + v_{t+dt}}{v_t}$ which we can rewrite as $r_t = \delta\theta$, a condition which is verified in our equilibrium. If we are in the constrained regime with $q_{t,t+dt}^- > 0$, then we must have $\beta_t^{N-} = -\rho \beta_t^{Nv} v_{t+dt}^-$ and a necessary condition for an interior solution in β_t^N and β_t^{Nv} (which must be the case in equilibrium) is $\frac{1}{q_{t,t+dt}} = \frac{\delta X dt + v_{t+dt}}{-\rho v_{t+dt}^- q_{t,t+dt}^- + v_t}$ which we can rewrite as $\frac{1}{q_{t,t+dt}} = \frac{\delta\theta dt + 1}{-\rho\mu^- q_{t,t+dt}^- + 1}$ or to a first order in dt, $r_t = \delta\theta + \rho\mu^-(r_t - r_t^K)$, a condition which is verified in our equilibrium.

A.2 Helicopter Money and Fiscal Capacity

One may wonder why not directly address the shortage of safe assets by printing money. Here we show that this is entirely equivalent to issuing public debt and hence it is subject to the same fiscal constraints.

Let us start backwards. In order to buy back the money stock after the bad Poisson shock, the government undertakes an open market operation immediately after the realization of the shock, swapping the extra supply of money $M^\varepsilon - M^{\varepsilon-}$ for debt D where

$$D = M^\varepsilon - M^{\varepsilon-},$$

and the interest payment associated to this debt is financed by a tax τ^- on the dividends of trees, where

$$D = \tau^- \mu^- \frac{X}{\theta}.$$

Consider what happens when the government issues additional money $\hat{M}^\varepsilon > M^\varepsilon = \frac{X}{\theta}$ in a safety trap, but maintains an adequate supply of money $M^{\varepsilon-} = \frac{\mu^- X}{\theta}$ after the bad Poisson shock. This stimulates output to

$$\hat{\xi} = \frac{\rho\mu^- + \frac{\varepsilon}{1-\varepsilon}\hat{M}^\varepsilon \frac{\theta}{X}}{\rho\mu^- + \frac{\varepsilon}{1-\varepsilon}}\xi > \xi.$$

This is exactly the same effect as that which would be achieved by issuing additional short-term debt in the amount $\varepsilon(\hat{M}^\varepsilon - M^\varepsilon)$, which is intuitive given that money and short-term debt are perfect substitutes at the zero lower bound. And exactly like this debt issuance policy, it requires that the government be able to increase taxes $\hat{\tau}^- > \tau^-$ after the bad Poisson shock where

$$\left(\hat{\tau}^- - \tau^-\right)\mu^- \frac{X}{\theta} = \varepsilon(\hat{M}^\varepsilon - M^\varepsilon).$$

Consider next what happens when the government issues additional money $\hat{M}^\varepsilon > M^\varepsilon = \frac{X}{\theta}$ in a safety trap, but keeps an excessive supply of money $\hat{M}^{\varepsilon-} > \frac{\mu^- X}{\theta}$ after the Poisson shock occurs (perhaps because it doesn't have the fiscal capacity to retire the extra money), while maintaining an interest rate of $\delta\theta$. In this case output is above potential at $\zeta\mu^- X$ where

$$\zeta = \hat{M}^{\varepsilon-}\frac{\theta}{\mu^- X}.$$

Hence the value of private safe assets is increased to

$$\frac{\rho\mu^-\zeta X\left(1-\varepsilon\right)}{\theta},$$

resulting in a mitigation of the recession before the Poisson shock when the economy is in a safety trap, increasing the value of ξ to $\hat{\xi}$ where

$$\hat{\xi} = \frac{\rho\mu^- \hat{M}^{\varepsilon-}\frac{\theta}{\mu^- X} + \frac{\varepsilon}{1-\varepsilon}\hat{M}^\varepsilon\frac{\theta}{X}}{\rho\mu^- + \frac{\varepsilon}{1-\varepsilon}}\xi > \frac{\rho\mu^- + \frac{\varepsilon}{1-\varepsilon}\hat{M}^\varepsilon\frac{\theta}{X}}{\rho\mu^- + \frac{\varepsilon}{1-\varepsilon}}\xi > \xi.$$

Thus issuing money while the economy is in a safety trap and not taking it away when the economy exits the safety trap further mitigates the recession associated with the safety trap. However, this extra effectiveness is not a free lunch, as it comes with the important cost of excessively stimulating the economy when it exits the safety trap.

A.3   Derivations for Section 6

We analyze the planning problem corresponding to the case of rigid prices. The first order conditions are

$$-\frac{d\hat{\nu}_t}{dt} = (\lambda_t^K - \lambda_t^N)\frac{\rho'(\hat{\jmath}_t)\mu^-}{\theta} - \lambda_t^N\frac{1}{\theta} + \nu_t\frac{d}{d\hat{\jmath}_t}\{\frac{\theta}{\rho'(\hat{\jmath}_t)\mu^-}[\alpha(1-\delta) - \frac{\rho(\hat{\jmath}_t)\mu^-}{\hat{\xi}_t}]\hat{\xi}_t\},$$

$$0 = \lambda_t^N \frac{1}{\theta} + \hat{\nu}_t \{ \frac{\theta}{\rho'(\hat{\jmath}_t)\mu^-}[\alpha(1-\delta) - \frac{\rho(\hat{\jmath}_t)\mu^-}{\hat{\xi}_t}]\} - \hat{\eta}_t,$$

$$\hat{\nu}_t \frac{\rho(\hat{\jmath}_t)}{\rho'(\hat{\jmath}_t)} + \hat{\gamma}_t = 0,$$

$$\lim_{t\to\infty} \hat{\nu}_t = 0,$$

$$\hat{\eta}_t(1 - \hat{\xi}_t) = 0,$$

$$\hat{\gamma}_t \hat{r}_t^K = 0.$$

Can we find $\lambda_t^K > 0$ and $\lambda_t^N > 0$ such that the solution coincides with the steady state of the competitive equilibrium? If the steady state of the competitive equilibrium does not feature a safety trap ($r^K > 0$ and $\xi = 1$), then taking $\lambda_t^K = \lambda_t^N[1 + \frac{r - r^K}{\theta}]$ and $\lambda_t^N > 0$ arbitrary such that $\int \lambda_t^N dt < \infty$, the solution of the planning problem coincides with the the competitive equilibrium, showing that the competitive equilibrium is constrained Pareto efficient.

But if the steady state of the competitive equilibrium does feature a safety trap ($r^K = 0$ and $\xi < 1$), then for any weights $\lambda_t^N > 0$ and $\lambda_t^K > 0$, the solution of the planning problem is different from the competitive equilibrium, showing that the competitive equilibrium is not constrained Pareto efficient. This would require $\hat{\eta}_t = 0$ and $\lambda_t^N = 0$, a contradiction. This shows that the competitive equilibrium is not constrained Pareto efficient.

Now continue to assume $r^K > 0$ and $\xi < 1$, and take $\lambda_t^K = \lambda_t^N[1 + \frac{r_t - r_t^K}{\theta}]$ and $\lambda_t^N > 0$, i.e. the Pareto weights that rationalize the competitive equilibrium outside of a safety trap. And take $\lambda_t^N = e^{-\phi t}$ so that the integrals converge. Renormalizing the Hamiltonian (and the multipliers), we get

$$\phi \hat{\nu}_t - \frac{d\hat{\nu}_t}{dt} = \frac{r - r^K}{\theta} \frac{\rho'(\hat{\jmath}_t)\mu^-}{\theta} - \frac{1}{\theta} + \hat{\nu}_t \frac{d}{d\hat{\jmath}_t} \{ \frac{\theta}{\rho'(\hat{\jmath}_t)\mu^-}[\alpha(1-\delta) - \frac{\rho(\hat{\jmath}_t)\mu^-}{\hat{\xi}_t}]\hat{\xi}_t\},$$

$$0 = \frac{1}{\theta} + \hat{\nu}_t \{ \frac{\theta}{\rho'(\hat{\jmath}_t)\mu^-}[\alpha(1-\delta) - \frac{\rho(\hat{\jmath}_t)\mu^-}{\hat{\xi}_t}]\} - \hat{\eta}_t,$$

$$\hat{\nu}_t \frac{\rho(\hat{\jmath}_t)}{\rho'(\hat{\jmath}_t)} + \hat{\gamma}_t = 0,$$

$$\lim_{t\to\infty} \hat{\nu}_t e^{-\phi t} = 0,$$

$$\hat{\eta}_t(1 - \hat{\xi}_t) = 0,$$

$$\hat{\gamma}_t \hat{r}_t^K = 0.$$

Assume that the solution converges to a non-degenerate steady state (including multipliers), then we necessarily have

$$\frac{\theta}{\rho'(\hat{j}_\infty)\mu^-}[\alpha(1-\delta) - \frac{\rho(\hat{j}_\infty)\mu^-}{\hat{\xi}_\infty}]\hat{\xi}_\infty = 0,$$

$$(r - r^K)\frac{\rho'(\hat{j}_\infty)\mu^-}{\theta} = 1 + \theta(\sigma + \theta)\hat{\nu}_\infty \leq 1,$$

where the last inequality is strict if $\nu_\infty < 0$ (which we can show holds by contradiction, as it would imply $j_\infty = j$, $\eta_\infty = \frac{1}{\theta} > 0$ and $\xi_\infty = \xi = 1$, which is impossible). This shows that in the long run, as long as we converge to a non-degenerate steady state (including multipliers), we have

$$\hat{j}_\infty > j.$$

Using

$$\hat{r}_\infty = \delta\theta + (1-\delta)\theta\frac{\alpha - \hat{j}_\infty - \frac{\rho(\hat{j}_\infty)\mu^-}{\hat{\xi}_\infty}}{1 - \hat{j}_\infty - \frac{\rho(\hat{j}_\infty)\mu^-}{\hat{\xi}_\infty}} < r$$

and

$$r^K_\infty = 0,$$

we can also show that

$$(\hat{r}_\infty - \hat{r}^K_\infty)\frac{\rho'(\hat{j}_\infty)\mu^-}{\theta} < (r - r^K)\frac{\rho'(\hat{j}_\infty)\mu^-}{\theta} < 1.$$

A.4   Derivations for Section 7

To find the expression for r, we can write

$$(g + \theta - r)\frac{X}{\theta}[1 - \frac{\theta V^S(\frac{\theta B^-}{\mu^- X^-})}{X}] = (1 - \alpha)(1 - \delta)X + g\frac{X}{\theta} - r^K B^S - rB^R,$$

$$(g + \theta)\frac{X}{\theta}[1 - \frac{\theta V^S(\frac{\theta B^-}{\mu^- X^-})}{X}] - (1 - \alpha)(1 - \delta)X - g\frac{X}{\theta} + r^K B^S = r\frac{X}{\theta}[[1 - \frac{V^S(\frac{\theta B^-}{\mu^- X^-})}{\frac{X}{\theta}}] - \frac{\theta B^R}{X}],$$

$$r = \frac{(g + \theta)[1 - \frac{\theta V^S(\frac{\theta B^-}{\mu^- X^-})}{X}] - (1 - \alpha)(1 - \delta)\theta - g + r^K\frac{\theta B^-}{X}}{[1 - \frac{V^S(\frac{\theta B^-}{\mu^- X^-})}{\frac{X}{\theta}}] - \frac{\theta B^R}{X}}.$$