Trade Dynamics with Sector-Specific Human Capital

Adam Guren David Hémous Morten Olsen*

May 2, 2013

Abstract

This paper develops a dynamic Heckscher Ohlin Samuelson model with sector-specific human capital and overlapping generations to characterize the dynamics and welfare implications of gradual labor market adjustment to trade. Our model is tractable enough to yield sharp analytic results, that complement and clarify an emerging empirical literature on labor market adjustment to trade. Existing generations that have accumulated specific human capital in one sector can switch sectors when the economy is hit by a trade shock. Nonetheless, the shock induces few workers to switch, generating a protracted adjustment that operates largely through the entry of new generations. This results in wages being tied to the sector of employment in the short-run but to the skill type in the long-run. Relative to a world with general human capital, welfare is improved for the skill group whose type-intensive sector shrinks, and policies intended to aid trade adjustment for affected workers can be counterproductive.

JEL: E24, F11, F16, J24

Keywords: sector-specific human capital, trade shock, transitional dynamics, worker mobility.

*Guren: Harvard University, guren@fas.harvard.edu. Hémous: INSEAD, david.hemous@insead.edu. Olsen: IESE Business School, University of Navarra, molsen@iese.edu. We thank Pol Antràs, Elhanan Helpman, Lawrence Katz, Nathan Nunn, and seminar participants at Harvad University, INSEAD, and Sciences-Po for helpful suggestions and remarks.

1 Introduction

The growth of North-South trade over the last 15 years—particularly due to the emergence of China—has sparked renewed interest in the consequences of inter-industry trade and its effects on labor reallocation and income inequality (Krugman, 2000, Autor et al., forthcoming, and Haskel et al., 2012). In addition to the effects of trade on relative factor rewards, concern has been raised over the welfare costs of protracted labor reallocation and of the idle/lost expertise for workers whose sector is hit by import competition. More generally, the dynamics of an economy's adjustment to trade shocks are critical to understanding the benefits and distributional consequences of both trade liberalizations and trade shocks.

Yet most models assume perfect factor mobility or complete immobility even though empirical results suggest that—owing to short- and medium-run adjustment costs—both assumptions are often too extreme for analyzing the impact of trade shocks on the labor market. Although recent work has increasingly incorporated labor market rigidities into classic models of trade, our theoretical understanding of the short-run dynamics and welfare implications of adjustment to trade remains limited.

We address this gap by adding sector-specific human capital to an otherwise classic Heckscher Ohlin Samuelson (HOS) model. This simple extension endogenously generates little immediate reallocation of labor in response to a trade shock and leads to a protracted transition, providing a better fit with the empirical findings. Moreover, the model has unexpected implications for welfare and policy. For instance, trade adjustment assistance that helps low-skill workers negatively affected by trade shock switch sectors can reduce the total real income of low-skill workers.

The model is an overlapping generations HOS model in which new workers of both low- and high-skill types enter the economy each period as old generations die. Both skill types are essential in both sectors, but the sectors differ in their skill intensities. Workers accumulate human capital that is specific to the sector of their employment. The empirical relevance of sector-specific human capital has been demonstrated most notably by Neal (1995), Parent (2000), and Kletzer (2001). Because our focus is on sector-specific human capital and sectoral reallocation we keep the neoclassical assumption of perfectly competitive markets and we consider an economy with homogeneous firms. This makes our analysis complementary with recent work emphasizing within-industry reallocation, most notably Helpman et al. (2010).

In steady state, workers never switch sectors and the model replicates the standard HOS model. Yet when prices and wages adjust in response to a trade shock, sector-specific human capital generates endogenous rigidities. Although all workers have the opportunity to switch sectors, not all do so and wages do not immediately equilibrate across sectors. Young workers with little accumulated sector-specific human capital find the higher relative wages of the expanding sector attractive enough to switch, whereas older workers with more accumulated human capital find it optimal to stay.

Our main finding is that most of the adjustment occurs not through immediate labor reallocation but rather through the entry of new generations of workers. Intuitively, the wage benefits of relocating to the expanding sector diminish as the economy adjusts to its new steady state, while human capital accumulated in the sector of previous employment is permanently idled if a worker switches. Consequently, even workers with a relatively small amount of accumulated specific human capital in the shrinking sector find it optimal not to switch. Technically, we use approximation methods to prove that the number of people who switch in response to a shock is second order in the price change whereas the length of adjustment is first order in the price change. Perhaps surprisingly, the transition can be slower when human capital accumulates faster. Given the small amount of labor reallocation that occurs upon impact, the immediate effect of a trade shock on factor rewards is tied to sector of employment and not (as in the standard model) to skill type. As the economy moves toward the new steady state, the standard Stolper–Samuelson result emerges whereby real wage changes are in fact tied to skill type.

To relate our model to the current debate over the consequences of imports of low-skill labor-intensive products, we consider a shock that lowers the price of goods produced by the low-skill–intensive sector. First, although sector specificity prevents some individual low-skill workers in the shrinking sector from taking advantage of the higher wages in the expanding sector, overall the slower adjustment benefits low-skill workers because factors of production are kept longer in the low-skill–intensive sector. Second, a policy that subsidizes low-skill workers who switch sectors would reduce the welfare of some of the low-skill workers who do not move by accelerating the transition.[1] This general equilibrium impact can be large enough to decrease the aggregate lifetime income of all low-skill workers alive at the time of the shock. When the subsidy is small enough, this case arises with constant elasticity of substitution (CES) production functions if the elasticity of substitution between high-skill and low-skill workers is the same in both sectors. This result continues to hold if one considers a retraining program that allows workers to keep part of their sector-specific human capital when switching sectors. Finally, there are distributional consequences across generations. For instance, low-skill workers in the high-skill intensive-sector who are old enough benefit from the decrease in the price of the low-skill-intensive good.

In two extensions we include physical capital and nonpecuniary sector preferences that generate bidirectional labor reallocation. Our results are robust in both cases. We also show that larger gross flows further delay the transition to the new steady state but cause more reallocation upon the shock's impact.

To illustrate the workings of our model, we calibrate a numerical version to data from the

[1] Because there are no inefficiencies in the economy, such a subsidy also reduces output.

United States. We divide US manufacturing into two sectors of similar size according to their skill intensity, and we simulate a trade shock that reduces the price of the low-skill sector's product by 1 percent. The numerical results show a relatively long transition: it takes 2.11 years for low-skill wages and 7.41 years for high-skill wages to be equalized again. Moreover, the number of workers switching sectors in response to the trade shock is very small: only low-skill (resp. high-skill) workers with experience less than 0.04 years (resp. 0.27 years) switch sectors. We also simulate a policy that subsidizes low-skill workers who switch sectors. A subsidy equal to 1 percent of the first-year income of a new low-skill worker decreases the total discounted lifetime income of all low-skill workers alive at the time of the trade shock because the negative general equilibrium impact of the subsidy is twice as large as the positive direct impact for the workers who receive it.

Our results relate to a large empirical literature, typically based on the HOS-model, on the distributional consequences of exposure to international trade, both in developing and developed countries. Goldberg and Pavcnik (2007) survey the literature on developing countries and document that in general labor market adjustments are sluggish and the trade liberalizations have not led to the reductions in income inequality predicted by the Stolper-Samuelson Theorem.[2] As for developed countries, Slaughter (2000) surveys an extensive literature of the 1990s on the role of international trade in explaining rising US inequality by correlating changes in the relative producer prices of low-skill intensive goods with relative wages of low-skilled workers as predicted by the Stolper-Samuelson theorem. He documents a limited support for the Stolper-Samuelson predictions especially in the 1970s, but argues that the methodology used is too limited to make firm conclusions. Wood (1995) argues that international trade plays a central role in the increase in inequality in the developed world. Though the limited labor mobility seems to contradict the central tenets of HOS theory and to undermine the empirical relevance of the HOS theory, our model suggests that a lack of labor reallocation and the presence of sector wage premia on impact are fully consistent with a HOS framework that incorporates rigidities. In fact, Robertson (2004) estimates the Stolper-Samuelson predictions emerge starting 3-5 years after a trade shock.[3] Our model provides some guidance for evaluating the time horizon at which Stolper-Samuelson effects might become important.

The model presented here also relates to a literature that examines the short-run dynamics of trade adjustment. Mayer (1974), Mussa (1978) and Neary (1978) analyze limited capital mobility, and Matsuyama (1992) analyzes sluggish labor market adjustment when workers are exogenously prevented from switching sectors. Yet only recently have efforts been made to in-

[2] In fact, in Brazil inequality increased during the period of liberalization, though as Helpman et al. (2012) demonstrate almost all of the change in inequality occurs within occupations, effects for which the HOS framework is silent.

[3] In addition, Robertson uses industry-specific tariff reductions. This addresses a potential bias in the estimation of wage effects from trade liberalizations as tariff reductions are often larger for low-skill intensive industries.

corporate sluggish labor adjustment into theoretical trade models. Most of these efforts – some of which include sector-specific human capital – focus on structurally estimated or calibrated models. For instance, Artuc et al. (2010) structurally estimate a dynamic rational expectations model of labor adjustment in which nonpecuniary idiosyncratic shocks in moving costs are the sole source of rigidities. Their model does not feature entering generations and sector-specific human capital, which (as we show) can endogenously generate rigidities for pecuniary reasons. Kambourov (2009) shows in a calibrated model that, in the presence of sector-specific human capital, firing costs reduce the benefits from trade liberalization. Closer to our work, Coşar (2011) calibrates a model with overlapping generations, sector-specific human capital, and job search, and Dix-Carneiro (2011) estimates a structural model with overlapping generations, sector-specific human capital, and switching costs. We focus on deriving sharp analytical predictions from a parsimonious dynamic HOS model in which the only impediment to labor mobility is sector-specific human capital. We discuss in more detail how these two papers compare and relate to our work in the main text.

The paper is organized as follows. In Section 2 we outline the model and derive the steady-state equilibrium. In Section 3 we analyze the transitional path, and in Section 4 we discuss the welfare implications of sector-specific human capital and its impact on the role of a trade adjustment policy. Section 5 presents two extensions featuring physical capital and nonpecuniary sector preferences. In Section 6, we calibrate and simulate the model, and Section 7 concludes. The main proofs and details on the calibration can be found in Appendix A, the remaining proofs are in Appendix B, which is available online.

2 The model

2.1 Production technology

We build a dynamic version of the standard, small open economy, HOS model. Time is continuous. In each period, two goods (indexed by $i = 1, 2$) are produced competitively using two factors of production: low-skill and high-skill human capital. We denote the stock of low-skill and high-skill human capital in sector i by L_i and H_i, with per-unit wages of w_i and v_i, respectively.[4] We assume that the production functions $Y_i = F_i(L_i, H_i)$ are concave, exhibit constant returns to scale (CRS), are twice differentiable, and have weakly positive cross partial derivatives ($\partial^2 F_i / \partial L_i \partial H_i \geq 0$). We use F_{ij} to denote the derivative of the production function in sector i with respect to factor j.

Sector 1 is assumed to be high-skill intensive at every wage ratio. Let good 1 be the numéraire and let the price of good 2 be p, which is set exogenously at the world price. Competitive labor markets imply that human capital is paid its marginal product, and competitive

[4] Although w_i and v_i technically denote the returns to a unit of human capital, we will abuse language slightly and refer to them as "low-skill wages" and "high-skill wages", respectively.

goods markets imply that prices equal marginal costs.

To this standard framework we add overlapping generations of workers who accumulate nontransferable sector-specific human capital in their sector of employment. The stock of specific human capital for an individual worker in a particular sector is given by the (weakly) increasing function $x_Z(a) \geq 0$, $Z \in \{L, H\}$, where a is the amount of time for which the worker has accumulated human capital in a given sector.[5] Note that the accumulation functions are different across types but the same across sectors. However, our results can be generalized without affecting any of the qualitative results to accumulation functions which are also different across sectors. The wage of a Z-skill worker of experience a in sector $i = 1, 2$ is thus $w_i x_Z(a)$. Complete nontransferability implies that a sector switcher must start over from $x_Z(0)$, although the worker could employ human capital accumulated in his previous sector if he moved back. Labor within a given skill type is perfectly substitutable, so the total stock of human capital is the sum of the human capital for all workers employed in the sector.

This setup is motivated by findings in the labor economics literature that sector-specific human capital is important. Neal (1995) uses US data from displaced worker surveys to compare workers who are displaced and switch sectors with those who do not. He finds that the semi-elasticity of the wage loss at displacement with respect to tenure is 2–3 times as high for industry switchers. Neal also shows that workers who switch jobs but stay in the same sector are rewarded for their previous tenure as if it were seniority within their new firm, providing further evidence that an important component of human capital is sector-specific. Similarly, Parent (2000) demonstrates that much of the measured return to firm seniority loads on industry tenure when it is included in a regression, and Kletzer (2001) shows that displaced workers' earnings losses rise with tenure and age but are lower for workers who stay in the same sector. In addition, Dix-Carneiro (2011) structurally estimates that the returns to seniority are imperfectly transferable across sectors in Brazil.[6]

Each overlapping generation lives for T periods, and the population grows at the rate of $\eta > 0$. Without loss of generality, we normalize the size of the population of low-skill and high-skill workers born at time $t = 0$ to 1 and $\overline{H}$ (respectively). For each type of worker $Z \in \{L, H\}$, we denote by $Z_i(t)$ the mass of human capital of workers of skill type Z who work in sector $i \in \{1, 2\}$. To solve for the model in a convenient form, we defined the normalized mass of human capital in sector $z_i(t)$ (with $z = l$ for low-skill workers and $z = h$ for high-skill workers) as the mass of human capital in sector i normalized by the size of the population

[5] There is some debate in labor economics over the relative importance of sector and firm-specific human capital. If we were to include firm-specific human capital, then issues of bargaining would arise. Since we deliberately adhere closely to assumptions of the original HOS model—including that of perfect competition in the labor market—we focus solely on sector-specific human capital. Yet the effects derived here would be found also in a model with firm-specific human capital.

[6] Kambourov and Manovskii (2009) find a substantial return to occupational tenure. To the extent that finely defined occupations differ across sectors, occupation-specific human capital can be reinterpreted as a form of sector-specific human capital.

of low-skill workers born at time t; thus, $z_1(t) = Z_1(t)/e^{\eta t}$. Let $n_Z(t)$ be the fraction of workers of type Z who enter sector 1 at time t. If nobody has moved during their lifetime, then $l_1(t) = \int_0^T n_L(t-\tau) e^{-\eta\tau} x_L(\tau) d\tau$ and $h_1(t) = \overline{H} \int_0^T n_H(t-\tau) e^{-\eta\tau} x_H(\tau) d\tau$ and with analogous expressions for sector 2. Competitive labor markets and CRS production functions imply that we can write wages as a function of normalized factors:

$$w_1(t) = F_{1L}(l_1(t), h_1(t)) \text{ and } w_2(t) = pF_{2L}(l_2(t), h_2(t)), \tag{1}$$

$$v_1(t) = F_{1H}(l_1(t), h_1(t)) \text{ and } v_2(t) = pF_{2H}(l_2(t), h_2(t)). \tag{2}$$

2.2 Preferences

In a natural extension of the static HOS model, all workers have identical time-separable preferences with discount rate δ. The lifetime utility of worker i at time t of age α with consumption profile $[C_{1i}(\tau), C_{2i}(\tau)]_{\tau=t}^{t+T-\alpha}$ is given by

$$\int_t^{t+T-\alpha} e^{-\delta(\tau-t)} u(C_{1i}(\tau), C_{2i}(\tau)) d\tau,$$

where $u(C_1, C_2)$ is assumed to be homogeneous of degree 1 (a worker is of age 0 when he enters the labor force). The consumption profile is indexed by individual i because it can, in principle, depend on the history of an individual's sectoral employment. Let $P(t)$ be the ideal price index associated with utility function $u(\cdot)$ and the prices of consumption goods in period t. Workers choose their sector of employment each period to maximize lifetime utility, which with income $[W_i(\tau)]_{\tau=t}^{t+T-\alpha}$ is

$$\int_t^{t+T-a} e^{-\delta(\tau-t)} \frac{W_i(\tau)}{P(\tau)} d\tau.$$

If prices are expected to be fixed over the lifetime horizon, then this choice is equivalent to choosing the sector with the highest discounted lifetime income at labor market entry.[7]

2.3 Steady state

As is standard, we consider only equilibria without specialization so that parameter values are such that production is active in both sectors. Because the skill accumulation functions are identical across sectors, incomplete specialization implies that steady-state wages are equalized across sectors at w^{ss} and v^{ss} for low-skill and high-skill workers, respectively. This, in turn, means that workers never switch sectors, as doing so would result in a loss of human capital without a higher wage per effective unit (therefore, the experience of a worker in his sector of employment is the same as his age). The total stock of normalized human capital is then

[7] Our assumption that the utility function is homogeneous of degree 1 simplifies the analysis by pinning down the interest rate to the pure time-discount rate δ. For small price changes, which are the focus of our analysis, this assumption is innocuous (provided there is a domestic assets market). See footnote 7.

$l^{\max} = \int_0^T e^{-\eta\tau} x_L(\tau)\, d\tau$ for low-skill workers and $h^{\max} = \overline{H} \int_0^T e^{-\eta\tau} x_H(\tau)\, d\tau$ for high-skill workers. Wage equalization across sectors implies that

$$w^{ss} = F_{1L}\left(n_L l^{\max}, n_H h^{\max}\right) = pF_{2L}\left(\left(1-n_L\right) l^{\max}, \left(1-n_H\right) h^{\max}\right), \tag{3}$$

$$v^{ss} = F_{1H}\left(n_L l^{\max}, n_H h^{\max}\right) = pF_{2H}\left(\left(1-n_L\right) l^{\max}, \left(1-n_H\right) h^{\max}\right). \tag{4}$$

This steady state of the normalized model is isomorphic to the HOS model. Hence, the Stolper–Samuelson theorem implies that, if p falls, then the new steady state will feature an increase in production in sector 1, an increase in the relative factor rewards to high-skill workers v/w, and an increase in the relative use of low-skill workers in both sectors. These results are stated formally as follows.

Lemma 1 *The steady state equilibrium described by* (3) *and* (4) *is isomorphic to the HOS model's equilibrium with $l^{\max}$ and $h^{\max}$ endowments of factors. In particular, the Stolper–Samuelson theorem is replicated such that, for a price change from p to p' with $p' < p$,*

$$\frac{w^{ss\prime} - w^{ss}}{w^{ss}} < \frac{p' - p}{p} < 0 < \frac{v^{ss\prime} - v^{ss}}{v^{ss}},$$

where $v^{ss\prime}$ and $w^{ss\prime}$ are the respective steady-state values of high- and low-skill wages for price p'.

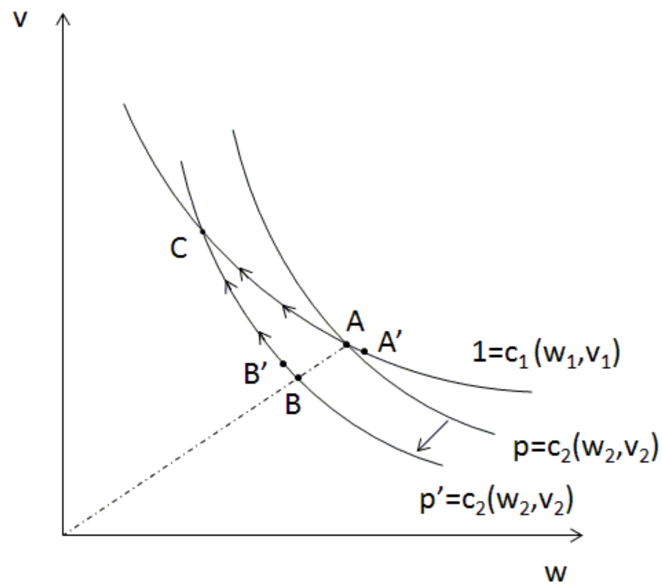
3 Transitional dynamics

3.1 Description

This paper's principal contribution consist of analyzing the transition between the two steady states. For expositional clarity, we consider an unexpected instantaneous and permanent downward shift (at time 0) in the price of the good produced by the low-skill–intensive sector (sector 2) from p to p'.

We conduct our analysis with the aid of Figure 1, which plots the isocost curves defined by $1 = c_1(w_1, v_1)$ and $p = c_2(w_2, v_2)$ as implied by the zero-profit conditions. The initial steady state is at point A, where $v_1 = v_2 = v^{ss}$ and $w_1 = w_2 = w^{ss}$. A well-known property of such cost curves is that the perpendicular vector at a given point gives the relative use of factors; the flatter slope of the vector associated with sector 2 reflects this sector's being more low-skill intensive than sector 1. A drop in prices in sector 2 to p' moves the isocost curve associated with sector 2's zero-profit condition southwest; hence, for a given allocation of labor, wages for both types in sector 2 decline proportionally. The eventual steady state is then given by point C, which implies higher relative wages for high-skill workers (i.e., the standard Stolper–Samuelson theorem described previously).

Figure 1: Transition paths along the cost curves



Notes: The wage paid to a unit of low-skill human capital is w and the wage paid to a unit of high skilled human capital is v. The economy is originally in a steady state at point A, at the intersection of the two loci along which price equals marginal cost in each sector. Sector 1 is high-skill intensive. A trade shock causes the price of the good produced by sector 2 to drop. On impact, wages in sector 2 shift to point B′(and wages in sector 1 to point A′). As new generations enter, the economy transitions along the two cost curves (as indicated by the arrows) to reach the new steady state at point C.

The following proposition characterizes the transitional phase for a small price change from p to p'.

Proposition 1 *For a (small) price drop from p to p' (with $dp \equiv p' - p < 0$), there exists an equilibrium fully described by the wage paths $(w_1(t), w_2(t), v_1(t), v_2(t))_{t=0}^{\infty}$ and the tuple $\{t_1, t_2, a_L, a_H\}$, where the following statements hold.*

- *There is a worker of age a_Z ($Z \in \{L, H\}$) in sector 2 who is indifferent between moving and not moving. All workers of type Z who are younger than a_Z in sector 2 move to sector 1; all older workers remain.*
- *Workers move only on impact of the trade shock at $t = 0$.*
- *The time at which wages are equalized first is given by $w_1(t_1) = w_2(t_1)$ and $v_1(t_2) = v_2(t_2)$. Moreover, $w_1(t) = w_2(t)$ for all $t \geq t_1$ and $v_1(t) = v_2(t)$ for all $t \geq t_2$.*
- *The equilibrium maximizes the present value of production.*

Proof. See Section A.1 in the Appendix. ■

The proof is given for marginal price changes and requires a positive population growth $\eta > 0$.[8] In Section 6, we establish that the same equilibrium exists for reasonable parameter values with nonmarginal price changes. We provide intuition for the structure of the equilibrium here.

First, the transition to the new steady state cannot be immediate. If it were, then a sufficient number of workers would switch sectors for wages to equalize across sectors. In that case, some workers would experience a loss in human capital without an offsetting increase in wages and so the move for them would not have been optimal. Second, there will be some sector switching at time 0. This is because the youngest workers have little human capital to lose by switching from sector 2 to sector 1, so a discrete difference in wages, (as implied by noninstantaneous adjustment) will lead some workers to move.

The equilibrium is efficient in the sense that it maximizes the present value of production from time 0 to infinite (if $\delta > \eta$, this present value of production is infinite, but the equilibrium still maximizes the present value of production from time 0 up until any time $t \geq T$).

These two points can also be illustrated using Figure 1. As already mentioned, point A gives the original steady state. During the transition, the economy will be described by two points, one for each sector on its corresponding isocost curve, until wages are again equalized. If there were no immediate reallocation, then wages in sector 2 would be given by point B and there would be a proportional drop of dp/p in both low- and high-skill wages in that sector. Instead, since there is some reallocation on impact, the economy jumps to point B′, which is "near" point B in a sense to be made precise shortly (although B′ is to the northwest of B in figure 1, the opposite positions are possible). Wages in sector 1 are described by a point A′ near A. The equilibrium wages in the two sectors eventually transition along each sector's respective isocost curve until point C, the new steady state.[9] By the logic of our preceding argument, the equilibrium point in sector 2 must always lie weakly southwest of the equilibrium point in sector 1; therefore, $v_1(t) \geq v_2(t)$ and $w_1(t) \geq w_2(t)$ at all points.

Define the ages of the low-skill and high-skill workers who are indifferent to moving as a_L and a_H, respectively. These ages are given by:

$$\int_0^{T-a_L} w_1(\tau)\, x_L(\tau)\, e^{-\delta\tau} d\tau = \int_0^{T-a_L} w_2(\tau)\, x_L(a_L+\tau)\, e^{-\delta\tau} d\tau, \quad (5)$$

$$\int_0^{T-a_H} v_1(\tau)\, x_H(\tau)\, e^{-\delta\tau} d\tau = \int_0^{T-a_H} v_2(\tau)\, x_H(a_H+\tau)\, e^{-\delta\tau} d\tau; \quad (6)$$

[8] A positive growth rate is required for mostly technical reasons. After $T - \max\{a_L, a_H\}$ time periods, when the first movers are dying out, the loss of human capital from dying generations takes a discrete jump for which new entering generations must compensate. Some population growth ensures that the entering generation is large enough (relative to the loss of human capital from dying generations) that no worker would want to switch sectors again; in our simulations, a growth rate of 2 percent was sufficient. Alternatively, some gross flow between the sectors (for instance because of stochastic nonpecuniary sector preferences of workers) could be used to circumvent the problem.

[9] Point C describes the wages of the new steady-state, but these wages are reached before all variables reach their new steady state levels. The Rybczynski theorem guarantees that, once wages are equalized, they will remain so until human capital reaches its maximum level and the new steady state is reached.

here the left-hand (resp., right-hand) side equals the lifetime earnings associated with switching to sector 1 and (resp., staying in sector 2). A worker older than the indifferent worker will lose more sector-2–specific capital and would have fewer years to enjoy the higher wages in sector 1; hence he will remain in sector 2. Similar logic implies that all workers younger than the indifferent worker will switch.

Because wages are not completely equalized on impact, all new workers will enter sector 1 for some time. Low-skill (resp. high-skill) workers will do so until $w_1(t) = w_2(t)$ (resp. $v_1(t) = v_2(t)$) which by definition occurs first at $t = t_1$ (resp. $t = t_2$). Without loss of generality, we consider parameter values for which $t_1 < t_2$. Doing so implies that the normalized stock of low-skill human capital in sector 1 at time t can be written as the sum of the mass of existing workers prior to time 0 (term 1), the mass of workers that move at time 0 (term 2), and newly born workers who all enter sector 1 until wages are equalized at time t_1 (term 3):

$$l_1(t) = \underbrace{n_L \int_t^T e^{-\eta\tau} x_L(\tau)\, d\tau}_{\text{term 1}} + \underbrace{e^{-\eta t}(1 - n_L)\, x_L(t) \int_0^{a_L} e^{-\eta\tau} d\tau}_{\text{term 2}} + \underbrace{\int_0^t e^{-\eta\tau} x_L(\tau)\, d\tau}_{\text{term 3}}, \tag{7}$$

for $0 \leq t \leq t_1$. The mass of low-skill human capital in sector 2 is given by the mass of stayers, $l_2 = (1 - n_L) \int_{t+a_L}^T e^{-\eta\tau} x_L(\tau)\, d\tau$. Equivalent expressions hold for high-skill workers whose wages are equalized at $t = t_2$.

The transition can therefore be split into three phases. In phase I ($t < t_1$) we have $w_1(t) > w_2(t)$ and $v_1(t) > v_2(t)$, and new workers enter only sector 1. In phase II ($t_1 \leq t < t_2$), $w_1(t) = w_2(t)$ and $v_1(t) > v_2(t)$; in this phase, low-skill workers enter both sectors (and so keep wages equal across sectors) while high-skill workers enter only sector 1. In phase III ($t_2 \leq t$) we have $w_1(t) = w_2(t)$ and $v_1(t) = v_2(t)$, and the allocation of entering workers across sectors ensures that wages remain equalized for both types.

It is worth noting that, in each phase, the model is isomorphic to a series of well-studied models in trade theory. During phase I, the model is isomorphic to a series of models with completely specific capital. During phase II, it is isomorphic to a series of Jones (1971) models. Finally, in phase III it is isomorphic to a series of HOS models.

3.2 Adjustment through new generations

To assess the extent to which the adjustment to the new steady state occurs by workers switching sectors versus new generations entering the workforce in one sector only, we use a Taylor expansion to obtain explicit expressions for the age of the indifferent workers as well as the time until wages are equalized. We formalize the results as follows.

Proposition 2 *Given the price change described in Proposition 1, the following statements hold.*

• *The times until equalization of wages t_1 and t_2 are of first order in dp. If $t_1 < t_2$, then t_1 is given by*

$$t_1 = \frac{w^{ss}}{(1-n_L)\,x_L(0)\,[w_{1L}+w_{2L}] + (1-n_H)\,x_H(0)\,\overline{H}\,[w_{1H}+w_{2H}]}\frac{dp}{p} + o\,(dp)\,, \tag{8}$$

where w_{iZ} denotes the derivative of low-skill wages in sector $i = 1,2$ with respect to labor type $Z \in \{L, H\}$.

• *The ages of the indifferent workers a_L and a_H are of second order in dp. If $t_1 < t_2$, then a_L is given by:*

$$a_L = -\frac{x_L(0)\,t_1}{2\int_0^T e^{-\delta\tau} x_L'(\tau)\,d\tau}\frac{dp}{p} + o\left(dp^2\right). \tag{9}$$

Proof. See Section A.1. ■

Similar expressions hold for the age a_H of the indifferent high-skill worker and the time t_2 at which high-skill wages are equalized; these expressions are derived in the Appendix.[10] Symmetric expressions hold when $t_1 > t_2$. The age of the indifferent worker is of second order whereas the time until wages are equalized again is of first order, which implies that most of the adjustment is driven by entry. In other words, whereas Matsuyama (1992) exogenously imposes that no workers can reallocate, we endogenously derive that very few will do so. The endogenous choice of reallocation has the additional benefit of enabling us to analyze policy designed to increase the number of workers that reallocate on impact (see Section 4.3).

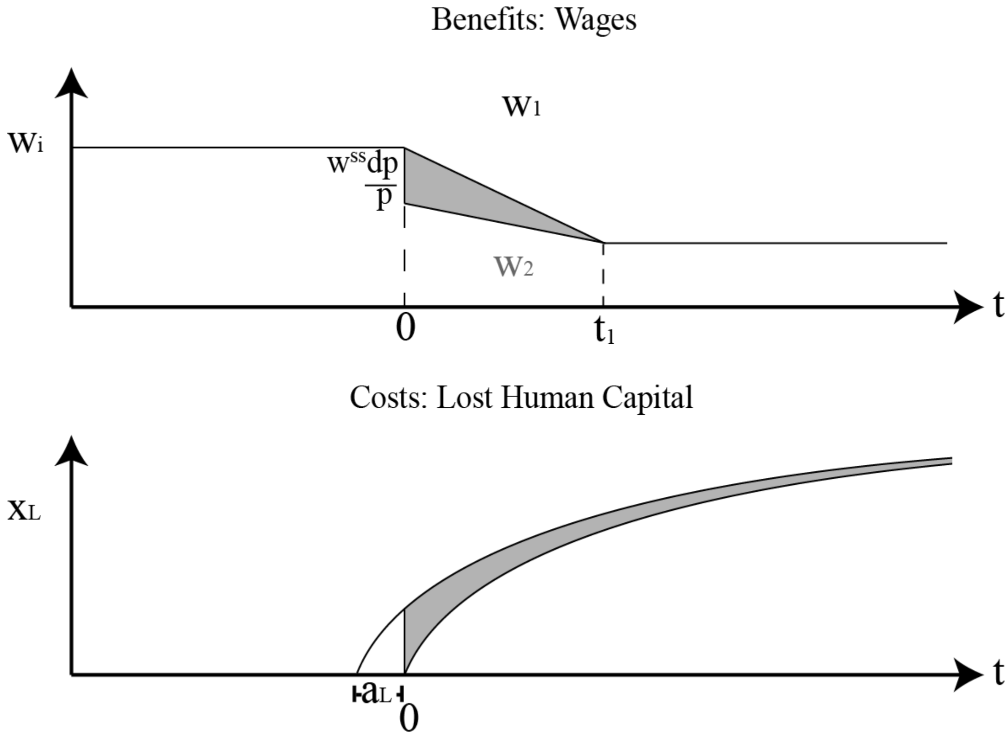
To understand the intuition behind this result, consider the indifferent low-skill worker's costs and benefits of moving to sector 1, which are plotted in Figure 2. The benefits are a higher wage per unit of human capital until time t_1, when wages are again equalized. Because the wage difference and the time until wages equalize are both first order in the price change, these benefits will be second order in that price change. The costs are a lower level of sector-specific human capital, and—since a worker has no incentive to switch back— they represent, in effect, a permanent loss of this human capital. The costs are thus first order in the age of the worker at the time of the trade shock. The age of the indifferent worker equates costs and benefits; therefore, whereas t_1 is of first order in the price drop, the age of the indifferent worker is second order in that price change.[11] The assumption of rational expectation plays a crucial role here: it is because workers correctly anticipate that the wage gap will quickly close that very few workers move. Alternate assumptions about expectations may make the mass of switchers first order.

Equation (8) follows from noting that the low-skill worker wage differential created on impact is given by $w^{ss}dp/p$ and that the denominator in (8) captures the effect on this wage

[10] With a more general homothetic function and domestic asset markets, the proposition still holds if one replaces δ with the steady-state interest rate in equation (9).

[11] The result that few people move does not depend on the permanent and unanticipated nature of the price shock we are considering. If the price change were perceived to be temporary then the incentive to move would be even lower.

Figure 2: Intuition For a_L Being of Second Order



Notes: The shaded area in the upper panel shows the benefits of switching (a higher wage in sector 1 relative to sector 2) while the shaded are in the lower panel shows the costs (forgone accumulated human capital). This explains why few individuals move on impact: the benefits are second order in the price change but the costs are first order. For the shaded areas to be equal, the number of switchers must be small.

differential of the inflow of new generations. The adjustment time depends on the share of people already allocated to sector 2, the production function, and the human capital accumulation function.

Perhaps surprisingly, more rapid specific human capital accumulation can have a negative effect on the speed of adjustment. To see this, consider the special case in which the high-skill and low-skill capital accumulation functions are proportional—that is, $x_H = \gamma x_L$ for γ a constant; then replace the low-skill capital accumulation function with some $\widehat{x}_L(a) \geq x_L(a)$, where $\widehat{x}_L(0) = x_L(0)$ and $\widehat{x}_L(T) = x_L(T)$, and replace the high-skill capital accumulation function with $\widehat{x}_H = \gamma \widehat{x}_L$. Such a function implies the same initial and terminal levels of human capital, but faster accumulation. Since $w_{1L} = F_{1LL}(n_L l^{\max}, n_H h^{\max})$ is the second derivative of a CRS production function, it is homogenous of degree -1. Hence, the change in capital accumulation function from (x_L, x_H) to $(\widehat{x}_L, \widehat{x}_H)$ will increase $l^{\max}$ and $h^{\max}$ proportionally and thereby decrease $w_{1L}, w_{2L}, w_{1H}, w_{2H}$ but *increase* the time until wages are equalized. Intuitively, faster accumulation of human capital translates into more effective factor endowment in the economy. With a concave production function, the implication is that the factor reallocation necessary to reequalize wages after the trade shock is larger. Because most of the adjustment occurs through entry, the transition period must be longer. This comparative static extends to t_2, the time at which wages of high-skill workers are equalized.[12]

Equation (9) results from noting that a first-order approximation to the accumulated wage difference is $1/2 \times t_1 w^{ss} dp/p$ per unit of human capital (which is close to $x_L(0)$ for the indifferent worker) while a first-order approximation to the loss is given by $a_L w^{ss} \int_0^T e^{-\delta\tau} x_L'(\tau)\, d\tau$ since, for every subsequent period, the worker's human capital will be lower by $x_L'(t)\, a_L$. Faster accumulation of human capital has an ambiguous effect on the number of people moving. As explained previously, it increases t_1 but also increases the denominator of (8) if the time discount rate is positive: $\int_0^T e^{-\delta\tau} \widehat{x}_L(\tau)'\, d\tau > \int_0^T e^{-\delta\tau} x_L(\tau)'\, d\tau$ if $\delta > 0$. When human capital increases faster, losing a given level of experience represents a bigger loss of human capital in the short run and a smaller loss in the long run; with positive discounting, the initial bigger loss matters more. Even so, the initial adjustment's share of total labor reallocation increases when the learning curve becomes steeper.[13]

Further insight into the transition process can be gained by considering the special case of CES production functions. When the elasticity of substitution, σ, is the same in both sectors, equation (8) can be written as

$$t_1 = \frac{-\sigma \frac{dp}{p}}{\frac{(1-n_L)x_L(0)}{l^{\max}}\left[\frac{\theta_{1H}}{n_L} + \frac{\theta_{2H}}{1-n_L}\right] - \frac{(1-n_H)x_H(0)\overline{H}}{h^{\max}}\left[\frac{\theta_{1H}}{n_H} + \frac{\theta_{2H}}{1-n_H}\right]} + o\left(dp\right), \tag{10}$$

where θ_{iH} is the factor share of high-skill workers in sector $i = 1, 2$. Consider the first term in the denominator. Each period, a fraction $(1 - n_L)x_L(0)/l^{max}$ of low-skill human capital is reallocated from sector 2 to sector 1 through the death of old and the entry of new generations. This reduces low-skill wages in sector 1 and increases them in sector 2. The relative importance of these two effects in closing the low-skill wage gap across sectors is captured by the relative importance of θ_{1H}/n_L and $\theta_{2H}/(1 - n_L)$. As is standard, the effect on low-skill wages from

[12] An analogous argument demonstrates that the length of the transition is decreasing in the rate of population growth η when the accumulation function is identical across sectors.

[13] To see this formally, note that the total amount of labor reallocation can be written as $dl_1^{ss} = \frac{v^{ss} - w^{ss}\frac{w_{1H}+w_{2H}}{v_{1H}+v_{2H}}}{w_{1L}+w_{2L}-\frac{(v_{1L}+v_{2L})(w_{1H}+w_{2H})}{v_{1H}+v_{2H}}}\frac{dp}{p}$ and that the mass of human capital moving upon shock is given by $a_L\left(1-n_L\right)x_L\left(0\right)$; hence the initial adjustment's share of total labor reallocation, χ_L, is given by

$$\chi_L = \frac{-x_L\left(0\right)}{2\int_0^T e^{-\delta\tau}x_L'\left(\tau\right)d\tau}\frac{w^{ss}\left(w_{1L}+w_{2L}-\frac{(v_{1L}+v_{2L})^2}{v_{1H}+v_{2H}}\right)}{\left(v^{ss}-w^{ss}\frac{w_{1H}+w_{2H}}{v_{1H}+v_{2H}}\right)\left(w_{1L}+w_{2L}+\frac{(1-n_H)x_H(0)}{(1-n_L)x_L(0)}\overline{H}\left(w_{1H}+w_{2H}\right)\right)}\frac{dp}{p}+o\left(dp\right).$$

The transformation of x_L into $\widehat{x}_L$ does not affect the second fraction, so χ_L increases. One can easily demonstrate that χ_H also increases when the learning curve becomes steeper.

changes in relative factors depends on the factor share of high skill workers, θ_{iH}, but the original allocation of low-skill workers is crucial: if n_L is close enough to 1 that most low-skill labor was initially allocated to sector 1, then the reallocation from sector 2 has little effect on sector-1 wages and so most of the adjustment in the wage gap comes from sector 2. The second term in the denominator captures the reallocation of high-skill workers. The interpretation is analogous except that the term is negative because the reallocation of high-skill workers to sector 1 widens the low-skill wage gap. Since the adjustment transpires through changes in factor intensity, the elasticity of substitution, σ, plays a crucial role. For higher σ, any change in factor intensity is associated with a smaller change in wages and thus, a longer adjustment time.

A large and recent empirical literature has argued against the importance of Heckscher–Ohlin forces by pointing out the strikingly little labor movement observed in the wake of trade liberalizations.[14] Such a critique strikes at the very core of Heckscher–Ohlin theory: although the presence of multiple countries and factors could explain other violations of its predictions, almost all factor proportions–based models predict substantial factor reallocation in response to a trade shock. Our model shows that the absence of short-run labor reallocation does not mean that Heckscher–Ohlin forces are unimportant. In addition, the model predicts that young workers are more responsive to trade shocks, which is consistent with the data. In Coşar (2011)'s calibration, when comparatively advantaged sectors begin to expand, many of the jobs are taken by younger workers. Similarly, Kletzer (2001) shows that workers with low tenure are considerably more likely to be displaced as a result of product competition from imports. In addition, workers displaced by import competition suffer larger income losses if they had higher tenure or if were then reemployed in a different sector. More generally, Kambourov (2009) find that industry mobility decline sharply with age in the United States: over the time period 1969-1997, they estimate a probability of switching industry at the 2-digit level of 21.3% per year for 23-28 year old high-skill workers (that is workers with at least some college education), but a probability of switching of 4% for 47-69 year old high-skill workers (for unskilled workers the corresponding numbers are 25% and 4.8%).

Note that the result that reallocation on impact is second order in the price change would apply in more general settings than that of a Heckscher–Ohlin model. Any model with overlapping generations and perfect foresight where reallocation costs are not proportional to the price change would feature this result.

[14] For instance, Wacziarg and Wallack (2004) consider 25 episodes of liberalization across many countries and find, over 2–5 year horizons, no evidence of labor reallocation at the 1-digit level and only weak evidence at the 3-digit level. Menezes-Filho and Muendler (2011) study Brazil's trade liberalization using linked employer–employee data and find that trade liberalization induces job displacement; however, exporters in comparative advantage sectors hire fewer workers in the short term, which results in a slow labor reallocation process. Goldberg and Pavcnik (2007) review the literature on trade liberalization in the developing world and show that, in almost every case, labor reallocation in the short run was extremely limited.

4 Welfare implications

The structure of the model allows us to conduct welfare analysis across sectors, skill types, and generations. We begin in Section 4.1 by analyzing the effects on real factor rewards. In Section 4.2 we turn to a welfare analysis that compares our model economy to one in which human capital is not sector specific. Finally, in Section 4.3, we consider the role of trade adjustment assistance.

4.1 Real factor rewards

Since few workers switch sectors in the immediate aftermath of the trade shock and since our model replicates the HOS model in the long run, the following corollary holds.

Corollary 1 *Consider a price change and equilibrium as described in Proposition 1. Then, for small price changes:*
- *in the short run, wages are tied to sector of occupation and move proportionally with price;*
- *in the long run, wages are tied to the type of skill and so the Stolper–Samuelson theorem applies.*

Consequently just after a price shock, the real wage of workers in sector 1 (including those who moved) will be higher than without the shock, irrespective of their skill level. Similarly, all workers in sector 2 will have a lower real wage than without the trade shock. Once wages are equalized, however, the Stolper–Samuelson result applies; therefore starting at some time before wages are equalized for both skill types, all high-skill workers will have a higher real wage than without the price shock and the opposite will hold for low-skill workers. Nevertheless, since the transition time is first-order and we are considering a small price change, the impact of the trade shock on welfare will not depart much from the Stolper–Samuelson theorem: only the very oldest low-skill workers in sector 1 will benefit from the price shock, and similarly only the very oldest high-skill workers in sector 2 will lose. In fact, several papers have reported that real wages do not follow Stolper–Samuelson's prediction in the short run (see the survey by Goldberg and Pavcnik, 2007). Our results show that this finding does not preclude the Stolper–Samuelson theorem from accounting accurately for the welfare consequences of trade liberalization in most of the population.

4.2 Comparison with a model of general human capital

In order to identify the winners and losers from the nontransferability of human capital, we compare our economy with one in which any accumulated human capital is general and can costlessly be utilized in both sectors. Such an economy features instantaneous adjustment to the new steady state and, since all human capital is fully transferable, the model is isomorphic to the standard HOS model at all times. We define the aggregate welfare of a generation as the sum of the discounted lifetime income of all its members. We compare the transition paths of the specific with the general human capital economies in the following proposition.

Proposition 3 *Following an unanticipated and permanent price shock, the aggregate welfare of a given generation born at time $t < 0$ is lower under sector-specific than under general human capital. The difference in total welfare is third order in the price change.*

In the context of a model with general human capital, the allocation of the sector-specific human capital model is equivalent to a misallocation of factors. This proposition stipulates that the resulting aggregate welfare loss is extremely small. The intuition is as follows. By the envelope theorem, a small factor misallocation has only a second-order effect on the total value of production, and the factors are misallocated for only a short period of time (until wages are equalized); hence the overall effect is of third order. The sector-specific human capital economy suffers also from the loss of effective units of human capital that results when workers who switch sectors must begin anew to accumulate human capital in their new sector. However, this loss is only fourth order because the mass of switchers is second order and each switcher will have accumulated only a second-order amount of human capital. Therefore, it is the indirect consequence—namely, the lack of mobility across sectors—that explains most of the cost of human capital's nontransferability.

Proposition 4 *For an unanticipated permanent price drop in the low-skill–intensive sector's product:*
- *all low-skill workers are better-off in the economy with sector-specific human capital than in the economy with general human capital; and*
- *all high-skill workers are worse-off in the sector-specific human capital economy than in the economy with general human capital.*

Proof. See Section A.2. ■

The transition created by the nontransferability of human capital "protects" low-skill workers.[15] The logic behind this result is based on Corollary 1: in the sector-specific human capital economy, wages for low-skill workers are at their lowest point in the long run (when the Stolper–Samuelson theorem applies); in a general human capital economy, however, the steady state is reached immediately. Proposition 4 suggests a qualification to the typical argument that slow adjustment is costly for those in a sector adversely affected by trade shocks. If we seek to make this argument for the low-skill workers, then sector-specific human capital and other

[15] For a specific generation of a given skill type, however, the difference between its welfare in the sector-specific human capital economy and its welfare in the general human capital economy is second order.

factor rigidities are insufficient. One would need to add other elements—such as unemployment and search frictions, from which this model abstracts—in order to generate a decline in the welfare of low-skill workers due to a slow transition.

4.3 Trade adjustment assistance policy

To build further on this point, we next consider the impact of a relocation program for low-skill workers willing to switch to sector 1; similar programs are studied in Coşar (2011) and Dix-Carneiro (2011). More specifically, we assume that the government distributes some income with a present value S to all low-skill workers who permanently switch from sector 2 to sector 1 at time $t = 0$.[16] The program is financed through lump-sum taxation on high-skill workers. Because there are no inefficiencies in our economy, such a program has a negative impact on output and so will hurt the economy as a whole. In fact, the program may even hurt the subgroup of low-skill workers, it may have been intended to assist.

Proposition 5 *Consider a small price change, and assume that the amount S given by the government is first order in the price change and small enough that full adjustment is not reached. Then the following statements hold.*
(i) *Workers who receive the payment are better-off than without the relocation program, workers who were already in sector 1 are worse-off, workers who stay in sector 2 are better- or worse-off depending on the parameters, and new low-skill workers are worse-off.*
(ii) *For a sufficiently small relocation program S, the aggregate present value of lifetime income of all low-skill workers alive at $t = 0$ is reduced if $(1 - n_L)\, w_{2L} - n_L w_{1L} > 0$ for $t_1 \leq t_2$ or if $(1 - n_L)\, v_{2L} - n_L v_{1L} < 0$ for $t_1 > t_2$; this condition is met for any CES production function in which the elasticity of substitution is the same for both sectors.*

Proof. See Section B.2. ■

The government program encourages more low-skill workers to move on impact, and the wages of low-skill workers are equalized more rapidly (t_1 is lower). The result is a reduction in w_1, which (a) hurts not only low-skill workers already in sector 1 but also incoming generations and (b) has an ambiguous impact on low-skill workers who remain in sector 2.[17] By contrast,

[16] For simplicity we assume that the government does not differentiate between low-skill workers of different ages, although doing so would not change our results. We assume that workers who switch back to sector 2 must reimburse the government; we could instead assume that payments are distributed over time in such a way that workers never want to move back. Finally, we assume that the income is distributed over time so that old workers don't move simply to benefit from the subsidy just before dying.

[17] More precisely: for as long as neither the high-skill nor the low-skill wages are equalized, the subsidy increases the wage of low-skill workers in sector 2 because the subsidy reduces the amount of low-skill workers in that sector *without* having a first-order effect on the number of high-skill workers. If low-skill workers' wages are the first to equalize, then the subsidy has no first-order effect on the low-skill wage after equalization. If high-skill workers' wages are the first to equalize, then the subsidy reduces the wage of low-skill workers in sector 2 after the equalization of high-skill wages, because in that case fewer high-skill workers enter sector 2.

workers who receive the payment necessarily benefit from the relocation program: it has a direct, first-order positive effect on their lifetime income but only an indirect, second-order negative impact via its general equilibrium effect. Even though the program is entirely financed by high-skill workers, there are parameter values for which the negative impact on sector-1 low-skill workers dominates the benefits enjoyed by movers and (possibly) sector-2 low-skill workers, so that low-skill workers as a whole lose.[18] Proposition 5 gives a simple condition for the subsidy to be counterproductive in this sense. Observe that the condition $(1 - n_L)\, w_{2L} - n_L w_{1L} > 0$ is equivalent to the elasticity of low-skill wages in sector 2 (with respect to low-skill human capital) being *lower* than the elasticity of low-skill wages in sector 1 (with respect to low-skill human capital)—where both are evaluated at the original steady state. This equivalence implies that, if wages are more sensitive to low-skill labor movement in sector 1 (where low-skill wages are dropping as low-skill human capital enters) than in sector 2, then the overall wage effect will be negative. This is the case with CES production functions as long as the elasticity of substitution is the same in both sectors. It should also be clear that, although this conclusion depends crucially on the relative skill intensities of the two sectors, it does *not* hinge on sector-specific human capital being the source of the rigid adjustment. In a Heckscher–Ohlin framework, any subsidy program that succeeds in more rapidly shifting the economy's resources to the skill-intensive sector entails a general equilibrium effect that is detrimental to the welfare of low-skill workers. More generally, this result demonstrates the importance of bearing in mind the long-term effects of trade shocks when assessing the implications of a subsidy for switching sectors. In the United States, where trade shocks are usually considered to be detrimental to low-skill workers in the long-run, such program might be counterproductive.

The intuition and relevance of our results are best seen by relating this paper to Dix-Carneiro (2011) and Coşar (2011), who both consider the welfare effects of a subsidy in more complex models using numerical methods. We see the parsimonious structure of our model as complementary.

Dix-Carneiro (2011) structurally estimates a dynamic Roy model with high- and low-skill labor, multiple sectors, capital, a labor supply decision, moving costs, and human capital that is imperfectly transferable across sectors. He matches gross flows across sectors by including idiosyncratic productivity and taste shocks. The model is estimated using matched employer–employee data from Brazil. Dix-Carneiro finds that sector-specific human capital (along with moving costs) plays an important role in explaining wage patterns. He simulates a trade shock of a 30 percent reduction in tariffs on the high-tech industry and finds a relatively fast labor reallocation, with 80 percent adjustment after only three years. He also finds that, although a switching subsidy reduces overall welfare by introducing distortions, it does increase the

[18] Low-skill workers' losses from subsidy implementation do not stem from the distortion that the subsidy creates: the distortion is second order, whereas the distributional effects emphasized in our discussion are first order.

welfare of low-skill workers.

In light of our results, it is important to understand the difference between this paper and that of Dix-Carneiro. First, he considers a relatively large price shock of 30 percent—although he finds a slower adjustment when he considers a 10 percent price shock. Second, the faster reallocation predicted by his model arises in part from the trade experiment being performed on the sector for which human capital is most easily transferred to other sectors. More importantly, Dix-Carneiro's conclusions about the moving subsidy actually supports our analysis. To see this, note that he considers a negative price shock to high-tech manufacturing—a sector that is relatively high-skill intensive—whereas we consider a shock to the low-skill–intensive sector. If we had considered a negative price shock to the skill-intensive sector, then the analogue of Proposition 5 would likewise have carried through; we would have found a negative effect on high-skill workers and a positive effect on low-skill workers, just as Dix-Carneiro does.[19]

Coşar (2011) builds an overlapping generations model, which features both search frictions and sector-specific human capital, and then calibrates this model using aggregate data from Brazil. The model's key feature is an externality whereby workers do not capture the full social benefit of their human capital; instead some of the returns to sector-specific skill acquisition is captured by future employers. This makes workers inefficiently reluctant to accept jobs, which slows down the adjustment period following a trade shock. Coşar's quantitative results suggest that sector-specific human capital is critical to explaining the sluggishness of the transition and the effects of trade liberalization on different generations, just as in our analysis. He also considers a sector-switching subsidy that is similar to ours and finds *positive* welfare effects for two reasons. First, he does not consider the distinction between low- and high-skill workers, so the unintended distributional consequences at the heart of our model are absent. Second, job search inefficiency can be partly overcome by the switching subsidy, which implies an increase in overall efficiency from such a subsidy. This rationale for a subsidy is not, however, specific to trade-displaced workers; it applies equally to any subsidy that encourages more search, a general point that is emphasized by Kletzer (2001).

The logic of Proposition 5 can be extended to a retraining program. Assume that the government can costlessly implement a retraining program that allows low-skill workers to transform their sector 2-human capital into sector 1-human capital up to some experience level $\overline{a}$ (that is a low-skill worker who has worked a years in sector 2 and 0 in sector 1 will start with an experience level equal to $\min(a, \overline{a})$ in sector 1). Further we assume that $\overline{a}$ is first-order in the price change but small enough that full adjustment for low-skill workers is not reached.

Remark 1 (i) Workers who were already in sector 1 and new low-skill workers are worse-off with the retraining programs, workers who stay in sector 2 may be better- or worse-off

[19] The two subsidy programs are not completely comparable because Dix-Carneiro allows both worker types to obtain the switching subsidy. Such an addition here would not qualitatively alter Proposition 5; it would simply encourage an even more rapid movement of resources to sector 1.

depending on parameters, and even workers who switch sectors with the retraining program but would not have without it are worse-off as a result of the retraining program for some parameters.
(ii) The aggregate present value of lifetime income of all low-skill workers alive at $t = 0$ is reduced if $(1 - n_L) w_{2L} - n_L w_{1L} > 0$ for $t_1 \leq t_2$ or if $(1 - n_L) v_{2L} - n_L v_{1L} < 0$ for $t_1 > t_2$.

Proof. See Section B.2. ■

In contrast to the subsidy, the retraining program increases the present value of production. In addition, beneficiaries from the program themselves might lose from it: in the limit where high-skill workers do not accumulate any human capital and where the retraining program allows to keep a large amount of accumulated human capital, workers's income under the retraining program is the same as their income under general human capital. The cautionary tale of considering long-term effects of trade adjustment policies therefore extends to this type of retraining program, which help low-skill workers acquire experience in a different sector.

5 Extensions

In this section we show that our results are robust to introducing an additional factor of production, or nonpecuniary sector preferences that generate bilateral flows of workers across sectors.

5.1 Physical capital

In this extension we allow for physical capital as a factor of production. The production functions are now given by

$$Y_i = F_i(L_i, H_i, K_i) \text{ for } i \in \{1, 2\},$$

where K_i is the physical capital employed in sector i. We assume that both functions are CRS with positive cross partial derivatives. The total amount of physical capital increases proportionally with population. We study two different cases, one where physical capital is entirely sector specific and one where it is fully transferable.

An equilibrium analogous to the one studied so far still exists in both cases. In particular, the number of workers switching sectors upon impact is second order whereas the time at which wages are equalized is first order. The Stolper–Samuelson theorem no longer holds, however. When physical capital is sector specific, the expressions derived in the case with no physical capital for the time t_1 of adjustment (8) and for the mass a_L of low-skill workers who switch sectors (9) are still valid, and so are the expressions derived for t_2 and a_H. When capital is fully transferable, these expressions become (respectively)

$$t_1 = \frac{\left(1-\frac{(w_{1K}+w_{2K})}{(r_{1K}+r_{2K})}\frac{r^{ss}}{w^{ss}}\right) w^{ss}\frac{dp}{p} + o\left(dp\right)}{\left(\begin{array}{c}\left(w_{1L}+w_{2L}-\frac{(w_{1K}+w_{2K})}{(r_{1K}+r_{2K})}\left(r_{1L}+r_{2L}\right)\right)\left(1-n_L\right)x_L\left(0\right)\\ +\left(w_{1H}+w_{2H}-\frac{(w_{1K}+w_{2K})}{(r_{1K}+r_{2K})}\left(r_{1H}+r_{2H}\right)\right)\left(1-n_H\right)x_H\left(0\right)\overline{H}\end{array}\right)}, \tag{11}$$

$$a_L = -\frac{\left(1-\frac{(w_{1K}+w_{2K})}{(r_{1K}+r_{2K})}\frac{r^{ss}}{w^{ss}}\right)t_1}{2\int_0^T x_L'\left(\tau\right)e^{-\delta\tau}d\tau}\frac{dp}{p} + o\left(dp^2\right). \tag{12}$$

Here r^{ss} denotes the rental rate of capital in steady state, and $r_{1X} = \frac{\partial^2 F_1}{\partial K_1 \partial X_1}$ and $r_{2X} = \frac{p\partial^2 F_2}{\partial K_2 \partial X_2}$ for $X \in \{L, H, K\}$ are the derivatives of the rental rate of capital in each sector.

When the allocation of capital *prior* to the price shock is identical in the sector-specific and fully transferable cases, comparing (8) and (11) shows that the transition time is longer when capital is transferable; also, comparing (9) and (12) shows that more low-skill workers switch sectors immediately after the trade shock (and it is similarly easy to show that more time is required for high-skill wages to equalize). It is intuitive that the transfer of capital from sector 2 to sector 1 increases the marginal product of the other factors, so more high-skill and low-skill workers need to reallocate in order to equalize wages. This directly increases the length of the transition period and the number of low-skill workers who switch sectors upon impact of the trade shock.[20]

5.2 Nonpecuniary sectoral preferences

There is empirical evidence that gross flows across sectors—that is, flows in both directions between sectors in the absence of trade shocks—significantly outweigh net flows (see e.g. Davis and Haltiwanger, 1992). In this section we augment our model with nonpecuniary sector preferences that generate gross flows and show that doing so does not significantly affect our qualitative results.

Workers of both types can be in one of three different "states": biased states 1 and 2 and a normal state 0. Workers in the biased state i receive a nonpecuniary benefit $b > 0$ per unit of time from working in sector i (these nonpecuniary benefit may originate, for instance, from geographical preferences if the two goods are produced in different places). We assume that workers in state 0 move to either of the biased states according to a Poisson process at a rate of $\lambda/2$ for each state; workers in a biased state (1 or 2) move to the normal state at a Poisson rate 1/2. To keep the problem tractable, we change the specification slightly. First, workers do not have a fixed lifetime and do not discount the future, but die at the Poisson rate δ. Second,

[20] Dix-Carneiro (2011) finds that the adjustment takes about the same amount of time whether physical capital is mobile or sector specific. However, his case is relatively specific because it involves full specialization in the home country when capital is mobile.

workers who switch sectors lose all the sector-specific human capital accumulated so far; for example, if a worker from sector 2 with an accumulated experience equal to a in that sector moves to sector 1 and later moves *back* to sector 2 then his sector-2 human capital reverts to nothing. There is no population growth, the flow of newborn workers is of size 1, and the accumulation functions are such that $x_Z(t)\,e^{-\delta t}$ has a finite integral over $[0,\infty)$ for $Z \in \{H, L\}$. We also assume that the economy is initially in a steady state in which the fraction of people in each state (normal or biased) is the same across ages. This assumption implies that the share of entrants in the normal state 0 is given by $1/(1+2\lambda)$ and the share in each of the biased states by $\lambda/(1+2\lambda)$. Finally, we assume that the nonpecuniary benefits b are large enough that—in the equilibrium considered—all workers in state 1 work in sector 1 and all workers in state 2 work in sector 2.

In steady state, workers in the normal state choose a sector at birth and remain in that same sector until they die or reach the biased state corresponding to the other sector. The share of workers switching sectors per unit of time is given by $\lambda/(2(1+2\lambda))$, so a higher λ is associated with larger gross flows. Following a small one-time, unanticipated price change, the equilibrium described in Proposition 1 still exists but with a_L and a_H now referring to the *experience* of the indifferent worker (instead of to the age, since the two can now differ). It is still the case that if workers in the normal state switch they will do so only at the time of the shock. The full solution to the problem is given in Appendix B.3. When $t_1 < t_2$, the time until wages are equalized is given by

$$t_1 = \frac{w^{ss}(1+2\lambda)}{(1-n_L)\,x_L(0)\,[w_{1L}+w_{2L}] + (1-n_H)\,x_H(0)\,\overline{H}\,[w_{1H}+w_{2H}]}\frac{dp}{p} + o(dp)\,. \tag{13}$$

This expression is identical to (8) except for the term $(1+2\lambda)$. The direct effect of a greater likelihood of moving from one sector to another for nonpecuniary reasons is an increase in the transition time t_1. The reason is that only workers in the normal state can respond to the incentive of a wage differential, and the share of such workers decreases with the rate λ at which workers leave the normal state.[21] This analysis also applies to the expression for t_2 and the case where $t_2 < t_1$.

When $t_1 < t_2$, the experience of the indifferent low-skill worker is given by

$$a_L = -\frac{x_L(0)\,t_1}{\int_0^\infty x'_L(s)\left(\left(1+\left(1+4\lambda^2\right)^{-\frac{1}{2}}\right)e^{-\lambda_1 s} + \left(1-\left(1+4\lambda^2\right)^{-\frac{1}{2}}\right)e^{-\lambda_2 s}\right)ds}\frac{dp}{p} + o\left(dp^2\right), \tag{14}$$

where $\lambda_1 \equiv \frac{1}{4}+\delta+\frac{1}{2}\lambda-\frac{1}{4}\left(1+4\lambda^2\right)^{\frac{1}{2}}$ and $\lambda_2 \equiv \frac{1}{4}+\delta+\frac{1}{2}\lambda+\frac{1}{4}\left(1+4\lambda^2\right)^{\frac{1}{2}}$. The denominator of this expression is decreasing in λ, so a higher probability of switching sectors plays a role similar

[21] There are indirect effects of λ as well which operate through its influence on the steady-state allocation n_L and n_H of newborn workers and on the steady-state mass of workers in each sector. Typically, an increase in λ means that a larger share of entrants needs to be allocated to the larger sector so as to compensate for the future sector switches caused by preference shocks.

to a higher death rate (or discount rate in the previous exercise): the loss of human capital resulting from a sector switch is less costly when the worker is likely to switch sectors again for nonpecuniary reasons. Thus the experience of an indifferent worker necessarily increases in the frequency of nonpecuniary shocks when t_1 is increasing in λ. Overall, this exercise suggests positive associations between larger gross flows (a large λ), slower transitions (higher t_1 and t_2), and more workers reallocating upon impact (greater a_L and a_H). Crucially, however, this extension does not alter our conclusion that the number of workers who switch sectors due to the price change is of second order in the price change.

In our set-up, an increase in λ is associated with a larger likelihood of being in a biased state, which leads to larger intersectoral gross flows. It is possible to dissociate the two by assuming that workers leave a biased state at a Poisson rate $\lambda/2$, in which case, the share of workers in each state is independent of λ in steady-state. Equation (13) holds but replacing $(1+2\lambda)$ by 3, so that an increase in λ only affects the time until which wages are equalized indirectly through its impact on n_L and n_H.

6 Simulations

6.1 Calibration

We supplement our analysis of a marginal price change by parameterizing the model to fit US data and then simulating the effects of a non-marginal price change. For the quantitative analysis, it is important to recognize that not all human capital is sector specific. Therefore, in this section, we incorporate accumulation of general human capital into the model by generalizing the human capital accumulation functions to $x_L(a,\theta)$ and $x_H(a,\theta)$, both of which are functions of a (a worker's experience in her current sector) and θ (her general experience). Both x_L and x_H are weakly increasing in both arguments. An equilibrium analogous to the one described previously exists and is characterized by a cutoff value of sector-specific human capital a_L (i.e., for low-skill workers) given by

$$\int_0^{T-a_L} w_1(\tau)\, x_L(\tau,\tau+a_L)\, e^{-\delta\tau} d\tau = \int_0^{T-a_L} w_2(\tau)\, x_L(\tau+a_L,\tau+a_L)\, e^{-\delta\tau} d\tau;$$

this equality replaces (5), with a similar expression for a_H (the cutoff for high-skill workers). Our previous analysis generalizes to this case (in particular, the accumulated human capital of the indifferent worker is, as before, of second order).[22]

[22] The analytical expressions given in (8) and (9) for the time until the wages of low-skill workers are equalized again (at t_1) and for the sector-specific human capital of the indifferent worker (a_L) must be updated as follows:

$$t_1 = \frac{w^{ss}}{(1-n_L)\, x_L(0,0)\,[w_{1L}+w_{2L}] + (1-n_H)\, x_H(0,0)\,\overline{H}\,[w_{1H}+w_{2H}]}\frac{dp}{p} + o(dp),$$

$$a_L = -\frac{x_L(0,0)\, t_1}{2\int_0^T e^{-\delta\tau}\frac{\partial x_L}{\partial a}(\tau,\tau)\, d\tau}\frac{dp}{p} + o\left(dp^2\right).$$

Before proceeding, we briefly describe our calibration (see Section A.3 for additional details). The production functions are assumed to be CES in both sectors with the same elasticity of substitution but different factor intensities, and the productivity of sector 1 is normalized to 1. So $F_1 = \left(\alpha_1 H_1^{\frac{\sigma-1}{\sigma}} + (1-\alpha_1) L_1^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$ and $F_2 = A_2 \left(\alpha_2 H_2^{\frac{\sigma-1}{\sigma}} + (1-\alpha_2) L_2^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$, with $\alpha_1 > \alpha_2$ since we have assumed that sector 1 is high-skill intensive. We choose $\sigma = 2$, which is in the range of commonly estimated values for the elasticity of substitution between high-skill and low-skill workers in the United States. To identify a high-skill–intensive and a low-skill–intensive sectors we rank each two-digit-SIC US manufacturing industry by the share of its total wage bill accruing to college-educated workers (which we identify as high-skill workers) based on data from EU KLEMS (March 2008 release) for 2000. We define the industries with the highest wage bill share for high-skill workers sector 1 and those with the lowest share as sector 2; the cutoff is chosen so that the two sectors generate approximately the same value added. The gross output of sector 1 is \$2.27 trillion (US) with an average wage bill share of 0.49 for high-skill workers, while the total output of sector 2 is \$2.19 trillion with an average high-skill wage bill share of 0.24. We obtain the ratio of high-skill workers to low-skill workers ($\bar{H}$) from the same data.

We assume that the flow utility function is Cobb–Douglas, $u(C_1, C_2) = C_1^{\nu} C_2^{1-\nu}$, and define a unit of good as what can be purchased for \$1 dollar in each sector (therefore $p = 1$ initially). We identify ν from the ratio of these two goods' consumption (we derive consumption in each sector by combining output data in EU KLEMS with trade data for 2000 by SIC from Schott, 2010). A unit of time corresponds to a year, and we fix T (the length of a lifetime of work) to be 40 years. Finally, we set the discount rate $\delta = 0.05$ and the growth rate $\eta = 0.02$.

For the human capital accumulation functions, we assume that $x_L(a, \theta)$ and $x_H(a, \theta)$ are proportional to each other; we base our parameterization on Neal (1995), who estimates wages both for displaced workers who stay in the same industry and for those who switch industries based on worker experience and tenure. These regressions allow us to define $x_L(a, \theta)$ and $x_H(a, \theta)$ up to the constants $x_L(0, 0)$ and $x_H(0, 0)$, as described in the Appendix. To derive α_1, α_2, A_2, $x_L(0, 0)$, and $x_H(0, 0)$, we use the following moments and constraints: the total share of high-skill workers in the wage bill of both sectors, the share of high-skill and of low-skill workers in sector 1 (both from EU-KLEMS), the output in each sector, and the constraints of wage equality for both high-skill and low-skill workers. There are eight moments and constraints for seven unknowns (the parameters plus the initial values for n_L and n_H), so we choose the parameters that come closest to fulfilling the constraints as measured by an equally weighted distance function. After calibration, in steady state the allocation of low- and high-skill workers to sector 1 is 41 and 66 percent, respectively.

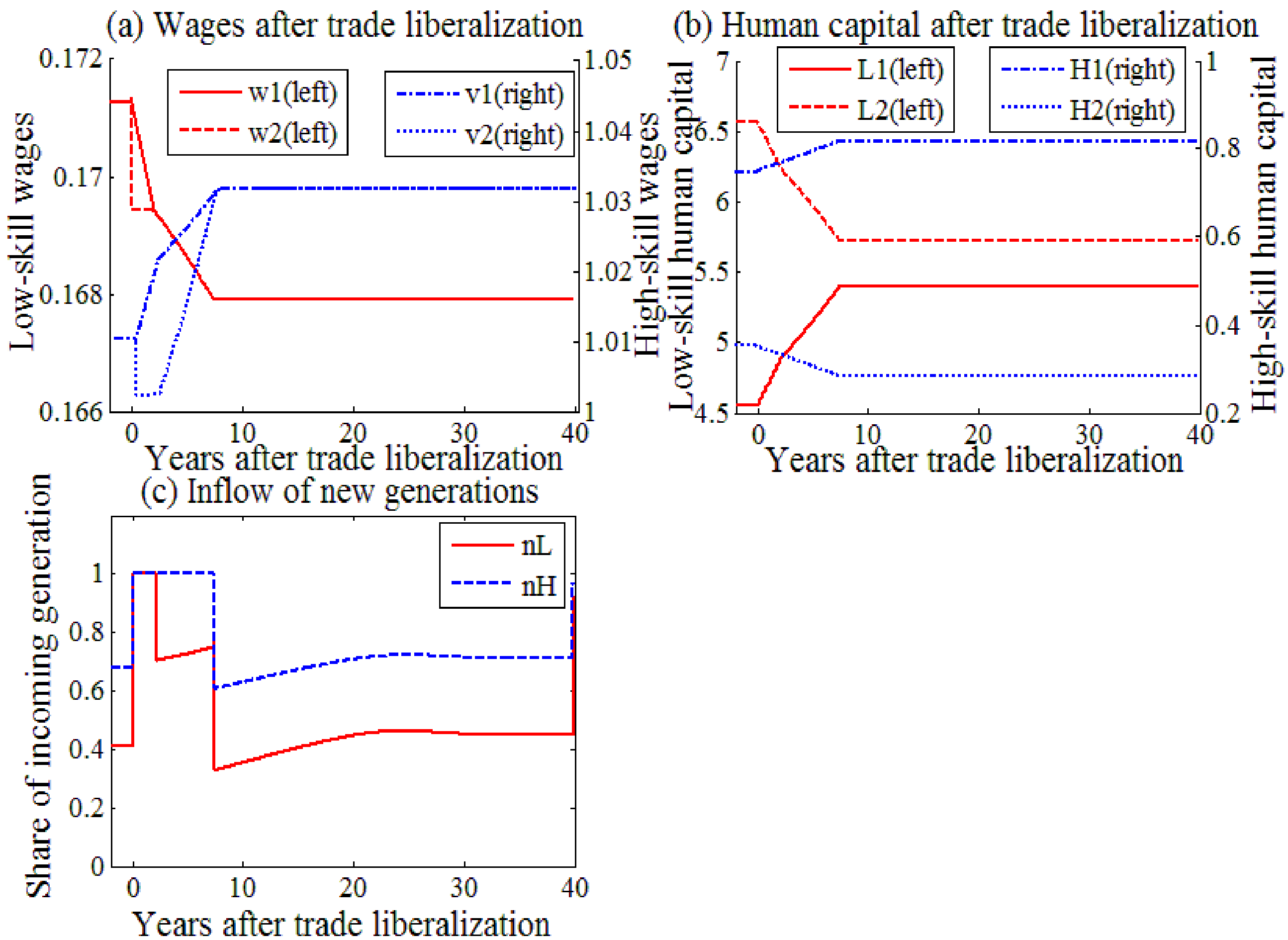


Figure 3: Simulated Transition After Trade Shock

6.2 Results

We simulate a trade shock by considering a 1 percent price drop in sector 2. The transition path is illustrated in Figure 3. Upon impact, low-skill and high-skill workers (respectively) younger than 0.041 and 0.27 years move from sector 2 to sector 1. Since only few workers switch and since these workers have hardly any sector-specific human capital, the initial loss of low-skill (respectively high-skill) human capital in sector 2 is only 0.1 (respectively 0.62) percent and the gain in sector 1 is only 0.14 (respectively 0.3) percent; the total loss in human capital of either type is even smaller (less then a hundredth of a percent). Because of this minuscule amount of immediate reallocation, wage changes are initially sector-dependent, as illustrated in Figure 3(a). New incoming generations will all enter sector 1, and human capital in this sector grows as shown in Figure 3(b). Low-skill wages are equalized after 2.11 years and high-skill wages after 7.41 years. Eventually, the total stock of human capital in sector 1 will have increased by 18.58 and 9.31 percent for (respectively) low-skill and high-skill workers.

Even for this small price change, the transition turns out to be quite long. The reason is that, with these calibrated parameters, factor intensity does not differ much across the two sectors; hence the mass of high-skill human capital that must move from sector 2 to sector 1 is fairly large. This is easily seen from equation (10), which gives the time until low-skill wages are equalized for the special case of a CES production technology. By assumption, the shape of the capital accumulation function is identical for the two skill types and so incoming generations represent the same share of human capital for high-skill and low-skill workers ($x_L(0)/l^{\max} = x_H(0)\overline{H}/h^{\max}$). If the original factor intensities are similar across sectors, then n_L and n_H are similar in size and the simultaneous reallocation of low-skill and high-skill human capital has little effect on factor intensity and hence on wages. As a result, the term in large parentheses in equation (10) turns out to be small, which makes the transition protracted.

To illustrate the accuracy of our approximation technique, Figure 4 shows the numerically computed values for t_1, t_2, a_L, and a_H and compares them to the values obtained with the Taylor approximations for different price changes. The approximation does quite well for a_L, t_1, and a_H. For t_2, the fit worsens significantly as the price change increases and t_2 becomes a relatively large number. This figure also shows that the times until which wages are equalized depend linearly on the size of the price change (for t_2, this holds only as long as the price change is sufficiently small) and that the age of the indifferent worker depends quadratically on the size of the price change.

We investigate numerically how the speed of human capital accumulation affects the transition by using accumulation functions $\widetilde{x}_Z(a, \theta) = x_Z(a/2, \theta/2)$, so that specific human capital is accumulated half as fast as in the baseline scenario. As theorized, we indeed find that the transition is slightly shorter: it takes 2 years for wages of low-skill workers to equalize (instead of 2.11) and 7.24 years (instead of 7.41) for those of high-skill workers. More workers switch sectors on impact—namely, low-skill workers whose age is less than 0.054 (instead of 0.041) and high-skill workers whose age is less than 0.32 (instead of 0.27). The discount rate is sufficiently large to ensure that, in the long-run, the smaller cost of switching dominates the impact of a shorter period of wage differences on the number of workers switching sectors.

Next we turn to the welfare implications. Because the immediate wage impact is tied to sector of employment, the oldest workers in sector 1 gain from the drop in sector 2-prices; in contrast, the effect of a trade shock on the youngest workers will be dominated by standard HOS effects in the long run. Figure 5 shows the welfare gains from trade liberalization for low-skill workers originally in sector 1 and 2 as a function of age and also for low-skill workers in the alternative model with general human capital. Welfare gains are expressed according to

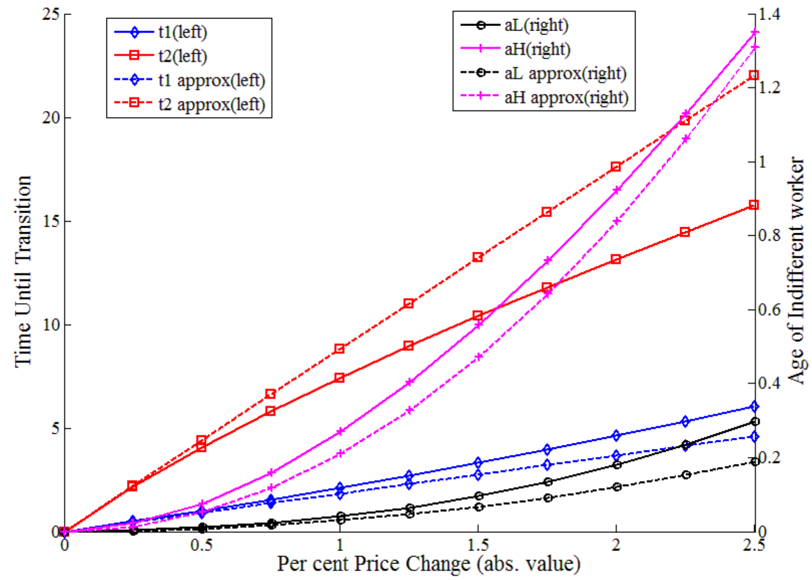


Figure 4: Times until wage equalization and ages of indifferent workers for different price changes (simulation results and approximation results)

the equivalent variation measure in percentage gains in consumption (that is the figure displays for each type of worker the percentage change of consumption without the trade shock which yields the same welfare as the trade shock). Although all low-skill workers are better-off than they would have been in a fully flexible world, only the oldest low-skill workers in sector 1 benefit from the trade shock. A corresponding graph for high-skill workers would demonstrate that, although all high-skill workers are worse-off than they would be in a world of complete capital mobility, only the oldest high-skill workers in sector 2 are hurt by the trade shock.

Finally, we examine the effects of a subsidy program (as in Section 4.3) that taxes high-skill workers to subsidize all low-skill workers who switch sectors; the subsidy has a net present value of up to 6 percent of a newly incoming low-skill worker's first year pay in sector 1. Figure 6 plots the total discounted income (including versus excluding the subsidy) of all low-skill workers (alive at $t = 0$) as a function of the subsidy's size. The total discounted income is computed at $t = 0$ and expressed as percentage gains relative to the no-subsidy case. In line with the results reported in Section 4, a subsidy initially lowers the overall income of low-skill workers by encouraging a faster adjustment toward the high-skill–intensive sector 1; for instance, a subsidy of 1 percent of the first-year pay reduces the total lifetime income of all low-skill workers (who are alive at time 0) by 0.003 percent. This subsidy itself amounts to 0.0035 percent of the lifetime income of all low-skill workers, so the negative general equilibrium effect exceeds the subsidy's direct positive effect by a factor of 2. For higher subsidy levels, however,

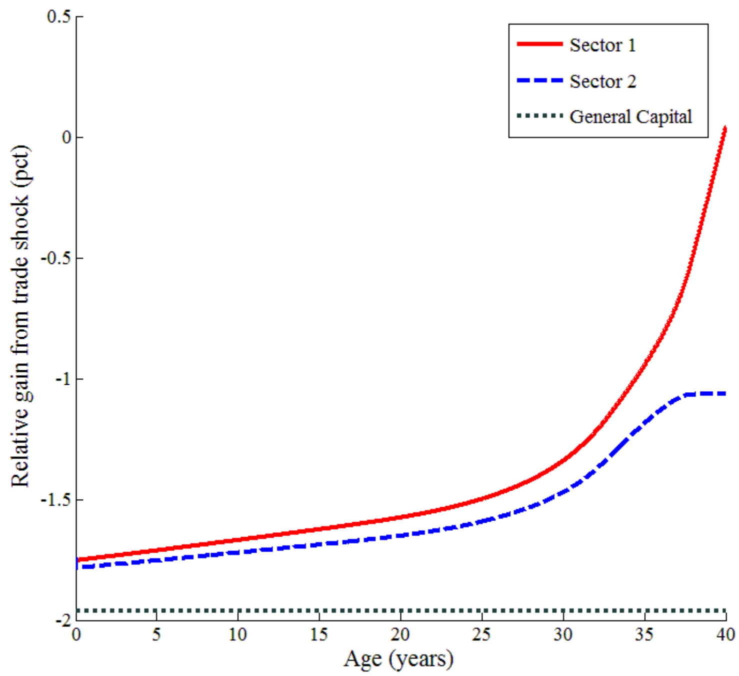


Figure 5: Welfare gains (in percentage of consumption) to low-skill workers, by age, who are alive at time 0.

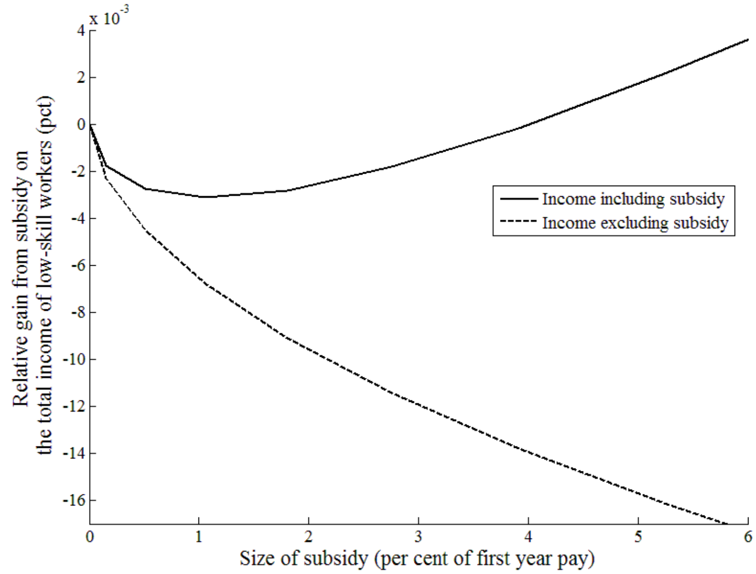


Figure 6: Effect of a switching sector subsidy for low-skill workers on discounted lifetime income of low-skill workers alive at time 0. The subsidy is financed by high-skill workers.

the direct effect of the subsidy outweighs the negative general equilibrium effect.

7 Conclusion

The mobility of factors is crucial for understanding the welfare effects of trade shocks. This paper adds sector-specific human capital to an otherwise classic dynamic HOS model. Our model replicates the standard HOS model in steady state but it differs during the transitional phase. In particular, our model endogenously generate (i) low levels of worker reallocation immediately after a trade shock and (ii) a protracted period of adjustment before wages reequilibrate. This model replicates previous empirical findings that mostly young people switch sectors, that most of the adjustment happens through the entry of new generations, and that wages after the shock are tied to sector and not to skill type. We also show that the model's qualitative predictions are unaltered by the inclusion of either general human capital, physical capital or gross flows from nonpecuniary sectoral preferences.

Moreover, the model delivers some surprising results: a faster accumulation of human capital can make the transition *longer* and all low-skill workers *benefit* from rigid labor markets when the low-skill–intensive sector is hit by a negative price shock. This last point is crucial for assessing the welfare effects of a subsidy for switching sectors. Although such a moving subsidy directly benefits the low-skill workers who receive it, the subsequent faster reallocation of resources to the high-skill–intensive sector hurts low-skill workers as a group. For a wide range of parameter values, this latter effect dominates; thus the total income of low-skill workers is actually reduced by the introduction of a subsidy intended specifically to assist some of them. The intuitions of the model are illustrated with a simulation, which reveals that the approximation methods are accurate for discrete price changes.

This paper employs the analytical approach of extending a classic trade model to analyze the interaction between labor market rigidities and international trade; it therefore complements the literature that studies similar questions using estimated and numerically solved models. The analytical approach has several virtues: it allows for greater generality, it provides linkages to well-understood models in trade, and it is easy to extend. These advantages open up several paths for future research. For instance, one could add firms and firm-specific human capital (or occupations and occupation-specific capital) to the model as a means for assessing the importance of the type of human capital specificity. Such models might build on the literature that addresses firm heterogeneity and the intraindustry reallocations triggered by trade liberalization and perhaps could illuminate why little interindustry labor reallocation is observed in the short run despite substantial intraindustry reallocations.

References

Artuc, E., Chaudhuri, S., McLaren, J., 2010. Trade shocks and labor adjustment: A structural empirical approach. The American Economic Review 103 (3), 1008 –1045.

Autor, D., Dorn, D., Hanson, G., Forthcoming. The china syndrome: Local labor market effects of import competition in the united states. American Economic Review.

Cosar, A. K., 2011. Adjusting to trade liberalization: Reallocation and labor market policies, mimeo.

Davis, S., Haltiwanger, J., 1992. Gross job creation, gross job destruction and employment reallocation. The Quarterly Journal of Economics 107 (3), 819–863.

Dix-Carneiro, R., 2011. Trade liberalization and labor market dynamics, staff working paper ERSD 2011-19.

Goldberg, P. K., Pavcnik, N., 2007. Distributional effects of globalization in developing countries. Journal of Economic Literature 45 (1), 39–82.

Haskel, J., Lawrence, R., Leamer, E., Slaughter, M., 2012. Globalization and u.s. wages: Modifying classic theory to explain recent facts. Journal of Economic Perspectives 26, 119–140.

Helpman, E., Itskhoki, O., Muendler, M.-A., Redding, S., 2012. Trade and inequality: From theory to estimation. NBER Working Paper.

Helpman, E., Itskhoki, O., Redding, S., 2010. Inequality and unemployement in a global economy. Econometrica 78 (4), 1239–1283.

Jones, R., 1971. A three-factor model in theory, trade, and history. In: Trade, Balance of Payments, and Growth: Papers in International Economics in Honor of Charles P. Kindleberger.

Kambourov, G., 2009. Labour market regulations and the sectoral reallocation of workers: The case of trade reforms. The Review of Economic Studies 76 (4), 1321–1358.

Kambourov, G., Manovskii, I., 2009. Occupational specificity of human capital. International Economic Review 50(1), 63–115.

Kletzer, L. G., 2001. Job Loss from Imports: Measuring the Costs. Institute for International Economics, Washington, DC.

Krugman, P. R., 2008. Trade and wages, reconsidered. Brookings Papers on Economic Activity, 103–154.

Matsuyama, K., 1992. A simple model of sectoral adjustment. The Review of Economic Studies 59 (2), 375–387.

Mayer, W., 1974. Short-run and long-run equilibrium for a small open economy. The Journal of Political Economy 82 (5), 955–967.

Menezes-Filho, N., Muendler, M., 2011. Labor reallocation in response to trade reform, nBER Working Paper No. 17372.

Mussa, M., 1978. Dynamic adjustment in the heckscher-ohlin-samuelson model. The Journal of Political Economy 86, 775–791.

Neal, D., 1995. Industry-specific human capital: Evidence from displaced workers. Journal of Labor Economics 13 (4), 653–677.

Neary, J. P., 1978. Short-run capital specificity and the pure theory of international trade. The Economic Journal 88, 488–510.

Parent, D., 2000. Industry-specific capital and the wage profile: Evidence from the national longitudinal survey of youth and the panel study of income dynamics. Journal of Labor Economics 18 (2), 306–323.

Robertson, R., 2004. Relative prices and wages inequality: Evidence from mexico. Journal of Economic Literature 64, 387–409.

Schott, P., 2010. U.s. manufacturing exports and imports by sic or naics category and partner country, 1972 to 2005, technical Note.

Slaughter, M., 2000. What are the results of product-price studies and what can we learn from their differences? The Impact of International Trade on Wages, edited by Robert C. Feenstra.

Timmer, M., O'Mahony, M., van Ark, B., March 2008. Eu klems database. In: The EU KLEMS Growth and Productivity Accounts: An Overview. University of Groningen & University of Birmingham, downloadable at www.euklems.net.

Wacziarg, R., Wallack, J. S., 2004. Trade liberalization and intersectoral labor movements. Journal of International Economics 64 (2), 411–439.

Wood, A., 1995. How trade hurt unskilled workers. Journal of Economic Perspectives 9, 57–80.

A Appendix

A.1 Existence of the equilibrium

This section proves Propositions 1 and 2. First we derive the times until equalization of high-skill wages and of low-skill wages as well as the ages of the indifferent high-skill and low-skill worker (t_1, t_2, a_H, and a_L, respectively) in an equilibrium which has the structure described in Proposition 1. Second, we show that workers switch sectors only at time 0. In the online Appendix B, we show that for η sufficiently large, it is possible to keep wages equalized forever once they have been equalized once; and we show that the equilibrium maximizes the present value of production.

A.1.1 Ages of the indifferent workers and times until wage equalization

Equations (5) and (6) pin down the indifferent workers. For a marginal price change, the difference between w_1 and w_2 is at most first order,[23] and workers whose age is nonmarginal will not switch sector (for these workers, $w_1(\tau)\, x_L(\tau) < w_2(\tau)\, x_L(a+\tau)$). Hence only the workers whose age is at most first order in the price change may be willing to move. We can therefore take a first-order expansion of (5) with respect to $(a_L, w_1(t), w_2(t))$ around $a_L = 0$ and $w_1(t) = w_2(t) = w^{ss}$ and then simplify to obtain

$$a_L = \frac{\int_0^{t_1} (dw_1(\tau) - dw_2(\tau))\, x_L(\tau)\, e^{-\delta\tau} d\tau}{w^{ss} \int_0^{T} x_L'(\tau)\, e^{-\delta\tau} d\tau} + o(t_1 dp), \tag{A.1}$$

where $dw_i(t) \equiv w_i(t) - w^{ss}$ and analogous definitions are used for high-skill wages. Since t_1 and $dw_i(t)$ will both be of at most first order in dp, it follows that a_L is at most second order. An analogous expression holds for a_H.

Now define $g_H(t) \equiv dv_1(t) - dv_2(t)$ and $g_L(t) \equiv dw_1(t) - dw_2(t)$ such that t_1 (resp. t_2) denotes the lowest t for which $g_L(t_1) = 0$ (resp. $g_H(t_2) = 0$). Define $dl_i(t) = l_i(t) - l_{ss}$. Given that a_L is at most second order, one can differentiate equation (7) and an analogous expression for l_2 to obtain:

$$dl_1(t) = -dl_2(t) = (1-n_L)\, x_L(0)\, t + o(dp) \;\text{ for } 0 \le t \le t_1,\, t_2; \tag{A.2}$$

similarly,

$$dh_1(t) = -dh_2(t) = (1-n_H)\, x_H(0)\, \overline{H} t + o(dp) \;\text{ for } 0 \le t \le t_1,\, t_2. \tag{A.3}$$

Taking a first-order expansion of $g_H(t)$ around $dp = 0$ and then using equations (A.2) and (A.3), we find that

$$g_H(t) = t\left[(v_{1L} + v_{2L})(1-n_L)\, x_L(0) + (v_{1H} + v_{2H})(1-n_H)\, x_H \overline{H}(0)\right] - \frac{v^{ss}}{p} dp + o(dp) \;\text{ for } 0 \le t \le t_1,\, t_2,$$

[23] An economic variable that is "at most n^{th} order" in the price change is one that can be of order m^{th} for $m \ge n$.

where $v_{iZ} = \frac{\partial v_i}{\partial Z}$ at the steady-state value (prior to the price change), and similarly that

$$g_L(t) = t\left[(w_{1H}+w_{2H})(1-n_H)x_H(0)\overline{H} + (w_{1L}+w_{2L})(1-n_L)x_L(0)\right] - \frac{w^{ss}}{p}dp + o(dp) \text{ for } 0 \le t \le t_1,\ t_2,$$

where $w_{iZ} = \frac{\partial w_i}{\partial Z}$. Since both F_1 and F_2 are CRS, it follows that $(v_{1L}+v_{2L})(1-n_L)x_L(0) + (v_{1H}+v_{2H})(1-n_H)x_H(0)\overline{H}$ or $(w_{1H}+w_{2H})(1-n_H)x_H(0)\overline{H} + (w_{1L}+w_{2L})(1-n_L)x_L(0)$ (or both) must be strictly negative.[24] Consider the case where

$$\begin{aligned} & \frac{1}{v^{ss}}\left((v_{1L}+v_{2L})(1-n_L)x_L(0) + (v_{1H}+v_{2H})(1-n_H)x_H(0)\overline{H}\right) \\ > \ & \frac{1}{w_{ss}}\left((w_{1H}+w_{2H})(1-n_H)x_H(0)\overline{H} + (w_{1L}+w_{2L})(1-n_L)x_L(0)\right), \end{aligned} \tag{A.4}$$

which implies $(w_{1H}+w_{2H})(1-n_H)x_H(0)\overline{H} + (w_{1L}+w_{2L})(1-n_L)x_L(0) < 0$.[25] Then g_L is positive but decreases over time until $t_1 > 0$; over the same time period, g_H remains strictly positive (and may increase or decrease). Therefore, if (A.4) holds then wages are equalized for low-skill workers first: $t_1 < t_2$. The Appendix focuses on this case; if $t_1 > t_2$ then symmetric expressions would obtain.

For $t \in (t_1, t_2)$, low-skill workers are allocated such that their wages are equalized across sectors but (A.3) still holds. At first order, $dl_1 + dl_2 = 0 + o(dp)$ because the number of low-skill workers who switch is of second order at most; therefore, $dw_1 = dw_2$ implies that

$$dl_1(t) = -\frac{w_{1H}+w_{2H}}{w_{1L}+w_{2L}}(1-n_H)x_H(0)\overline{H}t + \frac{w^{ss}}{w_{1L}+w_{2L}}\frac{dp}{p} + o(dp) \text{ for } t_1 \le t \le t_2. \tag{A.5}$$

Using this equation and (A.3), we can write $g_H(t)$ as

$$g_H(t) = \left(v_{1H}+v_{2H} - \frac{(v_{1L}+v_{2L})(w_{1H}+w_{2H})}{w_{1L}+w_{2L}}\right)(1-n_H)x_H(0)\overline{H}t + \left(\frac{v_{1L}+v_{2L}}{w_{1L}+w_{2L}}w^{ss} - v^{ss}\right)\frac{dp}{p} + o(dp),$$

for $t_1 \le t \le t_2$, which is negative and increasing in t when (A.4) holds. Hence t_2 is defined by

$$t_2 = \frac{v^{ss} - \frac{v_{1L}+v_{2L}}{w_{1L}+w_{2L}}w^{ss}}{(1-n_H)x_H(0)\overline{H}\left(v_{1H}+v_{2H} - \frac{(v_{1L}+v_{2L})(w_{1H}+w_{2H})}{w_{1L}+w_{2L}}\right)}\frac{dp}{p} + o(dp). \tag{A.6}$$

Using (A.2), (A.3), and that $dw_1 = dw_2$ for $t > t_1$, we can rewrite (A.1) as (9). Since $dw_1 > dw_2$ on $(0, t_1)$ and $dw_1 = dw_2$ from t_1, the only low-skill workers who switch from sector

[24] Assume that this is *not* the case then both

$$(F_{1LH} + pF_{2LH})(1-n_L)x_L(0) \ge -(F_{1HH} + pF_{2HH})(1-n_H)x_H(0)\overline{H}$$

and

$$(F_{1LH} + pF_{2H})(1-n_H)x_H(0)\overline{H} \ge -(F_{1LL} + pF_{2LL})(1-n_L)x_L(0),$$

with strict inequality for at least one of the two expressions. This implies, since F_1 and F_2 are CRS, that $\frac{F_{1HH}F_{2LL}+F_{2HH}F_{1LL}}{F_{1LH}F_{2LH}} < 2$. However the properties of CRS functions dictate that $\frac{F_{1HH}F_{2LL}+F_{2HH}F_{1LL}}{F_{1LH}F_{2LH}} = \frac{H_2}{L_2}\frac{L_1}{H_1} + \frac{L_2}{H_2}\frac{H_1}{L_1}$, which is strictly greater than 2 if F_1 and F_2 have different factor intensities.

[25] We rule out the case where (A.4) holds with equality. The same logic would apply, but then t_1 and t_2 would differ only at second order.

2 are those who are younger than a_L. In an analogous manner we can use (A.2), (A.3), (A.5), and the counterpart of (A.1) for high-skill workers to solve for a_H as follows:

$$a_H = -\frac{x_H(0)}{2\int_0^T x_H'(\tau)\, e^{-\delta\tau} d\tau}\frac{dp}{p}\left(t_2 - \frac{w^{ss}(v_{1L}+v_{2L})}{v^{ss}(w_{1L}+w_{2L})}(t_2 - t_1)\right). \tag{A.7}$$

Similarly, the only high-skill workers who switch from sector 2 are those who are younger than a_H.

Note that, in the opposite case where wages of high-skill workers are equalized first ($t_1 > t_2$), one can analogously derive the following expressions:

$$\begin{aligned}
t_1 &= \frac{w^{ss} - \frac{w_{1H}+w_{2H}}{v_{1H}+v_{2H}} v^{ss}}{(1-n_L)\, x_L(0)\left(w_{1L}+w_{2L} - (w_{1H}+w_{2H})\frac{v_{1L}+v_{2L}}{v_{1H}+v_{2H}}\right)}\frac{dp}{p} + o(dp), \\
t_2 &= \frac{v^{ss}}{(1-n_L)\, x_L(0)\,[v_{1L}+v_{2L}] + (1-n_H)\, x_H(0)\, \overline{H}\,[v_{1H}+v_{2H}]}\frac{dp}{p} + o(dp); \\
a_L &= -\frac{x_L(0)}{2\int_0^T x_L'(\tau)\, e^{-\delta\tau} d\tau}\frac{dp}{p}\left(t_1 - \frac{v^{ss}(w_{1H}+w_{2H})}{w^{ss}(v_{1H}+v_{2H})}(t_1 - t_2)\right) + o\left(dp^2\right), \\
a_H &= -\frac{x_H(0)\, t_2}{2\int_0^T e^{-\delta\tau} x_H'(\tau)\, d\tau}\frac{dp}{p} + o\left(dp^2\right).
\end{aligned}$$

To establish that Proposition 1 describes an equilibrium, we must still show that (i) workers will switch sectors only once and only time $t = 0$, and (ii) after t_1 (resp. t_2) it is always possible to adjust the flow of entrants such that wages of low-skill (resp. high-skill) workers remain equalized. We focus on low-skill workers below in what follows but the same reasoning applies as well to high-skill workers.

A.1.2 Workers switch only once

We begin by noting that low-skill workers who enter sector 1 will never switch because this sector always has wages that are weakly higher than those of sector 2. Furthermore, workers will not switch after time t_1 because then wages are equalized; workers will always remain in the sector where they have accumulated the most experience until time t_1. Therefore, the only workers who may switch are those born before $t = 0$ who entered sector 2, and they may switch only during the time period $[0, t_1]$. Let us consider such a worker. We denote her age by a, the time she spends in sector 1 during the time period $[0, t_1]$ by μ_1, and the time spent in sector 2 during that same time period by $\mu_2 = t_1 - \mu_1$. We seek to show that if such a worker were to stick to the same sector during the time period $(0, t_1]$, she would be better-off.

First consider the case where, at time t_1, the total experience accumulated in sector 1 is weakly greater than the total experience accumulated in sector 2, that is $\mu_1 \geq \mu_2 + a$ (which implies that a is at most of first order since t_1 is first order). Therefore from time t_1 onward, this worker would (weakly) prefer working in sector 1. We compare the welfare of this worker under this strategy to her welfare under the alternative strategy where she switches to sector 1 at time 0 (when the trade shock hits). During the time interval $[0, t_1]$, the worker benefits from a higher wage under the alternative strategy for periods where she works in sector 2 in the original strategy, but she suffers from a lower level of human capital. The loss in human capital is bounded above by $x_L(\mu_2 + a) - x_L(0)$, and it is suffered during a time period of length μ_2; hence this loss is at most of the same order as $(\mu_2 + a)\mu_2$. For periods where she works in sector 1 in the original strategy, she benefits from a higher level of human capital under the alternative strategy. The gain is equal to $x_L(\tau) - x_L(\tau - \mu_2)$. This gain is endured for a nonnegligible period of time and so is of the same order as μ_2. Hence the gain is of a higher order than the lower, and this worker would be better-off switching to sector 1 upon impact.

Now consider the opposite case where $\mu_1 < \mu_2 + a$ (at time t_1 the total experience accumulated in sector 1 is smaller than the total experience accumulated in sector 2) and the alternative strategy where the worker stays in sector 2 forever. During the time interval $[0, t_1]$, when the worker is employed in sector 1 under the original strategy, she suffers from a lower wage in the alternative strategy; the resulting welfare loss is at most of the same order as $\mu_1 dp$. For time periods where she works in sector 2 in the original strategy, she benefits from a higher level of human capital under the alternative strategy. In particular, from period t_1 onward, her human capital is higher by $x_L(\tau + a) - x_L(\tau + a - \mu_1)$. This gain lasts a nonnegligible period of time, so that the welfare gain is of the same order as μ_1. In this case, then, the gains are larger than the losses and so the worker is better-off under the alternative strategy, staying in sector 2 all along. This establishes point (i).

In Appendix B.1.1, we show that $n_L(t)$ and $n_H(t)$ are in $(0, 1)$ for η sufficiently large, which achieves the proof of existence of the equilibrium.

A.2 Proof of Proposition 4

The argument is most easily made with reference to Figure 1. Along the transition path, wages in sector 1 must remain weakly higher than wages in sector 2 ($w_1(t) \geq w_2(t)$ and $v_1(t) \geq v_2(t)$). From the figure it follows that $w_1(t), w_2(t) \geq w^{ss\prime}$ and $v_2(t), v_1(t) \leq v^{ss\prime}$; therefore, any low-skill worker who does not switch industries (and so does not lose any human capital) will benefit from the rigidity engendered by sector-specific human capital.

Consider, moreover, a low-skill worker of $\hat{t} \leq a_L$ who switches from sector 2 to sector 1. The lifetime income of this worker obeys

$$\int_0^{T-\hat{t}} w_1(\tau) x_L(\tau) e^{-\delta\tau} d\tau \geq \int_0^{T-\hat{t}} w_2(\tau) x_L(\hat{t} + \tau) e^{-\delta\tau} d\tau \geq \int_0^{T-\hat{t}} w^{ss\prime} x_L(\hat{t} + \tau) e^{-\delta\tau} d\tau.$$

Here the first inequality follows from equation (5) and the second from $w_1(t) \geq w^{ss\prime}$. Since lifetime income is higher under the rigid regime yet prices are the same, even those workers who switch are better-off. An analogous argument demonstrates that all high-skill workers would be better-off if human capital were *not* sector-specific.

A.3 Calibration details

In this appendix we provide some details on our calibration: the list of industries in each group, how we parameterize the accumulation function from Neal (1995), and how we derive the parameters α_1, α_2, A_2, $x_L(0,0)$, and $x_H(0,0)$.

As described in the main text, we split the 2-digit industries from EU KLEMS for 2000 into a high-skill group and a low-skill group of equal value-added size. "High skill" is defined as college graduate or above, and "low skill" is defined as some college or below (i.e., the sum of low and medium skills in the EU KLEMS's US classification). The high-skill industries—in decreasing order of their wage bill devoted to high-skill workers—are: office, accounting and computing machinery; medical, precision, and optical instruments; chemicals and chemical products; transport equipment; printing, publishing and reproduction, electrical engineering; and coke, refined petroleum, and nuclear fuel. The low-skill industries (in decreasing order of the wage bill share of high-skill workers) are tobacco; manufacturing not otherwise classified; food and beverage; pulp and paper; machinery not otherwise classified; textiles; rubber and plastics; nonmetallic minerals; basic metals; fabricated metal; wood; and leather and footwear. The high-skill wage bill share for the cutoff industries are 40.2% and 34.4%.

We base our estimate for the human capital accumulation function on columns 2 and 3 of Table 4 in Neal (1995). Neal regresses the log wage of displaced workers on experience (pre-displacement), experience squared, tenure (in the firm prior to the displacement), and tenure squared (plus a constant and some control variables that include education). He runs this regression separately for displaced workers who switch 2-digit industries and for displaced workers who stay in the same 2-digit industry. Our coefficients for experience, experience squared, tenure, and tenure squared will be denoted as γ_e^s, $\gamma_{e^2}^s$, γ_t^s, and $\gamma_{t^2}^s$ for workers staying in the same sector and as γ_e^a, $\gamma_{e^2}^a$, γ_t^a, and $\gamma_{t^2}^a$ for workers who switch to another sector. Since our model does not distinguish between experience in a sector and experience in a specific firm in a given sector, we add up the coefficients for experience and tenure. Then we identify the coefficient for switchers as the impact of general human capital on wages, and identify the difference between the coefficients for stayers and switchers as the effect of sector-specific human capital on wages. Following the specification of this regression, we posit capital accumulation functions of the form

$$x_Z(a,\theta) = x_{Z0}\exp\left(\varphi_a \min(a,m_a) + \varphi_{a^2}\min(a,m_a)^2 + \varphi_\theta \min(\theta,m_\theta) + \varphi_{\theta^2}\min(\theta,m_\theta)^2\right) \text{ for } Z = L, H,$$

Here a is the sector-specific experience, θ is the total experience, and m_a is the sector-specific experience level for which $\varphi_a a + \varphi_{a^2} a^2$ is maximized (and similarly for m_θ); that is, we flatten the accumulation functions once they reach their maxima. We obtain

$$
\begin{aligned}
\varphi_a &\equiv \gamma_e^s + \gamma_t^s - (\gamma_e^a + \gamma_t^a) = 0.027, \\
\varphi_{a^2} &\equiv \gamma_{e^2}^s + \gamma_{t^2}^s - \left(\gamma_{e^2}^a + \gamma_{t^2}^a\right) = -0.0007, \\
\varphi_\theta &\equiv \gamma_e^a + \gamma_t^a = 0.03, \\
\varphi_{\theta^2} &\equiv \gamma_{e^2}^a + \gamma_{t^2}^a = -0.0007, \\
m_a &= 19.28, \\
m_\theta &= 21.42.
\end{aligned}
$$

Observe that, in this specification, nearly half of total human capital is sector specific.

We find that the consumption share for sector 1 is given by $\nu = 0.5087$ and that $\overline{H} = 0.286$. The empirical estimate for the wage bill in sector 1 is given by $\widehat{\alpha_1} = 0.487$ and in sector 2 by $\widehat{\alpha_2} = 0.242$. The estimates for output—where one unit of good in each sector corresponds to \$1 trillion of output in the data—are $\widehat{Y_1} = 2.275$ and $\widehat{Y_2} = 2.194$ in sectors 1 and 2, respectively. The estimates of the share of high-skill and low-skill workers are $\widehat{n_H} = 0.618$ and $\widehat{n_L} = 0.341$.

Let $\xi \equiv \int_0^{40} e^{-\eta t} \frac{x_L(t,t)}{x_{L0}} dt$. Then we can express steady-state output in the model as

$$
\begin{aligned}
Y_1 &= \xi \left(\alpha_1 \left(n_H x_{H0} \overline{H} \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha_1) \left(n_L x_{L0} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \\
Y_2 &= A_2 \xi \left(\alpha_2 \left((1-n_H) x_{H0} \overline{H} \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha_2) \left((1-n_L) x_{L0} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},
\end{aligned}
$$

where n_H and n_L denote the endogenous steady-state allocations of workers of each type in sector 1. Moreover, wage equalization in both sectors imposes that

$$
\begin{aligned}
C_L &\equiv \frac{(1-\alpha_1) \left(\alpha_1 \left(\frac{n_H x_{H0} \overline{H}}{n_L x_{L0}} \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha_1) \right)^{\frac{1}{\varepsilon-1}}}{(1-\alpha_2) A_2 \left(\alpha_2 \left(\frac{(1-n_H) x_{H0} \overline{H}}{(1-n_L) x_{L0}} \right)^{\frac{\varepsilon-1}{\varepsilon}} + 1 - \alpha_2 \right)^{\frac{1}{\varepsilon-1}}} - 1 = 0, \\
C_H &\equiv \frac{\alpha_1}{\alpha_2 A_2} \frac{\left(\alpha_1 + (1-\alpha_1) \left(\frac{n_L x_{L0}}{n_H x_{H0} \overline{H}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}}}{\left(\alpha_2 + (1-\alpha_2) \left(\frac{(1-n_L) x_{L0}}{(1-n_H) x_{H0} \overline{H}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}}} - 1 = 0.
\end{aligned}
$$

We pin down the parameters α_1, α_2, x_{L0}, x_{H0}, and A_2 by solving for

$$
\min_{\alpha_1, \alpha_2, x_{L0}, x_{H0}, n_L, n_H, A_2} M\left(\alpha_1, \alpha_2, x_{L0}, x_{H0}, n_L, n_H, A_2\right),
$$

where M is the following distance function:

$$
M \equiv (\alpha_1 - \widehat{\alpha_1})^2 + (\alpha_2 - \widehat{\alpha_2})^2 + (n_H - \widehat{n_H})^2 + (n_L - \widehat{n_L})^2 + \left(\frac{Y_1}{\widehat{Y_1}} - 1 \right)^2 + \left(\frac{Y_2}{\widehat{Y_2}} - 1 \right)^2 + C_H^2 + C_L^2.
$$

We thus obtain the following parameters:

$$\begin{aligned}
\alpha_1 &= 0.705, \\
\alpha_2 &= 0.5779, \\
x_{L0} &= 0.2537, \\
x_{H0} &= 0.0878, \\
A_2 &= 0.7089.
\end{aligned}$$

With these parameters, the model predicts a steady-state allocation of $n_H = 0.68$ and $n_L = 0.41$

B Online Appendix

B.1 Rest of the proof of Proposition 1

B.1.1 $n_L(t)$ and $n_H(t)$ are in $(0,1)$ for η sufficiently large

We now seek to show that, once wages have been equalized, it is possible to adjust the flow of entrants $n_L(t)$, $n_H(t)$ in sector 1 so as to keep wages equalized for η sufficiently large ($\eta > \frac{1}{T}\ln\left(\frac{x_L(T)}{x_L(0)}\right)$ is a sufficient condition). We denote by $n_Z(t)$ the mass of workers of type $Z = L, H$ entering sector 1; to avoid confusion, we rewrite the original steady-state value as n_Z^{ss} instead of n_Z. Once wages have been equalized, the normalized mass of low-skill workers in sector 1 can be written as

$$\begin{aligned} l_1(t) &= n_L^{ss}\int_{\min(t,T)}^{T} e^{-\eta\tau}x_L(\tau)\,d\tau + (1-n_L^{ss})\,e^{-\eta t}x_L(t)\int_0^{\min(a_L,|T-t|)} e^{-\eta\tau}d\tau \qquad \text{(B.1)} \\ &+ \int_{\min(t-t_1,T)}^{\min(t,T)} e^{-\eta\tau}x_L(\tau)\,d\tau + \int_0^{\min(t-t_1,T)} e^{\eta(-\tau)}n_L(t-\tau)\,x_L(\tau)\,d\tau \text{ for } t \geq t_1. \end{aligned}$$

Compared with equation (7), there is a new term representing the entrants who have arrived since the equalization of wages (the first three terms are generalized to the case where t is sufficiently large that workers who switched start to die off). Similarly, the normalized mass of low-skill workers in sector 2—and of high-skill workers in sectors 1 and 2—can be written as follows:

$$l_2(t) = (1-n_L^{ss})\int_{\min(t+a_L,T)}^{T} e^{-\eta\tau}x_L(\tau)\,d\tau + \int_0^{\min(t-t_1,T)} e^{\eta(-\tau)}\left(1-n_L(t-\tau)\right)x_L(\tau)\,d\tau \text{ for } t \geq t_1, \qquad \text{(B.2)}$$

$$\begin{aligned} \frac{h_1(t)}{\overline{H}} &= n_H^{ss}\int_{\min(t,T)}^{T} e^{-\eta\tau}x_H(\tau)\,d\tau + e^{-\eta t}x_L(t)\,(1-n_H^{ss})\int_0^{\min(a_H,|T-t|)} e^{-\eta\tau}d\tau \qquad \text{(B.3)} \\ &+ \int_{\min(t-t_2,T)}^{\min(t,T)} e^{-\eta\tau}x_H(\tau)\,d\tau + \int_0^{\min(t-t_2,T)} e^{\eta(-\tau)}n_H(t-\tau)\,x_H(\tau)\,d\tau \text{ for } t \geq t_2, \end{aligned}$$

$$\frac{h_2(t)}{\overline{H}} = (1-n_H^{ss})\int_{\min(t+a_H,T)}^{T} e^{-\eta\tau}x_L(\tau)\,d\tau + \int_0^{\min(t-t_2,T)} e^{\eta(-\tau)}\left(1-n_H(t-\tau)\right)x_H(\tau)\,d\tau \text{ for } t \geq t_2. \qquad \text{(B.4)}$$

The functions $n_L(t)$ and $n_H(t)$ must satisfy $v_1(t) = v_2(t)$ for $t \geq t_2$ and $w_1(t) = w_2(t)$ for $t \geq t_1$, and our goal is to show that the solutions to these equations fall in the range $(0,1)$. Summing (B.1) and (B.2) and then summing (B.3) and (B.4), we obtain:

$$l(t) = l_1(t) + l_2(t) = l^{eff} - (1-n_L^{ss})\,e^{-\eta t}\int_0^{\min(a_L,|T-t|)} e^{-\eta\tau}\left(x_L(\tau+t) - x_L(t)\right)d\tau = l^{eff} + o\left(dp^4\right),$$

$$h(t) = h_1(t) + h_2(t) = h^{eff} - \overline{H}\,(1-n_H^{ss})\,e^{-\eta t}\int_0^{\min(a_H,|T-t|)} e^{-\eta\tau}\left(x_H(\tau+t) - x_H(t)\right)d\tau = h^{eff} + o\left(dp^4\right);$$

these equalities show that, at forth order, the masses of normalized factors are constant.

First, we consider the case where $t \in (t_1, t_2)$, so that only wages of low-skill workers are equalized and t is small. Equation (A.3) still holds, and differentiating (B.1) and (B.2) for small price changes and small t yields

$$dl_1(t) = -dl_2(t) = ((t - t_1)(n_L(t) - n_L^{ss}) + t_1(1 - n_L^{ss}))\, x_L(0) + o(dp).$$

Plugging these expressions into $dw_1 = dw_2 + o(dp)$ and using (8), the result is

$$(t - t_1)\left((w_{1H} + w_{2H})(1 - n_H^{ss})\, x_H(0)\,\overline{H} + (w_{1L} + w_{2L})(n_L(t) - n_L^{ss})\, x_L(0)\right) = o(dp);$$

therefore,

$$n_L(t) - n_L^{ss} = -\frac{(w_{1H} + w_{2H})(1 - n_H)\, x_H(0)\,\overline{H}}{(w_{1L} + w_{2L})\, x_L(0)} + o(1).$$

Since $(w_{1H} + w_{2H})(1 - n_H)\, x_H(0)\,\overline{H} + (w_{1L} + w_{2L})(1 - n_H)\, x_L(0) < 0$ if $t_1 < t_2$, it follows that $n_L(t)$ is in $(0, 1)$ for small price changes when $t \in (t_1, t_2)$.

For $t > t_2$, wages are equalized in both sectors. This implies, absent any factor intensity reversal, that wages must be at the new steady-state value: $w_2(t) = w_1(t) = w^{ss\prime}$ and $v_2(t) = v_1(t) = v^{ss\prime}$. Hence factor intensity in each sector must likewise be at the new steady state values, so we must have

$$l_1(t) = \frac{l(t) - \frac{l_2^{ss\prime}}{l_2^{ss\prime}} h(t)}{\left(1 - \frac{l_2^{ss\prime}}{h_2^{ss\prime}} \frac{h_1^{ss\prime}}{l_1^{ss\prime}}\right)}$$

(as well as similar expressions for $h_2(t)$, $l_1(t)$, and $l_2(t)$). Furthermore, observe that $\frac{dl(t)}{dt} = O\left(dp^2\right)$, where $O\left(dp^2\right)$ is a function of time and dp, which satisfies $\left|O\left(dp^2\right)\right| < M dp^2$ for some positive real M; similarly, $\frac{dh(t)}{dt} = O\left(dp^2\right)$. Therefore $\frac{dl_1(t)}{dt} = O\left(dp^2\right)$ and similarly for $\frac{dl_2(t)}{dt}$, $\frac{dh_1(t)}{dt}$, and $\frac{dh_2(t)}{dt}$.

More specifically, assume that $t_2 < t < T - \max(a_H, a_L)$. Then

$$\begin{aligned} \frac{dh_1(t)}{dt}\frac{1}{\overline{\overline{H}}} &= -e^{-\eta t} n_H^{ss} x_H(t) + (1 - n_H^{ss}) \frac{d}{dt}\left(e^{-\eta t} x_H(t)\right) \int_0^{a_H} e^{-\eta\tau} d\tau + e^{-\eta t} x_H(t) \\ &\quad - e^{\eta(t - t_2)} x_H(t - t_2) + n_H(t)\, x_H(0) + \int_0^{t - t_2} n_H(t - \tau) \frac{d\left(e^{-\eta\tau} x_H(\tau)\right)}{d\tau} d\tau, \end{aligned}$$

$$\frac{dh_2(t)}{dt}\frac{1}{\overline{\overline{H}}} = -e^{-\eta(t + a_H)}(1 - n_H^{ss})\, x_H(t + a_H) + (1 - n_H(t))\, x_H(0) + \int_0^{t - t_2} (1 - n_H(t - \tau)) \frac{d\left(e^{-\eta\tau} x_H(\tau)\right)}{d\tau} d\tau.$$

Similar expressions can be derived for low-skill workers:

$$\begin{aligned} \frac{dl_1(t)}{dt} &= -e^{-\eta t} n_L^{ss} x_L(t) + (1 - n_L^{ss}) \frac{d}{dt}\left(e^{-\eta t} x_L(t)\right) \int_0^{a_L} e^{-\eta\tau} d\tau + e^{-\eta t} x_L(t) \\ &\quad - e^{-\eta(t - t_1)} x_L(t - t_1) + n_L(t)\, x_L(0) + \int_0^{t - t_1} n_L(t - \tau) \frac{d\left(e^{-\eta\tau} x_L(\tau)\right)}{d\tau} d\tau, \end{aligned}$$

$$\frac{dl_2(t)}{dt} = -e^{-\eta(t+a_L)}\left(1-n_L^{ss}\right)x_L(t+a_L)+\left(1-n_L(t)\right)x_L(0)+\int_0^{t-t_1}\left(1-n_L(t-\tau)\right)\frac{d\left(e^{-\eta\tau}x_L(\tau)\right)}{d\tau}d\tau.$$

Since $\frac{dl_1(t)}{dt} = O\left(dp^2\right)$, we have

$$-e^{-\eta t}n_L^{ss}x_L(t)+n_L(t)x_L(0)+\int_0^{t-t_1}n_L(t-\tau)\frac{d\left(e^{-\eta\tau}x_L(\tau)\right)}{d\tau}d\tau = O(dp).$$

Because t_1 and t_2 are first order, this equation can be rewritten as

$$\left(n_L(t)-n_L^{ss}\right)x_L(0)+\int_0^{t-t_2}\left(n_L(t-\tau)-n_L^{ss}\right)\frac{d\left(e^{-\eta\tau}x_L(\tau)\right)}{d\tau}d\tau = O(dp). \tag{B.5}$$

Note that $n_L(t) = n_L^{ss}$ for $t \in (t_2, T-\max(a_H, a_L))$ is a solution to this functional equation when the right-hand side is exactly equal to 0. Hence there must be a solution of (B.5) that can be written as $n_L(t) = n_L^{ss} + O(dp)$, where $O(dp)$ is a function of time and dp, which satisfies $|O(dp)| < Ndp$ for some number N and all t. Then, for dp sufficiently small, $n_L(t)$ will be close to n_L^{ss}; in particular, $n_L(t)$ belongs to $(0,1)$. The same reasoning holds for $n_H(t)$.

Without loss of generality, we assume that $a_H > a_L$ and examine the case where $T - a_L \leq t < T - a_H$. The expressions for the derivatives of $h_1(t)$ and $h_2(t)$ remain identical, but those for the derivatives of $l_1(t)$ and $l_2(t)$ now become

$$\begin{aligned}\frac{dl_1(t)}{dt} &= -e^{-\eta t}n_L^{ss}x_L(t)+\left(1-n_L^{ss}\right)\int_0^{T-t}e^{-\eta\tau}d\tau\frac{d}{dt}\left(e^{-\eta t}x_L(t)\right)-\left(1-n_L^{ss}\right)x_L(t)e^{-\eta T}\\ &\quad +e^{-\eta t}x_L(t)-e^{\eta(t-t_1)}x_L(t-t_1)+n_L(t)x_L(0)+\int_0^{t-t_1}n_L(t-\tau)\frac{d\left(e^{-\eta\tau}x_L(\tau)\right)}{d\tau}d\tau,\end{aligned}$$

$$\frac{dl_2(t)}{dt} = \left(1-n_L(t)\right)x_L(0)+\int_0^{t-t_1}\left(1-n_L(t-\tau)\right)\frac{d\left(e^{-\eta\tau}x_L(\tau)\right)}{d\tau}d\tau. \tag{B.6}$$

Using a similar argument as the one for (B.5) and given that $T-t$ is second order for this time interval, we obtain

$$-\left(1-n_L^{ss}\right)e^{-\eta T}x_L(T)+\left(n_L(t)-n_L^{ss}\right)x_L(0)+\int_0^{t-t_2}\left(n_L(t-\tau)-n_L^{ss}\right)\frac{d\left(e^{-\eta\tau}x_L(\tau)\right)}{d\tau}d\tau = O(dp).$$

This equation implies, since $n_L(t) = n_L^{ss} + O(dp)$ (except for $t \leq t_2$, that is for a time period of duration $O(dp)$), that

$$n_L(t) = n_L^{ss}+\left(1-n_L^{ss}\right)e^{-\eta T}\frac{x_L(T)}{x_L(0)}+O(dp). \tag{B.7}$$

Therefore, $n_L(t) \in (0,1)$ for dp sufficiently small provided

$$\eta > \frac{1}{T}\ln\left(\frac{x_L(T)}{x_L(0)}\right).$$

The same reasoning holds when $T - a_H \leq t < T$ but now it applies to both high-skill and low-skill workers. Suppose now that $T \leq t < T + t_1$. Then

$$\frac{dl_1(t)}{dt} = -e^{\eta(t-t_1)} x_L(t-t_1) + n_L(t) x_L(0) + \int_0^{t-t_1} n_L(t-\tau) \frac{d\left(e^{-\eta\tau} x_L(\tau)\right)}{d\tau} d\tau,$$

$$\frac{dh_1(t)}{dt} \frac{1}{\overline{\overline{H}}} = -e^{\eta(t-t_2)} x_H(t-t_2) + n_H(t) x_H(0) + \int_0^{t-t_2} n_H(t-\tau) \frac{d\left(e^{-\eta\tau} x_H(\tau)\right)}{d\tau} d\tau;$$

at the same time, (B.6) and the equivalent equation for $\frac{dh_2}{dt}$ hold. In this case, $\frac{dh(t)}{dt} = \frac{dl(t)}{dt} = 0$ and so

$$-\left(1 - n_L^{ss}\right) e^{\eta(t-t_1)} x_L(t-t_2) + \left(n_L(t) - n_L^{ss}\right) x_L(0) + \int_0^{t-t_1} \left(n_L(t-\tau) - n_L^{ss}\right) \frac{d\left(e^{-\eta\tau} x_L(\tau)\right)}{d\tau} d\tau = 0;$$

therefore (B.7) still holds. Hence for $\eta > \frac{1}{T} \ln\left(\frac{x_L(T)}{x_L(0)}\right)$ and sufficiently small dp, we have $n_L(t) \in (0,1)$. The same reasoning applies to high-skill workers and extends to low-skill workers when $T + t_1 < t < T + t_2$.

Finally if $T + t_1 \leq t$ then the derivatives for high-skill workers can be written as

$$\begin{aligned} \frac{dh_1}{dt} \frac{1}{\overline{\overline{H}}} &= -\frac{dh_2}{dt} \frac{1}{\overline{\overline{H}}} = -\left(n_H(t-T) - n_H^{ss}\right) x_H(T) e^{-\eta T} d\tau + \left(n_H(t) - n_H^{ss}\right) x_H(0) \\ &\quad + \int_0^T \left(n_H(t-\tau) - n_H^{ss}\right) \frac{de^{-\eta\tau} x_H(\tau)}{d\tau} d\tau; \end{aligned}$$

as a result, the counterpart of (B.5) is now

$$\left(n_H(t) - n_H^{ss}\right) x_H(0) = \left(n_H(t-T) - n_H^{ss}\right) x_H(T) e^{-\eta T} d\tau - \int_0^T \left(n_H(t-\tau) - n_H^{ss}\right) \frac{de^{-\eta\tau} x_H(\tau)}{d\tau} d\tau.$$

From this expression it follows directly that a solution exists where $n_H(t)$ belongs to $(0,1)$ when $x_H(T) e^{-\eta T} / x_H(0) < 1$. Moreover, $n_H^{ss}(t)$ is close to n_H^{ss} except for a set of measure $O(dp)$. For $T + t_2 \leq t$, the reasoning extends to low-skilled workers. This completes the proof of existence

B.1.2 Production efficiency

We now prove that this equilibrium (when it exists) maximizes the present value of production (if it is finite) or the present value of production up to some date $t \geq T$. At any instant $t > T$, the competitive equilibrium described reproduces the outcome of a general human capital economy and therefore it does indeed maximize production. Therefore, we only have to show that the competitive equilibrium maximizes the discounted value of production from 0 to T.

First, we compare the present value of production in the competitive equilibrium described above with the present value of production in the (unique) competitive equilibrium obtained in a model with general human capital. Note that with general human capital, the competitive equilibrium maximizes the present value of production. Relative to the general human capital allocation, the allocation of the sector-specific human capital model corresponds to a misallocation of factors and a decrease in the endowments. The misallocation of factors is first order in the price change. By the envelope theorem, it has a second-order effect on the value of production at a given time. This misallocation of factors, however, only last for a period of time which is first order in the price change (until wages are equalized). Therefore, it has a third order effect on the present value of production. The loss of effective units of human capital occurs as workers switch sectors. Since only a second order mass of workers switch, and since these workers have accumulated only a second order amount of human capital, this loss only has a forth order impact. Therefore, the difference between the maximal present value in the general human capital case and the present value obtained in the competitive equilibrium with sector specific-human capital is third order in the price change. As the maximal present value of production with general human capital must be weakly greater than its counterpart with sector-specific human capital, it follows that their difference must be at most third order in the price change.

Since the steady-state allocation maximizes the value of output before the trade shock, the allocation of factors which maximizes the present value of production differs from the steady-state allocation at most at first order.[26] Therefore the marginal product of factors across sector differs at most at first order. Moreover, they may only differ at first order for a first order period of time (otherwise the difference in the present value of production relative to the case with general human capital will be more than third order).

Let us then consider a (low-skill or high-skill) worker born after the trade shock. Denote by μ_1 the time he spends in sector 1 and by μ_2 the time he spends in sector 2 in the present value maximizing allocation. Without loss of generality, assume that $\mu_1 \geq \mu_2$. Let us consider an alternative allocation where he would spend all of his time in sector 1 instead. This worker would then provide a higher level of human capital which translates into a gain in the present value of production of the same order as μ_2. The potential loss arises from a (possible) lower value of the marginal product of his type of human capital in sector 1 than in the sector 2. This loss is at most of second order (since marginal products are identical at first order except potentially for a first order period of time - where they may differ at first order). Therefore, this allocation of the worker's time can only be part of the social optimum if μ_2 is second order. The potential loss is then at most of the same order as $\mu_2 dp$, and must be smaller than the benefit. As a result, in the optimal allocation, newborn workers must remain in the same sector. Similarly workers who were already born at the time of the shock may only switch at the time of the shock and their accumulated experience at that time can only be second order.

[26] Technically this only holds almost everywhere, that is everywhere except for a time period of mass 0. All identities in the following are similarly holding "almost everywhere".

Given that the optimal number of workers switch at the time of the trade shock, no workers would subsequently switch sectors, the problem of finding the optimal allocation at $t > 0$ is simply one of finding the share of workers entering each sector (the endowments in effective units of human capital are fixed). Maximizing the present value of production requires to have workers being allocated to the sector with the largest marginal product of their human capital up until there are no more new workers to allocate or the marginal products are equalized across sectors. Since the initial switch is second order, marginal factor products are initially higher in sector 1 and both types of workers are initially allocated to sector 1. Our analysis of the competitive equilibrium shows that once marginal products have been equalized, it is possible to ensure that they remain equalize all along (provided that η is sufficiently large and the price change is small). Therefore marginal products in sector 1 must remain higher than in sector 2, and the initial switch only concerns workers of sector 2.

As a result, the optimal allocation takes the same form as the competitive allocation described above, with the share of entrants being allocated to the different sectors solving the same equations, but the mass of workers switching sectors at time 0 may still be different. To find this mass, we solve:

$$\max_{a_L, a_H} \int_0^T e^{-(\delta-\eta)t} \left(y_1(t) + p y_2(t)\right),$$

where y_1 and y_2 are the normalized amount of output, with the following constraints:

$$y_1(t) = F_1\left(l_1(t), h_1(t)\right) \text{ and } y_2(t) = F_2\left(l_2(t), h_2(t)\right),$$

$$\begin{aligned} l_1(t) &= n_L^{ss} \int_t^T e^{-\eta\tau} x_L(\tau)\, d\tau + \left(1 - n_L^{ss}\right) e^{-\eta t} x_L(t) \int_0^{\min(a_L, T-t)} e^{-\eta\tau} d\tau \\ &\quad + \int_{\max(0, t-t_1)}^{t} e^{-\eta\tau} x_L(\tau)\, d\tau + \int_0^{\max(0, t-t_1)} e^{-\eta\tau} n_L(t-\tau)\, x_L(\tau)\, d\tau. \end{aligned} \tag{B.8}$$

(this expression comes from (7) and (B.1)),

$$l_2(t) = \left(1 - n_L^{ss}\right) \int_{t+a_L}^T e^{-\eta\tau} x_L(\tau)\, d\tau + \int_0^{\max(0, t-t_1)} e^{-\eta\tau} \left(1 - n_L(t-\tau)\right) x_L(\tau)\, d\tau, \tag{B.9}$$

with similar constraints for $h_1(t)$ and $h_2(t)$, and where t_1, t_2, $n_L(t)$, $n_H(t)$ are defined as in the competitive case.

The first order equation with respect to a_L leads to:

$$\int_0^T e^{-(\delta-\eta)t} \left(w_1(t) \frac{\partial}{\partial a_L} l_1(t) + w_2(t) \frac{\partial}{\partial a_L} l_2(t) \right) = 0,$$

where $w_1(t)$ is the marginal product of low-skill labor in sector 1 (and similarly $w_2(t)$ in sector 2). Computing the integral separately over the intervals $(0, t_1)$, $(t_1, T - a_L)$ and $(T - a_L, T)$

and using that $w_1(t) = w_2(t) = w(t)$ after t_1 gives:

$$\begin{aligned}
&\int_0^{t_1} e^{-(\delta-\eta)t} \left(w_1(t) e^{-\eta t} x_L(t) e^{-\eta a_L} - w_2(t) e^{-\eta(t+a_L)} x_L(t+a_L) d\tau \right) \\
&+ \int_{t_1}^{T-a_L} e^{-(\delta-\eta)t} w(t) e^{-\eta(t+a_L)} (x_L(t) - x_L(t+a_L)) d\tau \\
= \;& 0,
\end{aligned}$$

which, in turn, can be rewritten as (5). The same holds for a_H. Therefore the equilibrium allocation maximizes the present value of production.

B.2 Trade adjustment assistance policy

This section provides details about the adjustment program. Assume that low-skill workers who switch sectors receive, over their lifetime, a sum with present value S (regardless of their age). We assume that S is first order in the price change and small enough that full adjustment is not reached. The indifferent worker is now characterized by

$$\int_0^{T-a_L} w_1(\tau) x_L(\tau) e^{-\delta\tau} d\tau + S = \int_0^{T-a_L} w_2(\tau) x_L(a_L+\tau) e^{-\delta\tau} d\tau \tag{B.10}$$

instead of by (5). Even for a marginal price change, an extremely old worker may be willing to switch sectors in order to claim the subsidy before dying: despite $w_1(\tau) x_L(\tau) < w_2(\tau) x_L(a_L+\tau)$ holding throughout, if the remaining lifetime of the worker is of order 1, then he might switch sectors. We therefore assume that the subsidy is distributed over a time period of nonnegligible duration so that very old workers do not switch

Taking a first-order approximation gives:

$$a_L = S \left(\int_0^T w^{ss} x_L(\tau)' e^{-\delta\tau} d\tau \right)^{-1} + o(dp),$$

so the number of low-skill workers who switch is now first order. Assuming that S is sufficiently small, full adjustment will not be reached and we still have $w_1(0) > w_2(0)$. In this case the structure of the equilibrium stays the same, in particular the mass of high-skill workers who switch sectors is still second order. To prove Proposition 5, we first derive a general expression for the change in the present value of lifetime income (at time 0) of workers relative to steady state and then address, in turn, the cases where $t_1 > t_2$ and $t_1 < t_2$. We then prove remark 1.

B.2.1 Present value of lifetime income

Denote by $W(t,i)$ the present value of lifetime income at time 0 for low-skill workers of age t who initially worked in sector i, and denote by $dW(t,i)$ the difference relative to steady state. For those who were initially in sector 1, we get

$$dW(t,1) = \int_0^{T-t} x_L(t+\tau) e^{-\delta\tau} dw_1(\tau) d\tau.$$

For those who were initially in sector 2, we get

$$dW(t,2) = \int_0^{T-t} x_L(t+\tau) e^{-\delta\tau} dw_2(\tau)$$

when their age t obeys $t > a_L$; otherwise, for $t < a_L$, we get

$$\begin{aligned} dW(t,2) &= \int_0^{T-t} x_L(\tau) e^{-\delta\tau} w_1(\tau) - x_L(t+\tau) e^{-\delta\tau} w^{ss} + S \\ &= dw^{ss} \int_0^{T-t} x_L(t+\tau) e^{-\delta\tau} d\tau + S + o(dp). \end{aligned}$$

Here dw^{ss} is the difference in the wages of low-skill workers between the new and the old steady states. Because the steady-state wage is not affected by the subsidy, the last equality shows that if S is first order in dp then workers who switch necessarily benefit from doing so.

B.2.2 Case where $t_1 > t_2$

Assume that $t_1 > t_2$ with the subsidy. Then (A.2) becomes

$$dl_1(t) = -dl_2(t) = (1-n_L) x_L(0)(a_L + t) \text{ for } t \le t_2$$

while (A.3) still holds. As a result,

$$t_2 = \frac{v^{ss}\frac{dp}{p} - a_L(v_{1L}+v_{2L})(1-n_L)x_L(0)}{(1-n_L)x_L(0)[v_{1L}+v_{2L}] + (1-n_H)x_H(0)\overline{H}[v_{1H}+v_{2H}]} + o(dp) \tag{B.11}$$

and so the wages of high-skill workers are equalized later. For $t \in (0, t_2)$, we have

$$\begin{aligned} dw_1 &= w_{1L}(1-n_L)x_L(0)(a_L+t) + w_{1H}(1-n_H)x_H(0)\overline{H}t + o(dp), && \text{(B.12)} \\ dw_2 &= -w_{2L}(1-n_L)x_L(0)(a_L+t) - w_{2H}(1-n_H)x_H(0)\overline{H}t + w^{ss}\frac{dp}{p} + o(dp) && \text{(B.13)} \end{aligned}$$

Therefore, a higher S (which implies a higher a_L) reduces wages in sector 1 but increases wages in sector 2 during that phase.

For $t \in (t_2, t_1)$, one can derive from $dv_1 = dv_2$ that

$$dh_1(t) = -\frac{v_{1L}+v_{2L}}{v_{1H}+v_{2H}}(1-n_L)x_L(0)(t+a_L) + \frac{v^{ss}}{v_{1H}+v_{2H}}\frac{dp}{p} + o(dp) \text{ for } t_1 \le t \le t_2.$$

This expression yields

$$t_1 = \frac{w^{ss} - \frac{w_{1H}+w_{2H}}{v_{1H}+v_{2H}} v^{ss}}{(1-n_L)x_L(0)\left(w_{1L}+w_{2L} - (w_{1H}+w_{2H})\frac{v_{1L}+v_{2L}}{v_{1H}+v_{2H}}\right)}\frac{dp}{p} - a_L + o(dp)$$

and so, with a higher S, wages are equalized sooner for low-skill workers. Note that, if $t_2 < t_1$ with the subsidy, then the same inequality must hold without the subsidy. Wages are now given by

$$dw_1 = \left(w_{1L} - w_{1H}\frac{v_{1L}+v_{2L}}{v_{1H}+v_{2H}}\right)(1-n_L)x_L(0)(t+a_L) + w_{1H}\frac{v^{ss}}{v_{1H}+v_{2H}}\frac{dp}{p} + o(dp) \text{ for } t_2 \le t \le t_1$$

$$dw_2 = \left(w_{2H} \frac{v_{1L} + v_{2L}}{v_{1H} + v_{2H}} - w_{2L} \right) (1 - n_L) \, x_L (0) \, (a_L + t) - w_{2H} \frac{v^{ss}}{v_{1H} + v_{2H}} \frac{dp}{p} + w^{ss} \frac{dp}{p} + o \, (dp) \text{ for } t_2 \le t \le t_1$$

Because $w_1(t), w_2(t) \ge w^{ss\prime}$ during the transition, we must have $w_{1L} - w_{1H} \frac{v_{1L}+v_{2L}}{v_{1H}+v_{2H}} < 0$ and $w_{2H} \frac{v_{1L}+v_{2L}}{v_{1H}+v_{2H}} - w_{2L} < 0$. Thus a higher S implies lower wages in both sectors during the second phase. Low-skill workers from sector 1 (and also incoming generations) are always hurt by a positive S. In contrast workers in sector 2 benefit during the first phase but are hurt during the second phase; hence the net impact depends on the length of each phase. More specifically, one can show that

$$\frac{\partial \left(dW \, (t, 2) \right)}{\partial a_L} = (1 - n_L) \, x_L (0) \left(-w_{2L} t_2 + w_{2H} \frac{v_{1L} + v_{2L}}{v_{1H} + v_{2H}} (t_1 - t_2) \right).$$

Since $w_{2H} \frac{v_{1L}+v_{2L}}{v_{1H}+v_{2H}} - w_{2L} < 0$, the left-hand side of this equality is negative when t_1 is sufficiently larger than t_2. In that case, workers who remain in sector 2 and are still alive at t_1 are hurt by the subsidy.

The aggregate net present value of income for all low-skill workers alive at $t = 0$ can be written as follows:

$$\begin{aligned} dW &= \int_{a_L}^{T} (1 - n_L) \, dW \, (\theta, 2) \, e^{-\eta\theta} d\theta + \int_0^{a_L} (1 - n_L) \, dW \, (\theta, 2) \, e^{-\eta\theta} d\theta + \int_0^T n_L dW \, (\theta, 1) \, e^{-\eta\theta} d\theta \\ &= \left(\begin{array}{c} \frac{w_{2H}(1-n_L) - w_{1H} n_L}{v_{1H}+v_{2H}} \left((1 - n_L) \, x_L (0) \, [v_{1L} + v_{2L}] + (1 - n_H) \, x_H (0) \, \overline{H} \, [v_{1H} + v_{2H}] \right) \frac{t_2^2}{2} \\ + \left((1 - n_L) \left(w_{2L} - w_{2H} \frac{v_{1L}+v_{2L}}{v_{1H}+v_{2H}} \right) + n_L \left(w_{1H} \frac{v_{1L}+v_{2L}}{v_{1H}+v_{2H}} - w_{1L} \right) \right) (1 - n_L) \, x_L (0) \, \frac{t_1^2}{2} \end{array} \right) \\ &\quad \times \int_0^T x_L (\theta) \, e^{-\eta\theta} d\theta + (1 - n_L) \, a_L S + dw^{ss} \int_0^T \left(e^{\delta\theta} \widetilde{X}_L (T) - e^{\delta\theta} \widetilde{X}_L (\theta) \right) e^{-\eta\theta} d\theta + o \left(dp^2 \right), \end{aligned}$$

where $\widetilde{X_L}$ is an indefinite integral of $x_L (\tau) \, e^{-\delta\tau}$. Since t_1, a_L, and t_2 all depend linearly on S at first order, this is a quadratic expression of S. Recall that $w_{2L} - w_{2H} \frac{v_{1L}+v_{2L}}{v_{1H}+v_{2H}} > 0$ and $w_{1H} \frac{v_{1L}+v_{2L}}{v_{1H}+v_{2H}} - w_{1L} > 0$, which means that the term in front of t_1^2 is positive. We also have $v_{iH} < 0$ and, when wages of high-skill workers are equalized first, $(1 - n_L) \, x_L (0) \, [v_{1L} + v_{2L}] + (1 - n_H) \, x_H (0) \, \overline{H} \, [v_{1H} + v_{2H}] < 0$. Thus if $w_{2H} (1 - n_L) - w_{1H} n_L < 0$ then the term in front of t_2^2 is negative. Note also that $\frac{dt_1}{dS}|_{S=0} < 0$, $\frac{dt_2}{dS}|_{S=0} > 0$, and $\frac{da_L S}{dS}|_{S=0} = 0$; hence, for a sufficiently small S, we find that dW is lower *with* the adjustment assistance policy than *without* it (and that, for a sufficiently small S, $t_2 < t_1$ with the subsidy if that inequality already held without it).

B.2.3 Case where $t_1 < t_2$

Reasoning as before, we obtain

$$t_1 = \frac{\frac{w^{ss}}{p} dp - (w_{1L} + w_{2L}) \, (1 - n_L) \, x_L (0) \, a_L}{(w_{1H} + w_{2H}) \, (1 - n_H) \, x_H (0) \, \overline{H} + (w_{1L} + w_{2L}) \, (1 - n_L) \, x_L (0)} + o \, (dp),$$

$$t_2 = \frac{\left(v^{ss} - \frac{w^{ss}(v_{1L}+v_{2L})}{(w_{1L}+w_{2L})}\right)}{(1-n_H)\,x_H(0)\,\overline{H}\left((v_{1H}+v_{2H})-(v_{1L}+v_{2L})\frac{(w_{1H}+w_{2H})}{(w_{1L}+w_{2L})}\right)}\frac{dp}{p} + o(dp)\,.$$

Hence t_1 is decreasing in S (wages of low-skill workers are equalized faster) and t_2 is independent of S at first order. For $t < t_1$, (B.12) and (B.13) still hold; therefore, a higher subsidy reduces wages in sector 1 but increases wages in sector 2. For $t \in (t_1, t_2)$,

$$dw(\tau) = \frac{w_{1L}w^{ss}}{(w_{1L}+w_{2L})}\frac{dp}{p} + \left(w_{1H} - w_{1L}\frac{(w_{1H}+w_{2H})}{(w_{1L}+w_{2L})}\right)(1-n_H)\,x_H(0)\,\overline{H}t + o(dp)\,,$$

which is independent of S at first order. As a result, newcomers and workers staying in sector 1 must always lose from the subsidy—and thus also lose when the subsidy is large enough to induce $t_2 > t_1$ when the parameter values are such that $t_2 < t_1$ without a subsidy. Workers staying in sector 2, however, now benefit from it.

We can write the change in the net present value of aggregate lifetime income of low-skill workers alive at time $t = 0$ as

$$\begin{aligned} dW \;=\; & \left(\int_0^T x_L(\theta)\,e^{-\eta\theta}d\theta\right)\frac{t_1^2}{2}\frac{(w_{1H}+w_{2H})(1-n_H)\,x_H(0)\,\overline{H}+(w_{1L}+w_{2L})(1-n_L)\,x_L(0)}{w_{1L}+w_{2L}} \\ & \times((1-n_L)\,w_{2L} - n_L w_{1L}) + a_L(1-n_L)\,S \\ & + \left(\frac{w_{1L}w^{ss}}{(w_{1L}+w_{2L})}\frac{dp}{p}t_2 + \left(w_{1H} - w_{1L}\frac{(w_{1H}+w_{2H})}{(w_{1L}+w_{2L})}\right)(1-n_H)\,x_H(0)\,\overline{H}\frac{t_2^2}{2}\right)\int_0^T x_L(\theta)\,e^{-\eta\theta}d\theta \\ & + dw^{ss}\int_0^T\left(e^{\delta\theta}\widetilde{X}_L(T) - e^{\delta\theta}\widetilde{X}_L(\theta)\right)e^{-\eta\theta}d\theta + o(dp)\,, \end{aligned}$$

where t_2 is independent of S at first order and t_1 depends negatively on S at first order. Since $\frac{(w_{1H}+w_{2H})(1-n_H)x_H(0)\overline{H}+(w_{1L}+w_{2L})(1-n_L)x_L(0)}{w_{1L}+w_{2L}} > 0$ it follows that, when wages of low-skill workers are equalized first, dW is negative for a sufficiently small S provided that $(1-n_L)\,w_{2L} - n_L w_{1L} > 0$.

B.2.4 Proof of remark 1

Under the retraining program, we get $a_L = \overline{a} + o(dp)$, therefore the analysis above holds exactly except that the term $an_L(1-n_L)\,S$ is not present in any of the equations.

Consider the case where wages of high-skill workers are nearly immediately equalized ($x_H(0)$ and $x_H(T)$ are close), then for $\overline{a}$ large enough, low-skill wages can be equalized (at their new steady-state value) after an arbitrarily small amount of time, so that all low-skill workers lose relative to a situation where $\overline{a} = 0$.

B.3 Gross flows model

In this appendix we solve the extended model with gross flows due to nonpecuniary sector preferences (as first presented in Section 5). First, we derive the mass of entrants in each state in a steady-state where the share of workers in a biased state is the same across all ages and then compute the gross flows. Second, we derive the system of partial derivatives equations that the utilities of workers must satisfy in equilibrium, and solve it in steady-state. Third, we derive the equilibrium following a small price shock.

B.3.1 Step 1: Number of entrants and gross flows

Denote by $g_i(t)$ the mass of high-skill workers in state $i = 0, 1, 2$ at time t (that is workers with no sectoral preference, workers biased towards sector 1 and workers biased towards sector 2), and denote by $\widetilde{g_i}(t)$ the flow of entrants in state i at time t. Workers die with a Poisson rate δ, switch from state 0 to each of the biased state with a Poisson rate $\lambda/2$, and switch from each of the biased state to state 0 with a Poisson rate 1/2. We then have the following system of differential equations:

$$\begin{aligned}
\frac{dg_0}{dt}(t) &= -(\lambda+\delta) g_0(t) + \frac{1}{2} g_1(t) + \frac{1}{2} g_2(t) + \widetilde{g_0}(t), \\
\frac{dg_1}{dt}(t) &= \frac{\lambda}{2} g_0(t) - \left(\frac{1}{2}+\delta\right) g_1(t) + \widetilde{g_1}(t), \\
\frac{dg_2}{dt}(t) &= \frac{\lambda}{2} g_0(t) - \left(\frac{1}{2}+\delta\right) g_2(t) + \widetilde{g_2}(t).
\end{aligned}$$

In steady state, all $g_i(t)$ and $\widetilde{g_i}(t)$ are constant. Furthermore, if the share of biased individuals is the same across ages then $g_i(t) = \frac{1}{\delta}\widetilde{g_i}(t)$. (Note that, for a death rate δ, the total mass of high-skill workers alive is given by $\delta^{-1}\overline{H}$, where $\overline{H}$ is the size of an incoming generation.) We can now solve the system to obtain that, in steady state,

$$\widetilde{g_0} = \frac{1}{1+2\lambda},\ \widetilde{g_1} = \widetilde{g_2} = \frac{\lambda}{1+2\lambda}.$$

The workers who switch sectors between t and $t + dt$ are the normal workers who become biased toward working in a different sector than the one in which they are employed in at time t. Therefore, a share $\frac{\lambda}{2}\frac{1}{1+2\lambda}$ (per unit of time) of workers switches sectors.

B.3.2 Step 2: System satisfied by the value functions of workers

Recall that our assumption that nonpecuniary benefits are sufficiently large that a biased worker in state i always works in sector i. Denote by $W_0^j(t, a)$ the value function of a low-skill worker of experience a at time t in the *normal* state in sector j, and denote by $W_i(t, a)$ the value function of a low-skill worker of age a at time t in the *biased* state $i \in \{1, 2\}$ (and so, by assumption, in sector i). The set of value functions must then satisfy

$$W_1(t,a) = (w_1(t)\, x_L(a) + b)\, dt + \left(1 - \frac{1}{2}dt - \delta dt\right) W_1(t+dt, a+dt) + \frac{1}{2} dt W_0^1(t+dt, a+dt). \tag{B.14}$$

Intuitively, biased workers in state 1 work in sector 1, receive wages $w_1(t)\,x_L(a)$ and enjoy the nonpecuniary benefit b, die with probability δdt, and revert back to the normal state with probability $\frac{1}{2}dt$ (without switching sectors). Similarly, we have

$$W_2(t,a) = (w_2(t)\,x_L(a) + b)\,dt + \left(1 - \frac{1}{2}dt - \delta dt\right) W_2(t+dt, a+dt) + \frac{1}{2}dt W_0^2(t+dt, a+dt). \tag{B.15}$$

Workers in the normal state can become biased with probability $\frac{\lambda}{2}dt$ for each biased state. If they remain unbiased, they decide rationally whether or not switch sectors. Irrespective of the reason, a worker who switches sectors loses all her accumulated experience. Hence the value function for unbiased workers in sector 1 is

$$\begin{aligned} W_0^1(t,a) &= w_1(t)\,x_L(a)\,dt + (1 - \lambda dt - \delta dt)\max\left(W_0^1(t+dt, a+dt), W_0^2(t+dt, 0)\right) \\ &\quad + \frac{\lambda}{2}dt W_1(t+dt, a+dt) + \frac{\lambda}{2}dt W_2(t+dt, 0), \end{aligned} \tag{B.16}$$

and similarly for sector 2:

$$\begin{aligned} W_0^2(t,a) &= w_2(t)\,x_L(a)\,dt + (1 - \lambda dt - \delta dt)\max\left(W_0^1(t+dt, 0), W_0^2(t+dt, a+dt)\right) \\ &\quad + \frac{\lambda}{2}dt W_1(t+dt, 0) + \frac{\lambda}{2}dt W_2(t+dt, a+dt). \end{aligned} \tag{B.17}$$

In steady state, $w_1(t) = w_2(t) = w$, the problem is stationary and normal workers never switch sectors. Define $\overrightarrow{W} \equiv \begin{pmatrix} W_1 & W_0^1 & W_2 & W_0^2 \end{pmatrix}$, and add a superscript ^{ss} for the steady-state values ($\overrightarrow{W^{ss}}$ depends only on experience, not on time). The problem then simplifies to

$$\frac{d}{da}\overrightarrow{W^{ss}}(a) = B\overrightarrow{W^{ss}}(a) - \begin{pmatrix} wx_L(a) + b & \frac{\lambda}{2}W_2^{ss}(0) + wx_L(a) & wx_L(a) + b & \frac{\lambda}{2}W_1^{ss}(0) + wx_L(a) \end{pmatrix}^T,$$

where

$$B = \begin{pmatrix} \frac{1}{2} + \delta & -\frac{1}{2} & 0 & 0 \\ -\frac{\lambda}{2} & \lambda + \delta & 0 & 0 \\ 0 & 0 & \frac{1}{2} + \delta & -\frac{1}{2} \\ 0 & 0 & -\frac{\lambda}{2} & \lambda + \delta \end{pmatrix}.$$

This matrix is diagonalizable with eigenvalues given by λ_1 and λ_2, where $\lambda_1 \equiv \frac{1}{4} + \delta + \frac{1}{2}\lambda - \frac{1}{4}\left(1 + 4\lambda^2\right)^{\frac{1}{2}}$ and $\lambda_2 \equiv \frac{1}{4} + \delta + \frac{1}{2}\lambda + \frac{1}{4}\left(1 + 4\lambda^2\right)^{\frac{1}{2}}$ are both positive. The solution to such a linear system of differential equations can be written as

$$\overrightarrow{W^{ss}}(a) = e^{Ba}\overrightarrow{W^{ss}}(0) - we^{Ba}\int_0^a x_L(s)\,e^{-Bs}\overrightarrow{1}ds + B^{-1}\left(I - e^{Ba}\right)\begin{pmatrix} b & \frac{\lambda}{2}W_2^{ss}(0) & b & \frac{\lambda}{2}W_1^{ss}(0) \end{pmatrix}^T,$$

where $\overrightarrow{1} = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}^T$. We assume that the accumulation function x_L is such that $\lim_{s\to\infty}\left\|x_L(s)\,e^{-Bs}\right\| = 0$, and we define $\overrightarrow{X_L}(a) \equiv -\int_a^\infty x_L(s)\,e^{-Bs}\overrightarrow{1}ds$. Since $\overrightarrow{W^{ss}}$ must remain bounded and e^{Ba} is unbounded, we get the following condition:

$$\overrightarrow{W^{ss}}(0) = -w\overrightarrow{X_L}(0) + B^{-1}\begin{pmatrix} b & \frac{\lambda}{2}W_2^{ss}(0) & b & \frac{\lambda}{2}W_1^{ss}(0) \end{pmatrix}^T. \tag{B.18}$$

Solving for this system defines $\overrightarrow{W^{ss}}(0)$, after which we can write

$$\overrightarrow{W^{ss}}(a) = -we^{Ba}\overrightarrow{X_L}(a) + B^{-1}\begin{pmatrix} b & \frac{\lambda}{2}W_2^{ss}(0) & b & \frac{\lambda}{2}W_1^{ss}(0) \end{pmatrix}^T. \tag{B.19}$$

B.3.3 Step 3: Equilibrium following a small price shock

We assume the existence of an equilibrium with a structure similar to the equilibrium described in Proposition 1—that is, in which normal workers switch only upon impact and the wages of high-skill (resp. low-skill) workers are equalized after a time t_2 (resp. t_1). A continuous version of the system of equations (B.14), (B.15), (B.16), and (B.17) exists and can be written as

$$\begin{aligned}
&\frac{\partial}{\partial a}\overrightarrow{W}(t,a) + \frac{\partial}{\partial t}\overrightarrow{W}(t,a) \qquad\qquad \text{(B.20)}\\
= \; & B\overrightarrow{W}(t,a) - B\overrightarrow{W}(t,a)\\
& -\begin{pmatrix} w_1(t)\,x_L(a) + b & \frac{\lambda}{2}W_2(t,0) + w_1(t)\,x_L(a) & w_2(t)\,x_L(a) + b & \frac{\lambda}{2}W_1(t,0) + w_2(t)\,x_L(a) \end{pmatrix}^T.
\end{aligned}$$

The worker from sector 2 who is indifferent between staying in that sector or switching will have an experience level given by a_L, which satisfies

$$W_0^2(0,a_L) = W_0^1(0,0).$$

As before, after $\max(t_1,t_2)$ all wages are constant at their steady-state value; hence $\overrightarrow{W}$ is also constant over time at its new steady-state value. Without loss of generality, we assume that $t_1 < t_2$. Then

$$\begin{aligned}
W_0^1(t_2,t_2) - W_0^1(0,0) &= \int_0^{t_2}\left(\frac{\partial W_1}{\partial t} + \frac{\partial W_1}{\partial a}\right)d\tau\\
&= \int_0^{t_2}\left((\lambda+\delta)\,W_0^1(\tau,\tau) - \frac{\lambda}{2}W_1(\tau,\tau) - \frac{\lambda}{2}W_2(\tau,0) - w_1(\tau)\,x_L(\tau)\right)d\tau,
\end{aligned}$$

where we have used (B.20). A first-order Taylor expansion of $W_0^1(\tau,\tau)$, $W_1(\tau,\tau)$, and $W_2(\tau,0)$ of order 1 around (t_2,t_2), (t_2,t_2), and $(t_2,0)$, respectively, gives

$$\begin{aligned}
&W_0^1(t_2,t_2) - W_0^1(0,0)\\
= \; & t_2\left((\lambda+\delta)\,W_0^1(t_2,t_2) - \frac{\lambda}{2}W_1(t_2,t_2) - \frac{\lambda}{2}W_2(t_2,0)\right)\\
& -\left((\lambda+\delta)\left(\frac{\partial W_0^1}{\partial t}(t_2,t_2) + \frac{\partial W_0^1}{\partial a}(t_2,t_2)\right) - \frac{\lambda}{2}\left(\frac{\partial W_1}{\partial t}(t_2,t_2) + \frac{\partial W_1}{\partial a}(t_2,t_2)\right) - \frac{\lambda}{2}\left(\frac{\partial W_2}{\partial t}(t_2,0)\right)\right)\frac{t_2^2}{2}\\
& -\int_0^{t_2} w_1(\tau)\,x_L(\tau)\,d\tau + o\left(t_2^2\right).
\end{aligned}$$

After t_2, $\overrightarrow{W}$ is independent of time: $\frac{\partial W_0^1}{\partial t}(t_2, t_2) = \frac{\partial W_1}{\partial t}(t_2, t_2) = \frac{\partial W_2}{\partial t}(t_2, 0) = 0$. Using superscript $^{ss\prime}$ to denote the new steady state, we can now rewrite the previous equation as

$$W_1^0(t_2, t_2) - W_1(0,0) = t_2\left((\lambda+\delta) W_0^{1ss\prime}(t_2) - \frac{\lambda}{2} W_1^{ss\prime}(t_2) - \frac{\lambda}{2} W_2^{ss\prime}(0)\right)$$
$$- \left((\lambda+\delta)\frac{dW_0^{1ss\prime}}{da}(t_2) - \frac{\lambda}{2}\frac{dW_1^{ss\prime}}{da}(t_2)\right)\frac{t_2^2}{2} - \int_0^{t_2} w_1(\tau)\, x_L(\tau)\, d\tau + o\left(t_2^2\right).$$

Similarly, we have

$$\begin{aligned}
& W_0^2(t_2, t_2+a) - W_0^2(0,a) \\
= \; & t_2\left((\lambda+\delta) W_0^{2ss\prime}(t_2+a) - \frac{\lambda}{2} W_2^{ss\prime}(t_2+a) - \frac{\lambda}{2} W_1^{ss\prime}(0)\right) \\
& - \left((\lambda+\delta)\frac{dW_0^{2ss\prime}}{da}(t_2+a) - \frac{\lambda}{2}\frac{dW_2^{ss\prime}}{da}(t_2+a)\right)\frac{t_2^2}{2} - \int_0^{t_2} w_2(\tau)\, x_L(\tau+a)\, d\tau + o\left(t_2^2\right).
\end{aligned}$$

Next we take the difference between the two equations and use that $W_0^{1ss\prime} = W_0^{2ss\prime}$ and $W_1^{ss\prime} = W_2^{ss\prime}$ to obtain

$$\begin{aligned}
& W_0^1(t_2, t_2) - W_1(0,0) - \left(W_0^2(t_2, t_2+a) - W_0^2(0,a)\right) \\
= \; & t_2\left((\lambda+\delta)\left(W_0^{1ss\prime}(t_2) - W_0^{1ss\prime}(t_2+a)\right) - \frac{\lambda}{2}\left(W_1^{ss\prime}(t_2) - W_1^{ss\prime}(t_2+a)\right)\right) \\
& + \left(\left((\lambda+\delta)\left(\frac{dW_0^{1ss\prime}}{da}(t_2+a) - \frac{dW_0^{1ss\prime}}{da}(t_2)\right) - \frac{\lambda}{2}\left(\frac{dW_1^{ss\prime}}{da}(t_2+a) - \frac{dW_1^{ss\prime}}{da}(t_2)\right)\right)\right)\frac{t_2^2}{2} \\
& + \int_0^{t_2}\left(w_2(\tau)\, x_L(\tau+a) - w_1(\tau)\, x_L(\tau)\right) d\tau + o\left(t_2^2\right).
\end{aligned}$$

Consider the indifferent worker. Since a_L must be at most first order and since $W_1(0,0) = W_0^2(0, a_L)$ by definition, it follows that

$$\begin{aligned}
W_0^{1ss\prime}(t_2) - W_0^{1ss\prime}(t_2+a_L) \; = \; & -t_2 a_L\left((\lambda+\delta)\frac{dW_0^{1ss\prime}}{da}(t_2) - \frac{\lambda}{2}\frac{dW_1^{ss\prime}}{da}(t_2)\right) \\
& + \int_0^{t_2}\left(w_2(\tau)\, x_L(\tau+a_L) - w_1(\tau)\, x_L(\tau)\right) d\tau + o\left(t_2^2\right) + o\left(a_L t_2\right).
\end{aligned}$$

Now if a_L were first order, then the LHS would have the same order as a_L while the RHS would be of order 2 at most (because $w_2(\tau)\, x_L(\tau+a) - w_1(\tau)\, x_L(\tau)$ is of first order in the price change). Since this is impossible, a_L must be of second order. As a result, the previous equation simplifies to

$$W_0^{1ss\prime}(t_2) - W_0^{1ss\prime}(t_2+a_H) = \int_0^{t_2}\left(w_2(\tau)\, x_L(\tau+a_L) - w_1(\tau)\, x_L(\tau)\right) d\tau + o\left(dp^2\right).$$

Now using that t_2 is at most first order—so that $t_2\left(\frac{dW_0^{1ss\prime}}{da}(0) - \frac{dW_0^{1ss\prime}(a_L)}{da}\right)$ would be of third order and wages would be equalized after t_1—we have

$$W_0^{1ss\prime}(0) - W_0^{1ss\prime}(a_L) = x_L(0)\int_0^{t_1}\left(w_2(\tau) - w_1(\tau)\right) d\tau + o\left(dp^2\right). \qquad \text{(B.21)}$$

Equation (B.19) yields

$$\begin{aligned}\overrightarrow{W^{ss\prime}}(a_L) - \overrightarrow{W^{ss\prime}}(0) &= w^{ss\prime}\left(\int_0^\infty \left(x_L(s+a_L) - x_L(s)\right) e^{-Bs}\overrightarrow{1}\,ds\right) \\ &= a_L w^{ss\prime}\left(\int_0^\infty x_L'(s)\, e^{-Bs}\overrightarrow{1}\,ds\right) + o\left(dp^2\right)\end{aligned} \tag{B.22}$$

Matrix B can be decomposed as $B = PDP^{-1}$ for

$$P \equiv \begin{pmatrix} 1 & 1 & 0 & 0 \\ \frac{1}{2} - \lambda + \left(\frac{1}{4}+\lambda^2\right)^{\frac{1}{2}} & \frac{1}{2} - \lambda - \left(\frac{1}{4}+\lambda^2\right)^{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & \frac{1}{2} - \lambda + \left(\frac{1}{4}+\lambda^2\right)^{\frac{1}{2}} & \frac{1}{2} - \lambda - \left(\frac{1}{4}+\lambda^2\right)^{\frac{1}{2}} \end{pmatrix}$$

and

$$D \equiv \mathrm{Diag}\left(\lambda_1, \lambda_2, \lambda_1, \lambda_2\right),$$

so (B.22) leads to

$$\begin{aligned} & W_0^{1ss\prime}(a_L) - W_0^{1ss\prime}(0) \\ = \;& a_L w^{ss} \int_0^\infty x_L'(s)\, \frac{1}{2}\left(\left(1 + \left(1+4\lambda^2\right)^{-\frac{1}{2}}\right) e^{-\lambda_1 s} + \left(1 - \left(1+4\lambda^2\right)^{-\frac{1}{2}}\right) e^{-\lambda_2 s}\right) ds + o\left(dp^2\right). \end{aligned} \tag{B.23}$$

As in Section A.1, we can establish the first-order development of changes in the effective mass of low-skill workers in each sector to obtain

$$dl_1(t) = -dl_2(t) = (1-n_L)\,\frac{1}{1+2\lambda} x_L(0)\, t + o(dp) \text{ for } 0 \le t \le t_1,\, t_2. \tag{B.24}$$

The difference between (B.24) and (A.2) arises because only a fraction $(1+2\lambda)^{-1}$ of the new-born generation can be allocated to either sectors (the other workers are in biased states). A similar expression holds for dh_1 and dh_2. This directly leads to (13), and (B.21), (B.23), and (B.24) together imply (14). Following the same steps as in Section A.1, we can then derive

$$t_2 = \frac{\left(v^{ss} - \frac{v_{1L}+v_{2L}}{w_{1L}+w_{2L}} w^{ss}\right)(1+2\lambda)}{(1-n_H)\, x_H(0)\, \overline{H}\left(v_{1H} + v_{2H} - \frac{(v_{1L}+v_{2L})(w_{1H}+w_{2H})}{w_{1L}+w_{2L}}\right)} \frac{dp}{p} + o(dp),$$

$$a_H = -\frac{x_H(0)\left(t_2 - \frac{w^{ss}(v_{1L}+v_{2L})}{v^{ss}(w_{1L}+w_{2L})}(t_2 - t_1)\right)}{\int_0^\infty x_H'(s)\left(\left(1 + \left(1+4\lambda^2\right)^{-\frac{1}{2}}\right) e^{-\lambda_1 s} + \left(1 - \left(1+4\lambda^2\right)^{-\frac{1}{2}}\right) e^{-\lambda_2 s}\right) ds} \frac{dp}{p} + o\left(dp^2\right).$$

Finally, the same reasoning as in Section A.1 shows that this is indeed an equilibrium.