

Growth, Trade, and Inequality*

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Abstract

We introduce firm and worker heterogeneity into a model of innovation-driven endogenous growth. Individuals who differ in ability sort into either a research activity or a manufacturing sector. Research projects generate new varieties of a differentiated product. Projects differ in quality and the resulting technologies differ in productivity. In both sectors, there is a complementarity between firm quality and worker ability. We study the co-determination of growth and income inequality in both the closed and open economy, as well as the spillover effects of policy in one country to outcomes in others.

Keywords: endogenous growth, innovation, income distribution, income inequality, trade and growth

JEL Classification: D33, F12, F16, O41

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1 Introduction

The relationship between growth and income distribution has been much studied. Researchers have identified several channels through which inequality might affect growth, such as if rich and poor households differ in their propensity to save (Kaldor, 1955-56), if poor households face credit constraints that limit their ability to invest in human capital (Galor and Zeira, 1993), or if greater inequality generates more redistribution and thus a different incentive structure via the political process (Alesina and Rodrik, 1994; Persson and Tabellini, 1994). Growth might affect distribution if the activities that drive growth make more intensive use of skilled labor than do other activities in the economy (Grossman and Helpman, 1991).

In this paper, we propose a novel mechanism that links distribution to growth, one that has not previously been considered in the literature. In an environment with heterogeneous workers and heterogeneous firms, markets provide incentives for certain types of workers to *sort* to certain activities and for the workers in a sector to *match* with certain types of firms. The fundamental forces that drive growth also determine the composition of worker types in each activity and thereby influence the matching of workers to firms. In this way of thinking, growth does not cause inequality, nor does inequality influence growth, but rather the two outcomes are jointly determined. We examine several potential determinants of growth and inequality, such as the productivity of an economy's manufacturing operations, its capacity for innovation, and its policies to promote R&D. Since we know from previous work that the extent of international integration and the policies that govern trade can have important influences on growth, we also investigate how the mechanism of sorting and matching of heterogeneous workers operates in an open economy.

We introduce our mechanism in a simple and stylized setting—although we believe that it would operate as well in a wide variety of growth models with heterogeneous workers and heterogeneous firms. We imagine that the economy undertakes two distinct activities that we refer to abstractly as *idea creation* and *idea using*. Our mechanism rests on two key assumptions. First, among a group of workers with heterogeneous abilities, greater ability confers a comparative advantage in creating ideas relative to using ideas. This implies rather directly that the more able types will sort into the idea-creating activity. Second, when research or production takes place, there exists a complementarity between the quality of an idea and the ability of the workers that implement the idea. As a consequence, there is positive assortative matching between heterogeneous firms and heterogeneous workers in both sectors of the economy. The forces that affect the sizes of the two sectors also affect the composition of workers in each sector and thereby affect the matching of workers with firms.

In our model, as in Romer (1990), the accumulation of knowledge serves as the engine of growth. Knowledge is treated as a by-product of purposive innovation undertaken to develop new products. Our treatment of trade, international knowledge diffusion, and growth extends the simplest, one-sector model from Grossman and Helpman (1991).¹ The advantage of the framework we develop

¹In Grossman and Helpman (1991), we devote several chapters to models with two or more industrial sectors in order to address the impact of intersectoral resource allocation on growth and relative factor prices. By considering

here is that it focuses on the new mechanism and allows us to consider the entire distribution of earnings that emanates from a given distribution of worker abilities and firm productivity levels, and not just, say, the skill premium (i.e., the relative wage of “skilled” versus “unskilled” workers), which has been the focus of much of the existing theoretical literature.

In the next section, we develop the model in the context of a closed economy. A country has a fixed endowment of research capital and a fixed supply of labor with an exogenous distribution of abilities. The economy assembles a single consumption good from differentiated intermediate inputs. Blueprints for the intermediate goods result from R&D services that are purchased by firms that engage in monopolistic competition. These firms have access to different technologies and can hire workers of any ability. A firm’s total output is the sum of what is produced by its various employees and the productivity of any employee depends on his ability and on the firm’s technology. Ability and technology are complementary, so that, in equilibrium, the firms that have access to the better technologies hire the more able workers.

Innovation drives growth. Entrepreneurs rent research capital to pursue their research ideas. Once an entrepreneur has done so, she learns the quality of her project. An entrepreneur accrues “R&D services” at a rate that depends on the quality of her project, the ability of the researchers she employs, and the stock of knowledge capital available in the economy. Knowledge accumulates with research experience and is non-proprietary, as in Romer (1990). R&D services can be converted into designs for new varieties of the differentiated project. Each design comes with a random draw of a production technology, so that some manufacturing firms ultimately operate sophisticated technologies and others simpler technologies. There is free entry in both sectors of the economy. Expected returns are zero, although the lucky entrepreneurs (those that draw above average research ideas) and the lucky manufacturers (those that draw above average production technologies) earn positive profits, while the others do not fully cover their fixed costs.

In equilibrium, all individuals with ability above some endogenous cutoff level sort into the research sector. They are hired there by the heterogeneous labs according to their ability. Similarly, for those who enter the manufacturing sector, there is endogenous matching between firms and employees. The complementarity between ability and technology delivers positive assortative matching in both sectors. These competitive forces of sorting and matching dictate the economy’s wage distribution. We focus the analysis on the resulting inequality of wages.

After developing the model, we show how the long-run growth rate and wage distribution are co-determined in a long-run equilibrium. More specifically, we derive a pair of equations that jointly determine the steady-state growth rate in the number of varieties and the cutoff ability level that divides manufacturing workers from inventors. Once we know the growth rate of intermediate varieties, we can calculate the growth rate of final output and the growth rate of wages. Once we know the cutoff ability level, we can calculate the entire distribution of relative wages.

In Section 3, we compare growth rates and wage inequality across countries that differ in their

here a model with one industrial sector, we neglect this important, additional channel for trade to influence growth and income distribution.

technological parameters and policy choices. In this section, we focus on isolated countries that do not trade and do not capture any knowledge spillovers from abroad. We show that Hicks-neutral differences in labor productivity in manufacturing that apply across the full range of ability levels do not generate long-run differences in growth rates or wage inequality, although they do imply differences in income and consumption levels. In contrast, differences in “innovation capacity” generate differences in growth and inequality. Innovation capacity reflects a parameter that measures the size of a country’s labor force, a parameter that reflects its ability to convert research experience into knowledge capital, a parameter that reflects inventor’s productivity in generating new ideas, and a parameter that measures a country’s stock of research capital. A country with greater innovation capacity grows faster in autarky but experiences greater wage inequality. Subsidies to R&D financed by proportional wage taxes also contribute to faster growth but greater inequality.

Section 4 addresses the impacts of globalization. Here, intermediate inputs are tradable subject to arbitrary iceberg trading costs and import tariffs. We follow Grossman and Helpman (1991) by introducing international sharing of knowledge capital and, in fact, allow for an arbitrary pattern of (positive) international spillovers. In particular, the knowledge stock in each country is a weighted sum of accumulated innovation experience in all countries including itself, with an arbitrary matrix of weighting parameters. We study a balanced-growth equilibrium in which the number of varieties of intermediate goods grows at the same constant rate in all countries. Even allowing for a wide range of differences in technologies and policies, we find that the long-run growth rate is higher in every country in the trading equilibrium than in autarky, but so too is the resulting wage inequality. Neither differences in manufacturing productivity, in trade frictions, or in innovation capacity generate any cross-nation differences in wage inequality. In fact, no matter what the pattern of international knowledge spillovers, if R&D subsidies are the same in a pair of countries and their inventors draw from the same technology distributions, their relative-wage distributions will converge in the long-run.² Differences in support for R&D do give rise to long-run differences in wage inequality, as a higher subsidy goes hand in hand with a greater spread in wages. In Section 4, we also examine how various policy and parameter changes affect long-run growth and inequality measures in the open economy both at home and abroad. For example, we show that an increase in the R&D subsidy rate in any country accelerates growth and raises inequality in all of them, as does an improvement in a country’s ability to absorb knowledge spillovers from abroad.

In this paper, we do not conduct any empirical tests for the operation of our mechanism, nor do we attempt to quantify its significance. In general, attempts to substantiate the operation of mechanisms linking inequality to growth have been hampered by inadequate data and methodological pitfalls. Kuznets (1955, 1963), for example, famously advanced the hypothesis that income inequality first rises then falls over the course of economic development. While the “Kuznets curve”—an inverted-U shaped relationship between inequality and stage of development—has been established for the small set of countries that Kuznets considered, subsequent studies using broader data sets

²Note, however, that the levels of all wages can vary across countries to reflect local conditions.

cast doubt on the ubiquity of this relationship.³ More generally, empirical assessment of the links between distribution and growth has proven elusive due to the fact that a country’s growth rate and its income inequality are jointly determined and there are few if any exogenous variables to serve as instruments for identifying causal relationships.⁴ It might be possible to calibrate a growth model to get a sense of the relative quantitative significance of various mechanisms that link distribution with growth, but the model that we have presented here is too simple for calibration purposes. We have chosen the simple (and familiar) specification in order to present starkly the mechanism that we have in mind, and leave quantification of the mechanism for future research.

2 The Basic Model

We develop a model of economic growth featuring heterogeneous workers, heterogeneous firms, and heterogeneous research opportunities. In the model, endogenous innovation drives growth. Workers that differ in ability engage either in creating ideas or using ideas. In keeping with the literature, we refer to the creation of ideas as “R&D” and the implementation of ideas as “manufacturing,” although we prefer not to interpret these terms too narrowly. Research generates new varieties of differentiated intermediate inputs. Firms that produce these inputs operate different technologies. In the equilibrium, the heterogeneous workers sort into one of the two activities and firms and research labs with different technologies hire different types of workers. The economy converges to a long-run equilibrium with a constant growth rate of final output and a fixed and continuous distribution of wage income.

We describe here the economic environment for a closed economy and defer the introduction of international trade until Section 4.

2.1 Demand and Supply for Consumption Goods

The economy is populated by a mass N of individuals indexed by ability level, a . The cumulative distribution of abilities is given by $H(a)$, which is twice continuously differentiable and has a positive density $H'(a) > 0$ on the bounded support, $[a_{\min}, a_{\max}]$.

Each individual maximizes a logarithmic utility function,

$$u_t = \int_t^\infty e^{-\rho(\tau-t)} \log c_\tau d\tau, \tag{1}$$

where c_τ is consumption at time τ and ρ is the common, subjective discount rate. The consumption good serves as numeraire; its price at every moment is normalized to one. It follows from the individual’s intertemporal optimization problem that

$$\frac{\dot{c}_t}{c_t} = \iota_t - \rho, \tag{2}$$

³See Helpman (2004, ch.4) for a survey of this evidence.

⁴A similar problem has plagued attempts to assess the relationship between trade and growth (see Helpman, 2004, ch.6).

where ι_t is the interest rate at time t in terms of consumption goods. Inasmuch as a varies across individuals, so does income and consumption.

Consumption goods are assembled from an evolving set Ω_t of differentiated intermediate inputs. Dropping the time subscript for notational convenience, the production function for these goods at a moment when the set of available inputs is Ω is given by

$$X = \left[\int_{\omega \in \Omega} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \quad (3)$$

where $x(\omega)$ is the input of variety ω . The elasticity of substitution between intermediate inputs is constant and equal to σ .

The market for consumption goods is competitive. It follows that the equilibrium price of these goods reflects the minimum unit cost of producing them. Since X is the numeraire, we have

$$\left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} = 1, \quad (4)$$

where $p(\omega)$ is the price of intermediate input ω .

2.2 Supply, Demand, Pricing, and Profits of Intermediate Goods

Once an intermediate good has been invented, it is produced by monopolistically-competitive firms using labor as the sole input. Firms that manufacture these goods are distinguished by their technology, φ . A firm with a higher φ is more productive, no matter what type(s) of workers it hires. Consider a firm that produces variety ω using technology φ and that hires a set L_ω of workers types with densities $\ell_\omega(a)$. In such circumstances, the firm's output is

$$x(\omega) = \int_{a \in L_\omega} \psi(\varphi, a) \ell_\omega(a) da, \quad (5)$$

where $\psi(\varphi, a)$ is the productivity of workers of type a when applying technology φ . Notice that productivity (given φ) is independent of ω .

We suppose that more productive technologies are also more complex and that more able workers have a comparative advantage in operating the more complex technologies. In other words, we posit a complementarity between the type of technology φ and the type of worker a in determining labor productivity. Formally, we adopt

Assumption 1 The productivity function $\psi(\varphi, a)$ is twice continuously differentiable, strictly increasing, and strictly log supermodular.

Assumption 1 implies $\psi_{\varphi a} > 0$ for all φ and a .

As is known from Costinot (2009), Eeckhout and Kircher (2012), Sampson (2014) and elsewhere, the strict log supermodularity of $\psi(\cdot)$ implies that, for any upward-sloping wage schedule $w(a)$, each manufacturing firm hires the particular type of labor that is most appropriate given its technology

φ , and there is *positive assortative matching* (PAM) between firm types and worker types. We denote by $m(\varphi)$ the ability of workers employed by firms that produce a variety of intermediate by operating a technology φ ; PAM is reflected in the fact that $m'(\varphi) > 0$.

Shephard's lemma gives the demand for any variety ω as a function of the prices of all available intermediate goods, namely

$$x(\omega) = X \left[\int_{v \in \Omega} p(v)^{1-\sigma} dv \right]^{\frac{\sigma}{1-\sigma}} p(\omega)^{-\sigma}.$$

In view of (4), demand for variety ω can be expressed as

$$x(\omega) = X p(\omega)^{-\sigma} \quad \text{for all } \omega \in \Omega. \quad (6)$$

Each firm takes aggregate output of final goods X as given and so it perceives a constant elasticity of demand, $-\sigma$. As is usual in such settings, the profit-maximizing firm applies a fixed percentage markup to its unit cost.

Considering the optimal hiring decision, a firm that operates a technology φ has productivity $\psi[\varphi, m(\varphi)]$ and pays a wage $w[m(\varphi)]$. Hence, the firm faces a minimal unit cost of $w[m(\varphi)]/\psi[\varphi, m(\varphi)]$. The firm's profit-maximizing price is given by⁵

$$p(\varphi) = \left(\frac{\sigma}{\sigma - 1} \right) \frac{w[m(\varphi)]}{\psi[\varphi, m(\varphi)]}. \quad (7)$$

This yields an operating profit of

$$\pi(\varphi) = \sigma^{-\sigma} (\sigma - 1)^{(\sigma-1)} X \left\{ \frac{w[m(\varphi)]}{\psi[\varphi, m(\varphi)]} \right\}^{1-\sigma}. \quad (8)$$

2.3 Inventing New Varieties

Entrepreneurs hire workers and capital to pursue research projects. When an entrepreneur contemplates a new project, she does not know its quality, q . At this stage, she perceives q as being drawn from some cumulative distribution function for project types, $G_R(q)$, with $G'_R(q) > 0$ on a bounded support $[q_{\min}, q_{\max}]$. Each project requires f units of research capital. Once an entrepreneur has rented the requisite capital to undertake her project, she discovers its quality. She then hires some number $\ell_R(a)$ of workers of some ability level a to carry out the research, paying the equilibrium wage, $w(a)$.

Projects generate ‘‘R&D services.’’ The volume of services that results from a project depends upon its quality, the number of researchers engaged in the project, their ability, and the state of knowledge in the economy. We follow Romer (1990) in assuming that knowledge accumulates as a by-product of research experience. The knowledge stock at time t is $\theta_K M_t$, where M_t is the mass

⁵We henceforth index intermediate goods by the technology with which they are produced (φ) rather than their variety name (ω), since all varieties are symmetric except for their different technologies.

of varieties that have developed before time t and θ_K is a parameter that reflects how effectively the economy converts cumulative research experience into applicable knowledge. The output of a research project of quality q that employs $\ell_R(a)$ workers with ability in the interval $[a, a + da]$ when the state of knowledge is $\theta_K M$ is given by $\theta_K M \psi_R(q, a) \ell_R(a)^\gamma da$, where $\psi_R(q, a)$ captures a complementarity between project quality and worker ability in determining innovation productivity. In particular, we adopt the following assumption, analogous to Assumption 1.

Assumption 2 Research productivity $\psi_R(q, a)$ is twice continuously differentiable, strictly increasing, and strictly log supermodular.

In equilibrium, the workers with type $m_R(q)$ work on projects of quality q . Assumption 2 ensures PAM in the research sector, so that $m'_R(q) > 0$.

Let K_R be the economy's fixed endowment of research capital and define $R \equiv K_R/f$. Then R gives the measure of active research projects at any point in time. This fixed quantity does not pin down the innovation rate in the economy, however, because the scale and productivity of the research labs are determined endogenously in equilibrium.

Manufacturing firms buy R&D services from research entrepreneurs at the price p_R . One unit of R&D services generates a design for a differentiated intermediate good along with an independent draw from a cumulative technology distribution, $G(\varphi)$, as in Melitz (2003). The technology parameter φ determines the complexity and productivity of the technology, as described in Section 2.2 above.

2.4 Free Entry

Entrepreneurs and firms can enter freely into research and manufacturing. A research entrepreneur must pay rf to rent the capital needed to carry out a project, where r is the equilibrium rental rate. The investment yields an expected return of $E\pi_R$, where

$$E\pi_R = \int_{q_{\min}}^{q_{\max}} \pi_R(q) dG_R(q)$$

and

$$\pi_R(q) = \max_{a, \ell_R} [p_R \theta_K M \psi_R(q, a) \ell_R^\gamma - w(a) \ell_R]$$

is the maximal profit for a research lab of quality q . Since entrepreneurs with projects of quality q hire researchers with ability $m_R(q)$, we have⁶

$$\pi_R(q) = (1 - \gamma) \gamma^{\frac{\gamma}{1-\gamma}} \{p_R \theta_K M \psi_R[q, m_R(q)] w[m_R(q)]^{-\gamma}\}^{\frac{1}{1-\gamma}}. \quad (9)$$

⁶We derive the maximal research project for an entrepreneur with a project of quality q by choosing $\ell_R(q, a)$ according to the first-order condition,

$$\ell_R(q, a) = \left[\frac{\gamma p_R \theta_K M \psi_R(q, a)}{w(a)} \right]^{\frac{1}{1-\gamma}},$$

and substituting this expression for optimal employment into the expression for operating profits.

Free-entry by entrepreneurs implies

$$rf = (1 - \gamma) \gamma^{\frac{\gamma}{1-\gamma}} (p_R \theta_K M)^{\frac{1}{1-\gamma}} \int_{q_{\min}}^{q_{\max}} \psi_R [q, m_R(q)]^{\frac{1}{1-\gamma}} w [m_R(q)]^{-\frac{\gamma}{1-\gamma}} dG_R(q),$$

which determines r .

Similarly, a manufacturing firm pays p_{Rt} to purchase the R&D services needed to introduce a variety of intermediate good at time t . If it draws a manufacturing technology φ , it will earn a stream of profits $\pi_\tau(\varphi)$ for all $\tau \geq t$. We have derived the expression for operating profits and recorded it (with time index suppressed) in (8). On a balanced-growth path, wages of all types of workers grow at the common rate g_w and final output grows at a constant rate g_X . Final output serves only consumption, so, by (2), $g_X = \iota - \rho$. Operating profits also grow at a constant rate g_π , independent of φ , and, by (8), $g_\pi = g_X - (\sigma - 1)g_w$. Finally, (4) and (7) imply that, in a steady state, $(\sigma - 1)g_w = g_M$. Combining these long-run relationships, the expected discounted profits for a new manufacturing firm at time t can be written as

$$\int_t^\infty e^{-\iota(\tau-t)} \int_{\varphi_{\min}}^{\varphi_{\max}} \pi_\tau(\varphi) dG(\varphi) d\tau = \frac{\int_{\varphi_{\min}}^{\varphi_{\max}} \pi_t(\varphi) dG(\varphi)}{\rho + g_M}.$$

Equating the cost of R&D services to the expected discounted value of a new product, and again dropping the time subscript, we have

$$p_R = \frac{\int_{\varphi_{\min}}^{\varphi_{\max}} \pi(\varphi) dG(\varphi)}{\rho + g_M}. \quad (10)$$

2.5 Sorting, Matching, and Labor-Market Equilibrium

Individuals choose employment in either research or manufacturing. In so doing, they compare the wages they can earn (given their ability) in the alternative occupations. Let $w_M(a)$ be the wage paid to employees in the manufacturing sector and let $w_R(a)$ be the wage paid to those entering research. To identify the equilibrium sorting pattern, we make use of two lemmas that characterize the wage schedules in the two sectors. First, we have

Lemma 1 *Consider any closed interval of workers $[a', a'']$ that is employed in the manufacturing sector in equilibrium. In the interior of this interval, wages must satisfy*

$$\frac{w'_M(a)}{w_M(a)} = \frac{\psi_a [m^{-1}(a), a]}{\psi [m^{-1}(a), a]} \text{ for all } a \in (a', a''), \quad (11)$$

where $m^{-1}(\cdot)$ is the inverse of $m(\cdot)$.

The lemma reflects the requirement that, in equilibrium, the firm with productivity φ must prefer to hire the worker with ability $m(\varphi)$ than any other worker. The lemma follows from the first-order condition for the profit-maximizing choice of $a = m(\varphi)$; it says that, the shape of the wage

schedule mirrors the rise in productivity as a function of ability, with productivity evaluated at the equilibrium match. In the event, no firm will have any incentive to upgrade or downgrade its labor force.

The second lemma applies to the research sector, and has a similar logic.

Lemma 2 *Consider any closed interval of workers $[a', a'']$ that is employed in the R&D sector in equilibrium. In the interior of this interval, the wage schedule must satisfy*

$$\frac{w'_R(a)}{w_R(a)} = \frac{\psi_{Ra} [m_R^{-1}(a), a]}{\gamma \psi_R [m_R^{-1}(a), a]} \text{ for all } a \in (a', a''). \quad (12)$$

This lemma expresses a preference on the part of each entrepreneur for the type of researcher that she hires in equilibrium compared to all alternatives. It comes from the first-order condition for maximizing research profits in (9). The shape of the wage schedule in R&D is slightly different from that in manufacturing, because the R&D sector has diminishing returns to employment in a given lab with its fixed research capital, whereas the manufacturing sector exhibits constant returns to scale. The entrepreneur's choice of researcher type reflects not only the direct effect of ability on the productivity shifter, but also the fact that different types imply different employment levels and therefore different diminishing returns; see Grossman et al. (2015) for further discussion of this point in a related setting.

We assume that high-ability workers enjoy a comparative advantage in R&D; in particular, we make

Assumption 3 $\frac{\psi_{Ra}(q,a)}{\gamma \psi_R(q,a)} > \frac{\psi_a(\varphi,a)}{\psi(\varphi a)}$ for all q, φ , and a .

Assumption 3, together with Lemmas 1 and 2, dictate the equilibrium sorting pattern. They ensure that there exists a cutoff ability level a_R such that all workers with ability above a_R are employed in the research sector and all workers with ability below a_R are employed in manufacturing.⁷ In a steady-state equilibrium with positive growth, $a_R < a_{\max}$. In any case, the equilibrium wage schedule, $w(a)$ satisfies

$$w(a) = \begin{cases} w_M(a) & \text{for } a \leq a_R \\ w_R(a) & \text{for } a \geq a_R \end{cases}, \quad (13)$$

with $w_M(a_R) = w_R(a_R)$.

We next derive a pair of differential equations that characterize the matching functions in the two sectors. In the manufacturing sector, the wages paid to all workers with ability less than or equal to some $\tilde{a} = m(\tilde{\varphi})$ matches what the firms with technology indexes less than or equal to $\tilde{\varphi}$ are willing to pay, considering their labor demands. This equation of labor supply and labor

⁷The wage schedule must be everywhere continuous, or else those paying the discretely higher wage will prefer to downgrade slightly. The two lemmas ensure that wages rise faster in the research sector just to the right of any cutoff point, and they rise slower in manufacturing just to the left of any cutoff point. It follows that there can be at most one such cutoff point.

demand implies

$$MX \left(\frac{\sigma}{\sigma - 1} \right)^{-\sigma} \int_{\varphi_{\min}}^{\tilde{\varphi}} \left\{ \frac{w_M [m(\phi)]}{\psi [\phi, m(\phi)]} \right\}^{1-\sigma} dG(\phi) = N \int_{a_{\min}}^{m(\tilde{\varphi})} w_M(a) dH(a) . \quad (14)$$

Differentiating this equation with respect to $\tilde{\varphi}$ yields

$$m'(\varphi) = \frac{MX}{N} \left(\frac{\sigma}{\sigma - 1} \right)^{-\sigma} \frac{w_M [m(\varphi)]^{-\sigma} G'(\varphi)}{\psi [\varphi, m(\varphi)]^{1-\sigma} H' [m(\varphi)]} \text{ for all } \varphi \in [\varphi_{\min}, \varphi_{\max}] . \quad (15)$$

Following Grossman et al. (2015), we show in Appendix A2.5 that this equation, together with the wage equation (11) and the boundary conditions,

$$m(\varphi_{\min}) = a_{\min}, \quad m(\varphi_{\max}) = a_R, \quad (16)$$

uniquely determine the matching function $m(\varphi)$ and the wage function $w_M(a)$ for workers in manufacturing, for a given cutoff value a_R .

The demand for R&D workers by projects with qualities between some \tilde{q} and q_{\max} is

$$R \int_{\tilde{q}}^{q_{\max}} \left\{ \frac{\gamma p_R \theta_K M \psi_R [z, m_R(z)]}{w_R [m_R(z)]} \right\}^{\frac{1}{1-\gamma}} dG_R(z)$$

and the wage paid to a worker by a project of quality z is $w_R [m_R(z)]$. Wage payments equal wage earnings. Therefore, labor-market clearing for this set of workers requires

$$R \int_{\tilde{q}}^{q_{\max}} w_R [m_R(z)] \left\{ \frac{\gamma p_R \theta_K M \psi_R [z, m_R(z)]}{w_R [m_R(z)]} \right\}^{\frac{1}{1-\gamma}} dG_R(z) = N \int_{m_R(\tilde{q})}^{a_{\max}} w_R(a) dH(a) . \quad (17)$$

Differentiating this equation yields a differential equation for the matching function in the research sector,

$$m'_R(q) = \frac{R}{N} \left\{ \frac{\gamma p_R \theta_K M \psi_R [q, m_R(q)]}{w_R [m_R(q)]} \right\}^{\frac{1}{1-\gamma}} \frac{G'_R(q)}{H' [m_R(q)]} , \quad (18)$$

with boundary conditions

$$m_R(q_{\min}) = a_R, \quad m_R(q_{\max}) = a_{\max} . \quad (19)$$

The differential equation (18) together with (12) and the boundary conditions (19) uniquely determine the matching function $m_R(q)$ and the wage function $w_R(a)$ for a given cutoff a_R . The proof is similar to the proof of uniqueness for the matching and wage functions in the manufacturing sector.

The solution to the two differential equations (15) and (18) give us matching functions for the two sectors that are parameterized by the cutoff point, a_R , which enters through the boundary conditions (16) and (19). To emphasize this dependence on a_R , we write the solutions as $m(\varphi; a_R)$ and $m_R(q; a_R)$. Note that the matching functions do not depend directly on N , R , X , θ_K , p_R or M . As shown in Grossman et al. (2015), the wage *ratios* in manufacturing—that is, the ratio of

wages paid to any pair of workers employed in that sector—are also uniquely determined by a_R , independently of N , X or M . Similarly, the relative wages of R&D workers are uniquely determined by a_R , independently of N , R , θ_K , p_R or M . We define relative wage functions $\lambda(a; a_R)$ and $\lambda_R(a; a_R)$ that describe inequality among workers in each sector as

$$\left. \begin{aligned} \lambda(a; a_R) &= \frac{w_M(a)}{w_M(a_{\min})} && \text{for } a \in [a_{\min}, a_R] \\ \lambda_R(a; a_R) &= \frac{w_R(a)}{w_R(a_R)} && \text{for } a \in [a_R, a_{\max}] \end{aligned} \right\} . \quad (20)$$

We note that the *levels* of the wages—for example, of $w_M(a_{\min})$ and $w_R(a_R)$ —do depend on parameters and variables like N , X , R , θ_K , p_R , and M that determine the momentary equilibrium.

2.6 The Steady-State Equilibrium

In this section, we derive a pair of equations that jointly determine the growth rate in the number of varieties and the cutoff ability level that separates researchers from production workers in a steady-state equilibrium. The first curve can be understood as a kind of resource constraint; the more workers that sort to R&D in equilibrium, the more new varieties are invented. The second relationship combines the free-entry condition for manufacturing with the labor-market-clearing condition for that sector. Once we have the steady-state values of g_M and a_R , we can calculate the other variables of interest, such as the growth rates of output and consumption and the distribution of income.

The growth in varieties reflects the aggregate output of the research sector. In steady state,

$$g_M = \theta_K R \int_{q_{\min}}^{q_{\max}} \psi_R[q, m_R(q)] \ell_R[q, m_R(q)]^\gamma dG_R(q) ,$$

where $\ell_R[q, m_R(q)]$ is steady-state employment by projects of quality q . In the appendix, we derive what we call the *RR* curve by substituting the labor-market-clearing condition for the research sector (17) into the expression for g_M . The *RR* curve is given by

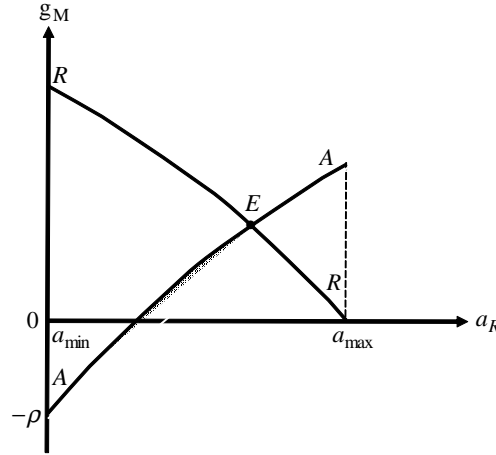
$$g_M = \theta_K N^\gamma R^{1-\gamma} \Phi(a_R) \int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a) , \quad (21)$$

where

$$\Phi(a_R) \equiv \left\{ \frac{\int_{q_{\min}}^{q_{\max}} \psi_R[q, m_R(q; a_R)]^{\frac{1}{1-\gamma}} \lambda_R[m_R(q; a_R); a_R]^{-\frac{\gamma}{1-\gamma}} dG_R(q)}{\int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)} \right\}^{1-\gamma} .$$

Notice that the right-hand side of (21) depends only on the cutoff value a_R and on exogenous parameters, inasmuch as the cutoff fully determines matching in the research sector and relative wages there. In the appendix, we show that the *RR* curve slopes downward, as depicted in Figure 1, despite the fact that $\Phi'(a_R) > 0$. The *RR* curve is a resource constraint, indicating that faster growth in the number of varieties requires that more resources be devoted to R&D and hence a lower cutoff ability level for the marginal research worker. Given the cutoff a_R , (21) indicates that

Figure 1: Equilibrium growth rate and ability cutoff



the growth rate will be higher the more productive is experience in generating knowledge capital, the larger is the population of workers, and the larger is the stock of research capital, which allows that more research projects can be undertaken.

Next we substitute the expression for profits of an intermediate good producer in (8) into the free-entry condition (10) and combine the result with the labor-market clearing condition for manufacturing (14), evaluated with $\tilde{\varphi} = \varphi_{\max}$. The result can be written as

$$\rho + g_M = \frac{1}{\sigma - 1} \frac{N}{p_R M} \int_{a_{\min}}^{a_R} w(a) dH(a) .$$

Again we can use (17), the labor-market-clearing condition for the research sector, together with the definition of the relative wages $\lambda(a; a_R)$ and $\lambda_R(a; a_R)$ to eliminate $p_R M$, so that we can write a second steady-state relationship involving only g_M and a_R . This is the AA curve depicted in Figure 1, and it is given by

$$\rho + g_M = \frac{\gamma}{\sigma - 1} \theta_K N^\gamma R^{1-\gamma} \Phi(a_R) \frac{\int_{a_{\min}}^{a_R} \lambda(a; a_R) dH(a)}{\lambda(a_R; a_R)} . \quad (22)$$

In the appendix, we prove that the AA curve must slope upward, as drawn.

The figure shows a unique balanced-growth equilibrium at point E .⁸ Once we know the long-run rate of growth in the number of intermediate goods, we can calculate the growth in consumption and wages from $g_w = g_c = g_M / (\sigma - 1)$.⁹ Once we know the steady-state cutoff level of ability a_R , we can compute the long-run distribution of relative wages using the wage structures dictated by Lemmas 1 and 2.

⁸If the AA curve falls below the horizontal axis for all $a_R \leq a_{\max}$, then no workers are employed in the research sector in the steady state. In such circumstances, growth rates of varieties, final output, consumption and wages are all zero.

⁹Consumption grows in the long run at the rate of income growth.

3 Growth and Inequality in Autarky Equilibrium

In this section, we compare growth rates and wage inequality in a pair of closed economies. We consider countries i and j that are basically similar but differ in some technological or policy parameters. We focus on balanced-growth equilibria, such as those described in Section 2. In the next section, we will perform similar cross-country comparisons for a set of open economies and examine how the opening of trade affects growth and wage inequality around the globe.

3.1 Productivity in Manufacturing

We begin by supposing that the countries differ only in their productivity in manufacturing, as captured by a Hicks-neutral technology parameter θ_{Mc} . In country c , a unit of labor of type a applied in a firm with technology φ can produce $\psi_c(\varphi, a) = \theta_{Mc}\psi(\varphi, a)$ units of a differentiated intermediate good. For the time being, the other characteristics of the countries are the same, including their sizes, their distributions of ability, their distributions of firm productivity, their discount rates and the efficiency of their knowledge accumulation.

In these circumstances, the matching function $m(\varphi; a_R)$ in the manufacturing sector is common to both countries; i.e., a difference between θ_{Mi} and θ_{Mj} does not affect matching in the manufacturing sector for a given a_R .¹⁰ Therefore, the relative-wage function $\lambda(a; a_R)$ also will be the same in both countries if they have the same cutoff point. But then the solution to (21) and (22) is the same for any values of θ_{Mi} and θ_{Mj} . In other words, countries that differ only in the (Hicks-neutral) productivity of their manufacturing sectors share the same long-run growth rate and the same marginal worker in manufacturing. It follows that relative wages for any pair of ability levels are also the same. We summarize in

Proposition 1 *Suppose that countries i and j differ only in manufacturing labor productivity $\psi_c(\cdot)$ and that these differences are Hicks-neutral; i.e., $\psi_c(\cdot) = \theta_{Mc}\psi(\cdot)$ for $c = i, j$. Then in autarky, both countries grow at the same rate in a balanced-growth equilibrium and both share the same structure of relative wages and the same degree of wage inequality.*

3.2 Capacity to Innovate

In our model, a country's capacity for innovation is described by four parameters: population size, which determines the potential scale of the research sector; the productivity of research workers; the efficiency with which research experience is converted into knowledge capital; and the endowment of

¹⁰To see this, differentiate the labor-market clearing condition, (15) with respect to φ , to derive the second-order differential equation,

$$\frac{m''(\varphi)}{m'(\varphi)} = (\sigma - 1) \frac{\psi_\varphi[\varphi, m(\varphi)]}{\psi[\varphi, m(\varphi)]} - \sigma \frac{\psi_a[\varphi, m(\varphi)]}{\psi[\varphi, m(\varphi)]} + \frac{G''(\varphi)}{G'(\varphi)} - \frac{H''[m(\varphi)]m'(\varphi)}{H'[m(\varphi)]}.$$

The productivity parameter θ_{Mc} appears in the numerator and the denominator of ψ_φ/ψ and of ψ_a/ψ , and so it does not affect matching for a given a_R .

research capital or, equivalently, the measure of research projects that can be undertaken simultaneously. In this section, we compare autarky growth rates and wage distributions in countries that differ in labor force, N_c , in efficiency of knowledge accumulation, θ_{Kc} , in research capital K_{Rc} and thus in the measure of active research projects, $R_c \equiv K_{Rc}/f$, and in the productivity of research workers, as captured by a Hicks-neutral shift parameter θ_{Rc} , where $\psi_{Rc}(q, a) = \theta_{Rc}\psi_R(q, a)$.

The RR curve in Figure 1 is defined by equation (21). In this equation, the right-hand side is proportional to $\theta_{Kc}N_c^\gamma R_c^{1-\gamma}\theta_{Rc}$, for a given a_R . The same expression also appears in equation (22) for the AA curve. We observe that $\theta_{Kc}N_c^\gamma R_c^{1-\gamma}\theta_{Rc}$ is a sufficient statistic for the innovation capacity in country c ; variation in this term explains cross-country variation in (autarky) long-run growth rates and wage distributions, all else the same.¹¹

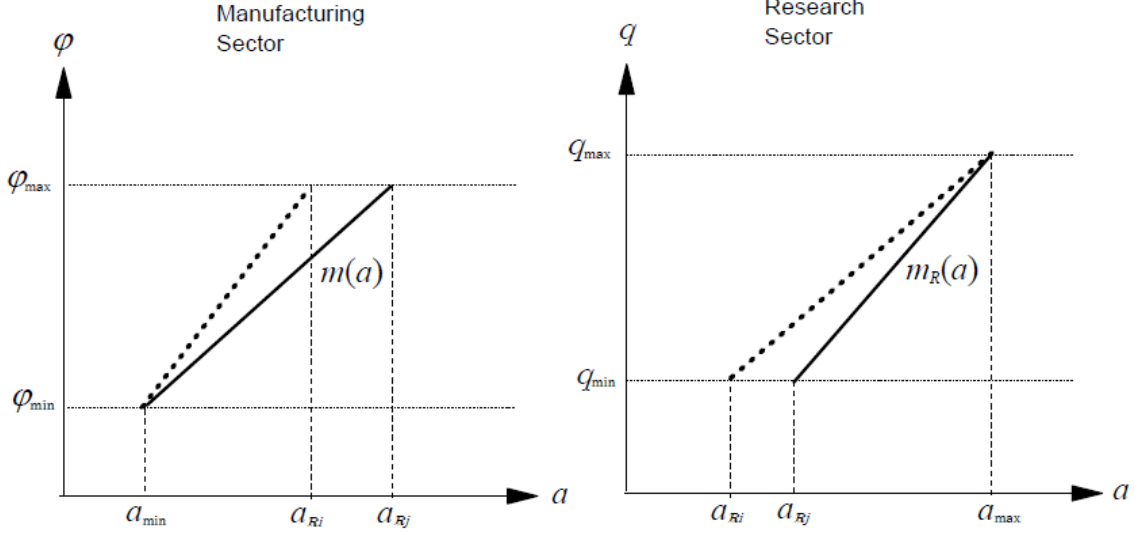
Consider two countries i and j that differ only in their innovation capacities, such that $\theta_{Ki}N_i^\gamma R_i^{1-\gamma}\theta_{Ri} > \theta_{Kj}N_j^\gamma R_j^{1-\gamma}\theta_{Rj}$. Under these circumstances, the AA and RR curves for country i lie above those for country j . But relative to the equilibrium cutoff point a_{Rj} in country j , the AA curve in country i passes above the RR curve in that country.¹² It follows that the equilibrium point for country i lies above and to the left of that for country j ; i.e., country i devotes more resources to R&D and it grows at a faster rate in the long run.

To compare wage inequality in the two countries, we first need to compare the matching of workers with firms and research projects that takes place in each. In Figure 2, the left panel depicts matching of firms and workers in the manufacturing sector. The solid curve represents the matching function $m_j(a) \equiv m(a; a_{Rj})$ in country j . The firms with the simplest technologies, namely, those with indexes φ_{\min} , hire the least-able workers, namely, those with indexes a_{\min} . The firms with the most sophisticated technologies, namely, those with indexes φ_{\max} , hire the most-able workers employed in the manufacturing sector, namely, those with indexes a_{Rj} . There is positive assortative matching in the sector and thus the matching function slopes upward. Now compare the matching function for country i , represented by the broken curve. Recall that $a_{Ri} < a_{Rj}$. In this country, too, the firms with technology φ_{\min} hire the workers with ability a_{\min} . And the firms with technology φ_{\max} hire the best workers in that country's manufacturing sector, who have index, a_{Ri} . Since we show in Appendix A2.5 that a pair of solutions to (11) and (15) that apply for different boundary conditions can intersect at most once, and since the curves for the two countries intersect at their common lower boundary, they cannot intersect elsewhere. It follows that the broken curve lies everywhere above the solid curve, except at the leftmost endpoint. This implies that a worker in country i with some ability level $a < a_{Ri} < a_{Rj}$ matches with a more productive firm than does his counterpart with similar ability in country j .

¹¹The reader may have noticed that the relative-wage function for R&D, $\lambda_R(a; a_R)$ appears under an integral in both equations, and the relative wage function for manufacturing, $\lambda(a; a_R)$ appears under an integral in (22). However, none of the four parameters under consideration affects the solution for the matching function in research or in manufacturing, given the cutoff ability a_R that appears in the boundary conditions. Given that the matching functions are not affected by these parameters except through a_R , the same is true of the relative-wage functions.

¹²An increase in $\theta_K N^\gamma R^{1-\gamma}\theta_R$ of some proportion shifts every point on the RR curve vertically upward by that same proportionate amount, but it shifts the AA curve up more than in proportion. Therefore, the new AA curve must pass above the new RR curve at the initial equilibrium value of a_R , and the new steady-state equilibrium must fall to the left and above point E in Figure 1.

Figure 2: Matching in manufacturing and research



The right panel of Figure 2 depicts the matching between researchers and research projects in the two countries. In both countries, the best projects, namely, those with indexes q_{\max} , hire the most-able researchers, namely, those with indexes a_{\max} . The solid curve again represents matching in country j . Here, entrepreneurs that find themselves with the least productive research projects hire the researchers with ability a_{Rj} , who are the least able among those employed in the R&D sector. The broken curve represents the matching in country i , where the least-able researchers have ability $a_{Ri} < a_{Rj}$. By a similar argument as before, the solid and dashed curves cannot intersect except at their common extreme point. It follows that a researcher in country i with some ability $a > a_{Rj} > a_{Ri}$ pursues a higher quality research project than his counterpart in country j with the same ability.

The different matching in the two countries translates into differences in wage inequality. Consider first inequality in the manufacturing sectors. We have seen in Figure 2 that manufacturing workers of any ability level in country i are paired with firms that have access to better technologies than the firms that hire their similarly-talented counterparts in country j . The better technology pairings boost the productivity of workers in i relative to those in j at all ability levels. But the complementarity between technology and ability implies that the productivity gain is relatively greatest for those who have more ability. This translates into a relative wage advantage for the more able of a pair of manufacturing workers in the country with the greater capacity for innovation. We have¹³

¹³Given the ability cutoff a_R and the matching function $m(\varphi; a_R)$ the wage equation for manufacturing implies

$$\ln \lambda(a; a_R) = \int_{a_{\min}}^a \frac{\psi_a[m^{-1}(v; a_R), v]}{\psi[m^{-1}(v; a_R), v]} dv \text{ for } a \in [a_{\min}, a_R] .$$

By Assumption 1, a deterioration in the match for the worker with ability v reduces the expression under the integral. It therefore reduces the relative wage of the worker with greater ability among any pair of workers employed in the

Lemma 3 *Suppose $a_{\min} < a_{Ri} < a_{Rj} < a_{\max}$. Then*

$$\frac{\lambda(a''; a_{Ri})}{\lambda(a''; a_{Rj})} > \frac{\lambda(a'; a_{Ri})}{\lambda(a'; a_{Rj})} \text{ for all } a'' > a' \text{ and } a', a'' \in [a_{\min}, a_{Ri}].$$

Now consider inequality in the research sector. Research workers also achieve better matches in country i than in country j , as illustrated in the right panel Figure 2. The relative research productivity of the more able in any pair of researchers is greater in country i than in country j , due to the complementarity between project quality and worker ability that we posited in Assumption 2. Akin to Lemma 3, we have

Lemma 4 *Suppose $a_{\min} < a_{Ri} < a_{Rj} < a_{\max}$. Then*

$$\frac{\lambda_R(a''; a_{Ri})}{\lambda_R(a''; a_{Rj})} > \frac{\lambda_R(a'; a_{Ri})}{\lambda_R(a'; a_{Rj})} \text{ for all } a'' > a' \text{ and } a', a'' \in [a_{Rj}, a_{\max}].$$

Finally, consider an individual who has an ability level $a'' \in [a_{Ri}, a_{Rj}]$. Such a worker sorts to the research sector in country i , but to the manufacturing sector in country j . If a'' were to work in manufacturing in country i , he would already earn a relatively higher wage in that country compared to some $a' \in [a_{\min}, a_{Ri}]$, thanks to the better technologies that all manufacturing workers access there. The fact that this individual instead chooses employment in the research sector implies that the wage offer there is even better than what he could earn in manufacturing. It follows that a'' earns relatively more compared to a' in country i than in country j . By the same token, if we compare the relative wages of $a'' \in [a_{Ri}, a_{Rj}]$ to $a''' \in [a_{Ri}, a_{\max}]$ in the two countries, a''' would earn relatively more in i than in j even if a'' were to work in the research sector in country j . The fact that this worker prefers to work in manufacturing in country j only strengthens the relative advantage for these lower-ability workers from residing in the country with the relatively smaller research sector.

Putting all the pieces together, we can compare the relative wages paid to any pair of workers of similar ability levels in the two countries. We have established

Proposition 2 *Suppose countries i and j differ only in their capacities for innovation, with $\theta_{Ki} N_i^\gamma R_i^{1-\gamma} \theta_{Ri} > \theta_{Kj} N_j^\gamma R_j^{1-\gamma} \theta_{Rj}$. In autarky, country i grows faster than country j in a balanced-growth equilibrium and it has greater inequality throughout its wage distribution. That is, $g_{Mi} > g_{Mj}$, and for any pair of workers $a', a'' \in [a_{\min}, a_{\max}]$ such that $a'' > a'$,*

$$\frac{w_i(a'')}{w_i(a')} > \frac{w_j(a'')}{w_j(a')},$$

where $w_c(a)$ is the equilibrium wage schedule in country c .

The proposition implies that, when countries differ only in their capacity for innovation, fast growth and wage inequality go hand in hand. A greater innovation capacity generates a relatively manufacturing sector.

larger research sector and therefore a lower cutoff ability level for the marginal worker who is indifferent between employment in the two sectors. The fact that $a_{Ri} < a_{Rj}$ means that manufacturing workers access better production technologies in country i than in country j and that research workers work on better projects there. In both cases, the better matches favor the relatively more able among any pair of ability levels, due to the complementarity between ability and technology on the one hand, and between ability and project quality on the other. Finally, the fact that ability confers a comparative advantage in R&D reinforces the tendency for the more able (and better paid) workers to earn relatively higher wages in the country that conducts more research.

3.3 Support for R&D

Next we examine the role that research policy plays in shaping growth and inequality, focusing specifically on cross-country differences in R&D subsidies. We consider symmetric countries i and j that differ only in their subsidy rates, s_i and s_j . The subsidy applies to the purchase of R&D services by manufacturing firms, so that the private cost of a product design and its associated technology draw becomes $(1 - s_c)p_{Rc}$ in country c . The subsidy is financed by a proportional tax on wages or on research capital.

With a subsidy in place, the equation for the AA curve in Figure 1 is replaced by

$$(1 - s_c)(\rho + g_{Mc}) = \frac{\gamma}{\sigma - 1} \theta_K N^\gamma R^{1-\gamma} \Phi(a_{Rc}) \frac{\int_{a_{\min}}^{a_{Rc}} \lambda(a; a_{Rc}) dH(a)}{\lambda(a_{Rc}; a_{Rc})} .$$

Since the relationship between the resources invested in R&D and the growth rate is not affected by the subsidy, neither is the RR curve that depicts this relationship..

It follows immediately that, if $s_i > s_j$, the AA curve for country i lies above and to the left of that for country j . Not surprisingly, the subsidy draws labor into the research sector and, thereby, stimulates growth. The link to the income distribution should also be clear. With $a_{Ri} < a_{Rj}$, the technology matches are better for manufacturing workers of a given ability in country i than in country j , and the project matches are better for the researchers there as well. The larger size of the research sector in country i also contributes to its greater inequality, because ability is more amply rewarded in R&D than in manufacturing. Together, these forces generate a more unequal distribution of wages in both sectors of country i compared to country j , and in the economy as a whole.

Proposition 3 *Suppose that countries i and j differ only in their R&D subsidies and that $s_i > s_j$. Then, in autarky, country i grows faster than country j in a balanced-growth equilibrium and it has more inequality throughout its wage distribution. That is, $g_{Mi} > g_{Mj}$, and for any pair of workers $a', a'' \in [a_{\min}, a_{\max}]$ such that $a'' > a'$,*

$$\frac{w_i(a'')}{w_i(a')} > \frac{w_j(a'')}{w_j(a')} .$$

In Section 4.4, we will revisit the effects of R&D subsidies for an open economy and will address the spillover effects of such subsidies on growth and inequality in a country's trading partners. We will see that R&D subsidies increase wage inequality not only in the economy that applies them, but also around the globe.

4 Growth and Inequality in a Trading Equilibrium

In this section, we introduce international trade among a set of countries that differ in size, in research productivity, in manufacturing technologies, in capacity to create and absorb international knowledge spillovers, and in their innovation and trade policies. First, we examine the effects of trade on growth and income inequality in a typical country. Then, we allow countries to differ along one dimension at a time and ask how each difference is reflected in the cross-country comparison of their income distributions. We also explore the spillover effects of policies and parameters in one country on growth and income inequality in its trading partners.

Our trading environment has C countries indexed by $c = 1, \dots, C$. In country c , there are N_c workers with a distribution of abilities, $H(a)$. A worker with ability a who applies a technology φ in country c can produce $\theta_{Mc}\psi(\varphi, a)$ units of any intermediate good, where $\psi(\varphi, a)$ again has the complementarity properties described by Assumption 1. We assume that manufacturing firms in all countries draw production technologies from a common distribution $G(\varphi)$.

All existing varieties of intermediate goods are internationally tradable subject to trading frictions. We model these frictions as a combination of iceberg trading costs and *ad valorem* tariffs, so that the delivered price of any intermediate good imported from country j and delivered in country c is τ_{jc} times as great as the price received by the exporter in the source country. The budget surplus generated by tariff revenue net of the cost of any R&D subsidies is redistributed by a proportional subsidy on wages.

Final goods are not tradable. Let q_c represent the price of the final good in country c , $p_{jc}(\omega)$ the price there of variety ω of an intermediate good imported from country j , and Ω_j the set of intermediate goods produced in country j . Competitive pricing of final goods implies that

$$\left\{ \sum_{j=1}^C \left[\int_{\omega \in \Omega_j} p_{jc}(\omega)^{1-\sigma} d\omega \right] \right\}^{\frac{1}{1-\sigma}} = q_c ,$$

while the choice of numeraire allows us to set any one of these prices equal to one. We denote by X_c the output of final goods in country c .

In the research sector, a team of researchers of size ℓ_R and with ability a who work on a project of quality q has productivity $\theta_{Rc}\psi_R(q, a) K_c \ell_R^\gamma$, where θ_{Rc} reflects the overall research productivity in country c and K_c is the national stock of knowledge capital. Assumption 2 again describes a complementarity between the researchers' abilities and quality of the project. Entrepreneurs in country c must hire f units of local research capital at the rental rate r_c in order to draw a research

project from the common distribution of project qualities, $G_R(q)$. Once the project quality is known, each entrepreneur hires local researchers to produce R&D services. R&D services are not internationally tradable, so the price p_{Rc} of these services may vary across countries.

The national knowledge stock in country c reflects the country's cumulative experience in R&D, its ability to learn from that experience, and the extent of knowledge spillovers from abroad. The evidence surveyed by Helpman (2004, ch.5) points to the existence of significant but incomplete international R&D spillovers. Coe et al. (2009) find, for example, that a country's researchers benefit differentially from domestic and foreign R&D experience and that the capacity to absorb domestic and foreign knowledge depends on a country's institutions and in particular on its regime for protection of intellectual property rights and the quality of its tertiary education. To capture this reality, we assume that the stock of knowledge in country c is given by

$$K_c = \sum_{j=1}^C \theta_{Kjc} M_j, \quad (23)$$

where θ_{Kjc} is a parameter that reflects the extent to which cumulative research experience in country j contributes to inventors' productivity in country c . We assume that $\theta_{Kjc} > 0$ for all j and c , so that every country reaps some spillover benefits from research that takes place anywhere in the world. Note that θ_{Kcc} measures the effectiveness with which country c converts its own research experience into usable knowledge; this parameter is the same as what we denoted by θ_K in Section 2.3 above. The special case of complete international spillovers into country c can be represented by setting $\theta_{Kjc} = \theta_{Kc}$ for all j . If spillovers are complete and countries are symmetric in their abilities to absorb knowledge, then $\theta_{Kjc} = \theta_K$ for all j and c .

4.1 The Effects of Trade on Growth and Inequality

To solve the open-economy model, we make use of a separability property of the dynamic equilibrium. First note that, along a balanced-growth path, the number of differentiated varieties grows at the same rate in all countries; i.e., $\dot{M}_c/M_c \equiv g_{Mc} = g_M$ for all c . In our one-sector model, this implies a convergence also in growth rates of per capita income.¹⁴ The output of final goods, X , in the closed-economy expression for the profits of a typical intermediate good (8) and in the labor-market clearing condition (14), is replaced in the open economy by the *market access* \bar{X}_c facing a typical producer of intermediates in country c , where

$$\bar{X}_c = \sum_j \tau_{jc}^{1-\sigma} q_j^\sigma X_j.$$

This variable, as defined by Redding and Venables (2004), scales the aggregate demand facing an intermediate good producer in country c (given its price), considering the production of final goods

¹⁴As we know from Grossman and Helpman (1991), growth rates of per capita income can vary across countries if there are multiple industries that produce final goods and if countries differ in the compositions of their long-run production patterns.

in each market, the cost of overcoming the trade barrier specific to the market, and the competition the firm faces from other intermediate goods sold in that market (as reflected in the price index for intermediate goods). Since this variable enters multiplicatively on the left-hand side of (14), the form of the matching function in the manufacturing sector, as described by the differential equation (15), remains the same for the open economy as for the closed economy.

We can solve for the growth rate of varieties in country c and the cutoff point for labor allocation a_{Rc} using two equations analogous to (21) and (22). In place of the former, we have

$$g_{Mc} = \kappa_c \theta_{Rc} N_c^\gamma R_c^{1-\gamma} \Phi(a_{Rc}) \int_{a_{Rc}}^{a_{\max}} \lambda_R(a; a_{Rc}) dH(a), \quad (24)$$

where $\kappa_c \equiv K_c/M_c$ is the ratio of the knowledge stock in country c to the country's own cumulative experience in research and

$$\Phi(a_{Rc}) \equiv \left\{ \frac{\int_{q_{\min}}^{q_{\max}} \psi_{Rc}[q, m_{Rc}(q; a_{Rc})]^{-\frac{1}{1-\gamma}} \lambda_R[m_{Rc}(q; a_{Rc}); a_{Rc}]^{-\frac{\gamma}{1-\gamma}} dG_R(q)}{\int_{a_{Rc}}^{a_{\max}} \lambda_R(a; a_{Rc}) dH(a)} \right\}^{1-\gamma},$$

as before (except that now we add a country-specific index, c). In place of the latter (and taking into account the R&D subsidy), we have

$$(1 - s_c)(\rho + g_{Mc}) = \frac{\gamma}{\sigma - 1} \kappa_c \theta_{Rc} N_c^\gamma R_c^{1-\gamma} \Phi(a_{Rc}) \frac{\int_{a_{\min}}^{a_{Rc}} \lambda(a; a_{Rc}) dH(a)}{\lambda(a_{Rc}; a_{Rc})}. \quad (25)$$

Notice the similarity between (24) and (25) and the equations that jointly determine steady-state equilibrium in the closed economy; the new equations incorporate the parameters θ_{Rc} and s_c that we have introduced to represent Hicks-neutral differences in researcher productivity and the R&D subsidy rates, respectively, and they include κ_c in place of θ_K (or what we now denote by θ_{Kcc}). Similar arguments as before imply that the RR curve for the open economy slopes downward and the AA curve slopes upward. Using (24) and (25), we can solve for the long-run values of g_{Mc} and a_{Rc} as a function of κ_c . Then, we can use a_{Rc} and the differential equations for wages in each sector to solve for the distribution of relative wages in country c . Separately, we can use a set of trade balance conditions and labor-market clearing conditions to solve for the relative prices of final goods and the wage *levels* in each country.

A key observation is that $\kappa_c > \theta_{Kcc}$ for all c . That is, in an open economy, researchers anywhere can draw not only on their own country's accumulated research experience when inventing new products, but also to some extent on the research experience that has accumulated outside their borders. No matter what the extent of international knowledge spillovers, so long as they are positive, a research team in any country can be more productive in the open economy than in autarky. This greater productivity translates a given labor input into greater innovation by (24) and it reduces the cost of R&D that is embedded in the zero-profit condition in (25).

Now we are ready to compare (24) and (25) to their analogs that describe the closed-economy equilibrium (with R&D subsidies). Note that the bigger κ_c appears in place of the smaller θ_{Kc} (i.e., θ_K) in each equation. Thus, the RR curve for the open economy lies proportionately above that for the closed economy, whereas the AA curve for the open economy lies more than proportionately above that for the closed economy. The two curves that determine the open-economy equilibrium in country c cross above and to the left of the intersection depicted in Figure 1. Thus, in a trade equilibrium, every country devotes more labor to research than in autarky and it invents new varieties at a faster rate. The expansion of the research sector (fall in a_{Rc}) exacerbates wage inequality, both as a reflection of the re-matching that takes place in both sectors (i.e., workers match with better firms and projects) and of the reallocation of labor to R&D, where ability is more amply rewarded. Meanwhile, the acceleration of innovation generates faster growth of wages and final output. We have established

Proposition 4 *Suppose that intermediate goods are tradable. Countries may differ in their manufacturing productivities, their research productivities, their labor supplies, their R&D subsidies, and their import tariffs. In a balanced-growth equilibrium, every country grows faster with trade than in autarky and every country has a more unequal wage distribution with trade than in autarky.*

4.2 Differences in Manufacturing Productivity and Trade Barriers

Suppose now that countries differ only in their manufacturing productivities, as parameterized by θ_{Mc} , and in their trade barriers, as reflected in τ_{jc} . For the moment, we assume they are equal in size ($N_c = N$ for all c), equal in research productivity ($\theta_{Rc} = \theta_R$ for all c), have similar R&D subsidies ($s_c = s$ for all c) and benefit symmetrically from complete international knowledge spillovers ($\theta_{Kjc} = \theta_K$ for all j and c). In these circumstances, a balanced-growth path with $g_{Mc} = g_M$ requires $\kappa_c = \kappa$ and $a_{Rc} = a_R$ for all c , per equations (24) and (25). It follows that not only do the long-run growth rates converge internationally, but so too do the sizes and compositions of the research sectors. Then, matching between technologies and production workers in manufacturing and between research projects and researchers in R&D is the same in all countries. Consequently the structure of relative wages is the same in all countries. The differences in manufacturing productivity and import tariff rates generate cross-country heterogeneity only in wage *levels*. We summarize in

Proposition 5 *Suppose that intermediate goods are tradable and countries differ only in manufacturing productivities and import tariffs. Then all countries grow at the same rate in a balanced-growth equilibrium and all have the same wage inequality in the long run.*

It is also clear that, in these circumstances, the long-run value of κ is independent of any θ_{Mc} and τ_{jc} , in which case (24) and (25) imply that changes in manufacturing productivities or in trade frictions do not affect the long-run growth rate or relative wages in any country.¹⁵ Moreover, κ_c

¹⁵With $\theta_{Kjc} = \theta_K$ for all j and c , (23) yields $K_c = \theta_K \sum_{j=1}^C M_j$ for all c , and thus $\kappa_c = \theta_K \left(\sum_{j=1}^C M_j \right) / M_c$ for all c . Then (25) and the fact established above that $a_{Rc} = a_R$ for all c imply that $\kappa_c = \kappa = \theta_K C$. Clearly, κ is

would be independent of θ_{Mc} and τ_{jc} (albeit not necessarily common across countries) if countries were of different sizes, had different R&D subsidies, had different research productivities, or had different capacities to generate or absorb international R&D spillovers. The parameters θ_{Mc} and τ_{jc} do, of course, affect income levels and consumer welfare.

4.3 Differences in Innovation Capacity and in Ability to Create and Absorb Knowledge Spillovers

Now suppose that all countries have equal R&D subsidy rates ($s_c = s$ for all c). They may differ in size (N_c), in their research productivity (θ_{Rc}), and in their numbers of active research projects ($R_c = K_{Rc}/f$). Moreover, there may be differences in their abilities to absorb R&D spillovers from abroad and in their abilities to convert research experience (their own and foreign) into usable knowledge that facilitates subsequent innovation. Such differences are reflected in the arbitrary matrix $\Theta_K = \{\theta_{Kjc}\}$ of spillover parameters that determines knowledge capital in country c , according to (23). Finally, as in Section 4.2, they may face or impose different trade barriers τ_{jc} and operate with different manufacturing productivities, θ_{Mc} . In all of these cases, (24) and (25) imply

$$\frac{g_{Mc}}{\rho + g_{Mc}} = (1 - s) \frac{\sigma - 1}{\gamma} \frac{\int_{a_{Rc}}^{a_{\max}} \frac{\lambda_R(a; a_{Rc})}{\lambda_R(a_{Rc}; a_{Rc})} dH(a)}{\int_{a_{\min}}^{a_{Rc}} \frac{\lambda(a; a_{Rc})}{\lambda(a_{Rc}; a_{Rc})} dH(a)} \quad \text{for all } c. \quad (26)$$

It is clear from (26) that, since all countries converge on the same long-run growth rate of varieties, they must also have the same ability cutoff level $a_{Rc} = a_R$. Then, all share a common long-run relative wage profile. It is interesting to note that international integration generates a convergence in income inequality around the globe, whereas differences in innovation capacity give rise to different degrees of inequality in autarky.

Although relative wages are the same in all countries, wage levels are not equalized internationally. We show in Appendix 4.3, for example, that if intermediate goods are freely traded ($\tau_{jc} = 1$ for all j and c) and knowledge spillovers are complete ($\theta_{Kjc} = \theta_{Kc}$ for all c), the relative wages of workers of any common ability level in countries i and j hinges on a comparison of innovation capacities per capita in these countries; i.e., on $\theta_{Ki}\theta_{Ri}(R_i/N_i)^{1-\gamma}$ versus $\theta_{Kj}\theta_{Rj}(R_j/N_j)^{1-\gamma}$. The greater is a country's ability to convert cumulative experience in R&D into usable knowledge, θ_{Ki} , or the greater is the productivity of its workers in R&D, θ_{Ri} , or the larger is its endowment of research capital relative to its labor force, R_j/N_j , the greater is its wage level. If trade is not free, a country's size can boost the level of its wages due to a home-market effect that expands market access for its producers.

Next observe that with $a_{Rc} = a_R$ for all c , (24) implies that $\zeta_c \equiv \kappa_c N_c^\gamma R_c^{1-\gamma} \theta_{Rc}$ takes a common

independent of any θ_{Mc} or τ_{jc} .

value across all countries, i.e., $\zeta_c = \zeta$ for all c . Substituting ζ into (23), we have

$$\zeta \mu_c = \sum_{j=1}^C \gamma_{jc} \mu_j ,$$

where $\mu_c \equiv M_c / \sum_j M_j$ is the share of country c in the total number of varieties of intermediate goods in the world economy and $\gamma_{jc} \equiv \theta_{Kjc} N_c^\gamma R_c^{1-\gamma} \theta_{Rc}$ is a measure of innovation capacity in a setting in which knowledge spillovers are not complete. We recognize ζ as being a characteristic root of the matrix $\mathbf{\Gamma} = \{\gamma_{jc}\}$, with associated characteristic vector $\boldsymbol{\mu} = \{\mu_c\}$. Moreover, by the assumption that $\theta_{Kjc} > 0$ for all j and c , all elements of $\mathbf{\Gamma}$ are strictly positive. Then the Perron-Frobenius Theorem implies that all elements of $\boldsymbol{\mu}$ can be positive (as they must be) only if ζ is the *largest* characteristic root of $\mathbf{\Gamma}$. Finally, the envelope theorem implies that ζ must be increasing in every element γ_{jc} of $\mathbf{\Gamma}$.¹⁶

We have thus established that an increase in any spillover parameter θ_{Kjc} , in any country size N_c , in any R&D productivity parameter θ_{Rc} , or in any country's research capital K_{Rc} , shifts upward the RR curve and the AA curve for every country, and the former by more (at the initial a_R) than the latter. The result is an increase in the common rate of long-run growth and an increase in wage inequality in every country.

We record our findings in

Proposition 6 *Suppose that intermediate goods are tradable and all countries have the same R&D subsidy s . Then all countries grow at the same rate in a balanced-growth equilibrium and all have the same wage inequality in the long run. An increase in any spillover parameter θ_{Kc} , in any country size N_c , in any R&D productivity parameter θ_{Rc} , or in any country's endowment of research capital K_{Rc} leads to faster growth and greater wage inequality in every country.*

4.4 Differences in R&D Subsidies

Suppose that international knowledge spillovers are complete and that countries are similar in all ways except in their R&D subsidies and in the proportional wage taxes used to finance these subsidies.¹⁷ When $N_c = N$, $R_c = R$, and $\theta_{Rc} = \theta_R$ for all c and when long-run growth rates

¹⁶Multiplying the characteristic equation by μ_c and summing over all c yields

$$\zeta = \frac{\sum_{c=1}^C \sum_{j=1}^C \gamma_{jc} \mu_j \mu_c}{\sum_{c=1}^C (\mu_c)^2} .$$

The largest characteristic root is found by maximizing the right hand side with respect to $\{\mu_c\}$. By the envelope theorem, the largest ζ is an increasing function of every γ_{jc} .

¹⁷It is relatively easy to verify that the implications of differences in research support would be the same as we describe here, even if we allowed for cross-country differences in innovation capacity and in tariff rates. However, we assume that these features are common in order to simplify the exposition.

converge to g_M , (24) and (25) imply

$$(1 - s_c) \frac{\rho + g_M}{g_M} = \frac{\gamma}{\sigma - 1} \frac{1}{\lambda(a_{Rc}; a_{Rc})} \frac{\int_{a_{\min}}^{a_{Rc}} \lambda(a; a_{Rc}) dH(a)}{\int_{a_{Rc}}^{a_{\max}} \lambda_R(a; a_{Rc}) dH(a)}.$$

We show in the appendix that the right-hand side of this equation is increasing in a_{Rc} . Therefore, if $s_i > s_j$, $a_{Ri} < a_{Rj}$; i.e., the country with the larger R&D subsidy devotes more of its labor force to research activities. This does not generate faster long-run growth in i than in j , but it does spell a more unequal long-run wage distribution there.

Although wage profiles do not converge in the presence of (differential) R&D subsidies, such policies do affect growth and inequality throughout the world. To examine these spillover effects of innovation policy, we treat (24) and (25) as a system of $C + 1$ equations that determines the C cutoff ability levels and the common growth rate, g_M . We prove in Appendix A4.4 that an increase in an arbitrary subsidy rate s_i leads to an expansion of the research sectors in all countries. In other words, $da_{Rj}/ds_i < 0$ for all $i, j \in \{1, \dots, C\}$. It follows that an increase in a single subsidy rate contributes not only to faster innovation throughout the world economy, but also to a spreading of the long-run wage distribution everywhere. We summarize in

Proposition 7 *Suppose that intermediate goods are tradable, that international knowledge spillovers are complete, and that countries differ only in their R&D subsidy rates. Comparing any two countries, the long-run wage distribution is more unequal in the one with the greater subsidy rate. An increase in any subsidy rate raises the common long-run growth rate and generates a spread in the distribution of wages in every country.*

The main lessons from this section are threefold. First, international integration affords researchers access to a larger knowledge stock, which raises research productivity worldwide and leads to an acceleration of innovation and growth. At the same time, the expansion of each country's research sector spells a ubiquitous increase in wage inequality. Second, national conditions that create differential incentives for research versus manufacturing generate long-run differences in wage distributions, whereas conditions that affect a country's ability to contribute to or draw on the world's stock of knowledge capital lead to a convergence in wage distributions but with cross-country differences in wage levels. Finally, technological conditions or government policies that cause an expansion of the research sector in one country typically have spillover effects abroad. In particular, when the incentives for R&D rise somewhere, the induced expansion in knowledge capital generates a positive growth spillover for other countries and a tendency for wage inequality to rise everywhere.

5 Concluding Remarks

In this paper, we have focused on one mechanism that links income distribution to long-run growth. The mechanism operates via sorting and matching in the labor market. We posit that the most able

individuals in any economy specialize in creating ideas and that innovation is the engine of growth. Among those that conduct research, the most able are relatively more proficient at performing the most promising research projects. Among those that use ideas rather than create them, the most able are relatively more proficient at using the most sophisticated technologies. In each case, the complementarity between worker ability and firm productivity dictates positive assortative matching. In the long run, the size of what we call the research sector determines not only the pace of innovation, but also the composition of the two sectors and the matches that take place.

Our model highlights an important mechanism in the simplest imaginable economic environment. We have abstracted from diversity in manufacturing industries, from team production activities that involve multiple individuals in both research and manufacturing, from capital inputs that may be complementary to certain worker or inventor types, and from a host of market frictions that can impede job placement and financing for innovation. In this simple setting, faster growth typically goes hand in hand with greater wage inequality. In response to events that encourage faster growth, the research sector expands by drawing the most able workers from the manufacturing sector, who then become the least able researchers. The expansion of the research sector at the extensive margin generates a re-matching between researchers and research projects that brings the relatively greatest benefit to those with greatest ability. Meanwhile, the contraction of the manufacturing sector generates re-matching between production workers and technologies that also favors relatively most those in this sector with greatest ability. The complementarity between ability and technologies implies an increase in wage inequality. This effect is strengthened by the fact that those with most ability have comparative advantage in the activity that underlies growth.

By allowing for international trade and international knowledge spillovers, we introduced links between inequality measures in different countries. Generally, we find that within-country income inequality is exacerbated by globalization. The mechanism is not the usual one, however, i.e., that trade leads to specialization in sectors that differ in factor intensity, but rather that international knowledge sharing makes innovation more productive and so creates incentives for expansion of the idea-generating portion of the economy worldwide. As the research sector expands in every country so too does the relative pay for the most able individuals (who engage in innovation) as well as for the more able individuals among those that sort to each sector. The more able researchers benefit relatively more from the improved matching with research projects while the most able workers in manufacturing benefit relatively more from the improved matching with technologies. Our treatment of the open economy also allows us to study the links between conditions and policies in one country and growth and distributional outcomes in its trade partners. For example, we find that an R&D subsidy in one country accelerates growth in all countries and increases within-country income inequality throughout the globe. While previous work on endogenous growth emphasized cross-country dependence in growth rates (e.g., Grossman and Helpman 1991), our model also features cross-country dependence in wage inequality. Moreover, while long-run growth rates converge, cross-country differences in wage distributions can persist even along a balanced-growth path.

Numerous possible extensions of our model come to mind. Additional elements of interdependence would arise if production functions involved multiple factors of production (or teams of individuals) and if sectors differed in their relative factor intensities. We also suspect that investment in ideas has more dimensions of uncertainty than just the productivity of the resulting technology, and that the prospects for success in innovation and the range of reachable technologies depend on the abilities of the individuals who generate the new ideas. Imperfect information about worker characteristics and frictions in labor markets undoubtedly impede the smooth, assortative matching that features in our model. Similarly, asymmetric information about research ideas and financing constraints impede investment in innovation and bias technological outcomes. All of these extensions would be interesting.

We view our contribution in this paper not as a final word on the link between growth and inequality, but as an exploration of a core mechanism that will play a role in richer economic environments. The empirical importance of this mechanism remains to be settled, although at this stage it is not obvious how to do so in light of the limited availability of historical data and the endogeneity of the variables of interest. Yet we are convinced that a better understanding of the relationship between growth and inequality can be obtained by studying economies in which both are endogenously determined.

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Appendix

A2.5 Uniqueness and Single Crossing of the Matching Function

In Section 2.5 we stated that the solution to the pair of differential equations (11) and (15) that satisfies the boundary conditions (16) is unique, and later that the matching functions of two solutions to (11) and (15) that apply for different boundary conditions can intersect at most once. Here, we prove these statements by adapting Lemma 2 in the appendix of Grossman et al. (2015) to the present circumstances.

We begin with the latter claim. As in Grossman et al. (2015), let $[m_{\varkappa}(\varphi), w_{\varkappa}(a)]$ and $[m_{\varrho}(\varphi), w_{\varrho}(a)]$ be solutions to the differential equations (11) and (15), each for different boundary conditions,

$$m(\varphi_{\min}) = a_{z,\min} \text{ and } m(\varphi_{\max}) = a_{z,\max}, \quad z = \varkappa, \varrho. \quad (27)$$

Let the solutions intersect for some $\varphi = \varphi_0$ and $a = a_0$. Without loss of generality, suppose that $m'_{\varrho}(\varphi_0) > m'_{\varkappa}(\varphi_0)$. We will now show that $m_{\varrho}(\varphi) > m_{\varkappa}(\varphi)$ for all $\varphi > \varphi_0$ and $m_{\varrho}(\varphi) < m_{\varkappa}(\varphi)$ for all $\varphi < \varphi_0$ in the overlapping set of (φ, a) .

To see this, suppose to the contrary there exists a $\varphi_1 > \varphi_0$ such that $m_{\varrho}(\varphi_1) \leq m_{\varkappa}(\varphi_1)$. Then differentiability of $m_z(\cdot)$, $z = \varkappa, \varrho$, implies that there exists a φ_2 with $\varphi_2 > \varphi_0$ such that $m_{\varrho}(\varphi_2) = m_{\varkappa}(\varphi_2)$, $m_{\varrho}(\varphi) > m_{\varkappa}(\varphi)$ for all $\varphi \in (\varphi_0, \varphi_2)$ and $m'_{\varrho}(\varphi_2) < m'_{\varkappa}(\varphi_2)$. This also implies that $m_{\varrho}^{-1}(a) < m_{\varkappa}^{-1}(a)$ for all $a \in (m_{\varrho}(\varphi_0), m_{\varrho}(\varphi_2))$, where $m_z^{-1}(\cdot)$ is the inverse of $m_z(\cdot)$. But then (15) implies that $w_{\varrho}[m_{\varrho}(\varphi_0)] < w_{\varkappa}[m_{\varrho}(\varphi_0)]$ and $w_{\varrho}[m_{\varrho}(\varphi_2)] > w_{\varkappa}[m_{\varrho}(\varphi_2)]$, and therefore

$$\ln w_{\varkappa}[m_{\varrho}(\varphi_2)] - \ln w_{\varkappa}[m_{\varrho}(\varphi_0)] < \ln w_{\varrho}[m_{\varrho}(\varphi_2)] - \ln w_{\varrho}[m_{\varrho}(\varphi_0)].$$

On the other hand, (11) implies that

$$\ln w_z[m_{\varrho}(\varphi_2)] - \ln w_z[m_{\varrho}(\varphi_0)] = \int_{m_{\varrho}(\varphi_0)}^{m_{\varrho}(\varphi_2)} \frac{\psi_a[m_z^{-1}(a), a]}{\psi[m_z^{-1}(a), a]} da, \quad z = \varkappa, \varrho.$$

Together with the previous inequality, this gives

$$\int_{m_{\varrho}(\varphi_0)}^{m_{\varrho}(\varphi_2)} \frac{\psi_a[m_{\varkappa}^{-1}(a), a]}{\psi[m_{\varkappa}^{-1}(a), a]} da < \int_{m_{\varrho}(\varphi_0)}^{m_{\varrho}(\varphi_2)} \frac{\psi_a[m_{\varrho}^{-1}(a), a]}{\psi[m_{\varrho}^{-1}(a), a]} da.$$

Note, however, that the strict log supermodularity of $\psi(\cdot)$ and $m_{\varrho}^{-1}(a) < m_{\varkappa}^{-1}(a)$ for all $a \in (m_{\varrho}(\varphi_0), m_{\varrho}(\varphi_2))$ imply the reverse inequality, which establishes a contradiction. It follows that $m_{\varrho}(\varphi) > m_{\varkappa}(\varphi)$ for all $\varphi > \varphi_0$. A similar argument shows that $m_{\varrho}(\varphi) < m_{\varkappa}(\varphi)$ for all $\varphi < \varphi_0$.

The fact that the matching functions for different boundary conditions can cross at most once immediately implies the uniqueness of the solution to (11) and (15) for a given set of boundary conditions, $m(\varphi_{\min}) = a_{\min}$ and $m(\varphi_{\max}) = a_R$. If there were two different solutions for these

boundary conditions, the resulting matching functions would have to intersect at least twice, which is not possible.

A2.6 The RR Curve

We derive now the equation for the RR curve and establish that it is downward sloping. In steady state,

$$g_M = \theta_K R \int_{q_{\min}}^{q_{\max}} \psi_R [q, m_R(q)] \ell_R [q, m_R(q)]^\gamma dG_R(q),$$

where $\ell_R [q, m_R(q)]$ is employment for a project of quality q . From footnote 7 we have

$$\ell_R [q, m_R(q)] = \left[\frac{\gamma p_{RM} \psi_R [q, m_R(q)]}{w_R(a)} \right]^{\frac{1}{1-\gamma}},$$

and therefore

$$g_M = \theta_K^{\frac{1}{1-\gamma}} (\gamma p_{RM})^{\frac{\gamma}{1-\gamma}} R \int_{q_{\min}}^{q_{\max}} \psi_R [q, m_R(q)]^{\frac{1}{1-\gamma}} w_R [m_R(q)]^{-\frac{\gamma}{1-\gamma}} dG_R(q).$$

Next, substituting (17) with $\tilde{q} = q_{\min}$ into this equation yields

$$g_M = \frac{N}{\gamma p_{RM}} \int_{a_R}^{a_{\max}} w_R(a) dH(a). \quad (28)$$

This is a version of the RR curve.

From (15) and (20), we obtain:

$$\begin{aligned} p_{RM} &= \frac{1}{\gamma \theta_K} \left\{ \frac{N \int_{a_R}^{a_{\max}} w_R(a) dH(a)}{R \int_{q_{\min}}^{q_{\max}} \psi_R [z, m_R(z)]^{\frac{1}{1-\gamma}} w_R [m_R(z)]^{-\frac{\gamma}{1-\gamma}} dG_R(z)} \right\}^{1-\gamma} \\ &= \frac{w(a_R; a_R)}{\gamma \theta_K} \left\{ \frac{N \int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)}{R \int_{q_{\min}}^{q_{\max}} \psi_R [q, m_R(q)]^{\frac{1}{1-\gamma}} \lambda_R [m_R(q; a_R); a_R]^{-\frac{\gamma}{1-\gamma}} dG_R(q)} \right\}^{1-\gamma}, \end{aligned}$$

and therefore

$$p_{RM} = \frac{w(a_R; a_R)}{\gamma \theta_K} \left(\frac{N}{R} \right)^{1-\gamma} \frac{1}{\Phi(a_R)}, \quad (29)$$

where

$$\Phi(a_R) \equiv \left\{ \frac{\int_{q_{\min}}^{q_{\max}} \psi_R [q, m_R(q; a_R)]^{\frac{1}{1-\gamma}} \lambda_R [m_R(q; a_R); a_R]^{-\frac{\gamma}{1-\gamma}} dG_R(q)}{\int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)} \right\}^{1-\gamma}. \quad (30)$$

Substituting this expression into (28) yields the modified RR curve,

$$g_M = \theta_K N^\gamma R^{1-\gamma} \Phi(a_R) \int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a). \quad (31)$$

We now prove

Lemma 5 *The function $\Phi(a_R)$ is increasing while the product $\Phi(a_R) \int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)$ is decreasing in a_R . Therefore the RR curve slopes downward.*

First, note that, in view of (12),

$$\log \lambda_R(a; a_R) = \int_{a_R}^a \frac{\psi_{Ra} [m_R^{-1}(z; a_R), z]}{\gamma \psi_R [m_R^{-1}(z; a_R), z]} dz \quad \text{for } a > a_R$$

and therefore

$$-\frac{\lambda_{Ra_R}(a; a_R)}{\lambda_R(a; a_R)} = \frac{\psi_{Ra}(q_{\min}, a_R)}{\gamma \psi_R(q_{\min}, a_R)} - \int_{a_R}^a \frac{\partial}{\partial a_R} \left\{ \frac{\psi_{Ra} [m_R^{-1}(z; a_R), z]}{\gamma \psi_R [m_R^{-1}(z; a_R), z]} \right\} dz.$$

The derivative under the integral on the right-hand side of this equation is negative, because an increase in a_R worsens each worker's match (see Figure 2), i.e., $m_R^{-1}(z; a_R)$ is declining in a_R and $\psi_{Ra}(q, z)/\psi_R(q, z)$ is increasing in q due to Assumption 2. Together with equation (12) and Assumption 3, this implies:

$$-\frac{\lambda_{Ra_R}(a; a_R)}{\lambda_R(a; a_R)} > \frac{\psi_{Ra}(q_{\min}, a_R)}{\gamma \psi_R(q_{\min}, a_R)} > \frac{\psi_a(\varphi, a_R)}{\psi(\varphi, a_R)} > 0 \quad \text{for all } \varphi \text{ and all } a > a_R.$$

From (11) we obtain:

$$\log \lambda(a; a_R) = \int_{a_{\min}}^a \frac{\psi_a [m^{-1}(z; a_R), z]}{\psi [m^{-1}(z; a_R), z]} dz \quad \text{for } a < a_R$$

and therefore

$$\frac{\lambda_a(a; a_R)}{\lambda(a; a_R)} = \frac{\psi_a [m^{-1}(a; a_R), a]}{\psi [m^{-1}(a; a_R), a]} > 0 \quad \text{for all } a < a_R.$$

Thus, we have

Lemma 6

$$-\frac{\lambda_{Ra_R}(a; a_R)}{\lambda_R(a; a_R)} > \frac{\lambda_a(a_R; a_R)}{\lambda(a_R; a_R)} = \frac{\psi_a(\varphi_{\max}, a_R)}{\psi(\varphi_{\max}, a_R)} \quad \text{for all } a > a_R.$$

Next, consider the definition of $\Phi(a_R)$ in (30); it can be expressed as

$$\begin{aligned} \log \Phi(a_R) &= (1 - \gamma) \log \left\{ \int_{q_{\min}}^{q_{\max}} \psi_R [q, m_R(q; a_R)]^{\frac{1}{1-\gamma}} \lambda_R [m_R(q; a_R); a_R]^{-\frac{\gamma}{1-\gamma}} dG_R(q) \right\} \\ &\quad - (1 - \gamma) \log \left\{ \int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a) \right\}. \end{aligned}$$

Differentiating this equation yields

$$\begin{aligned} \frac{\Phi'(a_R)}{\Phi(a_R)} &= -\gamma \int_{q_{\min}}^{q_{\max}} \omega_G(q, a_R) \frac{\lambda_{Ra_R} [m_R(q; a_R); a_R]}{\lambda_R [m_R(q; a_R); a_R]} dq - (1 - \gamma) \int_{a_R}^{a_{\max}} \omega_H(a, a_R) \frac{\lambda_{Ra_R}(a; a_R)}{\lambda_R(a; a_R)} da \\ &\quad + \frac{(1 - \gamma) \lambda_R(a_R; a_R) H'(a_R)}{\int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)}, \end{aligned} \tag{32}$$

where

$$\omega_G(q, a_R) = \frac{\psi_R[q, m_R(q; a_R)]^{\frac{1}{1-\gamma}} \lambda_R[m_R(q; a_R); a_R]^{-\frac{\gamma}{1-\gamma}} G'_R(q)}{\int_{q_{\min}}^{q_{\max}} \psi_R[q, m_R(q; a_R)]^{\frac{1}{1-\gamma}} \lambda_R[m_R(q; a_R); a_R]^{-\frac{\gamma}{1-\gamma}} dG_R(q)}$$

and

$$\omega_H(a, a_R) = \frac{\lambda_R(a; a_R) H'(a)}{\int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)}$$

are weights that satisfy

$$\int_{q_{\min}}^{q_{\max}} \omega_G(q, a_R) dq = \int_{a_R}^{a_{\max}} \omega_H(a, a_R) da = 1.$$

Lemma 6 implies

$$\begin{aligned} -\frac{\lambda_{Ra_R}[m_R(q; a_R); a_R]}{\lambda_R[m_R(q; a_R); a_R]} &> \frac{\lambda_a(a_R; a_R)}{\lambda(a_R; a_R)} \text{ for all } q, \\ -\frac{\lambda_{Ra_R}(a; a_R)}{\lambda_R(a; a_R)} &> \frac{\lambda_a(a_R; a_R)}{\lambda(a_R; a_R)} \text{ for all } a > a_R, \end{aligned}$$

and since the last term in (32) is positive, we have

Lemma 7

$$\frac{\Phi'(a_R)}{\Phi(a_R)} > \frac{\lambda_a(a_R; a_R)}{\lambda(a_R; a_R)} > 0,$$

The lemma establishes that $\Phi(a_R)$ is an increasing function.

Although, as shown above, $\Phi(a_R)$ is an increasing function and $\int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)$ is a decreasing function of a_R , their product is decreasing in a_R , and therefore the RR curve slopes downward. To see this, note from the definition of $\Phi(a_R)$ that

$$\begin{aligned} &\Phi(a_R) \int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a) \\ = &\Phi(a_R)^{-\frac{\gamma}{1-\gamma}} \int_{q_{\min}}^{q_{\max}} \psi_R[q, m_R(q; a_R)]^{\frac{1}{1-\gamma}} \lambda_R[m_R(q; a_R); a_R]^{-\frac{\gamma}{1-\gamma}} dG_R(q). \end{aligned}$$

Since $\Phi(a_R)$ is increasing in a_R and the expression under the integral on the right-hand side is declining in a_R , it follows that the right-hand side of this equation is declining in a_R and therefore that $\Phi(a_R) \int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)$ is declining in a_R . Consequently, the RR curve slopes downward.

A2.6 The AA Curve

In this section, we derive the equation for the AA curve and establish that the curve is upward sloping. Equations (8) and (10) yield

$$p_R = \sigma^{-\sigma} (\sigma - 1)^{(\sigma-1)} X \frac{\int_{\varphi_{\min}}^{\varphi_{\max}} \left\{ \frac{w_M[m(\varphi)]}{\psi[\varphi, m(\varphi)]} \right\}^{1-\sigma} dG(\varphi)}{\rho + g_M}. \quad (33)$$

while (14) yields

$$MX \left(\frac{\sigma}{\sigma - 1} \right)^{-\sigma} \int_{\varphi_{\min}}^{\varphi_{\max}} \left\{ \frac{w_M [m(\varphi)]}{\psi [\varphi, m(\varphi)]} \right\}^{1-\sigma} dG(\varphi) = N \int_{a_{\min}}^{a_R} w_M(a) dH(a). \quad (34)$$

Therefore:

$$\rho + g_M = \frac{1}{\sigma - 1} \frac{N}{p_R M} \int_{a_{\min}}^{a_R} w_M(a) dH(a)$$

This is a version of the *AA* curve. Using (20) and (29), this can be expressed as:

$$\rho + g_M = \frac{\gamma}{\sigma - 1} \theta_K N^\gamma R^{1-\gamma} \Phi(a_R) \int_{a_{\min}}^{a_R} \frac{\lambda(a; a_R)}{\lambda(a_R; a_R)} dH(a),$$

which is the *AA* curve in the text (see (22)). We now show that $\Phi(a_R) \int_{a_{\min}}^b \frac{\lambda(a; b)}{\lambda(a_R; b)} dH(a)$ is an increasing function of both a_R and b , for $b \rightarrow a_R$, and therefore the *AA* curve slopes upwards.

From (11) we obtain

$$\log \left[\frac{\lambda(a; b)}{\lambda(a_R; b)} \right] = - \int_a^{a_R} \frac{\psi_a [m^{-1}(z; b), z]}{\psi [m^{-1}(z; b), z]} dz \quad \text{for } a < a_R.$$

Due to Assumption 2 the right-hand side of this equation is rising in b , because an increase in b reduces the quality of matches for manufacturing workers (see Figure 2), i.e., $m^{-1}(z; b)$ is declining in b . Therefore $\Phi(a_R) \int_{a_{\min}}^b \frac{\lambda(a; b)}{\lambda(a_R; b)} dH(a)$ is rising in b . In addition, Lemma 7 implies that $\Phi(a_R) \int_{a_{\min}}^b \frac{\lambda(a; b)}{\lambda(a_R; b)} dH(a)$ is rising in a_R for $b \rightarrow a_R$, which establishes that the *AA* curve slopes upward.

A4.3 Cross-Country Wage Levels with Differences in Innovation Capacity

Here we consider the cross-country differences in wage levels that result from asymmetries in innovation capacity. We assume equal R&D subsidy rates and complete international knowledge spillovers; i.e., $s_j = s$ and $\theta_{Kjc} = \theta_{Kc}$ for all j . Note that this allows for international differences in capacities to convert knowledge capital into new varieties, as captured by θ_{Kc} . We also allow for differences in country size, N_c , in active research projects R_c (which is proportional to the country's research capital) and for differences in research productivity, θ_{Rc} .

We have seen in Section 4.3 that under these circumstances the cutoff ability levels a_{Rc} are the same in all countries, and therefore so are relative wages of workers with different ability levels. We represent the wage schedule in country c by $w_c(a) = \omega_c w(a)$ and refer to ω_c as the wage level in country c . Moreover, (24) implies that, in this case, $\kappa_c \theta_{Rc} N_c^\gamma R_c^{1-\gamma} = \zeta$ for all countries and therefore $\zeta M_c = \theta_{Rc} N_c^\gamma R_c^{1-\gamma} \theta_{Kc} \sum_j M_j$. As a result:

$$\frac{M_i}{M_j} = \frac{\theta_{Ri} N_i^\gamma R_i^{1-\gamma} \theta_{Ki}}{\theta_{Rj} N_j^\gamma R_j^{1-\gamma} \theta_{Kj}}.$$

In addition, free trade in intermediate inputs, i.e., $\tau_{jc} = 1$ for all j and c , implies $\bar{X}_c = \bar{X}$ for all

countries, so that market potential does not vary across countries. Using this result together with (34), which holds in every open economy with X replaced by \bar{X} , implies

$$\left(\frac{\omega_i}{\omega_j}\right)^\sigma = \frac{M_i/N_i}{M_j/N_j} = \frac{\left(\theta_{Ri}N_i^\gamma R_i^{1-\gamma}\theta_{Ki}\right)/N_i}{\left(\theta_{Rj}N_j^\gamma R_j^{1-\gamma}\theta_{Kj}\right)/N_j}.$$

It follows that wages are higher in country i than country j if and only if $\left(\theta_{Ri}N_i^\gamma R_i^{1-\gamma}\theta_{Ki}\right)/N_i > \left(\theta_{Rj}N_j^\gamma R_j^{1-\gamma}\theta_{Kj}\right)/N_j$, i.e., if and only if country i has a higher innovation capacity per person.

A4.4 Spillover Effects of National R&D Subsidies

In this appendix, we examine the effects of changing an R&D subsidy in one country on growth and inequality in that country and in all trading partners. We suppose that international knowledge spillovers are complete and that countries are similar in all ways except in their R&D subsidies and in the proportional wage taxes used to finance these subsidies. That is, we assume $\theta_{Kcj} = \theta_K$ and $\theta_{Rc} = \theta_R$ for all c and j , and $N_c = N$ and $R_c = R$ for all c . These assumptions focus attention on variations in R&D subsidies.

The equations for the RR and AA curves, (24) and (25), can be expressed in this case as:

$$g_M = \kappa_c \theta_R N^\gamma R^{1-\gamma} \Phi(a_{Rc}) \int_{a_{Rc}}^{a_{\max}} \lambda_R(a; a_{Rc}) dH(a), \quad (35)$$

$$(1 - s_c)(\rho + g_M) = \frac{\gamma}{\sigma - 1} \kappa_c \theta_R N^\gamma R^{1-\gamma} \Phi(a_{Rc}) \frac{\int_{a_{\min}}^{a_{Rc}} \lambda(a; a_{Rc}) dH(a)}{\lambda(a_{Rc}; a_{Rc})}, \quad (36)$$

where g_M is the same in all countries in the steady state. Dividing (36) by (35) yields:

$$(1 - s_c) \frac{\rho + g_M}{g_M} \Omega(a_{Rc}) = \Lambda(a_{Rc}), \quad (37)$$

where

$$\Omega(a_{Rc}) \equiv \Phi(a_{Rc}) \int_{a_{Rc}}^{a_{\max}} \lambda_R(a; a_{Rc}) dH(a)$$

is a decreasing function, as shown above (recall that RR slopes downward), and

$$\Lambda(a_{Rc}) \equiv \frac{\gamma}{\sigma - 1} \frac{\int_{a_{\min}}^{a_{Rc}} \lambda(a; a_{Rc}) dH(a)}{\lambda(a_{Rc}; a_{Rc})} \Phi(a_{Rc})$$

is an increasing function, as shown above (recall that AA is sloping upwards). It follows from this equation that countries with higher R&D subsidies have lower cutoffs a_{Rc} and employ more workers in R&D. Moreover, multiplying (35) by M_c , recalling that $\kappa_c = \theta_K \left(\sum_{j=1}^C M_j\right)/M_c$, and summing up, we obtain:

$$g_M = \theta_K \theta_R N^\gamma R^{1-\gamma} \sum_{j=1}^C \Omega(a_{Rj}).$$

Substituting this equation into (37) then yields:

$$(1 - s_c) \frac{\rho + \theta_K \theta_R N^\gamma R^{1-\gamma} \sum_{j=1}^C \Omega(a_{Rj})}{\theta_K \theta_R N^\gamma R^{1-\gamma} \sum_{j=1}^C \Omega(a_{Rj})} \Omega(a_{Rc}) = \Lambda(a_{Rc}). \quad (38)$$

There are C equations like (38), one for each country, and they allow us to solve the cutoffs a_{Rc} .

Now, proportionately differentiate this system of equations and write the (matrix) equation for the proportional changes as

$$\mathbf{A}_s \mathbf{a}_s = \mathbf{b}_s,$$

where

$$\mathbf{a}_s = \begin{pmatrix} \hat{a}_{R1} \\ \hat{a}_{R2} \\ \cdot \\ \cdot \\ \cdot \\ \hat{a}_{RC} \end{pmatrix}, \quad \mathbf{b}_s = \begin{pmatrix} \widehat{(1 - s_1)} \\ \widehat{(1 - s_2)} \\ \cdot \\ \cdot \\ \cdot \\ \widehat{(1 - s_C)} \end{pmatrix},$$

and a “hat” over a variable represents a proportional rate of change; i.e., $\hat{a}_{Rc} = da_{Rc}/a_{Rc}$ and $\widehat{(1 - s_c)} = d(1 - s_c)/(1 - s_c)$.

We note that the matrix \mathbf{A}_s has positive diagonal elements and negative off-diagonal elements. In particular, in row j , the diagonal element is $\varepsilon_{\Lambda j} + (1 - \eta_j) \varepsilon_{\Omega j}$, where $\varepsilon_{\Lambda j} > 0$ is the elasticity of $\Lambda(\cdot)$ evaluated at a_{Rj} , $\varepsilon_{\Omega j} > 0$ is minus the elasticity of $\Omega(\cdot)$ evaluated at a_{Rj} , and

$$\eta_j = \left[\frac{\rho}{\rho + \theta_K \theta_R N^\gamma R^{1-\gamma} \sum_{i=1}^C \Omega(a_{Ri})} \right] \frac{\Omega(a_{Rj})}{\sum_{i=1}^C \Omega(a_{Ri})} < 1.$$

For $j \neq c$, the off-diagonal element in column j is $-\eta_j \varepsilon_{\Omega j} < 0$.

Inasmuch as \mathbf{A}_s has only negative off-diagonal elements, we recognize that it is a Z -matrix. Moreover, there exists a diagonal matrix \mathbf{D}_s such that $\mathbf{A}_s \mathbf{D}_s$ is diagonally dominant in its rows. To see this, consider the diagonal matrix \mathbf{D}_s that has a diagonal entry in row j given by $1/\varepsilon_{\Omega j}$. Then the diagonal element in row c and column c of $\mathbf{A}_s \mathbf{D}_s$ is given by $\varepsilon_{\Lambda c}/\varepsilon_{\Omega c} + (1 - \eta_c)$ and the off-diagonal element in row c and column j is given by $-\eta_j$. Summing the entries in any row c gives $\varepsilon_{\Lambda c}/\varepsilon_{\Omega c} + 1 - \sum_{j=1}^C \eta_j > 0$, where the inequality follows from the fact that $\sum_{j=1}^C \eta_j < 1$.

Having established that \mathbf{A}_s is a Z -matrix and there exists a diagonal matrix \mathbf{D}_s such that $\mathbf{A}_s \mathbf{D}_s$ is diagonally dominant in its rows, it follows that \mathbf{A}_s is an M -matrix (see Johnson, 1982). Then its inverse, \mathbf{A}_s^{-1} , has only positive elements. We conclude that an increase in any subsidy rate (i.e., a reduction in any $1 - s_c$) reduces every cutoff point a_{Rj} , $j = 1, \dots, C$. Since more individuals are hired as researchers in every country, every country grows faster and experiences greater income inequality.