Modeling Insurance Markets

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Modeling Competition Insurance Markets is Tough

- There is no well-agreed upon model of competitive insurance markets
  - Despite 50 years of research!
- Standard notions of pure strategy competitive equilibria break down
  - Preferences/Demand are related to cost
- Insurers can manipulate not only price but also the design of contracts to affect their own (and others) costs
  - Leads to unraveling!
Akerlof (1970): Cars lose value the day after they’re sold...

- Argued that market for health insurance above age 65 does not exist because of adverse selection
  - Market unraveled because of adverse selection “death spiral”

Problem with model: single contract traded, so competition only on price

- Rothschild and Stiglitz (1976) + 1000+ other papers...
  - Compete on more than 1 dimension of the contract
  - Standard game-theoretic notions of (pure strategy) equilibria may not exist -> “Market unraveling”
Clarify when the standard competitive model goes wrong (and hence we have to choose amongst competing game-theoretic models)

- Clarify what we mean by “unraveling”

Discuss 2 classes of “solutions” to non-existence

- Miyazaki-Wilson-Spence (Reach the constrained pareto frontier)
- Riley (1979) (Don’t reach the frontier)

Context: Binary insurance model with uni-dimensional type distribution
Model Environment

- Unit mass of agents endowed with wealth $w$
- Face potential loss of size $l$ with privately known probability $p$
  - Distributed with c.d.f. $F(p)$ with support $\Psi$
    - Could be continuous, discrete or mixed
    - Rothschild and Stiglitz (1976): $p \in \{p_L, p_H\}$ (2 types)
  - Let $P$ denote random draw from population (c.d.f. $F(p)$)
- Agents vNM preferences
  \[ pu(c_L) + (1 - p) u(c_{NL}) \]
• Insurance structure: Rothschild and Stiglitz (1976) with menus

• There exists a set of risk-neutral insurance companies, \( j \in J \) seeking to maximize expected profits by choosing a menu of consumption bundles:

\[
A_j = \left\{ c^j_L (p), c^j_{NL} (p) \right\}_{p \in \Psi}
\]

• First, insurers simultaneously offer a menu of consumption bundles

• Given the set of available consumption bundles,

\[
A = \bigcup_j A_j
\]

• Individuals choose the bundle that maximizes their utility
An allocation \( A = \{c_L(p), c_{NL}(p)\}_{p \in \Psi} \) is a **Competitive Nash Equilibrium** if

1. **A is incentive compatible**

\[
p u(c_L(p)) + (1 - p) u(c_{NL}(p)) \geq p u(c_L(\tilde{p})) + (1 - p) u(c_{NL}(\tilde{p})) \quad \forall p, \tilde{p}
\]

2. **A is individually rational**

\[
p u(c_L(p)) + (1 - p) u(c_{NL}(p)) \geq p u(w - l) + (1 - p) u(w) \quad \forall p \in \Psi
\]

3. **A has no profitable deviations** [Next Slide]
No Profitable Deviations

For any other menu, $\hat{A} = \{\hat{c}_L (p), \hat{c}_{NL} (p)\}_{p \in \Psi}$, it must be that

$$\int_{p \in D(\hat{A})} [p (w - l - c_L (p)) + (1 - p) (w - c_{NL} (p))] dF (p) \leq 0$$

where

$$D (\hat{A}) = \left\{ p \in \Psi \left| \max_{p} \{pu (\hat{c}_L (p)) + (1 - p) u (\hat{c}_{NL} (p))\} > pu (c_L (p)) + (1 - p) u (c_{NL} (p)) \right. \right\}$$

- $D (\hat{A})$ is the set of people attracted to $\hat{A}$
- Require that the profits earned from these people are non-positive
Two Definitions of Unraveling

- **Akerlof unraveling**
  - Occurs when demand curve falls everywhere below the average cost curve
  - Market unravels and no one gets insurance

- **Rothschild and Stiglitz unraveling**
  - Realize a Competitive Nash Equilibrium may not exist
  - Market unravels a la Rothschild and Stiglitz when there does not exist a Competitive Nash Equilibrium
Theorem

The endowment, \( \{ (w - l, w) \} \), is a competitive equilibrium if and only if

\[
\frac{p}{1 - p} \frac{u'(w - l)}{u'(w)} \leq \frac{E[P|P \geq p]}{1 - E[P|P \geq p]} \quad \forall p \in \Psi \setminus \{1\} \tag{1}
\]

where \( \Psi \setminus \{1\} \) denotes the support of \( F(p) \) excluding the point \( p = 1 \).

- The market unravels à la Akerlof when no one is willing to pay the pooled cost of worse risks (Hendren 2013)
  - Theorem extends Akerlof unraveling to set of all potential traded contracts, as opposed to single contract
  - No gains to trade \( \rightarrow \) no profitable deviations by insurance companies
Akerlof Unraveling

\[ \frac{E[P|P>p]}{1 - E[P|P>p]} \]

\[ pu'(w-l) \]

\[ (1-p)u'(w) \]
Akerlof Unraveling (2)

\[ \frac{E[P|P>p]}{1 - E[P|P>p]} \]

\[ pu'(w-l) \]

\[ (1-p)u'(w) \]
Akerlof Unraveling (3)

\[
\frac{E[P|P>p]}{1 - E[P|P>p]} = pu'(w-l) \quad \text{and} \quad (1-p)u'(w)
\]
Aside: High Risks

- Corollary: If the market fully unravels a la Akerlof, there must exist arbitrarily high risks:

\[ F(p) < 1 \quad \forall p < 1 \]

- Need full support of type distribution to get complete Akerlof unraveling
  - Can be relaxed with some transactions costs (see Chade and Schlee, 2013)
When does a Competitive Nash Equilibrium exist?

Here, I follow Rothschild and Stiglitz (1976) and Riley (1979)

Generic fact: Competition $\rightarrow$ zero profits

Key insight of Rothschild and Stiglitz (1976): Nash equilibriums can’t sustain pooling of types
Rothschild and Stiglitz: No Pooling
Rothschild and Stiglitz: No Pooling (2)

Diagram showing Good Risk and Bad Risk regions with a 45-degree line and a point labeled 'Pooled' between them.
Rothschild and Stiglitz: No Pooling (3)
Regularity condition

- No pooling + zero profits $\Rightarrow$ No cross subsidization:

$$pc_L(p) + (1 - p)c_{NL}(p) = w - pl \quad \forall p \in \Psi$$

- Insurers earn zero profits on each type

A Regularity Condition

Suppose that either:

1. There exists an interval over which $P$ has a continuous distribution
2. $P = 1$ occurs with positive probability

Satisfied if either $F$ is continuous or $F$ is discrete with $p = 1$ in the support of the distribution

Can approximate any distribution with distributions satisfying the regularity condition
Result #2: Exhaustive of Possible Occurrences

**Theorem**

Suppose the regularity condition holds. Then, there exists a Competitive Nash Equilibrium if and only if the market unravels a la Akerlof (1970)

- Either no one is willing to cross-subsidize -> no profitable deviations that provide insurance
- Or, people are willing to cross-subsidize -> generically, this can’t be sustained as a Competitive Nash Equilibrium
- Proof: Need to show that Nash equilibrium does not exist when Akerlof unraveling condition does not hold
  - Case 1: \( P = 1 \) has positive probability
    - Risks \( p < 1 \) need to subsidize \( p = 1 \) type in order to get insurance
  - Case 2: \( P \) is continuous and bounded away from \( P = 1 \).
    - We know Akerlof unraveling condition cannot hold
    - Follow Riley (1979) – shows there’s an incentive to pool types -> breaks potential for Nash equilibrium existence
Generic No Equilibrium (Riley)
Generic No Equilibrium (Riley) (2)
Generically, either the market unravels a la Akerlof or Rothschild and Stiglitz

- No gains to trade $\rightarrow$ unravels a la Akerlof
  - No profitable deviations $\rightarrow$ competitive equilibrium exists

- Gains to trade $\rightarrow$ no unraveling a la Akerlof
  - But there are profitable deviations
  - Generically, no Competitive Equilibrium (unravels a la Rothschild and Stiglitz)

We don’t have a model of insurance markets!

- Generically, the standard Nash model generically fails to make predictions precisely when there are theoretical gains to trade
Two classes of models in response to non-existence

Consider 2-stage games:

Stage 1: firms post menu of contracts

Stage 2: Assumption depends on equilibrium notion:

- Miyazaki-Wilson-Spence: Firms can drop unprofitable contracts
  - Formalized as dynamic game in Netzer and Scheuer (2013)
- Riley: Firms can add contracts
  - Formalized as dynamic game in Mimra and Wambach (2011)

Then, individuals choose insurance contracts
Miyazaki (1979); Wilson (1977); Spence (1978)

Two Stage Game:

- Firms choose contracts
  - Menus (Miyazaki)
  - Single contracts (Wilson / Spence)
- Firms observe other contracts and can drop (but not add) contracts/menus
  - In Miyazaki, firms have to drop the entire menu
- Individuals choose insurance from remaining set of contracts
Reaching the Pareto frontier requires allowing some contracts to run deficits/surplus

- Individuals generically are willing to “buy off” worse risks’ incentive constraints

Miyazaki Wilson Spence allows for this if the good types want to subsidize the bad types

- If you try to steal my profitable contract, I drop the corresponding negative profit contract and you get dumped on!

MWS equilibrium maximizes welfare of best risk type by making suitable compensations to all other risk types to relax IC constraint

- Fully separating solution in Miyazaki
- Can be pooling in Wilson / Spence
Predicts “fully separating” contracts with no cross-subsidization across types

- IC constraint + zero profit constraints determine equilibrium

Why no cross-subsidization?

- If cross-subsidization, then firms can add contracts.
- But, firms forecast this response and therefore no one offers these subsidizing contracts

Predicts no trade if full support type distribution
Other non-game-theoretic approaches

- **Walrasian:**
  - Bisin and Gotardi (2006)
    - Allow for trading of choice externalities \(\rightarrow\) reach efficient frontier/MWS equilibrium (pretty unrealistic setup...)
  - Azevedo and Gottlieb (2015) \(\rightarrow\) reach inefficient Riley equilibria

- Search / limited capacity / limited liability / cooperative solutions / etc.
  - Guerrieri and Shimer (2010) \(\rightarrow\) reach inefficient Riley equilibria
Empirical Question?

- Need theory of a mapping from type distributions to outcomes
  - Standard model works if prediction is no trade
    - This happens for those with “pre-existing conditions” in LTC, life, and disability insurance (Hendren 2013)
  - But, standard model fails when market desires cross-subsidization
    - Key debate: can competition deliver cross-subsidization?
    - Should be empirical question!?

- In short, insurance markets are fun because no one agrees about how to model them!