Static versus Dynamic Informational Robustness in Intertemporal Pricing

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Abstract. We revisit the classic problem of a durable goods monopolist selling a product over time with commitment (as studied by Stokey (1979)). We explicitly assume that the buyer does not observe her value perfectly, but instead observes only a noisy signal of it, according to some information structure. The seller is uncertain about the information structure, and chooses prices to maximize the worst case profit. If information can arrive arbitrarily over time, the seller can do no better than if he had a single period to sell. On the other hand, if the buyer receives information only in the first period, the seller can improve upon the single period benchmark by offering a declining price path. These results extend to cases where multiple buyers may arrive over time, or additional information can be released by the seller. Our analysis illustrates the potential value of intertemporal price discrimination in settings with endogenous information.

Suppose a monopolist has developed a completely new durable product and is deciding how to set prices to maximize profit. Consulting the literature on intertemporal pricing, the monopolist may at first think that if the pool of potential consumers does not change over time, profit would be maximized by charging the optimal static price in each period. However, if the product is completely new, then the monopolist should consider the possibility that consumers will learn

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1See Stokey (1979), Bulow (1981), Conlisk, Gerstner and Sobel (1984), among others. These papers show that a seller with commitment does not benefit from choosing lower prices in later periods. Another strand of literature studies
something about their value after pricing decisions have been made. For example, the Apple Watch, Amazon Echo, and Google Glass were all new technologies that most consumers had little prior experience to inform their willingness-to-pay. On top of this, how much journalists or product reviewers write about the product may depend on what price path the monopolist sets. In general, this possibility of information arrival can make the monopolist’s problem quite complicated. This paper develops an intertemporal pricing model where a buyer observes some noisy signal of her value, possibly over time. We assume that the seller does not know the information structure that informs the buyer of her value, and chooses prices as if the information structure were the worst possible given the prices posted.

We show that if learning is unrestricted, then the seller achieves the optimal profit with a constant price path. One possible explanation for this result is as follows: in each period, the adversarial nature/competitor could release information that minimize the profit in that period. Doing so would make the seller’s problem separable across time, eliminating potential gains from decreasing prices. This intuition is however incomplete, because the worst case information structures for different periods need not be consistent, in the sense that past information may prevent the adversary from minimizing profits in the future. A key step of our argument is to show that the worst case information structure in any period takes a partitional form, so that consistency can be satisfied.

If the monopolist is able to rule out learning, then we show that intertemporal price discrimination has value. Specifically, Proposition 3 shows that if the buyer receives all information in the first period, then the seller can improve upon the one period benchmark by charging different prices in different periods. As far as we are aware, the potential for the pricing decision to interact with the availability of information provides a novel explanation for intertemporal price discrimination.

In the applications listed above, our results imply that when information is concentrated around the release date—say, through the Consumer Reports review—then intertemporal price discrimination may be optimal. But if information can arrive at later dates—say, through Amazon ratings or even word of mouth—then the optimality of the static price is restored.

We are not the first to consider the application of dynamic pricing with new products, and in fact there is by now a sizable literature that uses maxmin objectives in these settings. However, intertemporal pricing without commitment and derives the Coase conjecture, which predicts decreasing prices and vanishing profits when agents are patient and the selling horizon is long (see Fudenberg, Levine, Tirole (1985) and Gul, Sonnenschein, Wilson (1986)). In this paper, we are entirely concerned with a committed seller. See Section 1.1 for further discussion of this assumption.

Ambiguity is one general justification for using our maxmin objective. For this particular story, another justification is that a competitor may be interested in minimizing Apple’s profits (even in cases where the competitor does not have a direct competing product to the Apple Watch). Our framework would be appropriate if other firms were able to release information on the product in a way that is outside of Apple’s control.
all papers we are aware of consider ambiguity over the value distribution itself, and not over information structures. While Carrasco et. al. (2015) and Handel and Misra (2014) study the case where the buyer can purchase multiple units (whereas our buyer purchases at most a single unit), the case of durable goods has been studied by Caldentey, Liu, Lobel (2016), Liu (2016) and Chen and Farias (2016). These papers do not relate optimal pricing to learning and information arrival, as we do here. Information arrival places restrictions on how the value evolves, and rules out the main cases considered in the literature.3

Outside of the intertemporal pricing literature, mechanism design under ambiguity over information has indeed been studied in prior and ongoing work. Along these lines, a contemporaneous paper of Bergemann, Brooks and Morris (2016) examines a static auction environment with common values. Though their setting is quite different from ours, we share similar general motivation for considering informational robustness. However, we are not aware of any papers in this literature that consider the effect of learning on the designer’s maxmin decision problem, though this has been considered in the related literature of information design—see Ely (2017).

We proceed as follows. The next section sets up the model, defines different informational robustness concepts and discusses our commitment assumption. In Section 2 we consider the single period benchmark, and in Section 3 we show that when learning is unrestricted, more periods do not help the seller. We then restrict learning and show the optimality of intertemporal price discrimination in Section 4. Section 5 considers two natural extensions: arriving buyers and seller information disclosure. Section 6 concludes. All omitted proofs and additional results can be found in the Appendix.

1. MODEL

A seller (he) sells a durable good to a buyer (she) at time \( t = 1, 2, \ldots, T \), where \( T \leq \infty \). Both the seller and the buyer discount the future at rate \( \delta \). The product is costless for the seller to produce, while the buyer has unit demand and obtains discounted lifetime utility from purchasing the object equal to \( v \). The value \( v \) has distribution \( F \) supported on \( \mathbb{R}_+ \), with \( 0 < \mathbb{E}[v] < \infty \). Let \( v \) denote the minimum value in the support of \( F \). The distribution \( F \) is fixed and common knowledge, and the buyer’s value for the object does not change over time.

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3Several Bayesian settings allow for the value to change over time without explicitly modeling information arrival. Stokey (1979) assumes the value changes deterministically given the initial type. Deb (2014) and Garrett (2016) allow for stochastically changing values, but in these papers the value changes in ways that violate the martingale condition for expectations. Deb (2014) assumes the value is independently redrawn upon Poisson shocks. For Garrett (2016), the value follows a two-type Markov-switching process.

4Our analysis is unchanged in the case of a continuum of buyers who do not know their true values, as the worst case would simply involve each buyer being given the same information structure. The case with arriving buyers is considered in Section 5.1.
However, the buyer does not directly know $v$; instead, she observes some signal which gives information about $v$. An information structure is a function which maps true values into realizations of signals. To be precise, we define a static information structure to be a set $S$ and a function $I : \mathbb{R}_+ \to \Delta(S)$, with the interpretation that the buyer observes a signal $s$ at time 1, prior to making a purchase, and the conditional distribution of $s$ depends on the true value $v$. A dynamic information structure is a sequence of sets $(S_t)_{t=1}^T$ and functions $I_t(s^{t-1}) : \mathbb{R}_+ \to \Delta(S_t)$, with the interpretation that the agent observes $s_t$ at time $t$, which could depend on the entire history $s^{t-1}$ of realized past signals.

The timing of the model is as follows. At time 0, the seller commits to a price path $(p_t)_{t=1}^T$, where we allow $p_t = \infty$ to mean that the seller refuses to sell in period $t$. Then nature chooses an information structure $\mathcal{I} \in \Phi$, where $\Phi$ denotes either the set of static information structures or the set of dynamic information structures. Having observed both the price path and the information structure, the buyer chooses a strategy to maximize expected discounted value less price:

$$\tau^* \in \arg \max_{\tau} \mathbb{E} \left[ \delta^{\tau-1}(\mathbb{E}[v|s_1, \ldots, s_\tau] - p_\tau) \right].$$

Here $\tau$ represents a stopping time adapted to the informal arrival process under $\mathcal{I}$, meaning that the buyer purchases the good whenever $\tau$ dictates her to stop, i.e. purchase, given the signals observed so far. We allow $\tau$ to take any positive integer value $\leq T$, or $\tau = \infty$ to mean that the buyer does not ever buy.

We consider the case where the seller only knows that nature chooses some (static or dynamic) information structure, but does not know anything more than this. The seller evaluates payoffs as if the information structure chosen were the worst possible, given his own choice of the price path $(p_t)_{t=1}^T$ and buyer’s optimizing behavior. Hence the seller’s payoff is:

$$\sup_{(p_t)_{t=1}^T} \inf_{\mathcal{I} \in \Phi, \tau^*} \mathbb{E}[\delta^{\tau^*-1} p_{\tau^*}] \text{ s.t. } \tau^* \text{ is optimal given } (p_t)_{t=1}^T \text{ and } \mathcal{I}.$$ 

Note that when the buyer faces indifference, ties are broken against the seller. Breaking indifference in favor of the seller would not change our results, but would add cumbersome details.\(^5\)

1.1. Discussion of Assumptions

An important assumption of the model is that the monopolist has committed to a price path before the information structure is chosen. One may wonder whether a more realistic assumption might

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\(^5\)When ties are broken against the seller, it follows from our analysis that the $\sup \inf$ is achieved as $\max \min$. This would not be true if ties were broken in favor of the seller.
involve a seller who chooses prices after the information structure has been picked. The single period case of such a model has been considered by Roesler and Szentes (2016). With multiple periods, however, adapting the price dynamically in response to information arrival could be interpreted as sacrificing commitment. If we insist that nature cannot condition the information structure on prices in the future, then these prices should also be hidden from the buyer, bringing us to a non-commitment setting. The Coase conjecture implies that a patient seller facing a long selling horizon would rather commit to a price path at time 0 as opposed to changing prices over time.

Another important assumption of the model is that the monopolist is restricted to choosing price paths. In particular, we are ruling out mechanisms which utilize random prices. Bergemann and Schlag (2011) and Carrasco et al. (2015) show that in the single period benchmark, these random mechanisms may improve beyond a single posted price. The contrast with these papers arises because in our model, nature can condition the worst case information structure on the realized prices, eliminating possible gains from randomization. Whether the restriction to price paths is appropriate depends very much on the technology of the environment in practice. Our approach follows the intertemporal pricing literature, where posted price mechanisms are typically assumed.

2. ONE PERIOD BENCHMARK

We start with the case where the seller only has one period to sell the object. To solve this problem, we introduce a particular function that assists with the analysis. Informally, imagine $\tilde{v}$ is drawn according to $F$, and consider the random variable $\mathbb{E}[v \mid v \leq \tilde{v}]$. We let $G$ denote the resulting cumulative distribution function. Defining this formally requires some care, which we relegate to Appendix A.

It turns out that given the seller’s price $p$, nature chooses the worst case information structure in a way that a buyer who does not buy has expected value exactly $p$. Otherwise one could find an information structure that makes a non-purchasing buyer slightly more optimistic about her value and decreases the probability of sale. This implies that $G(p)$ is the highest probability that the object remains unsold, among all information structures nature can choose. We obtain the following proposition.

**Proposition 1.** In the one period model, the maxmin optimal price $p^*$ solves the following maximization problem:

$$p^* \in \arg\max_p p(1 - G(p)).$$  \hspace{1cm} (1)

For future reference, denote by $\Pi^* = \max_p p(1 - G(p))$ the one period maxmin profit.
It is worth comparing the optimization problem (1) to the standard model without ambiguity. If the buyer knew her value, the seller would maximize $p(1 - F(p))$. In our setting, the difference is that the transformed distribution $G$ takes the place of $F$, which will be useful for the analysis in later sections. The following example illustrates:

**Example 1.** Let $v \sim \text{Uniform}[0,1]$, so that $G(p) = 2p$. Then $p^* = \frac{1}{3}$ and $\Pi^* = \frac{1}{5}$. The information structure chosen by nature results in the buyer purchasing if and only if $v > 1/2$. Relative to the case where the buyer knows her value, the monopolist charges a lower price and obtains a lower profit under informational ambiguity. In Appendix B, we show that this comparative static holds generally.

### 3. MULTIPLE PERIODS WITH DYNAMIC INFORMATION STRUCTURES

In this section we show that having multiple periods to sell does not improve the seller’s profit when nature can choose amongst dynamic information structures. Since the seller can always sell exclusively in the first period, the single period benchmark $\Pi^*$ forms a lower bound for the seller’s maxmin profit.

To show that $\Pi^*$ is also an upper bound, we explicitly construct an information structure for any given price path, such that the seller’s profit against this information structure decomposes into a convex combination of single period profits. Our proof takes advantage of the partitional form of worst case information structures to show that nature can minimize the seller’s profit period by period.

**Proposition 2.** For any price path $(p_t)_{t=1}^T$, there is a dynamic information structure (and a corresponding optimal stopping time) that yields expected profit no more than $\Pi^*$. Hence the seller’s maxmin profit against all dynamic information structures is $\Pi^*$, irrespective of the time horizon $T$ and the discount factor $\delta$.

To prove this proposition, we first review the sorting argument when the buyer knows her value. In this case, given a price path $(p_t)_{t=1}^T$, we can find time periods $1 \leq t_1 < t_2 < \cdots < T$ and value cutoffs $v_{t_1} > v_{t_2} > \cdots \geq 0$, such that a buyer with $v \in [v_{t_j}, v_{t_{j-1}}]$ optimally buys in period $t_j$ (as in Stokey (1979)). This implies that in period $t_j$, the object is sold with probability $F(v_{t_{j-1}}) - F(v_{t_j})$.

Inspired by the one period problem, we construct an information structure under which in period $t_j$, the object is sold with probability $G(v_{t_{j-1}}) - G(v_{t_j})$ (that is, where $G$ replaces $F$). The following information structure $\mathcal{I}$ has this property:

- In each period $t_j$, a buyer is told whether or not her value is in the lowest $G(v_{t_j})$ percentile.
- In all other periods, no information is revealed.
We describe the buyer’s optimal stopping behavior under \( \mathcal{I} \) in the following lemma (with the proof in Appendix A):

**Lemma 1.** An optimal stopping time \( \tau^* \) under \( \mathcal{I} \) involves the buyer buying in the first period \( t_j \) when she is told her value is not in the lowest \( G(v_{t_j}) \) percentile.

Using this lemma, we can now prove the proposition by computing the seller’s profit under the information structure \( \mathcal{I} \) and the stopping time \( \tau^* \):

**Proof.** Since a buyer with value \( v \) in the percentile range \( (G(v_{t_j}), G(v_{t_{j+1}})] \) buys in period \( t_j \), the seller’s discounted profit is given by:

\[
\Pi = \sum_{j \geq 1} \delta^{t_j - 1} \cdot p_{t_j} \cdot (G(v_{t_{j-1}}) - G(v_{t_j}))
\]

\[
= \sum_{j \geq 1} (\delta^{t_j - 1}p_{t_j} - \delta^{t_{j+1} - 1}p_{t_{j+1}}) \cdot (1 - G(v_{t_j}))
\]

\[
= \sum_{j \geq 1} (\delta^{t_j - 1} - \delta^{t_{j+1} - 1})v_{t_j} \cdot (1 - G(v_{t_j}))
\]

\[
\leq \delta^{t_1 - 1} \cdot \Pi^*,
\]

where the second line is by Abel summation, the third line is by \( v_{t_j} \)’s indifference between buying in period \( t_j \) or \( t_{j+1} \), and the last inequality uses \( v_{t_j}(1 - G(v_{t_j})) \leq \Pi^*, \forall j. \]

One may wonder which price paths allow the seller to achieve \( \Pi^* \) as worst case profit. The answer is given by the following corollary:

**Corollary 1.** When the one period maxmin optimal price \( p^* \) is unique, the optimal pricing strategies of the seller are those with \( p^* = p_1 \leq p_t, \forall t. \)

### 4. MULTIPLE PERIODS WITH STATIC INFORMATION STRUCTURES

We now show that the seller can improve upon the single period benchmark if nature is restricted to choosing amongst static information structures. Since the seller can set \( p_t = p_2 \) for \( t > 2 \), it suffices to show this result for \( T = 2 \), which we assume throughout this section. Static information structures are much simpler to represent than dynamic information structures, since it is equivalent.
to work with the distribution of posterior expected values. If $D$ is the distribution of posterior expected values under some information structure, the seller’s profit is given by:

$$
\Pi = p_1(1 - D(v_1)) + \delta p_2(D(v_1) - D(v_2)) = (1 - \delta)v_1(1 - D(v_1)) + \delta v_2(1 - D(v_2)),
$$

where $v_1 = \frac{p_1 - \delta p_2}{1 - \delta}$ and $v_2 = p_2$ are the threshold values for buying in period one and period two respectively, assuming $p_1 \geq p_2$. We will find $v_1$ and $v_2$ such that $\Pi > \Pi^* + \varepsilon$ for all distribution $D(\cdot)$ that can arise under some information structure.

To do this, we introduce a result of Rothschild and Stiglitz (1970) (elaborated on by Gentzkow and Kamenica (2016)), which yields a tractable representation of $D$:

**Lemma 2** (Rothschild and Stiglitz (1970), Gentzkow and Kamenica (2016)). If $D$ is a distribution of posterior expected values, then

$$
\int_0^x D(s)ds \leq \int_0^x F(s)ds, \ \forall 0 \leq x \leq 1.
$$

This result is useful because it allows us to give a joint upper bound of $D(v_1)$ and $D(v_2)$. Our analysis from the one period benchmark shows that $D(v_i) \leq G(v_i)$ (because $G(v_i)$ maximizes the probability of no sale). By using (4), we can further show that for an appropriate choice of $(v_1, v_2)$, the inequality $D(v_i) \leq G(v_i)$ must be sufficiently slack at some $v_i$.\(^7\) Together with (3), we will be able to conclude $\Pi > \Pi^* + \varepsilon$.

**Proposition 3.** Suppose $T = 2$ and $p^* > v$.\(^8\) Then for all $\delta \in (0, 1)$, there exist prices $p_1, p_2$ and $\varepsilon > 0$, such that the seller’s profit against any static information structure is at least $\Pi^* + \varepsilon$.

**Proof.** We take $v_2 = p^*$, with $v_1 > p^*$ to be determined. From (4), for all $x > v_1 > p^*$ it holds that

$$
\int_0^x F(s)ds \geq \int_0^x D(s)ds \geq (v_1 - p^*)D(p^*) + (x - v_1)D(v_1),
$$

where the second inequality follows from the monotonicity of $D$.

Let us choose $x$ to be the buyer value corresponding to the $G(p^*)$ percentile under $F$, i.e. $F(x) = G(p^*)$. Since $p^* > v$, we have $G(p^*) > 0$ and $x > p^*$. Using integration by parts as well

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\(^7\)Intuitively, if this were not the case, $D$ would have to arise from a bipartite information structure with two different thresholds (corresponding to $v_1$ and $v_2$).

\(^8\)The assumption $p^* > v$ is not dispensable. For example, suppose $F \sim \text{Uniform}[0, 5, 1]$. Then Stokey (1979) implies that in the known value case, the seller can do no better than setting $p_1 = p_2 = 0.5$. Since nature can always inform the buyer perfectly, the seller’s maxmin profit is the same under either of our robustness concepts.
as the choice of $x$, we also have
\[
\int_0^x F(s) \, ds = xF(x) - \int_0^x s \, dF(s) = xF(x) - p^* F(x) = (x - p^*)G(p^*). \tag{6}
\]

The middle equality follow from $p^* = \frac{1}{F(x)} \int_0^x s \, dF(s)$, which means precisely that the lowest $F(x)$ percentile has expected value $p^*$. This crucial step of our argument is illustrated in Figure 1 above, for the distribution $F(x) = x^2$.

Combining (5) and (6), we deduce the following inequality:
\[
D(v_1) - G(p^*) \leq \frac{v_1 - p^*}{x - v_1} \cdot (G(p^*) - D(p^*)). \tag{7}
\]

Plugging this last inequality into the objective function (3), we see that
\[
\Pi = (1 - \delta)v_1(1 - D(v_1)) + \delta p^*(1 - D(p^*)) \\
> (1 - \delta)p^*(1 - D(v_1)) + \delta p^*(1 - D(p^*)) + \epsilon \\
= p^*(1 - G(p^*)) + p^* [\delta(G(p^*) - D(p^*)) - (1 - \delta)(D(v_1) - G(p^*))] + \epsilon \\
\geq p^*(1 - G(p^*)) + \epsilon \\
= \Pi^* + \epsilon. \tag{8}
\]
We note that the inequality in the second line above holds for \( D(v_1) < 1 \) and \( \epsilon \) sufficiently small. This is guaranteed by (7) and \( G(p^*) < 1 \), so long as we choose \( v_1 \) larger than but sufficiently close to \( p^* \). The inequality in the penultimate line holds by (7) whenever \( \frac{\delta}{1-\delta} \geq \frac{v_1-p^*}{x-v_1} \), which is again because \( v_1 \) can be chosen close to \( p^* \).

Since the seller can at most obtain \( \Pi^* \) by choosing \( p_2 \geq p_1 \), we obtain the following simple corollary:

**Corollary 2.** Suppose \( T = 2 \) and \( p^* > v \). Then against static information structures, any optimal pricing strategy of the seller consists of strictly decreasing prices.

## 5. Extensions

### 5.1. Arriving Buyers

Our analysis extends without change to a continuum of buyers (as argued in Footnote 4). One may wonder if the same conclusions would hold if buyers were to arrive over time, as in Conlisk, Gerstner and Sobel (1984), Sobel (1991), Board (2008) and Garrett (2016). In contrast to the single buyer case, selling only once is no longer optimal as the monopolist may want to capture buyers who only arrive in later periods. On the other hand, low prices in the future could allow nature to choose an information structure that delays purchase, which may be costly for the seller.

To comment on this possibility, we modify the model by assuming that in each period \( t \), a new buyer arrives and decides when to buy the object. To be precise, our timing is as follows:

- At time 0, the seller chooses a price path \((p_t)_{t=1}^T\).

- At time \( t \in \{1, \ldots, T\} \), a buyer arrives with value \( v(t) \) drawn from \( F \), independently from previous buyers.

- Once a buyer arrives at time \( t \), nature chooses an information structure \( \mathcal{I}(t) \) according to which this buyer learns her value.

- Given the price path and the information structure \( \mathcal{I}(t) \), the buyer arriving at time \( t \) chooses an optimal stopping time \( \tau(t) \) to purchase the object.

If every buyer learns only when she arrives, our analysis in the previous section directly implies that intertemporal price discrimination still has value. Hence we focus on dynamic information structures. If the seller were able to set personalized prices, Proposition 2 would directly imply that his maxmin profit is \( \frac{1-\delta^T}{1-\delta} \Pi^* \). We will show that the seller can achieve the same profit level by a constant price path of \( p^* \), even if he does not know which buyer arrives when.
Under known values, any arriving buyer facing a constant price path would buy upon arrival (if she were to buy at all), due to impatience. However, the promise of future information may induce delay. In the following lemma, we show that the seller can eliminate the potential damage of delayed purchase by committing to never decreasing the price.

**Lemma 3.** In the multi-period model with dynamic information structures (and one buyer), the seller can guarantee $\Pi^*$ with any price path $(p_t)_{t=1}^{T}$ satisfying $p^* = p_1 \leq p_t, \forall t$.

We present the intuition here and leave the formal proof to Appendix A. Let us fix a non-decreasing price path. For any dynamic information structure nature can choose, we consider an alternative static information structure that simply informs the buyer of her stopping time, in the first period. This replacement is in the spirit of the revelation principle; however, it differs due to the fact that we push nature’s recommendation to time 1. The proof shows that for non-decreasing prices, we can find a replacement such that the buyer still follows nature’s recommendation of whether or not to buy. This replacement has the property that the seller’s profit is decreased. Since the seller receives at least $\Pi^*$ against any static information structure, we obtain the lemma. We comment that more generally, for an arbitrary price path, the same worst case profit is obtained if one restricts to information structures that only reveal information after a price drop.

Armed with this lemma, we can show the following:

**Proposition 4.** In the multi-period model with dynamic information structures and arriving buyers, the seller can guarantee $\frac{1-\delta^T}{1-\delta} \Pi^*$ with a constant price path charging $p^*$ in every period. No other price path generates a higher worst case profit.

### 5.2. Seller Information Disclosure

Our model assumes that the seller has no control over the information the buyer receives. In practice, however, the seller may disclose information himself, for example via an advertising campaign (Johnson and Myatt (2006)). In this section we show that this possibility does not change our conclusions.

We modify the model in Section 1 by assuming that in addition, the buyer observes some signal $s_0 \in S_0$ at time 0. The signal space $S_0$ as well as the conditional distributions $H(\cdot \mid v)$ are common knowledge between the buyer and the seller, and we denote this initial information structure by $\mathcal{H}$. We allow nature to provide information conditional on $s_0$ but keep all other

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9 This contrasts with the obfuscation principle of Ely (2017), which implies that it is without loss to recommend an action at every time.

10 In principle, the seller may choose whether or not the buyer has access to $\mathcal{H}$. In Appendix B, we show that the seller always weakly prefers to release this information. Intuitively, releasing information limits the set of information structures with which nature can damage profits.
aspects of the model identical.

A signal $s_0$ induces a posterior distribution on the buyer’s value, which we denote by $F_{s_0}$. We define $G_{s_0}$ to be the analogue of $G$ for the distribution $F_{s_0}$ (in place of $F$). Then the same analysis as in Section 2 yields the following result:

**Proposition 1’.** In the one period model where the buyer observes initial information structure $\mathcal{H}$, the seller’s maxmin optimal price $p^*_\mathcal{H}$ is given by:

$$p^*_\mathcal{H} \in \arg \max_p (1 - \mathbb{E}_{s_0}[G_{s_0}(p)]).$$

(9)

We denote the maxmin profit in this case by $\Pi^*_\mathcal{H}$.

The expression (9) is familiar in two extreme cases: If $\mathcal{H}$ is perfectly informative, then $F_{s_0}$ is the point-mass distribution on $s_0$. This means $G_{s_0}(p)$ is the indicator function for $p \geq s_0$, so that $\mathbb{E}_{s_0}[G_{s_0}(p)] = F(p)$. In contrast, if $\mathcal{H}$ is completely uninformative, we return to Equation (1).

We now turn to the multi-period problem. Against dynamic information structures, our previous proof carries over and shows that the seller does not benefit from a longer selling horizon:

**Proposition 2’.** In the multi-period model where the buyer observes initial information structure $\mathcal{H}$, the seller’s maxmin profit against all dynamic information structures is $\Pi^*_\mathcal{H}$, irrespective of the time horizon $T$ and the discount factor $\delta$.

For the static information structure case, the analysis extends if the support of $F_{s_0}$ is independent of $s_0$. But in general, we need additional assumptions to ensure that the seller can improve upon the one period benchmark. The difficulty is related to the hypothesis $p^* > \underline{v}$ in the statement of Proposition 3. Below we present a version of such a result, leaving further discussion to Appendix B.

**Proposition 3’.** Define $\underline{v}_{s_0}$ to be the minimum value in the support of $F_{s_0}$. Suppose that the initial information structure $\mathcal{H}$ has finitely many signals, and that the one period maxmin optimal price $p^*_\mathcal{H}$ is distinct from all $\underline{v}_{s_0}$. Then for some $\epsilon > 0$, there exists a decreasing price path giving the seller profit $\Pi^*_\mathcal{H} + \epsilon$ against all static information structures.

6. CONCLUSION

In this paper, we have studied optimal monopoly pricing in relation to information availability, utilizing a robustness approach to provide a sharp answer on how they interact. A monopolist who has developed a completely new durable product should consider whether buyers can learn more about the product after the initial release date. In our model, if buyer learning is restricted,
then intertemporal price discrimination can improve profits. Without any restrictions, the single period benchmark (alternatively, a constant price path) is the best the monopolist can do.

The main lesson of this paper—that static and dynamic informational robustness may lead to qualitatively different optimal policies—is one that we hope will be scrutinized in other contexts and under other modeling assumptions. In particular, many of the models inspired by Stokey (1979) could be reevaluated using our techniques. The case of limited commitment seems compelling in our opinion, though there are technical difficulties associated with formalizing (seller) learning under ambiguity (see Epstein and Schneider (2007)). One could also ask these same questions in more general dynamic mechanism design settings, without restricting to posted prices.

This paper is among a growing literature which demonstrates the power of the maxmin approach in deriving sharp descriptions of optimal mechanisms: For instance, Chung and Ely (2007), Frankel (2014), Yamashita (2015) and Carroll (2015, 2017). In our setting, the space of information structures is difficult to work with, as is the set of optimal stopping times that could arise under some information structure. The maxmin approach allows us to focus on particular information structures—the partitional ones—with clear economic interpretations. While it is certainly worthwhile to analyze the Bayesian case, doing so would first require a similarly tractable restriction as the one we have provided here.

7. APPENDIX A: PROOFS OMITTED FROM MAIN TEXT

We first present a formal definition of the function $G(\cdot)$ introduced in Section 2.

**Definition 1.** Given a percentile $\alpha \in (0, 1]$, define $g(\alpha)$ to be the expected value of the lowest $\alpha$-percentile of the distribution $F$. In case $F$ is a continuous distribution, $g(\alpha) = \frac{1}{\alpha} \int_{0}^{F^{-1}(\alpha)} v dF(v)$.

But in general, $g$ is continuous and weakly increasing.

Let $v$ be the minimum value in the support of $F$. For $\beta \in (v, \mathbb{E}[v])$, define $G(\beta) = \sup\{\alpha : g(\alpha) \leq \beta\}$. We extend the domain of this inverse function to $\mathbb{R}_+$ by setting $G(\beta) = 0$ for $\beta \leq v$ and $G(\beta) = 1$ for $\beta > \mathbb{E}[v]$.\(^{11}\)

**Proof of Proposition 1.** Given a price $p$ set by the seller, minimum profit occurs when there is maximum probability of signals that lead the buyer to have posterior expectation $\leq p$. First consider the information structure $I$ that tells the buyer whether her value is in the lowest $G(p)$-percentile or above. By definition of $G$, the buyer’s expectation is exactly $p$ upon learning the former. This shows that, under $I$, the buyer’s expected value $\leq p$ with probability $G(p)$.

\(^{11}\)If $F$ does not have a mass point at $v$, $g(\alpha)$ is strictly increasing and $G(\beta)$ is its inverse function which increases continuously. If instead $F(v) = m > 0$, then $g(\alpha) = v$ for $\alpha \leq m$ and it is strictly increasing for $\alpha > m$. In that case $G(\beta) = 0$ for $\beta \leq v$, after which it jumps to $m$ and increases continuously to 1.
Now we show that $G(p)$ cannot be improved upon. To see this, note that it is without loss of generality to consider information structures which recommend that the buyer either “buy” or “not buy”. Hence nature seeks to choose an information structure minimizing the probability that the buyer buys. By Lemma 1 in Kolotilin (2015), this minimum is achieved by a partitional information structure, namely by recommending “buy” for $v > \alpha$ and “not buy” for $v \leq \alpha$. From this, it is easy to see that the particular information structure $I$ above is the worst case.

Thus, for fixed price $p$, the seller’s minimum profit is $p(1 - G(p))$. The proposition follows from the seller optimizing over $p$. ■

Proof of Lemma 1. Define “LOW” to be the signal where a buyer is told her value is below the $G(v_{t_j})$ percentile, and “HIGH” to be the opposite signal. We will show the following facts that together imply the lemma:

Fact 1. A buyer who has only received “LOW” signals in the past finds it optimal to delay her purchase.

Fact 2. A buyer receiving a “HIGH” signal finds it optimal to buy immediately.

To prove Fact 1, suppose $t < t_1$. Since no information has been revealed, the buyer’s expected value is $E[v]$, which clearly belongs to some interval $[v_{t_1}, v_{t_{j-1}}]$. Hence if no more information were to be revealed, such a buyer would buy in some later period $t_j > t$ (or never). Since more information could only improve the buyer’s payoff, in our setting the buyer still prefers to not buy at time $t$.

Suppose instead that the current period $t$ satisfies $t_k \leq t < t_{k+1}$. By assumption the buyer has received “LOW” signals in periods $t_1, t_2, \ldots, t_k$. According to $I$, this means her value is in the lowest $G(v_{t_k})$ percentile, so her expected value is simply $\min\{E[v], v_{t_k}\}$. Again, such a buyer would find it optimal to buy in some later period if no more information were to be revealed. Hence she also does not buy at time $t$.

To prove Fact 2, consider a period $t_j$ in which the buyer first receives a “HIGH” signal. This means $v$ is not in the lowest $G(v_{t_j})$ percentile. By definition, the lowest $G(v_{t_j})$ percentile has expected value $v_{t_j}$, so our buyer knows with certainty that her value $v$ is at least $v_{t_j}$. Since under known values, any buyer with value $v \geq v_{t_j}$ prefers to buy in period $t_j$ than at any later time, this also holds for our buyer who knows that $v$ belongs to the percentile range $(G(v_{t_j}), G(v_{t_{j-1}}))$. ■

Proof of Corollary 1. Sufficiency follows from Lemma 3. To prove necessity, note that the last inequality in (2) holds equal only when $t_1 = 1$ and every $v_{t_j}$ is a one period maxmin optimal price. Thus when $p^*$ is unique, there is no other $t_j$ except $t_1 = 1$. By definition, this means under known values, a buyer with $v > p^*$ optimally buys in the first period; otherwise he never buys. Thus $p_1 = p^*$ and $p_t \geq p^*$. ■
Proof of Lemma 3. Fix a dynamic information structure \((S_t, I_t)_{t=1}^T\) and an optimal stopping time \(\tau\) of the buyer. As discussed in the main text, we will construct a static information structure \((S, I)\) which weakly reduces the seller’s profit. We take \(S = \{\bar{s}, \underline{s}\}\), corresponding to the recommendation to buy and not to buy, respectively. To specify the distribution of these signals conditional on \(v\), let nature draw signals \(s_1, s_2, \ldots\) according the original dynamic information structure (and also conditional on \(v\)). If, along this sequence of realized signals, the stopping time \(\tau\) results in the buyer buying the object, let the buyer receive the signal \(\bar{s}\) with probability \(\delta^{\tau-1}\) at time 1. With complementary probability and when \(\tau = \infty\), let her receive the other signal \(\underline{s}\).

We claim that under this static information structure, a buyer receiving the signal \(\underline{s}\) has expected value at most \(p_1\). We actually show something stronger, namely that the buyer has expected value at most \(p_1\) conditional on the signal \(\underline{s}\) and any realized signal \(s_1\).\(^{12}\) To prove this, note that since stopping at time \(\tau\) is weakly better than stopping at time 1, we have

\[
E[v|s_1] - p_1 \leq E^{s_2,\ldots,s_T} [\delta^{\tau-1}(E[v|s_1, s_2, \ldots, s_\tau] - p_\tau)].
\]

(10)

Since \(p_\tau \geq p_1\), we obtain by simple algebra

\[
E[v|s_1] \leq E^{s_2,\ldots,s_T} [\delta^{\tau-1}E[v|s_1, s_2, \ldots, s_\tau] + (1 - \delta^{\tau-1})p_1].
\]

(11)

By noting \(E[v|s_1] = E^{s_2,\ldots,s_T} [E[v|s_1, s_2, \ldots, s_\tau]]\) from Doob’s Optional Sampling Theorem, we derive the following inequality:

\[
p_1 \geq \frac{E^{s_2,\ldots,s_T}[(1 - \delta^{\tau-1})E[v|s_1, s_2, \ldots, s_\tau]]}{E^{s_2,\ldots,s_T}[1 - \delta^{\tau-1}]}.
\]

(12)

On the other hand, we can compute that

\[
E[v|s_1, \underline{s}] = \frac{E^{s_2,\ldots,s_T}[(1 - \delta^{\tau-1})E[v|s_1, s_2, \ldots, s_\tau, \underline{s}]]}{E^{s_2,\ldots,s_T}[1 - \delta^{\tau-1}]} = \frac{E^{s_2,\ldots,s_T}[(1 - \delta^{\tau-1})E[v|s_1, s_2, \ldots, s_\tau]]}{E^{s_2,\ldots,s_T}[1 - \delta^{\tau-1}]}.
\]

(13)

The first equality follows directly from the definition of conditional expectation, as \(1 - \delta^{\tau-1}\) is the conditional probability of \(\underline{s}\) given \(s_1, s_2, \ldots, s_\tau\). The second equality holds because first, \(\underline{s}\) does not provide more information about \(v\) beyond \(s_1, s_2, \ldots, s_\tau\), and second,

\[
E^{s_2,\ldots,s_T}[(1 - \delta^{\tau-1})E[v|s_1, s_2, \ldots, s_\tau]] = E^{s_2,\ldots,s_T}[(1 - \delta^{\tau-1})E[v|s_1, s_2, \ldots, s_\tau]]
\]

because \(\tau\) is a stopping time.

\(^{12}\)Technically we will only consider those \(s_1\) such that \(\underline{s}\) occurs with positive probability given \(s_1\).
Comparing (12) and (13), we see that $\mathbb{E}[v|s_1, \bar{s}] \leq p_1$ as claimed. This implies that under the static information structure constructed above, it is optimal for the buyer to only buy upon receiving the signal $\bar{s}$. Moreover, all sale happens in the first period as $p_t \geq p_1, \forall t$. The probability of sale is at most $\mathbb{E}[\delta^{T-1}]$, and the seller’s profit is at most $\mathbb{E}[\delta^{T-1}] \cdot p_1$. This is no more than $\mathbb{E}[\delta^{T-1} \cdot p_t]$, the discounted profit under the original dynamic information structure. We have thus proved that with a non-decreasing price path, the seller’s profit is at least what he would obtain by selling only once at the price $p_1$. Taking $p_1 = p^*$ shows the lemma. 

Proof of Proposition 4. By the previous lemma, a constant price path $p^*$ delivers expected undiscounted profit $\Pi^*$ from each arriving buyer. However, Proposition 2 shows that the worst case profit obtained from this buyer is $\Pi^*$. The result follows.

Proof of Proposition 1’ and 2’. The proofs are direct adaptations of those presented in the main text, for the model without an initial information structure.

Proof of Proposition 3’. Define $x_{s_0}$ to be the buyer value corresponding to the $G(p^*_H)$ percentile under $F_{s_0}$. This is the analogue of $x$ from the proof of Proposition 3 in the main text. As with the proof there, we take $T = 2$ and consider a seller who chooses $v_2 = p_2 = p^*_H$ and $v_1 = \frac{p_1 - \delta p_2}{1 - \delta}$ slightly larger than $p^*_H$.

Fix any realized initial signal $s_0$. There are three possibilities, and in each case we will show that the seller prefers the above price path to selling only once:

1. $p^*_H \geq \mathbb{E}[v|s_0]$. By charging $p^*_H$, the seller obtains zero profit from such a buyer in the one period benchmark. The price path $p_1 > p_2 = p^*_H$ also yields zero profit.

2. $v_\delta < p^*_H < \mathbb{E}[v|s_0]$. In this case we can follow the argument in the proof of Proposition 3 to show that as long as $(v_1 - p^*_H)$ is sufficiently small compared to $(x_{s_0} - p^*_H)$, the seller does strictly better with two periods to sell. Since there are finitely many signals, we can satisfy this condition for every $s_0$.

3. $p^*_H < v_\delta$. Suppose we have chosen $v_1$ sufficiently close to $p^*_H$, so that $v_1 < v_\delta$. From the definition of $v_1$, we deduce that a buyer receiving the signal $s_0$ always prefers to buy in the first period, regardless of what additional information she might receive from nature. Since $p_1 > p^*_H$, the seller again does strictly better with two period to sell.

Since the seller’s one period maxmin profit is strictly positive, either of the latter two possibilities above occurs with positive probability. Hence the seller achieves profit strictly greater than $\Pi^*_H$ using a decreasing price path.
8. APPENDIX B: ADDITIONAL RESULTS

8.1. Ambiguity Leads to Lower Price

In Example 1, we mentioned that ambiguity over the information structure leads the seller to choose a lower price than if the buyer knew her value. The following proposition generalizes this observation to any continuous distribution.

**Proposition 5.** For any continuous distribution \( F \), let \( \hat{p} \) be an optimal price under known values:

\[
\hat{p} \in \arg \max_p \quad p(1 - F(p)).
\]

Then any maxmin optimal price \( p^\star \) satisfies \( p^\star \leq \hat{p} \). Equality holds only if \( p^\star = \hat{p} = v \).

**Proof of Proposition 5.** It suffices to show that the function \( p(1 - G(p)) \) strictly decreases when \( p > \hat{p} \), until it reaches zero. By taking derivatives, we need to show

\[
G(p) + pG'(p) = \frac{\partial}{\partial p}(F^{-1}(G(p))) \cdot F^{-1}(G(p)) \cdot F'(F^{-1}(G(p))) = G'(p) \cdot F^{-1}(G(p)).
\]

This enables us to write \( G'(p) \) in terms of \( G(p) \) as follows:

\[
G'(p) = \frac{G(p)}{F^{-1}(G(p)) - p}.
\]

Thus,

\[
G(p) + pG'(p) = \frac{G(p) \cdot F^{-1}(G(p))}{F^{-1}(G(p)) - p}.
\]

We need to show that the RHS above is greater than 1, or that \( F^{-1}(G(p)) < \frac{p}{1 - G(p)} \) whenever \( p > \hat{p} \) and \( G(p) < 1 \). This is equivalent to \( G(p) < F\left(\frac{p}{1 - G(p)}\right)\), which in turn is equivalent to

\[
\frac{p}{1 - G(p)} \cdot \left(1 - F\left(\frac{p}{1 - G(p)}\right)\right) < p.
\]
From the definition of $\hat{p}$, we see that the LHS above is at most $\hat{p}(1 - F(\hat{p})) \leq \hat{p} < p$, as we claim to show. Moreover, when $\hat{p} > v$, the last inequality $\hat{p}(1 - F(\hat{p})) < \hat{p}$ is strict. Tracing back the previous arguments, we see that $G(p) + pG'(p) > 1$ holds even at $p = \hat{p}$. In that case we would have the strict inequality $p^* < \hat{p}$ as desired. ■

### 8.2. More on Seller Information Disclosure

One way to interpret the release of information by the seller is the consider it as a restriction on the space of information structures that nature can minimize over. The setting in Section 5.2 is then one where the buyer has access to some information structure $H$ (which is common knowledge) and potentially more information chosen by nature (which the seller is uncertain about). By Blackwell (1951), we can equivalently consider nature choosing an information structure that is more information than $H$.

As the seller releases more informative signals, nature is minimizing over a smaller set of information structures. Thus the worse case profit of the seller can only go up. That is,

**Proposition 6.** Suppose $H$ is Blackwell more informative than $H'$. Then $\Pi_{H^*} \geq \Pi_{H'}^*$.

We conclude by studying an example in detail to illustrate the limits of Proposition 5 and Proposition 3'.

**Example 2.** In contrast to the conclusion of Proposition 5, if an additional signal is released by the seller, then the maxmin optimal price need not be lower than the known value price. Suppose $v \sim \text{Uniform}[0, 1]$, and the initial information structure $H$ tells the buyer whether $v \geq 3/5$ or not. Selling only to buyers who are told $v \geq 3/5$ results in profit equal to 6/25, which turns out to be the maxmin profit. Note that $p_{H^*} = 3/5 > 1/2 = \hat{p}$.

We show that the seller cannot improve upon 6/25 with multiple periods to sell, even if nature can only choose amongst static information structures. Suppose nature chooses the following static information structure (which is Blackwell more informative than $H$). There are 3 signals $s^1, s^2, s^3$, corresponding to $v \in [0, .1) \cup [.3, .4)$, $v \in [.1, .2) \cup [.4, .5)$ and $v \in [.2, .3) \cup [.5, .6)$, respectively; if $v \geq .6$, the value is revealed perfectly.

Given nature’s choice, the buyer’s distribution of posterior expectations has a $\frac{1}{3}$ mass at each of $v = .2, .3, .4$, while the remaining $\frac{2}{3}$ mass is uniformly distributed on $[.6, 1]$. By Stokey (1979), the seller’s maximum profit against this fixed distribution of values is achieved by selling only once. In our case, simple computation shows that $p = -.3, .4, .6$ are all optimal prices that yield a profit of 6/25, which is thus the seller’s maxmin profit. This suggests that the assumptions stated in Proposition 3’ are to some extent indispensable for the conclusion to hold.

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13See also Bergemann and Morris (2016) for an extension of Blackwell’s partial ordering to multi-agent settings.
References