AN ECONOMETRIC APPROACH TO GENERAL EQUILIBRIUM MODELING

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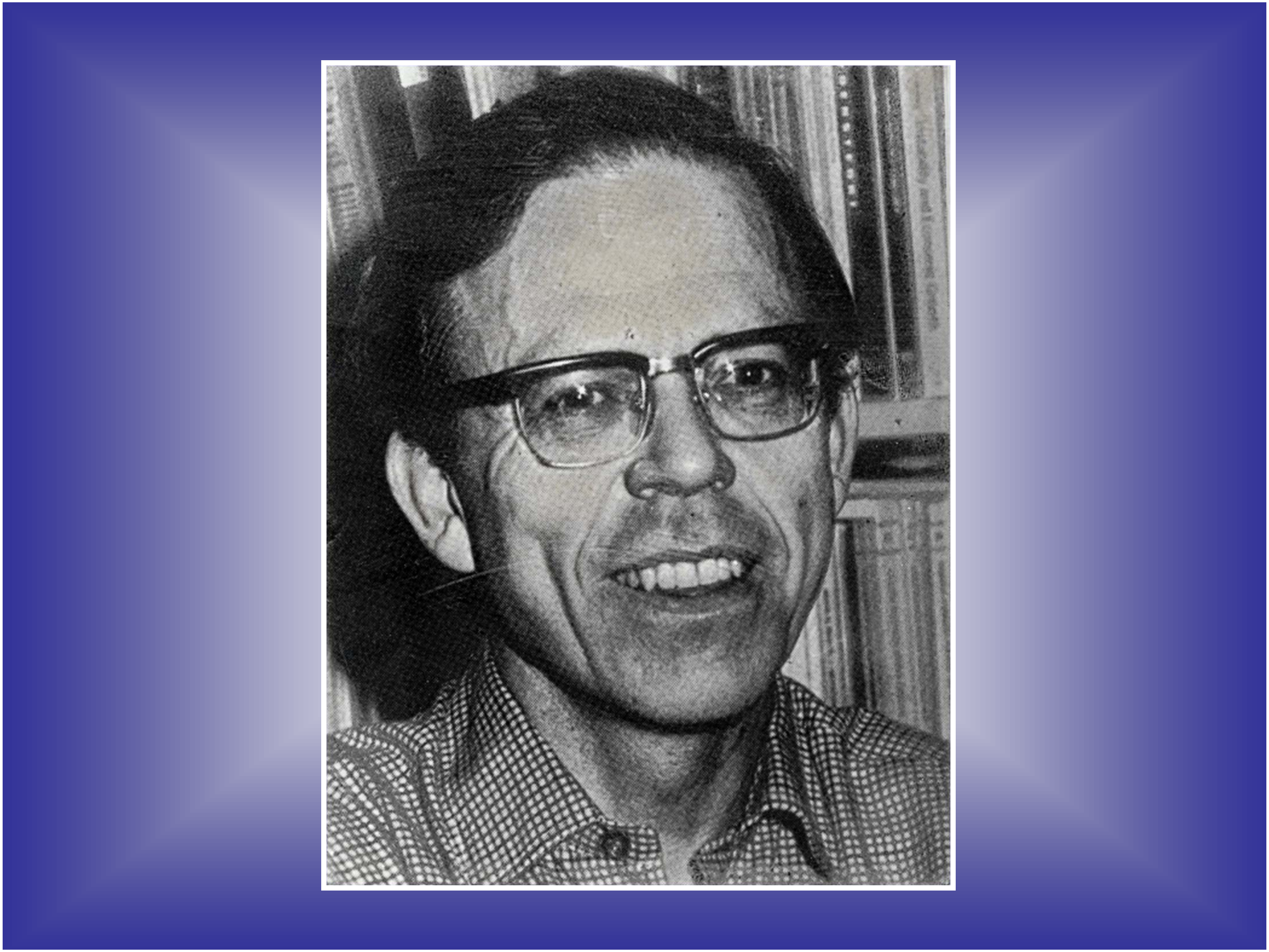
UNIVERSITY OF OSLO



AN ECONOMETRIC APPROACH TO GENERAL EQUILIBRIUM MODELING

Johansen's Multi-Sectoral Study of Economic Growth

Elements of the Econometric Approach:

Econometric Modeling of Producer Behavior: Jin and Jorgenson, 2010.

Econometric Modeling of Consumer Behavior:

Jorgenson, Lau, and Stoker, 1997; Jorgenson and Slesnick, 2008.

Intertemporal General Equilibrium Model

Version One: Jorgenson and Wilcoxen, 1990.

Version Sixteen: Jorgenson, Wilcoxen, Slesnick, Ho, and Goettle, 2010.

Application to Climate Policy: U.S. Environmental Protection Agency, 2009.

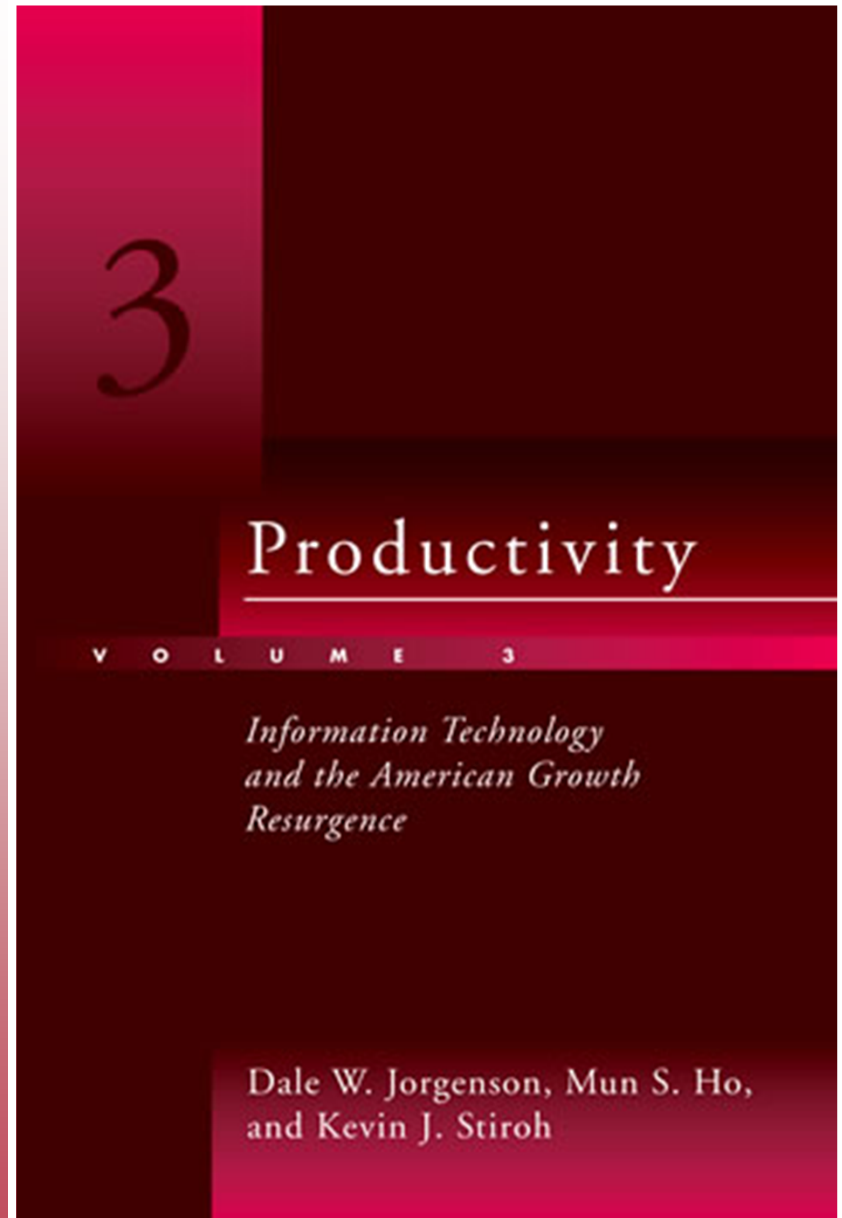
ECONOMETRIC MODELING OF TECHNICAL CHANGE

- Substitution vs. Technical Change
  Jorgenson (2000)

- Index Number Approach
  Jorgenson, Ho, and Stiroh (2005)

- Parametric Approach
  Jorgenson and Fraumeni (2000)

- State-Space Approach
  Jorgenson and Jin (2010)


3
Productivity
VOLUME 3
Information Technology and the American Growth Resurgence
Dale W. Jorgenson, Mun S. Ho, and Kevin J. Stiroh

STATE-SPACE MODELING OF TECHNICAL CHANGE

- Production Function

$$Q_j = f(K_j, L_j, E_j, M_j, t)$$

- Accounting Identity

$$P_{Qjt} Q_{jt} = P_{Kjt} K_{jt} + P_{Ljt} L_{jt} + P_{Ejt} E_{jt} + P_{Mjt} M_{jt}$$

- Price Function

$$P_{Qj} = p(P_{Kj}, P_{Lj}, P_{Ej}, P_{Mj}, t)$$

- Translog Price Function

$$\ln P_{Qt} = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln P_{it} + \tfrac{1}{2} \sum_{i,k} \beta_{ik} \ln P_{it} \ln P_{kt} + \sum_{i=1}^{n} \ln P_{it} f_{it} + f_{pt}$$

$$\text{i,k=\{K,L,E,M\}}$$

- Input Demand Equation

$$v_{Kt} = \frac{P_K K}{P_Q Q} = \alpha_K + \sum_{k} \beta_{Kk} \ln P_{kt} + f_{Kt}$$

SUMMARY: STATE-SPACE MODELING

- Stochastic Specification

$$\ln P_{Qt} = \alpha_0 + \boldsymbol{\alpha}'\ln \mathbf{p}_t + \tfrac{1}{2}\ln \mathbf{p}_t'\mathbf{B}\ln \mathbf{p}_t + \ln \mathbf{p}_t'\mathbf{f}_t + f_{pt} + \varepsilon_t^p$$

$$\mathbf{v}_t = \boldsymbol{\alpha} + \mathbf{B}\ln \mathbf{p}_t + \mathbf{f}_t + \boldsymbol{\varepsilon}_t^v$$

- Observable Variables

$$\ln \mathbf{p}_t = (\ln P_{Kt}, \ln P_{Lt}, \ln P_{Et}, \ln P_{Mt})'$$

$$\mathbf{v}_t = (v_{Kt}, v_{Lt}, v_{Et}, v_{Mt})'$$

$$\ln P_{Qt}$$

- Latent Variables

$$f_t = (f_{Kt}, f_{Lt}, f_{Et}, f_{Mt})'$$

$$f_{pt}$$

- Random Variables

$$\varepsilon_t^v = (\varepsilon_{Kt}, \varepsilon_{Lt}, \varepsilon_{Et}, \varepsilon_{Mt})'$$

$$\varepsilon_t^p$$

- Parameters

$$\alpha = (\alpha_K, \alpha_L, \alpha_E, \alpha_M)'$$

$$\mathbf{B} = [\beta_{jk}]$$

RESTRICTIONS FROM PRODUCTION THEORY

- Homogeneity

$$\alpha_K + \alpha_L + \alpha_E + \alpha_M = 1$$

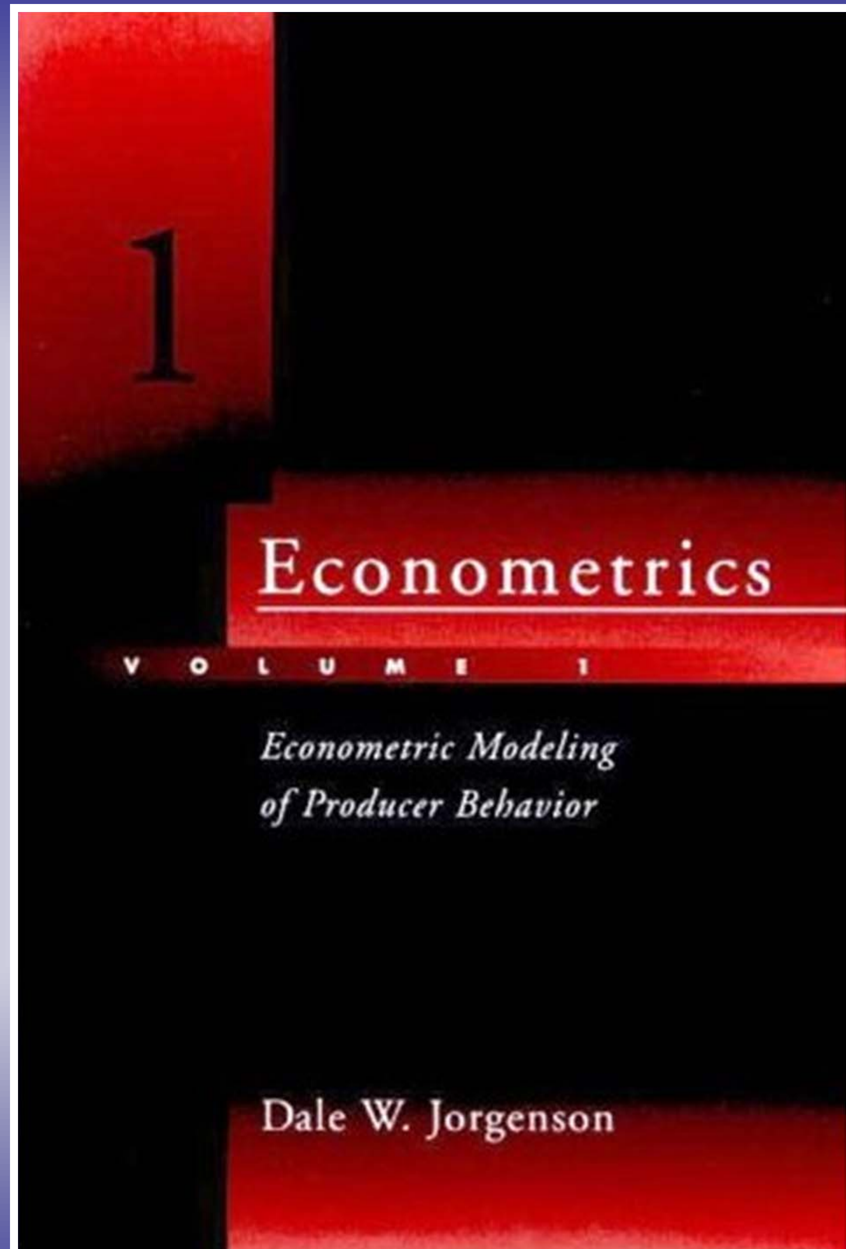
$$\sum_i \beta_{ik} = 0 \text{ for k = K, L, E, M}$$

- Symmetry

$$\beta_{ik} = \beta_{ki}$$

- Concavity

$$\mathbf{B} + \mathbf{v}_t \mathbf{v}_t ' - \mathbf{V}_t \text{ positive semi-definite}$$


1
Econometrics
VOLUME 1
Econometric Modeling
of Producer Behavior
Dale W. Jorgenson

DEFINITION OF TECHNICAL CHANGE

- Rate of Technical Change:

  Rate of change in the price of output, holding input prices constant

$$\Delta T_t = -\sum_{i=1}^{n} \ln P_{it}(f_{it} - f_{i,t-1}) - (f_{pt} - f_{p,t-1})$$

- Biases of Technical change

  Change in the value shares of inputs, holding input prices constant

$$\Delta v_t = f_t - f_{t-1}$$

- Transition Equation

$$\mathcal{F}_t = \mathbf{\Phi}\mathcal{F}_t + u_t$$

where:

$$\mathbf{F}_t = (1, f_{kt}, f_{lt}, f_{et}, \Delta f_{pt})'$$

is stationary. The transition equation is a vector auto-regressive scheme (VAR).

TWO-STEP KALMAN FILTER

- Kalman Filter, Hamilton (1994)

$$
\begin{array}{ccccccc}
 & & x_1 & & x_2 & & \cdots \\
 & & \downarrow & & \downarrow & & \\
 & & y_1 & & y_2 & & \cdots \\
 & & \uparrow & & \uparrow & & \\
\xi_0 & \rightarrow & \xi_1 & \rightarrow & \xi_2 & \rightarrow & \cdots
\end{array}
$$

- State Equation

$$\underset{(r\times 1)}{\xi_t} = \underset{(r\times r)}{F}\,\underset{(r\times 1)}{\xi_{t-1}} + \underset{(r\times 1)}{v_t}$$

where:

$$E(v_t v_\tau') = \begin{cases} \underset{(r\times r)}{Q} & t=\tau \\ 0 & otherwise \end{cases}$$

- Observation Equation

$$\underset{(n\times 1)}{y_t} = \underset{(n\times k)}{A'}\,\underset{(k\times 1)}{x_t} + \underset{(n\times r)}{H'}\,\underset{(r\times 1)}{\xi_t} + \underset{(n\times 1)}{w_t}$$

where:

$$E(w_t w_\tau') = \begin{cases} \underset{(n\times n)}{R} & t=\tau \\ 0 & otherwise \end{cases}$$

FILTERING AND SMOOTHING

- Filtering

$$\max_{\theta} l(\theta \mid Y_T) = \max_{\theta} \sum_{t=1}^{T} \log N(y_t \mid \hat{y}_{t|t-1}, V_{t|t-1})$$

where:

$$Y_t = (y_t', y_{t-1}', \ldots y_1', x_t', x_{t-1}', \ldots x_1')'$$

consists of observations up to time t and:

$$\hat{y}_{t|t-1} = E(y_t \mid Y_{t-1}); V_{t|t-1} = E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})']$$

ECONOMETRIC MODEL

$$y_t = \begin{bmatrix} v_{Kt} \\ v_{Lt} \\ v_{Et} \\ \ln\frac{P_{Qt}}{P_{Mt}} \end{bmatrix}, x_t = \begin{bmatrix} 1 \\ \ln\frac{P_{Kt}}{P_{Mt}} \\ \ln\frac{P_{Lt}}{P_{Mt}} \\ \ln\frac{P_{Et}}{P_{Mt}} \\ \frac{1}{2}(\ln\frac{P_{Kt}}{P_{Mt}})^2 \\ \frac{1}{2}(\ln\frac{P_{Lt}}{P_{Mt}})^2 \\ \frac{1}{2}(\ln\frac{P_{Et}}{P_{Mt}})^2 \\ \ln\frac{P_{Kt}}{P_{Mt}}\ln\frac{P_{Lt}}{P_{Mt}} \\ \ln\frac{P_{Kt}}{P_{Mt}}\ln\frac{P_{Et}}{P_{Mt}} \\ \ln\frac{P_{Lt}}{P_{Mt}}\ln\frac{P_{Et}}{P_{Mt}} \end{bmatrix}, A' = \begin{bmatrix} \alpha_K & \beta_{KK} & \beta_{KL} & \beta_{KE} & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_L & \beta_{KL} & \beta_{LL} & \beta_{LE} & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_E & \beta_{KE} & \beta_{LE} & \beta_{EE} & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_0 & \alpha_K & \alpha_L & \alpha_E & \beta_{KK} & \beta_{LL} & \beta_{EE} & \beta_{KL} & \beta_{KE} & \beta_{LE} \end{bmatrix}, \xi_t = \begin{bmatrix} 1 \\ f_{Kt} \\ f_{Lt} \\ f_{Et} \\ f_{pt} \\ f_{pt-1} \end{bmatrix},$$

where:

$$H' = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \ln\frac{P_{Kt}}{P_{Mt}} & \ln\frac{P_{Lt}}{P_{Mt}} & \ln\frac{P_{Et}}{P_{Mt}} & 1 & 0 \end{bmatrix}, w_t = \begin{bmatrix} \varepsilon_{Kt} \\ \varepsilon_{Lt} \\ \varepsilon_{Et} \\ \varepsilon_{pt} \end{bmatrix}, v_t = \begin{bmatrix} 0 \\ u_{Kt} \\ u_{Lt} \\ u_{Et} \\ u_{dpt} \\ 0 \end{bmatrix} F' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \chi_K & \delta_{KK} & \delta_{KL} & \delta_{KE} & \delta_{Kp} & -\delta_{Kp} \\ \chi_L & \delta_{LK} & \delta_{LL} & \delta_{LE} & \delta_{Lp} & -\delta_{Lp} \\ \chi_E & \delta_{EK} & \delta_{EL} & \delta_{EE} & \delta_{Ep} & -\delta_{Ep} \\ \chi_p & \delta_{pK} & \delta_{pL} & \delta_{pE} & \delta_{pp}+1 & -\delta_{pp} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

TWO-STEP KALMAN FILTER

- Instrumental Variables

$$\underset{(k\times 1)}{x_t} = \underset{(k\times m)}{\Pi}\ \underset{(m\times 1)}{z_t} + \underset{(k\times 1)}{\eta_t}$$

- New Observation Equation

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} A'\Pi \\ \Pi \end{bmatrix} z_t + \begin{bmatrix} H' \\ O \end{bmatrix} \xi_t + \begin{bmatrix} A'\eta_t + w_t \\ \eta_t \end{bmatrix}$$

or:

$$\underset{[(n+k)\times 1]}{\widetilde{y}_t} = \underset{[(n+k)\times m]}{\widetilde{A}'}\ \underset{(m\times 1)}{\widetilde{x}_t} + \underset{[(n+k)\times r]}{\widetilde{H}'}\ \underset{(r\times 1)}{\xi_t} + \underset{[(n+k)\times 1]}{\widetilde{w}_t}$$

- **Step One:** Estimate $\hat{\Pi} = XZ'(ZZ')^{-1}$ using OLS, which is a consistent estimator of Π, where X and Z represent the matrices of observations on x_t and z_t, t=1,2,…T.

- **Step Two:** Replace X in the standard Kalman filter with $\hat{X} = \hat{\Pi}Z$, that is, replace x_t with $\hat{x}_t$ at time t, and then use the standard filtering procedure to obtain the two-step MLE of the unknown parameters in the matrices A, H, F, R, Q.

- Wooldridge (2002, Chapter 12) shows that the estimator is consistent and asymptotically normal.

LIST OF SECTORS

Sector number	Sector Name
1	Agriculture
2	Metal Mining
3	Coal Mining
4	Petroleum and Gas
5	Nonmetallic Mining
6	Construction
7	Food Products
8	Tobacco Products
9	Textile Mill Products
10	Apparel and Textiles
11	Lumber and Wood
12	Furniture and Fixtures
13	Paper Products
14	Printing and Publishing
15	Chemical Products
16	Petroleum Refining
17	Rubber and Plastic
18	Leather Products
19	Stone, Clay, and Glass
20	Primary Metals
21	Fabricated Metals
22	Industrial Machinery and Equipment
23	Electronic and Electric Equipment
24	Motor Vehicles
25	Other Transportation Equipment
26	Instruments
27	Miscellaneous Manufacturing
28	Transport and Warehouse
29	Communications
30	Electric Utilities
31	Gas Utilities
32	Trade
33	Finance, Insurance, and Real Estate
34	Services
35	Government Enterprises

TESTS FOR OVER-IDENTIFICATION

sector	l_g	l	$2(l_g - l)$	p-value	p-value*35
1	545.23	533.74	22.98	0.003	0.12
2	397.19	389.25	15.88	0.044	1.55
3	472.23	465.80	12.85	0.117	4.10
4	471.55	470.86	1.38	0.995	34.81
5	523.07	519.08	7.98	0.435	15.24
6	670.18	666.67	7.02	0.535	18.71
7	696.27	694.53	3.50	0.900	31.48
8	565.98	563.63	4.71	0.788	27.57
9	646.94	645.71	2.47	0.963	33.71
10	650.35	646.54	7.62	0.472	16.50
11	591.04	585.43	11.23	0.189	6.61
12	662.38	660.57	3.62	0.890	31.14
13	635.90	631.93	7.95	0.438	15.34
14	683.52	678.03	10.97	0.203	7.11
15	585.34	576.20	18.28	0.019	0.67
16	475.49	475.32	0.35	1.000	35.00
17	634.98	633.03	3.90	0.866	30.32
18	601.50	594.70	13.59	0.093	3.25
19	635.55	627.89	15.31	0.053	1.87
20	559.01	552.91	12.20	0.143	4.99
21	655.88	651.01	9.74	0.284	9.94
22	651.09	648.44	5.28	0.727	25.45
23	606.36	598.98	14.76	0.064	2.24
24	615.70	610.40	10.61	0.225	7.87
25	650.03	646.27	7.51	0.482	16.88
26	631.77	624.36	14.82	0.063	2.20
27	629.95	625.20	9.51	0.301	10.55
28	554.27	550.80	6.93	0.544	19.03
29	679.91	668.13	23.57	0.003	0.09
30	582.87	580.88	3.97	0.860	30.09
31	572.67	569.65	6.04	0.643	22.49
32	682.83	676.87	11.91	0.155	5.43
33	719.33	713.18	12.30	0.138	4.84
34	681.84	679.19	5.30	0.725	25.38
35	511.34	510.08	2.53	0.960	33.61

Note:
(1) The number of degrees of freedom for the LR test for each sector is 8.
(2) The null hypothesis is that the instrumental variables are exogenous.
(3) High p-values indicate that we cannot reject the null hypothesis of exogeneity.
(4) The last column presents p-values adjusted for simultaneous inference.

TESTS OF VALIDITY OF THE INSTRUMENTAL VARIABLES

sector	LR	p-value
1	599.64	<0.001
2	414.64	<0.001
3	648.09	<0.001
4	608.54	<0.001
5	500.78	<0.001
6	502.97	<0.001
7	678.64	<0.001
8	579.19	<0.001
9	688.82	<0.001
10	639.39	<0.001
11	527.51	<0.001
12	633.63	<0.001
13	653.57	<0.001
14	638.36	<0.001
15	562.00	<0.001
16	503.73	<0.001
17	648.98	<0.001
18	477.98	<0.001
19	620.84	<0.001
20	432.61	<0.001
21	602.56	<0.001
22	633.47	<0.001
23	608.05	<0.001
24	537.26	<0.001
25	598.78	<0.001
26	631.35	<0.001
27	506.32	<0.001
28	442.42	<0.001
29	701.20	<0.001
30	676.44	<0.001
31	731.05	<0.001
32	724.93	<0.001
33	531.66	<0.001
34	715.32	<0.001
35	612.49	<0.001

Note:
(1) Number of degrees of freedom for the LR test for each sector is 99.
(2) The null hypothesis is that instrumental variables are uncorrelated with the endogenous independent variables.
(3) Low p-values indicate that we can reject the null hypothesis of no correlation.

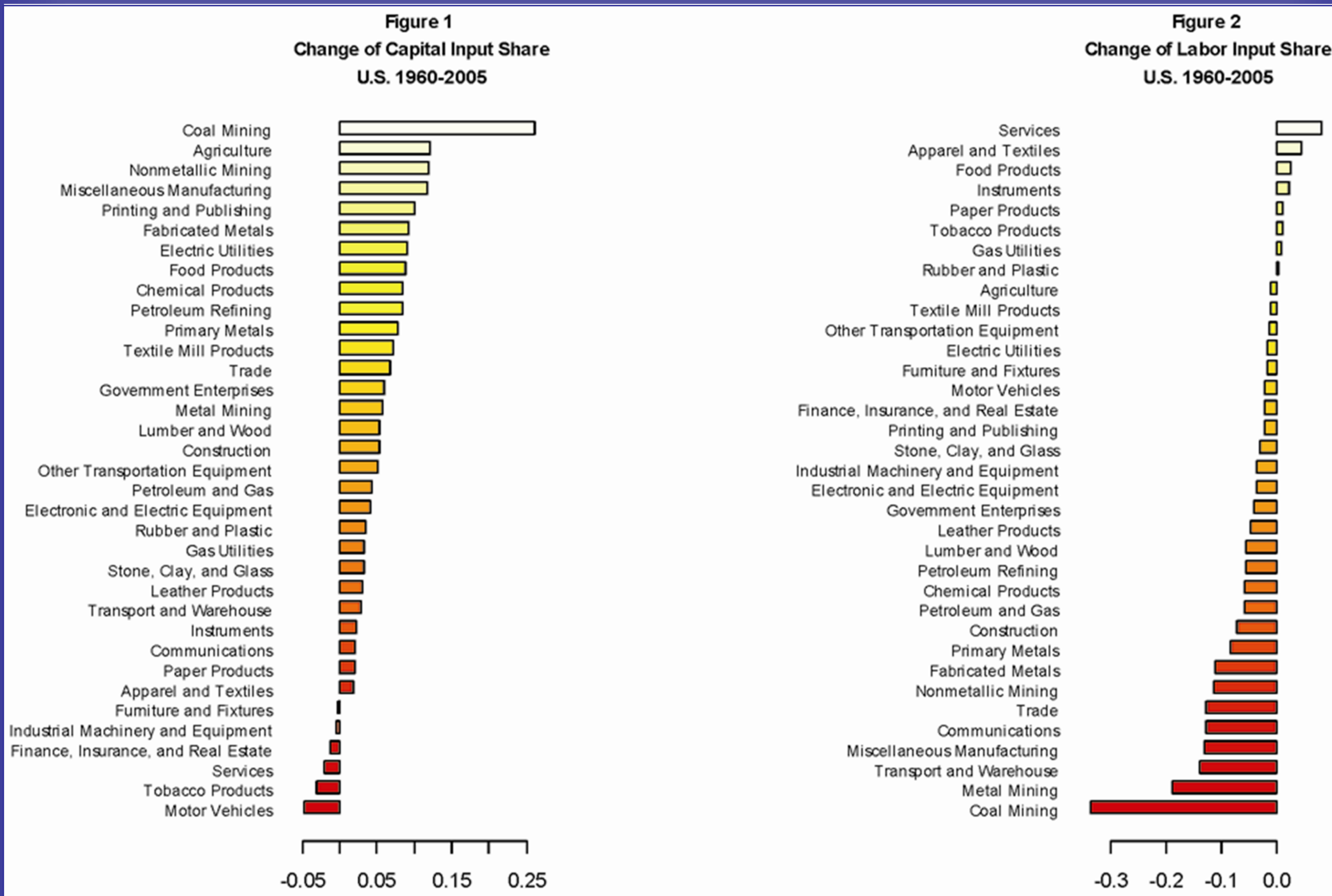


Figure 1
Change of Capital Input Share
U.S. 1960-2005
Coal Mining
Agriculture
Nonmetallic Mining
Miscellaneous Manufacturing
Printing and Publishing
Fabricated Metals
Electric Utilities
Food Products
Chemical Products
Petroleum Refining
Primary Metals
Textile Mill Products
Trade
Government Enterprises
Metal Mining
Lumber and Wood
Construction
Other Transportation Equipment
Petroleum and Gas
Electronic and Electric Equipment
Rubber and Plastic
Gas Utilities
Stone, Clay, and Glass
Leather Products
Transport and Warehouse
Instruments
Communications
Paper Products
Apparel and Textiles
Furniture and Fixtures
Industrial Machinery and Equipment
Finance, Insurance, and Real Estate
Services
Tobacco Products
Motor Vehicles
-0.05
0.05
0.15
0.25
Figure 2
Change of Labor Input Share
U.S. 1960-2005
Services
Apparel and Textiles
Food Products
Instruments
Paper Products
Tobacco Products
Gas Utilities
Rubber and Plastic
Agriculture
Textile Mill Products
Other Transportation Equipment
Electric Utilities
Furniture and Fixtures
Motor Vehicles
Finance, Insurance, and Real Estate
Printing and Publishing
Stone, Clay, and Glass
Industrial Machinery and Equipment
Electronic and Electric Equipment
Government Enterprises
Leather Products
Lumber and Wood
Petroleum Refining
Chemical Products
Petroleum and Gas
Construction
Primary Metals
Fabricated Metals
Nonmetallic Mining
Trade
Communications
Miscellaneous Manufacturing
Transport and Warehouse
Metal Mining
Coal Mining
-0.3
-0.2
-0.1
0.0


Figure 3
Change of Energy Input Share
U.S. 1960-2005
Transport and Warehouse
Nonmetallic Mining
Metal Mining
Government Enterprises
Agriculture
Leather Products
Textile Mill Products
Construction
Lumber and Wood
Trade
Furniture and Fixtures
Food Products
Miscellaneous Manufacturing
Fabricated Metals
Petroleum Refining
Stone, Clay, and Glass
Paper Products
Apparel and Textiles
Other Transportation Equipment
Finance, Insurance, and Real Estate
Printing and Publishing
Electronic and Electric Equipment
Industrial Machinery and Equipment
Motor Vehicles
Rubber and Plastic
Tobacco Products
Primary Metals
Instruments
Communications
Coal Mining
Services
Chemical Products
Electric Utilities
Gas Utilities
Petroleum and Gas
-0.20
-0.10
0.00

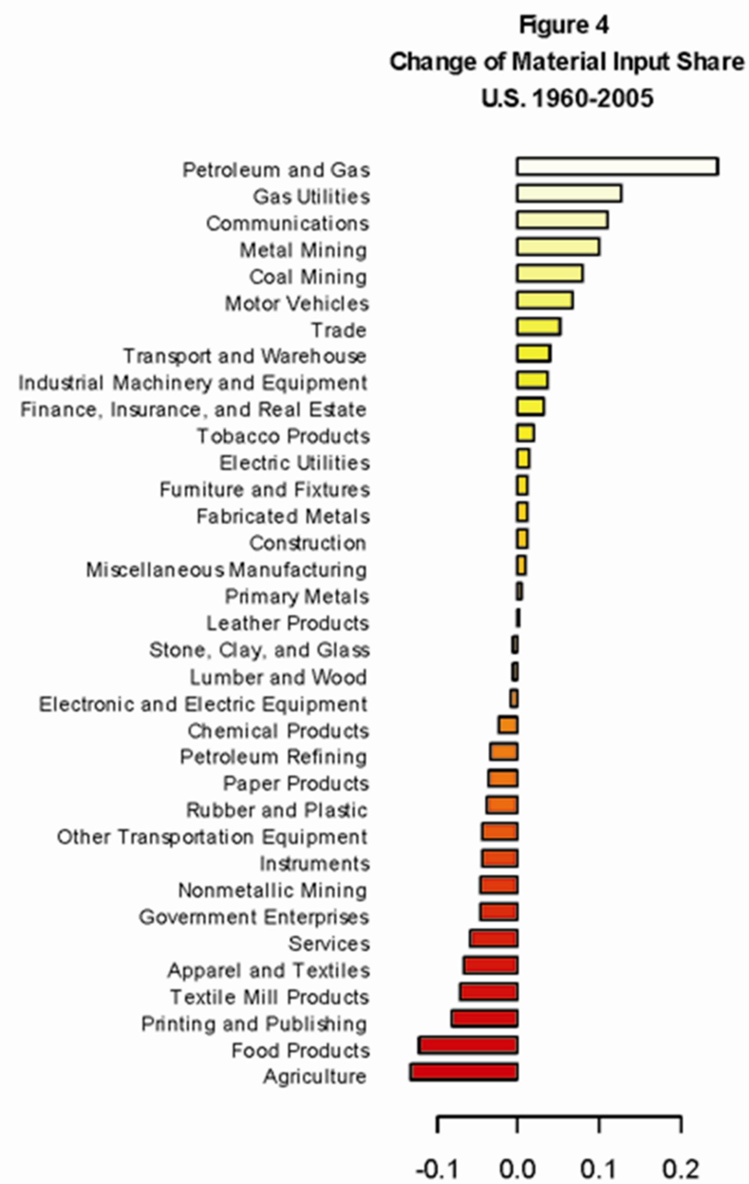


Figure 4
Change of Material Input Share
U.S. 1960-2005
Petroleum and Gas
Gas Utilities
Communications
Metal Mining
Coal Mining
Motor Vehicles
Trade
Transport and Warehouse
Industrial Machinery and Equipment
Finance, Insurance, and Real Estate
Tobacco Products
Electric Utilities
Furniture and Fixtures
Fabricated Metals
Construction
Miscellaneous Manufacturing
Primary Metals
Leather Products
Stone, Clay, and Glass
Lumber and Wood
Electronic and Electric Equipment
Chemical Products
Petroleum Refining
Paper Products
Rubber and Plastic
Other Transportation Equipment
Instruments
Nonmetallic Mining
Government Enterprises
Services
Apparel and Textiles
Textile Mill Products
Printing and Publishing
Food Products
Agriculture
-0.1
0.0
0.1
0.2

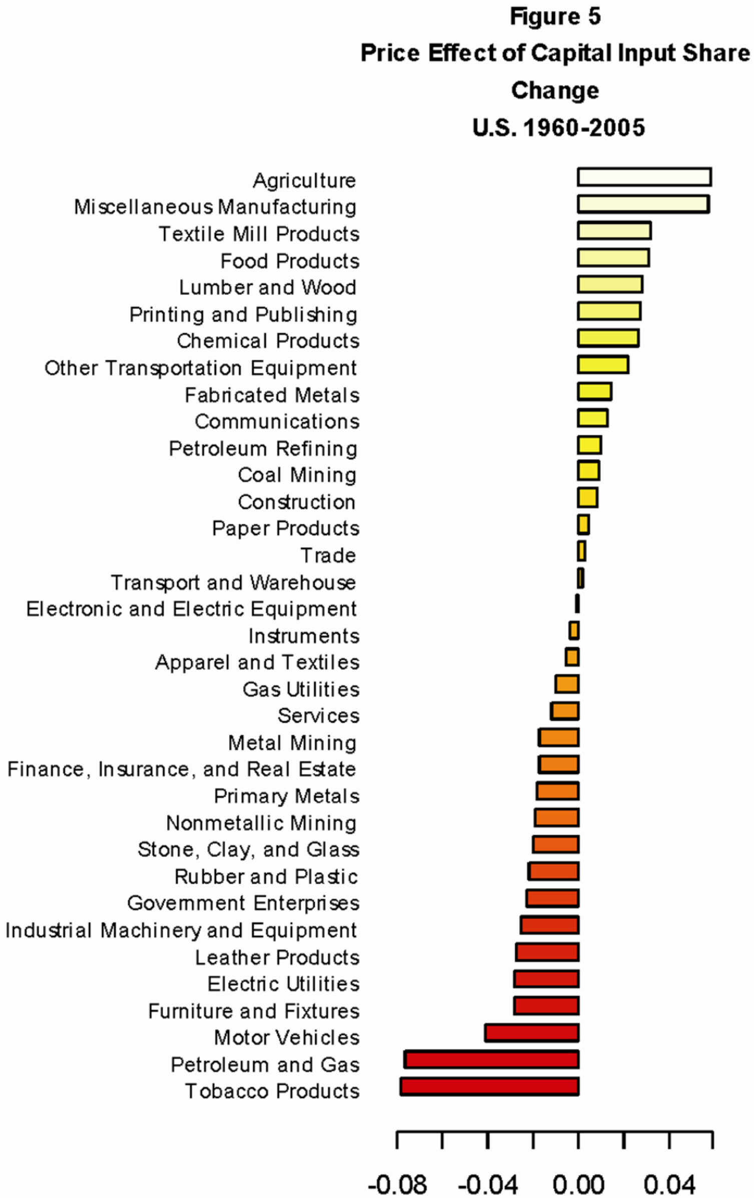


Figure 5
Price Effect of Capital Input Share
Change
U.S. 1960-2005
Agriculture
Miscellaneous Manufacturing
Textile Mill Products
Food Products
Lumber and Wood
Printing and Publishing
Chemical Products
Other Transportation Equipment
Fabricated Metals
Communications
Petroleum Refining
Coal Mining
Construction
Paper Products
Trade
Transport and Warehouse
Electronic and Electric Equipment
Instruments
Apparel and Textiles
Gas Utilities
Services
Metal Mining
Finance, Insurance, and Real Estate
Primary Metals
Nonmetallic Mining
Stone, Clay, and Glass
Rubber and Plastic
Government Enterprises
Industrial Machinery and Equipment
Leather Products
Electric Utilities
Furniture and Fixtures
Motor Vehicles
Petroleum and Gas
Tobacco Products
-0.08
-0.04
0.00
0.04

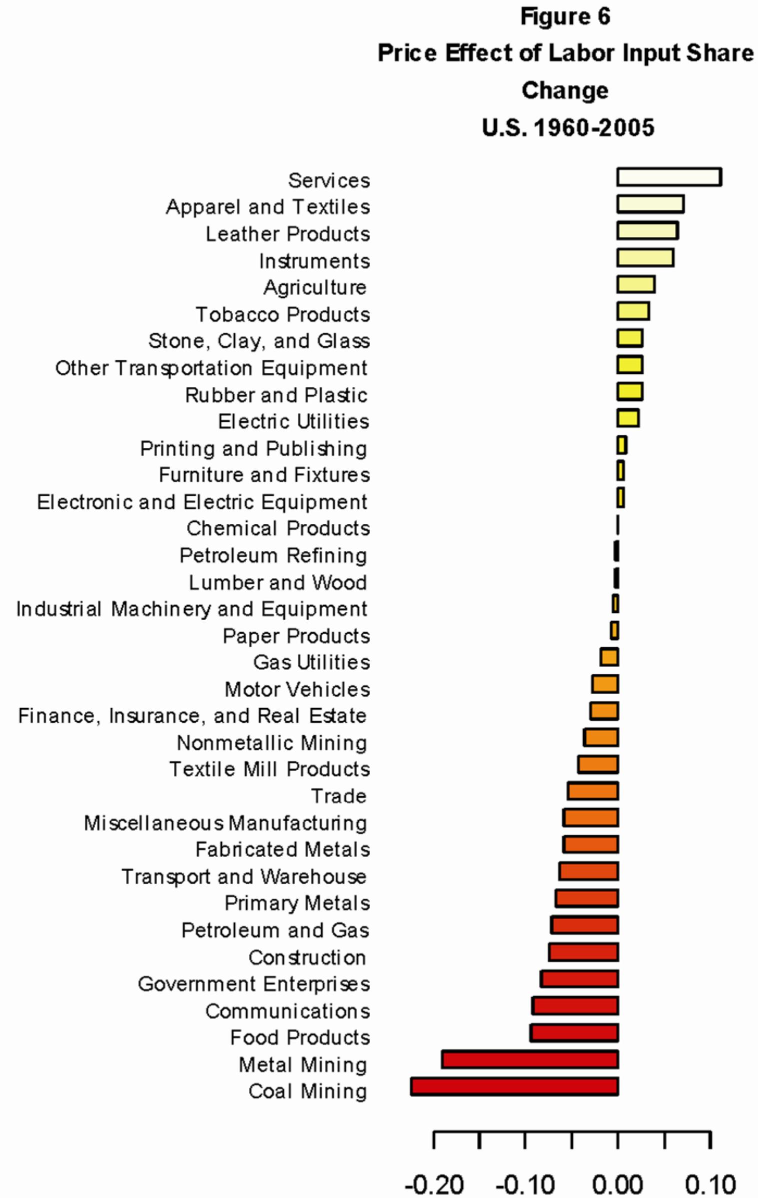


Figure 6
Price Effect of Labor Input Share
Change
U.S. 1960-2005
Services
Apparel and Textiles
Leather Products
Instruments
Agriculture
Tobacco Products
Stone, Clay, and Glass
Other Transportation Equipment
Rubber and Plastic
Electric Utilities
Printing and Publishing
Furniture and Fixtures
Electronic and Electric Equipment
Chemical Products
Petroleum Refining
Lumber and Wood
Industrial Machinery and Equipment
Paper Products
Gas Utilities
Motor Vehicles
Finance, Insurance, and Real Estate
Nonmetallic Mining
Textile Mill Products
Trade
Miscellaneous Manufacturing
Fabricated Metals
Transport and Warehouse
Primary Metals
Petroleum and Gas
Construction
Government Enterprises
Communications
Food Products
Metal Mining
Coal Mining
-0.20
-0.10
0.00
0.10

Figure 7
Price Effect of Energy Input Share Change
U.S. 1960-2005

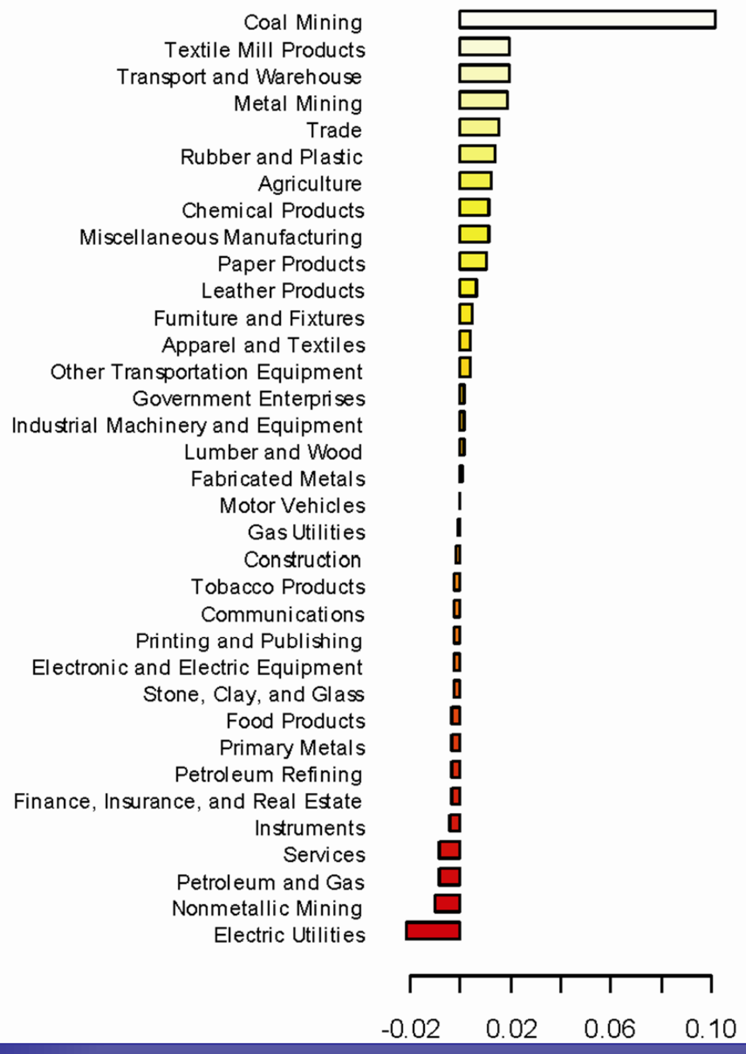


Figure 8
Price Effect of Material Input Share Change
U.S. 1960-2005

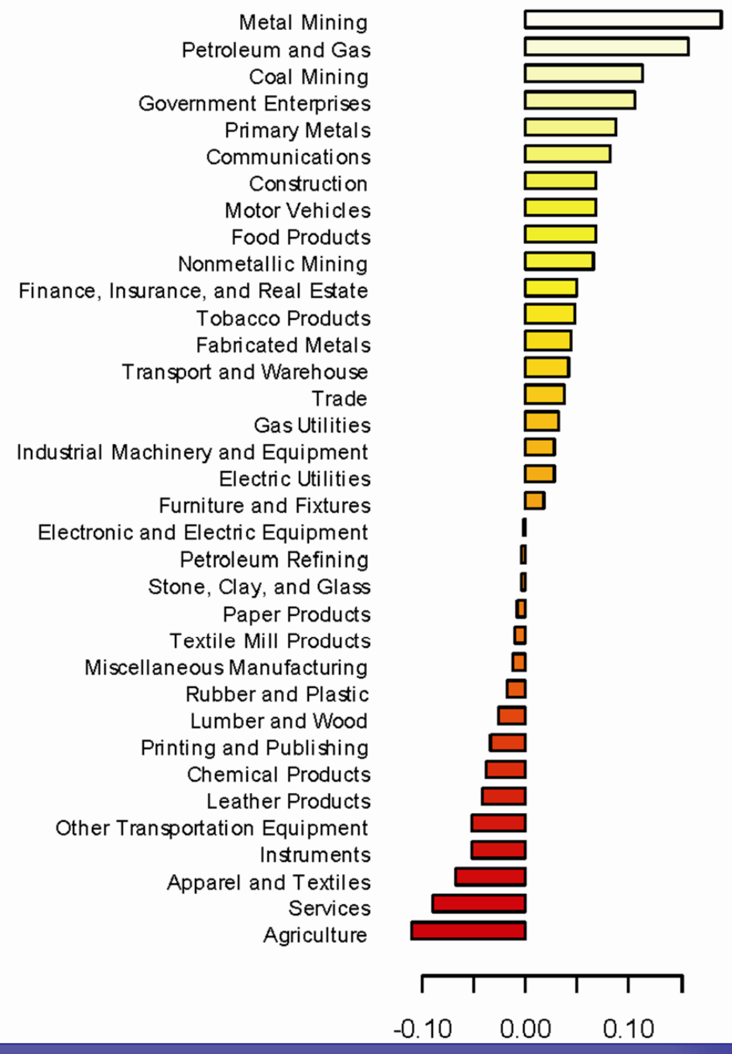


Figure 9
Bias of Technical Change for Capital Input
U.S. 1960-2005

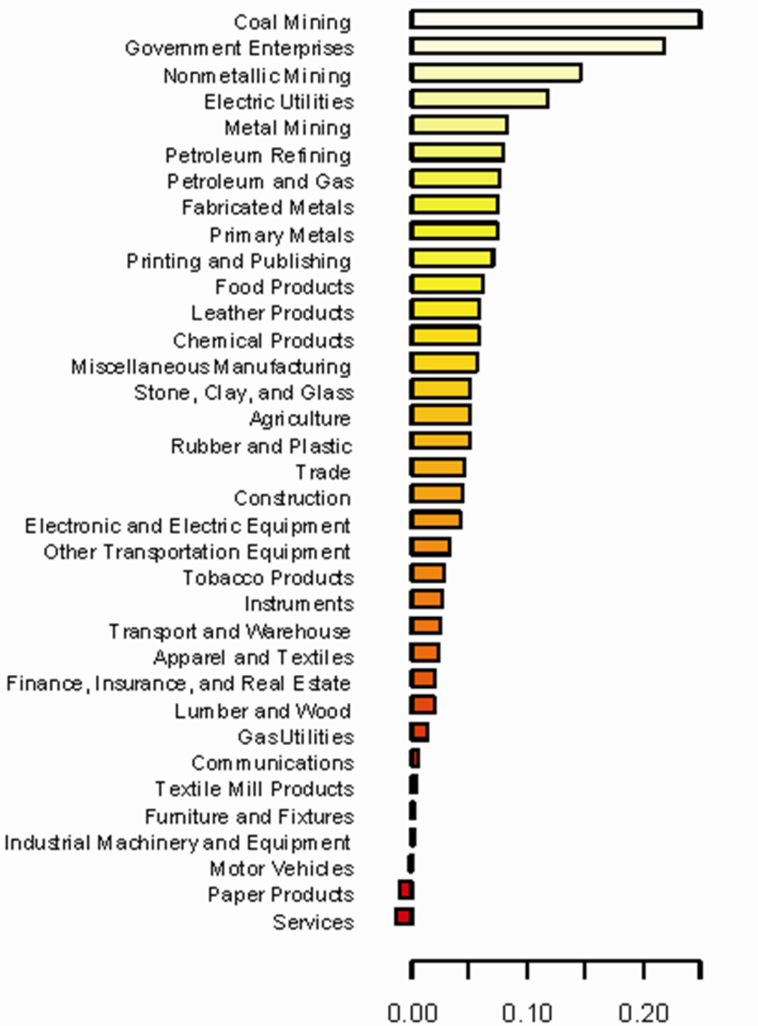


Figure 10
Bias of Technical Change for Labor Input
U.S. 1960-2005

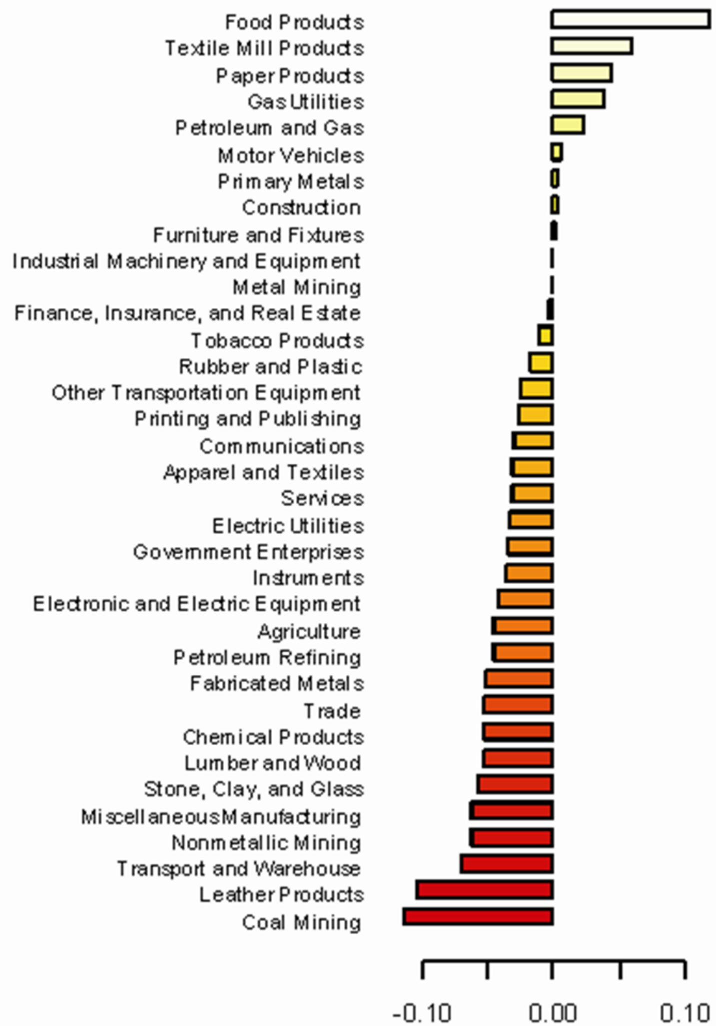


Figure 11
Bias of Technical Change for Energy Input
U.S. 1960-2005

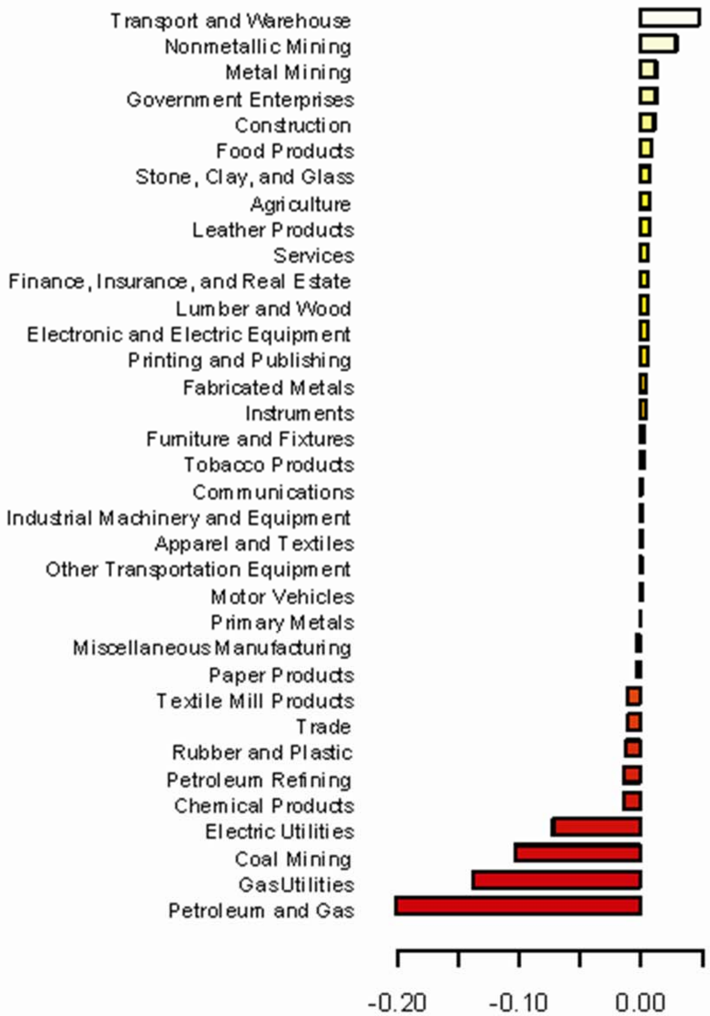


Figure 12
Bias of Technical Change for Material Input
U.S. 1960-2005

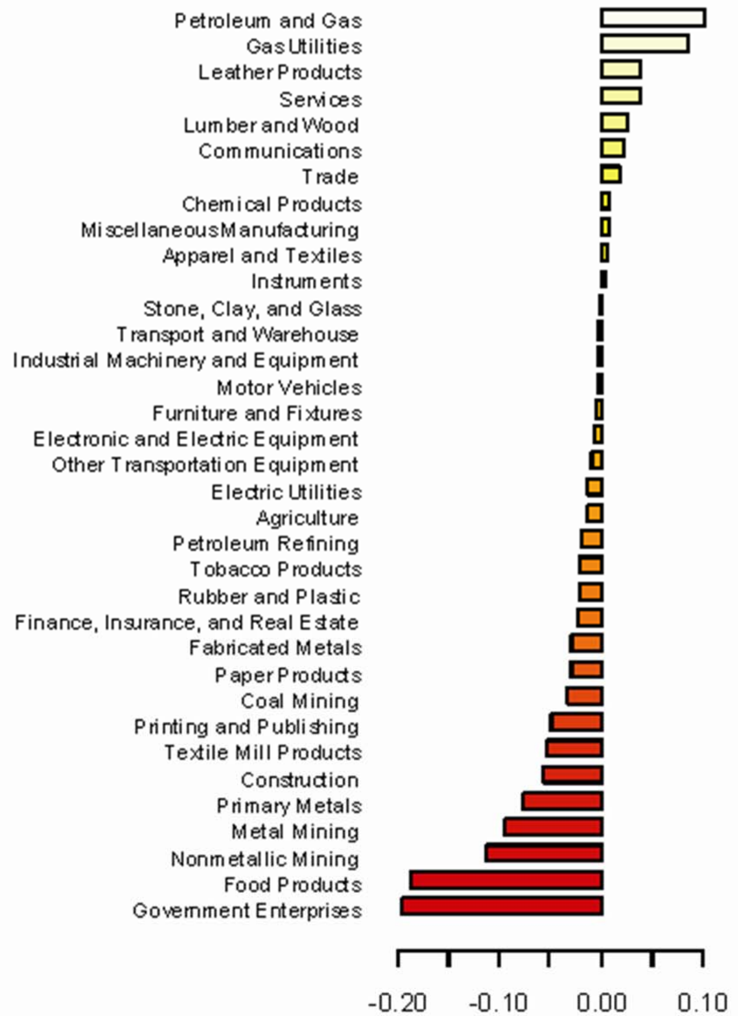


Figure 13
Reduction of Log Relative Ouput Price
U.S. 1960-2005

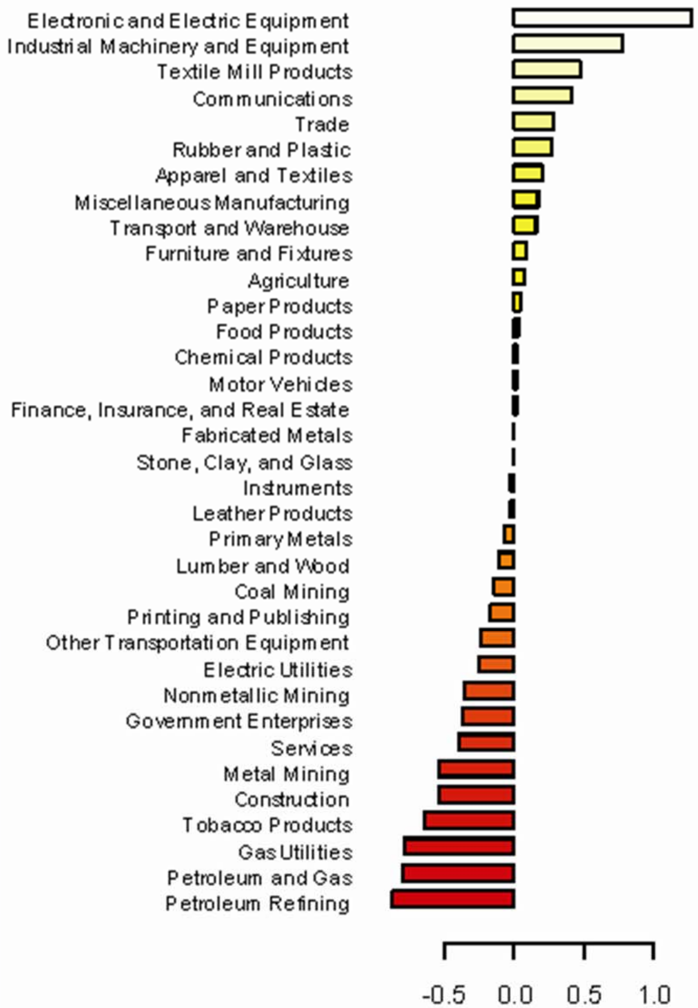
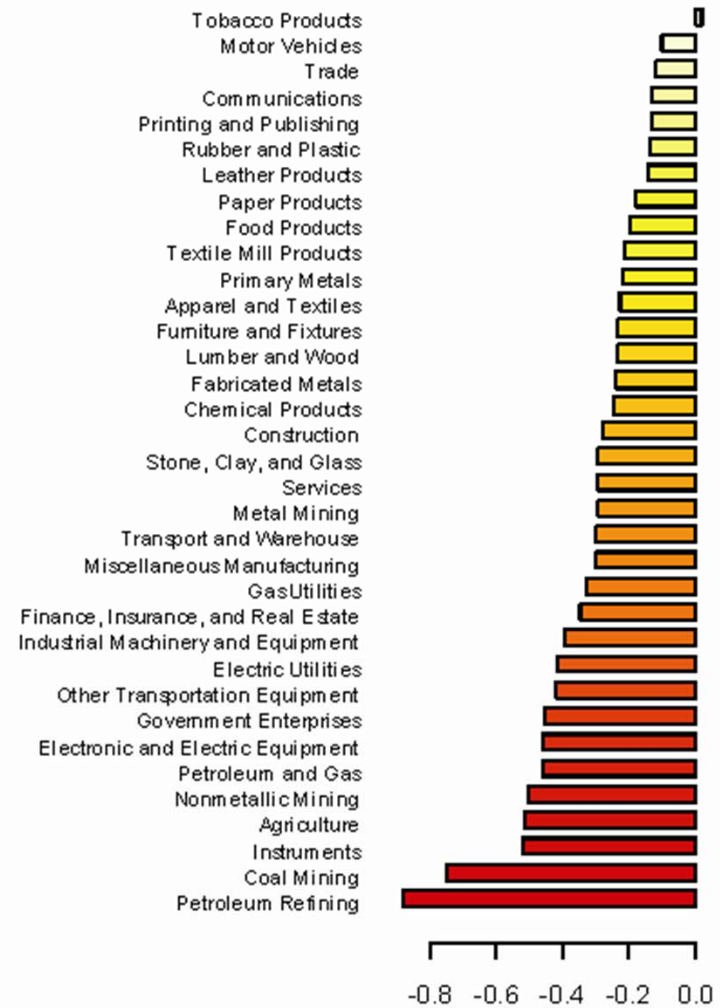


Figure 14
Price Effect of Log Relative Output
Price Change
U.S. 1960-2005



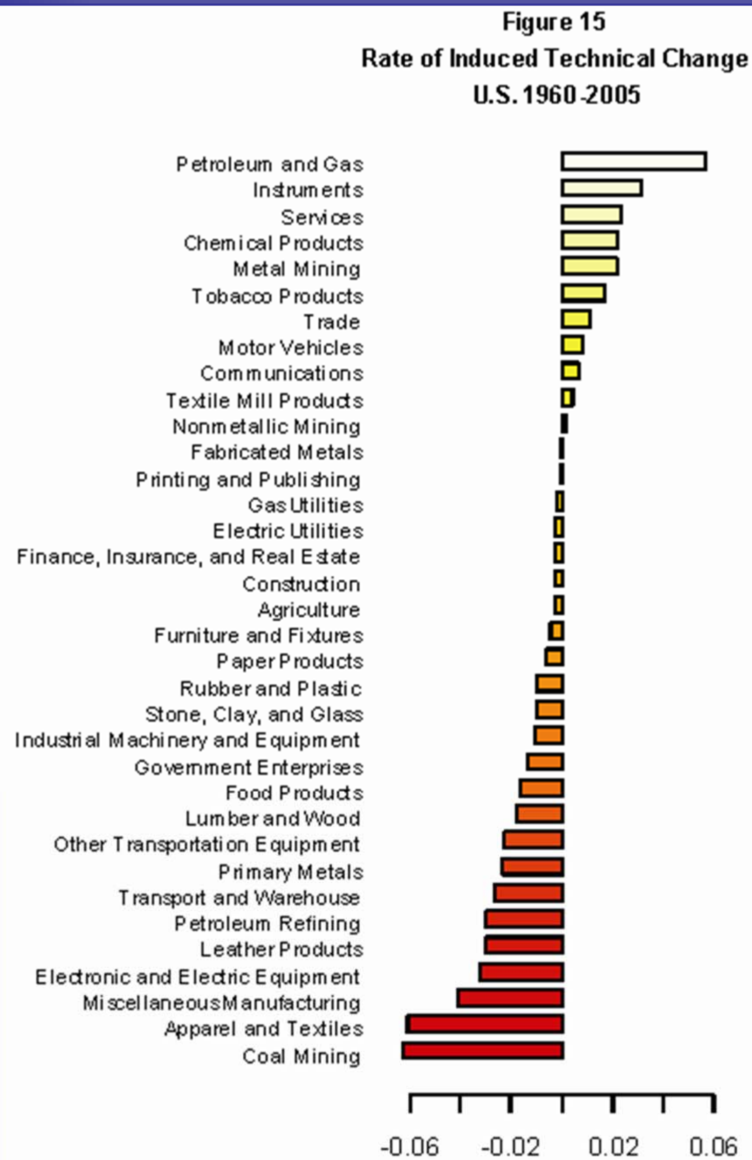


Figure 15
Rate of Induced Technical Change
U.S. 1960-2005
Petroleum and Gas
Instruments
Services
Chemical Products
Metal Mining
Tobacco Products
Trade
Motor Vehicles
Communications
Textile Mill Products
Nonmetallic Mining
Fabricated Metals
Printing and Publishing
Gas Utilities
Electric Utilities
Finance, Insurance, and Real Estate
Construction
Agriculture
Furniture and Fixtures
Paper Products
Rubber and Plastic
Stone, Clay, and Glass
Industrial Machinery and Equipment
Government Enterprises
Food Products
Lumber and Wood
Other Transportation Equipment
Primary Metals
Transport and Warehouse
Petroleum Refining
Leather Products
Electronic and Electric Equipment
Miscellaneous Manufacturing
Apparel and Textiles
Coal Mining
-0.06
-0.02
0.02
0.06

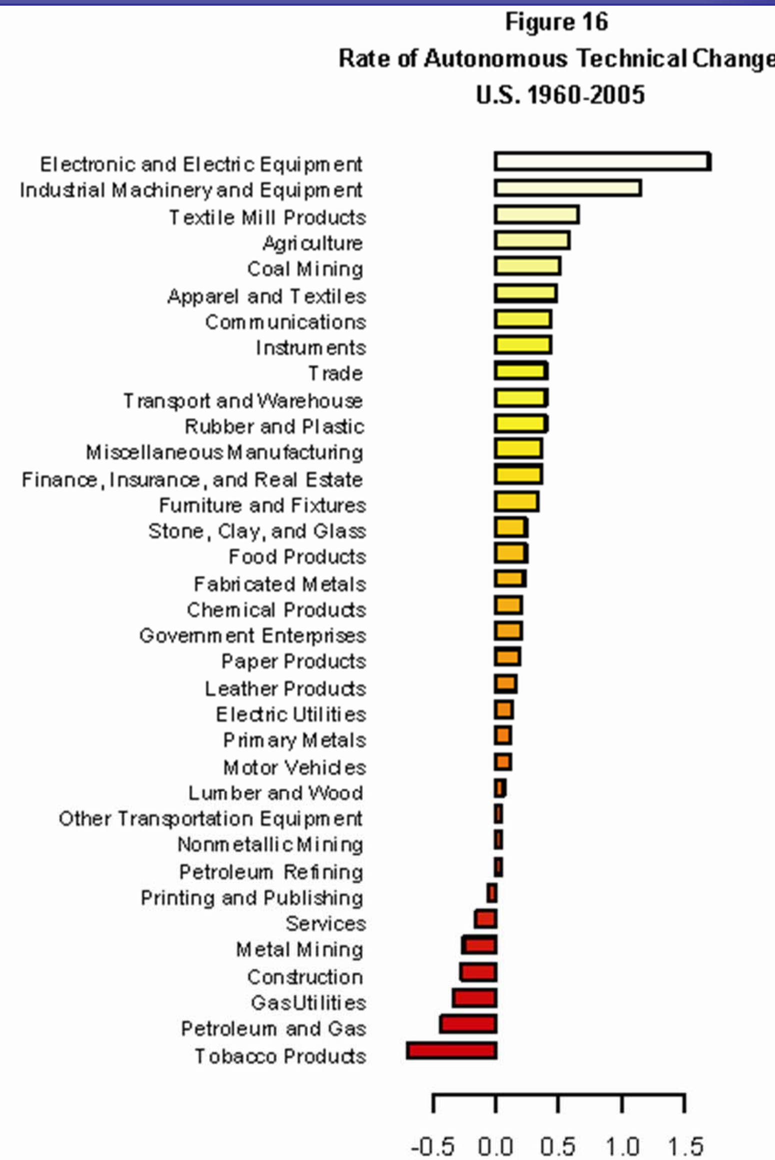


Figure 16
Rate of Autonomous Technical Change
U.S. 1960-2005
Electronic and Electric Equipment
Industrial Machinery and Equipment
Textile Mill Products
Agriculture
Coal Mining
Apparel and Textiles
Communications
Instruments
Trade
Transport and Warehouse
Rubber and Plastic
Miscellaneous Manufacturing
Finance, Insurance, and Real Estate
Furniture and Fixtures
Stone, Clay, and Glass
Food Products
Fabricated Metals
Chemical Products
Government Enterprises
Paper Products
Leather Products
Electric Utilities
Primary Metals
Motor Vehicles
Lumber and Wood
Other Transportation Equipment
Nonmetallic Mining
Petroleum Refining
Printing and Publishing
Services
Metal Mining
Construction
Gas Utilities
Petroleum and Gas
Tobacco Products
-0.5
0.0
0.5
1.0
1.5

ECONOMETRIC MODELING OF PRODUCER BEHAVIOR: SUMMARY.

Production Theory, Price Effects, and Share Elasticities.

Latent Variables, Rate, and Biases of Technical Change and the Kalman Filter.

Substitution and Technical Change Equally Important in Explaining Changes in Budget Shares.

Autonomous and Induced Technical Change. Generally Opposite in Sign; Autonomous Change Generally Positive; Induced Change Generally Negative and Much Less Important.

ECONOMETRIC MODELING
OF CONSUMER BEHAVIOR

Allocation of Full Consumption among Leisure and Different

Goods: Rank Two Versus Rank Three; Role of Human Capital; Cross-Section and Time-Series Variations in Prices; 154,180 Individual Observations on Expenditures, Including Leisure.

Exact Aggregation over Households; Role of Prices, Expenditures, and Demographics.

Compensated and Uncompensated Elasticities.

Allocation of Full Wealth among Time Periods; Synthetic Cohorts.

ECONOMETRIC MODELING OF CONSUMER BEHAVIOR

Rank three translog indirect utility function (Lewbel, 2001):

$$(\ln V_k)^{-1} = [\alpha_0 + \ln(\frac{\rho_k}{F_k})'\alpha^P + \frac{1}{2}\ln(\frac{\rho_k}{F_k})' B_{pp} \ln(\frac{\rho_k}{F_k})' B_{pA} A_k]^{-1} - \ln(\frac{\rho_k}{F_k})'\gamma_p \quad (1)$$

where: $B_{pp} = B'_{pp}, \iota' B_{pA} = 0, \iota' B_{pp} i = 0, \iota'\alpha_p = -1$ and $\iota'\gamma_p = 0$.

Define $\ln G_k$ as:

$$\ln G_k = \alpha_0 + \ln(\frac{\rho_k}{F_k})'\alpha^P + \frac{1}{2}\ln(\frac{\rho_k}{F_k})' B_{pp} \ln(\frac{\rho_k}{F_k}) + \ln(\frac{\rho_k}{F_k})' B_{pA} A_k \quad (2)$$

AGGREGATE DEMAND

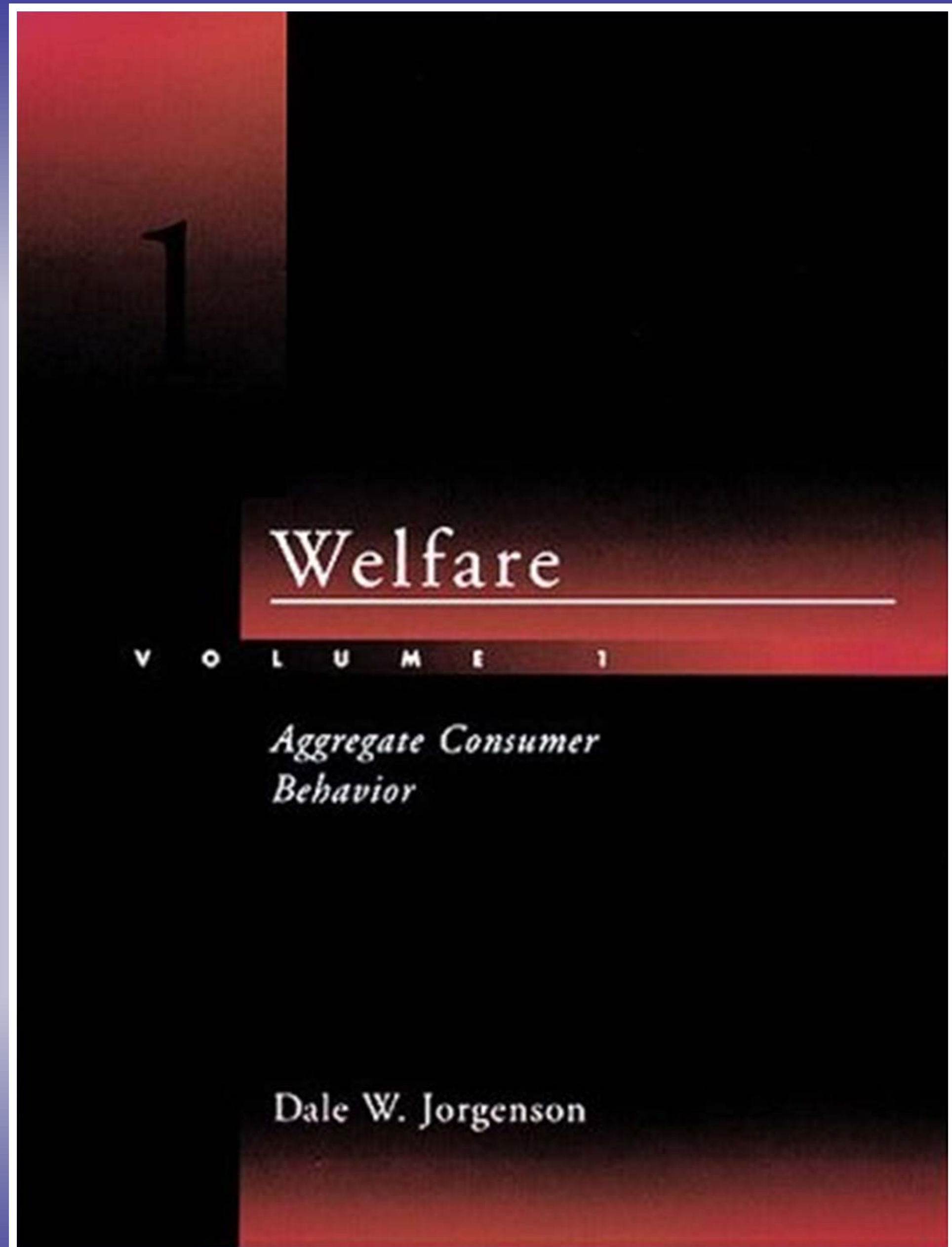
Roy's Identity yields budget shares:

$$w_k = \frac{1}{D(\rho_k)}(\alpha_p + B_{pp} \ln \frac{\rho_k}{F_k} + B_{pA} A_k + \gamma_p [\ln G_k]^2) \qquad (3)$$

where: $D(\rho_k) = -1 + \iota' B_{pp} \ln \rho_k$.

Exact aggregation (Jorgenson, Lau, and Stoker, 1997)

$$w = \frac{\sum_k F_k w_k}{\sum_k F_k} = \frac{1}{D(\rho)}[\alpha_p + B_{pp} \ln \rho - \iota' B_{pp} \frac{\sum F_k \ln F_k}{\sum F_k} + B_{pA} \frac{\sum F_k A_k}{F_k} + \gamma_p \frac{\sum F_k (\ln G_k)^2}{\sum F_k}].$$


1
Welfare
VOLUME 1
Aggregate Consumer Behavior
Dale W. Jorgenson

INTER-TEMPORAL ALLOCATION OF CONSUMPTION

Full expenditure F_{kt} allocated across time periods to maximize expected lifetime utility U_k:

$$\max_{F_{kt}} U_k = E_t \left\{ \sum_{t=1}^{T} (1+\delta)^{-(t-1)} \left[\frac{V_{kt}^{(1-\sigma)}}{(1-\sigma)} \right] \right\} \qquad (4)$$

subject to full wealth constraint W_k:

$$\sum_{t=1}^{T} (1+r_1)^{-(t-1)} F_{kt} \leq W_k$$

where: r_t nominal interest rate, σ intertemporal curvature parameter, δ rate of time preference.

EULER EQUATIONS

First-order conditions for optimization:

$$(V_{kt})^{-\sigma}\left[\frac{\partial V_{kt}}{\partial F_{kt}}\right] = E_t\left[(V_{k,t+1})^{-\sigma}\left[\frac{\partial V_{k,t+1}}{\partial F_{k,t+1}}\right]\frac{(1+r_{t+1})}{(1+\delta)}\right] \tag{5}$$

The estimating equation is:

$$(V_{kt})^{-\sigma}\left[\frac{\partial V_{kt}}{\partial F_{kt}}\right] = \left[(V_{k,t+1})^{-\sigma}\left[\frac{\partial V_{k,t+1}}{\partial F_{k,t+1}}\right]\frac{(1+r_{t+1})}{(1+\delta)}\right]\eta_{k,t+1} \tag{6}$$

where: η_{kt} expectational errors for household k at time t.

SIMPLIFYING THE ESTIMATING EQUATION

For rank 3 specification of V_k:

$$\frac{\partial V_{kt}}{\partial F_{kt}} = \frac{V_{kt}}{F_{kt}}(-D(\rho_{kt}))[1-(\gamma'_{\mathrm{P}\backslash} \ln \rho_{kt}) * G_{kt}]^{-2}$$

Last term approximately equal to one in the data;

taking logs:

$$\Delta \ln F_{k,t+1} = (1-\sigma)\Delta \ln V_{k,t+1} + \Delta \ln(-D(\rho_{k,t+1})) + \ln(1+r_{t+1}) - \ln(1+\delta) + \ln \eta_{kt} \quad (7)$$

DATA ISSUES

The Consumer Expenditure Survey (CEX), U.S. Bureau of Labor Statistics:

Data on expenditures on goods and services and labor supply.

CEX data for 1980-2006, 4000-8000 observations per year, 154,180 observations.

Consumer Price Index (CPI), U.S. Bureau of Labor Statistics:

Price data for four Census regions in all years.

WAGE EQUATION

Wage equation for worker *i*:

$$\ln P_{Li} = \sum_j \beta_j^z z_{ji} + \sum_j \beta_j^s (S_i^* z_{ji}) + \sum_j \beta_j^{nw} (NW_i^* z_{ji}) + \sum_l \beta_l^g g_{li} + \varepsilon_{it} \qquad (8)$$

where:

P_{Li} -- wage of worker i.

$\mathbf{z_i}$ -- vector including age, age squared, education, education squared.

S_i -- dummy variable female.

NW_i -- dummy variable nonwhite.

$\mathbf{g_i}$ -- vector of region-year interaction dummy variables.

Quality-adjusted wage for a worker in region s:

$$p_L^s = \exp(\hat{\beta}_s^g)$$

QUALITY-ADJUSTED LEISURE

Quality index for worker m:

$$q_{kt}^{m} = \frac{\mathrm{E}_{kt}^{m}}{p_{Lt} H_{kt}^{m}}$$

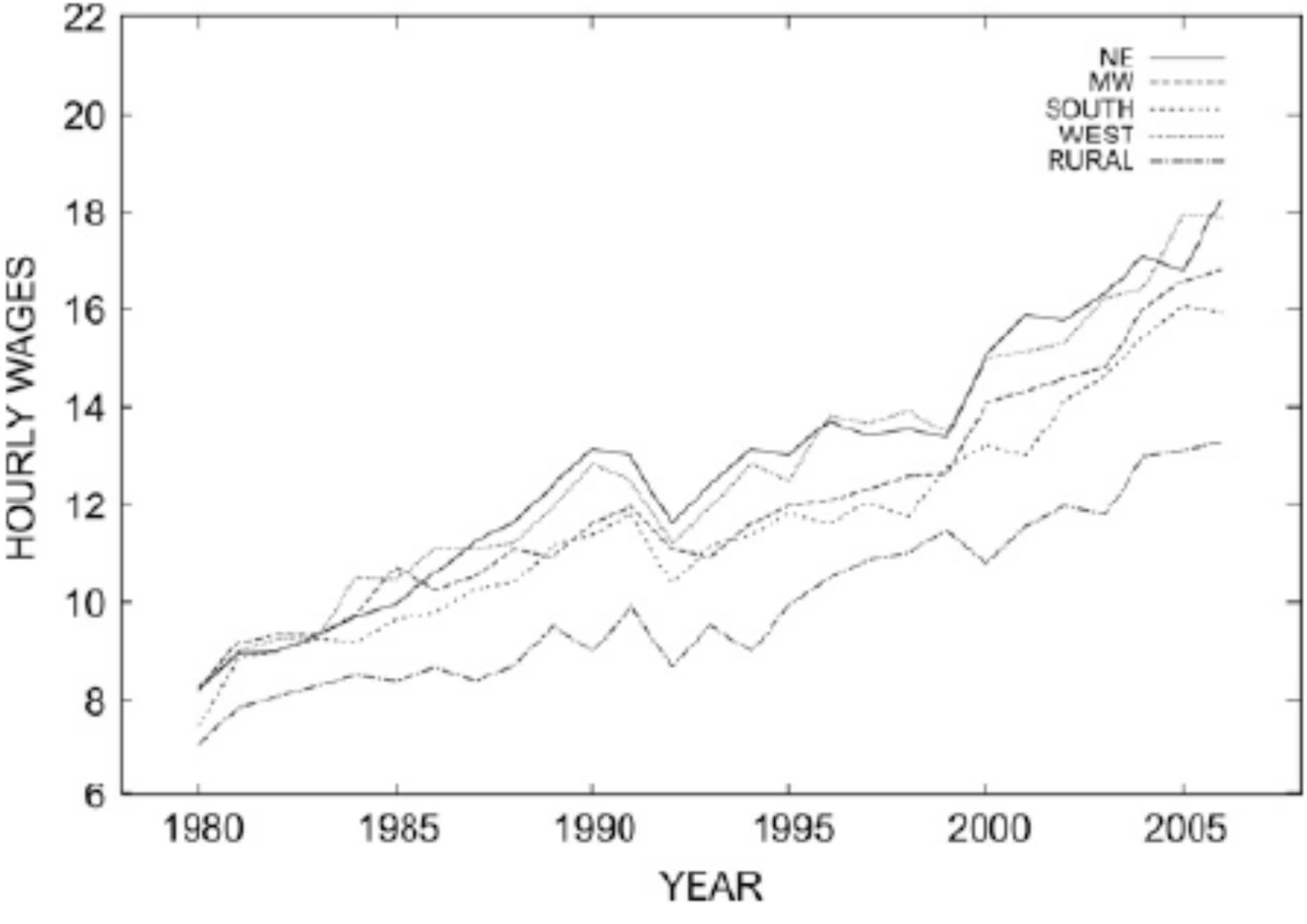
Time endowment in efficiency units $T_{kt}^{m} = q_{kt}^{m} * (14)$; leisure consumption:

$$R_{kt}^{m} = q_{kt}^{m}(14 - H_{kt}^{m}).$$

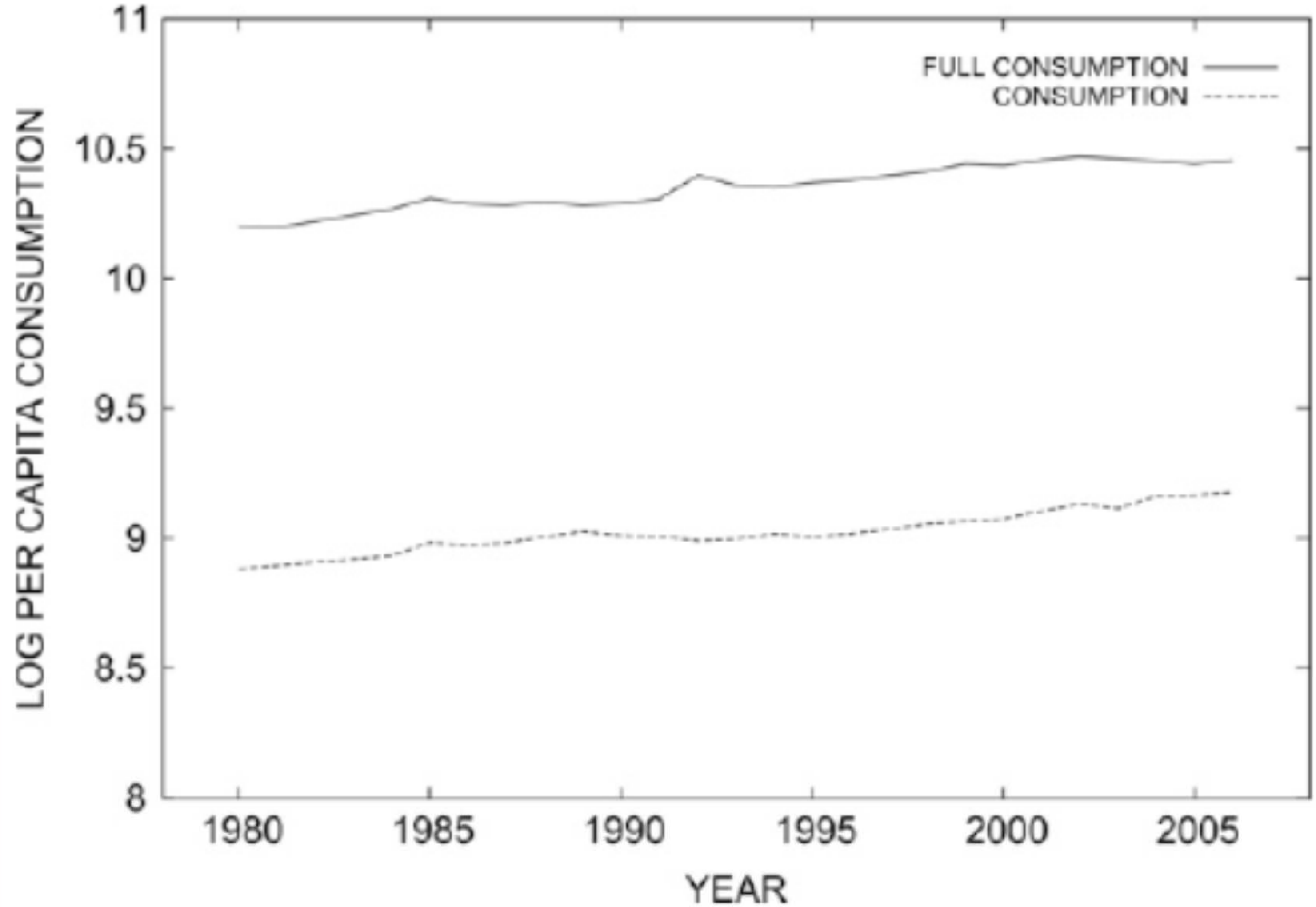
Full expenditure for household k:

$$F_{kt} = p_{Lt} R_{kt} + \sum_{i} p_{ik} x_{ik}$$

where $R_{kt} = \sum_{m} R_{kt}^{m}$ leisure summed over adult members.



Regional wages (current dollars).



Log consumption per capita (constant dollars).

Price and income elasticities (Reference household: Two adults, Two children, NE Urban, Male, White, Full expenditure = 100 K).

Good	Uncompensated price elasticity		Compensated price elasticity		Full expenditure elasticity	
	Rank 2	Rank 3	Rank 2	Rank 3	Rank 2	Rank 3
Nondurables	−0.918	−0.903	−0.822	−0.809	0.722	0.724
Capital services	−1.428	−1.432	−1.314	−1.319	0.926	0.930
Consumer services	−0.613	−0.614	−0.548	−0.548	1.088	1.096
Leisure	0.012	0.014	−0.323	−0.314	1.059	1.056
Labor supply	−0.026	−0.030	0.698	0.698	−2.289	−2.342

Full expenditure and household budget shares (Reference household: Two adults, Two children, NE Urban, Male, White).

Expenditure level	Rank 2	Rank 3	Rank 2	Rank 3
	Nondurables share		Capital services share	
7 500	0.227	0.268	0.147	0.183
25 000	0.183	0.192	0.136	0.145
75 000	0.143	0.140	0.126	0.125
150 000	0.117	0.116	0.120	0.119
275 000	0.095	0.100	0.114	0.119
350 000	0.086	0.095	0.112	0.120
	Consumer services share		Leisure share	
7 500	0.047	0.073	0.579	0.476
25 000	0.053	0.060	0.627	0.603
75 000	0.059	0.058	0.672	0.677
150 000	0.063	0.062	0.700	0.702
275 000	0.066	0.070	0.725	0.711
350 000	0.067	0.073	0.734	0.711

RANK THREE VERSUS RANK TWO

Disturbances of the demand equations are additive:

$$\mathbf{w_k} = \frac{1}{D(\rho_k)}(a_\mathbf{p} + B_{pp}\ln\frac{p_\mathbf{k}}{F_k} + B_{pA}\mathbf{A_k} + \gamma_\mathbf{p}[\ln G_k]^2) + \varepsilon_\mathbf{k}$$

Rank 2 translog demand system (Jorgenson, Lau and Stoker, 1997):

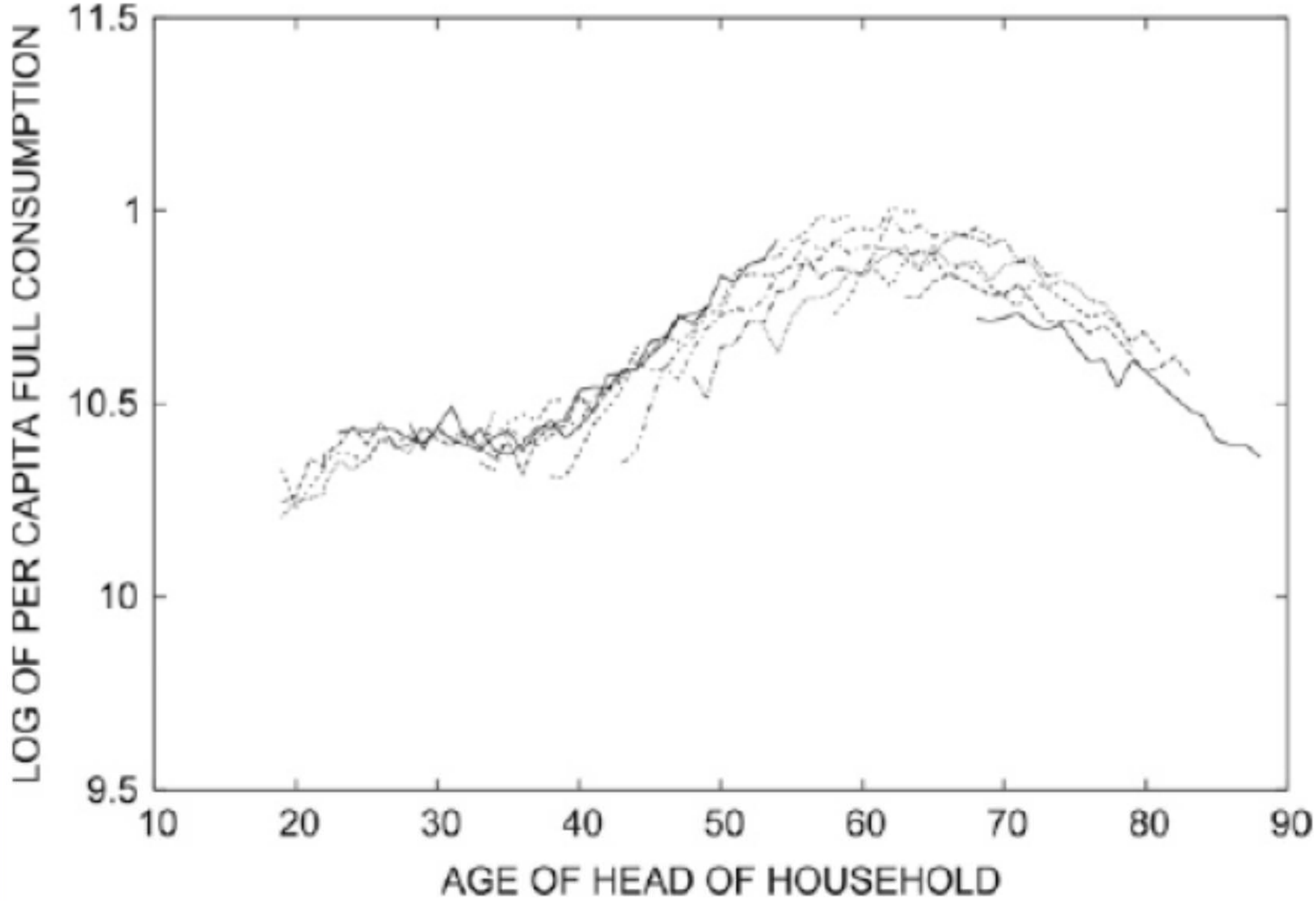
$$\mathbf{w_k} = \frac{1}{D(\rho_k)}(a_\mathbf{p} + B_{pp}\ln\frac{p_\mathbf{k}}{F_k} + B_{pA}\mathbf{A_k}) + \mu_\mathbf{k}$$

Aggregation factor:

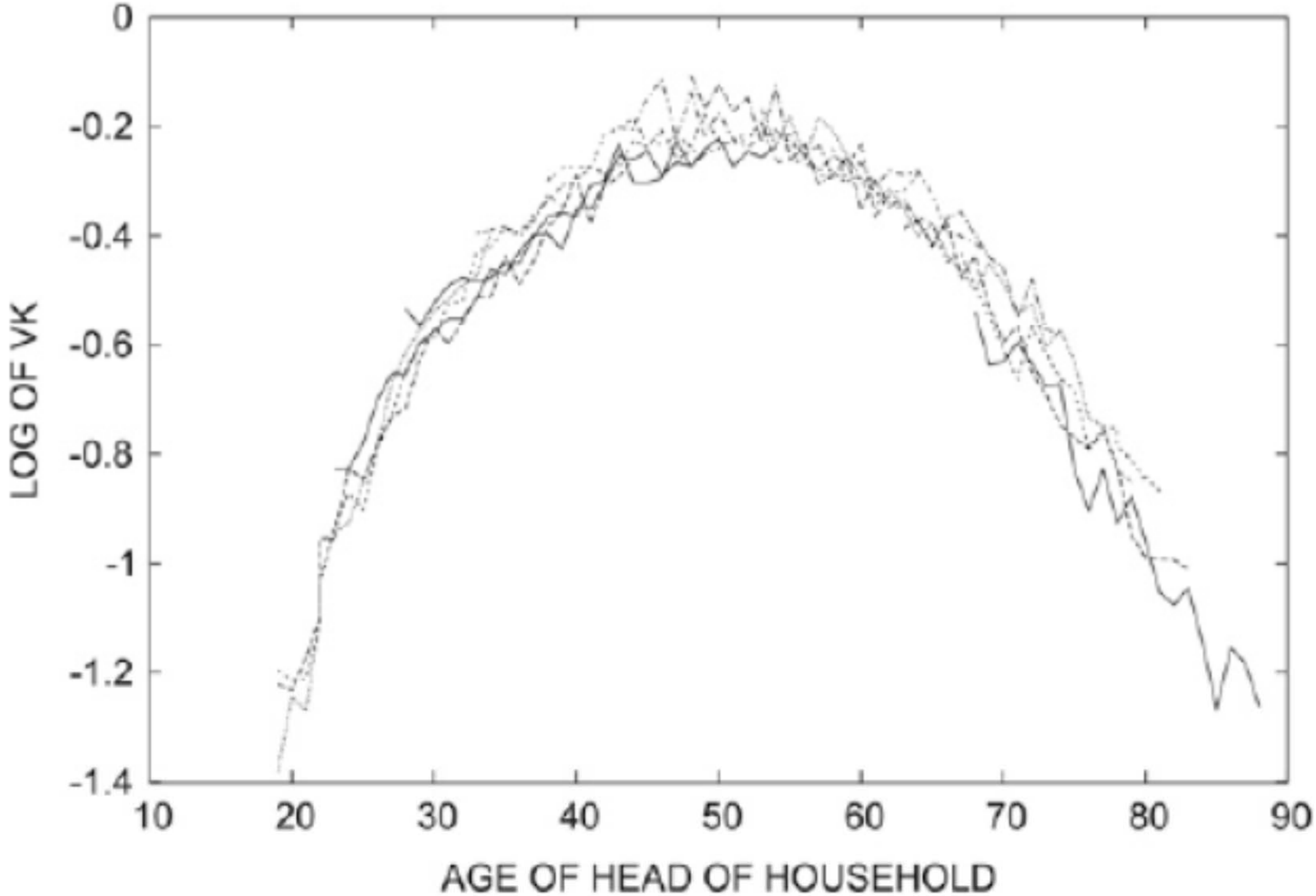
$$P_t = \frac{\sum_k F_{kt} D(\rho_\mathbf{kt})^{-1}(a_\mathbf{p} + B_{pp}\ln\rho_\mathbf{kt})}{\sum_k F_{kt}}$$

Aggregate budget shares.

Year	Sample shares	Fitted shares	R-squared	Aggregation factors		
				Price	Expenditure	Demographics
Nondurables						
1980–1981	0.1145	0.1074	0.1273	0.3985	−0.3009	0.0098
1985–1986	0.0993	0.1003	0.1609	0.4009	−0.3090	0.0084
1990–1991	0.0967	0.0990	0.1793	0.4051	−0.3141	0.0080
1995–1996	0.0898	0.0892	0.2198	0.3996	−0.3181	0.0077
2000–2001	0.0846	0.0852	0.1910	0.4011	−0.3235	0.0076
2005–2006	0.0864	0.0845	0.1806	0.4055	−0.3279	0.0068
Capital services						
1980–1981	0.0956	0.1162	0.0296	0.2100	−0.0141	−0.0797
1985–1986	0.1134	0.1178	0.1003	0.2103	−0.0143	−0.0782
1990–1991	0.1186	0.1213	0.1292	0.2132	−0.0145	−0.0774
1995–1996	0.1222	0.1240	0.1161	0.2161	−0.0150	−0.0771
2000–2001	0.1306	0.1272	0.1226	0.2193	−0.0154	−0.0766
2005–2006	0.1403	0.1344	0.1134	0.2255	−0.0155	−0.0756
Consumer services						
1980–1981	0.0566	0.0561	0.0018	−0.0439	0.1202	−0.0202
1985–1986	0.0626	0.0668	0.0111	−0.0370	0.1236	−0.0199
1990–1991	0.0706	0.0678	0.0317	−0.0379	0.1258	−0.0201
1995–1996	0.0734	0.0750	0.0318	−0.0326	0.1270	−0.0193
2000–2001	0.0724	0.0747	0.0420	−0.0350	0.1289	−0.0192
2005–2006	0.0748	0.0678	0.0245	−0.0434	0.1308	−0.0195
Leisure						
1980–1981	0.7333	0.7203	0.1506	0.4354	0.1948	0.0902
1985–1986	0.7247	0.7151	0.1532	0.4257	0.1997	0.0897
1990–1991	0.7141	0.7119	0.1804	0.4197	0.2028	0.0895
1995–1996	0.7146	0.7117	0.1791	0.4170	0.2060	0.0887
2000–2001	0.7124	0.7129	0.1758	0.4147	0.2100	0.0882
2005–2006	0.6985	0.7133	0.1458	0.4124	0.2126	0.0883



Age profile of per capita full consumption.



Age profile of $\ln V_k$.

Parameter estimates – intertemporal model.

Least squares estimates						
	OLS		Weighted OLS		Random effects	
Variable	Estimate	SE	Estimate	SE	Estimate	SE
δ	0.01471	0.0011	0.01185	0.0011	0.01460	0.0016
σ	0.08226	0.0194	0.11280	0.0218	0.10183	0.0202

Instrumental variables estimators						
	IV1		IV2		IV3	
Variable	Estimate	SE	Estimate	SE	Estimate	SE
δ	0.01253	0.0012	0.01251	0.0012	0.01249	0.0011
σ	0.03414	0.0357	0.05521	0.0350	0.08150	0.0337

ECONOMETRIC MODELING OF CONSUMER BEHAVIOR

Allocation of Full Consumption and Full Wealth; Elasticities.

Individual Observations on Expenditures, Including Leisure; Role of Human Capital.

Exact Aggregation.

Applications to Welfare Economics.

IGEM:

An Intertemporal Model of the U.S. Economy for Modeling Energy and Environmental Policy

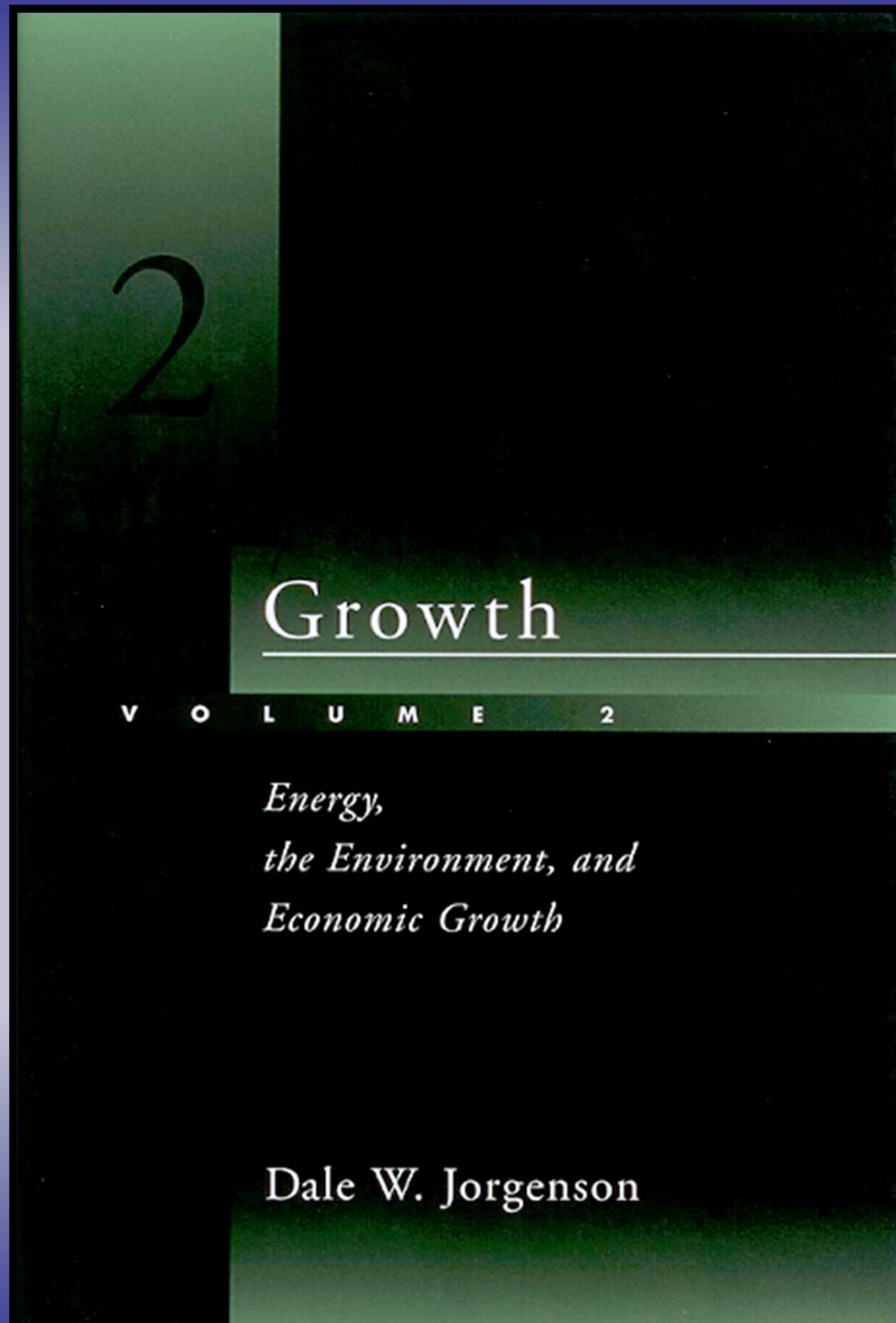
Household Model Incorporates Demography

Demand for Leisure and the Supply of Labor

Production Model Incorporates Technology

Endogenous Technical Change

Resources and Energy Supply


2
Growth
VOLUME 2
Energy,
the Environment, and
Economic Growth
Dale W. Jorgenson

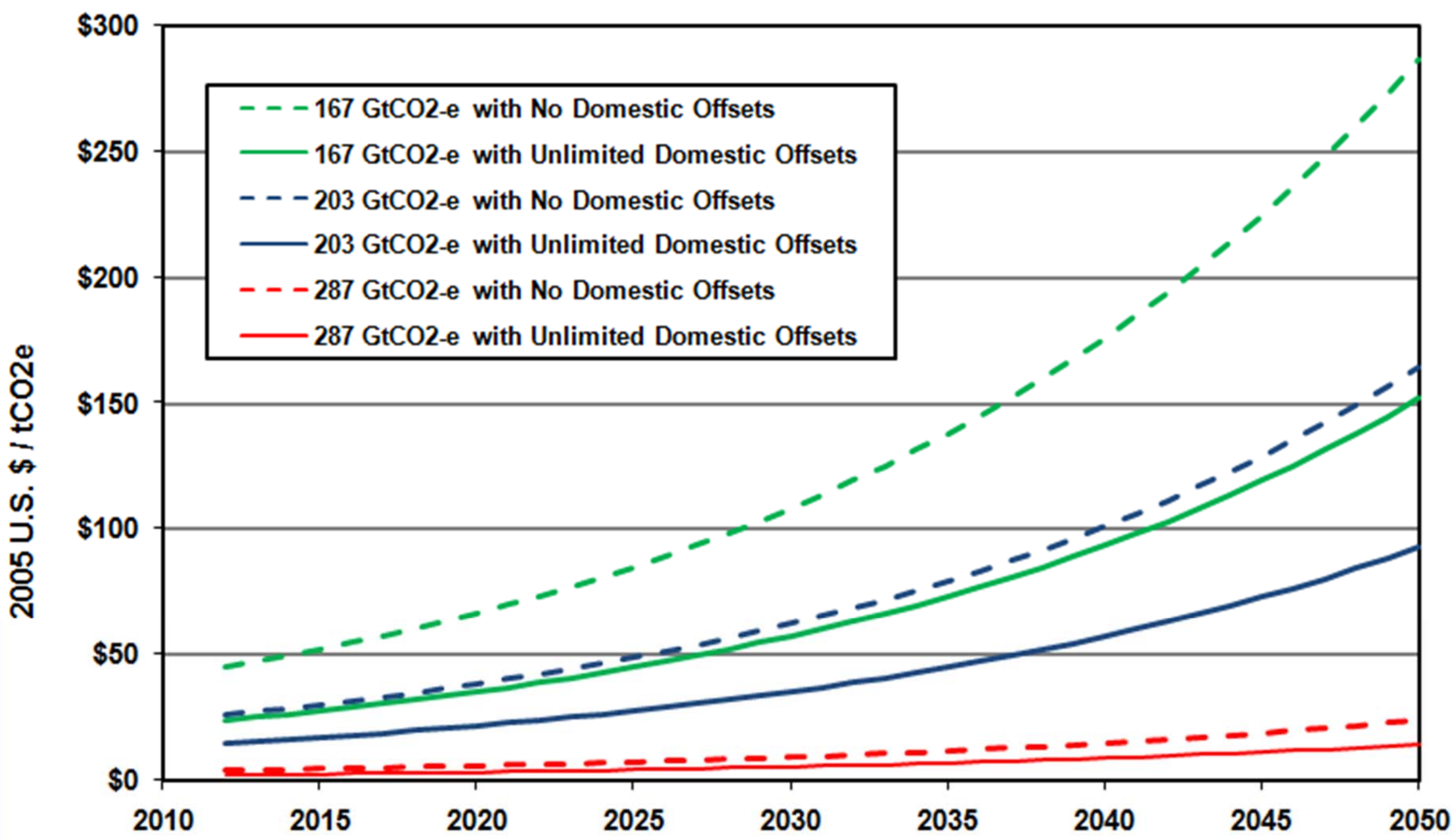
AGGREGATE IMPACTS OF CAP-AND-TRADE POLICIES

Establishing a Permit Price

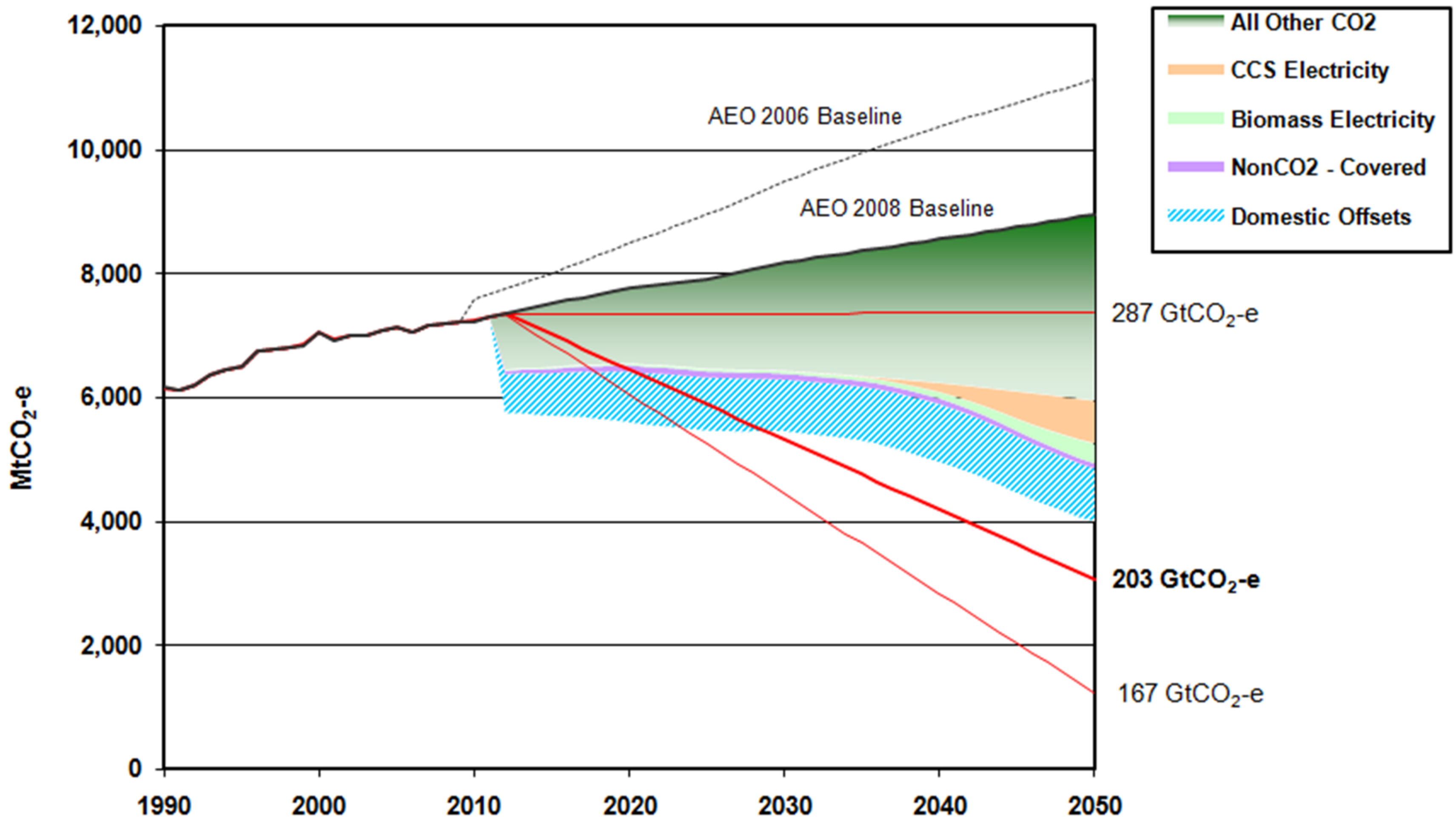
Emissions with and without Domestic Offsets

Impacts on GDP

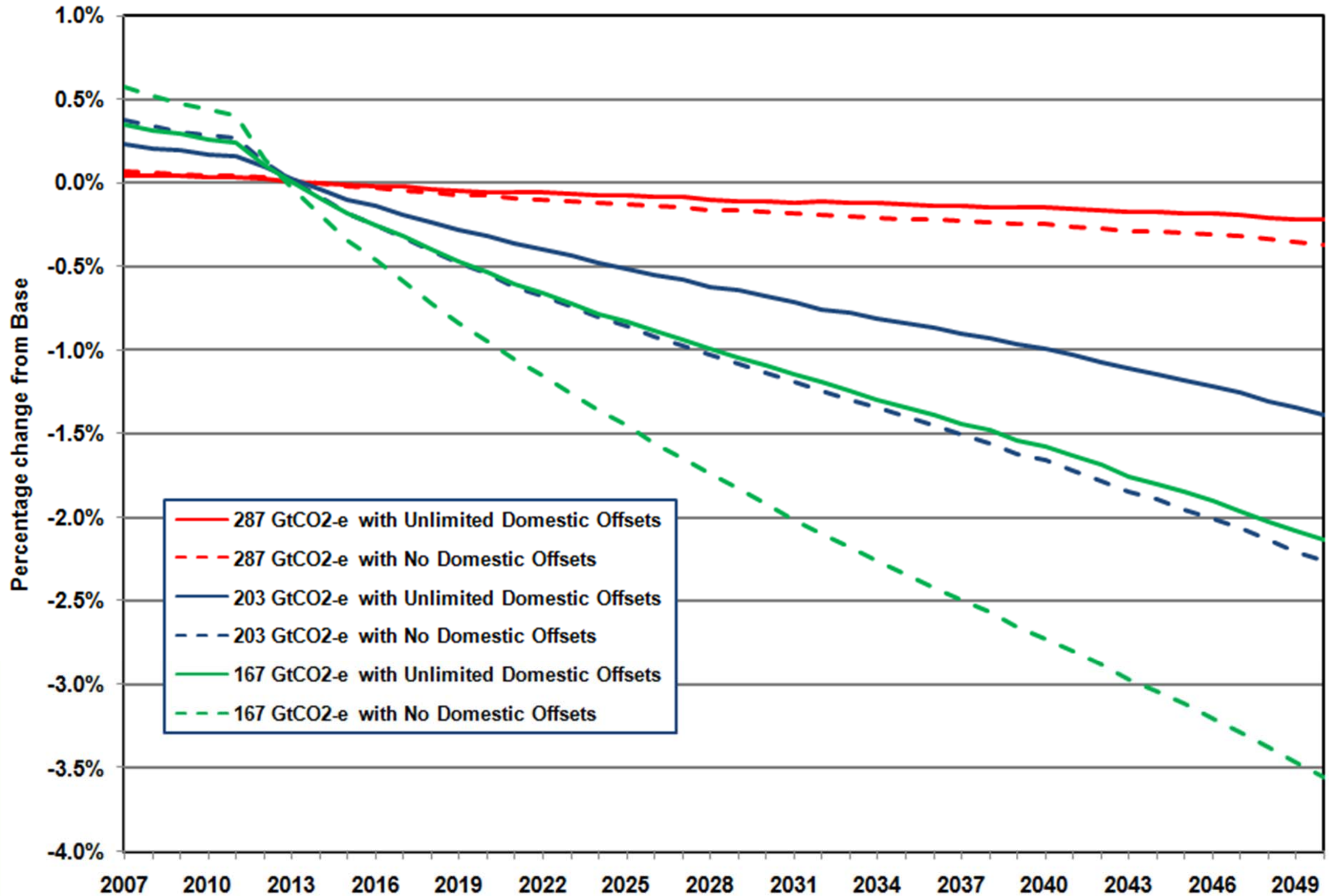
Impacts on Consumption



Allowance Prices



203 $GtCO_2$-e with Unlimited Domestic Offsets



Impacts on Real Consumption

INDUSTRY IMPACTS OF CAP-AND-TRADE POLICIES

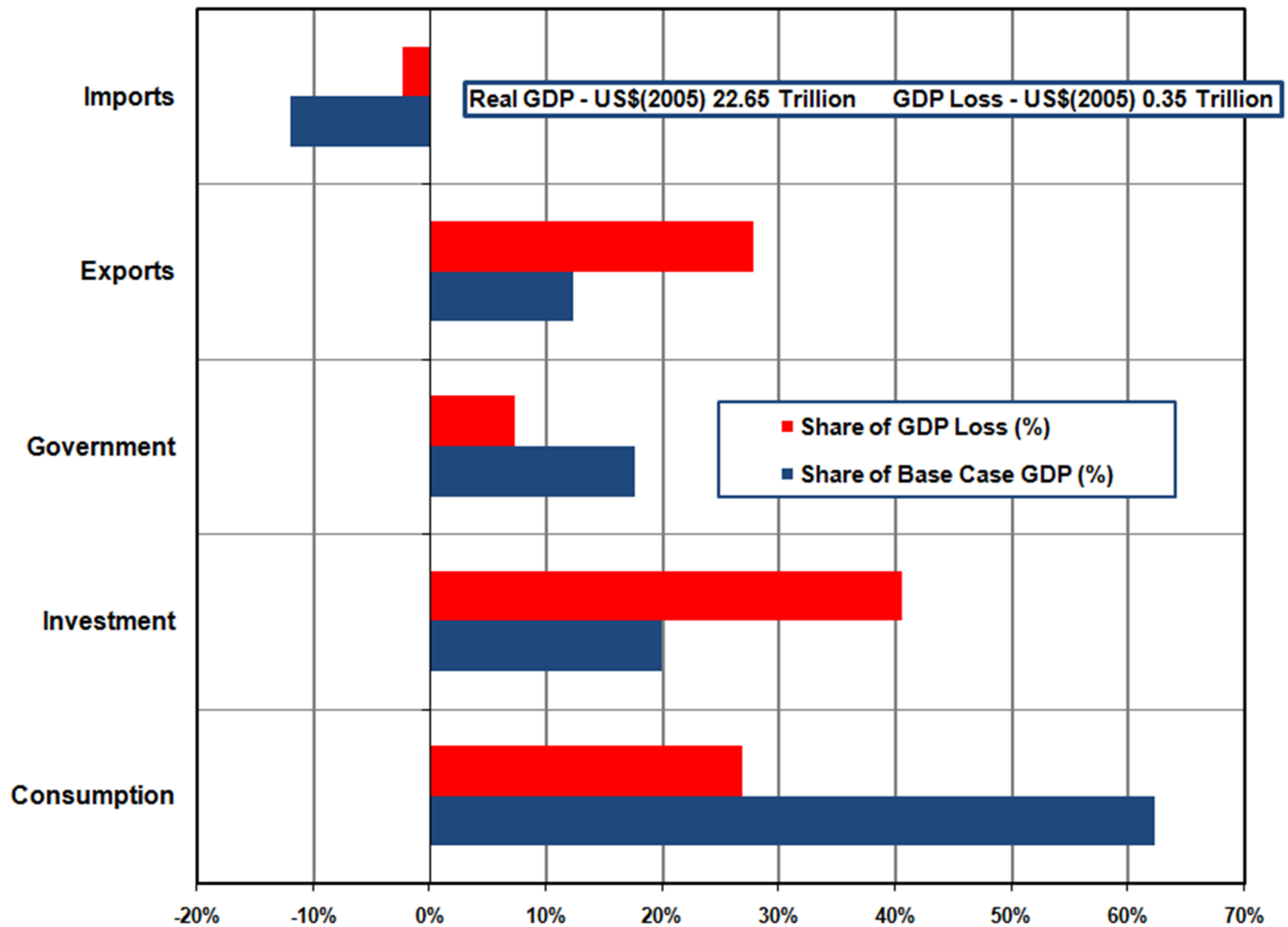
Changes in the Composition of GDP

Domestic Prices and Production

Composition of Output and Losses

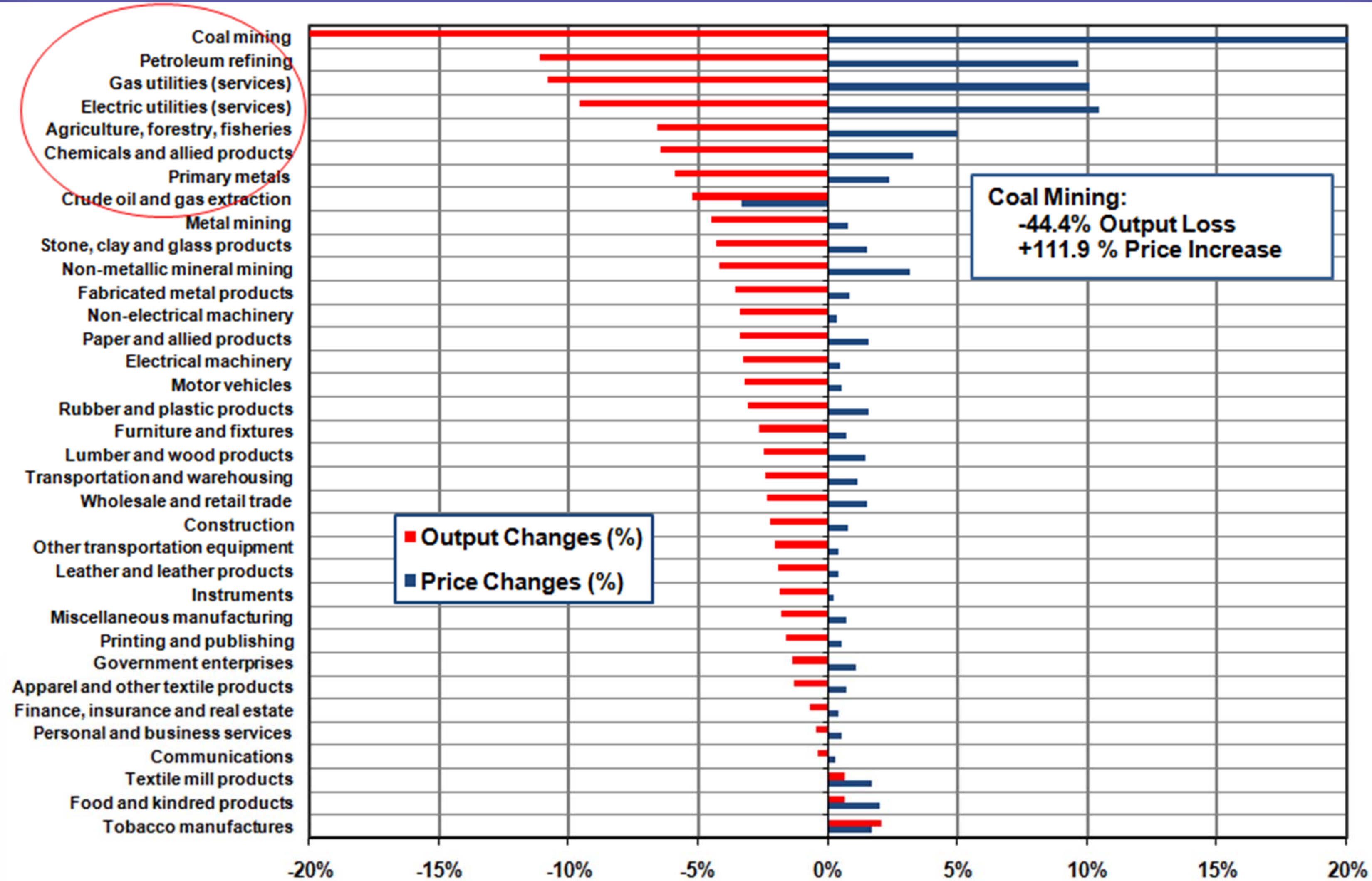
Revenues and Incomes

Input Demands and Labor Intensity



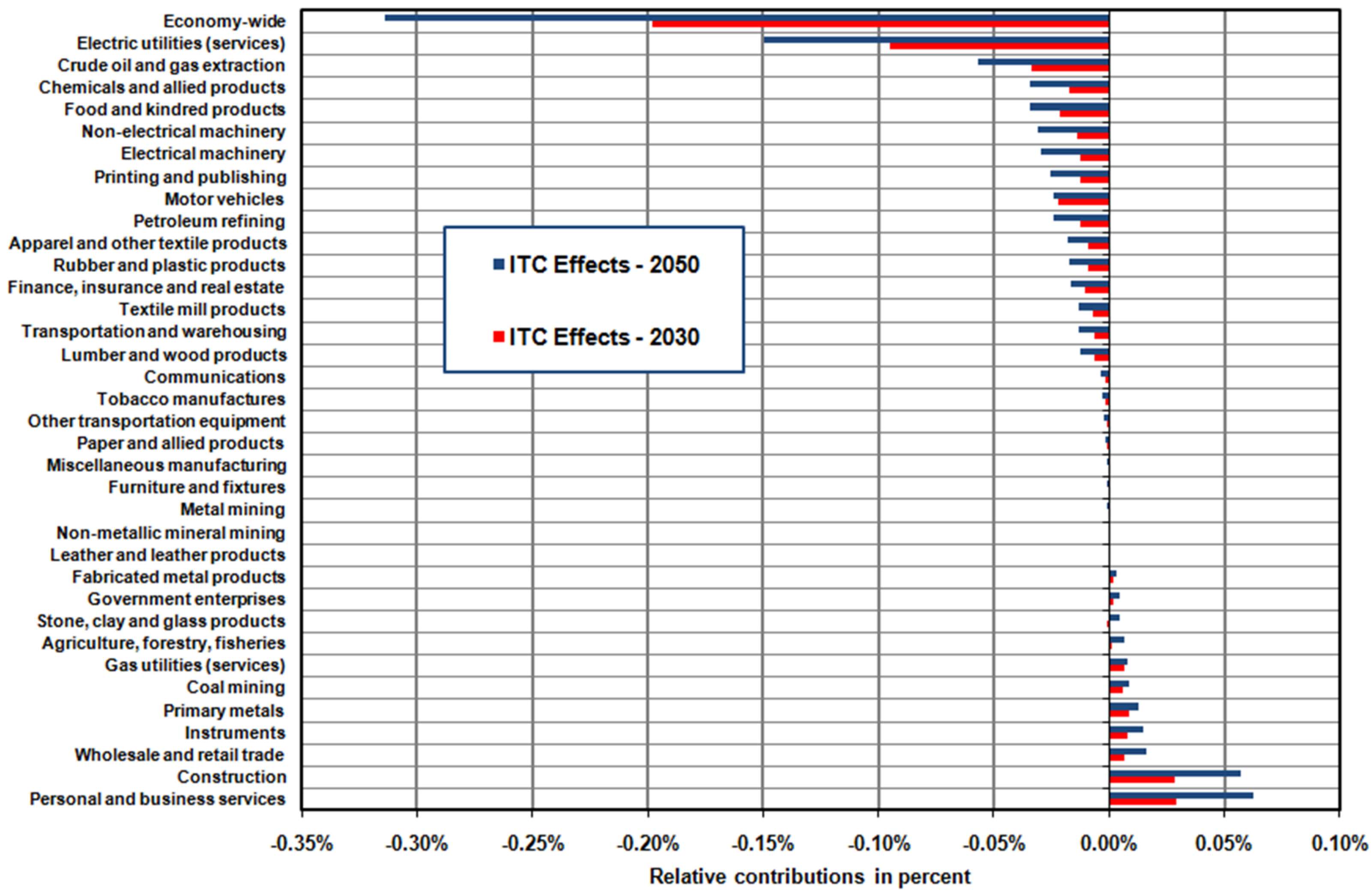
Composition of GDP and Losses, 2030

203 $GtCO_2$-e with Unlimited Domestic Offsets



Impacts on Domestic Prices and Production, 2030

203 $GtCO_2$-e with Unlimited Domestic Offsets



Endogenous Technical Change, 2030 and 2050

203 $GtCO_2$-e with Unlimited Domestic Offsets

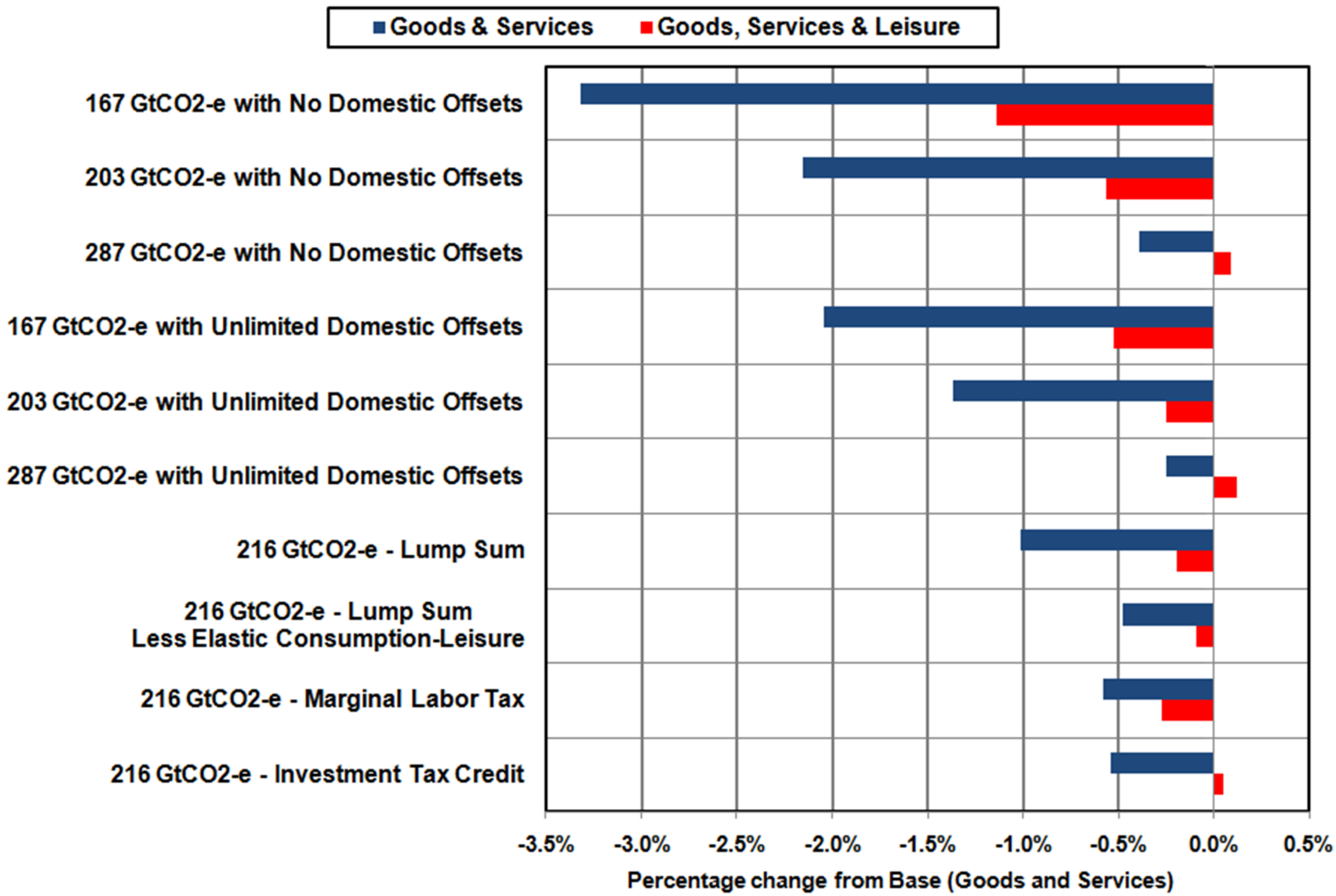
WELFARE IMPACTS OF CAP-AND-TRADE POLICIES

Equivalent Variations in Consumption

Equivalent Variations in Full Consumption

Revenue Recycling

The Role of Endogenous Technical Change



Equivalent Variations in Consumption and Full Consumption

AN ECONOMETRIC APPROACH TO GENERAL EQUILIBRIUM MODELING:

SUMMARY

Econometric Modeling of Producer Behavior: Substitution and Technical Change

Rate and Bias; Autonomous and Induced Technical Change

Econometric Modeling of Consumer Behavior: Demand for Goods and Leisure Intertemporal Allocation of Full Consumption

IGEM, Version 16: Controlling Emissions of Greenhouse Gases