WHY ROLL CALLS?
A MODEL OF POSITION-TAKING
IN LEGISLATIVE VOTING AND ELECTIONS

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Abstract

We develop a rationale for position-taking preferences in legislatures using a formal model of legislative vote-buying and elections. In our model, citizens and an interest group are motivated by policy, while legislators are motivated by holding office. The group may attempt to buy legislators’ votes by offering contracts based on their votes. If citizens cannot condition their re-election votes on legislators’ roll calls, then in equilibrium the group will buy its ideal policy and most legislators are voted out of office. If citizens can condition their votes on legislators’ roll calls, then policies are more moderate and more legislators are re-elected. Thus an endogenous preference for position-taking arises in a legislature with public roll calls, and both legislators and citizens will prefer such “open” proceedings ex ante.
1. Introduction

Members of the U.S. House and Senate devote much time and energy to “position-taking” activities—introducing and co-sponsoring bills, making speeches, and building roll-call records that are in tune with their constituents.\(^1\) These activities are not easily explained by appealing to members’ policy preferences, because they appear to engage in them regardless of the policy consequences. Members introduce bills that are sure to die in committee, they make long speeches on the floor even when the chamber is almost empty, and they carefully consider how to vote even on roll calls where the outcome is a foregone conclusion. Instead, it is widely believed that members engage in position-taking activities in order to improve their chances of reelection. And, numerous studies suggest that these efforts pay off. In particular, House and Senate members whose roll-call voting records do not fit their districts or are very extreme receive smaller vote-shares than other members.\(^2\)

Many rational choice theorists are puzzled by voters’ behavior in this story. Assuming that voters care about legislative outcomes, why should they punish or reward their individual representative at the polls depending on that representative’s roll call votes? Wouldn’t it be better for voters to punish or reward their legislators on the basis of what they care about, that is, outcomes?

This paper provides an explanation for voters’ behavior, based on vote-buying inside the legislature. The logic is as follows. Suppose that in addition to legislators and voters there is an interest group that faces no organized opposition. This group can observe roll calls and offer the legislators bribes based on their votes. If voters condition their voting decisions on outcomes, then they can induce outcome-based preferences in their legislators—that is, they can cause their legislators to act as if they had outcome-based preferences. However, if legislators only have outcome-based preferences, then the interest group can construct a set of bribes that will move policy anywhere it wants at virtually no cost. The group does this by exploiting the fact that it only needs the support of a majority of the legislature in order


to obtain its preferred policy.\textsuperscript{3} That is, outcome-based preferences are \textit{useless} for preventing an interest group from distorting policy in its preferred direction. In equilibrium, then, the group will “buy” policy equal to its ideal point.

If voters condition their voting behavior on their legislators’ voting records, then they can induce \textit{position-taking} preferences in their legislators—that is, they can cause their legislators to act as if they had position-taking preferences. And, if legislators have position-taking preferences, then the interest group must typically pay a large amount in bribes to move policy in its direction.\textsuperscript{4} This limits the extent to which an interest group distorts policy in equilibrium, resulting in legislative outcomes that most voters prefer to the group’s ideal point. Thus, voters can use position-taking-based voting strategies to prevent interest groups from dominating the legislature. In fact, they \textit{must} use such strategies.

To make the preceding argument precise, we employ a variant of the model in Snyder (1991), together with the insights on optimal bribes in Dal-Bo (2001). In addition to capturing the basic logic of the situation, the model generates several results that deserve mention.

First, unlike previous work where legislators have position-taking preferences, in our model legislators’ position-taking preferences are endogenous. Previous models focus on the legislative process, and simply \textit{assume} that legislators have position-taking preferences of some exogenously given form.\textsuperscript{5} We assume that players care about outcomes, and derive position-taking as part of the equilibrium, through voters’ electoral strategies and legislators’ desires to hold office. In the simplest case, the induced position-taking preferences imply that all legislators are equally costly for the lobbyist to bribe. This cost is larger, the more legislators value their offices.\textsuperscript{6}

Second, even when voters induce position-taking preferences, some legislators support the interest group’s legislation. Some do this because their constituency prefers it, while others take bribes and forfeit their seats (unless the value of office is very high). Thus, the

\textsuperscript{3}Snyder (1991) discusses this briefly. Dal-Bo (2001) lays out the argument fully, and characterizes a set of optimal bribes for the interest group. The argument does not require the legislature to operate under a simple majority rule, but holds for any supermajority rule short of unanimity.

\textsuperscript{4}This is shown in Snyder (1991), Groseclose (1996), Dal-Bo (2001), and elsewhere.

\textsuperscript{5}See, for example, Snyder (1991), Groseclose (1996), Groseclose and Snyder (1996), Diermeier and Myerson (1999), Dal-Bo (2001), and Groseclose and Milyo (2001). In addition, all of these papers except Snyder (1991) take the legislative agenda as exogenous.

\textsuperscript{6}This implies that the set of legislators supporting a group’s proposal, and the set of legislators opposing the group, need not be ideologically “connected” coalitions divided by a single cut-point.
median voter is typically unable to obtain her first-best outcome.

Third, legislators and citizens generally benefit from “open” proceedings. For any given interest, a majority of legislators will prefer the open legislature. With any uncertainty over the location of the interest group relative to the median, all legislators will prefer the open legislature ex ante. Moreover, risk averse voters also prefer the open legislature, since it produces more moderate outcomes. These results suggest that closed legislatures may not be sustainable in the long run. On the other hand, interest groups may be able and willing to pay bribes to prevent closed legislatures from becoming open.

Fourth, our model helps clarify how issue salience matters for legislative outcomes. Canes-Wrone (2001) argues that presidents “go public” in order to bring an issue to voters’ attention. This causes voters to pay more attention to what their congressmen are doing on the issue (e.g., how they vote), which gives interest groups less leverage over congress. Canes-Wrone assumes that groups have more leverage in the absence of this voter attention, but does not explain why this might be the case. Our model provides an explanation.

Before proceeding, three caveats concerning our analysis are in order.

First, we assume that there is only one organized group, or more generally, that all organized groups are on one side of the legislative median. Thus, our model is probably best applied to distributive issue areas such as agricultural policy (before the mid-1970s), veterans affairs, ship building construction subsidies and merchant marine operating subsidies, public funding of medical research, various tax preferences, public works projects such as water projects, airports and hospitals, and grazing on federal lands. It may also apply to some

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7See, for example, Ripley and Franklin (1980). In his analysis of agricultural policy making, Hansen (1991) notes: “By the 1960s, the farm lobby... had monopolized access to farm price support deliberations for an entire generation... The farm lobby faced no substantial competition for the loyalties of those in Congress who wrote the farm bills” (1991, p. 187). The farm lobby’s monopoly was not challenged until the 1970s. Wootton (1985) discusses veterans affairs, and quotes various observers: Steve Champlin, lobbyist for Vietnam Veterans of America, noted that the Veterans’ Administration is “a $21 billion world that nobody pays attention to;” and Bill Keller said veteran’s programs “are born and bred in an unusually closed system” (Wootton, 1985, p. 161). Oppenheimer (1975) and Wootton (1985) discuss the politics of the oil depletion allowance and “intangible” oil drilling expenses. As Wootton notes, “Several groups (AF of L, National Grange, American Bureau Federation) were from time to time mildly critical of percentage depletion, but systematic and sustained opposition was lacking” (1985, p. 255). With respect to the merchant marine, Price (1978) notes that while there is the potential for conflict, “the less-advantaged segments of the industry realized that, rather than questioning the benefits enjoyed by the subsidized lines, it was more fruitful to seek comparable benefits for themselves” (p. 558). See Strickland (1972) for an analysis of the narrow politics of medical research. Ferejohn (1974) describes the balance of forces on various rivers and harbors projects, and notes that flood control, water supply and beach erosion projects are particularly one-sided.
regulatory issues such as gun control and communications policy, at least during some eras.\(^8\)

Second, there are other ways to solve or partially solve the problems that arise due to the legislature’s lack of bargaining power vis-a-vis interest groups, even if voters are unable to induce position-taking preferences. Legislators might be able to collude, and raise their prices by acting as monopolists or oligopolists (Dal-Bo, 2001). This might take the form of “strong” parties, which we examine in Section 4.2. Alternatively, the legislature might endow some legislators with veto power or gatekeeping power over certain issues or types of bills, forcing interest groups to pay significant amounts to a subset of legislators (as in Diermeier and Myerson, 1999). This might take the form of strong committee chairmen. Or, the legislature might operate under unanimity rule. This is unlikely to be an attractive option, however, since unanimity rule is likely to be costly in other ways (e.g., Buchanan and Tullock, 1962). Competition between groups might also solve the problem. If there are two or more well-financed groups with preferences on opposite sides of the legislative median, then the cost of moving policy away from the median will typically be large (e.g., Snyder and Groseclose, 1996).

Third, there are other potential explanations for voter attention to position-taking activities. Voters might care directly about their members’ policy preferences, because they have veto power, gatekeeping power, or monopoly proposal power on some issues. Position-taking activities could then serve to signal these preferences (Snyder and Ting, 2002, 2003). Alternatively, they might care about their legislator’s preferences because he or she can use the legislative seat as a stepping stone to an executive office (in which capacity he or she will have veto power, gatekeeping power, or monopoly proposal power). Finally, Arnold (1990) argues that voters may focus on position-taking activities when deciding how to vote, because they are simply unable to evaluate the effect their legislator has on legislative outcomes. For scholars who agree with Arnold, our results are interesting because they show that voters may obtain the best attainable outcome even when they use strategies that appear unsophisticated.

\(^8\)In describing communications policy, Price (1978) notes that “the preeminence of the commercial broadcasters renders communications less effectively pluralistic than most of Commerce’s clientele centered areas” (p. 551). The distribution of PAC contributions provide one indication of how one-sided the gun control is: since 1990, gun rights groups (led by the National Rifle Association) have contributed ten times as much as gun control groups (see http://www.opensecrets.org/news/guns/).
The paper proceeds as follows. The next section lays out the two basic models of legislative vote-buying and elections. Section 3 derives the main results for both legislative organizations. Section 4 develops extensions for legislator policy preferences, “strong” parties, and repeated play. Section 5 concludes.

2. The Model

Our basic model has two variants, a closed one which does not feature publicly-known roll calls, and an open one which does. The only difference is that the latter allows voters to make their strategies contingent on their legislator’s voting record.\footnote{Alternatively, we could analyze the open game alone, and compare optimal voter strategies that condition on roll calls with sub-optimal strategies that do not. Our approach follows standard game-theoretic treatments, in which all players employ optimal strategies in whatever game they are playing.}

Environment and Players. In both games a legislature chooses a policy $x$ by majority rule from a convex set $X \subseteq \mathbb{R}$. Where convenient, we use $x^c$ and $x^o$ to denote the legislative outcomes of the closed and open games, respectively. There are three kinds of players: citizens, an interest group, and legislators. Each citizen $i$ cares about $x$ according to the single-peaked utility function $u^i : X \rightarrow \mathbb{R}$. We assume that the these utility functions satisfy the Gans-Smart single-crossing conditions. Citizens are divided into $N$ districts, with $M_j$ citizens in district $j$ ($M_j$ and $N \geq 3$ odd). Districts are denoted by the set $D \equiv \{d_j\}_{j=1,\ldots,N}$. Let $m_j$ denote the ideal policy of the median citizen in district $j$, and $m$ the median of $\{m_j\}$. For convenience we index districts in increasing order of $m_j$. In the model all agendas are binary, containing $m$ and some alternative $y$. This may be interpreted as the consequence of open amendment rule, or a status quo of $m$. We assume that citizens break ties in favor of $y$.

The interest group, $G$, also cares about outcomes and may attempt to influence legislators’ behavior by offering a payment, or bribe, for voting a certain way. These payments may represent campaign contributions, promises of future employment, or (in some environments) direct income transfers.\footnote{If the group is interpreted as being another legislator, then the payments could also represent within-chamber benefits such as committee membership or future supporting votes.} Let $l \in \{0, 1\}^N$ denote a roll call, where 1 denotes a vote for $m$. Let $b^j(l) \geq 0$ denote the bribe offered by $G$ to legislator $j$ under $l$. This bribe may depend
on the entire roll call, rather than only that legislator’s vote. G then receives
\[ u_G(x, g) - \sum_{j=1}^{N} b^j(l), \]
where \( u_G : X \times X \rightarrow \mathbb{R} \) is single-peaked in \( x \) and attains a maximum at \( g \in X \).

Legislators each represent a single district. We assume that they value both holding office \textit{per se} (and thus re-election) and bribes possibly offered by G. Legislator \( j \) receives \( wr^j + b^j(l) \), where \( w > 0 \) represents a fixed benefit from holding office (\textit{i.e.}, getting re-elected) and \( r^j \in \{0, 1\} \) indicates whether she is re-elected. Note that legislators do not have policy preferences.\(^{11}\)

\textit{Sequence.} The sequence of play is as follows.

1. \textit{Interest Group Draw.} Nature draws an interest group with ideal point \( g \sim F \), where \( F \) is a distribution on \( X \).

2. \textit{Vote Buying.} G offers an alternative \( y \in X \) to \( m \) and, for each legislator, a schedule \( b^j(l) \) of payments for voting for \( y \). All players observe \( y \). Legislators observe \( b \), the vector of bribe schedules, while citizens do not.

3. \textit{Legislative Voting.} Each legislator simultaneously casts a vote \( l^j \in \{0, 1\} \) over the agenda. G observes \( l \) and delivers payments according to \( b^j(l) \). The outcome \( x \) is determined by majority rule, where \( x = m \) if and only if \( \sum_j l^j \geq (N+1)/2 \). Citizens observe \( l \) only in the open game. All players observe \( x \).

4. \textit{Election Voting.} In each district, each citizen simultaneously casts a re-election vote \( c^i \in \{0, 1\} \), where 1 denotes a vote for the incumbent. The outcome is determined by majority rule, where \( r^j = 1 \) if and only if \( \sum_{i \in d_j} c^i \geq (M_j+1)/2 \).

Note that G would never need a bribe to achieve an outcome of \( m \). Thus bribes over the binary agenda can be characterized by a single payment (for \( y \)) for each possible roll call. Of course, G is assumed to be able to commit credibly to any promised bribe vector.

\textit{Strategies.} G’s strategy is a pair \( \{y, b\} \) consisting of a policy alternative \( y \in X \) and an \( N \)-dimensional bribe vector \( b \) with components \( b^j \in B \equiv \{\beta : \{0, 1\}^N \rightarrow 0 \cup \mathbb{R}_+\} \) mapping roll

\(^{11}\)As we will see in Section 4.1, policy preferences do not change the results of the basic model.
calls into non-negative payments for voting for $y$. Legislator $j$’s strategy $b^j : X \times B^N \rightarrow \{0, 1\}$ maps $y$ and the bribe schedules into a vote. Citizen $i$’s voting strategy in the open game is a function $c^i : \{0, 1\}^N \times X \rightarrow \{0, 1\}$ mapping the legislative voting record into vote for the incumbent or challenger in his district. Since the voting record is invisible to citizens in the closed game, strategies there are simply $c^i : X \rightarrow \{0, 1\}$.

### 3. Main Results: The Closed and Open Games

For both games, we characterize subgame perfect Nash equilibria, using weakly dominant strategies where possible. A further refinement is necessary since citizens are indifferent among all voting strategies, as legislators choose policy prior to the election. We therefore impose the following pivotal voting requirement: for citizen $i$, $c^i$ is chosen to maximize the legislator’s incentive to vote as the citizen would in the (possibly out of equilibrium) event that he is pivotal in that period’s election, given the information available to him. Thus, citizens preferring $m$ to $y$ will vote to re-elect their legislator if and only if the outcome is $m$ in the closed game, and if and only if the legislator voted for $m$ in the open game. We will refer to equilibria satisfying these requirements as PVSPE.

Loosely speaking, this refinement may be justified as follows. Consider a sequence of ‘trembled’ versions of each game in which each legislator and each citizen in her district are pivotal in their respective votes with positive (and diminishing) probability. Then the refinement is equivalent to the optimal voting contracts for citizens to offer to legislators. Alternatively, it selects the equilibrium that is optimal for pivotal legislators and voters (i.e., legislators are re-elected, and voters receive their optimal policy) in the trembled games. The pivotal voting rules then represent the limit of these contracts or equilibria as the trembles diminish to zero.

It will be convenient to define for each proposal $y$ a ‘supporting’ set $P(y)$ of districts with median voters that prefer $y$ to $m$. Formally,

$$P(y) = \left\{ d_j \mid |\{ i \in d_j \mid u^i(y) \geq u^i(m)\}| \geq (M_j+1)/2 \right\}.$$

The single-peakedness of voter preferences implies that if $y < m$, then $P(y)$ contains the ‘leftmost’ districts when it is non-empty (i.e., those with median voters $m_k$ for all $m_k \leq y$).

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12This might be implemented by having G tremble by not offering a bribe, and each legislator and citizen tremble by abstaining independently with positive probability.
$P(y)$ may also contain a district $k$ with median voter $m_k \in (y, m)$. It cannot contain any district $k$ with $m_k \geq m$. A symmetric logic holds for $y > m$. Thus, $0 \leq |P(y)| < (N+1)/2$ for all $y \neq m$.

3.1 The Closed Game

Since roll calls are not observable, the pivotal voting strategy for citizen $i$ (who lives in district $j$) is:

\[
e^i = \begin{cases} 1 & \text{if } x^c = y \text{ and } u^i(y) \geq u^i(m) \text{ or } x^c = m \text{ and } u^i(m) > u^i(y) \\ 0 & \text{otherwise.} \end{cases}
\]

(1)

Each district will re-elect its legislator if and only if the legislative outcome is preferred by a majority of citizens there. Because citizen preferences are single-peakedness and satisfy single-crossing, re-election occurs if the district median prefers the outcome to the defeated alternative. Since a majority of districts will prefer $m$ to any $y$, a majority of legislators will remain in office only if $y \neq m$ is defeated.$^{13}$

In the absence of bribes, these voting strategies present the legislator representing district $j$ with a unique weakly dominant strategy: $l^j = 0$ (i.e., vote for $y$) if and only if her district is in $P(y)$. This ensures that $m$ prevails against any $y$, and that legislators are defeated if and only if they represent districts in $P(y)$.

$G$ will therefore offer bribes if it can achieve a better outcome than $m$ at a sufficiently low cost. Suppose $b$ takes the following form for any $y$:

\[
b^j(l) = \begin{cases} w + \eta & \text{if } l^j = 0 \text{ and } \sum_{k \neq j} l^k = \frac{N-1}{2} \\ \epsilon & \text{if } l^j = 0 \text{ and } \sum_{k \neq j} l^k \neq \frac{N-1}{2} \\ 0 & \text{if } l^j = 1, \end{cases}
\]

for some $\eta > 0$ and $\epsilon > 0$.$^{14}$ That is, $G$ promises a payment large enough to compensate legislator $j$ for losing the election if and only if she votes for $y$ and is pivotal. Those voting against their district and not pivotal receive only $\epsilon$.

Given the voters’ responses, all legislators vote for $y$ (i.e., $l^j = 0$ for all $j$). To see why, note that no legislator is ever pivotal under this contract. Suppose that exactly $(N-1)/2$ legislators are voting for $y$. Then any legislator voting for $m$ would switch in order to collect

$^{13}$By (1), legislators are also re-elected if $y = m$, in which case $y$ technically defeats $m$.

$^{14}$These strategies are of the same form as those explored in Dal-Bo (2001). We employ a different refinement argument than he does.
$w + \eta$ and be removed from office rather than receive $w$ from re-election or 0 from being voted out. But if the number of legislators voting for $y$ is not $(N-1)/2$, then any legislator voting for $m$ would not be pivotal. She could switch her vote to $y$ and receive $\epsilon$ instead of zero if $y$ passes, or $w + \epsilon$ instead of $w$ if $y$ fails. Importantly, this logic holds for any $y$. Letting $y = g$ and $\epsilon \to 0$, $G$ can therefore achieve its ideal policy while paying zero bribes.$^{15}$

While the bribe defined by (2) is one of the simplest that works for any $y$, other bribes may also achieve the same result. It is easy to show, however, that the payoffs from the subgame induced by (2) are uniquely optimal for $G$. The following proposition summarizes the result.

**Proposition 1.** Policy and bribes in the closed game. In a PVSPE, $x^c = y^* = g$ and $b^{ij*} = 0$ for all legislators. ■

**Proof.** All proofs are in the Appendix. ■

Thus, in a world without roll calls, it is surprisingly easy for the group to receive its ideal policy. This occurs because $G$ has access to more observables than citizens, as well as the ability to take into account the externalities generated by legislators’ actions. Intuitively, these allow $G$ to write much more precise contracts to legislators than citizens can. The equilibrium clearly benefits districts with median voters that are at least as extreme as $g$ along with their legislators. But a majority of voters in a majority of districts (i.e., those not in $P(g)$) do worse in this world than one in which no bribes can be offered.

### 3.2 The Open Game

The stark results of the previous section are generated in part by the lack of observables for citizens to ‘contract’ on. We now turn to the case where roll calls can be used to serve this purpose.

As in the closed game, citizens are indifferent among all voting strategies. With observable roll calls, the pivotal voting strategy for citizen $i$ (who lives in district $j$) is:

$$c^i = \begin{cases} 1 & \text{if } l^j = 0 \text{ and } u^i(y) \geq u^i(m) \text{ or } l^j = 1 \text{ and } u^i(m) > u^i(y) \\ 0 & \text{otherwise.} \end{cases}$$

$^{15}$At $\epsilon = 0$, there are multiple equilibria (which feature minimum winning coalitions), but the one identified here is the unique one that features weakly dominant legislator voting strategies.
Thus, each district conditions on its legislator’s vote (and not the outcome), and re-elects her if and only if she votes for its median voter’s preferred policy.\textsuperscript{16} This implies that each legislator will have a reservation value of \( w \). Without bribes, legislators always vote with their district medians, \( m \) always prevails, and all legislators are re-elected.

Consider any arbitrary proposal \( y \). Legislators representing districts in \( P(y) \) have a dominant strategy of voting for \( y \), while given (3), others require a sufficiently large compensation for losing office. Thus to win the vote of a legislator in a district \( j \) where \( d_j \not\in P(y) \), \( b^j(l) \) must assume the following form:

\[
b^j(l) = \begin{cases} 
  w + \eta & \text{if } l = 0 \\
  0 & \text{otherwise}, 
\end{cases}
\]

for some \( \eta > 0 \). Legislator \( j \)'s best response to such a proposal \( \{y, b\} \) is therefore:

\[
l_j = \begin{cases} 
  0 & \text{if } d_j \in P(y) \text{ or } b^j(l) > w \\
  1 & \text{otherwise}. 
\end{cases}
\]

To achieve \( y \), \( G \) therefore must “buy” the votes of any combination of \( \left(\frac{N+1}{2} - |P(y)|\right) \) legislators, at a price of \( w+\eta \) each. \( G \) therefore solves:

\[
\arg \max_y u_G(y, g) - (w+\eta) \left[ \frac{N+1}{2} - |P(y)| \right].
\]

Clearly, the solution to this problem lies between \( m \) and \( g \). Suppose without loss of generality that \( g > m \). Two observations follow. First, \( P(y'') \subseteq P(y') \) for any \( y', y'' \in [g, m] \) and \( y'' > y' \), implying that \( |P(y)| \) is a weakly decreasing, integer-valued step function on \( [m, g] \), ranging from \( (N+1)/2 \) to \( |P(g)| \). Second, by single-peakedness, \( u_G(\cdot) \) is increasing on \( [m, g] \). It then follows that for any set of policies such that \( |P(y)| \) is constant, the optimal policy alternative for \( G \) to propose is either the maximum of that set, or \( g \) if it is contained within. Thus:

\[
y^* \in \left\{ g, \max \{ y \mid |P(y)| = k \} \right\}_{k=|P(g)|+1, \ldots, (N+1)/2}.
\]

The optimal policy proposal depends on \( w \) and the configuration of district median voter preferences.\textsuperscript{17} For \( w \) sufficiently large (e.g., \( w > u_G(g) - u_G(m) \)), \( G \) will propose \( y = m \)

\textsuperscript{16}With public roll calls many other voting strategies are possible, though none can do better than a pivotal voting strategy. For example, if all citizens conditioned on whether the legislator is pivotal, then \( G \) could achieve a result similar to that of the closed game.

\textsuperscript{17}The solution to this problem may not be unique. However, any configuration of preferences that leads to multiple solutions is not robust to small perturbations in voter preferences.
and zero bribes.\footnote{Note that if \(y = m\), by (3) all legislators vote for \(y\) and are re-elected.} Otherwise, as \(w\) decreases \(y^*\) moves closer to \(g\) and some legislators are bought. As \(w\) approaches zero, \(G\) proposes \(g\). Note that the legislators bought need not be from those districts with median voters nearest to \(g\). Since legislators do not have policy preferences, all legislators representing districts in \(D \setminus P(y)\) are equally attractive as coalition partners. However, voting records will be monotonic in district ideology if \(P(y)\) is non-empty and \(G\) randomizes among coalition partners in \(D \setminus P(y)\).

Letting \(\eta \to 0\), the following analog of Proposition 1 is easily established.

**Proposition 2.** Policy and bribes in the open game. In a PVSPE, \(x^o = y^* \in \begin{cases} [m, g] & \text{if } g \geq m \\ [g, m] & \text{if } g < m, \end{cases}\) and \(b^i = w\) for \((N+1)/2 - |P(y^*)|\) legislators. \(\blacksquare\)

Except in the limiting case where \(w\) is vanishingly small, the equilibrium legislative outcome and bribes differ substantially from those of the closed game. \(G\) must pay a strictly positive cost to achieve any non-median outcome. As a result, outcomes are closer to \(m\) in the open game than in the closed game. This leads to the following observation about citizen and legislator welfare in the two games.

**Comment 1.** For any \(g\): (i) A majority of voters in a majority of districts receive (weakly) higher payoffs in the open game than in the closed game; and (ii) All legislators receive (weakly) higher payoffs in the open game than in the closed game. \(\blacksquare\)

In the presence of a vote-buyer, citizens and legislators both tend to do better with open proceedings. By using the observable roll call votes in their voting strategies, citizens can induce a preference for ‘position-taking’ by their representatives. This effectively shields legislators from the externalities caused by other legislators’ votes, and makes contracts of the form in equation (2) impossible. As a result, all legislators in the open game expect to receive at least their value of re-election, \(w\). This raises the price of any non-median policy, thereby pushing outcomes closer to \(m\).

Note that Comment 1 speaks only to revealed values of \(g\). It is also useful to compare expected payoffs prior to the draw of \(G\). Given \(g\), not all legislators do strictly better in the open game because any legislator representing a district in \(P(g)\) would be re-elected with
zero bribes in both. In general, if the support of $F$ were such that a legislator’s district is always in $P(g)$, then that legislator would also receive identical payoffs in the two games. This condition cannot hold for any legislator, however, if $m$ is in the interior of $C_F$, the convex hull of the support of $F$. Thus under a wide variety of distributional assumptions legislators will have an *ex ante* preference for open proceedings. Additionally, sufficiently risk averse citizens also benefit *ex ante* when $m$ is in the interior of $C_F$.

Given the strong incentive for legislators (and voters) to adopt public roll calls, how might closed proceedings persist? The framework developed here suggests several possibilities. First, in some situations the benefits of open proceedings are limited. This is true if all interest groups are moderate; if competition between multiple interest groups raises the prices that legislators can charge for their votes; or if disciplined parties exist that charge monopoly prices. Second, supermajoritarian procedures over the choice of rules might obstruct the adoption of open proceedings. Finally, groups might buy votes at the stage where the choice of procedures is made. If voting at this stage is closed, then by Proposition 1 the same externality that allows groups to buy winning coalitions at zero cost allows them to keep the procedure closed at zero cost.

### 3.3 Numerical Example

The following simple numerical example illustrates the equilibria of the open game. Let $X = \mathbb{R}$ and $N = 11$. Assume that utilities for district median voters are quadratic, with $u^j(x) = -(x-j)^2$ in district $j$, so that $m_j = 0, 1, \ldots, 10$. G’s utility is also quadratic, with $u^G(x) = -(x-g)^2 - \sum_{j=1}^{N} b^j(l)$. The following table shows the equilibrium policy and bribe amounts as $g$ and $w$ vary.
Table 1
Equilibrium Policy and Bribes in the Open Game

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Since the median voters’ utility functions are symmetric, all districts such that \( m_j > (y+m)/2 \) (\( m_j < (y+m)/2 \)) will be in \( P(y) \) if \( y > m \) (\( y < m \)). Thus, all policies in the interval \((5+2k, 7+2k]\) for \( k = 0, \ldots, 4 \) command the support of exactly \( 5 - k \) legislators in the absence of bribes, and thus all “cost” \( w(k+1) \). \( G \) therefore chooses among \( m, g \), and the policy closest to \( g \) on each such interval contained in \( [m, g] \). Note that equilibrium policy approaches \( m \) as \( w \) increases, but bribes are not monotonic in \( w \). This is because an increase in \( w \) increases the cost of any given non-median policy, but also causes \( G \) to seek a more moderate position.

4. Extensions

4.1 Policy- and Office-Motivated Legislators

Given that the analysis above is simplified by the lack of policy preferences (thus creating a uniform “price” for each office), it is natural to ask whether the results are robust to the introduction of policy-motivated legislators. Thus, let legislator \( j \) receive \( wr^j + u_L^j(x) + b^j(l) \), where \( u_L^j : X \rightarrow \mathbb{R} \) is single-peaked and attains a maximum at \( m_j \). In other words, legislator \( j \)’s preferences match those of the median voter in her district.

It is easy to see that this specification does not change the result of the closed game: since the contract (2) causes no legislator to be pivotal, the policy component of each legislator’s utility is irrelevant to her voting decision. More importantly, the result of the open game is
also essentially unchanged. There are two cases. First, G may buy a coalition of \((N+3)/2\) (i.e., one more than a majority) legislators. Since no legislator is pivotal in this case, policy utility is again irrelevant and each legislator can be bought at price \(w\). Second, if \(w\) is sufficiently high, then G may prefer simply to compensate the least expensive \((N+1)/2 - |P(y)|\) legislators representing districts in \(D \setminus P(y)\) by the difference in utility between \(m\) and \(y\).

In both cases, policy utility increases somewhat the equilibrium cost of a winning coalition, thereby moderating policies relative to the open game.

**Comment 2.** With policy- and office-motivated legislators, the equilibrium outcomes and proposals satisfy: (i) \(x^c = y^* = g\) in the closed game; and (ii) \(x^{o'} = y^* \in \begin{cases} [m, x^o] & \text{if } g \geq m \\ [x^o, m] & \text{if } g < m \end{cases}\) in the open game.

Thus, the qualitative features of equilibria of the open and closed games are unchanged by policy-motivated legislators. However, the possibly greater moderation of policy further increases the desirability of open proceedings for most citizens and legislators in most districts.

### 4.2 Strong Parties

One way for legislators to gain leverage against the interest group is to raise their prices through collusion. Here we examine the possibility that cohesive “parties” can play this role. We restrict attention to the closed game, but briefly consider the open game later.

To keep the analysis tractable, we make very simple assumptions about the composition and behavior of parties. A party is an exogenous collection of legislators whose objective is to maximize the sum of its members’ payoffs (either through re-election or bribes). Each legislator may belong to at most one party, and each party must contain at least two members. We index parties by \(\pi\), and let \(N_\pi\) denote the size of party \(\pi\). Each party maintains perfect discipline, so that all members vote identically on any roll call. Thus, the closed game remain unchanged, with the exception that legislator \(j\) has no voting strategy \(l^j\) if she belongs to a party (say \(\pi\)). Instead, party \(\pi\) has a voting strategy \(p^\pi : X \times B^N \rightarrow \{\{0\}^{N_\pi}, \{1\}^{N_\pi}\}\) mapping a proposal \(y\) and a bribe vector \(b\) into \(N_\pi\) votes for or against \(y\).

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19 In this case, unless \(u^L_j(m) - u^L_j(y)\) are identical for all \(j\) and \(y\), G may not be indifferent among all coalition partners.
The analysis of the game is simplified by the following observation: without a majority party, equilibrium bribes and legislative outcomes are identical to those in Proposition 1. The result continues to hold because G can, by offering a bribe schedule identical to (2), cause no legislator or party to be pivotal for any proposal $y$. Each party $\pi$ will then vote for $y$ because doing so will result in a payment of $N_\pi \epsilon$ more than voting for $m$. A majority party is always pivotal, and thus can charge a non-zero price for delivering its votes. We thereby lose no generality by assuming the existing of a single, majority party that contains legislators from districts $D_{\pi}$.

To build a winning coalition, $G$ clearly needs only to secure the votes of the party. In the absence of bribes, for any proposal $y$, the party receives $\phi_{\pi}(y)w$ for voting for $y$, and $(N_\pi - \phi_{\pi}(y))w$ for voting for $m$, where $\phi_{\pi}(y) = |P(y) \cap D_{\pi}|$ is the number of districts represented by party legislators that prefer $y$ to $m$. The total cost to $G$ of receiving policy $y$ is therefore:

$$
\begin{cases}
0 & \text{if } \phi_{\pi}(y) \geq N_\pi/2 \\
(N_\pi - 2\phi_{\pi}(y))w & \text{if } \phi_{\pi}(y) < N_\pi/2.
\end{cases}
$$

As (7) makes clear, a cohesive majority party can raise the price of a winning coalition substantially. Two examples with legislators $1, \ldots, (N+1)/2$ belonging to the party illustrate the range of party influence. First, if $g > m$, then $\phi_{\pi}(y) = 0$ for any $y > m$ and $G$ must pay $(N+1)w/2$ to achieve any such $y$. $G$ therefore chooses between receiving $m$ at zero cost or $g$ at cost $(N+1)w/2$. Note that ideological diversity in the party lowers $G$’s cost, in the sense that adding a legislator from a district favoring $g$ over $m$ would lower the cost by $2w$. Second, if $m(N-1)/2 < g < m$, then $\phi_{\pi}(y) \geq N_\pi/2$ and the party will vote for $g$ without payment. It does so even though the legislator representing the median district will not be re-elected, because most party members will be re-elected.

These examples are special cases of the following result, which applies (7) to establish the conditions under which Proposition 1 holds.

**Comment 3.** In the closed game with a cohesive majority party, the equilibrium legislative outcome $x_{\pi}^e$ satisfies: (i) $x_{\pi}^e = g$ and $b_{j}^* = 0$ for all legislators if and only if $|\{d_j \in D_{\pi} \mid m_j \leq g \leq m \text{ or } m_j \geq g \geq m\}| \geq N_\pi/2$; and (ii) $x_{\pi}^e \in [g, m] \text{ or } x_{\pi}^e \in [m, g]$. 

Thus $x_{\pi}^e$ is always at least as moderate as $x^e$, and $g$ may be attained at zero cost only
if a majority of party district medians is both more extreme than G and on the same side as \( m \).\(^{20}\) Otherwise, the party can effectively charge a price that would allow it to receive its reservation utility (of approximately \( N \pi w/2 \)). This gives G an incentive to moderate its policy proposal. Under some conditions, G may settle for \( m \), though it may also “buy” \( g \) if it cared enough about policy.

The analysis here suggests three rationales for party cohesion in a world with closed proceedings. First, most voters weakly benefit from the more moderate legislative outcomes. Second, this moderation also weakly improves all legislators’ re-election prospects. These benefits exist even in districts not represented by the party. Finally, when bribes are strictly positive parties may redistribute them to their losing incumbents.

We note finally that party cohesion would not be as appealing to legislators in the analogous open game. This is because each legislator in the open game can guarantee herself a payoff of \( w \) when acting independently, while a party can collectively guarantee its members only about \( N \pi w/2 \). The model therefore provides one reason for expecting reduced party cohesion after roll calls are made public, or after voters pay more attention to roll calls.

4.3 Repeated Play

The infinitely repeated variants of the closed and open games serve two purposes. First, they establish that the strategies of the one-shot games are supportable as equilibria in an infinitely repeated game. In particular, they establish that pivotal voting strategies are optimal for securing long run policy benefits for citizens. Second, the policy results provide an additional rationale for open proceedings.

In both the closed and open games, each period is structurally identical to the corresponding stage games. We therefore preserve the notation of the previous sections, but now index periods with a subscript \( t \) where appropriate. Legislators are assumed to live for as long as they are re-elected, and each defeated legislator is replaced by a new one in the subsequent period.\(^{21}\) Citizens are infinitely-lived, while groups survive for only a single period each. The ideal points \( \{g_t\} \) of the groups \( \{G_t\} \) are i.i.d. draws from \( F \). In each period, all players receive the same utilities as in the one-shot game, and all players discount future

\(^{20}\)The party cannot induce an outcome more extreme than \( g \) because it lacks proposal power. In equilibrium, G exploits the fact that a party that preferred such an outcome would also prefer \( g \) to \( m \).

\(^{21}\)Recall that all legislators have identical utility functions, so the replacement mechanism is immaterial.
payoffs by a common factor $\delta$.

Strategies are now defined as follows. Let $H^P_t$ represent the history of publicly observable moves (i.e., group draws, citizen votes, and policy and election outcomes) prior to period $t$. Similarly, let $H^b_t$ and $H^l_t$, represent the histories of group offers and legislator votes, respectively. $G_t$’s strategy maps the entire game history into a policy and bribe vector: $y_t : H^P_t \times H^b_t \times H^l_t \to X$ and $b_t : H^P_t \times H^b_t \times H^l_t \to B^N$. Legislator $j$’s strategy $l^j_t : H^P_t \times H^b_t \times H^l_t \times X \times B^N \to \{0,1\}$ maps the histories and $G_t$’s current proposal into a vote. Citizen $i$’s voting strategy in the open game $c^i_t : H^P_t \times H^l_t \times \{0,1\}^N \times X \to \{0,1\}$ maps the publicly observable history and legislative voting record into a vote for the incumbent or challenger in his district. Finally, citizen strategies in the closed game map only public histories to a vote, or $c^i_t : H^P_t \times X \to \{0,1\}$.

As before, we focus on PVSPE. There are many such equilibria of these games. So, as is standard for games of this type, we simplify matters by imposing stationarity on equilibrium strategies. This requirement effectively rules out history-dependent strategies. Thus, let $v^i_c$ and $v^j_l$ denote the expected game values of citizen $i$ and legislator $j$ at the beginning of a period (prior to the draw of $G_t$), respectively.

The analysis is simplified considerably by the following observations. Because of stationarity and the fact that all legislators have identical preferences, citizens must be indifferent between candidates at each election. Thus, $v^i_c$ is independent of $c^i_t$. As in the single period games, all citizen voting strategies, including pivotal voting strategies, are thereby supportable in subgame perfect Nash equilibria. Further, the optimal expected equilibrium policies for each voter result from inducing the best “myopic” performance from his legislator in each period. This requires that $c^i_t$ reward the legislator for voting as the citizen would, in the event that a legislator is pivotal in determining $x_t$, and citizen $i$ is pivotal in determining whether she is re-elected. It then follows that in each period of the repeated games, citizens would wish to use pivotal voting strategies, as defined in (1) and (3).

Let $x^o(g)$ represent the legislative outcome in the open game when the group’s ideal point is located at $g$. The following result characterizes the equilibrium policy and bribes in a stationary PVSPE.

**Proposition 3.** Policy and bribes in the repeated games. In a stationary PVSPE: (i)
\( x_i^c = y_i^* = g_i \), and \( \sum_j b_j^* = 0 \) in the closed game; and (ii) \( x_i^o = y_i^* \in \begin{cases} 
[ m, x^o(g_i) ] & \text{if } g_i \geq m \\
[ x^o(g_i), m ] & \text{if } g_i < m, 
\end{cases} \)

and \( \sum_j b_j^* = \frac{w}{1-\delta} \left[ \frac{N+1}{2} - |P(y_i^*)| \right] \) in the open game.

Proposition 3 generally reinforces the desirability of public roll calls for citizens and legislators. It additionally provides a comparative static: exogenous electoral uncertainty or any other factor that reduces \( \delta \) hurts voters, but only in the open legislature.

The key intuition behind these results is that repetition may drive the reservation value—and hence price—of each legislator up from \( w \) to \( \frac{w}{1-\delta} \). In the closed game, this has no effect for the same reason as in the single period game. Groups can still exploit the externalities created by other legislators’ votes, causing no legislators to be pivotal and thereby buying their ideal policies at zero cost. In the open game, however, there exists a voting strategy (i.e., vote with their district) that gives each legislator the ability to stay in office forever. Hence, \( v_j^l = \frac{w}{1-\delta} \) and groups must pay the higher reservation values to each legislator. This reduces the relative attractiveness of the single-period solution (defined in (6)). Thus, policies can be no more extreme than in the one-shot open game.

Finally, it is worth noting that the results of Proposition 3 might change significantly if politicians were heterogeneous with respect to \( w \). In this case, citizen voting strategies would need to solve both “adverse selection” as well as moral hazard problems. As a result, citizens will have an incentive to keep “good” (i.e., high \( w \)) types in office, since these will be more expensive for groups to buy. But, if such types were always re-elected, groups could then buy their votes cheaply. A similar tension would exist if \( w \) remained homogeneous, but legislators were either term-limited or finitely-lived. In this case, citizens would always find “younger” legislators to be more desirable than older ones, and therefore the extent to which the former could be controlled by pivotal voting-like strategies will be limited.

5. Concluding Remarks

The model above shows that when voters base their voting decisions on their legislator’s individual roll call voting records, they can prevent interest groups from dominating legislative decision making. If voters do not employ such strategies, then interest groups can exploit the externalities that arise when bribes are possible and a legislature operates under majority rule, and obtain their most-preferred policy while paying almost no bribes in
equilibrium. Position-taking-based voting strategies prevent interest groups from exploiting this externality, and sharply raise the costs of obtaining favorable policies. Citizens therefore have an incentive to favor open procedures that make such strategies possible, and because of their effect on re-election rates, legislators do as well. It is worth emphasizing that these incentives exist even though players care about outcomes, and not position-taking per se.

It is tempting to ask how much voters might gain by this in practice. The following calculation, while highly speculative, provides some food for thought. Groseclose and Milyo (1999) and Diermeier, Keane, and Merlo (2002) estimate the value of a U.S. House seat to be roughly $700,000-$3,000,000. This is what it would cost to convince a sitting House member to leave congress rather than run for re-election, and therefore corresponds closely to the parameter \( w \) in our model. Suppose that the actual value is $1,000,000. If an interest group needs to buy 100 members to pass its bill, then the total cost is $100,000,000.\(^{22}\) Of course, there is also some chance that the congressmen and group are found guilty of accepting and offering bribes. Factoring in these probabilities can sharply increase the costs and decrease the expected benefits. For example, suppose the probability of being found guilty of bribery is \( 1-p \), and if this occurs then the bribes are forfeited to the government, policy reverts to the status quo (the median), and both the congressmen and interest group officials serve some time in jail. Then congressmen must be paid at least $1,000,000/p each, and the total cost is $100,000,000/p. Also, the interest group only achieves its policy goals with probability \( p \), so the value of the policy must be at least $100,000,000/p. For \( p = .25 \), this is $1,600,000,000, and for \( p = .1 \) it is $10,000,000,000. This is starting to look like real money, especially since the policy is only being “rented”—the current bribes do not bind future legislators, who will prefer to return policy to the median unless they, too receive sufficient bribes.

\(^{22}\)This does not seem unreasonable. If the distribution of district medians is uniform and the group’s ideal point is equal to the most extreme of the district medians, then lines 4 and 5 of Table 1 show that the group might be willing to buy 2/11-3/11 of the legislature. Scaled up by a factor of 435, this is about 80-120 representatives, so 100 is right in the middle.
Appendix

Proof of Proposition 1. Consider the subgame where $b(l)$ is as defined in (2) and $y = g$. Given legislator voting strategies $l^i(y, b(l)) = 0$ (resulting in outcome $x^e = y$), the citizen voting strategy

$$c^i(y, x^c) = \begin{cases} 
1 & \text{if } x^c = y \text{ and } u^i(y) \geq u^i(m) \text{ or } x^c = m \text{ and } u^i(m) > u^i(y) \\
0 & \text{otherwise.}
\end{cases}$$

(corresponding to equation (1)) is a best response satisfying the pivotal voting criterion. Given $c^i(y, x^c)$, there are three cases. First, if the number of legislators voting for $y$ is neither $(N - 1)/2$ nor $N$, then any legislator $j$ voting for $m$ could switch her vote to $y$ without changing the outcome. She would receive $\epsilon$ instead of 0 (if $x$ is not $d_j$’s preferred alternative), or $w + \eta$ instead of $w$ (if $x$ is $d_j$’s preferred alternative). Second, if $(N - 1)/2$ legislators vote for $y$, then any legislator voting for $m$ receives at most $w$. By switching to $y$, she receives $w + \eta$ instead. Finally, if $N$ legislators vote for $y$, then by switching a legislator receives 0 instead of $\epsilon$. Thus $l^i(y, b(l)) = 0$ is each legislator $j$’s best response to $y$ and $c^i(y, x^c)$ for any $\epsilon > 0$ and $\eta > 0$. In the limiting case where $\epsilon = 0$, the strategy is weakly dominant. (Voting for $y$ is dominant if exactly $(N - 1)/2$ legislators vote for each proposal.) Thus, $l^i(y, b(l))$ and $c^i(y, x^c)$ is the unique PVSPE for the subgame.

Note that under contract $b(l)$ and $b^j(l^*) = 0$ for all legislators. It is now sufficient to show that $x^c = g$ and $\sum_j b^j(l^*) = 0$ is optimal for $G$. This holds trivially by the single-peakedness of $u_G(\cdot)$ and quasilinearity of $G$’s utility. Thus $x^c = y^* = g$ and $b^j = 0$ for all legislators. ■

Proof of Proposition 2. Suppose without loss of generality that $m \geq g$. Consider any subgame in which $y \in [m, g]$, and $b(l)$ is as defined in (4). Given legislator voting strategies $l^i(y, b(l))$ corresponding to equation (5), the citizen voting strategy

$$c^i(y, 1, x) = \begin{cases} 
1 & \text{if } l^i = 0 \text{ and } u^i(y) \geq u^i(m) \text{ or } l^i = 1 \text{ and } u^i(m) > u^i(y) \\
0 & \text{otherwise.}
\end{cases}$$

(corresponding to equation (3)) is a best response satisfying the pivotal voting criterion. Given $c^i(y, 1, x^c)$, there are three cases for each legislator $j = 1, \ldots, N$. First, if $d_j \in P(y)$, then she receives $w$ by voting for $y$ and 0 otherwise, and so votes for $y$. Second, if $d_j \notin P(y)$
and \( b_j(l) = w + \eta \) if \( l_j = 0 \), then she votes for \( y \). Finally, if \( d_j \not\in P(y) \) and \( b_j(l) < w \) if \( l_j = 0 \), then she votes for \( m \). Thus, letting \( \eta \to 0 \), the strategy 
\[
\nu^*(y, b(l)) = \begin{cases} 
0 & \text{if } d_j \in P(y) \text{ or } b_j(l) \geq w \\
1 & \text{otherwise,}
\end{cases}
\]
corresponding to (5), is a best response to \( c^*(y, l, x^o) \). Thus, \( \nu^*(y, b(l)) \) and \( c^*(y, l, x^o) \) is the unique PVSPE for the subgame.

It is now sufficient to show that \( y^* \in [m, g] \). If \( y^* < m \), then by the single-peakedness of \( u_G(\cdot) \), G can obtain policy \( m \) with zero bribes: contradiction. If \( y^* > m \), then since \( |P(y)| \) is decreasing in \( y \) for \( y \geq m \) and \( u_G(\cdot) \) is single-peaked, G can obtain policy \( g \) at weakly lower cost: contradiction. Thus \( y^* \in [m, g] \). To attain a majority under legislator strategies given by \( \nu^*(y, b(l)) \), the payments must then satisfy \( b^j = w \) for \( N + 1/2 - |P(y^*)| \) legislators.

**Proof of Comment 1.** (i) Suppose without loss of generality that \( m \geq g \). Then the equilibrium policy of the closed game is \( x^c = g \) and the equilibrium policy of the open game is \( x^o \in [m, g] \). By the single-peakedness of voter preferences, a majority of voters in \( d_1, \ldots, d_{(M+1)/2} \) weakly prefer \( x^o \) to \( x^c \). Note that this preference is strict if \( x^o < g \).

(ii) In the open game, given the citizen voting strategies \( c^*(y, l, x^o) \), all legislators have a reservation utility of \( w \). In the closed game, legislators representing districts not in \( P(g) \) lose re-election and receive 0, while legislators representing districts in \( P(g) \) win re-election and receive \( w \). Thus all legislators do weakly better in the open game, and since the set \( D \setminus P(g) \) is non-empty, some do strictly better.

**Proof of Comment 2.** (i) For each legislator \( j \) and policy proposal \( y \), let \( \eta = \max\{0, u_L^j(m) - u_L^j(y)\} \). Then apply the proof of Proposition 1.

(ii) To achieve any \( y \), G has two options. First, it may buy any combination of \( (N + 3)/2 - |P(y)| \) legislators, at a price of \( w + \eta \) \( (\eta > 0) \) each, or:
\[
y' = \arg \max_y u_G(y, g) - (w + \eta) \left[ \frac{N + 3}{2} - |P(y)| \right].
\]
Second, it may buy a coalition \( C(y) \) consisting of the “cheapest” \( (N+1)/2 \) legislators, where legislator \( j \in C(y) \) only if \( u_L^j(m) - u_L^j(y) \leq u_L^j(k) - u_L^j(y) \) for any legislator \( k \not\in C(y) \). The price of each legislator is \( w + \max\{0, u_L^j(m) - u_L^j(y)\} + \eta \) \( (\eta > 0) \), where \( u_L^j(m) - u_L^j(y) < 0 \) if
$d_j \in P(y)$. Thus, G solves:

$$y'' = \arg \max_y u_G(y, g) - \sum_{C(y)} [w + \max \{0, u_L^i(m) - u_L^j(y)\} + \eta].$$

Clearly, $x'' \in \{y', y''\}$.

Now suppose without loss of generality that $g > m$. Clearly, $y' \in [m, g]$ and $y'' \in [m, g]$, so $y^* \in [m, g]$. In both cases, the cost of $C(y)$ for any $y$ is at least as high as its cost in the open game, where $(N+1)/2 - |P(y)|$ legislators are bought at a price of $w + \eta$. Thus, if $y' > y''$, then $y' = \arg \max_y u_G(y, g) - (w + \eta)[(N+1)/2 - |P(y)|]$, contradicting (6). If $y'' > y'$, then $u_G(y'', g) - \sum_{C(y')} (w + \max \{0, u_L^i(m) - u_L^j(y'')\} + \eta) > u_G(y'', g) - \sum_{C(y')} (w + \max \{0, u_L^i(m) - u_L^j(y'')\} + \eta)$. Since $\sum_{C(y)} \max \{0, u_L^i(m) - u_L^j(y)\}$ is increasing in $y$ and $\sum_{C(y)} (w + \eta) = (w + \eta)[(N+1)/2 - |P(y)|]$, this implies $u_G(y'', g) - (w + \eta)[(N+1)/2 - |P(y)|] > u_G(y', g) - (w + \eta)((N+1)/2 - |P(y)|)$, contradicting (6). We conclude that $y'' \in [m, y^o]$. 

**Proof of Comment 3.** (i) (Sufficiency) Note that $\{|d_j | D_\pi \mid m_j \leq g \leq m \text{ or } m_j \geq g \geq m\} \geq N_\pi/2$ implies $\phi_\pi(g) \geq N_\pi/2$. By (7), the party therefore votes for $g$ even if $G$ offers zero payments, establishing the result.

(Necessity) Suppose otherwise; i.e., $\{|d_j | D_\pi \mid m_j \leq g \leq m \text{ or } m_j \geq g \geq m\} < N_\pi/2$. This implies $\phi_\pi(g) < N_\pi/2$. By (7), the party votes for $g$ only if $G$ offers a total payment of $(N_\pi - 2\phi_\pi(g))w > 0$. Thus either $x^c_\pi \neq g$ or $b^*_j \neq 0$ for some legislator $j$: contradiction.

(ii) Suppose $g < m$; we show that $x^c_\pi \in [g, m]$. To show $x^c_\pi \leq m$, suppose $x^c_\pi > m$; then $G$ can propose $y = m$ and $b^i = 0$ for all legislators $j$, which yields strictly higher utility: contradiction. Now suppose $x^c_\pi < g$. Because citizen preferences are single-peaked and satisfy single-crossing, $\phi_\pi(y) \geq \phi_\pi(x^c_\pi)$ for any $y \in [x^c_\pi, m]$. Thus by (7), $G$ can propose $y = g$ and $b^*_j$ and receive a strictly better policy at the same cost: contradiction. A symmetric result holds for $g > m$. 

**Proof of Proposition 3.** Note first that by stationarity and the fact that all legislator preferences are identical, citizen continuation values $u_L^i$ are constant with respect to $c_L^i$. Thus, pivotal voting strategies are part of stationary subgame perfect equilibria.

(i) In the closed game, let voters adopt pivotal voting strategies $c^i(y_t, c_t)$ (corresponding to expression (1)) in each period. Suppose $G_t$ proposes some $y_t$ and the following contract
to each legislator $j$:

$$b^j_t(l_t) = \begin{cases} 
\frac{w}{1-\delta} + \eta & \text{if } l^j_t = y_t \text{ and } \sum_{k \neq j} l^k_t = \frac{N-1}{2} \\
\epsilon & \text{if } l^j_t = y_t \text{ and } \sum_{k \neq j} l^k_t \neq \frac{N-1}{2} \\
0 & \text{if } l^j_t = m,
\end{cases}$$

for some $\epsilon > 0$ and $\eta > 0$. Note that this contract is identical to that in (2), with the exception that pivotal legislators receive more than $w/(1-\delta) > v^j_t$ instead of $w$ to compensate for any feasible future value of holding office. Then as in the proof of Proposition 1, $l^j_t(y_t, b_t(l_t)) = 1$ is each legislator $j$’s best response to $y_t$ and $c^i(y_t, x^e_t)$. Given these strategies, $G_t$ proposes $y^*_t = g_t$. Letting $\epsilon \to 0$, equilibrium payments are $\sum_j b^j_t = 0$. Letting $\eta \to 0$, $G_t$ clearly can do no better with any other contract. Thus a stationary PVSPE is established.

(ii) In the open game, let voters adopt pivotal voting strategies $c^i(y_t, l_t, x^e_t)$ (corresponding to expression (3)) in each period. Suppose $G_t$ proposes some $y_t$ and the following contract to each legislator $j$:

$$b^j_t(l_t) = \begin{cases} 
\frac{w}{1-\delta} + \eta & \text{if } l^j_t = y_t \text{ and } d_j \notin P(y_t) \\
0 & \text{otherwise},
\end{cases}$$

for some $\eta > 0$. Note that this contract is identical to that in (4), with the exception that legislators receive more than $w/(1-\delta) = v^j_t$ instead of $w$ to compensate for their reservation values (legislators can win re-election in every period by voting with their district median’s preferred policy and thus receive $w/(1-\delta)$). Then as in the proof of Proposition 2, letting $\eta \to 0$,

$$l^j_t(y_t, b_t(l_t)) = \begin{cases} 
0 & \text{if } d_j \in P(y_t) \text{ or } b^j_t(l_t) \geq \frac{w}{1-\delta} \\
1 & \text{otherwise}
\end{cases}$$

is each legislator $j$’s best response to $y_t$ and $c^i(y_t, x^e_t)$. Given these strategies $G_t$ proposes $y^*_t = \arg\max_y u_G(y, g_t) - \frac{w}{1-\delta}[\frac{N+1}{2} - |P(y)|]$ and equilibrium payments are $\sum_j b^j_t = \frac{w}{1-\delta} \left[\frac{N+1}{2} - |P(y^*_t)|\right]$. It is straightforward to verify that since $w > w/(1-\delta)$, if $g_t \geq m$, then $y^*_t \leq x^o(g_t) = \arg\max_y u_G(y, g_t) - w[\frac{N+1}{2} - |P(y)|]$. By symmetry, if $g_t \geq m$, then $y^*_t > x^o(g_t)$. Thus a stationary PVSPE is established. $\blacksquare$
REFERENCES


