

# International Liquidity and Exchange Rate Dynamics

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Quarterly Journal of Economics (2015)

## Imperfect Finance and the Determination of Exchange Rates

*"One very important and quite robust insight is that the nominal exchange rate must be viewed as an asset price"* Obstfeld and Rogoff (1996)

- Exchange rates are disconnected from traditional macroeconomic fundamentals
- They are instead connected to financial forces: e.g. capital flows and financial conditions
- Demand and Supply of assets in different currencies is central to exchange rate determination
- Financial determination *in imperfect capital markets* is key for welfare analysis: floating exchange rates do not move to absorb real shocks as in Mundellian analysis

Important issues: framework is desirable, but has proven elusive

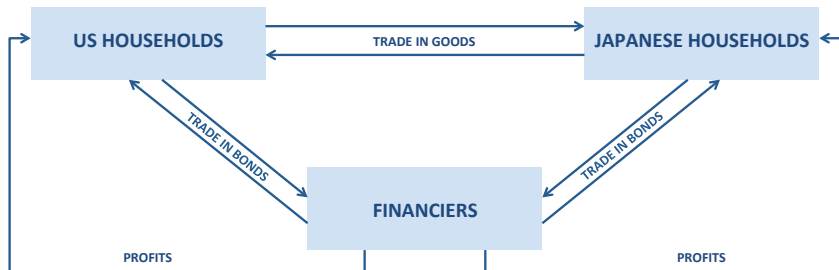
# Imperfect Finance and the Determination of Exchange Rates

We provide a basic framework of capital flows and exchange rates:

- Capital flows alter balance sheet of financiers who absorb resulting imbalances
- Financiers' balance sheets and risk bearing capacity determine the required compensation for absorbing unbalanced capital flows
- Such compensation determines both the level and dynamics of exchange rates
- Practical Example:  
US investors demand Brazilian Real bonds → Financiers provide these bonds in the short-medium run, Short Real and Long Dollar → To compensate financiers, the Real appreciates on impact and is expected to depreciate relative to the Dollar
- Our framework is a basic theory where a price, *the exchange rate*, has to move to *balance the demand/supply of assets in financial markets*

## Building up the Framework

- Real Model: basic exchange rate determination in a financial world



- Real effects of financial determination of exchange rates
  - Welfare and heterodox financial policies
- Monetary Model:
  - Nominal vs real exchange rates
  - Monetary shocks and exchange rate dynamics

## Exchange Rate Determination Frameworks

Two important papers in 1976: Dornbusch's "overshooting" model, and Kouri's "portfolio balance" model

- Obstfeld, Rogoff (1995) brought Mundell-Fleming-Dornbusch model into modern macroeconomics
- We provide a modern general equilibrium theory of the financial market forces first sketched by Kouri

Kouri's ideas:

- The demand and supply of assets denominated in different currencies as a determinant of exchange rates
- Key ingredients: domestic and foreign assets are imperfect substitutes, and imperfect capital markets
- Partial equilibrium framework
- Lack of foundations

## Basic Model

We present here the simplest model: real model, imperfect capital markets

- Two countries (US, Japan (\*)). Two periods ( $t = 0, 1$ )
- Unit measure of households in each country
- Four goods: 1 non-tradable (NT) and 1 tradable good in each country
- NT are endowments, tradables produced with int. immobile inelastically supplied labor
- NT good is the numéraire in each economy
- Incomplete Markets: two “risk-free” bonds that pay for sure one unit of the domestic numéraire (the NT good) for each economy
- Households borrow/lend in domestic “risk-free” bonds with the financiers
- Financiers absorb resulting imbalances in global capital flows

# The Household Problem

US households' consumption/saving decision:

$$\begin{aligned} \max_c \quad & \mathbb{E} [\theta_0 \ln C_0 + \beta \theta_1 \ln C_1] \\ \text{s.t.} \quad & \sum_{t=0}^1 \frac{C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t}}{R^t} \leq \sum_{t=0}^1 \frac{Y_{NT,t} + p_{H,t} Y_{H,t}}{R^t} \end{aligned}$$

where  $C_t \equiv [(C_{NT,t})^{\chi_t} (C_{H,t})^{a_t} (C_{F,t})^{l_t}]^{\frac{1}{\theta_t}}$ , and  $\theta_t = \chi_t + a_t + l_t$

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where  $C_t \equiv [(C_{NT,t})^{\chi_t} (C_{H,t})^{a_t} (C_{F,t})^{\iota_t}]^{\frac{1}{\theta_t}}$ , and  $\theta_t = \chi_t + a_t + \iota_t$

Corresponding Japanese households' problem:

$$\begin{aligned} \max_{c^*} \quad & \mathbb{E} [\theta_0^* \ln C_0^* + \beta^* \theta_1^* \ln C_1^*] \\ \text{s.t.} \quad & \sum_{t=0}^1 \frac{C_{NT,t}^* + p_{H,t}^* C_{H,t}^* + p_{F,t}^* C_{F,t}^*}{R^{*t}} \leq \sum_{t=0}^1 \frac{Y_{NT,t}^* + p_{F,t}^* Y_{F,t}^* + \pi_t}{R^{*t}} \end{aligned}$$

where  $C_t^* \equiv [(C_{NT,t}^*)^{\chi_t^*} (C_{H,t}^*)^{\xi_t} (C_{F,t}^*)^{a_t^*}]^{\frac{1}{\theta_t^*}}$ ; and  $\theta_t^* = \chi_t^* + a_t^* + \xi_t$



## Net Exports

US households' time  $t$  problem:

$$\max_{C_{i,t}} \chi_t \ln C_{NT,t} + a_t \ln C_{H,t} + l_t \ln C_{F,t} - \lambda_t (C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t})$$

Focus on two intra-temporal FOCs with respect to  $C_{NT,t}$  and  $C_{F,t}$ :

$$\frac{\chi_t}{C_{NT,t}} = \lambda_t; \quad \frac{l_t}{C_{F,t}} = \lambda_t p_{F,t}$$

Simplifying assumption:  $Y_{NT} = \chi_t \Rightarrow \lambda_t = 1$

Dollar value of US imports:  $p_{F,t} C_{F,t} = l_t$

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Dollar value of US imports:  $p_{F,t} C_{F,t} = l_t$

Similarly, Yen value of Japanese imports:  $p_{H,t}^* C_{H,t} = \xi_t$

So, Dollar value of US exports:  $\xi_t e_t$

where  $e_t$  is the exchange rate:  $e_t \uparrow$  is a Yen appreciation

Dollar value of US **net exports**:  $NX_t = \xi_t e_t - l_t$

## Interest Rates

US households' inter-temporal optimality condition (Euler Equation):

$$1 = \mathbb{E} \left[ \beta R \frac{U'_{1,C_{NT}}}{U'_{0,C_{NT}}} \right] = \mathbb{E} \left[ \beta R \frac{\chi_{1/C_{NT,1}}}{\chi_{0/C_{NT,0}}} \right] = \beta R,$$

Recall: simplifying assumption  $C_{NT} = Y_{NT} \equiv \chi_t$

Hence:  $R = \frac{1}{\beta}$

Likewise:  $R^* = \frac{1}{\beta^*}$

## Financiers' Asset Demand

- Unit measure of intermediaries, each financier runs one intermediary
- Agents are selected at random. Zero starting capital. Rebate all profits to households
- Trade Dollar and Yen bonds. Balance sheet:  $q_0 = -q_{F,0}e_0$
- Financiers maximize expected returns in dollars:

$$V_0 = \mathbb{E} \left[ \beta \left( R - R^* \frac{e_1}{e_0} \right) \right] q_0$$

*Intermediation Friction:* After taking positions, but before uncertainty is realized financiers can divert funds. If financiers divert, creditors recover  $\left(1 - \Gamma \left| \frac{q_0}{e_0} \right| \right)$  of their claims  $\left| \frac{q_0}{e_0} \right|$ :

$$\underbrace{\frac{V_0}{e_0}}_{\text{Intermediary Value in Yen}} \geq \underbrace{\left| \frac{q_0}{e_0} \right|}_{\text{Total Claims}} \underbrace{\Gamma \left| \frac{q_0}{e_0} \right|}_{\text{Diverted Portion}} = \underbrace{\Gamma \left( \frac{q_0}{e_0} \right)^2}_{\text{Total divertable Funds}}$$

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## Financiers' Asset Demand: Micro-foundations

Financiers' problem:

$$\max_{q_0} \quad V_0 = \mathbb{E} \left[ \beta \left( R - R^* \frac{e_1}{e_0} \right) \right] q_0 \quad \text{s.t.} \quad V_0 \geq \Gamma \frac{q_0^2}{e_0}$$

Optimality  $\Rightarrow$  Constraint always binds  $\Rightarrow$  Financiers' demand  $q_0$  dollar and  $-q_0/e_0$  yen, according to:

$$q_0 = \frac{1}{\Gamma} \mathbb{E} \left[ e_0 - \frac{R^*}{R} e_1 \right]$$

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$$Q_0 = \frac{1}{\Gamma} \mathbb{E} \left[ e_0 - \frac{R^*}{R} e_1 \right]$$

- $\Gamma \uparrow \infty$ : no amount of intermediation is possible  $\Rightarrow$  financial autarky
- $\Gamma = 0$ : any amount of intermediation is possible  $\Rightarrow$  Uncovered Interest Parity holds

This  $\Gamma$  demand function is key to the model: Basic Gamma model

With  $\Gamma = \gamma \text{Var}(e_1)^\alpha$ , and  $\alpha > 0 \Rightarrow$  UIP fails, but CIP holds

Simplifying assumption: financiers pay all profits to Japanese households



## Equilibrium Exchange Rate

From the three previous equations,

$$\xi_0 e_0 - \iota_0 + Q_0 = 0; \quad \xi_1 e_1 - \iota_1 - RQ_0 = 0.$$

$$Q_0 = \frac{1}{\Gamma} \mathbb{E} \left[ e_0 - \frac{R^*}{R} e_1 \right]$$

the equilibrium exchange rate follows (assume  $\xi_t = R = R^* = 1$ ):

$$e_0 = \frac{(1 + \Gamma) \iota_0 + \mathbb{E}[\iota_1]}{2 + \Gamma}; \quad \mathbb{E} \left[ \frac{e_0 - e_1}{e_0} \right] = \frac{\Gamma (\iota_0 - \mathbb{E}[\iota_1])}{(1 + \Gamma) \iota_0 + \mathbb{E}[\iota_1]}$$

### ► Derivation

- Financial Autarky ( $\Gamma \uparrow \infty$ ):  $e_0 = \iota_0$
- UIP ( $\Gamma \downarrow 0$ ):  $e_0 = \mathbb{E}[e_1] = \frac{\iota_0 + \mathbb{E}[\iota_1]}{2}$
- $\Gamma = \gamma \text{var}(\iota_1)^\alpha$

## Equilibrium Exchange Rate

Recall, the equilibrium exchange rate:

$$e_0 = \frac{(1 + \Gamma) \iota_0 + \mathbb{E}[\iota_1]}{2 + \Gamma}$$

- US net foreign assets:  $N_{0+} = e_0 - \iota_0 = \frac{\mathbb{E}[\iota_1] - \iota_0}{2 + \Gamma}$
- **Proposition:** When there is a financial disruption ( $\uparrow \Gamma$ ), countries that are net external debtors ( $N_{0+} < 0$ ) experience a currency depreciation ( $\uparrow e$ ), while the opposite is true for net-creditor countries. [▶ Derivation](#)
  - Suppose  $\iota_0 - \mathbb{E}[\iota_1] > 0$ , US runs a trade deficit and borrows in dollars
  - Financiers are long Dollar and short Yen ( $Q_0 > 0$ )
  - If financial conditions worsen ( $\uparrow \Gamma$ ), the Dollar depreciates ( $\uparrow e_0$ )
  - Empirical support: Della Corte, Riddiough and Sarno (2013)

## Gross Portfolio Flows

- So far households only traded bonds in *domestic* currency
- For simplicity, assume that some Japanese households have a noise demand  $f^*$  for Dollar bonds (financed in Yen bonds), then the equilibrium exchange rate follows:

$$e_0 = \frac{(1 + \Gamma) l_0 + \mathbb{E}[l_1] - f^* \Gamma}{2 + \Gamma}$$

► Derivation

- $\frac{\partial e_0}{\partial f^*} = -\frac{\Gamma}{2 + \Gamma}$ : if Japanese households demand Dollar bonds ( $f^* > 0$ ), then the Dollar appreciates ( $\downarrow e_0$ ): supply and demand of assets matters!
- This effect is absent both in complete market models or in models that assume UIP. Empirical support: Hau et al. (2010)
- Generalization: any portfolio demand that depends on fundamentals (but not on  $e$  directly) is tractable

## Flows not just Stocks Matter

- Recall:  $R = 1$
- US has an exogenous Dollar-denominated debt toward Japan.  $D_0$  due at time zero, and  $D_1$  due at time one
- Equilibrium exchange rate

$$e_0 = \frac{(1 + \Gamma) \iota_0 + \mathbb{E}[\iota_1]}{2 + \Gamma} + \frac{(1 + \Gamma) D_0 + D_1}{2 + \Gamma}$$

- When finance is imperfect ( $\Gamma > 0$ ):
  - The timing of repayment (a flow) matters, not just the stock of debt ( $D_0 + D_1$ )
  - The higher  $\Gamma$  the more weight on early repayment

## The Exchange Rate Disconnect

Consider two worlds: *Tranquil Times*, and *Distressed Times*

- Tranquil Times and Distressed Times have *identical macro fundamentals*, but...
- Financiers' risk bearing capacity:  $\Gamma_D > \Gamma_T$
- Financiers' balance sheet:  $Q_{0-}^T = -f < 0$ ;  $Q_{0-}^D = -f - \Delta_f$
- Equilibrium *exchange rates are different*:

$$e_0^T - e_0^D \propto (\Gamma_D - \Gamma_T)[\mathbb{E}[\iota_1] - \iota_0 + 2f] + \Gamma_D(2 + \Gamma_T)\Delta_f$$

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Intuition:

- $\mathbb{E}[\iota_1] - \iota_0 > 0$ : fundamental capital flows. US lends in Dollar
- $f > 0$ : starting balance sheet. Financiers short Dollar, long Yen

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- $\Delta_f > 0$ : increase financiers' imbalance. Shorter Dollar, longer Yen. Dollar appreciates
- $\Gamma_D(2 + \Gamma_T)$ : worse financial conditions reinforce the latter effect



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## Empirical Evidence

- Little connection between traditional fundamentals and exchange rates  
Meese, Rogoff (1983)
- More evidence that exchange rates are connected to flows in the medium run
  - Adrian, Etula, Groen (2011), Adrian, Etula, Shin (2013): financiers' balance sheet forecast USD FX
  - Hau, Massa, Peress (2010): inflows cause currency appreciation. Clean IV approach
  - Yogo, Hong (2012): CME speculators positions help predict currency returns
  - Froot, Ramodorai (2005): flows are associated with most of the variation in expected currency returns over medium horizons, fundamentals matter only at long horizon

## Currency Interventions: Welfare Consequences

Welfare analysis full details are in NBER working paper.

- Simple environment: uncertainty  $\mathbb{E}[\iota_1] = 1$ , sticky prices  $\bar{p}_{H,0}$
- The Japanese government buys  $q^*$  dollars and sells  $\frac{q^*}{e_0}$  yen at time 0

$$e_0 - \iota_0 - q^* + Q_0 = 0; \quad e_1 - \iota_1 + q^* - Q_0 = 0.$$

- $e_0(q^*) = 1 - \frac{\Gamma}{2+\Gamma} q^*$ : Dollar depreciates, creating employment
- Recall:  $Y_{H,0}(e_0) = \min\left(\frac{a_0 + e_0 \xi_0}{\bar{p}_{H,0}}, L\right)$
- US Welfare:

$$V(q^*) \equiv \mathbb{E}[U_0 + U_1] = V^{FB} + \ln \frac{Y_{H,0}(e_0(q^*))}{L} + O(q^{*2})$$

- **Proposition** If  $\Gamma > 0$  and  $Y_{H,0}(q^* = 0) < L$ , then welfare  $V(q^*)$  is increasing in intervention  $q^* \in [0, \bar{q}^*]$ , where  $e(\bar{q}^*)$  generates full employment
- Note that in this case there are no private incentives to intervene

## Rethinking Currency Interventions

Recent implementation on a *massive scale* of policies with similar rationale:

- **Switzerland:** In Sept. 2011 SNB set explicit floor for CHF/EUR at 1.2 and has accumulated CHF 450bn of reserves (80% of GDP)
- **US:** Starting in 2007 Fed provided Dollar liquidity via “unlimited” currency swaps, amounts outstanding reached \$600bn
- **Israel:** Bol has been intervening since 2008, accumulated reserves of 30% of GDP

*“I have no doubt that the massive purchases [of foreign exchange] we made between July 2008 and into 2010 [...] had a serious effect on the exchange rate which I think is part of the reason that we succeeded in having a relatively short recession.”* **Stanley Fischer** (WSJ 2010)

# Summary of other Financial Policies

- **Taxing international finance:**

- The government taxes financiers' profits at rate  $\tau$ , rebates lump sum
- Then, financiers' demand:  $Q_0 = \frac{\mathbb{E}[e_0 - e_1](1 - \tau)}{\Gamma} \equiv \frac{\mathbb{E}[e_0 - e_1]}{\Gamma^{eff}}$
- Policy warning: financiers matter, effect of the tax on ER depends on the sign of  $Q_0$  before the tax

- **Dilemma: joint monetary and FX policy**

- Use FX interventions ( $q$ ) or monetary policy ( $m_0$ )?
- **Proposition:** Suppose that at time zero  $\bar{p}_H$  is downwards rigid at a level inconsistent with full-employment, and that at time one it is either:
  - ① Flexible  $\Rightarrow$  use both FX and monetary policy. Rely more on the FX intervention when  $\Gamma$  is higher
  - ② Rigid  $\Rightarrow$  use only monetary policy. Currency intervention reduces welfare

## The Carry Trade: Financial risks

- We study carry trade returns:

$$R_1^c \equiv \mathcal{R}^* \frac{e_1}{e_0} - 1, \text{ with } \mathcal{R}^* \equiv \frac{R^*}{R}$$

- (To simplify the math take 3 periods with financial shocks:  $\Gamma_1$  stochastic, and with “very long” period 2):

$$e_0 = \frac{\Gamma_0 l_0 + \mathcal{R}^* \mathbb{E}_0 \left[ \frac{\Gamma_1 l_1 + l_2 \mathcal{R}^*}{\Gamma_1 + 1} \right]}{\Gamma_0 + 1};$$

$$e_1 = \frac{\Gamma_1 l_1 + \mathcal{R}^* \mathbb{E}_1 [l_2]}{\Gamma_1 + 1}; \quad e_2 = l_2.$$

- The carry trade does badly if there is a financial squeeze, i.e. if  $\Gamma_1$  goes up.:  $\frac{\partial R_1^c}{\partial \Gamma_1} < 0$ . (Brunnermeier, Nagel and Pedersen (2009)).
- Hence, the carry trade is exposed to “financial shocks”, not simply “fundamental shocks”

## The Carry Trade: Expected returns

- Carry trade return:

$$R_1^c \equiv \mathcal{R}^* \frac{e_1}{e_0} - 1, \quad \text{with } \mathcal{R}^* \equiv \frac{R^*}{R}$$

- (To simplify the math take 3 periods with financial shocks:  $\Gamma_1$  stochastic, and with “very long” period 2):
- Expected carry trade returns:

$$\mathbb{E}[R_1^c] = \frac{(\mathcal{R}^* - 1)\Gamma_0(\bar{\Gamma}_1 + 1 + \mathcal{R}^*)}{\bar{\Gamma}_1(\Gamma_0 + \mathcal{R}^*) + \Gamma_0 + (\mathcal{R}^*)^2}$$

$\frac{\bar{\Gamma}_1 + \mathcal{R}^*}{1 + \bar{\Gamma}_1} \equiv \mathbb{E}_0 \left[ \frac{\Gamma_1 + \mathcal{R}^*}{1 + \Gamma_1} \right]$ . Expected returns are higher:

- the higher is the interest rate differential  $\mathcal{R}^*$  (Lustig, Verdelhan (2007))
- the worse financial conditions are today ( $\uparrow \Gamma_0$ ), the better they are tomorrow ( $\downarrow \{\bar{\Gamma}_1, \Gamma_1\}$ ) (Brunnermeier, Nagel and Pedersen (2009))
- the more one invests in net-external-debtor countries' currencies (Della Corte, Riddiough and Sarno 2013)

## The Carry Trade: Fama regression

- Fama (1984) regresses:

$$\frac{e_1 - e_0}{e_0} = \alpha + \beta (R - R^*) + \varepsilon_1$$

- Under UIP,  $\beta = 1$ . Data shows  $\beta < 1$
- In the Gamma model:

$$\beta = \frac{1 + \bar{\Gamma}_1 - \Gamma_0}{(1 + \Gamma_0)(1 + \bar{\Gamma}_1)} < 1$$

- Note  $\beta < 0$  iff  $\bar{\Gamma}_1 + 1 < \Gamma_0$ , i.e. financial conditions are expected to improve



## UIP & CIP

Recall  $\Gamma_t = \gamma \text{Var}_t(e_{t+1})^\alpha$ , take  $\alpha > 0$

**Proposition** All replication trades are satisfied, hence **CIP holds**. Risky trades are affected by the constraint, hence **UIP fails**. Under mild boundedness of the shocks, all arbitrages are satisfied

- Previous propositions are about financial risk bearing capacity
- Constraint amplifies fundamental variance
- Model solution is still analytical

## Model extensions: Nominal ER, Portfolio Flows, Financial External Adjustment

The flow equations are now extended to be:

$$0 = m_0^* \xi_0 e_0 - m_0 l_0 + Q_0 + f^* - f e_0 - D^{US} + D^J e_0$$

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- Nominal:  $m_t$  and  $m_t^*$  are the US and Japanese money supplies
  - Money used domestically; nominal bonds traded internationally
  - Money is the numéraire in each country;  $P_t$  is the nominal price level
  - The US household problem:

$$\max_{\frac{M}{P}, C_{NT}, C_H, C_F} \quad \omega_t \ln \frac{M_t}{P_t} + \chi_t \ln C_{NT,t} + a_t \ln C_{H,t} + \iota_t \ln C_{F,t}$$

$$s.t. \quad M_t + p_{NT,t} C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t} \leq CE_t$$

- Optimality  $\Rightarrow M_t = \frac{\omega_t}{\lambda_t}$ . Let  $m_t \equiv \frac{M_t}{\omega_t}$
- Cash-less limit à la Woodford (1998):  
 $M_t \downarrow 0, \omega_t \downarrow 0, s.t. m_t \rightarrow \text{finite positive}$ , a policy variable

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- Nominal:  $m_t$  and  $m_t^*$  are the US and Japanese money supplies
- Capital flows:  $f$  and  $f^*$  are the demand for foreign bonds by the US and Japanese households
  - Flows that depend on all fundamentals, but not directly on  $e$  are tractable
  - E.g. Carry trade flows:

$$f = b + c(R - R^*)$$

$$f^* = d + g(R - R^*)$$

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  - $D^{US}$  and  $D^J$  are "legacy" positions
  - US net foreign assets:  $D^J e_0 - D^{US}$
  - Currency composition of gross asset and liabilities matters, not just net positions (Gourinchas and Rey (2007), Lane and Shambaugh (2010))



## Equilibrium Exchange Rate

**Proposition** The equilibrium exchange rate follows immediately from the Basic Gamma model by defining “pseudo” imports, exports, and risk bearing capacity, such that:

$$e_0 = \frac{\mathbb{E} \left[ \frac{\tilde{\iota}_0 + \frac{\tilde{\iota}_1}{R}}{\tilde{\xi}_1} \right] + \frac{\tilde{\Gamma} \tilde{\iota}_0}{R^*}}{\mathbb{E} \left[ \frac{\tilde{\xi}_0 + \frac{\tilde{\xi}_1}{R^*}}{\tilde{\xi}_1} \right] + \frac{\tilde{\Gamma} \tilde{\xi}_0}{R^*}}$$

$$\tilde{\iota}_0 \equiv m_0 \iota_0 + D^{US} - f^*;$$

$$\tilde{\xi}_0 \equiv m_0^* \xi_0 + D^J - f;$$

$$\tilde{\iota}_1 \equiv m_1 \iota_1 + R f^*;$$

$$\tilde{\xi}_1 \equiv m_1^* \xi_1 + R^* f;$$

$$\tilde{\Gamma} \equiv \Gamma / m_0^*$$

Next slide analyzes main properties of this more general economy

## Equilibrium Exchange Rate

The Dollar is weaker (i.e., the Yen is stronger):

**① Debts and their currency denomination:**

The higher the US net external liabilities in dollars are (higher  $D^{US}$ ); the lower the Japanese net external liabilities in Yen are (lower  $D^J$ )

**② Demand pressure:**

If  $\Gamma > 0$ , the lower the demand for the Dollar is (lower  $f^*$ )

**③ Interest rates:**

The higher the Japanese real interest rate is; the lower the US real interest rate is.

**④ Money supply:**

The higher the US present money supply is (high  $m_0$ ); the lower the Japanese present money supply is (low  $m_0^*$ ).

**⑤ Financial disruption:**

The higher  $\Gamma$  is, if the US is running a trade deficit.

## Infinite horizon

- **Proposition:** The exchange rate is:

$$e_t = \iota_* + \beta Q_t^- + \mathbb{E}_t \int_t^\infty e^{-\beta(s-t)} [\beta \hat{\iota}_s + \iota_* (r_s^* - r_s) - \beta f_s^*] ds$$

$$\beta = \frac{r + \sqrt{r^2 + 4\Gamma}}{2}$$

where demand shocks are  $f_s^*$ .

- Hence, the Yen is stronger if:
  - Japan is a creditor ( $Q_t^- > 0$ )
  - There is high demand import demand for Japanese goods ( $\hat{\iota}_s > 0$ )
  - Interest rates are higher in Japan than in the US ( $r_s^* - r_s > 0$ )
  - Noise traders (or governments) are selling the dollar ( $f_s^* < 0$ )

Paper also analytically extends to N countries.

## Take-Aways

We presented a basic model with:

- Imperfect capital markets: limited risk bearing, supply and demand of assets matters!
- Production: real effects of ER fluctuations, unemployment
- (Potentially) sticky prices: PCP, LCP, incomplete pass-through
- Welfare analysis: monetary policy, heterodox financial policies

**Key implications:** Exchange rates are a financial phenomenon determined by supply and demand of assets in different currencies. Financiers' balance sheets and risk tolerance are important determinants of ER

**Key take-away:** Floating exchange rate regimes can be the source of problems. Heterodox policies (interventions, capital controls) make sense when imbalances are big and financial markets distressed

## Derivation of Equilibrium Exchange Rate

Recall that we assume  $\xi_t = R^* = R = 1$ .

Adding the two flow equations,  $e_0 - \iota_0 + Q_0 = 0$  and  $e_1 - \iota_1 - Q_0 = 0$ , and taking expectations gives:

$$\mathbb{E}[e_1] = \iota_0 + \mathbb{E}[\iota_1] - e_0.$$

The financiers' demand simplifies to  $Q_0 = \frac{1}{\Gamma} (e_0 - \mathbb{E}[e_1])$  and from the time-0 flow equation,  $Q_0 = e_0 - \iota_0$ , so we have:

$$\mathbb{E}[e_1] = (1 + \Gamma) e_0 - \Gamma \iota_0$$

Combining the two equations gives the expression for the time-0 exchange rate:

$$e_0 = \frac{(1 + \Gamma)\iota_0 + \mathbb{E}[\iota_1]}{2 + \Gamma}$$

## Effect of financial disruptions on the exchange rate

- Recall the equilibrium exchange rate  $e_0 = \frac{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma}$  and US net foreign assets  $N_{0+} = \frac{\mathbb{E}[\iota_1] - \iota_0}{2+\Gamma}$ .
- The derivative of the exchange rate  $e_0$  with respect to financiers' risk bearing capacity  $\Gamma$  is

$$\begin{aligned}\frac{\partial e_0}{\partial \Gamma} &= \frac{(2+\Gamma)\iota_0 - (1+\Gamma)\iota_0 - \mathbb{E}[\iota_1]}{(2+\Gamma)^2} \\ &= \frac{\iota_0 - \mathbb{E}[\iota_1]}{(2+\Gamma)^2} \\ &= \frac{-N_{0+}}{2+\Gamma}.\end{aligned}$$

- When the US is a net external debtor ( $N_{0+} < 0$ ), the derivative is positive, so a financial disruption ( $\uparrow \Gamma$ ) causes the Dollar to depreciate ( $\uparrow e$ ).

## Gross portfolio flows and exchange rates

- Japanese households have a noise demand  $f^*$  for Dollar bonds, funded by an offsetting position  $-f^*/e_0$  in Yen bonds.
- The flow equations are now given by

$$\xi_0 e_0 - \iota_0 + Q_0 + f^* = 0, \quad \xi_1 e_1 - \iota_1 - RQ_0 - Rf^* = 0,$$

and the financiers' demand is still given by  $Q_0 = \frac{1}{r} \mathbb{E} \left[ e_0 - \frac{R^*}{R} e_1 \right]$ .

- Assume  $\xi_t = R = R^* = 1$  as before, and define  $\tilde{\iota}_0 \equiv \iota_0 - f^*$  and  $\tilde{\iota}_1 \equiv \iota_1 + f^*$ . Then the previous expression for the equilibrium exchange rate holds for the “tilde” economy:

$$\begin{aligned} e_0 &= \frac{(1 + \Gamma)\tilde{\iota}_0 + \mathbb{E}[\tilde{\iota}_1]}{2 + \Gamma} \\ &= \frac{(1 + \Gamma)(\iota_0 - f^*) + \mathbb{E}[\iota_1 + f^*]}{2 + \Gamma} \\ &= \frac{(1 + \Gamma)\iota_0 + \mathbb{E}[\iota_1] - \Gamma f^*}{2 + \Gamma} \end{aligned}$$

## Equilibrium Exchange Rate in the Extended Gamma Model

The flow equations,

$$0 = m_0^* \xi_0 e_0 - m_0 \iota_0 + Q_0 + f^* - fe_0 - D^{US} + D^J e_0$$

$$0 = m_1^* \xi_1 e_1 - m_1 \iota_1 - RQ_0 - Rf^* + R^* fe_1$$

and the financiers' demand

$$Q_0 = \frac{m_0^*}{\Gamma} \mathbb{E} \left[ e_0 - e_1 \frac{R^*}{R} \right]$$

can be expressed:

$$0 = \tilde{\xi}_0 e_0 - \tilde{\iota}_0 + Q_0, \quad 0 = \tilde{\xi}_1 e_1 - \tilde{\iota}_1 - RQ_0, \quad Q_0 = \frac{1}{\tilde{\Gamma}} \mathbb{E} \left[ e_0 - e_1 \frac{R^*}{R} \right],$$

where we define

$$\begin{aligned} \tilde{\iota}_0 &\equiv m_0 \iota_0 + D^{US} - f^*; & \tilde{\xi}_0 &\equiv m_0^* \xi_0 + D^J - f; \\ \tilde{\iota}_1 &\equiv m_1 \iota_1 + Rf^*; & \tilde{\xi}_1 &\equiv m_1^* \xi_1 + R^* f & \tilde{\Gamma} &\equiv \Gamma / m_0^* \end{aligned}$$

See the paper for the derivation of  $e_0$  in the “tilde” economy.