

## THE EQUITY PREMIUM AND THE CONCENTRATION OF AGGREGATE SHOCKS

N. Gregory MANKIW\*

*Harvard University, Cambridge, MA 02138, USA  
National Bureau of Economic Research*

Received June 1985, final version received November 1985

This paper examines an economy in which aggregate shocks are not dispersed equally throughout the population. Instead, while these shocks affect all individuals *ex ante*, they are concentrated among a few *ex post*. The equity premium in general depends on the concentration of these aggregate shocks; it follows that one cannot estimate the degree of risk aversion from aggregate data alone. These findings suggest that the empirical usefulness of aggregation theorems for capital asset pricing models is limited.

### 1. Introduction

Several recent and important studies have attempted to explain the joint behavior of asset returns and aggregate consumption using representative consumer models.<sup>1</sup> This empirical work raises the obvious question of whether it is valid to aggregate across consumers. In this paper I present a simple model economy in which aggregation is not valid and, in particular, obscures the economic forces underlying relative asset returns. I assume that aggregate shocks to consumption are not dispersed equally across all consumers. Instead, while all consumers are subject to adverse aggregate shocks *ex ante*, these shocks affect only some consumers *ex post*. I show that the concentration of aggregate shocks is a potentially important determinant of relative asset returns.

The model illustrates how the absence of certain contingent-claims markets can render representative consumer models largely ineffective as approximations to a complex economy with *ex post* heterogeneous consumers. Rubinstein (1974) and Grossman and Shiller (1982) prove aggregation theorems that do not require complete markets. The results I present here suggest that these theorems cannot be greatly extended. More important, they suggest

\*I am grateful to Fischer Black (the referee), Olivier Blanchard, Stanley Fischer, John Long, and David Romer for helpful comments.

<sup>1</sup>See, for example, Shiller (1982), Hansen and Singleton (1983), Mehra and Prescott (1985), and Dunn and Singleton (1986).

that these theorems do not fully justify the use of representative consumer models in empirical studies of asset pricing.

The general principle is that the absence of complete markets implies that individual consumption is more variable than per capita consumption, even if individuals are identical *ex ante*. Unless individuals have quadratic utility, so that the marginal utility schedule is linear, this extra variability generally affects both the mean of marginal utility and its covariance with asset returns. It is generally not possible to aggregate individuals' first-order conditions relating consumption and asset returns to a relation holding with per capita consumption data.

The model also suggests a possible solution to the equity premium 'puzzle' discussed by Mehra and Prescott (1985) among others. The nature of this puzzle can be seen using the consumption-beta relation Grossman and Shiller (1982) derive. They show that

$$E R_t = A \text{cov}(R_t, \Delta \ln C_t), \quad (1)$$

where  $R_t$  is the difference in return between any two assets,  $A$  is the harmonic mean of individuals' Arrow-Pratt coefficient of relative risk aversion, and  $C$  is aggregate consumption. This relation implies that

$$A \geq E R_t / \sigma(R_t) \sigma(\Delta \ln C_t). \quad (2)$$

In United States data, the equity premium is about six percent, the standard deviation of the realized equity premium is about twenty percent, and the standard deviation of the growth in consumption of non-durables and services is about three percent. The inequality in (2) therefore implies that the coefficient of relative risk aversion exceeds ten. Using (1) and noting that the correlation of relative market return and consumption growth is about one-third, we find that the implied coefficient of relative risk aversion is about thirty, which is generally considered implausible.

The model presented here suggests that the level of the equity premium is in part attributable to the role of incomplete markets in determining the equilibrium return on marketable assets. In particular, for any set of aggregate variables, the equity premium may be made arbitrarily large or small by changing the concentration of the aggregate shock among the population. This finding implies that one cannot judge the appropriateness of the equity premium from aggregate data alone.

## 2. A simple illustrative model

I illustrate the importance of the concentration of aggregate shocks using the simplest possible model. I first describe the aggregate economy and how an

observer might attempt to infer the degree of risk aversion from aggregate data. I then consider the disaggregate distribution of the aggregate shocks and the implications for relative asset returns.

### 2.1. *The aggregate economy*

There are two points of time in the model. At time zero, while the endowment of the consumption good is uncertain, portfolio choices are made. At time one, the endowment is realized and consumption takes place.

Per capita consumption in the economy takes on two values: a good value of  $\mu$ , and a bad value of  $(1 - \phi)\mu$ , where  $0 < \phi < 1$ . Each state occurs with probability  $\frac{1}{2}$ .

I examine a portfolio that pays  $-1$  in the bad state and pays  $1 + \pi$  in the good state, where  $\pi$  is the 'premium'. One can think of this portfolio as consisting of two assets: a short position in an asset that pays off in both states (Treasury bills) together with a long position in an asset that pays off only in the good state (equity).

Consider a representative consumer with utility function  $U(\cdot)$  deciding how much of the security to purchase. His goal is to maximize

$$E U(C), \quad (3)$$

where  $C$  is consumption. If  $R$  is the payoff of the portfolio, then the standard first-order condition is

$$E[RU'(C)] = 0 \quad (4)$$

The marginal utility weighted mean return is zero.

Given the distribution of per capita consumption, this first-order condition can be written as

$$(1 + \pi)U'(\mu) - U'((1 - \phi)\mu) = 0. \quad (5)$$

If it is valid to describe the economy as generated by this representative consumer, eq. (5) must hold at the equilibrium level of  $\pi$ . Eq. (5) therefore produces the following value of the premium:

$$\pi = -[U'(\mu) - U'((1 - \phi)\mu)]/U'(\mu). \quad (6)$$

For small values of  $\phi$ , the premium is approximately

$$\pi = -[\mu U''(\mu)/U'(\mu)]\phi = A\phi, \quad (7)$$

where  $A$  is the coefficient of relative risk aversion.

An economist observing the size of the aggregate shock ( $\phi$ ) and the premium on the portfolio ( $\pi$ ) might wish to estimate the degree of risk aversion. Using the approximation in (7) he would obtain

$$A = \pi/\phi. \quad (8)$$

Alternatively, he might explicitly parameterize the utility function as

$$U(C) = C^{1-A}/(1-A). \quad (9)$$

In this case, eq. (6) implies

$$A = -\log(1 + \pi)/\log(1 - \phi), \quad (10)$$

which is approximately the same as (8) for small  $\pi$  and  $\phi$ .

## 2.2. *Individuals and equilibrium*

Suppose there are an infinite number of individuals that are identical *ex ante*. That is, as of time zero, the distribution of consumption is the same for all individuals. I assume, however, that their consumption is not the same *ex post*. In particular, I assume that in the bad state, the fall in aggregate consumption of  $\phi\mu$  is concentrated among a fraction  $\lambda$  of the population.

The stochastic environment facing any given individual is therefore as follows. With probability  $\frac{1}{2}$ , a good state occurs: his consumption is  $\mu$  and the portfolio pays  $1 + \pi$ . With probability  $\frac{1}{2}$ , a bad state occurs. In the bad state, the portfolio pays  $-1$ ; his consumption is  $\mu$  with probability  $1 - \lambda$  and is  $(1 - \phi/\lambda)\mu$  with probability  $\lambda$ . I assume there do not exist contingent-claims markets through which individuals can diversify away this latter risk.<sup>2</sup>

The parameter  $\lambda$  measures the concentration of the aggregate shock. If  $\lambda = 1$ , then all individuals have the same consumption *ex post*. As  $\lambda$  approaches  $\phi$ , the aggregate shock becomes more highly concentrated. At  $\lambda = \phi$ , the aggregate shock is fully concentrated on a few individuals whose consumption falls to zero.

The first-order condition (4) holds for each individual, which implies

$$(1 + \pi)U'(\mu) - (1 - \lambda)U'(\mu) - \lambda U'((1 - \phi/\lambda)\mu) = 0. \quad (11)$$

The premium is therefore

$$\pi = \lambda \{ (U'[(1 - \phi/\lambda)\mu] - U'[\mu]) / U'[\mu] \}. \quad (12)$$

<sup>2</sup>It is this assumption that makes Rubinstein's (1974) aggregation theorem inapplicable. Rubinstein assumes that all risky assets are traded, so that the portfolio of risky assets is the same for all individuals.

The premium ( $\pi$ ) in general depends not only on the size of the aggregate shock ( $\phi$ ) but also on its distribution within the population ( $\lambda$ ).

### 2.3. *The implications of concentration*

I now consider how the concentration of the aggregate shock affects the size of the equity premium and the apparent degree of risk aversion that an observer might infer from aggregate data. I assume that the observer knows the size of the aggregate shock  $\phi$  and the size of the premium  $\pi$  and uses the results from the representative consumer model – that is, eqs. (9) and (10) – to estimate the coefficient of relative risk aversion.

The first result is:

*Proposition 1. If the utility function  $U(\cdot)$  is quadratic, then the premium is independent of the concentration of the aggregate shock. That is,  $\pi$  does not depend on  $\lambda$ .*

This result follows directly from eq. (12). It implies that if utility is quadratic, then the concentration of the aggregate shock does not affect the apparent degree of risk aversion. Hence, our observer is not led astray by his representative consumer model.

This result does not generalize, however, as the next proposition makes clear:

*Proposition 2. If the third derivative of the utility function is positive, then an increase in the concentration of the aggregate shock increases the premium. That is, if  $U''' > 0$ , then*

$$\frac{\partial \pi}{\partial \lambda} < 0.$$

*Proof.* By differentiating eq. (12), we obtain

$$\frac{\partial \pi}{\partial \lambda} = \frac{U'[(1 - \phi/\lambda)\mu] - U'[\mu] + (\phi\mu/\lambda)U''[(1 - \phi/\lambda)\mu]}{U'[\mu]}.$$

This can be rewritten as

$$\frac{\partial \pi}{\partial \lambda} = \frac{\int_{(1-\phi/\lambda)\mu}^{\mu} \{U''[(1 - \phi/\lambda)\mu] + U''[Z]\} dZ}{U'[\mu]}.$$

If  $U''' > 0$ , then the expression in the integral is negative over the range of integration. This completes the proof.

The condition of a positive third derivative is very plausible; indeed, it is even weaker than the condition of non-increasing absolute risk aversion.<sup>3</sup> The implication of Proposition 2 is that one cannot determine the size of the equity premium from aggregate data alone. It further suggests that our observer could be badly mistaken using a representative consumer model, that is, eq. (10). In particular, since our observer estimates the degree of risk aversion correctly if  $\lambda = 1$ , Proposition 2 implies that if  $\lambda < 1$ , our observer overestimates the degree of risk aversion.

The assumption that the concentrated shock is an adverse one is crucial to the direction of this bias. If, instead, we considered a model with a concentrated windfall, greater concentration would imply a smaller premium. The general case is discussed in section 3.

The next proposition shows that the error from using the representative consumer model in fact can be great:

*Proposition 3:* Suppose the utility function satisfies the Inada condition  $\lim_{C \rightarrow 0} U'(C) = \infty$ , then

$$\lim_{\lambda \rightarrow \phi} \pi = \infty.$$

Proposition 3 follows directly from eq. (12). It shows that regardless of the size of the aggregate shock, the equity premium can be made arbitrarily large by making the shock more and more concentrated. Thus, if the Inada condition is satisfied, one cannot place an upper bound on the equity premium from only the degree of risk aversion and the aggregate shock. Conversely, one cannot place a lower bound on the degree of risk aversion from the aggregate shock and the equity premium alone.

It may be instructive to apply some numbers to the model. Suppose  $\phi = 0.05$ , so that the aggregate endowment falls by five percent in the bad state. Table 1 presents the ratio of the true premium [eq. (12)] to the premium one would expect from the representative consumer model [eq. (7)]. Suppose  $\lambda = 0.2$ , so that twenty percent of the population experiences a fall in endowment of twenty-five percent in the bad state. If utility is logarithmic, then the true equity premium is 1.3 times what one would expect from a representative consumer model. If the constant relative risk aversion is six, then the equity premium is 2.6 times what one would expect. While the model is clearly

<sup>3</sup>This condition is related to the precautionary demand for saving; see Leland (1968) and Sandmo (1970). It is also related to skewness preference in asset demand; see Kraus and Litzenberger (1976) for some empirical support for the assumption of a positive third derivative.

Table 1  
 Ratio of the true premium to the premium inferred from the aggregate model:  $\phi = 0.05$ .<sup>a</sup>

	$A = 1$	$A = 3$	$A = 6$
$\lambda = 0.1$	1.9	4.2	17.5
$\lambda = 0.2$	1.3	1.6	2.6

<sup>a</sup> $A$  = coefficient of relative risk aversion,  $\phi$  = size of adverse aggregate shock,  $\lambda$  = fraction of population affected by aggregate shock.

too stylized to draw firm empirical conclusions, the numbers in table 1 do suggest that the concentration of aggregate shocks is a potentially important determinant of the equity premium.

### 3. Discussion

This section provides a less formal and perhaps more intuitive discussion of the effects highlighted in the model of section 2. As above, consider an economy in which all individuals are homogeneous *ex ante* but heterogeneous *ex post*. Let  $R$  be the difference in return between two tradable assets and  $C_i$  be the consumption of individual  $i$ . The first-order condition each individual satisfies is

$$E[RU'(C_i)] = 0. \tag{13}$$

Let  $\omega$  be the expectation of consumption. Since individuals are identical *ex ante*, this mean is the same for all individuals, and therefore does not require a subscript  $i$ . The second-order Taylor approximation of marginal utility around  $\omega$  is

$$U'(C_i) = U'(\omega) + U''(\omega)(C_i - \omega) + \frac{1}{2}U'''(\omega)(C_i - \omega)^2. \tag{14}$$

Substituting (14) into (13) yields

$$E(R) = -\frac{U''}{U'}E[R(C_i - \omega)] - \frac{1}{2}\frac{U'''}{U'}E[R(C_i - \omega)^2], \tag{15}$$

where the derivatives are evaluated at  $\omega$ . Now sum (15) over the individuals in the economy. Letting  $\bar{C}$  denote per capita consumption and  $N$  the number of individuals in the population, we obtain the following expression for the

expected excess return:

$$E(R) = -\frac{U''}{U'} E[R(\bar{C} - \omega)] - \frac{1}{2} \frac{U'''}{U'} E[R(\bar{C} - \omega)^2] \\ - \frac{1}{2} \frac{U'''}{U'} E\left[R\left\{\sum (C_i - \bar{C})^2 / N\right\}\right]. \quad (16)$$

The three terms in eq. (16) provide some insight into the determinants of relative asset yields.

If utility is quadratic, the second and third terms in eq. (16) disappear. Expected return then depends only on the covariance of per capita consumption with return. If the third derivative is positive, then expected return depends on the third cross-moment of per capita consumption with return, as represented in the second term of eq. (16).

The third term shows how the deviations of individual consumption from per capita consumption affect expected return. In particular, if  $U''' > 0$ , then expected return depends on the cross-moment of return with *ex post* heterogeneity. In the model of section 2, heterogeneity is great when return is low; this cross-moment is therefore negative, which exerts a positive influence on expected return. In general, however, non-diversifiable individual risk can exert either a positive or negative influence on the equity premium.

#### 4. Conclusion

The simple model presented here illustrates how one might be misled using a representative consumer model to estimate the degree of risk aversion from the size of the equity premium. Unless aggregate shocks to income affect all investors equally *ex post*, relative asset returns in general depend on the distribution of aggregate shocks among the population. It is therefore not possible to infer investors' risk aversion from aggregate data alone.

It seems plausible that the concentration of aggregate shocks is an important determinant of the equity premium. It is well-known that recessions do not affect all individuals equally; rather, they fall on a small fraction of the population that experiences very large losses in income. From 1929 to 1933, consumption of non-durables and services per capita fell only twenty percent. One suspects that certain investors experienced much larger drops in their standard of living.

The results obtained here require the absence of contingent-claims markets through which individuals can agree *ex ante* to spread this aggregate risk among themselves *ex post*. This assumption appears a reasonable approximation to observed behavior. Undoubtedly, the reason such markets do not exist is a combination of moral hazard and adverse selection considerations.

In light of these results, one might wonder whether representative consumer models remain a useful paradigm in empirical work. It is probably impossible to justify rigorously these models once we admit that many contingent-claims markets do not exist. Yet representative consumer models may nonetheless remain a useful approximation for applications in which the failure of Arrow–Debreu assumptions is not critical. Moreover, models using a ‘surrogate’ consumer with a hypothetical utility function may be useful for some purposes even if this surrogate cannot be interpreted as representative of actual individuals in the economy. Delineating the boundary between the (approximately) valid and invalid uses of representative consumer models is an important topic for future research.

## References

- Dunn, Kenneth B. and Kenneth J. Singleton, 1986, Modeling the term structure of interest rates under non-separable utility and durability of goods, *Journal of Financial Economics*, this issue.
- Grossman, Sanford J. and Robert Shiller, 1982, Consumption correlatedness and risk measurement in economies with non-traded assets and heterogeneous information, *Journal of Financial Economics* 10, 195–210.
- Hansen, Lars P. and Kenneth J. Singleton, 1983, Stochastic consumption, risk aversion, and the temporal behavior of asset returns, *Journal of Political Economy* 91, 249–265.
- Kraus, Alan, and Robert H. Litzenberger, 1976, Skewness preference and the valuation of risk assets, *Journal of Finance* 31, 1085–1100.
- Leland, Hayne E., 1968, Saving and uncertainty: The precautionary demand for saving, *Quarterly Journal of Economics* 82, 465–473.
- Mehra, Rajnish and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–162.
- Rubinstein, Mark, 1974, An aggregation theorem for securities markets, *Journal of Financial Economics* 1, 225–244.
- Sandmo, Agnar, 1970, The effect of uncertainty on saving decisions, *Review of Economic Studies* 37, 353–360.
- Shiller, Robert, 1982, Consumption, asset markets, and macroeconomic fluctuations, *Carnegie-Rochester Conference on Public Policy* 17.