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HALL'S CONSUMPTION HYPOTHESIS AND DURABLE GOODS

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Hall shows that consumption obeys an AR(1) process if the life cycle-permanent income hypothesis is true. This paper expands Hall's framework to show that expenditure on durable goods should be ARMA(1, 1) but not AR(1). Post-war U.S. data rejects the expanded model.

1. Introduction

In an important and innovative paper, Hall (1978) shows that consumption (C) must follow a first-order autoregressive process if the life cycle–permanent income hypothesis is true. That is, the only information available at time t useful in predicting C_{t+1} is C_t. No other variable known at t can increase the accuracy of the prediction. Intuitively, the reason is that consumers use all available information in the computation of permanent income and, thereby, of C_t. To the extent that information is available and relevant to C_{t+1}, it is already imbedded in C_t. The error term reflects new information regarding permanent income available at time $t+1$. If consumers form their estimates of permanent income rationally then this error must be serially uncorrelated. Thus, consumption obeys an AR(1) process.

Hall tests his 'random walk' hypothesis using quarterly per capita consumer expenditure on non-durables and services. He finds that the hypothesis is almost fully supported by the data. In particular, disposable income (Y_t, $Y_{t-1}, \ldots$,) and past consumption ($C_{t-1}, C_{t-2}, \ldots$) are not useful in predicting C_{t+1}, as the theory claims. He does find that stock market prices ($S_t, S_{t-1}, \ldots$) are statistically significant, although the increased predictive capability is very small. He argues that a slightly modified life cycle–permanent income model, in which 'some part of consumption takes time to adjust to a change in permanent income', is appropriate. Furthermore, he concludes that 'there seems little reason to doubt the life cycle–permanent income hypothesis'.[1]

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[1]Hall (1978, p. 985).

In the second section of this paper, I expand Hall's framework to deal with consumer expenditure on durable goods. This slightly generalized model implies that durable good expenditure should follow a mixed autoregressive-moving average process, ARMA(1, 1), but not AR(1). The third section examines the data. I find that consumer expenditure on durable goods does not follow the process predicted by the expanded model. In addition, after performing the same tests Hall performs, I conclude that the hypothesis that durable good expenditure is AR(1) cannot be rejected. This finding is inconsistent with Hall's version of the life cycle–permanent income hypothesis, and suggests that Hall is too hasty in closing the case on the theory of consumption.

2. Theory

Consider the slightly generalized version of Hall's model of consumption under uncertainty. The consumer maximizes:

$$\mathrm{E}_t \sum_{s=0}^{T-t} (1+\gamma)^{-s} U(K_{t+s}), \quad \text{subject to}$$

$$\sum_{s=0}^{T-t} (1+r)^{-s}(K_{t+s}-(1-\delta)K_{t+s-1}-w_{t+s})=A_t, \quad \text{where}$$

E_t = the mathematical expectation conditional on all information available in t,
γ = rate of subjective time preference,
r = real rate of interest, assumed constant over time,
$U(\ )$ = one-period utility function, strictly concave,
K = stock of goods providing services to the consumer,
δ = depreciation rate of the consumer's stock (K),
w = earnings, the only source of uncertainty,
A = assets apart from human capital.

If $\delta=1$, then the above model is exactly that of Hall, in which no goods are durable.

The budget constraint can be rewritten as

$$\sum_{s=0}^{T-t} (1+r)^{-s}(K_{t+s}-\phi w_{t+s})=\phi A_t+(1-\delta)\phi K_{t-1}-\frac{(1-\delta)\phi K_T}{(1+r)^{T-t+1}},$$

where

$$\phi=(1+r)/(\delta+r).$$

The maximization problem here is formally similar to Hall's, with the addition of initial and terminal stock terms. His theoretical results can be restated within this somewhat more general framework. I present the most important ones here:

Theorem 1. $\underset{t}{\mathrm{E}}\, U'(K_{t+1}) = \frac{1+\gamma}{1+r} U'(K_t)$.

Corollary 1. No information available in period t apart from K_t helps predict K_{t+1}, in the sense of affecting the expected value of marginal utility. In particular, income or wealth in periods t or earlier are irrelevant, once K_t is known.

Corollary 2. If the utility function is quadratic, then k obeys the exact regression:

$$K_{t+1} = a_0 + a_1 K_t + u_{t+1} \tag{1}$$

in which $a_1 = (1+\gamma)/(1+r)$. Again, no variable observed in period t or earlier will have a non-zero coefficient if added to this regression. In particular, u_t is not serially correlated.

Corollary 3. If the change in marginal utility from one period to the next is small, then K is AR(1).

Hall's paper provides a full derivation and discussion of these results.[2]

Having determined that K obeys (1), we can determine the stochastic structure of consumer expenditure (C). The fundamental identity between the stock K and the flow C is

$$K_{t+1} = (1-\delta)K_t + C_{t+1}. \tag{2}$$

(1) and (2) together imply

$$C_{t+1} = \delta a_0 + a_1 C_t + u_{t+1} - (1-\delta)u_t. \tag{3}$$

Thus, consumer expenditure on durables should obey an ARMA(1, 1) process, in which the moving average parameter is related to the rate of

[2]Theorem 1 and the subsequent corollaries can also be derived in a model with one durable good and one non-durable good if the utility function is additively separable and if the real interest rate in terms of the durable good is constant.

depreciation. If $\delta=1$, which is the special case Hall considers, then expenditure is AR(1).

One advantage of examining expenditure on durables, rather than non-durables, is that the time aggregation problem usually inherent in studies of this sort is avoided. The problem arises because Theorem 1 applies to K_t as measured at points in time. Studying non-durable consumption is difficult, since available data is measured as average consumption over an interval of time. But when studying durable goods, we can use the stock-flow identity (2) to examine expenditure, which is the change in the stock measured at points. Thus, we circumvent this problem of averaging over intervals.

Problems with this expanded model may arise because it does not take account of the illiquidity of consumer durables or possible stock adjustment costs. I expect that a more complete model would predict even greater serial correlation than exhibited in (3). For example, suppose that the *desired* stock K^* is proportional to permanent income, which follows a random walk, and that the actual stock adjusts according to the equation: $K_{t+1}=K_t+\lambda(K^*_{t+1}-K_t)$. Then it can be shown that C_{t+1} obeys an ARMA(2,1) process.[3] Thus, if adjustment costs are important, then (3) does not hold, but we still expect serial correlation when we run the regression of consumer expenditure on lagged expenditure.

3. Empirical results

To test the above theory, I attempt to parallel Hall's empirical work. I use seasonally adjusted quarterly per capita consumer expenditure on durable goods in 1972 dollars (C).[4] The estimation period is 1955:1 to 1980:1. I exclude the period from 1948:1 to 1954:4, which is included by Hall, since the Korean War might well have imposed constraints not taken into account in the theory. Nonetheless, using data beginning 1948:1, or beginning 1965:1, produces results qualitatively very similar to those reported here.

Before turning to the formal regression results, it is useful to examine the data casually. If $r=\gamma$, then $a_1=1$. In this case, the change in expenditure ΔC_{t+1} follows a first-order moving average process. The correlation coefficient ρ between ΔC_{t+1} and ΔC_t should be $(\delta-1)/(2-2\delta+\delta^2)$. which is negative, since $\delta<1$. If $\delta=0.05$, then $\rho=-0.48$. In fact, the estimated ρ is 0.06, which is the wrong sign.[5] It implies $\delta=1.07$. Expenditure on durables does not exhibit the autocorrelation suggested by the theory, a result confirmed below by direct estimation of eq. (3).

[3]This ARMA(2,1) process is AR(1) if $\delta=\lambda$. This special case is unlikely, as it would imply either an implausibly high depreciation rate or an implausibly low rate of adjustment.

[4]The NIA measure of consumer expenditure on durables includes such items as automobiles and furniture, but not residential structures.

[5]Even in the stock adjustment example considered above, $\rho<0$ so long as $\delta<\lambda$.

I first regress C_{t+1} on C_t using ordinary least squares. The result in regression (1) in table 1. The Durbin–Watson statistic suggests there is little serial correlation, contrary to the theory. I then use the method of unconditional least squares, which approximates maximum likelihood, to estimate the ARMA(1, 1) process, as predicted in eq. (3) in the last section. The result is regression (2) in table 1. The estimate of the quarterly depreciation rate (δ) is greater than one (1.038). The standard error of the estimate is 0.082. We cannot reject the null hypothesis that consumer durable expenditure is AR(1), i.e., $\delta = 1$. Furthermore, we can reject the null hypothesis that $\delta = 0.05$, or any other reasonable prior estimate for the depreciation rate. Thus, consumer expenditure on durables does not follow the ARMA process implied by the theory.

Regression (3) in table 1 is an OLS regression in which C_{t-1}, C_{t-2}, and C_{t-3} are included. The F statistic for the null hypothesis that these variables have zero coefficients is far below the critical F^* at the 95% level. Thus, I find that C_{t+1} cannot be predicted from its own past values beyond C_t. Consumer expenditure on durable goods does not follow a higher order autoregressive process.

Contrary to the theory, expenditure on durable goods follows an AR(1) process, the same simple stochastic process Hall finds for non-durables and

Table 1
ARMA estimates for expenditure on durables
Dependent variable — C_{t+1}.[a]

	(1)	(2)	(3)
Const	2.65 (5.04)	−4.37 (3.49)	3.48 (5.08)
C_t	1.002 (0.011)	1.015 (0.011)	1.044 (0.102)
C_{t-1}			0.096 (0.147)
C_{t-2}			−0.086 (0.147)
C_{t-3}			−0.056 (0.103)
δ		1.038 (0.082)	
s.e.	14.8	14.9	14.8
D.W.	1.89		1.99
F			0.853
F^*			2.70

[a]The numbers in parentheses are standard errors.

services.[6] After discovering this surprising similarity, it is natural to ask the same questions Hall poses regarding the predictability of expenditure, C_{t+1}, using information available at time t. The theory implies that expenditure on durables, unlike expenditure on non-durables, is predictable, since the error term in (3) contains u_t, which is in general correlated with information available at time t.

Can consumer expenditure be predicted from disposable income?

Let Y be per capita current dollar disposable income divided by the deflator for durable goods; this is the variable analogous to Hall's Y. Regression (4) in table 2 is an OLS regression that includes Y_t; regression (5) includes Y_t, Y_{t-1}, Y_{t-2}, and Y_{t-3}. In both cases the F statistic for the null hypothesis that all these coefficients are zero is below the critical F^* at 95%. We cannot reject the null hypothesis that disposable income is of no use in predicting C_{t+1}.

Can consumer expenditure be predicted from stock prices?

Hall finds that stock market prices are statistically significant in predicting expenditure on non-durables and services, but that the increased predictive capability is small. Let S be the Standard and Poor comprehensive index deflated by the implicit deflator for durables and then divided by the population. Regression (6) in table 2 reports the regression including S_t, S_{t-1}, S_{t-2}, and S_{t-3}. Unlike Hall, I cannot reject the null hypothesis that the coefficients of the S_{t-i} are all zero.

Can consumer expenditure be predicted from nominal interest rates?

Recent work by Grossman and Shiller (1981) indicates that asset returns may be useful in predicting consumption of non-durable goods. I do not attempt to develop in this paper the even more general case comprising a durable good and stochastic asset returns. Nonetheless, I do examine the usefulness of the nominal interest rate in forecasting consumer expenditure.

Let I_t be the average prime rate over the quarter t. Regression (7) in table 2 reports the OLS regression that includes I_t, I_{t-1}, I_{t-2}, and I_{t-3}. The F statistic for the null hypothesis that the coefficients of the I_{t-i} equal zero is well above the critical F^* at the 95% level (and even at the 99% level). Thus interest rates are indeed statistically significant predictors of consumer

[6]Expenditure on durable goods, though, is much more volatile than expenditure on non-durables and services, as measured by the standard error of the regression relative to the mean of the left-hand side variable.

Table 2
Predictive value of lagged information
Dependent variable — C_{t+1}.[a]

	Durable goods				Non-durables and services	
	(4)	(5)	(6)	(7)	(8)	(9)
Constant	−2.06 (5.70)	−1.42 (5.67)	17.95 (11.24)	−1.74 (4.63)	−9.11 (8.87)	−35.21 (9.86)
C_t	0.892 (0.066)	0.846 (0.073)	0.893 (0.068)	1.042 (0.017)	1.009 (0.003)	1.029 (0.005)
Y_t	0.016 (0.009)	0.103 (0.043)				
Y_{t-1}		−0.087 (0.059)				
Y_{t-2}		0.029 (0.058)				
Y_{t-3}		−0.023 (0.039)				
S_t			0.036 (0.020)			
S_{t-1}			0.004 (0.004)			
S_{t-2}			0.015 (0.058)			
S_{t-3}			−0.004 (0.040)			
I_t				−8.11 (2.54)		−6.53 (2.53)
I_{t-1}				2.96 (4.89)		0.37 (4.94)
I_{t-2}				−0.92 (5.21)		2.82 (5.27)
I_{t-3}				4.23 (2.93)		−0.19 (3.01)
s.e.	14.6	14.5	14.7	13.1	14.8	13.2
D.W.	1.75	1.80	1.76	2.51	1.61	1.98
F	2.90	1.84	1.20	7.65		7.15
F^*	3.94	2.46	2.46	2.46		2.46

[a]The numbers in parentheses are standard errors.

expenditure on durables. Furthermore, the increased predictive capability is not small: the standard error of the regression is reduced from \$14.8 to \$13.1. (Although not reported, adding a serial correlation correction — either moving average or autoregressive — reduces the standard error to \$12.6). The answer to the above question is, therefore, a resounding 'yes'.

Hall does not examine the usefulness of interest rates in predicting expenditure on non-durables and services. I, therefore, present the following results. Regression (8) is the equation Hall estimates; my results differ slightly from his because of the differing estimation period. Regression (9) includes I_t, I_{t-1}, I_{t-2}, and I_{t-3}. As with durable goods, interest rates are significant predictors of expenditure on non-durables and services. Furthermore, the increased predictive capability is substantial: the standard error of the regression decreases from \$14.8 to \$13.2. There is little reason to doubt that interest rates are useful in predicting consumer behavior.

The usefulness of nominal rates in forecasting consumer expenditure, although intriguing, is difficult to interpret. One might infer that the failure of the model is attributable to the assumption of a constant real interest rate. Yet there are two reasons not to trust that inference. First, the model also fails (and in much the same way) when restricting the sample to data between 1955 and 1971, a period for which Fama (1975) and Summers (1981) find a constant ex ante real interest rate.[7] Second, in Mankiw (1981), I derive a test allowing a variable and uncertain real interest rate for the case of non-durable goods. In that paper, I find that correcting for real interest rate changes leads to an even more pronounced rejection of the restrictions implied by the theory.

4. Conclusion

Hall shows that the life cycle-permanent income hypothesis implies a particular stochastic structure for consumption. And he finds that the data generally confirms his prediction. In this paper, I show that the model, when generalized to deal with durable goods, is inconsistent with post-war data. The theory implies expenditure on durable goods should follow a particular ARMA process. Yet the data soundly rejects that null hypothesis. In addition, the theory implies lagged information is useful in forecasting expenditure on durables, while this information is not useful in forecasting expenditure on non-durables and services. Examination of the data reveals no such difference between expenditure on durables and expenditure on non-durables and services.

[7]This fact is only suggestive, since the relevant rate is the after-tax real rate in terms of durable goods, while Fama and Summers examine the before-tax real rate in terms of a broader bundle of goods.

The source of the model's failure is difficult to pin down. The assumption of a constant real interest rate is certainly questionable. But as discussed above, it cannot in itself account for the results. The model also assumes that consumers can trade off present and future expenditure via capital markets and that the depreciation rate is constant. The empirical test requires that expenditure is measured accurately and that the seasonal adjustment does not greatly distort the data. These maintained hypotheses also may be to blame. Yet it is difficult to imagine that deviation from these assumptions is sufficiently great to lead to such an unequivocal rejection of the theory.

The failure may be attributable to restrictions placed upon the utility function. For example, the utility function may not be additively separable over time. Unfortunately, generating empirically testable hypotheses is difficult without this assumption. Alternatively, the utility function may not be additively separable among durable goods, non-durables and services, and leisure, as has been implicitly assumed. Relaxing this assumption appears a fruitful direction for future research.[8]

Probably the most enigmatic result is the similarity in stochastic structure between structure between expenditure on durables and expenditure on non-durables and services. It may be that those items classified as non-durables and services are partly durable. A new suit, for example, lasts the buyer longer than three months. And a once leaking faucet provides utility beyond the quarter in which it was repaired by the plumber. Durable goods, non-durable goods, and services differ only in their rate of depreciation. Assuming a depreciation rate of zero may be unrealistic for any category of consumption.

[8] I am currently pursuing this line of research with Lawrence Summers and Julio Rotemberg.

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