

Imperfect Information and Aggregate Supply*

N. Gregory Mankiw
Harvard University

Ricardo Reis
Columbia University

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1. Introduction

In his Nobel Prize lecture, George Akerlof (2002) said, “Probably the single most important macroeconomic relationship is the Phillips curve.” He is surely right that this relationship has played a central role in many business cycle theories over the past half century. At the same time, however, the Phillips curve has also been controversial and enigmatic.

As originally proposed by Phillips (1958), the eponymous curve entered macroeconomics as an empirical regularity—a mere correlation between a measure of inflation and a measure of economic activity. But soon thereafter, starting with Samuelson and Solow (1960), it was used to fill a need within macroeconomic theory. It explained how the Keynesian short run with sticky prices evolved in the classical long run with flexible prices. Today, in mainstream textbooks, the Phillips curve—or, equivalently, the aggregate supply relation—is the key connection between real and nominal variables. It explains why monetary policy, and aggregate demand more broadly, has real effects.

Once economists recognized the Phillips curve as a key relationship, they quickly started wondering what microeconomic foundation gave rise to this macroeconomic correlation. Friedman (1968) and Phelps (1968) suggested that imperfect information was the key. In the short run, some agents in the economy are unaware of some economic conditions, and this lack of knowledge gives rise to a short-run Phillips curve that, crucially, disappears in the long run.

This emphasis on imperfect information gave rise to more formal treatments of the Phillips curve and, more broadly, to the rational expectations revolution of the 1970s. Lucas (1972) formalized these ideas in a model in which some agents observe the prices of the goods they produce but not, contemporaneously, the prices of the goods they purchase. Because of this imperfect information, when households observe prices, they face a signal extraction problem to sort out movements in relative prices from movements in the overall price level. The result of this temporary confusion is a short-run Phillips curve.

Following Lucas, a large literature on imperfect information models developed. Some of it was empirical. Barro (1977), for instance, presented results suggesting that the distinction between anticipated and unanticipated movements in money was in fact crucial for explaining the real effects of money. Some of it was theoretical. Townsend (1983), for instance, emphasized how, under imperfect information, people can have different information and thus different expectations, and so forecasting the forecasts of others could be a central element of economic dynamics. In the 1990s, however, this literature went into hibernation. Other theories, including real business cycle models and new Keynesian sticky-price models, took center stage in discussions of economic fluctuations.

This chapter reviews the literature from the 2000s that revives imperfect information as a key to understanding aggregate supply and the Phillips curve. This work differs from the older work in three important, related ways. First, in the new models, information disseminates slowly rather than being perfectly revealed after some brief delay. The older literature assumed that the only obstruction to full information was the unavailability of data, whereas the new work starts from the realization that even when data on aggregates are available, it takes time and resources for people to process this

information so they will only gradually incorporate it into their actions. Second, the new work places a greater emphasis on the heterogeneity of expectations that comes with dispersion of information. It is the interaction between agents that are differentially informed that generates new theoretical questions. Third, whereas the older literature had limited strategic interactions, in the new work they take center stage.¹

We start in Section 2 by presenting a general equilibrium model of aggregate supply that allows for imperfect information. The model is deliberately simple and, but for one linearization, can be solved exactly in closed form. At the same time, it is quite general; many more complicated models have a similar reduced-form. Section 3 presents the foundations for most models of aggregate supply, including those that rely on imperfect information, introducing fundamental concepts such as menu costs and real rigidities.

Section 4 presents the two approaches to imperfect information models that we will study: partial and delayed information. Under partial information, individuals observe economic conditions subject to noise, whereas under delayed information, they observe conditions subject to a lag. We derive the common implications of these two approaches for three questions: the existence of a non-vertical aggregate supply curve, the persistence of the real effects of monetary policy, and the difference between idiosyncratic and aggregate shocks. We also compare imperfect information to the other leading model of aggregate supply, sticky prices.

Section 5 presents two implications of these two models that have led to new questions and data analysis. Delayed information models make sharp predictions for the dynamics of disagreement and have led to the use of survey data, while partial information models have shed new light on the debate over whether policy should be transparent.

Section 6 looks at the micro-foundations of the two approaches. Recent work on "rational inattention" (surveyed in the Sims chapter in this handbook) has been used to justify the assumption of partial information. In turn, models of "inattentiveness" have provided a micro-foundation for delayed information models.

Section 7 discusses more recent work that has taken these new approaches to imperfect information in different directions. These include the merging of imperfect information with sticky prices, the study of optimal policy, and the integration of these models with more conventional dynamic stochastic general equilibrium models. Section 8 concludes.

¹ Hellwig (2006) gives an alternative short survey of some of the topics covered in this chapter, and Veldkamp (2009) provides a book-length treatment of many other recent applications of imperfect-information models.

2. The baseline model of aggregate supply

We start with a model of monopolistic competition in general equilibrium, which is now standard in the study of monetary policy.²

2.1 The starting elements

To focus on the behavior of aggregate variables, we assume that there are complete insurance markets where all individual risks can be diversified. It is then only a small step to further assume that there is a representative agent that maximizes a utility function with a convenient functional form:

$$\max_{\{C_t, L_{it}, B_t\}_{t=0}^{\infty}} E \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \int_0^1 \left(\frac{L_{it}^{1+1/\psi}}{1+1/\psi} \right) di \right] \right\}. \quad (1)$$

The representative consumer has full information, $E(\cdot)$ denotes the statistical expectations operator, and $\beta \in (0,1)$ is the discount factor. There are many varieties of labor in this economy, referring to different skills and occupations, and the labor supplied by each is denoted by L_{it} , where i lies in the unit interval. ψ is the common Frisch elasticity of labor supply. Aggregate consumption, C_t , is a Dixit-Stiglitz aggregator of the consumption of many varieties of goods, also indexed by i , where $\gamma > 1$ governs the elasticity of substitution across varieties:

$$C_t = \left(\int_0^1 C_{it}^{(\gamma-1)/\gamma} di \right)^{\gamma/(\gamma-1)}. \quad (2)$$

The budget constraint at each date t is:

$$\int_0^1 P_{it} C_{it} di + B_t \leq \int_0^1 W_{it} L_{it} di + B_{t-1} (1 + R_t) - T_t + P_t \int_0^1 X_{it} di. \quad (3)$$

On the left-hand side are the uses of funds: spending on goods' varieties that each sells for P_{it} dollars, and saving an amount B_t in one-period bonds. On the right-hand side are the sources of funds. The first term is labor income, where W_{it} is the dollar wage that the i^{th} variety of labor earns. The second term is the return on savings, where R_t is the nominal interest rate. The two other terms are T_t , government lump-sum taxes, and X_{it} , the real profits from firm i .

There is a continuum of firms, where firm i hires labor variety i in a competitive market, taking W_{it} as given, but is the monopolistic supplier of good variety i . The maximand of each firm is its perceived real profits, as given by:

$$X_{it}(\cdot) = \hat{E}_{it} \left[(1 + \tau) P_{it} Y_{it} / P_t - W_{it} H_{it} / P_t \right], \quad (4)$$

² Blanchard and Kiyotaki (1987) present an early example. Gali (2008) gives a recent textbook presentation on these models in the context of aggregate supply.

where τ is a sales subsidy and Y_{it} is output produced using H_{it} units of labor. Because it is a monopolist, the firm takes into account that sales equal market demand, $Y_{it} = C_{it}$, together with the production function:

$$Y_{it} = A_{it}H_{it}. \quad (5)$$

Productivity A_{it} is stochastic and we denote its aggregate component by $A_t = \int A_{it}di$.

Note that the expectations of the firm are represented by the operator $\hat{E}_{it}(\cdot)$, which does not have to coincide with the full-information statistical operator $E(\cdot)$. If the firm had full information, then there would not be an expectation in expression (4) because all variables are known at date t when the firm makes its choices. The focus of this chapter is on the consequences of firms not having full information and having to form expectations of current prices, wages, and productivity.

The market-clearing conditions are $L_{it} = H_{it}$ in the labor market and $B_t = 0$ in the bond market. Fiscal policy simply taxes the consumer to pay for the sales subsidy: $T_t = \tau \int P_{it}Y_{it}di$. Monetary policy ensures that nominal income,

$$N_t = P_t Y_t, \quad (6)$$

follows an exogenous stochastic process. We refer to these shocks to N_t as "demand" shocks, while changes in productivity are "supply" shocks. We do not model the way in which monetary policy achieves the path for N_t , which may be directly via the money supply together with a cash-in-advance constraint in the consumer's problem, or via a nominal interest-rate rule with a very large response to deviations of N_t from $P_t Y_t$. The chapter by Friedman and Kuttner in this handbook focuses on these modeling and implementation issues.

2.2. The solution to the consumer's problem

Because the consumer's utility function is time separable and the aggregator across varieties is homothetic, the consumer problem breaks into two stages. In the first stage, for a given total consumption C_t , the consumer minimizes total spending subject to the constraint in equation (2). The solution to this problem delivers the demand function for each variety:

$$C_{it} = C_t (P_{it} / P_t)^{-\gamma}, \quad (7)$$

and the definition of the static cost-of-living price index:

$$P_t = \left(\int_0^1 P_{it}^{1-\gamma} di \right)^{1/(1-\gamma)}, \quad (8)$$

with the property that $\int P_{it}C_{it}di = P_t C_t$.

In the second stage, the consumer solves the intertemporal problem of choosing aggregate consumption and labor supply to maximize (1) subject to the sequence of budget constraints in (3). The solution is characterized by an Euler equation and a continuum of labor supply equations at each date in time:

$$1 = \beta E_t [(1 + R_{t+1}) P_t C_t / P_{t+1} C_{t+1}], \quad (9)$$

$$C_t L_{it}^{1/\psi} = W_{it} / P_t. \quad (10)$$

These conditions describe the consumer's decisions under both full information and imperfect information on the part of firms.

2.3. The full-information equilibrium

We first solve the model under the assumption of full information. In this special case, the firms' expectations $\hat{E}_{it}(\cdot)$ are identical to the full-information statistical operator $E(\cdot)$. This case is a standard benchmark against which we will compare the model with imperfect information.

Turning to the firm's problem, under full information maximizing (4) subject to (5) and (7) has a simple solution:

$$P_{it} = \left[\frac{\gamma}{(\gamma - 1)(1 + \tau)} \right] \left(\frac{W_{it}}{A_{it}} \right). \quad (11)$$

Firm i sets a price equal to a fixed markup over marginal cost, which equals the wage rate divided by labor productivity.

Combining all of the equations from (7) to (11), a few steps of algebra show that in equilibrium:

$$p_{it} = p_t + \mu + \alpha(y_t - a_{it}). \quad (12)$$

We have followed the convention that variables in small letters equal the natural logarithm of the same variable in capital letters. This equation states that the price of each firm increases one-to-one with the aggregate price level. The constant in this equation, $\mu = \ln [\gamma / (\gamma - 1)(1 + \tau)] / (1 + \gamma / \psi)$, reflects the markup. It is zero if price exactly equals marginal cost; more generally, it depends on the substitutability of the goods' varieties and the magnitude of the sales subsidy. Finally, the third term in the equation reflects the facts that higher output and consumption raise the marginal disutility of working and lower the marginal utility of consumption, thereby raising wages, marginal costs, and prices, while higher productivity lowers marginal costs and, therefore, prices.

The elasticity of the firm's price with respect to output is α , which equals $(\psi + 1) / (\psi + \gamma)$. This elasticity will play an important role, so let's pause and gauge its likely size. Because γ is greater than one, α must be smaller than one; α increases with the Frisch elasticity of labor supply and falls with the goods' elasticity of demand. Estimates of the labor supply elasticity ψ using micro data tend to be around 0.2, while macro estimates are closer to 1. Micro estimates of the goods' demand elasticity γ are around 4, while macro estimates are around 10.³ Therefore, α lies somewhere between 0.12 and 0.4. Our baseline preferred values are $\psi = 0.5$ and $\gamma = 7$, leading to $\alpha = 0.2$.

The monetary policy rule in (6) is exactly log-linear:

$$n_t = p_t + y_t, \quad (13)$$

but the price index in (8) is not. It has a simple log-linear approximation around the point where all prices are the same:

$$p_t = \int_0^1 p_i di. \quad (14)$$

This is the only approximation that we make in the full-information case.

Combining equations (12)-(14) gives the full-information equilibrium for output and prices:⁴

$$y_t^F = a_t - \mu / \alpha, \quad (15)$$

$$p_t^F = n_t - a_t + \mu / \alpha. \quad (16)$$

We are now in a position to define the object of our study: the *aggregate supply curve*. This is a map in (y, p) space that comes from varying the demand shock n_t . With full information, aggregate supply is vertical, as output is independent of monetary policy.⁵ It shifts to the right when productivity increases, and to the left if markups rise. The Pareto optimum in this economy has output equal to productivity, which is ensured by $\mu = 0$ or a constant subsidy $\tau = 1/(\gamma-1)$, and we will assume this case from now onwards (but most conclusions do not depend on this simplification).

2.4 The imperfect information equilibrium

Now consider the case in which firms have imperfect information about economic conditions. The consumer optimality conditions are still given by equations (9)-(10). For the firm though, optimal prices now satisfy:

³ See Rogerson and Wallenius (forthcoming) and Chetty (2009) for a discussion of micro and macro elasticities of labor supply, and Kimball and Shapiro (2008) for recent macro estimates. For macro estimates of the elasticity of goods' substitution see Hall (1988) and Basu and Fernald (1997), while for micro estimates see Broda and Weinstein (2006).

⁴ The model also has solutions for nominal interest rates, hours worked and consumption of different varieties, which can be derived using the equilibrium conditions. We do not focus on these.

⁵ Mathematically, the slope of the aggregate supply curve is defined as $(\partial y_t / \partial n_t) / (\partial p_t / \partial n_t)$.

$$\hat{E}_{it} \left[\left(\frac{P_{it}}{P_t} \right)^{-\gamma} \left(\frac{Y_t}{P_t} \right) \right] = \left[\frac{\gamma}{(\gamma-1)(1+\tau)} \right] \hat{E}_{it} \left[\left(\frac{P_{it}}{P_t} \right)^{-\gamma-1} \left(\frac{W_{it}}{A_{it}P_t} \right) \left(\frac{Y_t}{P_t} \right) \right] \quad (17)$$

If the firm has full information, this reduces to (11). Log-linearizing (17) around the non-stochastic case and using the assumption that $\mu = 0$ delivers the solution:

$$p_{it} = \hat{E}_{it} [p_t + \alpha(y_t - a_{it})] \quad (18)$$

The term inside the expectations is the nominal marginal cost of the firm. The firm must form expectations of the aggregate price level, output, and idiosyncratic productivity, because these are the three determinants of marginal costs. In this simple model, the firm would only have to see the wage it is paying its workers and their productivity to exactly measure marginal cost, but in the far-more complicated reality that the model is trying to capture, firms find it quite difficult to precisely measure their own marginal cost, as evidenced by the large sums spent every year in accounting systems and consultants.⁶

Equation (18) reflects the certainty-equivalence result that prices with imperfect information equal the expected price under full information in (12). Here it follows because a linearization of the optimality conditions is equivalent to a quadratic approximation of the objective function.⁷ This property has been used at least since Simon (1956) to make problems of incomplete information easier, and we will often (but not always) rely on it. The imperfect information equilibrium is defined as the values of y_t and p_t such that equations (13), (14) and (18) hold. To complete the model, the only ingredient that needs to be added is a specification of how firms form expectations.⁸

3. Foundations of imperfect-information and aggregate-supply models

If the firm has neither limits to its rationality nor any constraints on its ability to process information, then more information is better. The firm can always freely dispose of the information, and in general the ability to make more accurate forecasts will allow it to make decisions that yield higher expected profits.⁹ To justify why people do not have full information therefore requires the

⁶ A more realistic model would also take into account that production and delivery lags imply that the firm must make many decisions based on future marginal costs, so that forming expectations is unavoidable.

⁷ This result will also hold exactly if all variables are log-normal, but now with a different expression for μ .

⁸ While the model above is quite simple, it is also quite general. As Woodford (2003) shows, assuming that the preferences of the representative consumer are $u(C_t) - \int v(L_{it}) di$ each period or that the production function is $Y_{it} = A_{it} f(H_{it})$ leads to the same reduced-form after a log-linearization around a non-stochastic steady state. The only change is that the parameter α now depends on the curvature of these functions at the steady state, but reasonable calibrations lead to values not far from the 0.2 that we will work with.

⁹ It is possible that even though each firm individually is better off with more information, in equilibrium all are worse off. Hirshleifer (1971) is a classic example where the private return to inventors of racing to obtain information before others exceeds its social value.

presence of some information or rationality cost, k .¹⁰ The cost can be real resources or utility losses, may be variable or fixed, and may even be implicit in the form a shadow multipliers on an information constraint. Section 6, on the micro-foundations of imperfect information, is devoted to models of these costs. In this section, we discuss the choices that these information costs generate.

3.1 What to choose and plan?

With full information, we can think of the firm as either choosing the quantity of output to produce or the price to set. Choosing one of them instantly determines the other via the demand function. For instance, if the firm chooses its price, then using its information on aggregate output and the price level, it knows exactly the amount of output it will produce. With imperfect information, these two options are no longer equivalent. If the firm that chooses a price doesn't know aggregate output and the price level, it will not know how much output it will end up producing and selling at that price. An important component of an imperfect information model is the decision variable of the agent.

Reis (2006a) endogenized this choice by letting the firm choose *ex ante* its decision variable. If the firm chooses a plan for the price it charges, replacing the constraints into equation (4), its expected profits are:

$$\bar{X}_{it}^P = \max_{P_{it}} \hat{E}_{it} \left[(1 + \tau) P_{it}^{1-\gamma} P_t^{\gamma-1} Y_t - W_{it} A_{it}^{-1} P_{it}^{-\gamma} P_t^{\gamma-1} Y_t \right]. \quad (19)$$

A firm that instead chooses a plan for the output it produces expects to earn:

$$\bar{X}_{it}^Y = \max_{Y_{it}} \hat{E}_{it} \left[(1 + \tau) Y_{it}^{1-1/\gamma} Y_t - W_{it} A_{it}^{-1} P_t^{-1} Y_{it} \right] \quad (20)$$

Assuming there is no cost differential between planning prices and planning quantities, the firm will choose a price plan if $\bar{X}_{it}^P \geq \bar{X}_{it}^Y$ and a quantity plan otherwise.

To see what this decision entails, assume that all firms have full information, so the aggregate equilibrium is the full-information one described in Section 2.2 with $Y_t = A_t$ and $P_t = N_t/A_t$, and consider the marginal firm i that is choosing between price and quantity plans. Three cases highlight the different considerations at play. First, assume that there are no supply shocks ($A_{it} = 1$) and only demand N_t is stochastic, so that on aggregate $Y_t = 1$. In this case, manipulating equation (17) shows that the quantity plan involves choosing $Y_{it} = 1$, which is the full-information optimum. Quantity plans are preferred in this case, as the configuration of shocks makes the optimal quantity independent of news. Second, consider the case where monetary policy targets prices by the rule $N_t = A_t$, which ensures that on aggregate $P_t = 1$ and $Y_t = A_t$. Now, the optimal price for the marginal firm is $P_{it} = 1$, which can be achieved by a price plan since it requires no knowledge of news. Therefore, the price plan is preferred. Finally, consider the case where the A_{it} are idiosyncratic, with no aggregate shocks ($N_t = A_t = 1$). Some algebra shows that in this case, the firm is indifferent between price and quantity plans. Intuitively, with

¹⁰ There have been a few attempts at measuring this information cost directly. The most notable is Zbaracki et al (2004). By following a large industrial firm for a year, they measured the information costs of changing prices to be as large as 1% of revenue.

idiosyncratic productivity shocks, only the firm's idiosyncratic marginal costs are random. The demand for its good is fixed, so picking a price sets a quantity, and vice-versa, so the two give the same expected profits.

More generally, consider the case where the demand is an arbitrary function, $Y_{it} = Q(P_{it}, s_{it})$, with shocks s_{it} , while marginal costs are constant.¹¹ Then, a second-order approximation of the real profits under the two plans around the non-stochastic means of the shocks reveals that price plans are preferred if:

$$Q_s Q_{ps} + \left(-\frac{Q_s^2}{2Q_p} \right) Q_{pp} \leq 0 \quad (21)$$

To understand this result, consider the case depicted in figure 1 of a monopolist producing with zero marginal costs and facing a linear demand with slope one and additive shocks. Linear demand means that Q_{pp} is zero, and additive shocks that Q_{ps} is also zero, so equation (21) states that the firm should be indifferent between price and quantity plans. To see this graphically, the optimal price and quantity are Q^* and P^* if the shock equals its expected value, and because of the assumptions, the line segments from Q^* to O and from P^* to O are of the same length. If there is a positive shock to demand, then with a price plan, the new equilibrium will be at A , whereas with a quantity plan it will be at B . Because OA and OB have the same length, the firm is indifferent, confirming the mathematics.

Consider now the case where the shock hits the slope of the demand curve, so that when it shifts out, it becomes flatter. In this case, $Q_s Q_{ps} < 0$ so the result says that price plans are preferred. To see this graphically, note that OC is longer than OB so profits under a price plan are higher. Finally, say that when the demand curve shifts out, its slope on the horizontal dislocation is unchanged ($Q_{ps} = 0$) but the demand curve is now concave ($Q_{pp} < 0$). Again, because OC is longer than OB , price plans are preferred.

In the end, either price planning or quantity planning may be optimal for a firm facing imperfect information. But the determinants of this choice, like the shape of the firm's demand curve and the influence of the shocks on demand, are measurable so the theory provides sharp answers to guide the construction of models and can be tested using data.

3.2 Menu Costs

Consider the following question: if everyone has full information, will the marginal firm facing information costs k wish to pay this cost to obtain information? If the answer is no, then with these information costs the full information outcome is not a Nash equilibrium.

¹¹ For the case with a general cost function, see Reis (2006a).

This question is another way to pose the issue examined by Mankiw (1985) and Akerlof and Yellen (1985). Figure 2 plots the profit function for a marginal imperfectly-informed firm in a full-information economy, using equation (4), the functional forms in Section 2, and the extra assumption that there are only aggregate demand shocks, which are zero mean i.i.d log-normal with standard deviation σ . On the vertical axis are the profits with imperfect information relative to profits with full information, and on the horizontal axis is the standard deviation of the aggregate demand shock. Noticeably, the profit function is flat at the certainty case, so even a small cost k implies that the firm does not want to obtain information even for relatively high σ . Numerically, a cost of 1% of profits in the non-stochastic case leads to optimal individual inattentiveness for $\sigma \leq 0.0125$. In the post-war U.S., the standard error of nominal quarterly GDP growth is 0.01, which from the other perspective implies that as long as k exceeds 0.63% of profits, the firm will wish to become inattentive, and full information is not a Nash equilibrium.

This point can be made more generally using second-order log approximations. The firm's profits in equation (4), $X_{it}(p_{it} - p_t, \cdot)$ depend on the price it charges together with the other exogenous variables. With full information, the optimal choice is $p_{it} = p_t$, whereas without information the optimal p_{it} is some value p_{it}^* . The firm will choose to stay inattentive if:

$$X(0, \cdot) - X(p_{it}^* - p_t, \cdot) \leq k. \quad (22)$$

A second-order approximation around $p_{it}^* = p_t$ yields:

$$-X_p(0, \cdot)(p_{it}^* - p_t) - 0.5X_{pp}(0, \cdot)(p_{it}^* - p_t)^2 \leq k \quad (23)$$

The crucial insight, similar to that in Mankiw (1985), is that $X_p(0, \cdot) = 0$, since this is the necessary condition for the full-information price choice. Moreover, for small shocks to nominal income, p^* is close to p , and the second squared term is tiny. Even if k is a small cost of getting the information for updating a price menu, condition (23) will likely hold. This result is rooted in the envelope theorem: close to the maximum the profit function is flat, so small shocks have a second-order impact on profits. Hence, small informational costs may be sufficient to explain the failure of price setters to be fully informed.

3.3 Real rigidities

While the previous result shows that it is unlikely for full information to be a Nash equilibrium, the opposite question remains: is an equilibrium where all are uninformed a Nash equilibrium? The answer to this question is closely related to the concept, emphasized by Ball and Romer (1990), of real rigidities.

Focusing on the case with only demand shocks so the profit function is $X(p_{it} - p_t, n_t)$, then $X(0,0)$ are the profits without any shock to nominal income, $X(0, n_t)$ are the profits if the firm remains inattentive like all the other firms in the economy, and $X(p_{it}^*(n_t), n_t)$ are the profits if it obtains information, where $p_{it}^*(n_t)$ is the optimal price in this case as a function of the state of demand. Imperfect information will be a Nash equilibrium if:

$$X(p_{it}^*(n_t), n_t) - X(0, n_t) \leq k. \quad (24)$$

A second-order approximation of the expression on the left-hand side of (24) for n_t close to 0 yields:

$$0.5 \left[X_{pp}(0,0) \left(\frac{\partial p_{it}^*}{\partial n_t} \right) + 2X_{pn}(0,0) \right] \left(\frac{\partial p_{it}^*}{\partial n_t} \right) n_t^2 \leq k. \quad (25)$$

Because $p_{it}^*(n_t)$ is implicitly defined by the optimality condition $X_p(p_{it}^*(n_t), n_t) = 0$, the implicit function theorem gives the derivative: $\frac{\partial p_{it}^*}{\partial n_t} = -X_{pn}(0,0) / X_{pp}(0,0)$. But going back to the solution for $p_{it}^*(n_t)$ in equation (12), note that this is just the definition of the parameter α . Using it in the expression above gives the final condition:

$$-0.5\alpha^2 n_t^2 X_{pp}(0,0) \leq k. \quad (26)$$

Note that if α is small, this condition is more likely to be satisfied.¹²

Ball and Romer (1990) labeled the parameter α an index of real rigidities. In particular, a smaller α means more real rigidity. Note that α is a "real" parameter in that it depends on the properties of the real profit function. Ball and Romer's insight was that this real parameter influences the economy's nominal rigidity. Their result carries over to this setting: The more real rigidity there is, the more likely it will be that imperfect information on the part of price setters is a Nash equilibrium.¹³

3.4. Strategic complementarities

A concept closely related to real rigidity is the concept of strategic complementarity. Combining the expression for desired prices with the exogenous process for nominal income yields:

$$p_{it} = \hat{E}_{it} \left[(1 - \alpha) p_t + \alpha n_t - \alpha a_{it} \right]. \quad (27)$$

¹² The second-order condition for the optimum requires that $X_{pp}(0,0)$ is negative.

¹³ There are different mechanisms to generate real rigidities (Romer, 2008), as well as some challenges like the common finding that real rigidities induce firms to want to adjust more frequently in response to idiosyncratic shocks (Dotsey and King, 2005).

Cooper and John (1988) interpreted this expression as the best response by firm i to the other firms' actions, captured by the sufficient statistic p . Taking this game-theoretic perspective to the equilibrium of the model, $\alpha < 1$ implies that pricing decisions are strategic complements. That is, if other firms raise their prices, then firm i wishes to raise its price as well.

Strategic complementarities are important because with heterogeneity of information, there will be some firms that are better informed than others. If pricing decisions are strategic complements, then the better-informed firms will still not want to change their prices by much in order to keep them in line with the less-informed firms. Strategic complementarities therefore ensure that the aggregate supply curve is not too steep, so there is significant monetary non-neutrality. One illustration of this role is that two influential articles that found very steep aggregate supply curves (Chari, Kehoe and McGrattan, 2000, and Golosov and Lucas, 2007) both chose parameters that make α larger than one.

It is not entirely surprising that the same parameter α and condition $\alpha < 1$, are important for both real rigidities and strategic complementarities, even though these concepts start from different places. If the informed firm i does not want to change its price p_{it} much after a shock because it knows the other uninformed firms will not, then it will typically also be the case that the profit gain from obtaining information and changing p_{it} is small. Because of these similarities, the concepts of real rigidity and strategic complementarity are often used interchangeably in this literature, and we will do so in this chapter as well.¹⁴

4. Partial and delayed information models: common predictions

Having set out the basic framework in Section 2 and examined some foundational issues in Section 3, we now consider two models of imperfect information that have commanded attention in recent years. We call these the partial information model and the delayed information model.

Both of these models assume that people form expectations optimally but with incomplete information. The difference is the nature of the incompleteness. The delayed information model assumes that only a share λ of firms have up-to-date information, while the remaining have old information from previous periods. The partial information model assumes that firms observe a noisy signal with a relative precision τ . Both models introduce just one new parameter, λ or τ , which can be interpreted as an index of informational rigidities. By maintaining the assumption of optimal behavior subject to these new informational constraints, the tools used to solve these models are familiar to economists accustomed to rational expectations models.

To present the essence of these two approaches, consider our baseline model with only aggregate demand shocks that follow a random walk, so $n_t = n_{t-1} + v_t$, with v_t normally distributed with

¹⁴ With strategic complementarities comes the scope for multiple equilibria. Ball and Romer (1989) characterize the equilibrium multiplicity in their model with full information, while Morris and Shin (1998, 2001) and Heineman (2000) do it with partial information and Hellwig and Veldkamp (forthcoming) with delayed information.

mean zero and variance σ^2 .¹⁵ Combining (13), (14), and (18), the equilibrium price level solves the equation:

$$p_t = \int_0^1 \hat{E}_i [\alpha n_i + (1-\alpha)p_t] di. \quad (28)$$

The overall price level in the economy is an average across firms of their expectation of their optimal prices, which in turn are a weighted average the level of demand n_i and the price level p_t .

From this equation we can examine several features of imperfect information models that apply in both variants. First, we will show how incomplete information generates a non-vertical aggregate supply curve in a simple model where all information gets revealed after one period. Next, we will introduce gradual revelation of information to understand the persistence of the real effects of aggregate demand shocks. After a brief detour to compare imperfect information with sticky prices, we finally we consider the effects of idiosyncratic productivity shocks.

4.1. Non-vertical aggregate supply

Consider first the delayed information model. In this model, λ of agents have full information, so their subjective expectation of the contemporaneous values of aggregate demand and the price level coincides with the actual values of these variables. Suppose, for now, that the remaining $1-\lambda$ do not observe current shocks but do have full information on all variables one period before and form expectations optimally given this information. The equation describing the equilibrium for the price level becomes:

$$p_t = \lambda [\alpha n_t + (1-\alpha)p_t] + (1-\lambda)E_{t-1} [\alpha n_t + (1-\alpha)p_t]. \quad (29)$$

The key tool to solve this class of models is the "innovations representation" of the equation, sometimes also called the Wold representation. In particular, by rearranging terms, we can write the equation as:

$$p_t - E_{t-1}(p_t) = \alpha\lambda [n_t - E_{t-1}(n_t)] + (1-\alpha)\lambda [p_t - E_{t-1}(p_t)] + \alpha E_{t-1}(n_t - p_t). \quad (30)$$

Now, with the exception of the last term on the right-hand side, all other terms are uncorrelated innovations, and therefore have an expectation of zero as of the previous period. Taking expectations at $t-1$ of both sides of the equation shows that $E_{t-1}(p_t) = E_{t-1}(n_t) = n_{t-1}$, so the last term is zero. Solving for the innovation in prices as a function of the innovation in aggregate demand yields:

$$p_t = \left[\frac{\alpha\lambda}{1-(1-\alpha)\lambda} \right] (n_t - n_{t-1}) + n_{t-1}, \quad (31)$$

¹⁵ There is nothing special about the random walk beyond making the algebra slightly easier. The tools laid out below would apply to most other linear stochastic processes.

$$y_t = \left[1 - \frac{\alpha\lambda}{1 - (1-\alpha)\lambda} \right] (n_t - n_{t-1}). \quad (32)$$

Variation in the expected level of aggregate demand n_{t-1} leads to proportional changes in prices and no effect on output. However, shocks to aggregate demand $n_t - n_{t-1}$ increase both output and prices: the aggregate supply is no longer vertical.¹⁶ The slope of the aggregate supply curve falls with both α and λ ; that is, the stronger are informational or real rigidities, the flatter is the aggregate supply curve. Intuitively, uninformed firms do not adjust their price in response to a positive aggregate demand shock, which causes their sales to rise. Therefore, more uninformed firms leads to stronger monetary non-neutrality. In turn, for lower values of α , firms that do become informed want to set their prices closer to those of the uninformed firms, which leads their sales to rise and aggregate output to increase by more.

Now consider the partial information model. This model assumes that all firms have noisy signals of the state of aggregate demand. They observe $z_{it} = n_t + \varepsilon_{it}$, where the noise ε_{it} is independent across firms and time, normally distributed, has zero mean, and variance equal to σ^2/τ . The parameter τ plays the same role as λ did in the delayed-information model: a higher τ means less informational rigidities, because it implies that z_{it} is a more accurate signal of n_t .

A key feature of the partial information model is the absence of common knowledge. In particular, because each firm's signal is its private information, it cannot credibly transmit it to anyone else, so no one knows what others in the economy know.¹⁷ There is a role for higher-order beliefs, as each firm must form a belief of what other firms believe, as well as of what other firms believe that the firm believes, and so on. A consequence of this is that the law of iterated expectations does not hold in aggregate: in particular $\int \hat{E}_{it} \left[\int \hat{E}_{it}(\cdot) di \right] di \neq \int \hat{E}_{it}(\cdot) di$, or the second-order average belief is not equal to the first-order one. Successively taking expectations from the perspective of agent i , and averaging over all the agents, equation (28) becomes:

$$p_t = \alpha \sum_{j=1}^{\infty} (1-\alpha)^{j-1} \bar{E}_t^{(j)}(n_t). \quad (33)$$

We used the notation $\bar{E}_t(\cdot) \equiv \int \hat{E}_{it}(\cdot) di$, $\bar{E}_t^{(2)}(\cdot) \equiv \int \hat{E}_{it} \left[\int \hat{E}_{it}(\cdot) di \right] di$, and so on, as well the limiting condition that average infinite-order beliefs do not explode to infinity (which can be verified later).

The crucial tool to solve the partial information problem is the signal extraction formula. In particular, it is a standard result in statistics that:

¹⁶ We can also write the equilibrium in terms of an expectations-augmented Phillips curve as in Friedman (1968): $\Delta p_t = E_{t-1}(\Delta p_t) + [\alpha\lambda/(1-\lambda)]y_t$.

¹⁷ In contrast, in the delayed-information model, a share λ of firms have full information and so know exactly what the other firms know.

$$\hat{E}_{it}(n_t) = E_t(n_t | z_{it} = n_t + \varepsilon_{it}) = E_{t-1}(n_t) + \left(\frac{\tau}{1+\tau}\right) [z_{it} - E_{t-1}(n_t)]. \quad (34)$$

This equation gives up the first-order belief—the expectation of aggregate demand. The second-order belief is the expectation of others' expectations of aggregate demand. This is found by averaging equation (34) over all firms and then taking the expectation of the resulting expression, which yields:

$$\hat{E}_{it}^{(2)}(n_t) = E_{t-1}(n_t) + \left(\frac{\tau}{1+\tau}\right)^2 [z_{it} - E_{t-1}(n_t)]. \quad (35)$$

In this equation, each firm is using the signal it obtains to forecast other firms' signals and thus their expectations of demand. Note the signal obtains a smaller weight in this second-order belief than it did in the first-order belief. More generally, iterating over these two steps delivers the j -th order belief:

$$\bar{E}_t^{(j)}(n_t) = E_{t-1}(n_t) + [\tau / (1+\tau)]^j [n_t - E_{t-1}(n_t)]. \quad (36)$$

Combining this expression with equation (33) gives the solution:

$$p_t = \left(\frac{\alpha\tau}{1+\alpha\tau}\right)(n_t - n_{t-1}) + n_{t-1}, \quad (37)$$

$$y_t = \left(1 - \frac{\alpha\tau}{1+\alpha\tau}\right)(n_t - n_{t-1}). \quad (38)$$

Comparing with the solution for the delayed information model in (31)-(32), one finds that the models make very similar predictions. In particular, stronger real and nominal rigidities again imply a flatter aggregate supply curve. The intuition is clear in the limits. If $\tau = 0$, the signal is useless, the firms have no information on the current shocks, and prices are unchanged, so the aggregate supply curve is horizontal. If instead $\tau \rightarrow \infty$, firms have full information and the aggregate supply curve is vertical. In between, better information implies that prices adjust by more so the curve is steeper. The role of real rigidities arises because the smaller is α , the more firms want to charge what other firms are charging, and the more weight each gives to what others are thinking. In other words, more real rigidity gives a larger role to higher-order beliefs. The higher the order of the beliefs, the closer they are to $E_{t-1}(n_t)$ and the less they respond to the signal. Thus, in the partial information model, as in the delayed information model, more real rigidity means greater monetary non-neutrality.

4.2 Persistence

So far, because all information becomes known after one period, aggregate demand shocks moved output for only one period. Now, we relax this assumption by assuming that firms have imperfect information that may last for an extended span of time.

In Mankiw and Reis (2002), we proposed a model of persistent delayed information, which we called the sticky-information model. We assumed that every period, a fraction λ of firms gets

independently drawn from the population and receives full information.¹⁸ At any date, there will therefore be a share $\lambda(1-\lambda)^j$ of firms that last updated their information j periods ago. With this exponential distribution, the equilibrium price level now solves the equation:

$$p_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j} [\alpha n_t + (1-\alpha)p_t]. \quad (39)$$

The innovations representation for aggregate demand is $n_t = \sum_{k=0}^{\infty} v_{t-k}$, where the v_{t-k} are the uncorrelated innovations. Since the v_{t-k} are the only shocks in the model, it is a good guess that the innovations representation for the price level will depend on them as well: $p_t = \sum_{k=0}^{\infty} \varphi_k v_{t-k}$. Solving the model is to solve for the φ_k unknown coefficients. While the approach of the previous section will not work, a slight extension of it does: the method of undetermined coefficients.¹⁹ It relies on two observations: first, that $E_{t-j}(p_t) = \sum_{k=j}^{\infty} \varphi_k v_{t-k}$ and likewise for n_t , and second that the φ_k must be the same for all possible realizations of the shocks. Equation (39) then imposes the conditions:

$$\varphi_k = \lambda \left[\alpha \sum_{j=0}^k (1-\lambda)^j + (1-\alpha) \varphi_k \sum_{j=0}^k (1-\lambda)^j \right], \quad (40)$$

for every $k = 0, 1, \dots$. These equations yield the model's solution:

$$p_t = \sum_{k=0}^{\infty} \left[\frac{\alpha [1 - (1-\lambda)^{k+1}]}{1 - (1-\alpha) [1 - (1-\lambda)^{k+1}]} \right] v_{t-k}, \quad (41)$$

$$y_t = \sum_{k=0}^{\infty} \left[\frac{(1-\lambda)^{k+1}}{1 - (1-\alpha) [1 - (1-\lambda)^{k+1}]} \right] v_{t-k}. \quad (42)$$

On impact, a positive aggregate demand shock still leads to an increase in both prices and output, and stronger real and informational rigidities still enhance the response of output and attenuate the response of prices. Figure 3 plots the impulse responses of both output and inflation over time, with $\lambda=0.25$, so that firms update their information on average once per year.²⁰ Output only approaches zero asymptotically as the share of firms that have learned about the shock goes to 1, and the half-life of

¹⁸ An allegory for this model is to think of each firm having a stochastic alarm clock that every period rings with probability λ making it "wake up" and see what is going on.

¹⁹ There is a long tradition of using this method to solve macroeconomic models with optimal expectations. See Taylor (1985) for an early review. More recently, Mankiw and Reis (2007), Reis (2009b), and Meyer-Gohde (forthcoming) develop general algorithms to solve sticky-information models with many equations and variables.

²⁰ Khan and Zhu (2005) and Dopke, Dovern, Fritsche and Slacaleck (2008a) econometrically estimate Phillips curves with sticky information and find $\lambda = 0.25$ for the United States, France, Germany, and the United Kingdom, while $\lambda = 0.5$ for Italy.

the shocks is one-and-a-half years. The response of inflation is also delayed with two properties that have been emphasized in the empirical literature: (i) it is hump-shaped, and (ii) it peaks after output.²¹

Let's turn now to the partial information model. Its dynamic version is due to Woodford (2002), who called it the imperfect common knowledge model. This model assumes that each firm receives a private signal z_{it} of aggregate demand, just as before, but now never gets to learn what past aggregate demand was. As it receives new signals, the firm not only forms an expectation of the present circumstances, but also revises its views on the past. Therefore, as in the sticky-information model, all firms will eventually become informed about the value of a shock today.

The approach of the last section only works if we assume that after some large number of periods, shocks become common knowledge. Hellwig (2008) and Lorenzoni (2009, forthcoming) take this route and let the number of periods become larger and larger to obtain an approximation to the solution. Woodford (2002) instead proposed an alternative guess-and-verify method using dynamic signal-extraction tools.²² The guess is that:

$$p_t = (1-\theta)p_{t-1} + \theta n_t. \quad (43)$$

Writing this guess, together with the random-walk for nominal demand and the signal z_{it} in vectors, gives:

$$\begin{pmatrix} n_t \\ p_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \theta & 1-\theta \end{pmatrix} \begin{pmatrix} n_{t-1} \\ p_{t-1} \end{pmatrix} + \begin{pmatrix} 1 \\ \theta \end{pmatrix} v_t \Rightarrow s_t = Ms_{t-1} + cv_t, \quad (44)$$

$$z_{it} = (1 \ 0) \begin{pmatrix} n_t \\ p_t \end{pmatrix} + \varepsilon_{it} \Rightarrow x_{it} = es_t + \varepsilon_{it}. \quad (45)$$

Here we have defined the new matrices and vectors s_t , M , c , and e to write the s problem as a state-space system. The dynamic version of the signal extraction formula in Section 4.1 is the Kalman filter:

$$E_{it}(s_t) = ME_{it-1}(s_{t-1}) + \kappa [z_{it} - eME_{it-1}(s_{t-1})], \quad (46)$$

where $\kappa = (\kappa_1, \kappa_2)'$ is a 2x1 vector of Kalman gains (e.g., Hamilton, 1995: chapter 13). Integrating this expression over all agents, and using (44) then leads to:

$$\bar{E}_t(s_t) = \kappa e Ms_{t-1} + (M - \kappa e M) \bar{E}_{t-1}(s_{t-1}) + \kappa e cv_t. \quad (47)$$

²¹ Coibion (2006) thoroughly describes the features of the sticky-information model that generate hump shapes in inflation.

²² Other approaches to solving partial information models are Amato and Shin (2006), who truncate the problem going backwards at some date, Rondina (2008), who uses the Wiener-Kolmogorov formulae for signal extraction, and Kasa (2000) who attacks the problem in the frequency domain.

Next, note that equation (33) implies that $p_t = \alpha \bar{E}_t(n_t) + (1-\alpha) \bar{E}_t(p_t)$. Using equation (47) to replace for the average expectations of n_t and p_t , and performing the matrix algebra operations, this equation for the price level becomes:

$$p_t = (1-\theta)p_{t-1} + [\alpha\kappa_1 + (1-\alpha)\kappa_2]n_t + [\theta - \alpha\kappa_1 - (1-\alpha)\kappa_2] \bar{E}_{t-1}(n_{t-1}). \quad (48)$$

This verifies the original guess in (43) and shows that $\theta = \alpha\kappa_1 + (1-\alpha)\kappa_2$. The expressions for the Kalman gains are messy, but one can show that θ is the positive solution of the quadratic equation:

$$\theta^2 + \alpha\tau\theta - \alpha\tau = 0. \quad (49)$$

The partial information model again has similar predictions to its delayed information counterpart. There is still an upward-sloping aggregate supply curve, and the larger are the indices of real and informational rigidities then the larger and more persistent are the effects of nominal demand on output. Figure 4 has the impulse responses, and while the one for output is similar to figure 3, the one for inflation has a significant difference: there is no hump-shape.²³ While the absence of hump shapes is not a generic property of the partial information model (they appear with other stochastic processes for aggregate demand), this case shows that the two models are not observationally equivalent. With good enough data, we would be able to distinguish between them.

4.3. A digression on sticky prices

The main alternative to models of imperfect information and aggregate supply are models based on sticky prices. Indeed, in much of the recent business-cycle literature, the norm for explaining price adjustment is some version of the Calvo (1983) model. A full comparison of these approaches is beyond the scope of this chapter. But, because we have just been discussing persistence, it is worth noting one specific comparison regarding the dynamics of inflation. This particular difference between the approaches, at least in their simplest form, has motivated some recent work on imperfect information.

The Calvo model can be viewed as a special case of the sticky-information model in which the plan that firms set for prices must consist of a single number for all dates. Therefore, when a firm chooses its plan, it sets a price that is optimal on average over the duration of the plan. The optimal price to set at the adjustment date is then a weighted average of the expected optimal price at all dates in the future. This leads to front-loading: changes in expected future conditions affect prices today. This front-loading is the source of many empirical problems of the sticky-price model, described by Mankiw (2001), Mankiw and Reis (2002), Rudd and Whelan (2007), and many others.

²³ The value of τ was set to 0.005 so that the impact response of output is the same as in the delayed information model. The standard deviation of the noise is therefore fourteen times the standard deviation of the shock to demand. Whether this is realistic or not is hard to say; finding a direct empirical counterpart to the signal-to-noise ration in partial information models is a standing challenge.

The first problem comes from trend inflation. The weighted average that gives the optimal adjustment price will be too high relative to the optimum today, and too low relative to the optimum in the future, so that even if there is full information, the long-run aggregate supply curve will not be vertical. The second problem is that prices and inflation will jump in response to news today about future circumstances. In the data, however, estimated impulse responses of inflation to shocks are very sluggish and often hump-shaped. Ball (1994) put this problem in an elegant way: if the monetary authorities announce today a disinflation for the future, Calvo price-setters will cut their prices immediately, leading to a boom in economic activity. The experience of almost all disinflations in the OECD refutes this prediction.

Various solutions to the problems of the Calvo model have been suggested. Perhaps firms choose not prices but price deviations from a trend or target price index. Or perhaps firms automatically index their prices to past inflation. Or perhaps a fraction of firms follow simple rules of thumb when setting prices.

While these modifications of the Calvo model solve some of its empirical shortcomings, they come with two problems of their own. First, by assuming backward-looking behavior in ways that are not observed in the micro data, they effectively renounce the enterprise of micro-foundations. Firms do not seem to index their prices, nor does such indexation follow from even boundedly-rational behavior; if the goal is to just add whatever it takes to fit the macro data, then one might as well do this from the start, in the tradition of good reduced-form work. Imperfect information, in contrast, is a theory of optimal forward-looking behavior that does not imply front loading and therefore does not require these fixes to avoid its counterfactual implications.

The second problem with these fixes was highlighted by Reis (2006a). The sticky information model can account not only for the persistent inflation of the post-war United States, but also for the serially uncorrelated inflation of the pre-war era. The reason is that incompletely informed but optimizing agents adjust their behavior to the different monetary policies of those two periods. The many hybrid versions of the Calvo model, by rigging in automatic persistence to fight the front-loading behavior of the model, cannot fit the data from different policy regimes if their key parameters (such as the degree of automatic indexation or the share of rule-of-thumb agents) are truly structural and therefore invariant to the policy regime.

In addition to the Calvo model, there is another strand of models with sticky prices, in which firms choose every instant whether to change their prices subject to a fixed cost. These are sometimes called state-dependent models. An important difference between these models and models of imperfect information is the role of what is called the selection effect. In state-dependent pricing models, only those firms whose current price is very far from their optimal price will choose to adjust. Thus, when firms adjust, they do so by a large amount. This selection effect means that substantial movements in the overall price level can be consistent with many firms not adjusting at all. As a result, the aggregate supply curve can be very steep and the effects of monetary policy very small and transient. By contrast, with imperfect information, firms do not know for sure what their optimal price

would be. Therefore, this selection effect is mitigated, and all else equal, aggregate demand shocks have larger and more persistent effects.

Despite the problems of models with full information and sticky prices in fitting the aggregate data, the fact remains that most prices in the economy change infrequently. A more promising route than comparing sticky prices with imperfect information is instead to develop models that merge the two approaches. There is already some exciting work in this area, which we review in section 7.

4.4. Two sources of shocks

The models of imperfect information can also take into account many sources of information. In this section we show how by re-introducing the shocks to idiosyncratic productivity A_{it} . For simplicity, we revert to the assumption of Section 4.1 that information becomes known to all after one period.

One approach to deal with multiple shocks is to assume, following Mankiw and Reis (2006), that there is still only one source of information. In particular, in the delayed information model, there is a single parameter λ , and when firms obtain information, they observe both the aggregate and the idiosyncratic shocks. In the partial information model, the corresponding assumption is that there is only one noisy signal. Because the firm wants to set a price proportional to its nominal marginal cost, it would want its piece of information to be a single signal of this variable. If we restrict signals to exogenous variables, the component of nominal marginal cost is $n_t - a_{it}$ (see equation (18)), so the firm would choose to observe a noisy signal z_{it} on this.²⁴

Following the same steps as in Section 4.1, the solution for output and prices is exactly the same as in equations (31)-(32) and (37)-(38). Imperfect information on idiosyncratic shocks leads to more mistakes in the prices set by uninformed firms, but these are uncorrelated with the mistakes due to aggregate demand shocks. Even though the losses in profits from lack of information increase, the predictions for the slope of the aggregate supply curve are unchanged.²⁵

An alternative approach, following Carroll and Slacaleck (2007) and Mackowiak and Wiederholt (2008), is to assume that there are two sources of information. In terms of the delayed information model, this would imply that the share of firms receiving news about aggregate demand (call it λ^n) is different from the share of firms receiving information about idiosyncratic productivity (say λ^a). In the partial information model, the precision of information on the two shocks might be different, leading to two separate indices of informational rigidity, τ^n and τ^a .

Working through this version of the model, it is straightforward to show that again the same aggregate equilibrium holds, and that it is the rigidity of aggregate information, λ^n or τ^n , that affects the

²⁴ Models where agents receive signals from endogenous variables, like prices, are much harder to solve and so have unfortunately been little explored so far. Angeletos and Werning (2006) is an exception, but their focus is on the uniqueness of equilibrium.

²⁵ One difference from the model in of Section 4.1 is that now, like in Lucas (1973), the ratio of their variances of the aggregate and idiosyncratic shocks will affect the slope of aggregate supply.

aggregate supply curve. One virtue of this extension is that it is possible to have firms that are well informed about their local conditions, while being misinformed about the aggregate. Moreover, because the firm cares about marginal cost, which depends on $n_t - a_{it}$, if idiosyncratic shocks are much more volatile than aggregate shocks, firms will try to obtain more accurate information on a_{it} rather than on n_t . Since the benefits of obtaining more information on the more volatile idiosyncratic shocks are always larger than the benefits of more information on the aggregate shocks, as long as the cost of the two types of signals is the same, firms will get more information on the idiosyncratic conditions.²⁶

The virtue of allowing for two sources of information is that it is then possible to have individual prices being quite volatile in response to the closely-monitored idiosyncratic productivity shocks, while at the same time aggregate prices are sluggish in response to poorly-observed nominal demand shocks.²⁷ Mackowiak and Wiederholt (2008) and Nimark (2008) emphasize this point to match the large and frequent price changes that we observe in the micro data. Klenow and Willis (2007) find support in the micro data for the proposition that price changes only slowly incorporate past aggregate information on nominal demand.

5. Partial and delayed information models: novel predictions

Beyond addressing the long-standing questions about the slope of the aggregate supply curve and the persistence of economic fluctuations, the two classes of imperfect information models also have generated a variety of new applications.

5.1 Delayed information and time-varying disagreement

Mankiw, Reis and Wolfers (2004) emphasized the predictions of the sticky information model for disagreement. In this model, without news, everyone would have the same information and would make the same forecasts of the future. In response to news, some people learn about it and revise their forecasts, while others remain uninformed, so there is disagreement. As more people become informed, and more news happens, different groups emerge with different forecasts. In the delayed information model, disagreement is therefore an endogenous variable that co-moves with the other endogenous variables in response to the shocks. This prediction can be tested using survey data on people's expectations.²⁸

The most reliable large datasets on people's expectations concern inflation. The Michigan Survey of Consumer Attitudes and Behavior asks a cross-section of 500 to 700 members of the general

²⁶ Mackowiack and Wiederholt (2009) show this result for the partial information model using the rational inattention approach to model the costs of information. The same result holds for the delayed information model using the inattentiveness micro-foundation, as long as improving the accuracy of information on each of the two shocks has the same cost.

²⁷ One feature of reality that these models ignore is that information on some variables may be easier to obtain and understand than information on other variables.

²⁸ In contrast, while there is also disagreement in the partial information model, it is always equal to the exogenous variance of the signals.

public every month, the Livingston Survey collects the forecasts of 48 professional economists twice a year, and the Survey of Professional Forecasters surveys 34 professional forecasters every quarter. These surveys have long-term series (starting in 1946, 1946, and 1968 for the three surveys, respectively), and they expend considerable effort making sure that the respondents are providing answers on a particular common measure of inflation. While some care is always warranted in interpreting the results from surveys, these are perhaps the best available measures of disagreement.²⁹

In the delayed information model, define disagreement as the cross-sectional standard deviation of inflation expectations:

$$D_t = \sqrt{\lambda \sum_{i=0}^{\infty} (1-\lambda)^i \left[E_{t-i}(\Delta p_{t+1}) - \lambda \sum_j (1-\lambda)^j E_{t-j}(\Delta p_{t+1}) \right]^2}. \quad (50)$$

Taking the solution for the price level in equation (41) in Section 4.2, and using again the method of undetermined coefficients, a few steps of algebra show that this expression equals:

$$D_t = \sqrt{\sum_{i=0}^{\infty} \left\{ \lambda (1-\lambda)^i \left[1 - \lambda (1-\lambda)^i \right] \sum_{k=i}^{\infty} (\varphi_k - \varphi_{k-1}) \right\} \mathcal{V}_{t-i}^2}. \quad (51)$$

Figure 5 plots the impulse response of disagreement to a one-standard deviation shock to aggregate demand. On impact, disagreement increases by almost 0.12%, and at its peak by 0.18%. Mankiw, Reis and Wolfers (2004) find that in the data disagreement is indeed positively related with recent changes in inflation and output, and Coibion and Gorodnichenko (2008) find a positive relation between disagreement and oil price shocks, but are unable to statistically pin down the sign of the relationship between disagreement and other measures of shocks. Finally, Branch (2007) finds that the sticky information model can match many features of the distribution of inflation expectations in the Michigan survey.

Carroll (2003) takes a different approach that emphasizes the distinction between professional forecasters (in the SPF survey) and households (in the Michigan survey). He assumes that professionals have close to perfect information, while households have very sticky information. He finds that, just as the sticky-information model predicts, the expectations of households gradually converge to the expectations of professionals.³⁰

Mankiw, Reis and Wolfers (2004) noted that the model's predictions are broadly consistent with the U.S. experience in the first half of the 1980s, following the Volcker disinflation. As monetary policy contracted, inflation and output fell, while disagreement increased substantially. Moreover, as shown in

²⁹ There is fairly strong evidence that survey expectations are reliable and have useful information. Ang, Bekaert and Wei (2007) find that the median inflation expectation is the best available forecaster of inflation beating all econometric alternatives. Inoue, Kilian and Kiraz (2009) confirm that the surveys are backed by actions, by finding that household consumption growth responds to their perceived real interest rate, using their reported expectation of inflation, and this is stronger the higher is the education of the household.

³⁰ Dopke, Doornik, Fritsche and Slacalek (2008b) confirm Carroll's findings for France, Germany, Italy and the UK.

figure 6, disagreement in the data moved in striking agreement with the model. It is noticeable that the distribution of inflation expectations went from its usual bell shape to a bimodal distribution for a little over a year, as some people seemed to have updated their expectations while others had not.³¹

Taking the unconditional expectation of D_t in equation (49), we obtain a prediction for the average amount of disagreement. For our baseline parameters, $\alpha = 0.2$, $\lambda = 0.25$, $\sigma = 0.01$, predicted disagreement is 0.5%. This predicted value is well below the disagreement we observe in the data, but this discrepancy may be expected for at least two reasons. First, D_t in equation (51) ignores other sources of shocks, and in particular, aggregate productivity shocks. Second, there is more heterogeneity in the real world than just the differences in information sets that the model emphasizes.

Other empirical work using survey data has typically been supportive of imperfect information models more generally. In particular, Curtin (2009) added new questions to the Michigan survey asking people about their knowledge of current inflation. He found that knowledge of the present is as imperfect as forecasts of the future—a feature of the world that is perhaps the very essence of imperfect information models.

Moving forward, imperfect information models face the difficulty that sometimes slight changes in the information structure can change their predictions significantly. Berkelmans (2009) and Hellwig and Venkateswaran (2009) introduce multiple shocks in partial information models, similar to the one in Section 4.2, and find that the impulse responses of inflation to demand shocks can be quite different depending on which combinations of other shocks are allowed. Another example is the different predictions for the choice of portfolios by investors with rational inattention reached by van Niewerburgh and Veldkamp (forthcoming) and Mondria (forthcoming) from small differences in the specification of the available signals. One way out of this problem is to use data that directly disciplines the modeling of information. There is a wealth of data asking people about their expectations, and using these data in novel ways offers, in our view, the biggest promise in empirical work on imperfect information models in the near future.

Moreover, most of the work described above tries to explain the data on expectations using the data on aggregate variables. There is much less work attempting to explain macroeconomic variables using expectations data. We expect that this will be a fruitful topic of research in the years to come.

5.2. Partial information and optimal transparency

A classic issue is the role of transparency in monetary policy. Typically, economists have argued that more clarity on the part of central banks is desirable. As Morris and Shin (2002) have emphasized, partial information models provide some novel insights to study the optimal degree of transparency.

³¹ Dovern, Fritsche, and Slacalek (2009) find that in the G-7, countries with more independent central banks have less disagreement about inflation and nominal variables.

Within the 1-period partial information model of Section 4.1, assume that beyond the private signal z_{it} , there is also a public signal $m_t = n_t + v_t$, where v_t is normal, has mean zero, and variance σ^2/ω . One interpretation of this public signal is that it is a policy announcement by the central bank. If the authority is maximally transparent, then $\omega \rightarrow \infty$, whereas a completely opaque central bank makes no announcements, which corresponds to $\omega = 0$.

Given its two signals, the firm's optimal forecast now is:

$$\hat{E}_{it}(n_t) = E_{t-1}(n_t) + \left(\frac{\tau}{1+\tau+\omega}\right)[z_{it} - E_{t-1}(n_t)] + \left(\frac{\omega}{1+\tau+\omega}\right)[m_t - E_{t-1}(n_t)]. \quad (52)$$

Averaging over this and iterating on the expectations as before gives the solution:

$$p_t = \left(\frac{\alpha\tau + \omega}{1 + \alpha\tau + \omega}\right)[n_t - E_{t-1}(n_t)] + \left(\frac{\omega}{1 + \alpha\tau + \omega}\right)v_t + E_{t-1}(n_t), \quad (53)$$

$$y_t = \left(\frac{1}{1 + \alpha\tau + \omega}\right)[n_t - E_{t-1}(n_t)] - \left(\frac{\omega}{1 + \alpha\tau + \omega}\right)v_t. \quad (54)$$

Once again, the aggregate supply curve is non-vertical and flatter the stronger are real and informational rigidities. The public signal has two effects. First, as with the private signal, the more precise the public signal, the steeper is the aggregate supply curve. Second, shocks to this common information now generate fluctuations in prices and output. In particular, if the central bank's announcement misleads firms into believing aggregate demand is higher than it actually is ($v_t > 0$), they will raise prices and output will fall.

Imperfection of information creates welfare losses relative to the first best through two channels. First, because output would be constant with full information, any output variability is costly to the risk-averse consumer. Second, because all firms are identical, any price dispersion reflects a misallocation of resources. Using the equilibrium solution, these two measures are:

$$\text{Var}_{t-1}(y) \equiv E_{t-1}(y^2) = \frac{(1+\omega)\sigma^2}{(1+\omega+\alpha\tau)^2}, \quad (55)$$

$$\text{Var}_i(p_{it}) \equiv \int_0^1 (p_{it} - p_t)^2 di = \frac{\alpha^2\tau\sigma^2}{(1+\omega+\alpha\tau)^2}. \quad (56)$$

The first best is achieved here with maximal transparency, which occurs as $\omega \rightarrow \infty$. In this case, there is complete information. This case, however, is arguably of limited relevance, as the central bank can never be completely clear or completely certain that all agents in the economy will perfectly process the information it provides.

The more relevant question is whether, at the margin, increased transparency is good or bad. An improvement in transparency (higher ω) unambiguously lowers the cross-sectional dispersion of

prices. As firms have more precise common information, they coordinate more. However, more transparency has an ambiguous effect on output volatility. The reason is that with a more precise public signal, firms on the one hand decide to rely less on their private signals, undermining the information they reveal, and on the other hand are now exposed to fluctuations because of the public signal mistakes. Because of strategic complementarities, each firm would like the other firms to respond more to their private signals than they do, as this aggregates and reveals information. Increased transparency may exacerbate the inefficient use of information by firms and could potentially reduce welfare.

Depending on the relative weight of output and price stabilization in the policymaker's objective function, there may be a range of ω between 0 and some positive value, where raising ω actually lowers welfare. While complete transparency is the global optimum, if there is an upper bound on the precision of the public signals, it may be best to be less transparent than this upper bound. By picking different parameters, Morris and Shin (2002) argue this case is likely, while Svensson (2006) argues it is not.

Within the context of the specific aggregate supply model that we consider, Roca (2006) provided the unambiguous answer for all parameter values. He posited that a natural utilitarian measure of social welfare is Woodford's (2003) second-order approximation of the utility of the representative agent. In this case, the relative weight on the cross-sectional dispersion of prices vis-a-vis the variance of output is equal to γ/α . Because the elasticity of substitution across varieties is positive, $\gamma > 1$, simple algebra shows that this condition is sufficient for welfare to increase with higher transparency.

Outside of this particular model of aggregate supply, Angeletos and Pavan (2007) provided a general characterization of the inefficiency in using information, and a set of conditions for transparency to increase or decrease welfare. Amador and Weill (2008) recover the Morris and Shin (2002) result that transparency may be harmful by assuming that agents must distinguish between productivity and monetary shocks and use the distribution of prices in the economy to learn. Reis (2010) studies the optimal timing for releasing information by policymakers, asking how far in advance (if at all) should changes in policy be announced.

One conclusion from this literature seems robust: Increased transparency may reduce the incentive for people to rely on and thus reveal private information. The effects of this behavior on welfare, however, are more ambiguous and may depend on the particulars of the model. This literature has already succeeded in showing that the case for transparency is not as clear cut as it may have seemed just a decade ago. The hope is that future work using these tools and insights may lead to a better understanding of how authorities may wish to communicate with the public, a long-standing question in economics.

6. Micro-foundations of incomplete information

So far we have discussed two models of aggregate supply built on the assumption of imperfect information, but we have not addressed a more foundational question: Why is information imperfect in the first place? The theory of "inattentiveness" proposed by Reis (2006a, 2006b) has been used to justify delayed information, while the theory of "rational inattention" proposed by Sims (2003) has been used to justify why firms would have partial information.

6.1 Inattentiveness

For a firm to set a price reflecting the current state of the world requires incurring at least three costs. There is a cost of acquiring information, in the sense of obtaining all of the relevant bits of data that are informative. There is a cost of absorbing information, in the sense of interpreting all of this information and translating it into the sufficient statistics for the price decision. And finally, there is a cost of processing information in the sense of computing the map from the sufficient statistics to the optimal action on prices. The cost of acquiring information may be small, and may have fallen in this information age, but the costs of absorbing and processing information may be large and arguably higher today than in the past. In Reis (2006a), these various costs are modeled as a fixed cost that the firm has to pay whenever it wants to acquire information and become attentive. If it does not pay the cost, the firm remains inattentive, following a predetermined plan that may not be best for the current circumstances.³²

Letting the costs of planning be denoted by the fixed amount k , and the value of a firm at date t that has just obtained information on the random-walk demand shock by $V(n_t)$ then the Bellman equation for this problem is:

$$V_i(n_t) = \max_d E_t \left\{ \sum_{s=0}^{d-1} \beta^s \max_{p_{i,t+s}} [X_i(p_{i,t+s}, \cdot)] - \beta^d k + \beta^d V_i(n_{t+d}) \right\}, \quad (57)$$

where d is the number of time periods between information acquisition. The solution will be a function $d(n_t)$, so that while price adjustment is time-dependent, in that it does not depend on the state of the world at the date of the adjustment, it is recursively-state-dependent, since it depends on the state of the world at the last adjustment date.³³ In principle, this result should make it possible to distinguish between this model of inattentiveness and partial information models. Testing whether the fraction of firms adjusting their plans today does not depend on news today would test the inattentiveness model. However, because the fraction of adjusters depends on the past state of the world, and since most relevant variables are very persistent, in practice these tests will have little power.

This problem can be solved numerically, but to obtain an analytic solution, we make three simplifications. First, we work with a quadratic log-approximation of the profit function. This is the certainty equivalent approximation, and it implies that $X(p_{i,t+s}) = -\Xi (p_{i,t+s} - p_{i,t+s}^*)^2$ where Ξ is a

³² The assumption that attention is an all-or-nothing affair is extreme, but it could be relaxed. For instance, the model could be extended to allow firms to observe some information when inattentive.

³³ The issue of time versus state dependence is important, because the latter comes with a selection effect that greatly reduces the real impact of nominal shocks (Golosov and Lucas, 2007, Caballero and Engel, 2007).

scalar, and that the inner maximization has the solution $p_{it+s} = E_t(p_{it+s}^*)$. Second, we ignore the fact that d must be an integer and proceed to take derivatives and solve equations as if d could be a number in the real line. This approximation is not too damning; using quantum calculus, we could dispense with this assumption and obtain similar results. Third, we ignore strategic considerations by focusing on the $\alpha = 1$ case.³⁴

The first step to solving the problem is to realize that, using these assumptions:

$$E_t \left[\sum_{s=0}^{d-1} \beta^s \max_{p_{i,t+s}} [X_i(p_{i,t+s}, \cdot)] \right] = \sigma^2 \Xi \left[\frac{1 - d\beta^{d-1} + (d-1)\beta^d}{(1-\beta)^2} \right]. \quad (58)$$

Expected profits (at the time when the firm is making its pricing plan) do not depend on the state of aggregate demand. Thus, under these special conditions, the value function is a constant, and the optimal inattentiveness does not depend on n_t . It then follows from the problem in (57) that the necessary optimality conditions imply that d^* maximizes:

$$\frac{-\sigma^2 \Xi [1 - d\beta^{d-1} + (d-1)\beta^d] - (1-\beta)^2 \beta^d k}{1 - \beta^d}. \quad (59)$$

Using the implicit function theorem, it is straightforward to show that there is a unique positive d^* that is of order \sqrt{k} , and that it increases with k and falls with σ^2 . Therefore, inattentiveness is first-order long with second-order costs of planning, increases the more costly it is to plan, and falls the more volatile is the world.

The assumption of the sticky information model that information arrives as a Poisson process, implying an exponential distribution of uninformed price-setters, is harder to justify. Reis (2006a) provides some conditions under which it holds, but they are quite strict.³⁵ Carroll (2006) proposes an alternative, arguing that information spreads like a virus in a population with the rate of arrival of information λ being the analogous of an infection contact rate. However, this idea has not yet been formalized.

³⁴ Reis (2006a) provides two alternatives: an alternative case with an exact analytical solution by setting the problem in continuous time and assuming an iso-elastic profit function, and a general approximate solution to the problem using perturbation theory.

³⁵ Dupor and Tsuruga (2005) examine the predictions of a sticky information model in which all firms are inattentive for the same amount of time N and are perfectly staggered in their adjustment dates, so the distribution of inattentiveness is uniform. It turns out that the comparison between this model and the more standard model depends on how the two models are calibrated. If the mean duration of inattentiveness at the time of adjustment is the same for the two models ($N=1/\lambda$), then demand shocks are less persistent with a uniform distribution than with an exponential. But if, instead, the average age of plans within the economy at any moment is set to be the same ($0.5(N+1)=1/\lambda$), then the two models yield similar dynamics. Dixon and Kara (2006) argue that the latter is the better calibration.

6.2 Rational inattention

The chapter by Sims in this handbook reviews rational inattention theory in detail, so here we limit ourselves to its link to partial information models of aggregate supply. We start with a brief introduction to the two key concepts of rational inattention.

The first concept is entropy. For a variable n_t in the real line with probability density function $f(n_t)$, its entropy is:

$$H(n_t) = -\int f(n_t) \ln(f(n_t)) dn_t. \quad (60)$$

Entropy is analogous to variance in that it measures uncertainty, is non-negative, and equals zero if n_t is certain.³⁶ The second concept is mutual information, defined as:

$$I(n_t; z_{it}) = H(n_t) - H(n_t \parallel z_{it}). \quad (61)$$

The information that the signal z_{it} has on the variable n_t is therefore the reduction in entropy that results from having the conditional distribution of n_t on z_{it} instead of the unconditional distribution of n_t .

The rational inattention problem for a price-setting firm consists of picking the signals to maximize profits subject to the constraint on the amount of information it processes:

$$\max_{f(n_t \parallel z_{it})} \left[\max_{p_{it}} X(p_{it}, n_t) \right] \quad \text{subject to: } I(n_t; z_{it}) \leq k. \quad (62)$$

While this seems like a standard constrained maximization, several features make the problem unique. First, note that the choice variable is a conditional probability density function, not a scalar. Another way of stating this is that the signal is $z_{it} = n_t + \varepsilon_{it}$, and we are choosing the distribution of ε_{it} . Second, nothing in the structure of the problem guarantees that the solution is a known distribution or even that it has a smooth density.³⁷ Third, the constraint is that there is a fixed finite capacity k , so the firm is unable to expend resources in obtaining more capacity (e.g., more managers or consultants) even if the benefits from doing so were very large. Fourth, note that this is not an intertemporal problem (unlike in the inattentiveness theory) because it is assumed that the firm cannot trade capacity over time. In the theory of rational inattention, it is assumed that agents can only observe some signals of the world every period, but cannot choose to pay more attention at certain times.³⁸

³⁶ Entropy has some appealing properties including its link to the notion of information, data compression and descriptive complexity (Cover and Thomas, 1991), although it has been strongly criticized as a measure of risk (Aumann and Serrano, 2008).

³⁷ In fact, Matejka (2008) finds that rational inattention problems typically have discontinuous solutions with point-mass distributions.

³⁸ An exception is Moscarini (2004), who formulates a rational inattention problem in terms of choosing the discrete dates at which to observe continuously-arriving information.

Because this is a hard problem, three approaches have been followed in the literature to solve it. One approach is to solve the problem numerically (Sims, 2007). This work is still in its infancy as it seems that the needed numerical tools are not in the standard economist's toolkit.³⁹

A second approach is to constrain the set of admissible signal distributions to known distributions (van Nieuwerburgh and Veldkamp, forthcoming, Mondria, forthcoming). In particular, it is often assumed that the signals must be normally distributed, since then the functional problem reduces to choosing a single parameter, the variance of the noise.⁴⁰ In particular, using the definition of mutual information in (60) and (61) and the density of the normal distribution, a few steps of algebra show that the information constraint becomes a constraint on the precision of the signal:

$$0.5 \ln(1 + \tau) \leq k. \quad (63)$$

Because more precise signals raise expected profits, it is clear that this constraint will always bind at the optimum. Therefore, expression (63) holds as an equality, and it gives the optimal precision of signals τ as a monotonic function of information capacity k . Firms with higher capacity have more precise signals.

A third approach is to solve for the optimal distribution for some special cases of the profit function. One natural and simple case is when the profit function is quadratic, $X(n_{t+s}) = -\Xi (p_{it+s} - p_{it+s}^*)^2$, and nominal income is normally distributed. In this case, one can show that the optimal distribution function for the errors is the normal distribution. This is the only case where the exact analytic solution is known.

7. The research frontier

There has been much other recent work on imperfect information with implications for aggregate supply and the effect of aggregate demand. We review some of this work in this section.

7.1. Merging incomplete information and sticky prices

When one looks at the price path for many goods, three features stand out.⁴¹ First, prices change all the time, on average every three to four months. Second, many of these changes follow what seem like predetermined patterns that simple algorithms can spot; the actual re-setting of price plans

³⁹ Recent developments in the numerical solution of rational inattention models are in Matejka (2008), Lewis (2009), and Tutino (2009).

⁴⁰ It is an elementary result that the signal will be unbiased since changes in the mean have no effect on entropy and the firm would not benefit from any such bias.

⁴¹ See the chapter by Klenow and Malin in this Handbook, and the recent work of Eichenbaum, Jaimovich and Rebelo (2008).

reflecting new information seems to occur less often than once a year.⁴² Third, in a plot of prices over time, there are many horizontal segments, reflecting short-lived intervals when nominal prices are unchanged.

The first two features match the predictions of imperfect information models, and sticky information models in particular. The prevalence of what some researchers call predictable "sales" are precisely the price plans in these models and, as found by Klenow and Willis (2007), these plans seem to only slowly incorporate available information. The third feature, though, is puzzling to these models, because there is no reason why the predetermined plan would involve the exact same price over an interval of time. There are some attempts at explaining the prevalence of these prices using imperfect information, but a more common answer has been the presence of physical costs of changing prices in addition to the information costs, leading to sticky prices.⁴³

Bonomo and Carvalho (2004, forthcoming) assumed that the cost of changing price plans includes both an information cost and a physical price-adjustment cost. Thus, when firms update their information, they are constrained to pick a plan where a single price is chosen, unlike the time-varying plans in the sticky-information model. In a stationary environment, the result is the Calvo model of price adjustment, derived here as a special case of sticky information. The advantage of this information-interpretation of the Calvo model is that it leads to an endogenous choice of the frequency of price adjustment, along similar lines to the inattentiveness theory in Section 6.1.

Another approach is taken by Dupor, Kitamura, and Tsuruga (forthcoming), who merge sticky information with the Calvo model of price-adjustment. They assume that every period, each firm has a random chance of updating its information (as in the sticky-information model), while an independent random event determines whether the firm can reset its price (as in the Calvo model). They find that this model empirically dominates the hybrid Phillips curve of Gali and Gertler (1999) and others.

Others have merged partial information with the Calvo model. See, in particular, Morris and Shin (2006), Nimark (2008), and Angeletos and La'O (2009a). Nimark's (2008) results are similar to Dupor and Kitamura, while Morris and Shin (2006) and Angeletos and La'O (2009a) focus instead on the inertia of forward-looking expectations and the dynamics of higher-order beliefs.

Another branch of work has merged imperfect information with fixed costs of changing prices as in state-dependent pricing models. Knotek (2006) does this for the sticky-information model. He finds that the model can fit well the micro facts from the price data, while keeping most of the predictions for aggregate supply as in the sticky-information model. Gorodnichenko (2008) examines a state-dependent

⁴² The once-a-year adjustment matches the survey responses in Blinder, Canetti, Lebow and Rudd (1998), suggesting that perhaps firm managers were responding how often they adjusted their price plans, rather than the actual prices. This is plausible since many of the predetermined changes look like sales.

⁴³ Matejka (2008) shows that the optimal distribution of signals from a particular rational inattention problem has point masses so that a discrete set of signals and prices are chosen. Chen, Levy, Ray and Bergen (2008) document that price increases tend to be small while declines tend to be large, and after ruling out other explanations conclude in favor of information-based theories. Knotek (2008) finds that "convenient" prices are more likely in locations where transactions must be made quickly.

model with partial information. He emphasizes the positive externality from a price change: when a firm chooses to adjust its price, it releases some of its private information to other firms.

Another merger of these various models has been proposed by Woodford (2009). He assumes that firms can pay a fixed information cost at discrete times to perform a price review, and when they do so they obtain full information about the state of the economy at that moment, just as in delayed information models. At the same time, he assumes that between these adjustment dates, firms obtain signals as in partial information models. The cost of an information update is fixed as in the theory of inattentiveness, while the informativeness of the signals is determined by a limited-capacity channel as in the theory of rational inattention. Under the extra assumption that the calendar date is also a costly piece of information, so the price plan must consist of a single number, Woodford (2009) shows that this model generalizes the state-dependent pricing model. In the limit where the channel capacity is infinite, the model is exactly like a conventional state-dependent pricing model while when the channel capacity is zero the model becomes isomorphic to the Calvo model. In between, for intermediate levels the model reproduces the generalized Ss model of Caballero and Engel (1999).

7.2 Heterogeneity in the frequency of information adjustment

Haltiwanger and Waldman (1989) studied the properties of equilibrium in models where some agents are informed, and so respond to shocks, while others are not. They showed that with strong strategic complementarity, the non-responders have a disproportionate effect on the equilibrium. Intuitively, the firms that obtain information want their prices to stay close to the those that are not adjusting, so the equilibrium ends up mimicking the lack of information of the uninformed. This may be clearer in the limit: as $\alpha \rightarrow 0$, firms want their price to equal the aggregate price level, so even if only a small fraction of firms do not have information on current shocks, the equilibrium will involve no firm responding to shocks at all.

Carvalho and Schwartzman (2008) proposed a sticky-information model with many sectors, where the frequency of information adjustment is different across sectors. Their important finding is that demand shocks are much more persistent in this economy than in an equivalent single-sector economy with the average frequency of information adjustment. Because of strategic complementarities, the sector that adjusts less often has a disproportionate effect on the aggregate dynamics since the other sectors want to keep their prices close to theirs.⁴⁴

7.3. Optimal policy with imperfect information

Ball, Mankiw and Reis (2005) study optimal monetary policy in a simple sticky-information economy. They show that price-level targeting is better than inflation-targeting. Because firms choose

⁴⁴ Carvalho (2006) makes the same point in Calvo sticky-price models, and Nakamura and Steinsson (forthcoming) discuss the interaction between heterogeneity and strategic complementarity in a menu cost model.

plans for prices and want to minimize their forecast errors well into the future, price-level targeting dominates inflation targeting. That is, base drift is quite costly. The optimal policy is an elastic price standard: there is a deterministic target for the price level and the central bank deviates from it when output is expected to deviate from its full-information level.

Jinnai (2006) and Branch, Carlson, Evans and McGough (2009) examine how policy choices affect the optimal frequency of information updating. The latter show that if the central bank becomes more concerned with inflation relative to output, this makes the firm's forecasting problem easier. It therefore ends up lowering the variance of output together with that of inflation. This mechanism may partially explain the "Great Moderation", and suggests a fruitful avenue for future research to test models of inattentiveness using historical changes in the volatility of inflation and the business cycle

Reis (2009) characterizes optimal policy rules in an estimated medium-scale model with pervasive sticky information. Relative to models with rigidity in agents' actions such as habits by consumers, sticky prices by firms, sticky wages by workers, and adjustment costs by investors, sticky information leads to a larger focus on stabilizing real activity. This is true both in terms of the optimal variance of output relative to inflation as well as in terms of the optimal policy-rule coefficients.

Adam (2007) studies optimal monetary policy in a simple partial information economy similar to the one we presented in Section 4.2. He shows that in response to persistent shocks, policy should stabilize the output gap in the short run, focusing on stabilizing the price level only in the medium run. Adam (2009) shows that with partial information, discretionary policy can be much more costly relative to commitment than with full information. He also confirms the Branch et al (2009) result described above in partial information economies: an increased focus on price stability may lower the variance of both inflation and output. Lorenzoni (forthcoming) extends the analysis of optimal monetary policy to a setting where all price setters have a common signal on productivity (similar to the policy announcement in Section 6.2).

Finally, Angeletos and Pavan (2007, 2009) provide a more general, but also more abstract, characterization of efficiency and optimal policy with incomplete information. They focus on the externalities that one agent's use of information imposes on others. Angeletos and La'O (2008) characterize optimal fiscal and monetary policy over the business cycle in a partial information economy.

7.4 Other choices with imperfect information

The resurgence of work on imperfect information models has not been constrained to the study of pricing decisions by firms. At the same time, an equally large literature has sprung up using very similar ideas and often the same authors, but applied to different questions in economics.⁴⁵

⁴⁵ There is also an active parallel work in finance, surveyed in the book by Veldkamp (2009).

Mankiw and Reis (2005) and Koenig (2004) considered sticky information on the part of workers setting wages to explain unemployment. Carroll, Slacalek and Sommer (2007) and Luo (2008) focused on consumption choices with sticky and partial information, respectively. Reis (2006b) investigated the inattentiveness model for consumers, while Tutino (2009) and Lewis (2007) extended the rational inattention model to deal with the dynamic decision of consumers. Angeletos and Pavan (2004) considered physical investment decisions.

A fruitful line of work has applied the inattentiveness model to portfolio choice. Gabaix and Laibson (2001) emphasized the potential for delayed information to explain the equity premium. Building on Duffie and Sun (1990) and Reis (2006b), Abel, Eberly and Panageas (2007) provided a micro-founded inattentiveness model of delayed adjustment and characterized its implications for portfolio choice and asset prices. Abel, Eberly, and Panageas (2009) combined delayed information with transaction costs and showed a remarkable result: the behavior of the consumer converges to time-dependent adjustments with constant intervals of inattentiveness, as if the transaction costs were not present. Huang and Liu (2007) study portfolio choice with rational inattention.

In an important contribution, Lorenzoni (2009) shifts the focus of imperfect information from the demand to the supply shock. He shows that a common signal about productivity can generate business cycles that resemble those due to demand shocks. Angeletos and La'O (2009b) consider partial information about shocks on tastes, productivity and desired markups. Finally, La'O (2009) applies the partial information model to financial contracting.

Finally, in the open economy literature, Bacchetta and van Wincoop (2006) considered a simple partial information model for traders in currency markets and showed this could explain some of the puzzling disconnect between exchange rates and fundamentals. Crucini, Shintani and Tsuruga (2008) used instead a delayed information model and showed it can explain volatile and persistent real exchange rate movements both in the aggregate and at the sectoral level. Bacchetta and van Wincoop (forthcoming) find that a delayed information model can explain the forward discount puzzle.

7.5 DSGE models with imperfect information

Dynamic stochastic general equilibrium modeling, surveyed by Christiano in a chapter in this handbook, has been an active area of intersection between academic and central-bank researchers. The first DSGE models with imperfect information have recently appeared, and this is likely an area of much future work.

In a series of papers, Mankiw and Reis (2006, 2007) and Reis (2009, forthcoming) put forward a first DSGE model with sticky information in all markets.⁴⁶ In their model, firms when setting prices,

⁴⁶ There had been some previous attempts, by Trabandt (2004), Andres et al (2005), Kiley (2007), Laforte (2007), and Korenok and Swanson (2005, 2007) with sticky information only on the part of firms. Mankiw and Reis (2006) criticized this work and argued that information stickiness should be pervasive across all markets, both on grounds

households when choosing consumption, and workers when setting reservation wages are all allowed to be inattentive, and estimates using both Euro-area and U.S. data show that sticky information is pervasive across all of these markets. Their work also contributed algorithms to solve medium to large-scale models with sticky information, and to evaluate likelihood functions.⁴⁷

Mackowiack and Wiederholt (2010) propose a DSGE model with partial information. They show that the utility and profit losses from inattentive behavior are small even though the aggregate dynamics are significantly different than the full-information alternative. Moreover, by allowing for different shocks and different signals, as explained in Section 4.3, they find that these individual losses are significantly smaller than those in standard sticky-price models.

The models above still involve some simplifications to make the information heterogeneity manageable. In particular, it is often difficult to define equilibrium in markets where both sellers and buyers are inattentive. This is an active area of work.⁴⁸

8. Conclusion

Since the birth of business cycle theory, economists have struggled with one overarching question: What is the nature of the market imperfection, if any, that causes the economy to deviate in the short run from full employment and the optimal allocation of resources? Or, to put the question more concretely and more prosaically in terms of undergraduate macroeconomics, what friction causes the short-run aggregate supply curve to be upward sloping rather than vertical, giving a role to aggregate demand in explaining economic fluctuations? The theme of the literature surveyed here is that the answer is to be found in the natural uncertainty of economic conditions coupled with peoples' inherent limitations in obtaining and processing information.

The models described here build on much of traditional macroeconomics. In his 1936 classic *The General Theory*, John Maynard Keynes emphasized vast uncertainty as a key fact of economic life; his famous "beauty contest" parable relates closely to the common-knowledge problem we described earlier. Similarly, in his 1968 AEA presidential address, Milton Friedman stressed the failure of some agents to correctly perceive monetary conditions as an explanation for the short-run Phillips curve—a theme that pervades the models surveyed in this chapter.

These models are also tied to more recent themes in macroeconomic research. The models examined here are all solved using mathematical tools that economists developed during the rational expectations revolution of the 1970s. But in contrast to early rational expectation theory, these models

of methodological consistency and, more importantly, because such pervasive stickiness empirically helps to fit the U.S. data.

⁴⁷ Meyer-Gohde (forthcoming) has improved on these algorithms significantly, and his publically available programs make the solution and estimation of sticky-information models as easy as conventional rational-expectations models.

⁴⁸ Reis (2009b) discusses the existing open questions on micro-founding sticky information in general equilibrium.

typically assume agents make decisions based on a much more limited set of information. This assumption of restricted information has been made more palatable in recent years by the growth of behavioral economics, which has stressed imperfections in human cognition.

Despite building on a long tradition, models on imperfect information and aggregate supply are still in their infancy. Without doubt, much progress has been made in recent years, and we hope this chapter has given readers a taste of this research and some leads about where to learn more. This line of work still offers many attractive open questions concerning macro theory, empirics, and policy. We expect it to remain a fruitful area of research in the years to come.

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Figure 1. Choosing between price and quantity plans

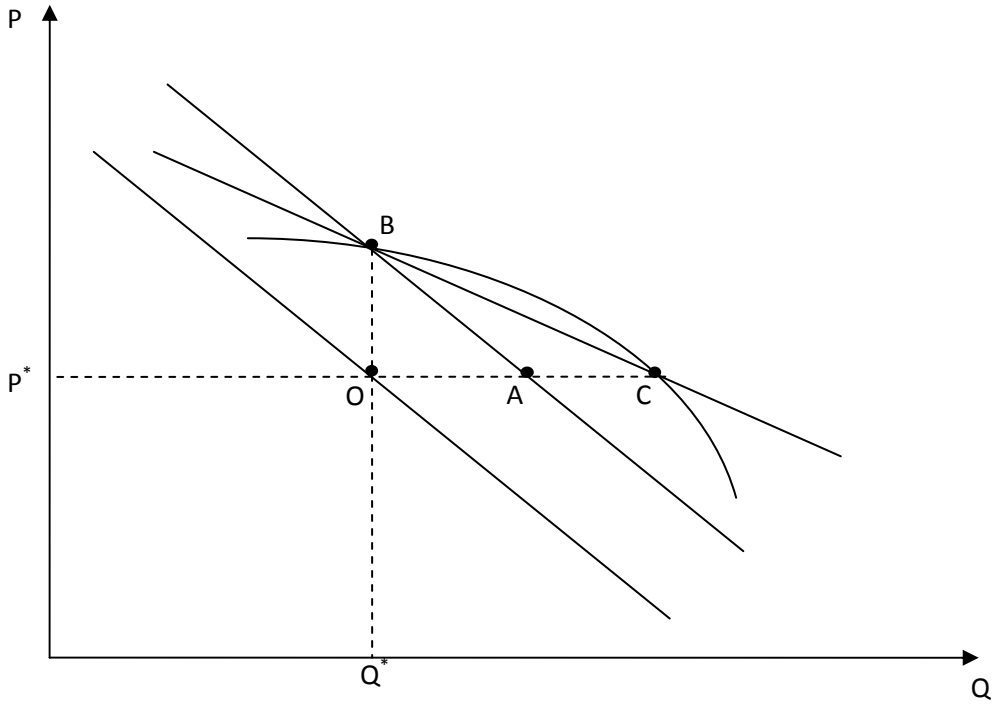


Figure 2. Profits if inattentive while all other firms are fully informed

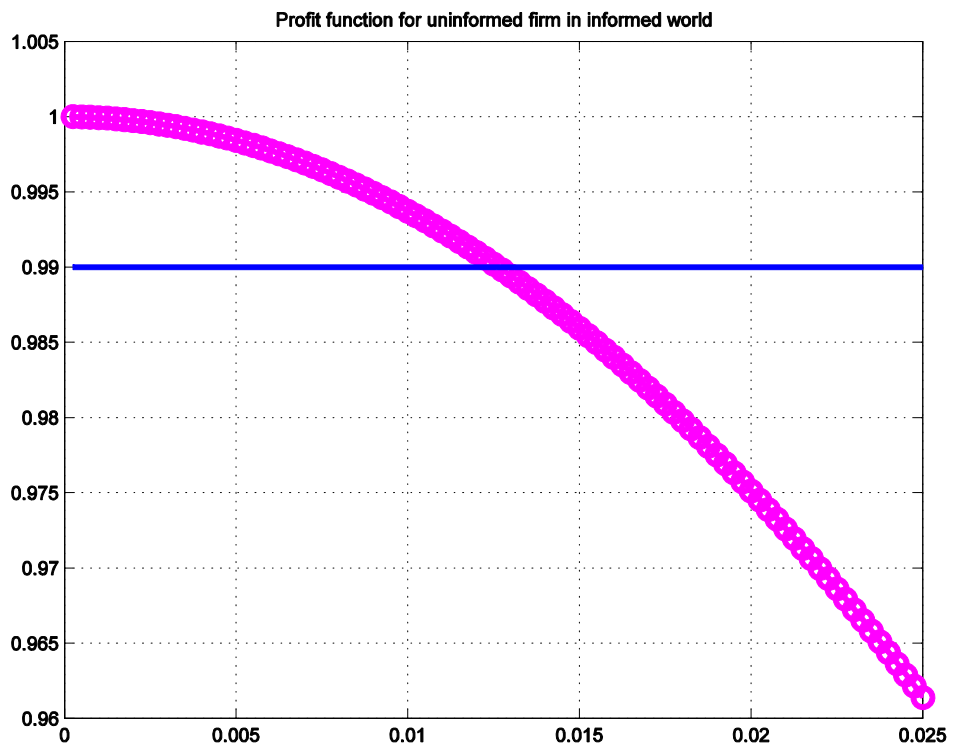


Figure 3. Impulse response of inflation and output to nominal demand shocks with delayed information

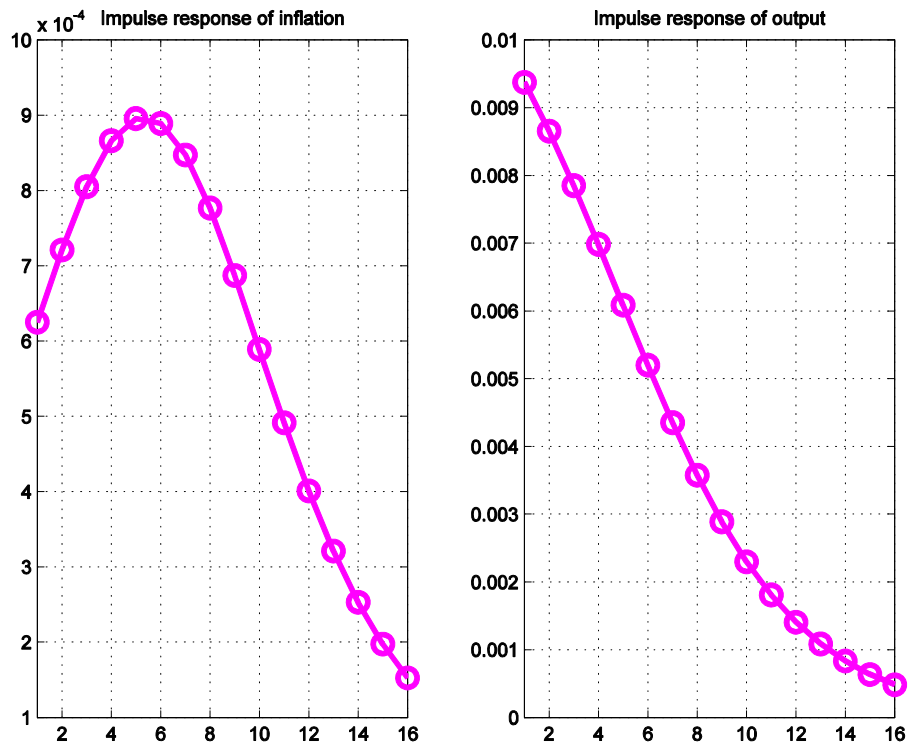


Figure 4. Impulse response of inflation and output to nominal demand shocks with partial information

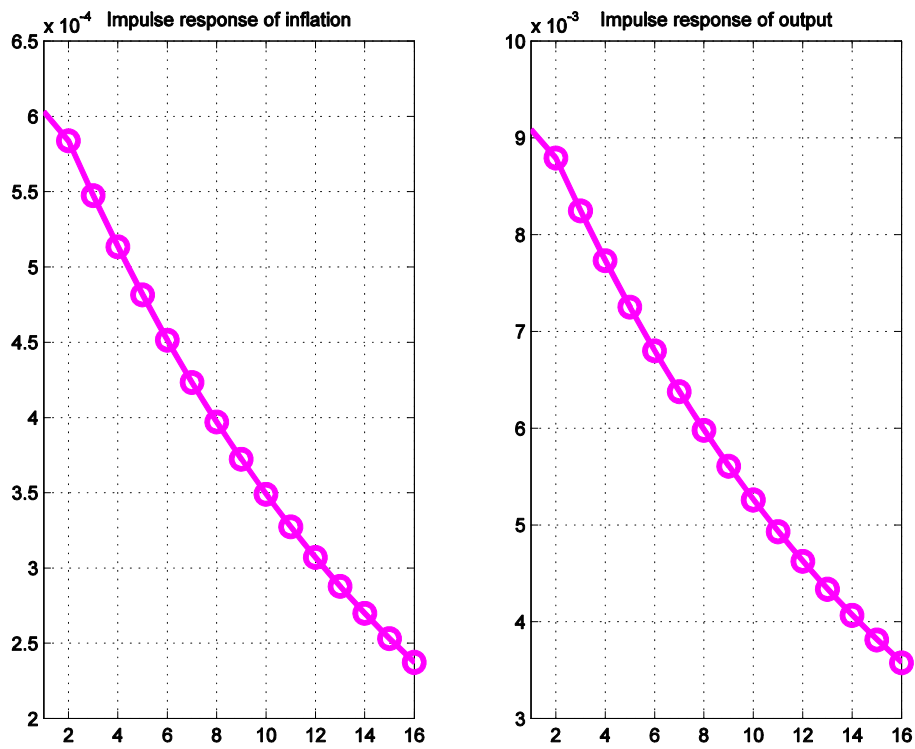


Figure 5. Impulse response of disagreement with delayed information

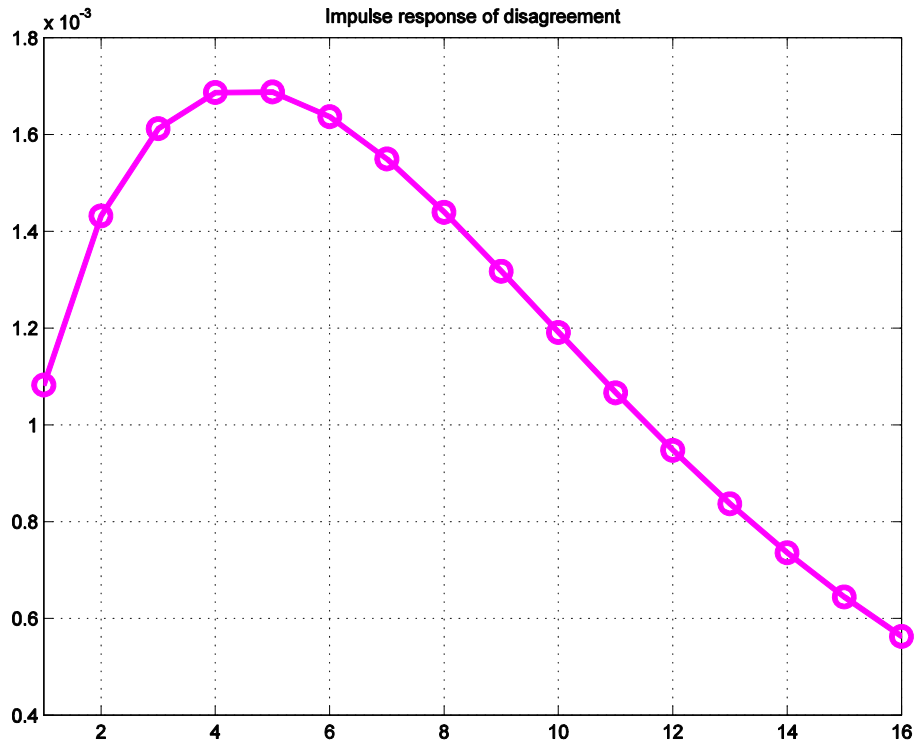


Figure 6. Disagreement during the Volcker years (from Mankiw, Reis and Wolfers (2004))

