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Trade Liberalization and Firm Dynamics

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1.0 Introduction

Firms operating in the same country and industry respond to globalization in different ways. Empirical work using microlevel data on firms or plants initially highlighted this contrast for export decisions and how this decision correlates with observable firm-performance measures such as size and productivity: Only a subset of relatively bigger and more productive firms export.[1] Subsequent work documented a wide-ranging set of other responses to globalization that consistently vary across firms in the same country and industry and also are strongly correlated with firm-level performance measures: the number and location of export destinations, entry and exit from the domestic market, range of products produced, international organization of production (including but not limited to multinational status and outsourcing/offshoring decisions), and innovation activities such as R&D.

This empirical work prompted the development of models with heterogeneous firms in open economies that capture how changes in the extent of globalization (i.e., indexing frictions to trade and foreign investment and the size of the global economy) influence those varied firm-level responses. These models highlight the composition effects that are induced by the different firm-level responses to globalization. To focus on those composition effects, the majority of models examine cases in which the firms' responses to globalization (i.e., export decisions, organization of production, and

[1] Throughout this chapter, we refer to firms as the microlevel unit of production. Some of the empirical evidence we reference is based on more disaggregated plant-level data, which differentiates between a firm's different production locations.

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innovation) do not change over time. This assumes a stable aggregate environment with no firm dynamics.

In this chapter, we focus on a relatively new direction in this literature, examining how firm responses evolve over time to changes in the extent of globalization, with an emphasis on trade liberalization. Recent empirical work documented the dynamic interactions among those firm responses, particularly between the export decision and innovation: Trade liberalization increases the return to both firm activities, and each activity additionally affects the returns to the other. Clearly, such interactions induce the mapping between firm characteristics and their export and innovation decisions to change over time. This provides an important rationale for the study of models that incorporate those dynamic interactions. Furthermore, the aggregation of those changing firm decisions, along with the evolution of entry and exit, generate substantial differences in the economy's overall response to globalization over time. This implies that an analysis of steady-state outcomes may give a misleading summary of the overall effects of liberalization. This is the second rationale for studying models that feature dynamic interactions in firm responses to liberalization: They are able to contrast the response of key economic aggregates to trade liberalization at different time horizons.

In the next section, we review theoretical models that capture some of these dynamics, and we summarize the associated empirical evidence. Our focus herein is to explain and motivate in a unified manner how key modeling ingredients of firm dynamics interact to generate endogenous aggregate-transition dynamics. We develop a range of variants of this type of model to isolate different interactions and mechanisms. We review some of the simpler variants analytically and computationally derive the transition dynamics for the others. We show how the response of innovative activities magnifies the productivity differences between exporters and nonexporters. We examine how the responses of average productivity and trade flows can be significantly different to those in static-trade models, and how these responses can differ substantially at short- and long-term horizons, depending on the responses of entry, innovation, and timing of trade liberalization.

We restrict our analysis to one type of international-market-participation decision – that is, export to a single destination – and one type of technology choice – that is, innovation intensity; we focus on the key interactions between these two firm decisions in response to trade liberalization. We incorporate fixed export costs (potentially sunk) but assume that trade liberalization takes the form of reductions in the per-unit export costs.

We separately consider two cases for innovation: One in which innovation is exogenous to the firms, so that firm productivity evolves stochastically but independently from any firm decisions; the other in which innovation is endogenous and differs across firms and responds to changes in the aggregate-trading environment. In all cases, incumbent firms also make endogenous exit decisions. When trade is liberalized, the aggregate-transition dynamics reflect a combination of all of those decisions by incumbent firms as well as the aggregate response of entry. We highlight how the interaction of both firm dynamics and endogenous export-market selection jointly induce long-lasting aggregate transition dynamics in response to a one-time unanticipated drop in that variable trade cost. We discuss how comparisons of consumption across steady states can significantly overstate or understate welfare measures that take into account transition dynamics. We also show how expectations regarding future trade costs give rise to their own transition dynamics: because the drop in the trade cost either is anticipated ex-ante or is not expected to last ex-post (i.e., the trade liberalization is temporary).

To highlight the importance of the interaction between firm dynamics and export-market selection, we develop analytical models that feature either one or the other, and we show how those models do not generate any endogenous transition dynamics in response to trade liberalization. These analytical cases show that the entrants' expectation regarding future potential export profits (relative to incumbent firms) is a key factor generating aggregate-transition dynamics. Endogenous innovation by firms further amplifies productivity differences between exporters and nonexporters and generates longer-lasting transition dynamics. We show that trade liberalization even induces some nonexporters to increase their innovation activities (despite facing stronger competition in their domestic market) because they anticipate exporting in the future and respond to the associated higher returns postliberalization. All of these dynamic effects imply large differences over time in the response of aggregate-trade volumes to trade liberalization: Long-run trade elasticities (relative to the fall in trade costs) are substantially higher than the corresponding short-run elasticities.

We show that firms' expectations regarding trade liberalization are particularly relevant when innovation is endogenous. If trade liberalization is expected to be only temporary, then the amplification effect of endogenous innovation is weakened – and is no longer significantly different than for the exogenous innovation case. Conversely, when trade liberalization is anticipated ex-ante, endogenous innovation induces a significant rise in innovation ahead of the drop in the trade cost. Differences in the response

of innovation between exporters and nonexporters also amplify (endogenously) productivity differences between the two groups of firms. Finally, we incorporate sunk-trade costs to examine how trade liberalization affects the option value of becoming an exporter. This leads to additional anticipation effects ahead of the change in trade costs.

The remainder of the chapter is organized as follows. Section 2.0 is a brief overview of the recent literature on trade liberalization and firm dynamics. Section 3.0 introduces our model and characterizes the equilibrium with two symmetric countries. Section 4.0 describes our model parameterization. Section 5.0 presents the theoretical and quantitative results for the different trade-liberalization scenarios, which are designed to isolate the separate roles of export-market participation, firm dynamics, endogenous innovation, sunk-export costs, and anticipation effects regarding trade liberalization. Section 6.0 is the conclusion. An appendix presents proofs of our analytic results.

2.0 Literature Overview

Workhorse general equilibrium models of heterogeneous firm dynamics in closed economies (e.g., Hopenhayn 1992; Atkeson and Kehoe 2005; and Luttmer 2007) recently have been extended to open-economy settings.[2] For example, Arkolakis (2011) and Impullitti, Irarrazabal, and Opromolla (2012) consider dynamics extensions of Melitz (2003), in which firms experience exogenous random shocks to their productivity, to account for salient features of the data on firm dynamics by domestic and exporting firms. Whereas these papers focus on stable aggregate environments, Alessandria and Choi (2007) and Ruhl (2008) examine the transition dynamics to trade liberalizations, focusing on the role of entry into domestic and export markets.[3] Alessandria and Choi (2007) show, as we do herein, that welfare calculations based on steady-state consumption comparisons can give significantly different answers than welfare calculations that take into account transition dynamics across steady states. They also consider physical capital, from which we abstract, as another source of endogenous dynamics as in the neoclassical growth model. Other papers examine more specifically how firms make joint decisions regarding both export status and technology choice. For example, Bustos (2011), and Yeaple (2005) consider static

[2] See Luttmer (2010) for a comprehensive survey of aggregate models of firm dynamics.

[3] Buera and Shin (2009) examined the transition dynamics of aggregate productivity in response to reforms, such as capital-account liberalizations and removal of various microdistortions in economies with underdeveloped financial markets.

models in which there is a binary technology choice, and they highlighted how firms jointly decide to enter export markets and to adopt the new technology (or to do neither).[4] Costantini and Melitz (2009) extend this type of joint decision to a dynamic framework in which firms face both idiosyncratic uncertainty and sunk costs for both exporting and technology adoption. The sunk costs and uncertainty combine to generate option values for export and exit decisions. We explore the effects of those option values in this chapter. Our model of innovation follows closely the model developed by Atkeson and Burstein (2010), which builds on Griliches's (1979) model of knowledge capital. As in Ericson and Pakes (1995) and more recently in Doraszelski and Jaumandreu (2011), the fruits of innovative activity are stochastic, so the model can account for simultaneous growth and decline and for entry and exit of firms in steady state. Whereas Atkeson and Burstein (2010) focus on the offsetting responses of exit, export, innovation, and entry decisions to permanent trade liberalizations and the offsetting effects of changes in these decisions on aggregate productivity and welfare, we focus on the effects of changes in these decisions on transition dynamics of average productivity, trade flows, and output, allowing for sunk-export costs considering both temporary and anticipated liberalizations.

In this chapter, we model firm dynamics arising from changes over time in productivity or product quality that affect firms' production for all markets simultaneously; however, a number of recent papers focus on demand dynamics as firms accumulate customers in foreign markets.[5] Ruhl and Willis (2008) introduce demand shifters that grow over time to explain the slow growth of exporters as they enter new markets. Eaton et al. (2008) model firms' investments directed at increasing foreign demand (e.g., searches for foreign buyers and maintaining existing relationships with current buyers), in response to export-market entry. Such demand-related investments have effects similar to innovation except that their returns affect only export profits as opposed to overall profits. Chaney (2011) models the growth of exporters as they meet foreign importers in international social networks.[6] There also are a number of recent papers featuring models of

[4] See also the related work of Ederington and McCalman (2008); Navas-Ruiz and Sala (2007); van Long, Raff, and Stähler (2011); and Rubini (2010).

[5] Foster, Haltiwanger, and Syverson (2010) document the importance of demand accumulation in accounting for U.S. plant dynamics in a number of U.S. manufacturing sectors.

[6] Drozd and Nosal (2012) present a macroeconomic model of customer accumulation to account for salient features of international relative prices and the dynamics of aggregate trade flows. Alessandria, Midrigan, and Kaboski (2010) present a model of trade and

firm dynamics driven by frictional labor markets (e.g., Cosar et al. 2010; Cosar 2011; Dix-Carneiro 2010; Fajgelbaum 2010; Kambourov 2009). These models can generate slow transitions from trade liberalization as workers reallocate across firms and sectors.

Empirical work using micro-level data confirms the importance of dynamics in explaining firms' export behavior. This behavior also is reflected in aggregate-export patterns: New exporters initially account for a small proportion of aggregate exports, but those exporters grow faster than both established exporters and nonexporters. They account for a substantial portion of aggregate-export growth over longer periods (i.e., more than 40 percent for the U.S. export growth from 1987 to 1992, and more than 50 percent for the export growth in both Colombia and Morocco from 1984 to 1991; Bernard and Jensen 2004a; Roberts and Tybout 1997). Hysteresis effects are a major driver of those aggregate-export dynamics: Past export experience explains a significant proportion of a firm's current and future export performance, even after controlling for all observable firm-performance indicators.[7] This type of hysteresis behavior is explained by a combination of sunk-export costs and firm-level uncertainty (e.g., a form of stochastic firm dynamics, which could be generated simply by shocks exogenous to the firm). Das, Roberts, and Tybout (2007) econometrically measure sizable sunk-export costs for Colombian exporters, which induce effects of firms' expectations regarding future export-market conditions on current export behavior. Ruhl (2008) shows that this combination of sunk-export costs and idiosyncratic firm uncertainty also explains how the elasticity of trade with respect to changes in trade costs or aggregate productivity can vary substantially depending on the perceived persistence of those changes. Bergin and Lin (2012) documents another example in which firms' expectations about the future aggregate-trading environment induces noticeable changes in firms' current export-market-entry decisions. They show that European firms enter export markets prior to the implementation of European Monetary Union (EMU) (and the associated decreases in trading frictions).

Another driver of the rapid export growth by new exporters is the link between export-market entry and firm innovation. Many recent papers

inventory management to account for the dynamics of aggregate-trade flows and prices in the aftermath of large devaluations.

[7] See Roberts and Tybout (1997) for evidence in Colombia; Bernard and Wagner (2001) in Germany; and Bernard and Jensen (2004a) in the United States.

using micro-level data document this link: Lileeva and Trefler (2010) for Canada; Verhoogen (2008) for Mexico; Bustos (2011) for Argentina; and Aw, Roberts, and Xu (forthcoming) for Taiwan:[8] They all found that a firm's export-market entry (driven by reductions in trade costs) is associated with increased innovation. Bloom, Draca, and Van Reenen (2008) document a similar relationship on the import competition side between changes in the trading environment and firm innovation and skill upgrading: Firms in European industries most exposed to increased import competition from China respond by increasing their innovation and information-technology intensity.[9]

3.0 Model Economy

In this section, we present our model of trade liberalization and firm dynamics. Time is discrete and each period is labeled $t = 0, 1, 2, \ldots$. We do not model any aggregate uncertainty. The economy has two symmetric countries: home and foreign (foreign variables are denoted with an asterisk superscript). Given the symmetry across countries, we focus on only the variables relevant for the home country. Households inelastically supply L units of labor and derive utility only from consumption. Production in each country is structured as follows. There is a final nontraded consumption good that is produced using a continuum of differentiated intermediate goods with a constant-returns-to-scale Constant Elasticity of Substitution (C.E.S.) technology with elasticity of substitution $\rho > 1$.[10] These intermediate goods can be traded internationally subject to fixed and variable trade costs. Firm productivity in the intermediate-goods sector is determined endogenously by a firm's choice regarding innovation intensity. Firm entry and exit are endogenous and determine the aggregate measure of intermediate goods available in each country.

[8] There is another strand of the empirical literature that documents a reduced-form relationship between export-market entry and subsequent firm-productivity growth (often labeled as learning by exporting). See Greenaway and Kneller (2007) and Lopez (2005) for recent surveys.

[9] Bernard, Jensen, and Schott (2006) document a similar effect for import competition on the capital and skill intensity of the affected U.S. firms (i.e., increases in competition from low-wage exporters associated with increases in both capital and skill intensity).

[10] By assuming a demand structure with constant markups, we do not allow for strategic considerations in pricing. This could generate interesting implications for different returns to innovation across firms (Aghion et al. 2005).

A firm in the home country with state z has productivity equal to $\exp(z)^{1/(\rho-1)}$ and produces output $y_t(z)$ with labor $l_t(z)$ according to the constant-returns-to-scale production technology:[11]

$$y_t(z) = \exp(z)^{1/(\rho-1)} l_t(z). \tag{1}$$

In addition, every operating firm must pay an overhead fixed cost equal to f units of labor in every period. We rescale firm productivity using the exponent $1/(\rho-1)$ for expositional convenience: As we explain herein, this rescales a firm's size, variable profits, and production employment (i.e., net of the overhead cost) to be proportional to $\exp(z)$.

Differentiated intermediate goods produced in home can be used for domestic production of the final good or exported to foreign for use in foreign's final goods production. We let $a_t(z)$ denote the domestic absorption of firm z's production. Similarly, we let $a_t^*(z)$ denote the quantity of firm z's production used in foreign's final goods production. Exports by firm z incur both a fixed cost f_X (measured in units of domestic labor) and a per-unit cost. The latter takes the form of an iceberg cost equal to $\tau - 1$ units for each unit of the good exported (with $\tau \geq 1$). Due to the fixed export cost, it is unprofitable for some firms to export. We let $x_t(z) \in \{0, 1\}$ denote the export indicator for firm z at home ($x_t = 1$ if the firm exports; 0 otherwise). Feasibility requires:

$$a_t(z) + x_t(z)\,\tau\, a_t^*(z) = y_t(z). \tag{2}$$

A firm in the foreign country with state z has the same production technology as the home firm but with output denoted $y_t^*(z)$, production labor $l_t^*(z)$, and domestic absorption $b_t^*(z)$. Exports to the home country, $b_t(z)$, are subject to both fixed and per-unit costs; hence, feasibility requires that $x_t^*(z)\,\tau\, b_t(z) + b_t^*(z) = y_t^*(z)$ and that f_X units of foreign labor be used to pay the fixed export cost for all foreign exporting firms (with $x_t^*(z) = 1$).[12]

The constant-returns-to-scale C.E.S. production technology for the final good implies that the total quantity of the good produced at home is

[11] Because we do not make any assumptions regarding physical quantity units for the differentiated intermediate goods, firm productivity in that sector can be directly reinterpreted as product quality. In this alternative variant, firms innovate to improve product quality rather than productivity. This reinterpretation does not change any of our findings. Our model also can be extended to include other forms of physical and human capital as variable factors of production. Consideration of these forms of capital would lead to the standard amplification of the impact of a change in productivity on aggregate output.

[12] Symmetry between home and foreign implies that $y_t(z) = y_t^*(z)$, $x_t(z) = x_t^*(z)$, $a_t(z) = b_t^*(z)$, and $a_t^*(z) = b_t(z)$.

given by:

$$Y_t = \left[\int a_t(z)^{1-1/\rho} M_t(z)\,\mathrm{d}z + \int x_t^*(z)\, b_t(z)^{1-1/\rho} M_t^*(z)\,\mathrm{d}z \right]^{\rho/(\rho-1)}, \tag{3}$$

where $M_t(z)$ is the distribution of operating firms in the home country over the productivity index z and M_t^* is the corresponding distribution in the foreign country. The total measure of operating firms in the home country is given by $\int M_t(z)\,\mathrm{d}z$. Production of the final good in the foreign country, Y_t^*, is defined analogously.

The final goods sector in both countries is competitive. We let P_t denote the final goods price in home; $p_{at}(z)$ and $p_{bt}(z)$ denote the prices of the domestic and imported intermediate goods in the home country. We choose labor as the numéraire good and normalize the wage to 1. Final goods producers take these prices as given; they also take all of the decisions by the intermediate-goods firms (which determine $x_t(z)$, $x_t^*(z)$, $M_t(z)$, $M_t^*(z)$) as given. The final goods price thus is given by its unit cost:

$$P_t = \left[\int p_{at}(z)^{1-\rho} M_t(z)\,\mathrm{d}z + \int x_t^*(z)\, p_{bt}(z)^{1-\rho} M_t^*(z)\,\mathrm{d}z \right]^{1/(1-\rho)}. \tag{4}$$

The demand for the intermediate goods at home then is given by:

$$\frac{a_t(z)}{Y_t} = \left(\frac{p_{at}(z)}{P_t} \right)^{-\rho} \quad \text{and} \quad \frac{b_t(z)}{Y_t} = \left(\frac{p_{bt}(z)}{P_t} \right)^{-\rho}. \tag{5}$$

Analogous equations hold for prices and quantities in the foreign country.

Intermediate-goods firms in each country are monopolistically competitive. A home firm with productivity index z faces a static profit-maximization problem involving the choice of labor input $l_t(z)$; prices $p_{at}(z)$; $p_{at}^*(z)$; quantities $a_t(z)$, $a_t^*(z)$; and an export decision $x_t(z)$. In doing so, the firms take as given the prices and quantities of the final goods in both countries (we recall that the wage is normalized to 1). Firm z's profit-maximization problem is:

$$\Pi_t(z) = \max_{\substack{y_t(z), l_t(z), p_{at}(z), p_{at}^*(z), \\ a_t(z), a_t^*(z), x_t(z)}} p_{at}(z) a_t(z) + x_t(z)\, p_{at}^*(z) a_t^*(z) - l_t(z) - x_t(z)\, f_X \tag{6}$$

subject to (1), (2), and (5). We define $\bar{z}_{Xt}$ as the export productivity cutoff: $\bar{z}_{Xt} \equiv \arg\min \{x_t(z) = 1\}$.

We now describe the productivity dynamics of individual firms. At the beginning of each period t, every existing firm faces an exogenous probability δ of incurring a "death" shock that would induce exit (i.e., exogenous exit unconditional on productivity). The remaining $1-\delta$ portion of firms still can choose to exit (i.e., endogenous exit conditional on productivity) or to continue to operate and pay the overhead cost f. Productivity for those firms then evolves over time depending on their investment in innovation, which stochastically improves productivity. We model the evolution of productivity for producing firms as follows: In the following period $(t+1)$, a firm with productivity index z has a probability q of having productivity $\exp(z+\Delta_z)^{1/(\rho-1)}$ and a probability $1-q$ of having productivity $\exp(z-\Delta_z)^{1/(\rho-1)}$. Hence, the expected growth of z is given by $(2q-1)\,\Delta_z$.[13] We examine a special case in which $\Delta_z = 0$ so that productivity is constant throughout a firm's life; this is the case with *no productivity dynamics*.

In the specification of our model with *endogenous innovation*, each firm invests in innovation by choosing the expected growth of its current productivity z (determined by q). A firm with productivity z choosing an innovation intensity q must hire $\exp(z)c(q)$ units of labor as its investment in innovation. We assume that $c(q)$ is increasing and convex in q.

With this evolution of firm productivity, the expected discounted present value of profits (abstracting from aggregate uncertainty) for a firm with initial productivity index z is:

$$V_t(z) = \max\left[0,\ V_t^o(z)\right], \tag{7}$$

where the value for operating firms is given by the following Bellman equation:

$$\begin{aligned} V_t^o(z) = \max_{q\in[0,1]} \Pi_t(z) - \exp(z)c(q) - f \\ + (1-\delta)\frac{1}{R_t}\left[q\,V_{t+1}(z+\Delta_z) + (1-q)\,V_{t+1}(z-\Delta_z)\right]. \end{aligned} \tag{8}$$

R_t is the world interest rate in period t (in units of labor). We let $q_t(z)$ denote the optimal innovation intensity of the firm referenced in (8). This choice of innovation intensity must satisfy the first-order condition:

$$\exp(z)\,c'(q) = (1-\delta)\frac{1}{R_t}\left[V_{t+1}(z+\Delta_z) - V_{t+1}(z-\Delta_z)\right]. \tag{9}$$

[13] We note also that if the time period is small, then our binomial-productivity process approximates a geometric Brownian motion in continuous time, as in the work of Luttmer (2007).

We note that with our scaling of the innovation-cost function, $\exp(z)$, we are assuming that the innovation cost required to increase the size of the firm by a fixed percentage scales with the size of the firm. This implies that for sufficiently large firms, the innovation decision and derived growth rate is independent of size, consistent with Gibrat's law, and the firm-size distribution has a Pareto right tail. We denote by $\bar{q}_t$ the innovation intensity for such very large firms (i.e., $\bar{q}_t = \lim_{z\to\infty} q_t(z)$).

In our quantitative analysis, we assume that the innovation-cost function has the form $c(q) = h\exp(bq)$ so that the curvature of this function is indexed by the parameter b. If this curvature parameter b is high (or low), then innovation is highly inelastic (or elastic) to changes in the incentives to innovate. With a very high curvature parameter b, innovation decisions of firms are effectively constant both across firms and over time, $q_t(z) = \bar{q}$. This is the *exogenous innovation* case.

Because the value function of operating firms $V_t^0(z)$ is strictly increasing in z, the endogenous exit or operate decision (7) must follow a cutoff rule: Firms with productivity at or above a cutoff $\bar{z}_t$ choose to operate, whereas the remaining firms choose the exit option. We note that if $f = 0$, then $V_t^o(z) = V_t(z)$ and $\bar{z}_t = -\infty$; hence, there is no endogenous exit.

New firms are created with an investment of f_E units of labor, which yields a new firm in the following period with an initial productivity index z drawn from a distribution $G(z)$. In any period in which new firms enter, free entry requires that:

$$f_E = \frac{1}{R_t}\int V_{t+1}(z)G(z)\,\mathrm{d}z. \tag{10}$$

We let M_{Et} denote the measure of new firms entering in period t that start producing in period $t+1$.

Households in the home country have preferences of the form $\sum_{t=0}^{\infty}\beta^t\log(C_t)$, where C_t is their consumption of the home final good in period t and $\beta \leq 1$ is their discount factor. Each household in the home country faces an intertemporal budget constraint of the form:

$$\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\frac{1}{R_{j-1}}\right)(P_tC_t - L) \leq \int V_0(z)\,M_0(z)\,\mathrm{d}z, \tag{11}$$

with $R_{-1} = 1$. In writing the budget constraint (11), we assume that the free-entry condition holds with equality in every period and that the representative consumer in each country owns only the firms in their respective

countries. The right-hand side thus represents the initial home-consumer wealth from the existing stock of home firms in period 0.

With balanced trade (which is immediate with symmetric countries), production and consumption of the final good are equalized in every period: $C_t = Y_t$. Without loss of generality, we normalize aggregate labor supply to 1. This labor is used for production of intermediate goods (L_{Pt}); innovation (L_{It}); and to cover the fixed costs for entry, overhead production, and export:[14]

$$L_{Pt} + L_{It} + M_{Et} f_E + \int [f + x_t(z)\, f_X]\, M_t(z)\, \mathrm{d}z = 1,$$

where

$$L_{Pt} = \int l_t(z) M_t(z)\, \mathrm{d}z \quad \text{and} \quad L_{It} = \int \left[\exp(z) c(q_t(z))\right] M_t(z)\, \mathrm{d}z. \tag{12}$$

The evolution of the distribution of operating firms M_t over time is given by the exogenous probability of exit δ, the decisions of operating firms to invest in their productivity $q_t(z)$, and the measure of entering firms in period t, M_{Et}. The distribution of operating firms $M_{t+1}(z')$ in the home country in period $t+1$ is equal to the sum of three inflows of firms: new firms that entered in period t; firms continuing from period t that draw positive productivity shocks (and thus had productivity equal to $z' - \Delta_z$ in period t); and firms continuing from period t that draw negative productivity shocks (and thus had productivity equal to $z' + \Delta_z$ in period t). We write this as follows:

$$M_{t+1}(z') = \begin{cases} M_{Et} G(z') + (1-\delta) q_t(z-\Delta_z) M_t(z-\Delta_z) \\ \qquad + (1-\delta)\left[1 - q_t(z+\Delta_z)\right] M_t(z+\Delta_z) & \text{for } z' \geq \bar{z}'_{t+1} \\ 0 & \text{for } z' < \bar{z}'_{t+1} \end{cases}. \tag{13}$$

An equilibrium in this economy is a collection of sequences of aggregate prices $\{R_t,\ P_t,\ P_t^*\}$ and prices for intermediate goods $\{p_{at}(z),\ p_{at}^*(z),\ p_{bt}(z),\ p_{bt}^*(z)\}$; a collection of sequences of aggregate quantities $\{Y_t,\ Y_t^*,\ C_t,\ C_t^*,\ L_{Pt},\ L_{Pt}^*, L_{It},\ L_{It}^*\}$ and quantities of the intermediate goods $\{a_t(z),\ a_t^*(z),\ b_t(z),\ b_t^*(z),\ l_t(z),\ l_t^*(z)\}$; and a collection of sequences of firm-value functions and profit, exit, export, and innovation decisions $\{V_t(z),\ V_t^*(z),\ V_t^o(z),\ V_t^{o*}(z),\ \Pi_t(s),\ \Pi_t^*(s),\ \bar{z},\ \bar{z}^*,\ x_t(z),\ x_t^*(z),\ q_t(z),\ q_t^*(z)\}$, together with distributions of operating firms and measures of entering

[14] We note that by assuming that fixed and innovation costs are incurred in terms of labor only, we assume that $\lambda = 1$ in the more general formulation of Atkeson and Burstein (2010).

firms $\{M_t,\ M_{Et},\ M_t^*,\ M_{Et}^*\}$. These equilibrium objects satisfy the following conditions: households in each country maximize their utility subject to their budget constraints, intermediate-good firms in each country maximize the net present value of their per-period profits, final-good firms in each country maximize profits, all of the feasibility constraints are satisfied, and the distribution of operating firms evolve as described herein.

As previously mentioned, we focus our analysis on symmetric equilibria across countries. To ensure this symmetry, we assume that the initial distribution of firms is identical across countries: $M_0(z) = M_0^*(z)$. This also ensures equality of initial wealth for home and foreign consumers. The symmetry between home and foreign then will be preserved in every subsequent period. Thus, all of the home-country variables introduced here will be equal to their foreign-country counterpart.

A steady state of our model is an equilibrium in which all of the aggregate variables are constant. In what follows, we omit time subscripts when discussing the steady state.[15] Depending on parameter values, there are two types of steady states in our model: one with and one without entry. The parameter restrictions required to have a steady state with entry imply that the equilibrium-innovation decision of large firms leads them to shrink in expectation.[16] Our results are derived in the case in which every period, there is positive firm entry.

Aggregation in a Symmetric Equilibrium

The equilibrium of our model with heterogeneous firms and endogenous innovation cannot be fully solved in closed form. Appendix A describes a simple algorithm to solve for the symmetric steady state. The online appendix of Atkeson and Burstein (2010) describes an algorithm to solve for the transition dynamics across steady states. We now present aggregate relations that hold in a symmetric equilibrium of our model that we use in our analytic results. We also define aggregate statistics that we report from our experiments.

[15] See Atkeson and Burstein (2010) for a detailed discussion of how a similar model of innovation by firms in a closed economy can be extended to allow for endogenous and semi-endogenous growth.

[16] A sufficient condition to obtain a steady state with entry is that $\lim_{z\to\infty}(1-\delta)\left\{q(z)\exp(\Delta_z)+[1-q(z)]\exp(-\Delta_z)\right\} < 1$. In a steady state without entry, this condition is satisfied with equality, and the expected growth of continuing firms is offset exactly by exit. In this case, aggregate variables are constant but the distribution of firms by size is not because production becomes concentrated in a shrinking number of large firms.

The solution to the static profit-maximization problem (6) in a symmetric equilibrium is:

$$\Pi_t(z) = \Pi_{Dt} \exp(z) + \max\left(\Pi_{Dt}\tau^{1-\rho} \exp(z) - f_X, 0\right), \tag{14}$$

where Π_{Dt} is a domestic market-demand index given by:

$$\Pi_{Dt} = \frac{P_t^{\rho} Y_t}{\rho^{\rho} (\rho - 1)^{1-\rho}}. \tag{15}$$

This market-demand index also proportionally scales production employment for all firms:

$$l_t(z) = (\rho - 1)\, \Pi_{Dt} \left[1 + x_t(z)\, \tau^{1-\rho}\right] \exp(z). \tag{16}$$

Given that firm revenues are proportional to firm employment, the ratio of exports to the aggregate value of production (for the intermediate-goods sector) is given by:

$$s_{Xt} = \frac{\tau^{1-\rho} \int x_t(z) \exp(z) M_t(z)\, \mathrm{d}z}{\int \exp(z) M_t(z)\, \mathrm{d}z + \tau^{1-\rho} \int x_t(z) \exp(z) M_t(z)\, \mathrm{d}z}. \tag{17}$$

Similarly, the ratio of exporters' revenues in the domestic market to total domestic revenues (i.e., the market share of exporters in their domestic market) is given by:

$$s_{Dt} = \frac{\int x_t(z) \exp(z) M_t(z)\, \mathrm{d}z}{\int \exp(z) M_t(z)\, \mathrm{d}z}.$$

This share of exporters in the domestic market differs from the share of exporters in total production in that it does not capture the direct impact of changes in trade costs on the size of exporters; instead, it reflects the reallocation of production between nonexporters and exporters in the domestic market.

Aggregating (16) across firms and using the expression for the aggregate price (4), we can express aggregate output Y_t as:

$$Y_t = Z_t^{\frac{1}{\rho-1}} L_{Pt}, \tag{18}$$

where:

$$Z_t = \int \left[1 + \tau^{1-\rho} x_t(z)\right] \exp(z) M_t(z)\, \mathrm{d}z \tag{19}$$

is an index of aggregate productivity. Average productivity for domestic production is given by:

$$\bar{Z}_t = \frac{\int \exp(z) M_t(z)\, \mathrm{d}z}{\int M_t(z)\, \mathrm{d}z}.$$

Because firm size on the domestic market is proportional to $\exp(z)$, this average productivity index also is proportional to the average firm size on the domestic market. Stated another way, for a given level of market demand Π_{Dt}, changes in average productivity $\bar{Z}_t$ are proportional to changes in average firm size on the domestic market.

4.0 Baseline Parameterization

To understand our model's aggregate dynamics in response to trade liberalization, we present both analytic and computational results. The latter are based on a calibration of the model that follows Atkeson and Burstein (2010). Here, we present an overview of this calibration, which is designed to target a number of salient features of U.S. data on firm dynamics, firm-size distribution, and international trade. We refer readers to Atkeson and Burstein (2010) for further details.

The parameters that we must choose are the steady-state real-interest rate $R = 1/\beta$, the distribution G of productivity draws of entrants, the parameters governing the variance of employment growth for surviving firms Δ_z, the exogenous-exit rate of firms δ, the marginal-trade costs τ, the fixed costs of operation f and entry f_E, the fixed costs of exporting f_X, the parameters of the innovation-cost function h and b, and the elasticity of substitution across intermediate goods in final output ρ.

Table 1 summarizes the targets and parameter values used in our baseline parameterization. We consider two choices of the curvature parameter b for the innovation-cost function: (1) b is sufficiently high that the innovation decision is effectively inelastic across firms and over time ($q_t(z) = \bar{q}$); and (2) $b = 10$ so that innovation decisions are quite elastic across firms and over time. We set β such that the steady-state interest rate (annualized) is 5 percent, and $\rho = 5$ consistent with estimates by Broda and Weinstein (2006). We choose the distribution G of productivity draws of entrants so that all firms enter with a common productivity index $z = 0$. (The cross-sectional distribution of z then is determined by the ergodic distribution generated by the productivity dynamics.) We choose Δ_z to match the standard deviation of the growth rate of employment of large U.S. firms. We choose the exogenous-exit rate δ to match the annual employment-weighted exit rate

Table 1. *Baseline parametrization*

Calibrated parameters	Exogenous (inelastic) innovation	Endogenous (elastic) innovation
Exogenous exit rate δ	0.005	0.005
Innovation step size Δz	0.25	0.25
Level of innovation cost function h	–	0.00108
. . . or implied Pareto coefficient (employment distribution of large firms)	−0.25	−0.25
Marginal-trade cost τ	0.231	0.231
Fixed costs of international trade f_x	1.4	0.285
Targets	**Data**	
Employment growth rate of large firms (standard deviation)	0.25	
Employment-based exit rate (firms with more than 500 employees)	0.0055	
Pareto coefficient for employment-based size distribution (1,000 to 5,000 employees)	−0.20	
Exports/gross output (intermediate goods in model)	0.075	
Employment share of exporters (production employment in model)	0.400	
Other parameters	**All Scenarios**	
Interest rate $1/\beta$	0.05	
Elasticity of substitution p	5	
Entry cost f_E	1	
Overhead production cost f	0.1	

Note: All time-related parameter values are annualized.

of large U.S. firms. We normalize entry costs $f_E = 1$, and we set the fixed costs of operation $f = 0.1$. We choose the parameters f_X, $\tau^{1-\rho}$, and h to match three observations in the United States: (1) the fraction of exports in gross output; (2) the fraction of total production employment accounted for by exporting firms; and (3) the shape of the right tail of the firm-size distribution.

5.0 Trade-Liberalization Scenarios

We restrict our analysis of trade liberalization to decreases in the per-unit trade cost τ. Throughout all scenarios, we consider the effects of a 3.5 percent reduction in τ, starting from the calibrated steady state described

previously.[17] We consider the following experiments. We first consider the effects of a permanent unanticipated reduction in the trade cost and describe how firm-productivity dynamics interact with endogenous export-market selection to generate endogenous aggregate-transition dynamics. We then examine the role of anticipated changes in trade policy. We first look at the case in which that same unanticipated drop in the trade cost is anticipated to revert back to the former steady state (i.e., so the trade liberalization is only temporary). We then look at the case in which the initial trade liberalization is anticipated and permanent thereafter. We present our computational results graphically, using figures that show the responses over time of key variables for the economy. In several cases, we also complement these numerical results with analytical propositions regarding those dynamic responses (the proofs are relegated to the Appendix).

To highlight the key interactions between firm-productivity dynamics and endogenous export-market selection, we start with two scenarios that exhibit only one of those features at a time, eliminating the other. The first scenario eliminates endogenous market selection (i.e., all firms export), and the second scenario eliminates firm-productivity dynamics (i.e., firm productivity remains constant postentry). In both cases, we show that permanent trade liberalization does not induce any endogenous transition dynamics. To motivate how firm-productivity dynamics and endogenous export-market selection interact to generate transition dynamics, we develop a simplified analytical variant of our model that nevertheless exhibits both of those features. This analytical model clarifies how differences in current and future export-market profitability (relative to the profitability of domestic sales) drives transition dynamics in response to a sudden, permanent trade liberalization (via its effect on firm entry). In the third scenario, we show that these analytic results carry over to our full model with exogenous innovation. In the fourth scenario, we add endogenous innovation and describe its effect on the transition dynamics – contrasting them to the previous scenario in which innovation is exogenous. In the fifth scenario, we examine the case of a temporary trade liberalization. We show how expectations regarding the permanence of trade liberalization affect the response of innovation. Last, we study the effects of an anticipated trade liberalization. We return to the case of a permanent decrease in trade costs, but we now assume that this decrease in trade cost is anticipated (ahead of any change in the trade cost). The sixth scenario captures those

[17] We choose this change in trade cost to ensure that our model with endogenous innovation produces a steady state with entry (i.e., with very large changes in trade costs, the increase in the growth rate of exporting firms induces a nonstationary firm-size distribution).

anticipation effects and shows how they are more pronounced when innovation is endogenous, relative to the exogenous-innovation case. The seventh and eight scenarios add sunk export-market costs, thus introducing partial irreversibility and option values associated with the export-entry decisions. We show how the perceived permanence of the trade liberalization shapes the strength of entry into exporting, and how anticipated trade liberalization affects the option values from entry into exporting, inducing transition dynamics ahead of the actual drop in the trade cost.

Scenario 1: No Export-Market Selection

In this scenario, we start with our baseline model but eliminate export-market selection by setting the fixed export cost to zero, $f_X = 0$ (i.e., all firms export regardless of their productivity z). The responses of the key variables of the model are depicted in Figure 1. Panel A reports the time path of the exogenous per-unit trade cost τ as a percentage of the old steady state, τ_t/τ_0 (i.e., 96.5 percent, reflecting the permanent 3.5 percent decrease in τ). Panels B–F report the other responses in elasticity format: $\log(X_t/X_0)/\log(\tau_t/\tau_0)$ for each variable X. Thus, those responses depict the percentage change in X per 1 percent change in τ. Panel B depicts the response of final output Y_t (equal to aggregate consumption C_t) and production labor L_{Pt}; Panel C depicts the response of entry M_{Et}; Panel D depicts the response of average productivity for domestic production $\tilde{Z}_t$; Panel E depicts the response of the revenue share of exporters in domestic revenues, s_{Dt}, and in total revenues, s_{Xt}; and Panel F depicts the response of the innovation intensity q_t for large firms (technically, $\lim_{z\to\infty} q_t(z)$).

As is apparent from those responses in Figure 1, the permanent trade liberalization does not induce any transition dynamics. Final output jumps up to its new steady-state level, along with the share of exports in total revenues. The change in the trade cost does not induce any further reallocations: The intensity of innovation, exit threshold (not shown in the figure), entry, and production labor remain unchanged; hence, there are no composition effects across firms (i.e., no change in average productivity $\tilde{Z}_t$). The economy's response to the lower trade cost $\tau < \tau_0$ is identical to the response in Krugman's (1980) model with representative firms: All firms reallocate their production toward export sales without varying total employment; there is no change in firm entry or exit; and final output (i.e., consumption) rises due to only the direct effect of the change in the trade cost (i.e., fewer units of output "melt" away). Proposition 1 in Appendix B derives these results analytically and shows that the proportional increase

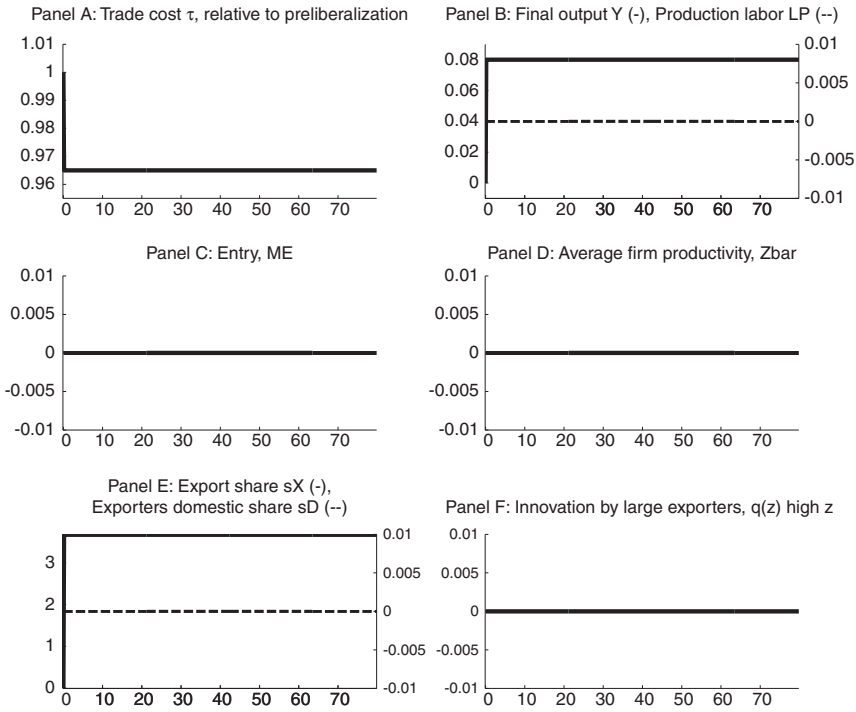

Figure 1. Scenario 1, no export-market selection.

in final output is:

$$\frac{Y}{Y_0} = \left(\frac{1 + \tau^{1-\rho}}{1 + \tau_0^{1-\rho}} \right)^{\frac{1}{\rho - 1}}$$

This is the same rise in final output as in an Armington world, where every country produces a single good with exogenous unit-labor requirements. The main intuition for the absence of any interfirm reallocation or change in innovation is that in a world with C.E.S. demand and constant markups, the increased export opportunities from lower trade costs are offset exactly by the reductions in domestic sales (driven by increased imports). All firms face those same exact trade-offs; thus, there is no scope for any interfirm reallocations and no motive for any change in innovation decisions. The expected value of an entering firm relative to the entry cost is unchanged; hence, there is no change in entry behavior.[18]

[18] If the cost of entry falls (e.g., if entry cost required both labor and final output), then entry rises, giving rise to endogenous transition dynamics across steady states. All other

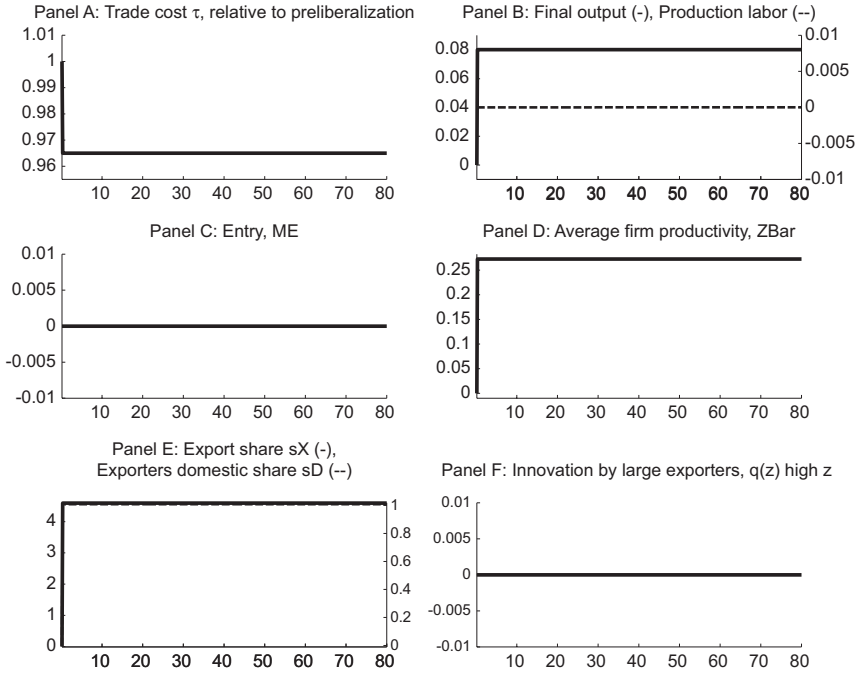

Figure 2. Scenario 2, no firm-productivity dynamics.

Scenario 2: No Firm-Productivity Dynamics

We now allow for export-market selection (i.e., the fixed cost $f_X > 0$ is calibrated to match the previously described aggregate export patterns) but we eliminate firm-productivity dynamics. This is equivalent to setting $\Delta_z = 0$. To match the right tail of the firm-size distribution of the model with firm dynamics, we assume that the initial firm productivity is drawn at entry from a Pareto distribution G with shape parameter $\theta > \rho - 1$.

The responses of the same key variables are depicted in Figure 2. Here, too, we see that the permanent trade liberalization does not induce any endogenous transition dynamics. As in the previous scenario, there is no response in entry and final output (i.e., consumption) immediately jumps to its new steady-state level. However, this scenario features some

results still hold (e.g., innovation and exit thresholds remain unchanged). Similarly, if innovation requires both labor and final output, innovation rises for all firms, giving rise to endogenous transition dynamics.

interfirm reallocation. The lower trade cost induces high-productivity firms to become exporters. Firms with low productivity cannot take advantage of the increased export opportunities; thus, they disproportionately suffer from the reductions in domestic sales. This induces the exit of the least-productive firms, and labor is reallocated from nonexporters and exiters toward exporters. The expansion of the exporters relative to the nonexporters and exiters induces an increase in average-firm productivity (see Panel D). Panel E shows how this expansion of exporters relative to nonexporters is reflected in the increased share of exporters in domestic revenues (which was flat in the previous scenario with no interfirm reallocations). The response of the share of exporters in total revenues is now higher (relative to the previous scenario) due to the effects of the lower trade cost on the extensive margin of trade (new exporters).[19]

This scenario features the same interfirm reallocations that would be observed in a static model of trade liberalization. In our dynamic version, those reallocations all take place immediately on the decline in trade costs. A key feature driving this result is that trade liberalization does not affect the incentives for entry. The exit- and export-market cutoffs then jump immediately to their steady-state levels, and there are no ensuing transition dynamics. These results are described in Proposition 2 in Appendix C but we note here that this result for entry is a dynamic version of the result derived by Arkolakis et al. (2010). In both cases, this result is driven by offsetting effects of lower trade costs on an entrant's expected profits. On the one hand, the lower trade cost increases the profitability of large exporters (i.e., high z); on the other hand, it also reduces the profitability of small firms (i.e., low z). The exact offsetting of these two forces is naturally specific to the assumed parametrization (especially C.E.S. demand and Pareto distribution for firm productivity).

Firm Dynamics and the Effect of Trade Liberalization on Entry: Building Intuition

We now show that adding firm-productivity dynamics completely changes the trade-offs described previously for the effects of trade liberalization on entry. We first build a simplified (and slightly modified) version of our model that motivates a simple analytic rule governing this trade-off. This rule is not tied to a specific parametrization and allows us to predict the

[19] In an alternative calibration, in which θ is chosen to be equal to $\rho - 1$, the trade elasticities in Scenarios 1 and 2 would be equal and our other qualitative results remain unchanged.

response of entry to trade liberalization for more general cases. We then show how it applies to our full model with both endogenous export-market selection and firm-productivity dynamics.

We consider the following version of our model that allows for time variation in the fixed export cost f_X. In particular, $f_X = \infty$ if the firm has entered $T \geq 0$ or fewer periods ago, and $f_X = 0$ if the firm has entered $T + 1$ or more periods ago. Hence, conditional on survival, firms start exporting after T periods of operation. We note that if $T = 0$, all firms export. We also assume that all entering firms have productivity index $z = 0$; there are no productivity dynamics ($\Delta_z = 0$); and there are no fixed overhead costs, $f = 0$ (therefore, exit takes place due to exogenous death only).

We denote by $\tilde{s}_X$ the ratio of discounted time-series revenues from exports in total discounted time-series revenues of entering firms. That is, whereas s_X measures the export share in the cross section of firms, $\tilde{s}_X$ measures the export share in the discounted revenues of entering firms.[20] If $\beta = 1$, then $\tilde{s}_X = s_X$; if $\beta < 1$ and $T > 0$, then exporters are back-loaded so $\tilde{s}_X < s_X$. More generally, if $\beta < 1$, then as T rises (so that it takes longer for an entrant to become an exporter), profits from exporting become a less important component of a firm's value on entry and $\tilde{s}_X$ falls relative to s_X.

Proposition 3 in Appendix D states that in this economy, a permanent reduction in marginal trade costs, τ, leads to a decline (increase) in the steady-state mass of entering firms, M_E, if and only if (iff) $\tilde{s}_X$ is strictly lower (higher) than s_X, and the steady-state mass of entering firms remains unchanged iff $\tilde{s}_X = s_X$. The intuition for this result is straightforward. When $\tilde{s}_X < s_X$, incumbent exporting firms benefit proportionally more than entering nonexporting firms from lower trade costs. Hence, trade liberalization makes entry less profitable and entry falls. This result suggests that the relative size of entering firms has an important role in shaping the change in entry and the aggregate transition dynamics in response to a trade liberalization.[21]

We now apply this logic to understand the response of entry and aggregate-transition dynamics in our full model with productivity dynamics.

[20] $\tilde{s}_X$ is calculated analogously to the export share s_X using (17), with a "discounted" distribution of firms $\tilde{M}(z)$ that is defined analogously to $M(z)$ in (13) – except that $\beta(1-\delta)$ replaces $(1-\delta)$ and $\beta G(z)$ replaces $G(z)$.

[21] Atkeson and Burstein (2010) extended this result to a more general setting for the structure of productivity and export-participation dynamics. Fattal-Jaef (2010) presented a related result in a model of firm dynamics and misallocation distortions.

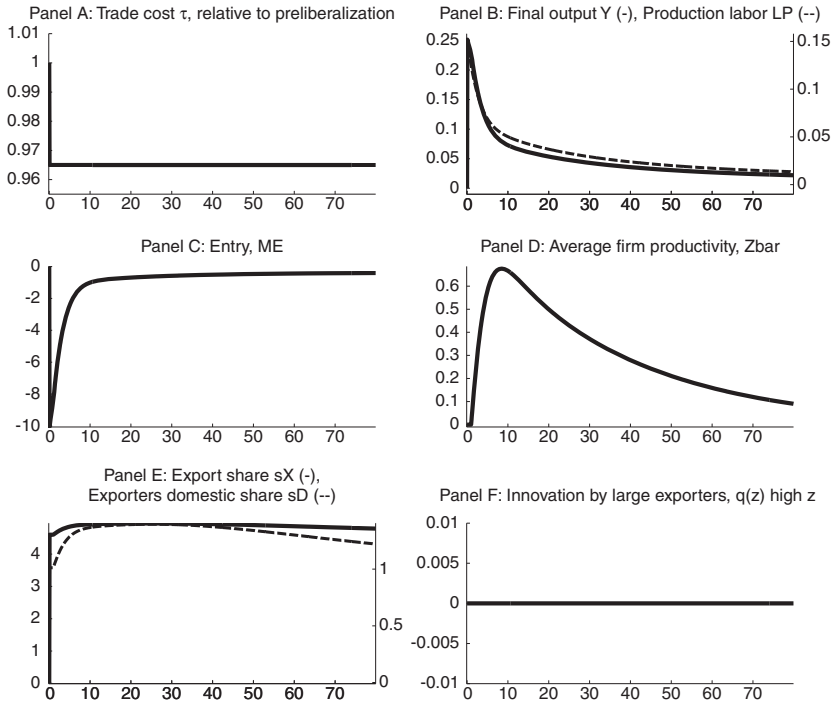

Figure 3. Scenario 3, export-market selection and firm dynamics with exogenous innovation.

Scenario 3: Export-Market Selection and Firm Dynamics with Exogenous Innovation

We return to our baseline model with both endogenous export-market selection and firm-productivity dynamics. In this scenario, we assume that the parameter b of the innovation-cost function is sufficiently high so that innovation intensity is constant across firms and over time: $q_t(z) = \bar{q}$. This is the exogenous-innovation case. The parameters that determine the evolution of firm productivity are such that entering firms, on average, are smaller than incumbents – and thus are less likely to export than incumbents.[22] This implies that with $\beta < 1$, $\tilde{s}_X < s_X$ from our previous analytical model.

The responses of the same set of variables for this scenario are depicted in Figure 3. A permanent trade liberalization makes entry less profitable

[22] This also matches the empirical regularity that most firms do not export immediately after entry.

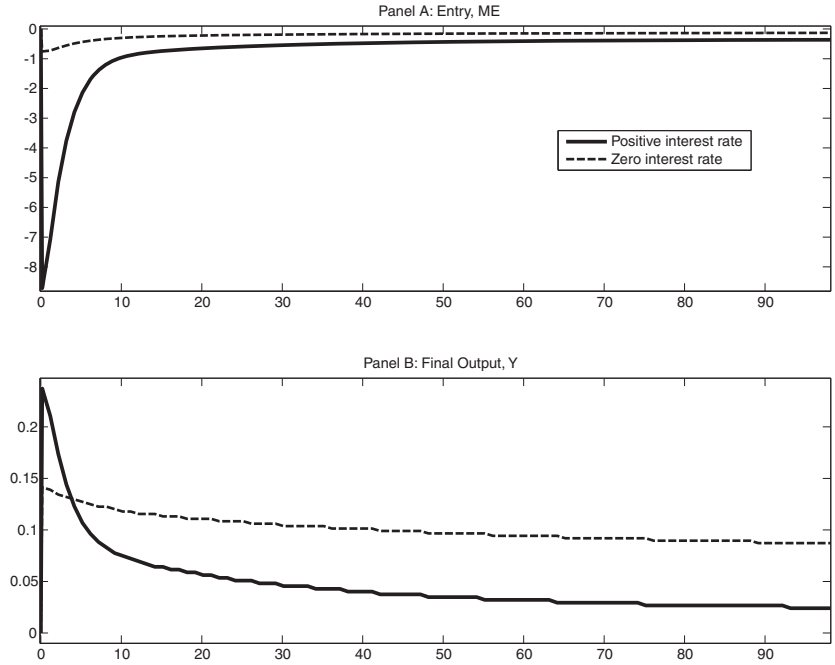

Figure 4. Scenario 3, export-market selection and firm dynamics with exogenous innovation, varying the interest rate.

because incumbent exporters benefit proportionally more than nonexporting entrants. As shown in Panel C, entry responds as our simple analytic model suggests: It drops on the reduction of the trade costs. Because the new steady state with lower trade costs features a smaller mass of producing firms (due to the smaller mass of entrants in steady state), there is an "overhang" of incumbent firms immediately after trade liberalization. This leads to an overshooting of the entry response because the mass of producing slowly decreases to its new lower steady-state level. The labor resources previously allocated to entry are available for production employment. There is a spike in production employment (inversely related to the downward spike in entry) and a similar spike in final output. Due to these transition dynamics, we see that comparing consumption across steady states understates the welfare gains from liberalization that accrue during transition as the mass of firms declines.

The reallocation of market share toward exporting firms leads to an increase in average firm productivity in the new steady state. Average productivity overshoots this new steady-state level over the transition due to

the overshooting of entry (because entrants are relatively less productive). The transition dynamics for the fraction of exporters' revenues in total and domestic revenues are not very pronounced. They behave similarly to the previous scenario with no firm-productivity dynamics.

To emphasize how the difference between the export profitability shares $\tilde{s}_X$ and s_X drive the response of entry, Figure 4 contrasts the response of entry and final output under our baseline interest rate and a zero interest rate ($\beta = 1$). In the latter case, the delay for entrants to grow and start exporting becomes inconsequential; their future expected revenues from exporting $\tilde{s}_X$ is equal to the cross-sectional export share s_X. Figure 4 shows how in this case, the response of entry becomes muted, as do the transition dynamics for final output. Similarly, if the model is calibrated so that entering firms grow faster (and, hence, become exporters sooner), the decline in entry also is muted relative to our baseline parameterization.

Scenario 4: Endogenous Export-Market Selection and Endogenous Innovation

We now switch to the more elastic parametrization for innovation. The economy's responses are reported in Figure 5. This is the first scenario to feature a response in firm innovation intensity to trade liberalization. This response is exhibited in Panel F for the innovation intensity of large exporting firms (i.e., all firms above a given large-size threshold). As in the case with exogenous innovation, the permanently lower trade costs increase the value of exporters relative to nonexporters. In the current case, those exporters respond by innovating relatively more. Appendix E derives analytically this reallocation of innovation from nonexporters to exporters and discusses how it depends on key model parameters.

The immediate response of entry is similar to the case of exogenous innovation, and for the same reasons: Trade liberalization makes entry less profitable as entrants' exporting profits are pushed back into the future. Average firm size and productivity steadily increases (see Panel D) from the combination of selection (i.e., reduced entry by smaller, less-productive firms) and increased innovation by exporters. Although the response in increased innovation intensity is immediate, its effects on the productivity and size of exporters takes a long time to unfold. As the average size and productivity of exporters grow relative to those of entrants, the expected profitability for that latter group falls farther, inducing a second dip in entry (see Panel C). The increase in relative size and productivity of exporters also leads to a steadily increasing share of exporters in domestic and total

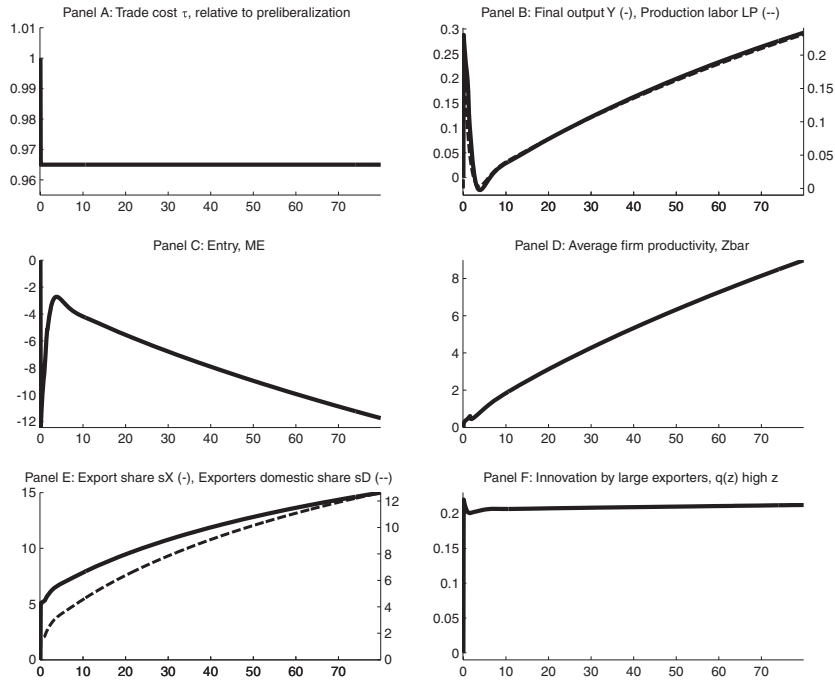

Figure 5. Scenario 4, endogenous export-market selection and endogenous innovation.

revenue (see Panel E). Hence, the elasticity of trade volumes to changes in trade costs is higher when it is calculated over longer time horizons.

The labor resources used for entry are reallocated to production (and some to the increased innovation activity). Production labor steadily rises, mirroring the decrease in entry (see Panel B). Those changes in production labor also are reflected in a rise in final output. As opposed to the exogenous innovation case, steady-state consumption now overstates the welfare gains from trade liberalization because the increase in average productivity takes many periods to materialize. The main message for welfare-comparative statics remains that transition dynamics strongly affect any comparison based on steady-state consumption.

We note that the increase of innovation to trade liberalization is not limited to current exporters: Trade liberalization also spurs the innovation response of nonexporters who anticipate exporting in the future (given the lower trade costs). In fact, from Equation (9), firms close to the export threshold have the biggest incentive to innovate because they have the highest marginal benefit from successful innovation (i.e., the difference

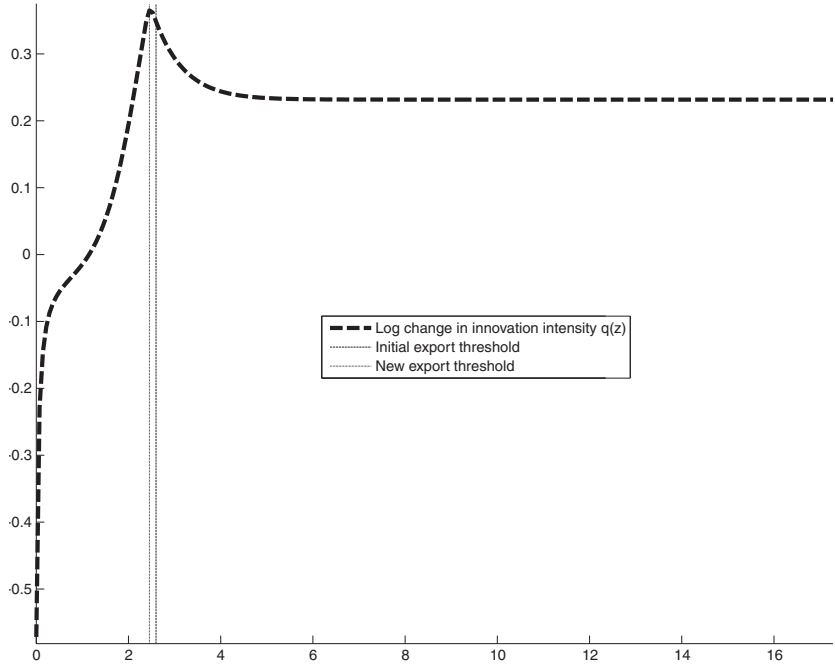

Figure 6. Scenario 4, endogenous export-market selection and endogenous innovation, change in innovation intensity.

between profits as an exporter relative to profits from serving only the domestic market). This result is highlighted in Figure 6, which shows the *difference* in innovation intensity, $\Delta q(z)$, across the two steady states (i.e., pre- and post-liberalization) as a function of z. In the figure, z is reported as the difference relative to the exit threshold $\bar{z}_X$. The figure also shows the change across the two steady states in the export threshold. All exporters increase their innovation intensity ($\Delta q(z) > 0$); however, firms around the export threshold – both current exporters and nonexporters that anticipate being exporters in the future – have the highest increase in innovation activity. Due to the increased competition from foreign producers and more-productive domestic producers, a subset of firms with the lowest productivity (that have the smallest probability of becoming exporters) innovates less in response to trade liberalization.

Figures 5 and 6 also highlight how increases in firm productivity immediately following export-market entry need not reflect learning by exporting; this observed empirical pattern also may reflect the outcome from the innovation activities undertaken by new exporters.

Scenario 5: Temporary Trade Liberalization

In the previous scenario, we saw how endogenous innovation can drastically affect the transition dynamics to trade liberalization – as long as the economy also features endogenous export-market selection (we recall the result from Scenario 1 that endogenous innovation does not induce transition dynamics in the absence of endogenous export-market selection). Those transition dynamics are driven by the response of innovation intensity by exporters and anticipated exporters, as well as the response of entry. In this scenario, we highlight another crucial ingredient needed to deliver this innovation response: Trade liberalization must be perceived to be long-lasting; otherwise, the incentives for innovation are strongly muted because its benefits accrue slowly over time. This issue of perceived persistence for trade liberalization is empirically consequential because it exhibits so much variation among trade liberalization episodes (both across countries and within countries across episodes). To highlight the role of this perceived persistence for innovation, scenario 5 examines the case of a trade liberalization (i.e., same unanticipated 3.5 percent drop in trade cost) that then is reversed gradually within a 10-year period. The initial trade liberalization occurs as in the previous scenario: a sudden unanticipated 3.5 percent drop in the per-unit trade cost. We assume that the trade cost remains at its "liberalized" level for four years and then linearly reverts back to its pre-liberalization level during the following five years. This unwinding of trade liberalization is anticipated by firms immediately after the initial drop in the trade cost.

We focus on this four-year period following the initial trade liberalization and contrast the innovation-intensity response with the previous scenario, in which trade liberalization is anticipated to be permanent (with the same assumptions regarding the endogenous-innovation process). We note that whatever *actually* happens to the trade cost after that four-year period is not critical for our analysis within that time window. Trade liberalization may end up not being permanent when anticipated as such and vice versa. Within the four-year window following trade liberalization, the only difference between the two scenarios is one of anticipation by firms regarding the permanence of the initial liberalization.

Figure 7 contrasts the innovation response for the permanent and temporary trade-liberalization scenarios. Panel A shows the divergence in the path of the trade cost four years beyond the initial liberalization, but it does not show the key difference in anticipation within the four-year window. The anticipation effect for the innovation response is clearly visible in Panel B: The response of innovation when trade liberalization is anticipated to be

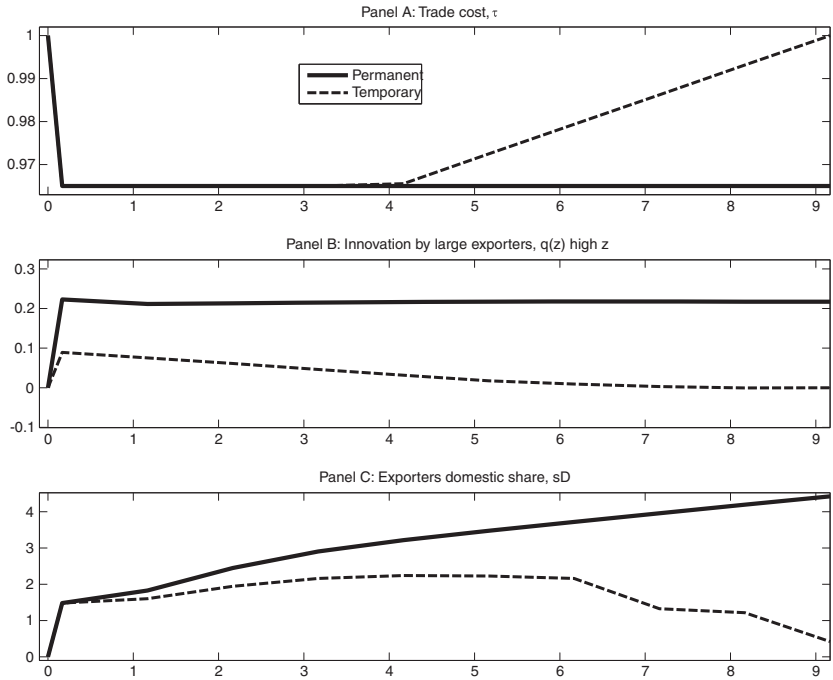

Figure 7. Scenario 5, temporary trade liberalization.

temporary is strongly muted following the initial liberalization. In that four-year period, the lower increase in innovation intensity translates into significant differences across scenarios in the growth rates of exporters. When trade liberalization is perceived to be temporary, the weaker increase in innovation by exporters leads to a smaller increase in the share of exporters' domestic revenue (see Panel C). Hence, endogenous innovation matters less for aggregate-transition dynamics when trade liberalization is expected to be temporary.

Scenario 6: Anticipated Trade Liberalization

In the previous scenario, we discussed how firms' expectations regarding the permanence of trade liberalization affect the response of endogenous innovation. In a similar way, the response of endogenous innovation also is sensitive to firms' anticipation regarding the initial onset of trade liberalization. To highlight this, we construct a scenario in which the initial trade liberalization is anticipated two years ahead to the actual decrease in trade costs, and we focus on that two-year period when the trade cost

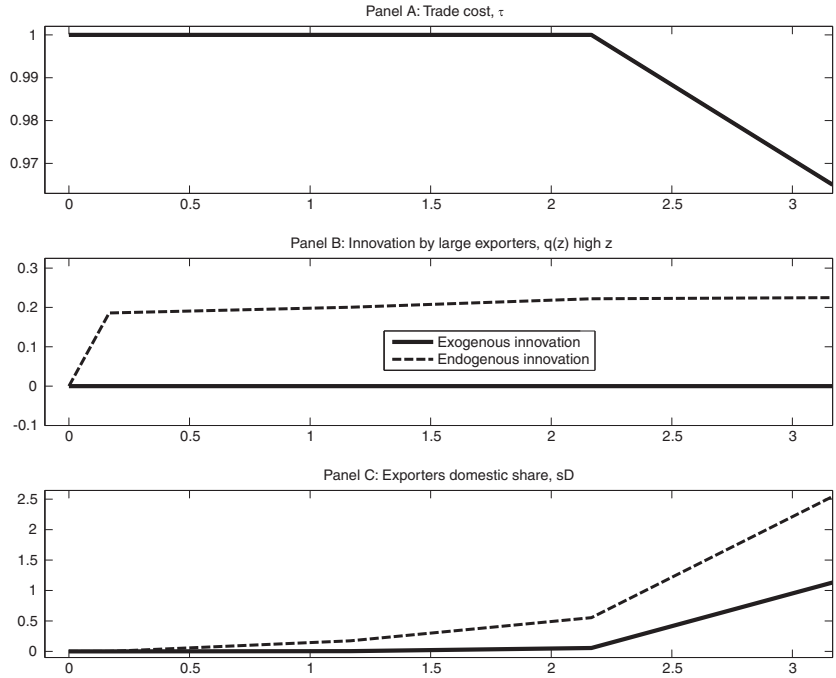

Figure 8. Scenario 6, anticipated trade liberalization.

has not yet decreased (but is anticipated to do so). Panel A of Figure 8 shows the time path of the trade cost, which is "announced" to the firms at $t = 0$. Panel B shows how exporters increase their innovation intensity on this announcement, ahead of the anticipated drop in the trade cost. This increase in innovation then is reflected in a higher growth rate for exporters, which results in an increasing share of exporters in domestic revenues in Panel C (ahead of the reduction in the trade cost).[23]

The anticipated response of innovation in this scenario is driven by the firms' desire to smooth their innovation activities over time in response to the higher benefits to innovate: Because the cost of innovation is convex, firms do not want to cluster their innovation activities immediately before the drop in the trade cost and prefer instead to spread them out ahead of the anticipated liberalization.

[23] Furthermore, the associated rise in the share of exporters in domestic revenues (in advance of the decline in trade costs) is larger than it is for the case of exogenous innovation (see the solid line in Panel E of Figure 8).

Scenario 7: Temporary Trade Liberalization and Sunk-Export Costs

In the two previous scenarios, we examine the consequences for the dynamics of trade liberalization that relate to the export-entry decision in the presence of sunk-export costs. We introduce (partial) irreversibility of the export decision by assuming that some of the fixed export costs are sunk. In particular, in addition to the fixed export costs f_X, we assume that a nonexporter that becomes an exporter must hire f_{XS} units of labor during the first period as an exporter. An exporter that stops exporting must incur these sunk costs to restart exporting.

With sunk-export costs, the choice to enter the export market is no longer a static decision as in Expression (14). Instead, current export-entry decisions are shaped by expected future exporting profits, subject to firms' expectations regarding their idiosyncratic productivity and the path of trade costs. This represents an additional channel that generates endogenous aggregate-transition dynamics from trade liberalization. Because this channel operates independently of the endogenous innovation response, we highlight its effect using the case of exogenous innovation. We first examine how export-entry decisions under sunk-export costs interact with the perceived persistence of trade liberalization. Panel A in Figure 9 depicts the path of trade costs under a temporary (i.e., lasting four years) and a permanent trade liberalization. In both cases, the liberalization is initially unanticipated.

As a reference point, we first consider our baseline model calibrated with only fixed export costs (and exogenous innovation). In this case, the perceived permanence of the reduction in trade costs has little bearing on export decisions because entry into exporting is a static decision based on the level of current profits from exporting. This implies, as shown in Panel B, that the increase in the share of exporters in domestic revenues during the first four years of the trade liberalization is similar in both the temporary and permanent cases.

Next, we designate a proportion of the fixed export cost to be sunk. In particular, we set the fixed export cost at a low level ($f_X = 0.2$) and choose f_{XS} to match our baseline calibration targets. In this case, entry of nonexporters into exporting is stronger when the reduction in trade costs is perceived to be permanent. This is shown in Panel C: In the first four years, the rise in the share of exporters in domestic revenues (and, hence, also the overall response of trade volumes) is larger in the permanent case than in the temporary case. That is, trade liberalization must be perceived

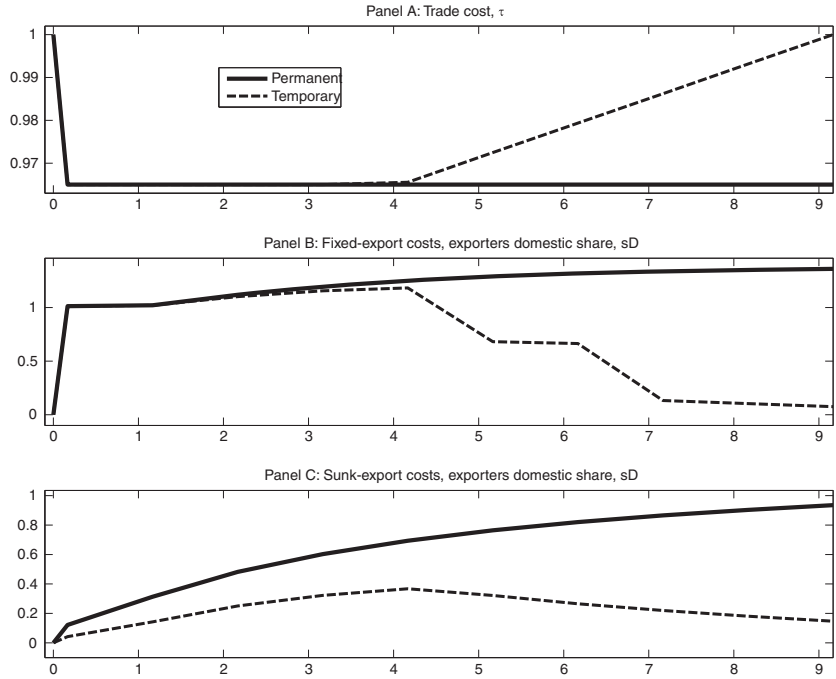

Figure 9. Scenario 7, temporary trade liberalization and sunk-export costs.

to be long-lasting to provide incentives for firms to pay the sunk-export cost.[24,25]

Scenario 8: Anticipated Trade Liberalization and Sunk-Export Costs

We now illustrate a second implication of export-entry decisions in the presence of sunk-export costs for the aggregate dynamics following trade liberalization. Here, we focus on an anticipated trade liberalization, as

[24] This result is similar to the one in Ruhl (2008). The latter shows how sunk-export costs induce a larger response of exports to permanent shocks (i.e., trade liberalization) than to temporary shocks (i.e., business cycles). See also the related work by Albuquerque and Rebelo (2000).

[25] We note also that the nature of fixed export costs shapes the strength of the short-run response of entry into export markets. In particular, comparing Panels B and C of Figure 9 for either the temporary or the permanent trade liberalization case, we see that the short-run increase in the share of exporters in their domestic market (and, hence, the overall response of trade volumes) is much larger under fixed export costs than under sunk-export costs. In contrast, the long-run responses of the trade volumes are similar under fixed or sunk export costs (projecting the responses beyond the 10 years displayed in the figures). The hysteresis band in export participation (described in the next scenario) has an important role in generating this difference in the short-run responses.

described in Scenario 6. In the presence of productivity dynamics, sunk-export costs generate an option value associated with the export-market-entry decision and an associated hysteresis band: There is a set of firms – associated with productivity z in a given range – whose export decision is determined by their prior export status. This option value is sensitive to the firms' expectations regarding future trade costs and thus responds to announcements regarding those future costs.

As a useful starting point, we consider first a version of our model with no productivity dynamics (i.e., $\Delta_z = 0$). In the initial equilibrium before trade liberalization is announced, the static profit gain from exporting for any nonexporting firm z must be lower than the sum of the fixed export cost, f_X, and the per-period flow value of the sunk cost, $\frac{R+\delta-1}{R} f_{XS}$:

$$\left(\tau^{1-\rho} - 1\right) \Pi_D \exp(z) < f_X + \frac{R + \delta - 1}{R} f_{XS}. \tag{20}$$

We now consider the periods after trade liberalization is announced but ahead of the actual reduction in trade costs τ. Abstracting from general equilibrium changes in the domestic-market-demand index, Π_D, and in the interest rate, R, Condition (20) remains unchanged. That is, before the reduction in trade costs, it is still the case that the static profit gains from exporting are lower than the annual fixed and sunk costs of export. Therefore, firms that were non-exporters before the announced trade liberalization will not find it optimal to start exporting ahead of the reduction in trade costs. Once the reduction in trade costs is materialized, Condition (20) will be reversed for some firms, which then will start exporting. Without productivity dynamics – and abstracting from general equilibrium considerations before the reduction in trade costs – sunk-export costs do not give rise to anticipated changes in export decisions. A similar argument can be applied to the innovation decision. Abstracting from general equilibrium changes in Π_D and R, there would be no incentive to innovate ahead of the reduction in trade costs (in contrast to the results in Scenario 6).

Figure 10 shows how the combination of sunk-export costs and productivity dynamics results in changes in export decisions ahead of the reduction in trade costs. The path and announcement regarding the trade costs are identical to Scenario 6 (again depicted in Panel A). We run one simulation using our previously calibrated model with only fixed export costs and then another one with sunk-export costs (parameterized as described in Scenario 7). Both cases include productivity dynamics (i.e, $\Delta_z > 0$) but exogenous innovation and take into account the general equilibrium changes in the domestic-market-demand index, Π_D, and in the interest rate, R.

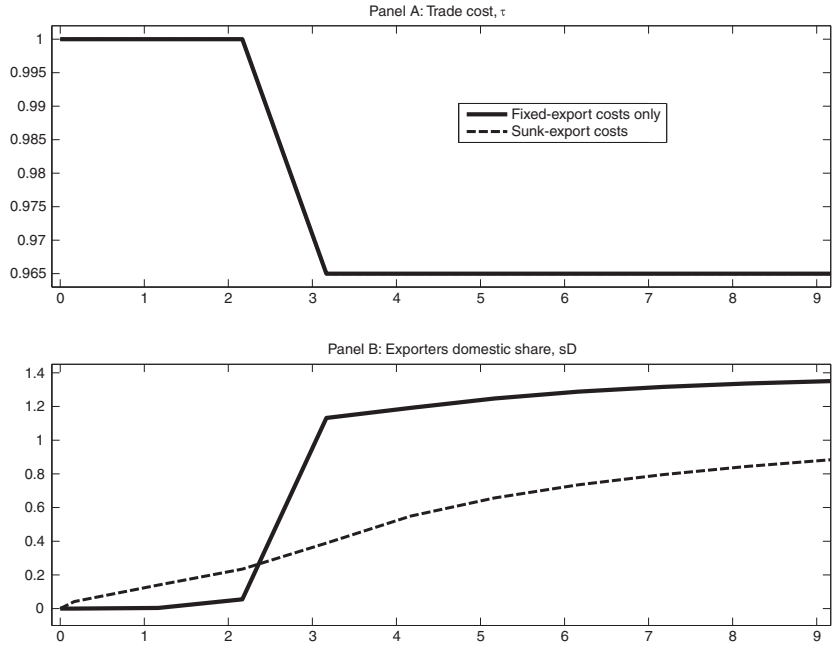

Figure 10. Scenario 8, anticipated trade liberalization and sunk-export costs.

Panel B shows how the option value associated with the sunk-export cost induces the entry of firms into the export market – captured by the rising share of exporters in domestic revenues – ahead of the decrease in the trade cost. In the preliberalization steady state, some firms just below the export-cost cutoff do not export due to the option value associated with waiting (to export): They want to mitigate potential losses from an adverse productivity shock following export-market entry. The announcement of future trade liberalization then substantially reduces this option value of waiting to export for those firms (i.e., the consequences of a future adverse shock are not as dire because those firms would still choose to export as long as the per-unit trading costs are reduced); those firms enter the export market ahead of the reduction in trade costs. This is reflected in the increased share of exporters in domestic revenues ahead of the trade liberalization. This increase is substantially larger in the model with sunk-export costs than in the model with only fixed-export costs.

Productivity dynamics are key for this result. Without productivity dynamics ($\Delta_z = 0$), there are no noticeable differences in the aggregate

response of the economies with fixed- and sunk-export costs – even considering the general equilibrium effects on Π_D and R from which we abstracted in the previous analytic derivation.[26]

6.0 Conclusions

In this chapter, we review the recent literature that incorporates firm dynamics in models of international trade. We characterize key model ingredients that generate substantial aggregate-transition dynamics stemming from endogenous shifts in firm-size distribution in response to trade liberalization. We show how two ingredients, export-market selection and firm dynamics, are needed jointly in order to generate any endogenous transition dynamics. We then examine the aggregate effects resulting from endogenous innovation responses and the anticipation effects regarding the timing of trade liberalization (i.e., when it is expected to begin and whether it is temporary or permanent). Our computational results show how the responses of trade volumes, innovation, and aggregate output vary greatly over time depending on the details assumed for firm dynamics, endogenous innovation, and the expected time path of trade liberalization. This has important consequences for many issues in international economics that rely on predictions for the effects of globalization over time on those key aggregate outcomes such as trade volumes, innovation, and aggregate productivity (i.e., an endogenous source of comparative advantage).[27] For empirical work, it is important to recognize that a measured response to globalization at a given point in time represents only one snapshot for the effects of globalization – one that may evolve considerably in the future (without any subsequent changes to economic fundamentals) and that is sensitive to many unobservables regarding firms' expectations and anticipations.

To keep our analysis tractable, we limit our modeling exercise to only one form of international market participation (i.e., exports) and one form of innovation. As noted in the introduction, the literature examines many other forms of international-market participation. Our choice should not be construed to imply that dynamics are more important for exports than for those other forms of market participation. We chose exports as an

[26] Bergin and Lin (2012) studied an alternative mechanism, based on heterogeneity in sunk-export costs across firms. In this setup, some firms also enter export markets in anticipation of future trade liberalization.

[27] Note that the consequences of those modeling assumptions for welfare outcomes are substantially more muted. This is discussed in greater detail in Atkeson and Burstein (2010).

example (it is very well understood) to highlight the importance of dynamic considerations. Similarly, we recognize that we model innovation in a coarse, reduced-form manner. First, we assume that only labor is used for innovative activities and thus do not capture how globalization may affect the cost of innovation via changes in other input costs or spillovers. Second, we did not specify explicitly the channels through which innovation and investment activities generate performance improvements – other than through a direct effect on labor productivity. Such channels include management quality, offshoring that breaks up the production chain, development of a firm's product range, changes in input usage due to the availability of imported intermediates, and marketing and relationship building with foreign buyers. Many of these factors apply differentially across a firm's destination markets and hence could result in destination-specific dynamics that differ from those of our model (where productivity improvements apply worldwide). These are all fruitful areas for future work.

APPENDIX: ANALYTIC RESULTS

A. Solving Symmetric Steady State

For a given value of Π_D, export decisions are determined by the static condition that variable profits from exports must exceed fixed costs of exporting, or:

$$x(z) = 1 \text{ iff } \Pi_D \tau^{1-\rho} \exp(z) \geq f_X \tag{A.1}$$

The export cutoff is defined by $\Pi_D \tau^{1-\rho} \exp(\bar{z}_X) = f_X$. To solve for firms' steady-state exit and innovation decisions, we must solve the firms' Bellman equation (7), removing the time subscripts from all variables and letting $R_t = 1/\beta$. Standard arguments state that this Bellman equation has a unique solution $V(z)$, corresponding to any given value of Π_D under appropriate parameter restrictions.

We use the free-entry condition (10) to solve for the equilibrium value of Π_D. Using standard arguments, we can show that a unique solution for Π_D exists because the value function of operating firms, $V^o(z)$, is strictly increasing in Π_D. The solution to this problem now gives us the firms' exit decisions $\bar{z}$, export decisions $x(z)$, and innovation decisions $q(z)$. These decisions, under certain parameter restrictions, imply from (13) a steady-state distribution of firms' productivities scaled by the mass of entering

firms, $\tilde{M}(z) = M(z)/M_E$. The mass of entering firms is the solution to:

$$M_E\left[\frac{L_P}{M_E} + f_E + \int\left[\exp(z)c(q(z)) + f + x(z)\, f_X\right]\tilde{M}(z)\,\mathrm{d}z\right] = 1 \tag{A.2}$$

where $L_P/M_E = \int l(z)\,\tilde{M}(z)\,\mathrm{d}z$ is determined using (16). We then can solve aggregate output and productivity using (18) and (19) and the price level as $P = \frac{\rho}{\rho-1}\frac{1}{Z}$.

B. No Export-Market Selection

We state our main result in the following proposition, which is straightforward to extend to asymmetric countries (Atkeson and Burstein 2010).

Proposition 1: *Consider a world economy with no fixed costs of trade ($f_X = 0$). In this economy, a permanent change in marginal-trade costs from τ_0 to τ has no impact on the equilibrium exit threshold, $\bar{z}$; innovation decisions of firms, $q(z)$; mass of entering firms, M_E; and ratio of exporters' revenues in the domestic market to total domestic revenues, s_D. Moreover, the economy transits immediately to a new steady state in which changes in the export share, s_X, aggregate productivity, Z, and output, Y, are:*

$$\frac{s_X}{s_{0X}} = \frac{\tau^{1-\rho}/\left(1+\tau^{1-\rho}\right)}{\tau_0^{1-\rho}/\left(1+\tau_0^{1-\rho}\right)}$$

$$\frac{Z}{Z_0} = \frac{1+\tau^{1-\rho}}{1+\tau_0^{1-\rho}}$$

$$\frac{Y}{Y_0} = \left(\frac{1+\tau^{1-\rho}}{1+\tau_0^{1-\rho}}\right)^{\frac{1}{\rho-1}}$$

Proof: We first solve for the equilibrium in a symmetric steady state. With $f_X = 0$ for all firms, (A.1) implies that all firms export and the variable profits of a firm with productivity z are $\Pi_D\left(1+\tau^{1-\rho}\right)\exp(z)$. Hence, under the assumption that all firms export, the Bellman equation in the steady state, (7), can be written with $\tilde{\Pi}\exp(z)$ replacing $\Pi_t(z)$, where $\tilde{\Pi} = \Pi_D\left(1+\tau^{1-\rho}\right)$. Our arguments in the previous section imply that a unique level of $\tilde{\Pi}$ exists that satisfies the free-entry condition (10), independent of the parameter τ. The corresponding exit thresholds, $\bar{z}$, and innovation decisions, $q(z)$, that solve the Bellman equation at this level of $\tilde{\Pi}$ are the equilibrium exit and innovation decisions. Hence, these

also are independent of τ. From (16), variable employment $l(z)$ for each firm is independent of τ, as is s_D and, from (A.2), the mass of entrants. The change in the other aggregate variables immediately follow from (17), (18), and (19). Given that firms' productivity distribution $M(z)$ is unchanged across the steady state, the adjustment to the new steady state is immediate. ■

C. No Productivity Dynamics

Proposition 2: *Consider a world economy with positive fixed costs of trade ($f_X > 0$) and no productivity dynamics ($\Delta_z = 0$); the distribution of entering firms G is such that the distribution of* $\exp(z)$ *is Pareto:*

$$\text{cdf of } G(z) = 1 - \left(\frac{\exp(z_0)}{\exp(z)}\right)^{\sigma}, \quad \text{for } z > z_0$$

where $\sigma > 1$. We suppose that both the exit and export cutoffs are interior, $\bar{z}_D > z_0$, $\bar{z}_X > z_0$. In this economy, a permanent reduction in marginal-trade costs, τ, leads to an increase in the steady-state exit cutoff, $\bar{z}$; a decline in the export cutoff, $\bar{z}_X$; and has no impact on the steady-state mass of entering firms, M_E. The economy transits immediately to a new steady state.

Proof: We first solve for the steady state. With the Pareto distribution of entering firms and no productivity dynamics, the free-entry condition (10) can be written as:

$$\frac{(1-\beta(1-\delta))}{\beta} f_E = \frac{\sigma \exp(z_0)^{\sigma}}{(\sigma-1)} \Pi_D \left[\exp(\bar{z})^{1-\sigma} + \exp(\bar{z}_X)^{1-\sigma} \tau^{1-\rho}\right] - (\exp(z_0))^{\sigma} \left[\exp(\bar{z})^{-\sigma} f + \exp(\bar{z}_X)^{-\sigma} f_X\right]$$

or using the cutoff definitions in the model without productivity dynamics, $\exp(\bar{z}) = f/\Pi_D$ and $\exp(\bar{z}_X) = \tau^{\rho-1} f_X/\Pi_D$:

$$\frac{(1-\beta(1-\delta))}{\beta} f_E = \frac{\exp(z_0)^{\sigma}}{\sigma-1} (\Pi_D)^{\sigma} \left(f^{1-\sigma} + f_X^{1-\sigma} \tau^{(1-\rho)\sigma}\right) \quad \text{(C.1)}$$

We note from this equation that a reduction in international-trade costs τ results in a decline in profitability Π_D and a rise in $\Pi_D \tau^{(1-\rho)}$. Hence, $\bar{z}_D$ rises and $\bar{z}_X$ falls.

The labor-market-clearing condition (A.2) can be written using (16) and the definition of the cutoffs as:

$$1 = M_E\left[f_E + \frac{\exp(z_0)^{\sigma}}{\delta}(\Pi_D)^{\sigma}\left(\frac{\sigma(\rho-1)}{\sigma-1}+1\right)\left(f^{1-\sigma}+f_X^{1-\sigma}\tau^{(1-\rho)\sigma}\right)\right] \tag{C.2}$$

Combining (C.1) and (C.2), we obtain the result that in steady state, M_E is independent of τ. In response to a permanent reduction in τ, with the mass of entering firms unchanged and the exit cutoff $\tilde{z}_D$ increasing, the transition to a new steady state is immediate. We note that in response to an increase in τ, even though the mass of entrants is unchanged, the exit cutoff falls; therefore, the transition to the new steady state is not immediate unless $\delta = 1$. ■

D. Not All Firms Export, with Time Variation in Fixed-Export Costs

We consider the modified version of our economy described in Section 5.0.

Proposition 3: *In this economy, a permanent reduction in marginal-trade costs, τ, leads to a decline (increase) in the steady-state mass of entering firms, M_E, iff $\tilde{s}_X$ is strictly lower (higher) than s_X; the steady-state mass of entering firms remains unchanged iff $\tilde{s}_X = s_X$.*

Proof: The free-entry condition in this economy is:

$$\begin{aligned} f_E &= \Pi_D\beta\sum_{t=0}^{T-1}\beta^t(1-\delta)^t + \Pi_D\left(1+\tau^{1-\rho}\right)\beta^{T+1}(1-\delta)^T\sum_{t=0}^{\infty}\beta^t(1-\delta)^t \\ &= \beta\frac{1-\beta^T(1-\delta)^T}{1-\beta(1-\delta)}\Pi_D + \frac{\beta^{T+1}(1-\delta)^T}{1-\beta(1-\delta)}\Pi_D\left(1+\tau^{1-\rho}\right) \end{aligned}$$

The share of discounted time-series revenues of exports in total discounted time-series revenues of entering firms is:

$$\tilde{s}_X = \frac{\frac{\beta^{T+1}(1-\delta)^T}{1-\beta(1-\delta)}\Pi_D\tau^{1-\rho}}{\beta\frac{1-\beta^T(1-\delta)^T}{1-\beta(1-\delta)}\Pi_D + \frac{\beta^{T+1}(1-\delta)^T}{1-\beta(1-\delta)}\Pi_D(1+\tau^{1-\rho})} = \frac{\beta^T(1-\delta)^T\tau^{1-\rho}}{1+\beta^T(1-\delta)^T\tau^{1-\rho}}$$

Given a mass of entrants, M_E; the mass of nonexporters, M_D; and the exporters, M_X, is:

$$M_D = M_E\frac{1-(1-\delta)^T}{\delta} \quad \text{and} \quad M_X = M_E\frac{(1-\delta)^T}{\delta}$$

The share of revenues accounted for by exports is:

$$s_X = \frac{M_X \tau^{1-\rho}}{M_D + M_X (1+\tau^{1-\rho})} = \frac{(1-\delta)^T \tau^{1-\rho}}{1+(1-\delta)^T \tau^{1-\rho}}$$

We note that if $\beta = 1$, then $s_X = \tilde{s}_X$. Differentiating the free-entry condition:

$$0 = \Delta \log \Pi_D + \frac{\beta^T (1-\delta)^T \left(1+\tau^{1-\rho}\right)}{1+\beta^T (1-\delta)^T \tau^{1-\rho}} \Delta \log \left(1+\tau^{1-\rho}\right), \text{ or}$$

$$\Delta \log \Pi_D = -\frac{1+\tau^{1-\rho}}{\tau^{1-\rho}} \tilde{s}_X \Delta \log \left(1+\tau^{1-\rho}\right)$$

Aggregate output and the price level are given by:

$$Y = \left[M_D + M_X \left(1+\tau^{1-\rho}\right)\right]^{1/(\rho-1)} L_p$$

$$\frac{1}{P} = \frac{\rho-1}{\rho} \left[M_D + M_X \left(1+\tau^{1-\rho}\right)\right]^{1/(\rho-1)}$$

Thus:

$$\Pi_D = \frac{P^\rho Y}{\rho^\rho (\rho-1)^{1-\rho}} = \frac{1}{\rho^\rho (\rho-1)^{1-\rho}} \frac{L_P}{M_E \left(\frac{M_X}{M_E} + \frac{M_D}{M_E}(1+\tau^{1-\rho})\right)}$$

and log-differentiating and using the expression for s_X:

$$\Delta \log \Pi_D = \Delta \log L_P - \Delta \log M_E - \frac{1+\tau^{1-\rho}}{\tau^{1-\rho}} s_X \Delta \log \left(1+\tau^{1-\rho}\right)$$

or substituting:

$$(s_X - \tilde{s}_X) \frac{1+\tau^{1-\rho}}{\tau^{1-\rho}} \Delta \log \left(1+\tau^{1-\rho}\right) = \Delta \log L_P - \Delta \log M_E$$

We have $L_P = 1 - f_E M_E$, or:

$$\Delta \log L_P - \Delta \log M_E = -\left(\frac{f_E M_E}{L_P} + 1\right) \Delta \log M_E$$

Combining:

$$\left(\frac{f_E M_E}{L_P} + 1\right) \Delta \log M_E = (\tilde{s}_X - s_X) \frac{1+\tau^{1-\rho}}{\tau^{1-\rho}} \Delta \log \left(1+\tau^{1-\rho}\right)$$

When trade costs fall, $\Delta \log \left(1+\tau^{1-\rho}\right) > 0$, and we obtain the result stated in the proposition. ■

E. Endogenous Innovation

Here, we provide analytics to understand the response of innovation decisions $q(z)$ to changes in trade costs. To simplify our analysis, we set fixed overhead costs to zero, $f = 0$, so that there is no endogenous exit, and we focus on steady states.

We first use the Bellman equation (7) and free-entry condition (10) to determine the impact of changes in trade costs on firm profitability Π_D in equilibrium. We consider the impact of a decline in the marginal cost of trade, τ, on Π_D. Because this raises $\tau^{1-\rho}$, Π_D must fall in equilibrium to restore the free-entry condition. We note as well that $\Pi_D \tau^{1-\rho}$ must rise if the free-entry condition is to be satisfied. Hence, for firms that do not switch export status, the profits of exporters – proportional to $\Pi_D \left(1 + \tau^{1-\rho}\right)$ – must rise relative to the profits of nonexporters. The export threshold falls so that some firms that previously did not export now start to export.

We note that the magnitude of the decline in Π_D in response to a decline in international-trade costs is determined in large part by the distribution G of productivities of newly entering firms. If newly created firms tend to be small nonexporters, then free entry requires that the discounted expected value of profits of these firms remain roughly constant. In this case, Π_D remains roughly constant and the profits of large exporting firms, $\Pi_D \left(1 + \tau^{1-\rho}\right)$, rise by roughly the change in $\left(1 + \tau^{1-\rho}\right)$. In contrast, if newly created firms tend to be large exporting firms, then free entry requires that $\Pi_D \left(1 + \tau^{1-\rho}\right)$ remains roughly constant and Π_D falls.

We now examine the impact of these equilibrium changes in firm profitability on the level of innovation. To do so, we solve for the innovation decisions $q(z)$ in (7) as a function of variable profits Π_D and the other parameters of the model. The Bellman equation (7) is a standard problem of valuing the profits of the firm together with an option: the option to start exporting (we recall that in this section, we are abstracting from the option to exit by assuming $f = 0$, so $V(z) = V^o(z)$). The following proposition characterizes the shape of the value function $V(z)$ and the innovation decision $q(z)$ in steady state.

Proposition 4: *Suppose that the fixed overhead costs are set to zero,* $f = 0$. *Then, the value function* $V(z)$ *that solves (7) in steady state has the form* $V(z) = A(z)\exp(z)$ *with* $\lim_{z\to\infty} A(z) = A_X$ *and* $\lim_{z\to-\infty} A(z) = A_D$, *and the optimal* $q(z)$ *has* $\lim_{z\to\infty} q(z) = \bar{q}_X$ *and* $\lim_{z\to-\infty} q(z) = \bar{q}_D$, *where*

A_D and $\bar{q}_D$ solve:

$$A_D = \frac{\Pi_D - c(\bar{q}_D)}{1 - (1-\delta)\beta\left[\bar{q}_D \exp(\Delta_z) + (1-\bar{q}_D)\exp(-\Delta_z)\right]} \tag{E.1}$$

$$c'(\bar{q}_D) = (1-\delta)\beta A_D\left[\exp(\Delta_z) - \exp(-\Delta_z)\right] \tag{E.2}$$

and A_X and $\bar{q}_X$ solve these two equations with the term Π_D in (E.1) replaced with $\Pi_D\left(1+\tau^{1-\rho}\right)$. These solutions have $A_X > A_D$ and $\bar{q}_X > \bar{q}_D$. Moreover, A_D, A_X, $\bar{q}_D$, and $\bar{q}_X$ are increasing in Π_D, whereas A_X and q_X are decreasing in τ.

Proof: The first part of the proposition follows by construction. The term $A_D \exp(z)$ with A_D given herein represents the expected discounted present value of variable profits of a firm that sets its innovation decision $q(z)$ to the constant $\bar{q}_D$ and that never exports. Likewise, the term $A_X \exp(z)$ with A_X given herein represents the expected discounted present value of variable profits of a firm that sets its process-innovation decision $q(z)$ to the constant $\bar{q}_X$, that always exports, and that is so large that the fixed cost of exporting f_X is a negligible portion of its variable profits.

For the second part of the proposition, we show that A_D is increasing in Π_D. Differentiating (E.1):

$$\frac{\partial A_D}{\partial(\Pi_D)} = \frac{1}{1 - (1-\delta)\beta\left[\bar{q}_D \exp(\Delta_z) + (1-\bar{q}_D)\exp(-\Delta_z)\right]} + \frac{\partial A_D}{\partial \bar{q}_D}\frac{\partial \bar{q}_D}{\partial \Pi_D} > 0$$

where we use the fact that innovation choice is optimal, $\partial A_D/\partial \bar{q}_D = 0$, and:

$$(1-\delta)\beta\left[\bar{q}_D \exp(\Delta_z) + (1-\bar{q}_D)\exp(-\Delta_z)\right] < 1$$

to guarantee that the discounted value of profits is finite. This same logic can be used to show that $A_X > A_D$, A_X is increasing in Π_D and A_X is decreasing in τ.

For the third part of the proposition, we show that $\bar{q}_D$ is increasing in A_D. Differentiating (E.2):

$$\frac{\partial \bar{q}_D}{\partial A_D}\frac{\partial A_D}{\partial \Pi_D} = \frac{(1-\delta)}{c''(\bar{q}_D)}\beta\left[\exp(\Delta_z) - \exp(-\Delta_z)\right]\frac{\partial A_D}{\partial \Pi_D} > 0$$

where we use $\frac{\partial A_D}{\partial \Pi_D} > 0$ and the assumption that $c''(\bar{q}_D) > 0$. The same logic is used to show that $\bar{q}_X$ is increasing in Π_D and decreasing in τ.

This proposition implies that for very small firms, the innovation decision $q(z)$ is constant at $\bar{q}_D$ across firms. These firms do not export and all grow at the constant rate $\left[\bar{q}_D \exp(\Delta_z) + (1 - \bar{q}_D)\exp(-\Delta_z)\right]$ in expectation. Likewise, for very large firms, the innovation decision $q(z)$ is constant at $\bar{q}_X$ across firms. These firms do export and all grow at the constant rate $\left[\bar{q}_X \exp(\Delta_z) + (1 - \bar{q}_X)\exp(-\Delta_z)\right]$ in expectation. The intuition for how A_i and $\bar{q}_i$ change with changes in profitability then is straightforward. If $\Pi_D \exp(z)$ or $\Pi_D \left(1 + \tau^{1-\rho}\right)\exp(z)$ rises, this raises the spread between the value of a firm that successfully innovates to $z + \Delta_z$ relative to the same firm that fails to innovate and falls to $z - \Delta_z$. From Equation (9), this increased spread in profits raises the incentives to engage in innovation.

We note that the responsiveness of very large and very small firms' innovation decisions $\bar{q}_X$ and $\bar{q}_D$ to changes in profitability and marginal-trade costs is determined by the curvature of the innovation-cost function as indexed by $c''(q)/c'(q)$. In particular, because the innovation choice is optimal, $\partial A_i/\partial \bar{q}_i = 0$; hence, the change in steady-state innovation with a change in profits is given by:

$$d\bar{q}_D = \frac{c'(\bar{q}_D)}{c''(\bar{q}_D)} \frac{d(\Pi_D)}{\Pi_D - c(\bar{q}_D)} \text{ and } d\bar{q}_X = \frac{c'(\bar{q}_X)}{c''(\bar{q}_X)} \frac{d\left(\Pi_D \left(1 + \tau^{1-\rho}\right)\right)}{\Pi_D \left(1 + \tau^{1-\rho}\right) - c(\bar{q}_X)}$$

If $c''(\cdot)/c'(\cdot)$ is very large, then innovation decisions and firm-growth rates are not very responsive to changes in firm profitability; if this curvature is small, then innovation decisions and firm-growth rates are very responsive to changes in profitability. By a similar argument, this curvature of the innovation cost function $c''(\cdot)/c'(\cdot)$ also controls the difference in the innovation decisions and implied growth rates of very large firms ($\bar{q}_X$) and very small firms ($\bar{q}_D$) in a steady state.

With this proposition, we obtain the following results regarding the impact of permanent changes in trade costs on the innovation decisions of very large firms (i.e., exporters) and very small firms (i.e., nonexporters). A reduction in the marginal costs of trade τ leads to a reduction in the innovation of very small firms and an increase in the innovation of very large firms relative to very small firms. This result follows directly from the fact that a reduction in the marginal costs of trade reduces Π_D and increases $\Pi_D \left(1 + \tau^{1-\rho}\right)$ relative to Π_D. The extent of reallocation of innovation from nonexporters to exporters depends in part on the size distribution of newly created firms. If they are small, then $\Pi_D \left(1 + \tau^{1-\rho}\right)$ increases and innovation in very large firms rises in absolute terms, whereas Π_D remains roughly constant, leaving innovation in small firms roughly unchanged. Conversely, if newly created firms are large exporting firms, then profits

$\Pi_D\left(1+\tau^{1-\rho}\right)$ and innovation in these firms remain roughly unchanged, whereas for small firms, profits and innovation fall.

We note that a reduction in the fixed costs of trade f_X, by lowering the equilibrium level of profitability Π_D, leads to a reduction in innovation in both very large and very small firms (i.e., these firms do not switch export status). Similar arguments state that a decline in the entry cost f_E results in a decline in innovation for both very large and very small firms.

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