Do Display Ads Influence Search?

Attribution and Dynamics in Online Advertising

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Abstract

As firms increasingly rely on online media to acquire consumers, marketing managers rely on online metrics such as click-through rate (CTR) and cost per acquisition (CPA). However, these standard online advertising metrics are plagued with attribution problems and do not account for synergy or dynamics. These issues can easily lead firms to overspend on some actions and thus waste money, and/or underspend in others, leaving money on the table.

We develop a multivariate time series model to investigate the dynamic interaction between paid search and display ads, and calibrate the model using data from a bank that uses online ads to acquire new checking account customers. We find that both search and display ads exhibit dynamics that improve their effectiveness and ROI over time. Moreover, our results suggest that display ads increase search conversion. However, display ads also increase search clicks, thereby increasing search advertising costs. After accounting for these three effects, we estimate that each $1 invested in display and search leads to a return of $1.24 for display and $1.75 for search ads. These ROI estimates are respectively 10% and 38% higher than those obtained by standard metrics, which may have led the company to under-invest. We use these results to show how optimal budget allocation may shift after accounting for attribution and dynamics. Although display benefits from synergy attribution, the strong dynamic effects of search call for an increase in search advertising budget share by up to 36% in our context.

Keywords: online advertising, attribution, dynamics, synergy, display, search, marketing metrics, vector error correction, time series.
INTRODUCTION

Firms are motivated to spend more of their marketing budget online as consumers increasingly use online media to find information. Worldwide digital advertising spending in 2012 was $103 billion, or about 20% of total money spent on advertising, and is expected to increase to $163 billion, or 25% of total advertising spend, by the end of 2016 (eMarketer 2013). In 2012, almost half of all digital ad dollars worldwide were spent on paid search, and 38% were used for display ads (ZenithOptimedia 2012).

The introduction of online metrics such as click through rate (CTR) and cost per acquisition (CPA) by Google and other online advertisers has made it easy for marketing managers to justify their online ad spend in comparison to the budgets used for television and other media. However, these metrics suffer from the fundamental problem of attribution, since they give credit to the last click and ignore the impact of other ad formats that may have contributed to sales.

Most managers recognize the attribution problem, and intuitively believe that display and search ads interact to influence consumers. Recently, analytical firms and ad agencies have started addressing this problem, but most of their solutions tend to be ad-hoc. For example, some industry models give equal weight or credit to all ad exposures received by a consumer in, say, a two week period; others give more weight to recent ad exposures and exponentially lower weight to past ads (Havas Digital 2010). Recent research offers better solutions based on individual-level data (Abishek et al. 2012, Li and Kannan 2014, Xu et al. 2014), which is often unavailable and not necessary for budget allocation decisions (in contrast to the more tactical targeting of online ads). However, aggregate-level modeling typically only includes same-period synergy in
an interaction term (e.g. Naik and Peters 2009). Dynamic synergy where display spending in current period increases the conversion of search to sales in future periods, should occur at the aggregate level if it is indeed the case that display ads stimulate subsequent visits through other advertisement formats (Xu et al. 2014). Moreover, online spending can show performance feedback (Dekimpe and Hanssens 1999), so that lower sales can induce managers to lower spending, further decreasing sales.

In this research we use modern time series models to infer the dynamic interactions between search and display ads and search and performance. Specifically, we address the following questions:

- Do display ads influence paid search clicks and vice versa?
- If so, how large are these effects and what dynamic patterns do they follow?
- Is there a long-term equilibrium among display ads, search clicks and performance?
- What are the implications for online marketing metrics and optimal budget allocation?

A key benefit of our approach is the estimation of dynamic interactions, not just among display and search (dynamic synergy) but also from sales to display and search spending (performance feedback). These phenomena are empirically demonstrated in the long-term equilibrium (cointegration) we find and the vector-error correction model we estimate. A key limitation of our data and methodology is that they can not uncover the mechanism by which higher display exposure may lead to higher search clicks and performance (such as consumer and website characteristics or their interactions).

This research draws on, and contributes to two streams of literature – online advertising effectiveness (specifically, display and search) and the spillover effects of online advertising.
In the context of display ads, researchers have studied the impact of ad exposure on click-through behavior (Chatterjee et al. 2003), long-term brand awareness (Drèze and Huss herr 2003), and repurchase decisions (Manchanda et al. 2006). Research has also explored the potential of targeted display advertising (Sherman and Deighton 2001, Shamdasani et al. 2001, Moore et al. 2005) and the consequences of its intrusiveness (Edwards et al. 2002, Goldfarb and Tucker 2011a, 2011b). Lewis and Riley (2011) use a randomized experiment to measure the causal effect of online display advertising on offline retail sales.

In the context of paid search, researchers have focused on understanding optimal advertising strategy in complex search engine environments. Ghose and Yang (2009) and Rutz et al. (2012) adopt a keyword-specific approach to understand the performance of individual keywords and guide optimal keyword investment decisions. Further work examined spillover within search. Yang and Ghose (2010) identify complementarities across organic and paid search listings, and Rutz and Bucklin (2011) find spillover effects from generic search to branded search. Wiesel et al. (2011) model consumer progression through the purchase funnel, and explain how online advertising may drive sales in the offline channel. In contrast to the above study, which mainly examines advertising effectiveness within a particular online channel, we study ad effectiveness taking into account the interaction and feedback between both search and display channels. Furthermore, we focus on the role of search and display advertising in customer acquisition in the commercial banking industry, where consumer decision process tends to be longer and more involved, and the attribution problem is more severe.

Several studies consider spillovers and synergies in online and offline consumer behavior. Naik and Peters (2009) propose a hierarchical model to capture synergies within the offline channel and across online and offline channels. Their model builds on earlier work (Naik and
Raman 2003), and argues that investing in offline and online advertising simultaneously generates greater revenues than investing in each channel individually.

Recent studies examine the interaction between paid search and display at the individual level. Abishek et al. (2012) estimate a Hidden Markov Model of individual consumer behavior to propose and find that display and search ads affect individual consumers differently based on their states in the decision process. Field experiments by Lewis and Nguyen (2011) and Papadimitrou et al (2012) find that exposure to a display ad increases the number of relevant search queries by respectively 27%-45% and 5%-25%. Li and Kannan (2014) show that display ads increase visits through later search. Likewise, Xu et al (2014) find that display ads have a very low impact on purchase conversion, but that they stimulate subsequent visits through other advertisement formats. These results are consistent with surveys showing that between 14% and 50% of consumers exposed to a display ad, perform a branded search (Fulgoni and Morn 2008, iProspect 2009, Malm and Hamman 2009).

As these studies suggest, display ads appear to influence the effectiveness of search ads. However, they lack two important elements that we consider in our study. First, most studies do not allow for sufficiently long dynamic effects of advertising. Studies that attempt to incorporate dynamics do so in an ad-hoc fashion. For example, Papadimitrou et al. (2012) use a 10-minute window while the ad agency for our data provider uses a two-week period (an ad-hoc assumption) to examine the effect of display on search. In contrast, our analysis shows that these dynamic effects are very strong and may last several weeks. Ignoring them can lead to significant underestimation of the effectiveness of online ads. Second, most of the previous studies used click-through rates or similar metrics to measure the impact of display ads on search. In contrast,
we examine how display ads influence search clicks, conversion and ultimately the profitability of the firm. This allows a more appropriate budget allocation between search and display.

The remainder of this article is organized as follows. First, we present our conceptual framework by explaining the relation between attribution and consumer funnel progression. Then, we present the data, modeling methodology, and empirical analysis. To conclude, we provide a set of attribution and dynamics adjusted marketing metrics, and discuss managerial implications.

Conceptual Framework

The consumer journey can be conceptualized as a conversion funnel. While consumer behavior tends to be “exploratory” at the initial stages, it turns to “goal directed” search in later stages (Novak and Hoffman 2003). A consumer may be exposed to a brand through display ads, she may click on these ads to get more information, and may eventually convert. This is the direct impact of display ads on conversion that is found to be small (e.g. Dinner et al. 2011, Manchanda et al. 2006). Alternatively, a consumer could be actively searching for a product online, where she encounters a search ad, clicks on it, and converts. Goal-directed consumers are closer to converting to purchase (Novak and Hoffman 2003) and thus the direct effect of search ads is usually larger than the direct effect of display ads (Manchanda et al. 2006, Wiesel et al. 2011, Dinner et al. 2011). It is common to measure these direct effects of display and search using online metrics such as CTR, CPC, and CPA.

Besides direct conversion, passive forms of advertising exposure may also influence consumers’ consideration sets, and subsequent active engagement with the firm moves consumers down the funnel towards conversion. Also in this scenario, display ads may influence consumers at the top or middle of the funnel while search ads may have more impact at the
bottom of the funnel. As, Xu et al. (2014) state: “Unlike the typical univariate approach in modeling the conversion of website visits, to study the conversion effect of various types of online advertisements, we need an holistic perspective to study their dynamic interactions.”

Figure 1 shows how the firm’s online advertising strategy may influence consumers’ purchase behavior and the firm’s budget allocation. In this framework, the firm allocates a budget between search and display ads that determines the number of ad impressions to consumers. These in turn affect display or search clicks, and eventually, conversion. Two important aspects of our framework should be noted. First, we expect strong interaction between search and display ad impressions and clicks. The empirical results will show if display ads indeed influence search ad effectiveness, and if so, by how much. Second, the system explicitly recognizes simultaneity, whereby the firm’s advertising budget influences consumers’ exposure and purchase behavior, which in turn affects how much the firm spends on advertising (known as ‘performance feedback’ in Dekimpe and Hanssens 1999).

[Insert Figure 1 about here]

Importantly, all the displayed effects in Figure 1 play out over time. We do not a priori know how for how long display and search advertising influence conversions, nor do we know the time span over which performance feedback materializes. This situation is typical in marketing and economics (e.g. Dekimpe and Hanssens 1999, Sims 1980). In dynamic environments, consumer and manager learning may give rise to the evolving business scenario (Dekimpe and Hanssens 1999), in which both marketing spending and consumer conversions evolve together in a long-term equilibrium. For instance, when the company cuts back marketing spending, less consumers convert, which in turn induces lower marketing spending through
budgeting rules such as % of sales allocations (Hulbert 1981). Alternatively, a virtuous spiral emerges from companies increasing spending and obtaining higher sales, which in turn justify higher spending. Such evolving business scenario has been demonstrated in the context of traditional advertising (Baghestani 1991) and regular price changes (Fok et al. 2006). In our context of online impressions, clicks and conversions, such equilibrium is especially intuitive: consumers can only click when companies serve impressions; and cookies track whether converted consumers were exposed to specific online ads².

DATA DESCRIPTION

We use data from a large commercial bank that operates mainly in the southern U.S. After the financial crisis, advertising to acquire new consumers became increasingly important given reduced margins and declining consumer confidence. The bank invests heavily in both paid search and display ads to acquire customers for its checking account.

For the calendar year 2010, the bank and its advertising agency provided us weekly data on the bank’s online marketing expenditure, search and display impressions and clicks, and the number of online applications completed by consumers for a new checking account. The bank invested about $1 million in online advertising, almost equally split between search and display.³ There are two limitations of our data set. First, the bank does not track if online advertising influences consumers to open a checking account in its retail branch. This means that we cannot investigate the impact of online advertising on offline behavior and vice versa. Second, our

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² We are not aware of any marketing theory that explicitly predicts such equilibrium, while economic theory abounds with equilibrium theories that, if they involve non-stationary variables, “require the existence of a combination of the variables that is stationary” (Enders, 2010, p.356).

³ To maintain the confidentiality of the client bank, we have disguised some of the data while maintaining the relationship between the variables of interest.
dataset consists of only aggregate levels of consumer behavior. Although individual-level data is necessary if the analysis aims to uncover the effects of individual exposure, managers routinely use aggregate data to assess the overall performance of marketing instruments and to allocate budgets amongst them.

Paid search data capture weekly spend, clicks, impressions, and the number of applications completed through the paid search ad’s landing page. The display data also contain information on weekly spend, clicks, and impressions. Using internet cookies, applications completed were attributed to display advertising if a consumer had seen a display ad at least one month before converting through the display ad network’s landing page using organic search or a direct link. However, the display advertising data excluded display-driven paid search conversions, as the paid search campaigns are overseen by platforms maintained by search engines unrelated to the display ad networks.

The bank invested in five search engines and eleven ad networks. We aggregate over search engines and ad networks to the week level to avoid over-parameterization as our primary interest lies in the interplay of search and display advertising, as opposed to the performance of individual search engines or display ad networks. Furthermore, the bank’s limited investment over a number of smaller ad networks and search engines makes it difficult to estimate the impact at the level of a search engine or ad network.

Table 1 presents a correlation matrix of the variables in our data. The notations used for the variables are indicated below:

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4 This is another form of ad-hoc attribution between display ads and other online media. However, we do not investigate this in our study due to the lack of individual-level data available to us.
5 In a robustness check, we separate the data for Google (50% of our search engine volume) versus other search engines and find similar results.
- $SA_t$: Checking account applications completed through paid search in week $t$.
- $DA_t$: Checking account applications completed after exposure to a display ad.
- $SI_t$: Paid search ad impressions.
- $SG_t$: Paid search ad clicks.
- $SE_t$: Weekly expenditure on paid search advertising.
- $DI_t$: Display ad impressions.
- $DC_t$: Display ad clicks.
- $DE_t$: Weekly expenditure on display advertising.

Table 1 shows that many variables are highly correlated, especially those related to the same marketing instrument. Display impressions exhibit a 0.98 correlation with clicks and a 0.94 correlation with spend. Therefore, we excluded display clicks and spend from the analysis. In the case of paid search, spend exhibits a high correlation (0.88) with impressions, so we exclude search advertising spend from the analysis to minimize possible collinearity.

Table 2 provides summary statistics of our data and Figure 2 shows the time series behavior of the resulting series. Besides their week-to-week variation (which allows model estimation and attribution inferences), all series also show a clear downward trend. We learned from management that the bank was exhausting its advertising budget, and hence decreased its
investments over time to avoid overspending. We incorporate this trend as a non-deterministic component of the model, allowing for the other endogenous variables to explain it.

Figure 2 also shows model-free indications of dynamic synergy from display to search. Observe that display impressions are substantially higher in the earlier period (up to week 33) than in the later period of the data (a slope difference test yields a p-value of 0.07). In the period with high display impressions, search applications are correlated 0.58 with search clicks one week earlier and 0.60 with search clicks two weeks earlier. In the period with low display impressions, search applications are only correlated 0.45 with search clicks one week earlier and 0.49 with search clicks two weeks earlier. These correlations are depicted graphically in Figure 3.

[Insert Figure 3 about here]

**METHODOLOGY AND ANALYSIS**

We use persistence modeling techniques to capture the complex dynamic interdependencies in online advertising (Dekimpe and Hanssens 1999). Persistence modeling extends multivariate time series methods into the domain of marketing, thereby enabling researchers to model the effects of spillover and feedback dynamics through a system of equations involving marketing actions and consumer response. Persistence modeling is particularly relevant in the context of online advertising as the associated multivariate time series techniques require no stringent a priori restrictions on model structure and allow all variables of interest to affect each other.

Persistence modeling involves several steps. A series of tests are used to determine the correct model specification. Granger causality tests are used to identify which variables enter the system endogenously. Unit root tests are done to determine which of the endogenous variables exhibit non-stationary behavior and should enter the model in differences. Next, cointegration
tests are used to identify stationary linear combinations of non-stationary endogenous variables that must be considered in the specification to correct for temporary deviations away from the implied long-run equilibria (see Murray 1994 for a concise and memorable illustration of cointegration and its relation to error-correction models).

Granger causality tests, conducted pair-wise for variable lag-lengths ranging from 1 to 20, suggest that all variables should enter the system endogenously. Figure 4 presents a schematic of the Granger causality results. For example, an arrow from $DI$ to $SC$ indicates that $DI$ is found to Granger-cause $SC$ for at least one of the lag-lengths considered. Interestingly, no arrow exists from display impressions to search applications, implying that if display does affect search, the effect travels through search impressions and search clicks. The complex nature of interdependencies depicted in Figure 4 points to the appropriateness of using a flexible approach, such as persistence modeling, to capture cross-ad spillovers and online advertising dynamics.

We conduct Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) unit root tests to determine if the endogenous variables are evolving or stationary. Table 3 summarizes the resulting statistics of the unit root tests. The KPSS test identifies all series as evolving, whereas the ADF test identifies all but $SI$ as evolving. We choose to include $SI$ as an evolving variable to guard against spurious regression, which is more damaging than differencing a stationary variable (Granger and Newbold 1986).

The Johansen cointegration trace test identifies three cointegrating relations. These relations can be interpreted as long-run equilibrium conditions which may arise as a result of
firm budgeting rules or consumer decision processes (Dekimpe and Hanssens 1999). Based on the outcomes of the Granger causality, unit root and cointegration tests, we specify a vector error correction model (VEC) with all variables as endogenous. The model is called ‘error-correction’ because a deviation (‘error’) from the long-term equilibrium will invites adjustment of the variables towards restoring the equilibrium. Different variables work stronger towards restoring the equilibrium, which is captured in their estimated ‘speed of adjustment’ coefficient. The interpretation of VEC models is particularly interesting from a substantive perspective. A long-term equilibrium links search and display applications to a combination of firm control variables (e.g. search and display ad impressions) and consumer actions (e.g. clicks on these ads). This time series behavior of the variables implies learning and action adjustment by managers and prospective consumers (Pauwels 2001). Consumers learn from marketing stimuli and adjust their application behavior. Managers observe these behavioral outcomes and adjust their marketing stimuli.

The general form of the VEC model with \( K \) lags is given by equation 1:

\[
\Delta Y_t = \Gamma_0 D_{1t} + \sum_{k=1}^{K} \Gamma_k \Delta Y_{t-k} + \alpha e_{t-1} + u_t, \\
\]

where \( e_{t-1} = \beta \begin{bmatrix} D_{2t} \\ Y_{t-1} \end{bmatrix} \),

and \( u_t \sim MVN(0, \Sigma) \),

In equation (1), \( Y_t \) is the vector of endogenous variables at time \( t \), \( D_{1t} \) and \( D_{2t} \) are vectors of deterministic components (e.g. intercept, trend), \( e_t \) is a matrix of cointegrating relations, \( \Gamma_0 \), \( \Gamma_1, ..., \Gamma_k \), \( \alpha \) and \( \beta \) are parameter matrices to be estimated, and \( \Sigma \) is the covariance matrix of the
multivariate-normally distributed error terms \( u_t \). The coefficients in \( \Gamma_1, \ldots, \Gamma_k \) capture the effects of past changes in the endogenous variables on their current deviations. The coefficients in \( \alpha \) reflect the speed of adjustment of the endogenous variables towards the equilibrium cointegrating relations defined in \( \mathbf{e}_{t-1} \). We refine the model further by allowing for an intercept in both \( D_{1t} \) and \( D_{2t} \). The intercept in the model specification allows for the possibility of a deterministic time trend to exist concurrently with the stochastic one implied by the error correction model. The intercept term in the cointegrating vector is included to account for the initial values of the endogenous variables. The Bayesian Information Criterion identifies a lag-length of 1 as optimal. The resulting model specification is indicated in equation (2):

\[
\begin{bmatrix}
\Delta SA_t \\
\Delta DA_t \\
\Delta SI_t \\
\Delta SC_t \\
\Delta DI_t
\end{bmatrix} =
\begin{bmatrix}
Y_{10} \\
Y_{20} \\
Y_{30} \\
Y_{40} \\
Y_{50}
\end{bmatrix} +
\begin{bmatrix}
Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} \\
Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} \\
Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} \\
Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} \\
Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55}
\end{bmatrix} \begin{bmatrix}
\Delta SA_{t-1} \\
\Delta DA_{t-1} \\
\Delta SI_{t-1} \\
\Delta SC_{t-1} \\
\Delta DI_{t-1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} \\
\alpha_{21} & \alpha_{22} & \alpha_{23} \\
\alpha_{31} & \alpha_{32} & \alpha_{33} \\
\alpha_{41} & \alpha_{42} & \alpha_{43} \\
\alpha_{51} & \alpha_{52} & \alpha_{53}
\end{bmatrix} \begin{bmatrix}
\beta_{10} & 1 & 0 & 0 \\
\beta_{20} & 0 & 1 & 0 \\
\beta_{30} & 0 & 0 & 1 \\
\beta_{41} & 1 \\
\beta_{52} & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 \\
\mathbf{u}_{SA,t} \\
\mathbf{u}_{DA,t} \\
\mathbf{u}_{SI,t} \\
\mathbf{u}_{SC,t} \\
\mathbf{u}_{DI,t}
\end{bmatrix}
\]

The parameters are recovered in two steps. First, Johansen’s procedure is used to estimate the cointegrating vectors. Then, the first differences of the endogenous variables are regressed on an intercept, their lags and the cointegrating vectors to recover the remainder of the coefficients.

Not all the coefficients in this model are identified. In particular standard errors cannot be recovered for \( \beta_{10}, \beta_{20} \) and \( \beta_{30} \). Furthermore, an arbitrary normalization is required to identify the remaining coefficients of the \( \beta \) matrix.
Table 4 shows the full set of parameter estimates and asymptotic standard errors. The model exhibits good fit for a model in differences\textsuperscript{6}, with individual equation $R^2$ statistics ranging from 0.27 to 0.45. Portmaneau tests fail to find significant evidence of residual autocorrelation and normality tests fail to reject normality of the residuals. Furthermore, generalized fluctuation tests for structural change fail to find significant evidence of parameter instability\textsuperscript{7}.

[Insert Table 4 about here]

It is difficult to directly interpret the parameters of persistence models (Sims 1980), so we proceed to derive implications by impulse response analysis.

\textit{The Effects of Search and Display Ads}

As recommended for multivariate time series models (Sims 1980), we use impulse response functions to analyze the impact of search and display advertising, and assess significance by applying a one standard error band to the impulse response coefficients\textsuperscript{8} (Sims and Zha 1999, Dekimpe and Hanssens 1999). Pesaran and Shin (1998) provide a derivation of the generalized impulse response function, which captures the impact of an unexpected shock to the endogenous variables in a VEC model by constructing two forecasts and taking their difference. One forecast takes the shock into consideration, while the other does not. The difference of the two forecasts

\textsuperscript{6} As detailed in the robustness section, the corresponding explanatory power for the model with all variables in levels is in the 0.80 to 0.95 range.

\textsuperscript{7} We test for structural change using generalized fluctuation tests to ensure that our model parameters remain stable over time. Generalized fluctuation tests derive an empirical fluctuation process for each endogenous variable based on either the residuals or the estimates of the VEC model.

\textsuperscript{8} We calculate confidence bands for the impulse response functions by simulating 1000 random draws from a multivariate normal distribution with mean zero and covariance matrix equal to the residual covariance matrix of the model, using these draws to perturb the data, and estimating the impulse response functions 1000 times on the resulting simulated datasets. Quantiles of the distributions of coefficients provide an indication of the accuracy of the impulse response functions. We take the 16\textsuperscript{th} and 84\textsuperscript{th} percentiles of the empirical distribution to approximate a one standard error band.
provides the incremental impact of the shock. Impulse response functions trace the impact of a shock to one endogenous variable through other endogenous variables, thereby providing a cumulative view of all dynamic interactions that took place.

Shocks unexpected by the model can represent e.g. a product launch, an extra price promotion or a TV ad campaign in previous applications of persistence modeling (e.g. Pauwels et al. 2004, Srinivasan et al. 2009). In our case, a shock to search impressions, clicks and display impressions, represents an increase in consumer viewership of the ads. In the case of search and display applications, a shock represents an increase in applications, holding display exposure and search clicks unchanged. It is common in practice to make budgeting decisions based on search clicks and CPC, and display impressions and cost per thousand (CPM) impressions. Therefore, we use search clicks and display impressions as the marketing variables of interest and interpret the forecasts that result from their shocks as the effects of increases in marketing investment. We apply one standard deviation shocks to the marketing variables and study their sustenance, implications for performance, and interaction between search and display.

Figure 5 presents the sustenance levels of search clicks and display impressions. Sustenance measures the response of a variable to a one standard deviation shock to itself. The plots suggest that persistent investment and complex consumer transitions between different channels of the conversion funnel lead to sustained levels of long-run exposure to marketing. Panel 5a shows that a shock of 4,000 search clicks wears-in after 8-10 weeks and stabilizes at about 900 clicks per week in the long run. Display impressions follow a similar pattern according to panel 5b. A shock of 5 million impressions wears-in over a period of 7-8 weeks and stabilizes at a sustained level of 1.4 million impressions per week.
Sustained levels of marketing activity are common in practice (e.g. Dekimpe and Hanssens 1999) and often represent the commitment of managers to continue investing in an action that was initially taken (Ghemawat 1991). It may also allow the firm to reap permanent performance benefits by repeating ‘short-term’ activities, as we investigate next.

The plots in Figure 6 show the performance impact of marketing. The top row captures the impact of initial shocks and persistence in marketing exposure on search applications. Panel 6a shows the impact of search clicks on search applications. A shock of 4,000 clicks generates 15 search applications initially. After a wear-in period of 4 weeks, 900 clicks (Figure 5a) generate 26 applications per week (Figure 6a). A smaller number of search clicks is required to maintain a higher level of search applications in the long run, suggesting that the effectiveness of an injection to search advertising increases as it persists over time. Thus, the sustained commitment to search ads (figure 5) leads to a sustained increase in search applications (figure 6). Dekimpe and Hanssens (1999) call this scenario ‘evolving business’.

Panel 6b shows the impact of display impressions on search applications. As expected, an initial increase in display impressions does not generate any search applications. However, after a period of two weeks, display impressions positively impact search applications. A sustained level of 1.4 million impressions (Figure 5b) generates about 20 search applications per week (Figure 6b). Consistent with our conceptual framework, display exposure appears to drive consumers to paid search over time.
The bottom row of Figure 6 captures the impact of online ads on display applications. A shock to search clicks does not affect display applications (Figure 6c), except for the initial period, which may point to consumers who would have applied through display substituting into the search channel. Panel 6d shows that the effect of display impressions on display applications is powerful and immediate. A shock of 5 million impressions (Figure 5b) generates 34 applications immediately. After one week, display applications dip and then stabilize at 28 applications per 1.4 million impressions (Figure 5b) in the long run.

To further understand the impact of advertising, we consider the interaction between search and display ads to see how increased levels of display ads may drive search impressions and clicks. Figure 7 plots the impact of a shock in display impressions on search impression and search clicks. In the short-run, we observe a decrease in both search impressions and clicks, which may be driven by consumer substitution across channels. In the long-run, a sustained increase in display impressions drives a significant increase in search impressions and clicks, suggesting that display exposure not only increases conversion through search, but also drives search visitation and search clicks. This finding, together with the lack of direct Granger-causality between display impressions and search applications, suggests that display advertising drives search applications through search impressions and clicks. Hence, in calculating the overall impact of display advertising, we must take into account the potential associated increases in costs from search advertising.

[Insert Figure 7 about here]

In sum, the impulse response analysis suggests that display advertising may drive search behavior. This synergy is confirmed in both Granger causality tests and in the Forecast Error
Variance Decomposition (FEVD), which tracks the percentage of forecast error variance in a variable that can be attributed to shocks to any endogenous variable in the system (Nijs et al. 2007). Past shocks to display impressions explain 40% of the forecast error variance in search impressions, 17% in search clicks, and 16% in search applications. All of these estimates are significantly different from zero and consistent with the inference that display impressions move consumers through search media.

Robustness Checks to Functional Specification and Selection Concerns

The estimated Vector-Error Correction functional form is driven by the outcome of unit root and cointegration tests. How sensitive are our substantive findings to the test outcomes?

First, we estimated a Vector Autoregressive (VAR) model in differences, thus eliminating the long-term equilibrium adjustment part in equations (1) and (2). The resulting impulse response functions show similar short-term effects of marketing actions on performance. They also capture persistent same-channel effects, but these are not statistically significant as a result of poor model fit and large standard errors. Incorporating cointegration (through the VEC specification) improves the fit of the model, lowers standard errors and introduces the cross-channel spillover effect of display advertising.

Second, we estimated a VAR model in levels, ignoring the unit root evidence that lead us to estimate the original model in differences. The VAR in levels model exhibits high explanatory power, as expected, with individual equation $R^2$ in the .80 – .95 range. The immediate (same-week) and wear-in (next weeks) effects are similar to those of the VEC. In contrast, the persistent effects are zero, as assumed by any model in levels.

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9 We apply orthogonalized FEVD with different logical variable orderings such as $DI, SI, SC, DA, SA$ and find similar results across orderings. The orthogonalized IRFs are also similar to the generalized IRFs.
In sum, the short-term effects of marketing actions are similar across functional forms. However, the proper tests and Vector Error Correction specification are needed to uncover the long-term cross channel effects. These results echo calls in economics (e.g. Enders 2010) that researchers should consider the econometric implications of their beliefs that certain variables are in long-term equilibrium.

Selection biases may result from paid ad reaction of consumers who would have applied anyway through other channels. In our context, the bank’s offer may be more suited to a specific subgroup of consumers, who are both more likely to get exposed to a bank’s ads and more likely to act on such exposure. Such a form of targeting would lead to an over-estimation of the performance effect of paid search. In principle this concern is thus severe, and mentioned by Li and Kannan (2014) as the first limitation of their model. Consumers may also self-select into using a particular search engine and searching for terms related to banking. As such, consumers with exposure to display and search ads may be different from consumers with exposure to display ads only, and these differences (unobserved in our data) may be confounded with display-search spillover effects.

In practice, how do we assess whether these selection concerns are severe in our data? First, if the consumers who click on display or search ads are much more interested a priori, then reducing online spending would hardly make a dent in performance, as observed for both eBay and a leading lodging brand, which shut off paid search (Blake et al. 2013, Li and Kannan 2014). For those very well-known brands, we would thus not find a long-term equilibrium between online display/search and performance. In contrast, our data provider, who reduces display and search spending in most of the data period (see Figure 2), observes a strong reduction in online

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10 We thank an anonymous reviewer for pointing us to this issue.
applications, as captured by the long-term equilibrium among display, search and applications.

Second, to address the self-selection concerns, we examine consumer conversions through different search engines. If self-selection concerns were potentially severe, we would expect to see vastly different estimates of the effects of display ads on search across search engines. Search engine characteristics differ, and so do the characteristics of consumers who use these search engines (Schwartz 2014). Similarity of estimates across search engines would suggest that features of the search engine or characteristics of the consumers who use these search engines do little to explain the estimated display-search spillovers in the main model. Figure 8 shows the estimated IRFs using data on impressions, clicks and conversions through the Google search engine, which accounts for around 50% of all completed applications, and Figure 9 shows the estimated IRFs using data on all search engines other than Google. The cross-channel IRFs for Google are directionally similar to those obtained for all other search engines, but marginally insignificant as a result of smaller effect magnitudes and an increased estimator variance. Google has a distinctive layout and is known to attract a different user demographic than other search engines. The similarity of effects across Google and other search engines suggests that the display-search spillover effects cannot be explained alone by user or search engine heterogeneity.

[Insert Figure 8 about here]

[Insert Figure 9 about here]

A concern remains that consumers who search after viewing a display ad may inherently have a higher propensity to complete a search application that cannot be attributed to characteristics of the search engine and its users. Absent experimental variation, we cannot rule
out this explanation with the data we have available. Independent from our research, the bank we study did conduct a randomized experiment\textsuperscript{11}. The results from the experiment showed that display ads improve search ad conversion by 15-20\% within two weeks for consumers who were exposed to a display ad previously. This finding helps assuage remaining concerns about self-selection, although the data used for this experiment and certain details of the experimental design are not available to us for analysis.

We acknowledge that the absence of experimental variation is a limitation of our work that prevents us from completely ruling out selection concerns. Future research should examine the extent to which selection interacts with brand familiarity (as suggested by Li and Kannan 2014) or to other characteristics of the product (e.g. involvement) or brand (Demirci et al. 2014).

\textit{MANAGERIAL IMPLICATIONS}

Standard online metrics such as CPA and static ROI are static measures that ignore attribution or dynamic effects. As suggested by the impulse response analysis, shocks to display advertising may increase both exposures to search marketing and search applications. Moreover, the performance effects are non-stationary and stabilize only 2-4 weeks after the initial marketing shock, implying that marketing metrics should take into account not only attribution,

\begin{footnotesize}
\textsuperscript{11} The experiment was conducted by the bank’s ad agency for two months by randomly assigning target consumers to test and control groups (the exact process of randomization used by the agency was not revealed to us). The agency tracked the conversion rate of both test and control groups 14 days after the display ad was shown – the 14 day time period was arbitrarily selected by the agency and our results later show that the dynamics of conversion are spread over a longer time frame. The results of the experiment showed that the control group clicked on the search ads 87,152 times, which led to 1,095 completed applications, or a conversion rate of 1.26\%. The test group (a smaller group) clicked 49,504 times on search ads within 14 days of their exposure to display ads, which led to 734 completed application for a conversion rate of 1.48\%. Based on these results the agency concluded that display ads improve search ad conversion by 1.48/1.26 or about 17\%, and this ratio varied between 15\%-20\% for different ad networks.
\end{footnotesize}
but also the dynamic effects of marketing. We proceed to develop adjusted marketing metrics, taking impulse response function estimates as the spillover and dynamic effects of online ads.

**CPA and ROI of Search Ads**

We begin with the implications for search metrics, where only dynamics need to be taken into account, since search clicks do not impact display applications (Figure 6c). Although a complicated pricing and bidding system drives cost per click (CPC), as a simplification we assume that CPC remains constant over the impulse response forecast.

Table 5 contrasts CPA and ROI calculated in a standard fashion with their dynamic counterparts as implied by the impulse response analysis. CPA is calculated as the total search expenditure divided by the sum of all search applications. The standard approach is a static measure of cost per acquisition (or application in our context) that is commonly used by most marketing managers and search engines like Google. In contrast, dynamic CPA incorporates the long-run effects implied by the impulse response functions.

[Insert Table 5 about here]

Using the bank’s annual expenditure on search ads and the total number of search applications, we find the standard CPA to be $73, a number that the bank and its ad agency used to assess the performance of its search ads. For estimating the dynamic effect of search, recall that a shock of 4,000 search clicks generates about 15 applications in the short run. Using the average CPC rate of $1.07 from our data, the immediate dynamic CPA is then $296. However, as discussed earlier, search clicks increase by 900 per week and generate about 26 search applications per week in the long run, implying a long run CPA of $38, 48% lower than the standard CPA.
Table 5 also provides a measure of ROI that shows the return for every $1 invested in search ads. According to the bank, about 80% of the customers who complete online applications are approved for a checking account, and two-thirds of the approved customers actually fund the account (i.e., put some money in their account within a month). In other words, 80% * 67% = 53.6% of the applications become active customers. In addition, both search and display ads were generally accompanied by a promotional offer that included a free iPod Nano, iPod Touch or $100-$150 cash. The bank estimates that on average the effective cost of these promotions is about $100 for each new active customer acquired through the online channel. The bank further estimates the average customer lifetime value (CLV) to be $300 for every active account.

Using this information we calculated the ROI for the standard and the dynamic approaches. For example, the standard CPA is $73, but the effective cost of getting an active account is $[(73/0.536)+100] = $236, and the benefit of this account in the long run is its CLV of $300. ROI is then simply the benefit ($300) divided by the effective cost of an active account ($236), or 1.27 for the standard approach.

The results show that accounting for long run dynamic effects reduces search CPA by 48% and increases its ROI by 38% compared to the standard metrics that ignore these dynamic effects. In other words, the firm may be underinvesting in search by relying on standard metrics.

**CPA and ROI of Display Ads**

Results for impulse response functions show that search ads do not affect display applications, but display impressions influence both search and display applications. Therefore, in addition to dynamics, display metrics should incorporate attribution. Moreover, shocks to display advertising not only increases search applications, but also increases search clicks, which
may lead to greater search cost. Display attribution must take into account not only the benefit, but also this additional cost of spillover into the search channel.

To make this point salient, Table 6 presents a comparison of three methods for calculating display CPA and ROI: (1) without attribution to search, (2) with attribution to search applications, but without accounting for additional search cost, and (3) with additional search applications and search cost both considered. For these calculations we use the average cost-per-thousand (CPM) impressions of $2.05 from our data. The calculations for CPA and ROI follow the same logic as before, except that now we also include the impact of display impressions on search applications and search cost.

[Insert Table 6 about here]

The long-run CPA for display is 35% lower than in the standard approach when we account for its impact on search applications but ignore the additional cost it may drive. However, even after accounting for the additional cost, long-run CPA for display is 14% lower than in the standard approach. ROI of display impressions exhibits a similar pattern – it is 28% higher than the standard ROI when only attribution to search applications is considered, and is about 10% higher when both additional search applications and extra search costs due to display ads are included.

Budget Allocation

It is clear from the previous analysis that search ads are more effective than display ads, even when we account for attribution effects of display ads. This is due to the fact that search ads show a significant dynamic effect, which is intuitive in the context of a bank checking account, where consumers are likely to take several weeks before making a decision. These results have
direct implications for budget allocation. How should the firm allocate its online advertising budget between search and display and how does this allocation compare to the firm’s current allocation?

Ideally, we would derive an optimal dynamic advertising policy for the bank, which would specify how much the bank should invest in search and display each week to maximize the expected discounted sum of future profits (Naik and Raman 2003). There are two challenges to using this approach in our specific context. First, the bank we observe is in an evolving business scenario. The impulse response functions contain information about the long-run effects of persistent marketing (ie the bank can expect 28 applications if it invests in 1.4 million display impressions per week), but not about the decay rate in applications if the bank shuts down its marketing. Second, the VEC model (required for consistent estimation with non-stationary time series) models differences in the response variables as a function of differences in the explanatory variables and a set of co-integrating relationships. As the VEC specification differences out intercepts, we cannot obtain estimates of how many applications the bank would have received had it not invested in advertising. Absent these components (baseline level of applications and decay rates of advertising) we cannot construct a per-period profit expression without additional assumptions.

An alternative approach is to use a static optimization rule. In a non-stationary scenario, the firm should allocate budget according to the long-run effectiveness of marketing instruments (Dekimpe and Hanssens 1999). Optimal budgeting would then allocate shares according to the ratio of display and search advertising long-term elasticities, assuming constant elasticity of advertising. This ‘ratio of elasticities’ rule is standard in marketing budget allocation and robust to popular profit functions in marketing (e.g. additive in Dorfman and Steiner 1954 and
multiplicative in Wright 2009). It requires neither the estimates of the baseline level of applications given no advertising, nor the decay rate of advertising. Moreover, past extensions of static to dynamic allocation (Nerlove and Arrow 1962, Eeckhoudt 1972) often result in optimal dynamic advertising policy that is largely similar to the optimal static policy12.

Specifically, we calculate the long run elasticity for search as \( \frac{\Delta \text{applications}}{\text{mean(applications)}} \times \frac{\Delta \text{clicks}}{\text{mean(clicks)} \times \frac{5}{1.07 \text{click}}} \), where $1.07 is the average cost per click. For display, we apply the same formula but take into account the additional applications from the search channel in the numerator, and the additional costs from search clicks in the denominator. Thus,

\[ \frac{\Delta \text{applications}}{\text{mean(applications)}} \times \frac{\Delta \text{impressions}}{\text{mean(impressions)} \times \frac{2.05}{1000 \text{impressions}}} = \frac{\Delta \text{clicks}}{\text{mean(clicks)} \times \frac{5}{1.07 \text{click}}} \times \frac{2.05}{1000 \text{impressions}} \],

where $2.05 is the average CPM for display. These calculations yield 0.96 and 0.57 as the respective long-term elasticities13 for search and display in table 7.

[Insert Table 7 about here]

Consistent with our previous results, we find that search elasticities are significantly higher than the display elasticities, suggesting that the firm would be better off spending more on search than its current 54% budget allocation. Display advertising yields a lower elasticity even after accounting for attribution. Wiesel et al. (2011) similarly find a high search advertising

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12 Nerlove and Arrow (1962) find that the optimal decision rule is “similar to a rule of thumb actually used by many firms” and involves setting advertising as a percentage of sales. Eeckhoudt (1972) finds that the Dorfman-Steiner rule holds “with only a minor modification” in a dynamic setting. The extent to which the dynamic allocation differs from the static allocation is directionally ambiguous and depends on the model assumptions and parameters.

13 The long-run elasticity reflect the percent change in the total number of display and search applications from a 1% change in investment for a particular marketing instrument, taking into account any additional costs it may drive. We use sample means instead of the last observation in the series as our data exhibit a decreasing tendency, whereas impulse response functions are calculated as averages over the data range. Therefore, using the last observation in the series may yield inconsistent elasticity estimates.
elasticity of 4.35 in the context of a multichannel furniture retailer. Dinner et al. (2011) find a long-run search elasticity of 0.49 and a display elasticity of 0.15. Manchanda et al. (2006) find a display elasticity of around 0.02 with respect to repeat purchase behavior. While our elasticity estimates are within the broad range of the estimates found in the previous studies, it is important to note that our context of bank applications and the $100 incentive offered by the bank makes direct comparisons across studies somewhat difficult.

Given the current advertising budget of the firm, the optimal allocation between search and display ads is given by the ratio of their elasticities. Table 8 shows the actual and proposed budget allocation.

[Insert Table 8 about here]

The firm is currently allocating 54% of its online ad budget on display advertising even though the standard metrics used by the firm show the search CPA ($73) to be about 20% lower than the display CPA ($88). The bank and its ad agency made this allocation recognizing that the standard metrics do not account for attribution. Given the nature of the product category they expected display to have significant impact on search applications. To test this hypothesis the ad agency conducted a field experiment and found that search effectiveness improved by about 20% when it was preceded by display ads. This factored into their budget allocation.

However, our model suggests that search should have 63% of the total budget, or almost 36% higher than the budget currently allocated by the firm, and display budget should be reduced. It may seem counterintuitive to reduce the budget allocation for display ads after accounting for its attribution (something that the firm is also trying to do through its experiment), but a simple attribution analysis ignores two important aspects. First, it ignores the additional
cost of search clicks that are accompanied by the search applications generated by display.

Second, and perhaps more important in our application, the firm is ignoring dynamic effects that are particularly strong for search ads.

CONCLUSIONS

Our goal in this study was to find out whether online display ads influence search (attribution problem), whether online advertising, more generally, exhibits dynamic effects, and if so, what implications this has for the firm’s budget allocation. We used persistence modeling on data from a bank that used online advertising to acquire new customers for its checking account. We found that display ads can have a significant impact on search applications, as well as clicks. The majority of this spillover was not instant, but took effect only after two weeks. On the other hand, search advertising did not lead to an increase in display applications.

Our findings suggest that simple static metrics, commonly used in the industry, may not accurately measure the effectiveness of online advertising. We propose dynamic versions of the classic metrics and find that search CPA is 48% lower than the static CPA, while search ROI is 38% higher than the static ROI. Similar pattern emerges for display advertising, where we also account for attribution. This made display CPA 14% lower and ROI 10% higher than their standard counterparts. Finally, we show that these revised measures of ad effectiveness lead to a very different budget allocation than the one used currently by the firm. Specifically, we find that even though our proposed allocation gives credit to display due to its effect on search applications, search ad budget should be increased by 36% from its current level due to its strong dynamic effects, and display ad budget should be decreased by 31%.
The strong dynamic effects of online advertising are consistent with a recent study conducted by Google (Lecinski 2011), which finds that in commercial banking, consumers are most influenced by the search they conduct 1-2 months prior to conversion. The study finds that consumers search 1-3 months prior for insurance and automotive products, but only 1-4 weeks prior for technology, and may search on the same day as they purchase for CPG products, pointing to a possible boundary condition on our findings. We expect strong dynamic effects of online advertising to persist, and overcome attribution effects, for products that require significant deliberation, such as financial investments, home or car purchases. Products such as consumer-packaged goods may not benefit from strong dynamic effects as much, as individuals tend to consider these products only over a short time horizon.

Prior research has seldom looked at advertising and performance data characterized by an evolving business scenario. In contrast to a stationary environment, advertising in an evolving business scenario may exhibit permanent effects. We find that in an evolving business scenario, advertising becomes more effective over time, as fewer search clicks or display impressions are required to sustain the same level of applications as at the start of a campaign. Furthermore, attribution effects may also be permanent, as we find for the spillover from display into search. In new product contexts, where both managers and consumers may be learning about advertising effectiveness and budgeting rules, dynamic effects can be strong and carry over from past advertising investments. Firms may be able to ride these waves and leverage the increased advertising effectiveness brought about by past advertising investments. Hence, in an evolving business scenario, more so than in a stationary environment, budget allocation needs to take into account past advertising campaigns, possibly even those implemented months in advance.
Our study has several limitations that can provide avenues for future research. We do not consider spillover effects of search and display into other channels. Future research may examine the effects of online ads on conversions and funnel progression in mobile and offline channels. We use aggregate data that does not allow us to untangle the mechanism that may be driving consumer decisions. Using disaggregate data, future research could provide richer insights into the consumer journey and progression, and the differential impact of various marketing instruments at various stages of the conversion funnel. Future studies may also wish to generalize our findings by examining multiple products and contexts. Finally, we call for further research addressing the targeting and self-selection concerns in online marketing.

Overall, our research suggests that managers should carefully consider the interaction and dynamic effects of search and display advertising. Our results show that classic metrics used in practice are highly biased since they do not account for these effects. As a result firms may be making suboptimal budget allocation decisions.
References


Table 1: Correlation Matrix

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Table 2: Summary statistics (per week)

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Table 3: Summary of unit root test results

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Note: Bold numbers indicate significant evidence of non-stationarity.
Table 4: VEC parameter estimates (asymptotic t-statistics in parentheses)

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<td>[-1.29520]</td>
<td>[-0.76186]</td>
<td>[1.48318]</td>
<td></td>
</tr>
<tr>
<td>$\Delta DI_t_t-1$</td>
<td>-3.72E-06</td>
<td>5.22E-06</td>
<td>-0.009798</td>
<td>-0.000447</td>
<td>0.144877</td>
</tr>
<tr>
<td>[-2.05150]</td>
<td>[2.68708]</td>
<td>[-2.71173]</td>
<td>[-2.39148]</td>
<td>[0.77352]</td>
<td></td>
</tr>
<tr>
<td>$constant$</td>
<td>-1.594333</td>
<td>-6.117405</td>
<td>-7426.962</td>
<td>-356.0680</td>
<td>-249869.3</td>
</tr>
<tr>
<td>[-0.45454]</td>
<td>[-1.62862]</td>
<td>[-1.06336]</td>
<td>[-0.98637]</td>
<td>[-0.69013]</td>
<td></td>
</tr>
</tbody>
</table>

$R^2$ 0.396648 0.381397 0.453215 0.370934 0.266994
Table 5: CPA and ROI for search ads

<table>
<thead>
<tr>
<th>CPA</th>
<th>Standard</th>
<th>Dynamic (immediate)</th>
<th>Dynamic (wear-in)</th>
<th>Dynamic (long-run)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPA</td>
<td>$73</td>
<td>$296</td>
<td>$52</td>
<td>$38</td>
</tr>
<tr>
<td>ROI</td>
<td>$1.27</td>
<td>$0.46</td>
<td>$1.52</td>
<td>$1.75</td>
</tr>
</tbody>
</table>

% Change: Standard vs. Dynamic

Table 6: CPA and ROI for display ads

<table>
<thead>
<tr>
<th>CPA</th>
<th>Standard</th>
<th>Dynamic (immediate)</th>
<th>Dynamic (wear-in)</th>
<th>Dynamic (long-run)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPA</td>
<td>$88</td>
<td>$298</td>
<td>$120</td>
<td>$99</td>
</tr>
<tr>
<td>ROI</td>
<td>$1.14</td>
<td>$0.46</td>
<td>$0.92</td>
<td>$1.05</td>
</tr>
</tbody>
</table>

% Change

Numbers in this table have been rounded.
Table 7: Advertising elasticities

<table>
<thead>
<tr>
<th></th>
<th>Ad Elasticity (immediate)</th>
<th>Ad Elasticity (wear-in)</th>
<th>Ad Elasticity (long-run)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Search</td>
<td>0.12</td>
<td>0.71</td>
<td>0.96</td>
</tr>
<tr>
<td>Display</td>
<td>0.19</td>
<td>0.46</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 8: Actual and proposed budget allocations

<table>
<thead>
<tr>
<th></th>
<th>Actual Budget Allocation</th>
<th>Proposed Budget Allocation</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Search</td>
<td>$542,000</td>
<td>$739,000</td>
<td>+36%</td>
</tr>
<tr>
<td>Display</td>
<td>$639,000</td>
<td>$442,000</td>
<td>-31%</td>
</tr>
</tbody>
</table>
Figure 1: Online advertising conversion funnel

Figure 2: Weekly trends
Figure 3: Descriptive evidence of search-display interaction

Figure 4: Granger causality graph
Figure 5: Sustenance levels of marketing variables

(a) Sustenance level of a shock in search clicks
(b) Sustenance level of a shock in display impressions

Weeks

Search Clicks

Display Impressions

-2e+06 0e+00 2e+06 4e+06 6e+06
Figure 6: Performance impact of marketing variables
Figure 7: Impact of display ads on search funnel progression
Figure 8: Impulse response functions for Google
Figure 9: Impulse response functions for search engines other than Google