What Courses to Take After Math 18, 19, 21

This document was compiled in the springs of 2015 and 2016 from student surveys and panels, and official course descriptions. Quotes are from students. There is more general advice at the end as well.

A course with a * in front of it has been considered a highlight by students.

In the Math Department

Good first courses after Math 18, 19, 21:

- **Math 101: Sets, Groups and Topology*

  Math 101 is both an introduction to proofs and a way to get a taste of three main branches of math – analysis, algebra, and topology. Students appreciate that the course "holds your hand in the beginning" to explain various proof methods and make you comfortable no matter what your background is.

  Math 101 was highly recommended as a first math course after 21; although you can go straight to *Math 112*, *Math 121* and *Math 130*, often students in those courses who haven't taken 101 find them very stressful because they feel like they're trying to catch up on how to prove things. Taking *Math 101* will help you feel more secure going into those other courses.

  "I recommend this course because I think it's a very enjoyable introduction to proof-based math. One of the things that I really enjoyed about 21b was the sense that we were abstracting away from the specifics that we might talk about in any particular problem, instead shifting the focus to the properties that might arise out of the structure of a problem (ex. kernel, image, eigenvalues), and seeing how these properties transfer to problems that might, on the surface look very different. I feel that Math 101 takes the opposite approach in some ways, structuring ideas from the ground up and examining the properties that arise from them. In this way I think that it complements what we learned in 21b very nicely."

- *Math 153: Mathematical Biology - Evolutionary Dynamics*

  Math 153 introduces basic concepts of mathematical biology and evolutionary dynamics: evolution of genomes, quasi-species, finite and infinite population dynamics, chaos, game dynamics, evolution of cooperation and language, spatial models, evolutionary graph theory, infection dynamics, somatic evolution of cancer. (Taken from the course website.)

  A lot of students who have taken *Math 153* have loved it so much they went on to take more courses related to the subject or to work with Professor Nowak's (the usual instructor) research group.

- **Math 121: Linear Algebra and Applications*

  Math 121 covers linear algebra in a more theoretical way; you prove many of the the things you learned about linear algebra in *Math 21b*.

  Although this class doesn't assume you have a proof background, students have recommended taking 101 before this if you haven't done any proofs before.

  "I felt like it was a great transition into proof-based mathematics."

- *Math 130: Classical Geometry*

  Math 130 presents several classical geometries, these being the affine, projective, Euclidean, spherical and hyperbolic geometries. They are viewed from many different perspectives, some historical and some very topical. Emphasis on reading and writing proofs. The only prerequisites are Math 21a,b, 23a, 25a or 55a, and those may be taken concurrently. (Taken from the course website.)

- **Math 157: Mathematics in the World*

  Math 157 is a great sampler course: it covers all sorts of topics, from elementary problem solving (similar to things you might find in high school math contests) to programming, number theory, probability, and algorithms.

  The course features lots of in-class problem solving.

Later courses:

- **Math 112: Introductory Real Analysis*

  Math 112 covers real analysis, which you can think of as the theoretical background to calculus. You look at ideas like limits and derivatives that are familiar from calculus, but now from a theoretical perspective; you can think of it as the class that proves all the things you know from single variable calculus.

  "Math 112 was my first proof-based math class. The more natural sequence is to take 101 after 21b before proceeding to higher-level math, but I found that 112 was manageable and served as a good chance to try out proofs. You should ask the professor offering it whether it will be accessible to those without a background in proofs. Having taken it helped me to better understand what I might find in other higher-level math courses. It's also an important prerequisite if you want to take Stat 210 (the grad version of Stat 110)."

- *Math 113: Analysis I: Complex Function Theory*

  In *Math 1b* and *Math 21b*, you've seen that complex numbers can be helpful even when studying problems in the real world. *Math 113* covers analytic functions of one complex variable, which is basically calculus using complex numbers.

- *Math 117: Probability and Random Processes with Economic Applications*

  Math 117 is a self-contained treatment of the theory of probability and random processes with specific application to the theory of option pricing. Topics: axioms for probability, calculation of expectation by means of Lebesgue integration, conditional probability and conditional expectation, martingales, random walks and Wiener processes, and the Black-Scholes formula for option pricing. Students will work in small groups to investigate applications of the theory and to prove key results. (Taken from the course website.)

- **Math 122: Algebra I: Theory of Groups and Vector Spaces*

  Math 122 is an introduction to abstract algebra. In *Math 21b*, we study linear spaces, which are sets that are closed under addition and scalar multiplication. Similarly, mathematicians study sets that are closed under other operations; that's what abstract algebra is all about. This course builds on the linear algebra ideas you learned in *Math 21b*. (This course does assume you have background with doing proofs.)

- *Math 152: Discrete Mathematics*

  Math 152 is an introduction to sets, logic, finite groups, finite fields, finite geometry, finite topology, combinatorics and graph theory. A recurring theme of the course is the symmetry group of the regular icosahedron. Elementary category theory will be introduced as a unifying principle. Taught in a seminar format: students will gain experience in presenting proofs at the blackboard. Not to be taken in addition to Computer Science 20, Mathematics 55 or Mathematics 122. Covers material used in Computer Science 121 and Computer Science 124. (Taken from the course website.)

  From the instructor: *Math 152* is a possibility for students who want to participate actively (it's a seminar style course) and who want to see diverse topics that are not in Math 21 (groups, rings and fields, logic, topology, category theory).

Outside the Math Department

Highly recommended:

- **Stat 110: Introduction to Probability*

  Stat 110, Introduction to Probability, will "change the way you think".

  "If you are interested in stat, you should take this class because Stat 110 is challenging in a good way. Stat 110 changes the way of thinking in a daily life and you will appreciate how stat can be applied to your normal life."

  "It changes how you think about numbers. All your typical intuition is wrong in this class, which is a challenge, but it's incredible the kinds of problems you can solve with it."

  "Its just a generally fantastic course that will teach you all about probability theory and statistical distributions, and it opens the door to so many other opportunities in statistics and econometrics"

  "While it doesn't relate directly to Math 21b, it definitely is a math course! Probability very relevant to a lot of fields, and the class is very well taught and very fun."

- *AM 105: Ordinary and Partial Differential Equations*

  AM 105 covers ordinary differential equations: power series solutions; special functions; eigenfunction expansions. Review of vector calculus. Elementary partial differential equations: separation of variables and series solutions; diffusion, wave and Laplace equations. Brief introduction to nonlinear dynamical systems and to numerical methods. (Taken from the course website.)

  "Differential equations was my favorite part of Math 21b, so 105 is great because we basically start where we left off in Math 21b."

  Some students have reported significant overlap between the material in *Math 21b* and in *AM 105*, so keep that in mind, and maybe ask the instructor about what will be covered.

- *AM 120: Applicable Linear Algebra*

  AM 120 presents an algorithmic approach to topics in matrix theory which arise frequently in applied mathematics: linear equations, pseudoinverses, quadratic forms, eigenvalues and singular values, linear inequalities and optimization, linear differential and difference equations. (Taken from the course website.)

  Here again, some students have reported significant overlap between the material in *Math 21b* and in *AM 120*, so keep that in mind, and maybe ask the instructor about what will be covered.

- *AM 121: Introduction to Optimization: Models and Methods*

  "Wonderfully organized course, interesting lectures, challenging psets. Professor Parkes was very clear and helpful."

  "Good application of linear algebra and interesting problems"

  AM 121 is a journey into the mathematical ideas and computational methods for solving deterministic and stochastic optimization problems. (Taken from the course website.)

- **CS 124: Algorithms*

  CS 124 covers the modern theory of algorithms, focusing on the themes of efficient algorithms and intractable problems. (Taken from the course website.)

  "This is pretty mathy in that you're often proving things, but it comes from a different perspective. In pure math, you're often content just to show that something exists; in CS 124, just knowing it exists isn't enough; you might want to actually find it."

- *PHIL 144: Logic and Philosophy*

  PHIL 144 presents three philosophically important results of modern logic: Gödel's incompleteness theorems; Turing's definition of mechanical computability; Tarski's theory of truth for formalized languages. It discusses both mathematical content and philosophical significance of these results. (Taken from the course website.)

  "*PHIL 144* is very interesting and presents the proof of Gödel's incompleteness theorem "done right"."

  "Gödel's incompleteness theorem basically proves that some things cannot be proven!!"

More to consider:

- *Stat 139: Statistical Sleuthing through Linear Models*

  Stat 139 is a serious introduction to statistical inference with linear models and related methods. Topics include t-tools and permutation-based alternatives, multiple-group comparisons, analysis of variance, linear regression, model checking and refinement, and causation versus correlation. Emphasis on thinking statistically, evaluating assumptions, and developing tools for real-life applications. (Taken from course website.)

- *Physical Sciences 10: Quantum and Statistical Foundations of Chemistry*

  Physical Sciences 10 is an introduction to the fundamental theories of quantum mechanics and statistical mechanics and their role in governing the behavior of matter. (Taken from the course website.)

  "This class is basically all linear algebra! It's also incredibly useful and relevant if you're at all interested in chemistry or physics. Also, it's a ton of fun and incredibly well taught." (taught by Adam Cohen and Logan McCarty)

- *OEB 242: Population Genetics*

  OEB 242 presents mathematical theory, experimental data, and history of ideas in the field, including analytical methods to study genetic variation with applications to evolution, demographic history, agriculture, health and disease.

- *AM 104: Series Expansions and Complex Analysis*

  AM 104 introduces fundamental concepts for solving real-world problems and emphasizes their applications through examples from the physical and social sciences. Topics: series expansions and their convergence; complex functions, mappings, differentiation, integration, residues, Taylor and McLaurin expansions; wave (Fourier) and wavelet expansions and transformations, and their uses in signal and image analysis and solving differential equations. (Taken from the course website.)

- *AM 106: Applied Algebra*

  AM 106 is an introduction to abstract algebra and its applications. Sets, subsets, and partitions; mappings, operations, and equivalence relations; groups, rings, and fields, polynomials, encryption, computer coding, application of modular arithmetic, combinatorial designs, lattices, application of trellis representation of lattices, fast algorithms.

- *AM 111: Introduction to Scientific Computing*

  "A great continuation for Math 21b. Students will be very familiar with all the concepts required, and it is a stress-free class that focuses on how linear algebra can be used in computational algorithms to make certain difficult calculations very easy. Students will learn MATLAB and implement some of MATLAB's functions. Class will culminate in a final project where students build their own algorithmic tool to solve a problem. For an example, I built a suite of functions that was able to determine whether a signature is forged or not, based on a test set of only 10 true signatures!"

- *AM 115: Mathematical Modeling*

  "AM 115 is an incredible applied math course to take: it covers a broad range of real-world math applications that will pull together all the different math techniques you've learned. The methods and information you've learned from Math 21b are critical to this course."

- *CS 20: Discrete Mathematics for Computer Science*

  "It's discrete math, so it's REALLY different from the calculus and linear algebra that most of us learned in high school and in our first years at Harvard. It teaches you to think about numbers in a different way, and it's structured around in-class problem solving that makes you learn the material on your feet. It also teaches some probability, and I think it's a fantastic precursor to Stat 110. If you're going into a math/tech field, a lot of interview questions relate to the discrete math and probability you'd learn in this course."

- *CS 181: Machine Learning*

  CS 181 provides a broad and rigorous introduction to machine learning, probabilistic reasoning and decision making in uncertain environments. (Taken from the course website.)

  "Cool applications of some of the concepts that that you learn at the beginning of 21b (projections, eigenvalues, etc.)"

- *Ec 1011a: Microeconomic Theory*

  Ec 1011a teaches the basic tools of economics and to apply them to a wide range of human behavior. Tools include consumer theory, optimization under uncertainty, game theory, welfare economics, incentive theory, and the economics of information. Topics include industrial organization, public finance, law and economics, the economics of the family, religion, and riots. (Taken from course website.)

Other things students have said about math courses

Why study math?

- Math is a "good language for problem solving" and developing your lateral thinking skills.
- On pure math vs. applied: Math courses give you a sense of "where does the math come from" rather than just "what it is". It's the difference between being able to use a machine vs. learning how to actually build the machine.
- One pure math concentrator said he "likes the feeling of being lost" and then figuring things out. He talked about how being able to do math is not something you're born with but rather a skill you can work hard at it and develop. He described it as a "good kind of challenging".

A couple of pieces of advice from students:

- Don't ever hesitate to ask for help! Students have related stories of feeling totally confused in the first few weeks of a course and assuming that they were the only one, only to discover when they asked for help that there were several other students in the same boat.
- If you're undecided among math, applied math, stats, and similar fields, take *Stat 110*, *CS 50*, and a theoretical math class like Math 101; taking all three will probably help you pick a direction.