Appendix A: How Grants May Affect Investor Decisions Through A Simple Signal Extraction Model

Motivated by stylized facts about the entrepreneurial financing market, I consider the grant mechanism through the lens of a signal extraction problem. In the real world, there is tremendous uncertainty about how a given technology idea will work in practice. Layered on an entrepreneur's own uncertainty are information asymmetries between entrepreneurs and investors (Gompers and Lerner 1999). Based on noisy signals, venture investors choose a few firms out of hundreds or even thousands of proposals (Metrick 2007). Investors must rely on heuristics to evaluate many firms quickly (Kirsch et al. 2009).

A grant might alleviate financial constraints for recipient firms either through (1) *certification*; or (2) *internal resources.* Certification is informational content in the grant decision itself that alleviates information asymmetries. It occurs if the grantor identifies or is perceived to identify better firms. The second channel is the money itself. There are two subcategories within internal resources: (2a) *prototyping* and (2b) *valuation.* First, by investing in prototyping (proof-of-concept work), grantees can demonstrate their technology's viability. Alternatively, non-diluting capital increases the entrepreneur's wealth, potentially allowing him to accept investment without relinquishing an excessive share of the company. This last channel is a mechanical decrease in financial frictions that is independent of whether the firm invests the grant money in R&D.

I capture these mechanisms in a simple signal extraction problem, drawing heavly on Phelps (1972) and Aigner and Cain (1977). Theoretical work on entrepreneurial financing has focused on extensions to Inderst and Muller's (2004) search model and on competitive dynamics in the staging of financing (Bergemann, Hege and Peng 2009, Nanda and Rhodes-Kropf 2012, Thiele 2014). The theory suggested here is closest to the literature on signaling in entrepreneurial finance. Leland and Pyle (1976) first noted that managers could signal commitment though equity. Patents have also been considered as signals, as in Hsu and Ziedonis (2008), Czarnitzki, Hall and Hottenrott (2013), and Conti, Thursby and Thursby (2013). My setup is somewhat similar to that in Tian and Wang (2014), where investors with varying tolerance for failure receive a series of signals about a firm's likelihood of success.

Suppose the startup posseses a uni-dimensional latent technology quality T_i reflecting all of these factors. In one sense, T can be considered the project's net present value. Let T be normally distributed with mean $\bar{t}$ and variance σ_T^2, so that each entrepreneur's quality is $T_i = \bar{t} + \tau_i$.

A venture investor is interested in evaluating startups. Although he knows the quality distribution, he receives only an error-ridden, unbiased signal about the quality $\tilde{T}_i = \bar{t} + \tau_i + \varepsilon_i$. The error $\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$ is independent of T. The investor calculates $\mathbf{E}\left(T_i \mid \tilde{T}_i\right)$. This expected quality is dependent on the reliability of the signal; if the signal is extremely noisy, the investor should place more weight on the mean $\bar{t}$, whereas if σ_ε^2 is relatively small, he should place more weight on the signal. The logical weight to place on the signal is:

$$\frac{Cov\left(\tilde{T}, T\right)}{Var\left(\tilde{T}\right)} = \frac{\sigma_T^2}{\sigma_\varepsilon^2 + \sigma_T^2} = \alpha \tag{1}$$

The expected technology quality is a weighted average of the signal and the underlying mean:

$$\mathbf{E}\left(T_i \mid \tilde{T}_i\right) = (1-\alpha)\bar{t} + \alpha\tilde{T}_i \tag{2}$$

where the first term is a "group" effect and the second term is an "individual" effect. α is the familiar slope coefficient of a linear regression of T on $\tilde{T}$ and a constant. This regression line is depicted in Figure 1.A.

The government, meanwhile, also receives a signal $\tilde{T}_i^G$ about the firm, but this signal is private to the government and unobservable to investors and entrepreneurs.[1] For the purposes of this toy model, I need not make any functional form assumptions about $\tilde{T}_i^G$. The government awards grants to a subset of firms located above a cutoff in the signal distribution. Whether a firm has a grant (g) or does not (n) is a truncated dichotomous version of the signal, $x \in \{g, n)$. The investor observes only this binary signal.[2]

The government also receives a signal about the firm, $\tilde{T}_i^G$, which neither the investor nor entrepreneurs observe.[3] The government awards grants to a subset of firms whose $\tilde{T}_i^G$ are located above a cutoff. Whether a firm has a grant (g) or does not (n) is a truncated dichotomous version of $\tilde{T}_i^G$. The investor observes this binary signal $x \in \{g, n)$.[4] The

[1] In practice, the objectives of VC investors and the stated DOE SBIR program objectives overlap, albeit incompletely. Investors typically judge startups along three dimensions: a) technology novelty, utility, and appropriability; b) market potential and future financing risk; and c) team experience and competence. DOE's review process is based on a) scientific merit; b) cost-effectiveness; and c) energy sector impact.

[2] Furthermore, the investor does not observe whether a non-grantee firm applied and lost or did not apply at all. The model is agnostic about whether the absence of a grant is a negative signal. However, this is not a central point.

[3] I need not make any functional form assumptions about $\tilde{T}_i^G$.

[4] The investor does not observe whether a non-grantee firm applied and lost or did not apply at all. The model is agnostic about whether the grant has a negative effect on losers (though this seems unlikely because the applicant firms form a small subset of the space of energy startups). In Section 4.1.3 I argue that negative

grant might affect the mean technology quality ($\bar{t}$), the quality variance (σ_T^2), and the signal variance (σ_ε^2). Any value of the grant money that is unrelated to its technology quality is μ_x, where $\mu_n = 0$ and $\mu_g \geq 0$. After the competition entrepreneurs have technology quality $T_{i,x} = \bar{t}_x + \mu_x + \tau_{i,x}$. Now $T_x \sim N\left(\bar{t}_x + \mu_x, \sigma_{T,x}^2\right)$, and the signal error becomes $\varepsilon_x \sim N\left(0, \sigma_{\varepsilon,x}^2\right)$.

Consider two firms i and j with the same noisy signal $\tilde{T}_i = \tilde{T}_j = \tilde{T}_k$, where one won a grant and the other did not. The difference between the two firms' expected qualities is:

$$\mathcal{D} = \mathbf{E}\left(T_i \mid \tilde{T}_i = k, x = g\right) - \mathbf{E}\left(T_j \mid \tilde{T}_j = k, x = n\right) \tag{3}$$

$$= (\bar{t}_g + \mu_g)\left(1 - \frac{\sigma_{T,g}^2}{\sigma_{\varepsilon,g}^2 + \sigma_{T,g}^2}\right) - \bar{t}_n\left(1 - \frac{\sigma_{T,n}^2}{\sigma_{\varepsilon,n}^2 + \sigma_{T,n}^2}\right) + \tilde{T}_k\left(\frac{\sigma_{T,g}^2}{\sigma_{\varepsilon,g}^2 + \sigma_{T,g}^2} - \frac{\sigma_{T,n}^2}{\sigma_{\varepsilon,n}^2 + \sigma_{T,n}^2}\right)$$

The regression discontinuity design in the following section will approximate this situation of two firms possessing the same signal, where one just wins a grant and another just loses. If $\mathcal{D} > 0$, the grant has a positive effect on investment. We can now identify the mechanisms that might drive this equation away from zero.

1. *Certification:* Suppose that the award process separates applicant firms into higher and lower technology quality types, but has no other effect. Now $\bar{t}_g > \bar{t}_n$, while $\mu_g = 0$, $\sigma_{T,g}^2 = \sigma_{T,n}^2 = \sigma_T^2$, and $\sigma_{\varepsilon,g}^2 = \sigma_{\varepsilon,n}^2 = \sigma_\varepsilon^2$. The difference in expected quality for two firms with the same signal $\tilde{T}$ is

$$\mathcal{D} = (\bar{t}_g - \bar{t}_n)\left(1 - \frac{\sigma_T^2}{\sigma_\varepsilon^2 + \sigma_T^2}\right) \tag{4}$$

   If the investor believes the government is choosing superior firms, this expression is positive. From the regression line perspective in equation 2, $\bar{t}_g > \bar{t}_n$ generates two parallel lines (Figure 1.B).

2. *Internal Resources*

   (a) *Equity Channel:* The grant increases the entrepreneur's internal resources, allowing him to maintain a larger share of the firm in exchange for a given external investment. The only difference between grantees and non-grantees, here, is μ. The \$150,000 investment is positive NPV from the investor perspective, but the equity that would need to be transferred would destroy entrepreneurial incentives. This manifests as a mean shifting effect for grantees (Figure 1.B). Now the

spillovers seem absent.

difference is:

$$\mathcal{D} = \mu_g \left(1 - \frac{\sigma_T^2}{\sigma_\varepsilon^2 + \sigma_T^2}\right) \tag{5}$$

(b) *Prototyping:* Suppose the grant permits firms to increase the precision of their signal $\tilde{T}$ by developing a prototype, so $\sigma_{\varepsilon,g}^2 < \sigma_{\varepsilon,n}^2$. The signal is more reliable for grantees. With all else held the same, $\bar{t}_g = \bar{t}_n = \bar{t}$, $\mu_g = 0$, and $\sigma_{T,g}^2 = \sigma_{T,n}^2 = \sigma_T^2$. In this case the difference is:

$$\mathcal{D} = \left(\bar{t} - \tilde{T}_k\right) \left(\frac{\sigma_T^2}{\sigma_{\varepsilon,n}^2 + \sigma_T^2} - \frac{\sigma_T^2}{\sigma_{\varepsilon,g}^2 + \sigma_T^2}\right) \tag{6}$$

The slope of the grantee regression line is steeper (Figure 1.C), because $\frac{\sigma_{T,}^2}{\sigma_{\varepsilon,g}^2 + \sigma_{T,}^2} > \frac{\sigma_{T,}^2}{\sigma_{\varepsilon,n}^2 + \sigma_{T,}^2}$. If an entrepreneur has a high-type technology ($\bar{t} + \tau_i > \bar{t}$), a signal-precision effect of the grant will be more valuable, whereas it will be harmful to a low-type ($\bar{t} + \tau_i < \bar{t}$).

i. This same effect occurs if grantees perform R&D work that alters their underlying technology quality τ_i such that grantee quality is more variable. Then $\sigma_{T,g}^2 > \sigma_{T,n}^2$, $\sigma_{\varepsilon,g}^2 = \sigma_{\varepsilon,n}^2 = \sigma_\varepsilon^2$ and $\bar{t}_g = \bar{t}_n = \bar{t}$. Now the difference is:

$$\mathcal{D} = \left(\tilde{T}_k - \bar{t}\right) \left(\frac{\sigma_{T,g}^2}{\sigma_\varepsilon^2 + \sigma_{T,g}^2} - \frac{\sigma_{T,n}^2}{\sigma_\varepsilon^2 + \sigma_{T,n}^2}\right) \tag{7}$$

The expression in the last set of brackets is positive, so we are again in Figure 1.C.

ii. In the alternative scenario where firms' underlying quality is pushed toward the mean ($\sigma_{T,g}^2 < \sigma_{T,n}^2$), the expression in the last set of brackets is now negative, and the regression line is steeper for non-grantees.[5]

iii. *Note on Risk Preferences:* This discussion assumes that the expected technology quality enters the VC firm's profit function linearly. If the VC firm is risk-averse, then the variances of the errors are costly. Although VCs pursue high-risk strategies, they often tolerate market risk more readily than technology risk, and seek technology validation prior to investment. With investor risk aversion, the improved signal precision for the grantee shifts its regression line upwards by some risk factor. Within the model, it is indistinguishable

[5] I do not allow the final scenario, where $\sigma_{\varepsilon,g}^2 > \sigma_{\varepsilon,n}^2$, as it seems unlikely that a grant would increase the signal's noise.

from a certification effect.

Given these possible mechanisms, I shift to the government perspective, and connect the model to the empirical design. Entrepreneurs have an ultimate observable quality T_i^O, which is a function of latent quality T_i and resources provided to the entrepreneur. Figure 11 shows the correlation of this outcome with the private government signal $\tilde{T}_i^G$. Applicant firms with $\tilde{T}_i^G$ to the right of the red cutoff line are awardees, while applicants to the left are losers. My regression discontinuity design measures this difference:

$$\mathcal{D} = \mathbf{E}\left(T_i^O \mid \tilde{T}_i^G = k, x = g\right) - \mathbf{E}\left(T_i^O \mid \tilde{T}_j^G = k, x = n\right) \tag{8}$$

First, consider the no-effect case, depicted in Figure 2.A. When the government signal is uninformative about outcomes and the grant has no funds effect, the observed outcome projected on the government signal is a horizontal line; here $\mathcal{D} = 0$.

Second, if the signal is informative about outcomes, the regression line is upward sloping (Figure 2.B).[6] Here, the grant acts as a binary signal about firm quality, which the market learns is informative, so we observe a jump at the discontinuity due to certification ($\mathcal{D} > 0$). Investors are more likely to finance grantees because they have higher mean expected quality ($\bar{t}_g > \bar{t}_n$), even if the money itself has no effect. Figure 2.B, which describes actual investment outcomes as a function of the government signal, maps to Figure 1. B, which shows how the government signal affects investor beliefs.

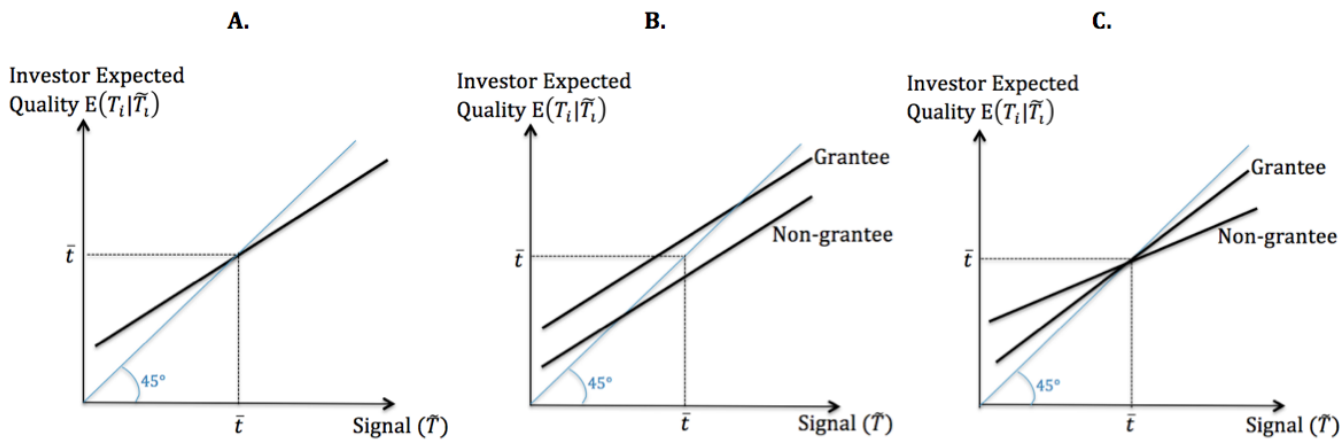
Finally, if $\tilde{T}_i^G$ is uninformative but the grant money itself benefits recipients through either funding or prototyping, we observe a horizontal line with a jump at the discontinuity, shown in Figure 2.C. Because the funding channel is a mean-shifting effect ($\mu_g > 0$), it maps to Figure 1.B from the investor perspective. With only a prototyping channel, the government signal is uninformative (Figure 2.C), but prototyping changes the variance of the signal to investors and so maps to Figure 1.C.

A matrix of hypotheses derived from this model is contained in Table 1. The left-hand column contains the basic questions that I ask in the estimation. In the five columns on the right, I write the answer to the question that best supports they hypothesis in that column. That is, the "Yes" or "No" answer is the one that that supports the hypothesis being the *primary driver* of the grant effect on subsequent VC investment. Multiple mechanisms may

[6] It is possible that the government signal is informative in the other direction; that is, it orders poor quality firms above higher quality firms on average. In this case the line will slope down, and we would expect a downward jump at the discontinuity.

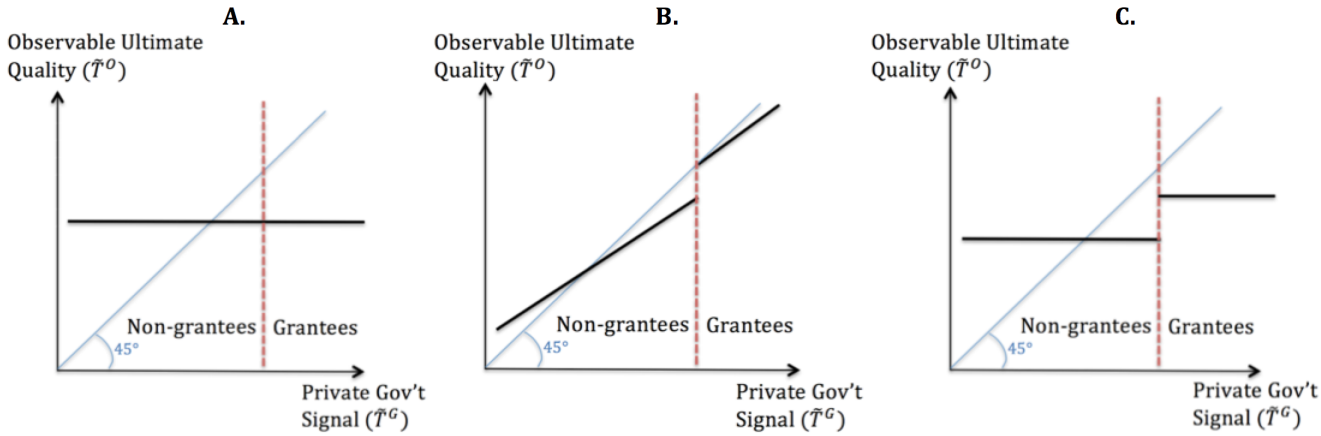
act at once, so my conclusions will be based on the balance of correct answers and the strength of the evidence for a given hypothesis. There are three important assumptions. First, VCs consider only relatively high signal type firms. Second, prototyping can only improve signal precision ($\sigma^2_{\varepsilon,g} \leq \sigma^2_{\varepsilon,n}$). Third, I can only rule out an internal resources hypothesis if I assume that the mechanical alleviation of financial frictions is not decreasing in the size of the award, as is standard in the literature cited above. However, if the relationship is non-monotonic, I cannot distinguish between early stage prototyping effects and an internal resources effect. In the empirical sections below, I argue that my evidence best supports prototyping via reduced signal noise.

Figure 1: Possible grant effects on investor expectations of quality given firms' noisy signal to investors



Note: Figure 11.A shows the investor's expected quality of the entrepreneur (y-axis) as a function of the noisy signal that the investor observes (x-axis). Figure 11.B shows that a certification or valuation effect increases the mean expected quality of grantees relative to non-grantees ($\bar{t}_g > \bar{t}_n$). Figure 11.C shows that a prototyping effect increases the slope of the grantee line relative to the non-grantee line. This occurs because the grant causes the grantee's signal to be more reliable, which for example may occur if prototyping decreases the variance of the noisy signal $\left(\sigma^2_{\varepsilon,g} < \sigma^2_{\varepsilon,n}\right)$.

Figure 2: Possible grant effects on firm outcome given firms' private signal to government



Note: Figure 12.A shows this observable outcome (y-axis) as a function of the signal that the government receives from the firm, which is private to the government (x-axis). In this case, the government signal $\tilde{T}^G$ is wholly uninformative about outcomes, so the line is flat, and there can be no certification effect with rational investors. In Figure 12.A, there is both no certification effect and no effect of the grant money itself, so there is no jump at the discontinuity between non-grantees and grantees. Figure 11.B shows a prototyping or valuation effect increasing outcomes for grantees relative to non-grantees in the absence of certification ($\tilde{T}^G$ uninformative). Figure 11.C. shows the certification case, in which $\tilde{T}^G$ is informative and thus correlated with outcomes. In the absence of a valuation or prototyping effect, we nonetheless observe a jump at the discontinuity as the market accounts for information in the private government signal $\tilde{T}^G$.

Table 1: Tests of Hypotheses for Phase 1 Grant Effect on Subsequent VC

Test	Certification $\bar{t}_g > \bar{t}_n$	Prototyping 1 $\sigma^2_{\varepsilon,g} < \sigma^2_{\varepsilon,n}$	Prototyping 2 $\sigma^2_{T,g} > \sigma^2_{T,n}$	I. Internal Resources
1. Does Phase 1 increase likelihood of subsequent VC?	**Y**	**Y**	**Y**	**Y**
2. Does Phase 2 increase likelihood of subsequent VC?	**Y**	**N**	**Y**	**Y**
3. Does Phase 1 increase subsequent patents?	**N**	**Y**	**Y**	**N**
4. Does Phase 1 increase subsequent citations?	**N**	**N**	**Y**	**N**
5. Do DOE applicant rankings contain positive information about outcomes?	**Y**	**N**	**N**	**N**

Note: This table relates the model in section 2 to empirical tests. Yes (Y) or No (N) is the answer to the Test question that best supports the hypothesis as the primary driver of the grant effect on subsequent VC. Blue indicates the answer my evidence supports, while red indicates the answer that my evidence does not support. The mean firm quality is $\bar{t}$, with individual qualities defined by the variance around this mean σ^2_T. The noise in the signal is cause by a mean-zero error with variance σ^2_ε. g indicates a firm is a grantee, while n indicates the firm is not a grantee.