Lecture 11: Capital Taxation and NDPF

Stefanie Stantcheva

Fall 2016
GOALS OF THIS LECTURE

(1) New model: A simpler framework for optimal capital tax theory.

(2) New Dynamic Public Finance: A toolbox.
The Need for a Simpler Model for Optimal Capital Taxation

1) Public debate centers around a simple equity-efficiency tradeoff:

   Is the distribution of capital fair? How does capital react to taxation?

2) Econ literature: disparate models and results (individual preferences, shocks, govt objective, policy tools)

Connect 1) and 2) by deriving robust optimal capital tax formulas in terms of estimable elasticities and distributional parameters

⇒ optimal K tax theory looks like optimal L tax theory.

Centered around equity-efficiency trade-off.

Highlights main forces + policy implications for K tax (often obscured).
Goals and Contributions

1) Start with dynamic model with linear utility for consumption and concave utility for wealth.

⇒ Transitional dynamics instantaneous ⇒ Simple, tractable theory.

Put simplicity to use: new formulas for policy-relevant cases (nonlinear tax, cross-effects, shifting, consumption tax, ..) and normative considerations.

2) Generalize to model with concave utility ⇒ Same optimal K tax formulas apply, with appropriately defined elasticity of the tax base.

Qualitatively: Lessons and intuitions from simpler model still valid.

Quantitatively: Sluggish adjustments reflected in elasticity.

The faster K adjustments, the closer to simpler model.

3) Numerically explore optimal taxation using U.S. IRS data.
A Simpler Model of Capital Taxation

For exposition: Exogenous and uniform labor income \( z \)

Heterogeneous discount rate \( \delta_i \) (assume \( \delta_i > r \))

Exogenous and uniform rate of return \( r \) on wealth \( k \), income: \( rk \)

Time invariant tax \( T_K(rk) \)

Initial wealth \( k_{i, init} \), exogenous.

Individual \( i \) has instantaneous utility \( u_i(c, k) = c + a_i(k) \)

linear in consumption \( c \) and increasing and concave in wealth \( k \).

Maximizes:

\[
U_i = \delta_i \cdot \int_{t=0}^{\infty} [c_i(t) + a_i(k_i(t))] e^{-\delta_i t}
\]

s.t. \( \frac{dk_i(t)}{dt} = rk_i(t) - T_K(rk_i(t)) + z_i(t) - c_i(t) \)
Solving the Individual’s Maximization Problem

\[ U_i = \delta_i \cdot \int_{t=0}^{\infty} [c_i(t) + a_i(k_i(t))] e^{-\delta_i t} \]

s.t. \[ \frac{dk_i(t)}{dt} = r_k(t) - T_K(r_k(t)) + z_i(t) - c_i(t) \]

Hamiltonian: \[ c_i(t) + a_i(k_i(t)) + \lambda_i(t) \cdot [r_k(t) - T_K(r_k(t)) + z_i(t) - c_i(t)] \]

FOC in \( c_i(t) \): \[ \lambda_i(t) = 1 \Rightarrow \text{constant multiplier} \]

FOC in \( k_i(t) \): \[ a'_i(k_i(t)) + \lambda_i(t) \cdot r \cdot (1 - T'_K) = -\frac{d\lambda_i(t)}{dt} + \delta_i \cdot \lambda_i(t) \]

\[ \Rightarrow a'_i(k_i(t)) = \delta_i - \bar{r} \quad \text{where} \quad \bar{r} = r \cdot (1 - T'_K) \]
Steady State

Utility for wealth puts limit on impatience to consume \((\delta_i > \bar{r})\)

\[ MU \text{ for wealth } a'_i(k) = \delta_i - \bar{r} = \text{value lost in delaying consumption} \]

Wealth accumulation depends on heterogeneous preferences \(a_i(\cdot), \delta_i,\) and net-of-tax return \(\bar{r}\) (substitution effects, no income effects)

\[ \Rightarrow \text{Heterogeneity in (non-degenerate) steady-state wealth.} \]

At time 0: jump from \(k_i^{\text{init}}\) to \(k_i(t)\) (consumption quantum Dirac jump):

\[
U_i = rk_i(t) - T_K(rk_i(t)) + z_i(t) + a_i(k_i(t)) + \delta_i \cdot (k_i^{\text{init}} - k_i(t))
\]

Dynamic model equivalent to a static model:

\[
U_i = c_i + a_i(k_i) + \delta_i \cdot (k_i^{\text{init}} - k_i) \quad \text{with} \quad c_i = rk_i - T_K(rk_i) + z_i
\]

Announced vs. unannounced tax reforms have same effect.
Wealth in the Utility

Technical reason: to smooth otherwise degenerate steady state ($\delta_i = \delta = \bar{r}$)

Possible, but more complicated is uncertainty (in paper).

Entrepreneurship: “cost” of managing wealth, $-h_i(k)$ (return $r_i > \delta_i$).

Wealth brings non-consumption utility flows: Weber’s “spirit of capitalism.”

Keynes (1919, 1931) “love of money as a possession”, “the virtue of the cake [savings] was that it was never to be consumed.”

Social status (measure of ability, performance, success)

Power and political influence.

Philanthropy and moral recognition, warm glow bequests.

Empirical evidence in favor of wealth in the utility:

Caroll (2000): helps explain top wealth holdings.
Isomorphism with Static Labor Taxation Model

\[ U_i = c_i + a_i(k_i) + \delta_i \cdot (k_i^{\text{init}} - k_i) \] \ with \ \[ c_i = r k_i - T_K(r k_i) + z_i \]

is mathematically isomorphic to static labor income model:

\[ U_i = c_i - h_i(z_i) \] \ with \ \[ c_i = z_i - T_L(z_i) \]

Optimal K tax analysis isomorphic to optimal L income tax theory.

Differences of degree rather than of kind, quantitative differences.

Key differences (e.g.: uncertainty, shocks to productivity vs. taste) reflected in estimable elasticities.

In general model, slow adjustment will be reflected in lower elasticity.

Bypasses transitional dynamics, greatly simplifies K tax analysis

Like labor supply decisions (not instantaneous, e.g. human capital investment).
Government Optimization

Government sets a time invariant budget balanced $T_K(\cdot)$ to maximize its social objective

$$\int g_i \cdot U_i(c_i, k_i) di$$

with $g_i \geq 0$ social marginal welfare weight

Optimal $T_K(\cdot)$ depends on three key ingredients:

1. **Social preferences:** $g_i = \text{value of$1 extra given to } i \left( \int g_i = 1 \right)$.

2. **Efficiency costs:** Elasticity $e_K = (\bar{r}/k) \cdot (dk/d\bar{r})$ measures how wealth $k$ responds to $\bar{r} = r \cdot (1 - T'_K)$

3. **Distribution of capital income:** $H_K(rk)$ (for nonlinear tax).
Optimal Linear Capital Taxation at rate $\tau_K$

\[ k^m(\bar{r}) \equiv \int k_i \, di \] average wealth (depends on $\bar{r}$ with elasticity $e_K$).

Revenues $\tau_K k^m(\bar{r})$ rebated lump-sum.

$\tau_K$ maximizes $SWF = \int g_i \cdot U_i(c_i, k_i) \, di$ with

\[ U_i = rk_i \cdot (1 - \tau_K) + \tau_K \cdot rk^m(\bar{r}) + z_i + a_i(k_i) + \delta_i \cdot (k_i^{init} - k_i) \]

Standard optimal tax derivation (using envelope thm for $k_i$):

\[ \frac{dSWF}{d\tau_K} = rk^m \cdot \int g_i \cdot \left(1 - \frac{k_i}{k^m}\right) - rk^m \cdot \frac{\tau_K}{1 - \tau_K} \cdot e_K \]

Mechanical Revenue net of Welfare Effect

Behavioral Effect

Optimal $\tau_K$ such that $dSWF / d\tau_K = 0.$
Optimal Linear Capital Tax $\tau_K$

$$\tau_K = \frac{1 - \bar{g}_K}{1 - \bar{g}_K + e_K} \quad \text{with} \quad \bar{g}_K = \frac{\int_i g_i \cdot k_i}{\int_i k_i} \quad \text{and} \quad e_K = \frac{\bar{r}}{k^m} \cdot \frac{dk^m}{d\bar{r}} > 0$$

Zero capital tax result: $\tau_K = 0$ only if:

- $\bar{g}_K = 1$ (no inequality in $rk$, or no redistributive concerns $g_i \equiv 1$), or
- $e_K = \infty$.

$\tau_K > 0$ as long as $g_i$ decreasing in $k_i$, or wealth concentrated among low $g_i$ agents.

$\tau_K = 1/(1 + e_K)$ is revenue-maximizing in Rawlsian case: $g_i = 0$ if $k_i > 0$.

Top revenue maximizing rate: $\tau_K = 1/(1 + a_{Ktop} \cdot e_{Ktop})$ with $a_{Ktop}$ the Pareto tail parameter for top bracket.
Optimal Nonlinear Capital Tax

\[ T'_K(rk) = \frac{1 - \bar{G}_K(rk)}{1 - \bar{G}_K(rk) + \alpha_K(rk) \cdot e_K(rk)} \]

1) \( \bar{G}_K(rk) \equiv \int_{\{i: r_{ki} \geq rk\}} \frac{g_i d_i}{P(r_{ki} \geq rk) \int_j g_j d_j} \) is the average \( g_i \) above capital income level \( rk \)

2) \( \alpha_K(rk) \) the local Pareto parameter of capital income distribution

3) \( e_K(rk) \) the local elasticity of \( k \) wrt to \( 1 - T'_K(rk) \) at income level \( rk \)

Capital income is very concentrated (top 1% capital income earners have 60%+ of total capital income)

⇒ Asymptotic formula:

\[ T'_K(\infty) = \frac{1 - G_K(\infty)}{1 - G_K(\infty) + \alpha_K(\infty) \cdot e_K(\infty)} \] relevant for most of the tax base
Equity Considerations: The Ant and the Grasshopper

Credit: Adelya Tumasyeva
Equity Considerations: The Ant and the Grasshopper

Credit: Adelya Tumasyeva
Equity Considerations: The Ant and the Grasshopper

Credit: Adelya Tumasyeva
Equity Considerations: The Ant and the Grasshopper

Credit: Adelya Tumasyeva
Equity Considerations: The Ant and the Grasshopper

Credit: Adelya Tumasyeva
Equity Considerations: The Ant and the Grasshopper

Credit: Adelya Tumasyeva
Equity Considerations for Capital Taxation: Generalized Welfare Weights

(1) Inequality in wealth deemed fair and wealth is not a tag

Equality of opportunity argument: grasshopper had same savings opportunities as ant, conditional on labor earnings.

Capital accumulated by sacrificing consumption, why punish saving behavior?

What if ant had higher work (grain harvesting) ability? $\rightarrow$ role for nonlinear labor income tax.

$\rightarrow g_i$ independent of and uncorrelated with $k_i \rightarrow \tau_K = 0.$
(2) Inequality in wealth viewed as unfair

Even conditional on labor earnings, high wealth comes from higher patience $\delta_i$ or higher valuation of wealth $a_i$ – unfair heterogeneity, like earnings ability.

or parental wealth ($k_i^{init}$) – ant’s parents left extra grain.

or higher returns $r_i$ (luck) – ant speculated on grain-forward derivatives.

$\rightarrow g_i$ decreasing in $k_i \rightarrow \tau_K > 0$. 
(3) Wealth as a tag

May or may not care about $k$ per se ($g_i$ may not depend on $k_i$ directly).

But wealth may be tag for aspects that enter $g_i$ negatively: parental background (see Saez-Stantcheva), ability.

Having more grain means more likely to come from rich family.

$\bar{G}_K(rk)$ is representation index of agents from poor background at income $rk$.

$\rightarrow \text{corr}(g_i, k_i) < 0 \rightarrow \tau_K > 0$. 
Adding in Labor Income Responses & Labor Taxation

Add in choice of labor income, with potentially arbitrary heterogeneity in disutility $h_i(z)$.

$$U_i = rk_i + z_i - T(rk_i + z_i) + a_i(k_i) + \delta_i \cdot (k_i^{\text{init}} - k_i) - h_i(z_i)$$

$$T'_L(z) = \frac{1 - \bar{G}_L(z)}{1 - \bar{G}_L(z) + \alpha_L(z) \cdot e_L(z)}$$

1) $\bar{G}_L(z) \equiv \frac{\int_{\{i: z_i \geq z\}} g_i d_i}{\mathbb{P}(z_i \geq z) \int_i g_i d_i}$ is the average $g_i$ above labor income level $z$

2) $\alpha_L(z)$ the local Pareto parameter of capital income distribution

3) $e_L(z)$ the local elasticity of $k$ wrt to $\bar{r}$ at income level $rk$

Separable labor and capital taxes each set according to Mirrlees (1971) and Saez (2001) formulas.
Joint Preferences in Capital and Labor and Cross-Elasticities

Agent’s dynamic problem is again equivalent to maximizing:

\[ U_i = c_i + v_i(k_i, z_i) + \delta_i(k_i^{init} - k_i) \quad \text{with} \quad c_i = \bar{r}k_i + z_i - T_L(z_i) \]

Choice \((c, k, z)\) is such that:

\[ v_{iz}(k_i, z_i) = 1 - T'_L(z_i), \quad v_{ik}(k_i, z_i) = \delta_i - \bar{r}, \quad c_i = \bar{r}k_i + z_i - T_L(z_i) \]

Optimal capital tax (at any, possibly non-optimal \(\tau_L\)):

\[ \tau_K = \frac{1 - \bar{g}_K - \tau_L \frac{z^m}{k^m} e_{Z,(1-\tau_K)}}{1 - \bar{g}_K + e_K} \]

with \[ \bar{g}_K = \int_i k_i g_i \quad \text{and} \quad e_{Z,(1-\tau_K)} = \frac{dz^m}{d(1 - \tau_K)} \frac{(1 - \tau_K)}{z^m} \]
Comprehensive nonlinear income taxation $T(rk + z)$

Govt uses solely comprehensive taxation $T(y)$ with $y_i \equiv rk_i + z_i$

$$U_i = rk_i + z_i - T(rk_i + z_i) + a_i(k_i) + \delta_i \cdot (k_i^{\text{init}} - k_i) - h_i(z_i)$$

Standard Mirrlees’ formula applies to comprehensive income tax problem

$$T'(y) = \frac{1 - \bar{G}_Y(y)}{1 - \bar{G}_Y(y) + \alpha_Y(y) \cdot e_Y(y)}$$

with $\bar{G}_Y(y) \equiv \frac{\int_{\{i:y_i \geq y\}} g_i d_i}{P(y_i \geq y) \int g_i d_i}$

$\alpha_Y(y)$ local Pareto parameter for $y$ distribution,

$e_Y(y)$ local elasticity of $y$ with respect to $1 - T'$. 
Tax shifting and Comprehensive Taxation

Suppose individual $i$ can shift $x$ dollars from labor income to capital income at utility cost $d_i(x)$

Reported labor income $z_L$ and capital income $z_K$ are elastic to tax differential $\tau_L - \tau_K$

If shifting elasticity is infinite, then $\tau_L = \tau_K$ is optimal

If shifting elasticity is finite, then optimal $\tau_L, \tau_K$ closer than they would be absent any shifting

If shifting elasticity is large then $e_K$ can appear large, but wrong to set $\tau_K$ at $1/(1 + e_K)$ in that case
Heterogeneous Returns

Heterogeneous returns \( r_i \) important in practice:

Same sufficient stats formula, but replace:

\[
\bar{g} = \frac{\int_i g_i \cdot r_i k_i}{\int_i r_i k_i} \quad \text{and} \quad e_K = \frac{(1 - \tau_K)}{\int_i r_i k_i} \cdot \frac{d \int_i r_i k_i}{d(1 - \tau_K)}
\]

Values of \( e_K \) (responsiveness of \( k \) to taxes) and \( \bar{g}_K \) (social judgement about capital income) could be affected.
Different Types of Capital Assets

Could have ≠ elasticities (housing vs. financial assets)

Different social judgments or distributional characteristics $g^j_K$.

Formulas hold asset by asset, determined by: $g^j_K$, $e^j_K$, and cross-elasticities $e_{Ks,(1-\tau^j_K)}$.

$$\tau^j_K = \frac{1 - g^j_K}{1 - \bar{g}^j_K + e^j_K}$$

$$g^j_K = \frac{\int g_i \cdot k^j_i}{\int k^j_i}, \quad e^j_K = \frac{\bar{r}^j}{k^{m,j}} \cdot \frac{dk^{m,j}}{d\bar{r}^j} > 0, \quad e_{Ks,(1-\tau^j_K)} = \frac{\bar{r}^j}{k^{m,s}} \cdot \frac{dk^{m,s}}{d\bar{r}^j}$$
Different Types of Capital Assets

Could have \( \neq \) elasticities (housing vs. financial assets)

Different social judgments or distributional characteristics \( \bar{g}_K^j \).

Formulas hold asset by asset, determined by: \( \bar{g}_K^j \), \( e_K^j \), and cross-elasticities \( e_{K^s,(1-\tau_K^j)} \).

\[
\tau_K^j = \frac{1 - \bar{g}_K^j - \sum_{s \neq j} \tau_K^s \frac{k_{m,s}^j}{k_{m,j}^i} e_{K^s,(1-\tau_K^j)}}{1 - \bar{g}_K^j + e_K^j}
\]

\[
\bar{g}_K^j = \frac{\int_i g_i \cdot k_i^j}{\int_i k_i^j}, \quad e_K^j = \frac{\bar{r}_j}{k_{m,j}^i} \cdot \frac{dk_{m,j}^i}{d\bar{r}_j} > 0, \quad e_{K^s,(1-\tau_K^j)} = \frac{\bar{r}_j}{k_{m,s}} \cdot \frac{dk_{m,s}^j}{d\bar{r}_j}
\]
Can a consumption tax be better than a wealth tax and more progressive than a tax on labor income?

Bill Gates: “Imagine three types of wealthy people. One guy is putting his capital into building his business. Then there’s a woman who’s giving most of her wealth to charity. A third person is mostly consuming, spending a lot of money on things like a yacht and plane. While it’s true that the wealth of all three people is contributing to inequality, I would argue that the first two are delivering more value to society than the third. I wish Piketty had made this distinction, because it has important policy implications.”
Consumption Taxation in our Model

Consider linear consumption tax at (inclusive) tax rate $\tau_C$ so that:

$$\frac{dk_i(t)}{dt} = r(1 - \tau_K)k_i(t) + z_i(t) - T_L(z_i(t)) - c_i(t)/(1 - \tau_C)$$

Agents care about real wealth $k^r = k \cdot (1 - \tau_C)$.

Even with wealth-in-utility, $\tau_C$ equivalent labor tax + tax on initial wealth (Kaplow, 1994, Auerbach, 2009).

Thought experiment: equal labor income.

With $\tau_C$, wealthy look like pay more taxes, but paid less when accumulated more nominal wealth. Real wealth inequality unaffected.

With 2-dim heterogeneity: labor tax not sufficient (Atkinson-Stiglitz).

$\Rightarrow \tau_C$ cannot address steady-state capital income inequality
Fact 1: K income more unequally distributed than L income
Fact 2: At the top, total income is mostly capital income
Fact 3: Two-dimensional heterogeneity, inequality in K income even conditional on L income
Methodology for Computing Optimal Tax Rates

Suppose constant elasticity of labor, capital, and total income ($e_L, e_K, e_Y$) and that choice at zero tax represents preference type: $(\theta_i, \eta_i)$.

Based on the IRS micro data, use pairs $(z_i, r_k_i)$ to invert individual choices to obtain $(\theta_i, \eta_i)$.

Non-parametrically fit type distributions and empirical Pareto parameters.

Solve for optimal $T'_K$, $T'_L$, and $T'_Y$ using sufficient stats formulas.

For capital – our simpler theory provides a much easier way to compute optimal tax rates based on the data.

Simulations set $g_i = \frac{1}{\text{disposable income}_i}$ and use several values for elasticities.
Optimal Labor Income Tax Rate $T'_{L}(z)$
Optimal Capital Income Tax Rate $T'_K(rk)$

![Graph showing the optimal capital income tax rate for different values of $e_K$. The x-axis represents capital income in $\$0,000$ steps from $\$200,000$ to $\$1,000,000$, and the y-axis represents the marginal tax rate. Three lines represent different values of $e_K$: $e_K = 0.25$, $e_K = 0.5$, and $e_K = 1$. Each line corresponds to a specific tax rate as capital income increases.]
Optimal Tax Rate on Comprehensive Income $T'_Y(y)$

<table>
<thead>
<tr>
<th>Total Income</th>
<th>Marginal Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$200,000$</td>
<td>0</td>
</tr>
<tr>
<td>$400,000$</td>
<td>0.1</td>
</tr>
<tr>
<td>$600,000$</td>
<td>0.2</td>
</tr>
<tr>
<td>$800,000$</td>
<td>0.3</td>
</tr>
<tr>
<td>$1,000,000$</td>
<td>0.4</td>
</tr>
</tbody>
</table>

\[
e_Y = 1
\]
\[
e_Y = 0.25
\]
\[
e_Y = 0.5
\]
The generalized model

Utility is

$$V_i(\{c_i(t), k_i(t), z_i(t)\}_{t \geq 0}) = \delta_i \cdot \int_{t=0}^{\infty} u_i(c_i(t), k_i(t), z_i(t)) e^{-\delta_i t} dt$$

with $u_i(c, k, z)$ concave in $c$, concave in $k$, concave in $z$

⇒ consumption smoothing ⇒ sluggish transitional dynamics (a sum of anticipatory and build-up effects).

Convergence to steady state no longer instantaneous:

$$u_{ik} / u_{ic} = \delta_i - \bar{r}, u_{ic} \cdot (1 - T'_L) = -u_{iz}$$ and $c = rk + z - T(rk, z)$.

Social welfare:

$$SWF = \int_i \omega_i V_i(\{c_i(t), k_i(t), z_i(t)\}_{t \geq 0})$$
Optimal Linear Capital Tax in the Steady State

Given $\tau_K$ and $\tau_L$, rebated lump-sum $\to$ convergence to steady state.

At time 0, start from steady state, consider unanticipated small reform $d\tau_K$, with elasticities:

$$e_K(t) = dk^m(t)/d\bar{r}(\bar{r}/k^m(t)) \to e_K.$$ 

$$e_L,(1-\tau_K) = dz^m/d\bar{r}(\bar{r}/z^m).$$

Optimal linear capital income tax in steady state:

$$\tau_K = \frac{1 - \bar{g}_K - \tau_L \frac{z^m}{k^m} e_{L,(1-\tau_K)}}{1 - \bar{g}_K + \bar{e}_K}$$

If fast responses $\bar{e}_K \approx e_K$, quantitative results of simpler model hold.

Slow adjustment: $\bar{e}_K < e_K$.

But is it reasonable to exploit short-run sluggishness?
Optimal Linear Capital Tax in the Steady State

Given $\tau_K$ and $\tau_L$, rebated lump-sum $\to$ convergence to steady state.

At time 0, start from steady state, consider unanticipated small reform $d\tau_K$, with elasticities:

$$e_K(t) = \frac{dk^m(t)}{d\bar{r}(\bar{r}/k^m(t))} \to e_K.$$\n
$$e_L, (1-\tau_K) = \frac{dz^m}{d\bar{r}(\bar{r}/z^m)}.$$\n
Optimal linear capital income tax in steady state:

$$\tau_K = \frac{1 - \bar{g}_K - \tau_L \frac{z^m}{k^m} e_L, 1-\tau_K}{1 - \bar{g}_K + \bar{e}_K}$$

If fast responses $\bar{e}_K \approx e_K$, quantitative results of simpler model hold.

Slow adjustment: $\bar{e}_K < e_K$. But is it reasonable to exploit short-run sluggishness?
Optimal Linear Capital Tax in the Steady State

Given $\tau_K$ and $\tau_L$, rebated lump-sum $\rightarrow$ convergence to steady state.

At time 0, start from steady state, consider unanticipated small reform $d\tau_K$, with elasticities:

$$e_K(t) = \frac{dk^m(t)}{d\bar{r}(\bar{r}/k^m(t))} \rightarrow e_K.$$  

$$e_L,(1-\tau_K) = \frac{dz^m}{d\bar{r}(\bar{r}/z^m)}.$$  

Optimal linear capital income tax in steady state:

$$\tau_K = \frac{1 - \bar{g}_K - \tau_L \frac{z^m}{k^m} e_L,1-\tau_K}{1 - \bar{g}_K + \bar{e}_K}$$

If fast responses $\bar{e}_K \approx e_K$, quantitative results of simpler model hold.

Slow adjustment: $\bar{e}_K < e_K$.
But is it reasonable to exploit short-run sluggishness?
Optimal Linear Capital Tax in the Steady State

Given $\tau_K$ and $\tau_L$, rebated lump-sum $\rightarrow$ convergence to steady state.

At time 0, start from steady state, consider unanticipated small reform $d\tau_K$, with elasticities:

$$e_K(t) = \frac{dk^m(t)}{d\bar{r}(\bar{r}/k^m(t))} \rightarrow e_K.$$

$$e_L, (1-\tau_K) = \frac{dz^m}{d\bar{r}(\bar{r}/z^m)}.$$

Optimal linear capital income tax in steady state:

$$\tau_K = \frac{1 - \bar{g}_K - \tau_L \frac{z^m}{k^m} e_L, 1-\tau_K}{1 - \bar{g}_K + \bar{e}_K} \quad \text{with} \quad \bar{e}_K = \int_i g_i \delta_i \int_0^\infty e_K(t) \cdot e^{-\delta_i t} dt.$$

If fast responses $\bar{e}_K \approx e_K$, quantitative results of simpler model hold.

Slow adjustment: $\bar{e}_K < e_K$.

But is it reasonable to exploit short-run sluggishness?
General analysis of reforms

Comparison to standard dynamic objective:
\[ SWF_d = \int_i \omega_i \cdot V_i(\{c_i(t), k_i(t), z_i(t)\}_{t \geq 0}) \]

Any reform can be summarized by:
\[ e_{total}^K = e_{ante}^K + e_{post}^K \]

Simpler model: \( e_{total}^K = e^K \).

Generalized model: \( \bar{e}_K = e_{total}^K = e_{ante}^K + e_{post}^K \) (if anticipated), \( \bar{e}_K = e_{post}^K \) if not anticipated.

In every model: difference between primitives vs. reform considered.
Comparison with Previous Dynamic Models

$e_K$ steady state: Chamley-Judd model:

Infinite (degenerate) steady state elasticity $e_K = \infty$.

Aiyagari and wealth-in-utility have $e_K < \infty$.

$e_K^{ante}$ anticipation elasticity:

If reform announced infinitely in advance, $e_{ante} = \infty$, always, with full certainty.

Reasonable?

$e_{ante} < \infty$ if uncertainty (Aiyagari).

$e_K^{post}$ adjustment to reform: sluggish in all models, except with no transitional dynamics (linear utility).
NEW DYNAMIC PUBLIC FINANCE: REFERENCES

Dynamic taxation in the presence of future earnings uncertainty

Recent series of papers following upon on Golosov, Kocherlakota, Tsyvinski REStud ’03 (GKT)

Principle can be understood in 2 period model: Diamond-Mirrlees JpubE ’78 and Cremer-Gahvari EJ ’95

Generalized to many periods by GKT and subsequent papers

Simple exposition is Kocherlakota AER-PP ’04

Two comprehensive surveys: Golosov-Tsyvinski-Werning ’06 and Kocherlakota ’10 book
So far: representative agents, ex ante heterogeneity, aggregate uncertainty

We now consider idiosyncratic uncertainty that is not only ex ante, but unfolds over time

Skill shocks or preference shocks

Start with finite horizon: $t = 0, 1$

Preferences $U(c_0, c_1(s), y(s)/s)$

Interpretation: skill shock $s$ realized in period 1. Consumption decision $c_0$ in period 0 is made before the shock is realized
SET UP: FAILURE OF A-S and RESOURCE CONSTRAINT

- Note difference to time-0 shock (ex ante heterogeneity) as considered so far. Preferences would be $U(c_0(s), c_1(s), y(s)/s)$
- Under separability $+$ homogeneity, the Atkinson–Stiglitz (1976) theorem would rule out the optimality of a capital tax.
- With the period-1 shock, we will find a downward distortion of saving to be optimal (positive capital tax).
- Technology: linear storage with rate of return $R^* = 1/q$, so that the aggregate resource constraint is

$$c_0 + q \sum_s c_1(s)p(s) \leq q \sum_s y(s)p(s) \quad (1)$$
FIRST BEST

\[
\max_{c_0, c_1(s), y(s)} \sum_s U(c_0, c_1(s), y(s)/s)p(s)
\]

s.t. (1)

FOCs for \([c_0]\)

\[\mathbb{E}[U_{c_0, c_1(s), y(s)/s}] = \lambda\]

and for \([c_1(s)]\)

\[U_{c_1(s)}(c_0, c_1(s), y(s)/s) = \lambda q\]
Hence,

\[ \mathbb{E}[U_{c_0}, c_1(s), y(s)/s] = R^* U_{c_1(s)}(c_0, c_1(s), y(s)/s) \quad \forall s \]

\[ \Rightarrow \text{Full Insurance} \]

Taking expectations on both sides

\[ \mathbb{E}[U_{c_0}, c_1(s), y(s)/s] = R^* \mathbb{E}[U_{c_1(s)}(c_0, c_1(s), y(s)/s)] \] (2)

For instance, if \[ U(c_0, c_1(s), y(s)/s) = u(c_0) + \beta u(c_1(s)) - h(y(s)/s) \],
then we obtain the usual Euler equation

\[ u'(c_0) = \beta R^* \mathbb{E}[u'(c_1(s))] \]

and \[ c_1(s) = c_1 \] for all \( s \).
FREE SAVING GIVEN INCOME TAX

• Free saving with non-linear income tax $T(Y)$:

$$\max_{c_0, c_1(s), y(s)} \sum_s U(c_0, c_1(s), y(s)/s) p(s)$$

s.t.

$$c_0 + k_1 \leq e$$

$$c_1(s) \leq y(s) - T(y(s)) + Rk_1 \quad \forall s$$

• FOCs and $R = R^*$ yields the Euler equation

$$u'(c_0) = \beta R^* \mathbb{E}[u'(c_1(s))]$$

• If agents can freely decide how much to save in a risk-free asset with return $R = R^*$, we obtain the Euler equation as in the first best...
PRIVATE INFORMATION AND INCENTIVE CONSTRAINTS

- Suppose $s$ is private information and agents make reports $r = \sigma(s)$, where $\sigma$ denotes the reporting strategy.
- Truth-telling: $\sigma^*(s) = s \quad \forall s$
- Denote
  \[ c_1^\sigma(s) = c_1(\sigma(s)) \]
  and
  \[ y^\sigma(s) = y(\sigma(s)) \]

- The set of incentive constraints is
  \[ \mathbb{E}[U(c_0, c_1(s), y(s)/s)] \geq \mathbb{E}[U(c_0, c_1^\sigma(s), y^\sigma(s)/s)] \quad \forall \sigma, s \]

- This is equivalent to
  \[ U(c_0, c_1(s), y(s)/s) \geq U(c_0, c_1(r), y(r)/s) \quad \forall r, s \]  (3)
The second best (dynamic Mirrlees) problem is

$$\max_{c_0, c_1(s), y(s)/s} \mathbb{E}[U(c_0, c_1(s), y(s)/s)]$$

s.t.

$$c_0 + q \sum_s c_1(s)p(s) \leq q \sum_s y(s)p(s) \quad (RC)$$

and

$$U(c_0, c_1(s), y(s)/s) \geq U(c_0, c_1(r), y(r)/s) \quad \forall r, s \quad (IC)$$
FEASIBLE VARIATIONS

1. (RC) and (IC) define the set $F$ of feasible allocations, i.e.

$$F \equiv \{(c_0, c_1(s), y(s)) \mid (c_0, c_1(s), y(s)) \text{ satisfies (RC) and (IC)}\}$$

2. Key question: Is free saving feasible? Formally, if $(c_0, c_1(s), y(s)) \in F$, does this imply that $(c_0 - \Delta, c_1(s) + R^*\Delta, y(s)) \in F$ as well, for some $\Delta \in \mathbb{R}$?

   In other words, if the agent saves a little in period 0 ($\Delta$) is she still willing to supply the same output (i.e. not lie about $s$)?

3. Depends on income effects in general

4. For instance, suppose

$$U(c_0, c_1(s), y(s)/s) = \hat{U}(c_0, c_1(s) - h(y(s)/s))$$

Then, given $c_0$, just maximize $c_1(s) - h(y(s)/s)$. There are no income effects due to quasilinearity, and the above variation is feasible.
FEASIBLE VARIATIONS

- Easier to see using (IC):
  \[
  \widehat{U}(c_0 - \triangle, c_1(s) + R^*\triangle - h(y(s)/s)) \geq \\
  \widehat{U}(c_0 - \triangle, c_1(r) + R^*\triangle - h(y(r)/s))
  \]

  if and only if

  \[
  c_1(s) + R^*\triangle - h(y(s)/s) \geq c_1(r) + R^*\triangle - h(y(r)/s)
  \]

  which is implied by the original allocation being feasible, i.e.
  \((c_0, c_1(s), y(s)) \in F\)

- But in general, saving in period 0 has a negative income effect on
  labor supply in period 1 (if leisure is a normal good)

- e.g. consider preferences

  \[
  U(c_0, c_1(s), y(s)/s) = u(c_0) + \beta u(c_1(s)) - h(y(s)/s)
  \]

  Additive separability + concavity of \(u(.)\) mean leisure is normal
  (output is “inferior”) and so the variation above is no longer feasible.
CAN WE FIND A FEASIBLE VARIATION?

- Free saving is not feasible with these preferences due to negative income effect on labor supply.

- Consider

\[(c_0 - \triangle, c_1(s) + \delta(\triangle, s), y(s))\]  \hspace{1cm} (5)

with \(\delta(\triangle, s)\) chosen such that (IC) is satisfied:

\[u(c_0 - \triangle) + \beta u(c_1(s) + \delta(\triangle, s)) = u(c_0) + \beta u(c_1(s)) + A(\triangle) \quad \forall s, \triangle\]  \hspace{1cm} (6)

for some \(A(\triangle)\), and such that it is resource neutral:

\[-\triangle + q \sum_m \delta(\triangle, s)p(s) = 0 \quad \forall \triangle\]  \hspace{1cm} (7)

- With the “free saving“ variation, we had \(\delta(\triangle, s) = -R^* \triangle\). What is key difference?
Verify Incentive Compatibility of Variation

Verify that the variation maintains incentive compatibility:

\[ u(c_0 - \triangle) + \beta u(c_1(s) + \delta(\triangle, s)) - h(y(s)/s) \geq \]

\[ u(c_0 - \triangle) + \beta u(c_1(r) + \delta(\triangle, r)) - h(y(r)/s) \]

if and only if (6)

\[ u(c_0) + \beta u(c_1(s)) + A(\triangle) - h(y(s)/s) \geq \]

\[ u(c_0) + \beta u(c_1(r)) + A(\triangle) - h(y(r)/s) \]

if and only if

\[ u(c_0) + \beta u(c_1(s)) - h(y(s)/s) \geq u(c_0) + \beta u(c_1(r)) - h(y(r)/s) \]

Is this true?

Key: Given separability, all that matters for incentive compatibility is the total utility from consumption.
INVERSE EULER EQUATION

Suppose the original allocation \((c_0, c_1(s), y(s))\) solves the second best problem. Then, since the variation \(\delta(\Delta, s)\) is feasible as just shown, it cannot improve the objective.

Formally,

\[
0 = \arg\max_{\Delta} \sum_s p(s)[u(c_0 - \Delta) + \beta u(c_1(s) + \delta(\Delta, s)) - h(y(s)/s)]
\]

\[
= \arg\max_{\Delta} \sum_s p(s)[u(c_0) + \beta u(c_1(s) + A(\Delta)) - h(y(s)/s)]
\]

\[
= \arg\max_{\Delta} A(\Delta),
\]

where we used

\[
u(c_0 - \Delta) + \beta u(c_1(s) + \delta(\Delta, s)) = u(c_0) + \beta u(c_1(s)) + A(\Delta) \quad \forall s, \Delta (8)
\]

\[\text{FOC } A'(0) = 0\]
INVERSE EULER EQUATION

- \( \delta(\triangle, s) \) satisfies (IC); differentiate w.r.t. \( \triangle \):

\[
-u'(c_0) + \beta u'(c_1(s)) \frac{\partial \delta(\triangle, s)}{\partial \triangle} \bigg|_{\triangle=0} = A'(0)
\]

rearrange (at optimum):

\[
\frac{\partial \delta(\triangle, s)}{\partial \triangle} \bigg|_{\triangle=0} = \frac{u'(c_0) + A'(0)}{\beta u'(c_1(s))} = \frac{u'(c_0)}{\beta u'(c_1(s))} \quad \forall s \tag{9}
\]

- Condition for resource neutrality of the variation implies:

\[
-1 + q \sum_s p(s) \frac{\partial \delta(\triangle, s)}{\partial \triangle} \bigg|_{\triangle=0} = 0
\]
INVERSE EULER EQUATION (II)

- Using expression for \( \frac{\partial \delta(\Delta, s)}{\partial \Delta} \bigg|_{\Delta=0} \):

\[
\frac{1}{u'(c_0)} = \frac{1}{\beta R^*} \sum_s \frac{p(s)}{u'(c_1(s))}
\]  \hspace{1cm} (10)

- With separable preferences, optimal allocation has to satisfy this inverse Euler equatoin (Diamond/Mirrlees 1978, Rogerson 1985, Golosov et al. 2003)

- Is this necessary and sufficient? (Think of optimality of \( y(s) \)).

- Implies that the Euler equation is violated.

\[
u'(c_0) = \beta R^* \sum_s p(s) u'(c_1(s))\]  \hspace{1cm} (11)

Is it always violated?
Inverse Euler implies that, at the optimum,

\[ u'(c_0) = \left[ \frac{1}{\beta R^*} \sum_s \frac{p(s)}{u'(c_1(s))} \right]^{-1} = \beta R^* \left( \mathbb{E} \left[ \frac{1}{u'(c_1(s))} \right] \right)^{-1} \]

By Jensen’s inequality and convexity of the function \( f(x) = 1/x \),

\[ u'(c_0) < \beta R^* \mathbb{E}[u'(c_1(s))] \]

The optimal allocation is incompatible with free saving. Is saving is discouraged or encouraged?

Intuition: saving in period 0 increases income in period 1 across all shocks \( s \rightarrow \) negative income effect on \( y \rightarrow \) is this good or bad for Planner?

Implications for capital taxation, but study distinction between the wedge derived here and actual implementations later.
TECHNICAL POINT: DUAL APPROACH

- Consider allocation in terms of utils

\[ u_0 \equiv u(c_0), \quad u_1(s) \equiv u(c_1(s)) \]

- Move from original allocation \((u_0, u_1(s))\) to variation \((\tilde{u}_0, \tilde{u}_1(s))\) such that

\[ u_0 + \beta u_1(s) = \tilde{u}_0 + \beta \tilde{u}_1(s) \quad \forall s \]

- In particular, set

\[ \tilde{u}_0 = u_0 - \beta \triangle \]

and

\[ \tilde{u}_1(s) = u_1(s) + \triangle \quad \forall s \]
Are incentive constraints affected?

\[ \tilde{u}_0 + \beta \tilde{u}_1(s) - h(y(s)/s) \geq \tilde{u}_1(r) + \beta \tilde{u}_1(r) - h(y(r)/s) \quad \forall r, s \]

if and only if

\[ u_0 - \beta \Delta + \beta (u_1(s) + \Delta) - h(y(s)/s) \geq u_1(r) - \beta \Delta + \beta (u_1(r) + \Delta) - h(y(r)/s) \quad \forall r, s \]

if and only if

\[ u_0 + \beta u_1(s) - h(y(s)/s) \geq u_1(r) + \beta u_1(r) - h(y(r)/s) \quad \forall r, s \]

is this true?

Variation by construction keeps total expected utility unchanged

Dual problem: minimize total resource cost of allocation

\[
\min_{\Delta} \left\{ C(u_0 - \beta \Delta) + q \sum_s p(s) C(u_1(s)) + \Delta \right\}
\]

where \( C(u) \) is the inverse function of \( u(c) \)
INVERSE EULER AGAIN

- If the original allocation \((u_0, u_1(s))\) is optimal, \(\Delta = 0\) must solve this problem.
- The FOC evaluated at \(\Delta = 0\) is:
  \[-C'(u_0)\beta + q \sum_s p(s)C'(u_1(s)) = 0\]

- Use \(C'(u) = 1/u'(c)\)
  \[\frac{1}{u'(c_0)} = \frac{q}{\beta} \sum_s \frac{p(s)}{u'(c_1(s))}\]
  which is the inverse Euler equation again (recall \(q = 1/R^*\))
- Alternative interpretation: \(1/u'(c)\) is resource cost of providing some given incentives
- IEE requires the equalization of the expected resource cost of providing incentives across both periods
INFINITE HORIZON

- General model with separable preferences

\[ \sum_{t,s^t} \beta^t [u(c(s^t)) - h(y(s^t)/s_t)] Pr(s^t) \]

and \( s^t = (s_0, s_1, ..., s_t) \)

- Agents have reporting strategies such that (why does it depend on \( s^t \) not \( s_t \)?)

\[ r_t = \sigma_t(s^t) \]

where the truth telling strategy is such that

\[ \sigma^*_t(s^t) = s_t \quad \forall s^t, t \]

- \( \sigma^t(s^t) \) denotes the history of reports induced by the strategy \( \sigma_t(s^t) \), i.e.

\[ \sigma^t(s^t) = (r_0, r_1, ..., r_t) = (\sigma_0(s_0), \sigma_1(s_1), ..., \sigma_t(s^t)) \]
DYNAMIC INCENTIVE CONSTRAINTS

- Dynamic incentive constraints

\[ \sum_{t,s^t} \beta^t [u(c(s^t)) - h(y(s^t)/s_t)/Pr(s^t)] \geq \sum_{t,s^t} \beta^t [u(c(\sigma^t(s^t))) - h(y(\sigma^t(s^t))/s_t)Pr(s^t)] \forall \sigma \]

- Pick some node \( s^t \). Then set

\[ \tilde{u}(s^\tau) = u(s^\tau) \]

for any \( s^\tau \neq s^t \) and \( s^\tau \neq (s^t, s_{t+1}) \). i.e. leave consumption utilities unchanged at any node that is not \( s^t \) or any of its direct successors.

- At \( s^t \), set

\[ \tilde{u}(s^t) = u(s^t) - \beta \triangle \]

and

\[ \tilde{u}(s^t, s_{t+1}) = u(s^t, s_{t+1}) + \triangle \quad \forall s_{t+1} \]
DYNAMIC INCENTIVE CONSTRAINTS

- Key: if initial allocation was incentive compatible, perturbed one is as well.
- Moreover, perturbed allocation does not change total expected utility (from any reporting strategy \( \sigma_t(s^t) \), thus also from truth-telling)
- Minimize expected resource cost of the perturbed allocation by choosing \( \Delta \)

\[
\min_{\Delta} \left\{ C(u(s^t) - \beta \Delta) + q \sum_{s^{t+1}|s^t} Pr(s^{t+1}|s^t) C(u_1(s^{t+1}) + \Delta) \right\}
\]

- If the initial allocation is optimal, this program must be solved at \( \Delta = 0 \) with FOC

\[
\frac{1}{u'(c(s^t))} = \frac{1}{\beta R^*} E \left[ \frac{1}{u'(c(s^{t+1}))} \mid s^t \right]
\]

General inverse Euler equation has to hold for all nodes \( s^t \)
GENERAL INVERSE EULER EQUATION

• Implies

\[ u'(c(s^t)) < \beta R^* \mathbb{E}\left[u'(c(s^{t+1})\right|s^t] \quad \forall s^t \]

i.e. savings need to be distorted downwards compared to the Euler equation from free saving

• May require individualized capital taxes that keep track of the entire history of skill shocks \(s^t\) such that

\[ u'(c(s^t)) = \beta \mathbb{E}\left[(1 + r^*(1 - \tau^k(s^{t+1})))u'(c(s^{t+1})\right|s^t] \quad \forall s^t \]

where \(r^* \equiv R^* - 1\)

• However, simple linear capital tax may not work

• Farhi and Werning (2011) show how to use this framework to evaluate the welfare gains from optimal saving distortions starting from some baseline allocation, e.g. the free saving allocation (Aiyagari 1994)
STEP BACK: What is implementation? Why was it not discussed before?!

Back to 2 period model, 2 shocks \( s \in \{H, L\} \)

Suppose have found the optimal allocation with consumption \( \{c_0^*, c_1^*(L), c_1^*(H)\} \)

It satisfies the inverse Euler equation

\[
\frac{1}{u'(c_0^*)} = \frac{1}{\beta R^*} \left[ \frac{p_L}{u'(c_1^*(L))} + \frac{p_H}{u'(c_1^*(H))} \right]
\]

Suppose we introduce a linear capital tax \( \tau^k \) such that the Euler equation is satisfied

\[
u'(c_0^*) = \beta R^*(1 - \tau^k)[p_H u'(c_1^*(L)) + p_H u'(c_1^*(H))]
\] (12)
IMPLEMENTATION: LINEAR CAPITAL TAX, NONLINEAR INCOME TAX

- Introduce a non-linear income tax system $T_0, T_1(y)$ so that the individuals' budget constraints become

$$c_0 + k_1 \leq e_0 - T_0$$

in period 0 and

$$c_1(s) \leq y(s) - T_1(y(s)) + (1 - \tau^k)R^* k_1$$

in period 1

- Note: Very restrictive tax system where the capital tax is linear and separable from the labor income tax

- Can we find a tax system $T_0, T_1, \tau^k$ such that $\{c^*_0, c^*_1(L), c^*_1(H)\}$ is incentive compatible? [What do you think?]

- If we could force the agent to choose $c^*_0$ and thus $k^*_1$? We'd be back to a standard Mirlees problem in period 1, so we can always find $T_1(y)$ that implements $c^*_1(s), y^*(s)$
PROBLEM WITH LINEAR CAPITAL TAX

- Suppose $H$'s incentive constraint is the binding one at the optimum (which means?)

\[ u(c^*(H)) - h(y^*(H)/H) = u(c^*(L)) - h(y^*(L)/H) \]  \hspace{1cm} (13)

i.e. if the agent saves optimally $k^*_1$, truth-telling is optimum

- Moreover, given truth-telling in period 1, Euler equation holds, so the agent finds it optimal to choose optimal savings $k^*_1$

- But: double-deviation $\sigma_1(s) = L$ for all $s \in \{H, L\}$ and $\tilde{k}_1 = k^*_1 + \epsilon$

- If $\sigma_1(s)$ for all $s$ and $k_1 = k^*_1$, then

\[ u'(c^*_0) < \beta R^*(1 - \tau^k)u'(c^*_1(L)) \]  \hspace{1cm} (14)

Why?

- It is optimal to deviate to $\tilde{k}_1 = k^*_1 + \epsilon$ with $\epsilon > 0$

- What is agent tempted to do here? Explain in words.
DOUBLE DEVIATION

- Profitable deviation: save more in period 0 and always claim to be low type in period 1 period

\[
\bar{U} = u(c_0^* - \epsilon) + \beta \left[ u(c_1^*(L) + R^*(1 - \tau^k)\epsilon) - p_L h(y^*(L)/L) - p_H h(y^*(L)/H) \right]
\]

\[
\approx \epsilon \left[ - u'(c_0^*) + \beta R^*(1 - \tau^k) u'(c_1^*(L)) \right]
\]

\[
+ u(c_0^*) + \beta \left[ u(c_1^*(L)) - p_L h(y^*(L)/L) - p_H h(y^*(L)/(H)) \right]
\]

\[
> u(c_0^*) + \beta \left[ p_H u(c_1^*(H)) - h(y^*(H)/H) \right] + p_L u(c_1^*(L)) - h(y^*(L)/L)) \]

- Where is first approximation coming from? Where did the second equality come from?
- Hence, the double deviation makes the agent better off than truth telling under the optimal allocation that we wanted to implement.
LINEAR CAPITAL TAX DOES NOT WORK – SOLUTIONS?

- State-dependent linear capital tax $\tau^k(s)$ so that
  $$u^'(c^*_0) = \beta R^*(1 - \tau^k(s))u^'(c^*_1(s)) \quad \forall s$$
  state by state (Kocherlakota 2005). Prevents profitable double-deviations.

  $$\tau^k(s) = 1 - \frac{u^'(c^*_0)}{\beta R^*u^'(c^*_1(s))}$$

  is high whenever $c^*_1(s)$ is low.

- What does this mean? Returns to saving are made risky so as to make savings unattractive.

- However: $\tau^k(s)$ is zero in expectation so that the government does not raise revenue with the capital tax.

- In general, capital taxes must be contingent on the entire history of shocks.
LINEAR CAPITAL TAX DOES NOT WORK – SOLUTIONS? (II)

- Albanesi and Sleet (2006): joint tax function on wealth and income $T(y, k)$ with iid shocks (wealth is a sufficient statistic for history of shocks with iid shocks)
- Werning (2010): non-linear capital tax works more generally, i.e. $T^k(R^*k_t, s^t)$ rather than $(1 - \tau^k(s^t))R^*k_t$. Moreover, with such a non-linear capital tax, one can make it history-independent, i.e. $T^k(R^*k_t)$, (in contrast to Kocherlakota’s implementation).
REFERENCES


Laroque, G. “Indirect Taxation is Superfluous under Separability and Taste Homogeneity: A Simple Proof”, Economic Letters, Vol. 87, 2005, 141-144. (web)


