Lecture 3: Optimal Income Taxation (II)

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GOALS OF THIS LECTURE

1) Illustration of structural vs. policy elasticities using the example of the linear top tax rate.

2) General non-linear tax derivation à la Saez (2001) without income effects.

3) Mechanism Design approach of Mirrlees (1971) and link between the two approaches (primitives vs. sufficient stats).

4) Adding income effects.

5) Extension 1: Migration effects

6) Extension 2: Rent-Seeking
Recap: The Envelope Theorem

When considering a tax change (small), the envelope theorem tells us that if all is regular, the direct welfare impact of the tax change on agent $i$ is the mechanical impact on his consumption times marginal utility.

This means behavioral responses (e.g.: the adjustment in labor supply) have no first-order impact on welfare if they are at the optimum level chosen by the agent to start with.

The social impact is the mechanical change in consumption times marginal social welfare weight.

But the behavioral responses do have a first-order impact on revenues. Those revenues are either rebated or valued at the marginal cost of public funds. Either way, this does have a first-order effect on welfare. (When is this not true?)
Elasticities: reduced-form vs. structural

Sometimes, it’s enough to express formulas in terms of the reduced-form elasticities, so-called “policy elasticities.”

Other times, interested in decomposing the reduced-form elasticity into primitive, structural elasticities, i.e., income and substitution effects.

Depends on the context and what you know from the data.

Let’s illustrate this with the top tax rate derivation.
Recall from last lecture: the top tax rate derivation.

We do not even specify a utility function.

Consider constant MTR $\tau$ above fixed $z^*$. Goal is to derive optimal $\tau$.

Assume w.l.o.g there is a continuum of measure one of individuals above $z^*$.

Let $z(1-\tau)$ be their average income [depends on net-of-tax rate $1-\tau$], with elasticity $e = \left[(1-\tau)/z\right] \cdot dz/d(1-\tau)$.

! Careful, what is $e$?

Note that $e$ is a mix of income and substitution effects (see Saez ’01).
Optimal Top Income Tax Rate (Mirrlees ’71 model)

Disposable Income
\[ c = z - T(z) \]

Market income \( z \)

Top bracket:
Slope \( 1 - \tau \)

Reform:
Slope \( 1 - \tau - d\tau \)

Source: Diamond and Saez JEP'11
Optimal Top Income Tax Rate (Mirrlees ’71 model)

Disposable Income $c = z - T(z)$

Market income $z$

$z^* - T(z^*)$

Mechanical tax increase:
$\frac{d\tau}{\tau} [z - z^*]$  

Behavioral Response tax loss:
$\tau \, dz = - \frac{d\tau \cdot e^z \cdot \tau}{1 - \tau}$

Source: Diamond and Saez JEP’11
OPTIMAL TOP INCOME TAX RATE

Consider small \( d\tau > 0 \) reform above \( z^* \).

1) Mechanical increase in tax revenue:

\[
dM = [z - z^*] d\tau
\]

2) Welfare effect:

\[
dW = -\bar{g} dM = -\bar{g} [z - z^*] d\tau
\]

where \( \bar{g} \) is the social marginal welfare weight for top earners.

3) Behavioral response reduces tax revenue:

\[
 dB = \tau \cdot dz = -\tau \frac{dz}{d(1 - \tau)} d\tau = -\frac{\tau}{1 - \tau} \cdot \frac{1 - \tau}{z} \cdot \frac{dz}{d(1 - \tau)} \cdot zd\tau
\]

\[\Rightarrow dB = -\frac{\tau}{1 - \tau} \cdot e \cdot zd\tau\]
OPTIMAL TOP INCOME TAX RATE

\[ dM + dW + dB = d\tau \left[ (1 - \bar{g})[z - z^*] - e\frac{\tau}{1 - \tau}z \right] \]

Optimal \( \tau \) such that \( dM + dW + dB = 0 \) implies

\[ \frac{\tau}{1 - \tau} = \frac{1 - \bar{g})[z - z^*]}{e \cdot z} = \frac{(1 - \bar{g})}{a \cdot e} \]

\[ \tau = \frac{1 - \bar{g}}{1 - \bar{g} + a \cdot e} \quad \text{with} \quad a = \frac{z}{z - z^*} \]

Optimal \( \tau \downarrow \bar{g} \) [redistributive tastes]

Optimal \( \tau \downarrow \) with \( e \) [efficiency]

Optimal \( \tau \downarrow a \) [thinness of top tail]
OPTIMAL TOP TAX RATE – STRUCTURAL FORMULA

Let’s now derive this in terms of the structural elasticities.

!! Change notation to map the Saez (2001) paper (easier for you). \( \bar{z} = z^* \) and \( z_m = z \).

Mechanical revenue effect (M) (at constant incomes) and the welfare effect (W) are naturally the same as above.

Behavioral response: change in marginal tax rate is \( d\tau \), change in virtual income is \( dR = \bar{z}d\tau \).

The change in an individual’s income at income \( z \) is:

\[
\frac{dz}{1 - \tau} d\tau + \frac{dz}{dR} dR = -\left( \varepsilon_u(z) - \eta(z)\bar{z} \right) \frac{d\tau}{1 - \tau}
\]
Sum over all individuals earning more than $\bar{z}$ and multiply by $\tau$ to get the revenue change:

$$B = -(\bar{\varepsilon}^u z_m - \bar{\eta} \bar{z}) \frac{\tau d\tau}{1 - \tau}$$

where

$$\bar{\varepsilon}^u = \int_{\bar{z}}^{\infty} \varepsilon^u(z) h(z) \frac{dz}{z_m}$$

is a weighted average of uncompensated elasticities. $\varepsilon^u(z)$ itself is the average uncompensated elasticity over individuals earning $z$ (not necessary to assume that agents have homogeneous elasticities at given $z$).

$$\bar{\eta} = \int_{\bar{z}}^{\infty} \eta(z) h(z) dz$$

is the average income effect for agents with income above $\bar{z}$. 
OPTIMAL TOP TAX RATE – STRUCTURAL FORMULA

Sum of $B + M + W = 0$ means:

$$\frac{\tau}{1 - \tau} = \frac{(1 - \bar{g})(z_m/\bar{z} - 1)}{\bar{\varepsilon}^u z_m/\bar{z} - \bar{\eta}}$$

Use Slutsky and definition of $a$ to rearrange:

$$\frac{\tau}{1 - \tau} = \frac{(1 - \bar{g})}{\bar{\varepsilon}^u + (a - 1)\bar{\varepsilon}^c}$$

Comparing to previous formula, we see that the reduced-form and structural elasticities are linked through:

$$a \cdot e = \bar{\varepsilon}^u + (a - 1)\bar{\varepsilon}^c$$

!! Careful: still not “primitive” elasticities (haven’t specified utility functions).
GENERAL NON-LINEAR INCOME TAX $T(z)$

(1) Lumpsum grant given to everybody equal to $-T(0)$

(2) Marginal tax rate schedule $T'(z)$ describing how (a) lump-sum grant is taxed away, (b) how tax liability increases with income

Let $H(z)$ be the income CDF [population normalized to 1] and $h(z)$ its density [endogenous to $T(.)$]

Let $g(z)$ be the social marginal value of consumption for taxpayers with income $z$ in terms of public funds [formally $g(z) = G'(u) \cdot u_c / \lambda$]: no income effects ⇒ $\int g(z)h(z)dz = 1$

Redistribution valued ⇒ $g(z)$ decreases with $z$

Let $G(z)$ the average social marginal value of $c$ for taxpayers with income above $z$ [$G(z) = \int_z^\infty g(s)h(s)ds/(1 - H(z))$]
Disposable Income \( c = z - T(z) \)

Pre-tax income \( z \)

Mechanical tax increase: \( d \tau dz \ [1-H(z)] \)

Social welfare effect: \( -d \tau dz \ [1-H(z)] G(z) \)

Behavioral response: \( \delta z = - d \tau e^z / (1-T'(z)) \)

\[ \rightarrow \text{Tax loss: } T'(z) \delta z h(z) dz \]

\[ = -h(z) e^z T'(z) (1-T'(z)) dz d\tau \]

Small band \((z, z+dz)\): slope \(1 - T'(z)\)

Reform: slope \(1 - T'(z) - d\tau\)

Source: Diamond and Saez JEP'11
GENERAL NON-LINEAR INCOME TAX

Assume away income effects $\varepsilon^c = \varepsilon^u = e$ [Diamond AER’98 shows this is the key theoretical simplification]

Consider small reform: increase $T'$ by $d\tau$ in small band $z$ and $z + dz$

Mechanical effect $dM = dzd\tau[1 - H(z)]$

Welfare effect $dW = -dzd\tau[1 - H(z)]G(z)$

Behavioral effect: substitution effect $\delta z$ inside small band $[z, z + dz]$: $dB = h(z)dz \cdot T' \cdot \delta z = -h(z)dz \cdot T' \cdot d\tau \cdot z \cdot e(z)/(1 - T')$

Optimum $dM + dW + dB = 0$
GENERAL NON-LINEAR INCOME TAX

\[ T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e(z)} \]

1) \( T'(z) \) decreases with \( e(z) \) (elasticity efficiency effects)

2) \( T'(z) \) decreases with \( \alpha(z) = (zh(z)) / (1 - H(z)) \) (local Pareto parameter)

3) \( T'(z) \) decreases with \( G(z) \) (redistributive tastes)

Asymptotics: \( G(z) \to \bar{g}, \alpha(z) \to a, e(z) \to e \Rightarrow \) Recover top rate formula
\[
\tau = \frac{(1 - \bar{g})}{(1 - \bar{g} + a \cdot e)}
\]
$a = \frac{z_m}{z_m - z^*}$ with $z_m = E(z | z > z^*)$

$\alpha = \frac{z^* h(z^*)}{1 - H(z^*)}$

Source: Diamond and Saez JEP'11
Negative Marginal Tax Rates Never Optimal

Suppose $T' < 0$ in band $[z, z + dz]$

Increase $T'$ by $d\tau > 0$ in band $[z, z + dz]$: $dM + dW > 0$ and $dB > 0$

because $T'(z) < 0$

⇒ Desirable reform

⇒ $T'(z) < 0$ cannot be optimal

EITC schemes are not desirable in Mirrlees ’71 model
MIRRLEES MODEL

The difference to before: we need to specify the structural primitives.

Key simplification is the lack of income effects (Diamond, 1998). We look into income effects next time.

Individual utility: $c - v(l)$, $l$ is labor supply.

Skill $n$ is exogenously given, equal to marginal productivity. Earnings are $z = nl$.

Density is $f(n)$ and CDF $F(n)$ on $[0, \infty)$.

Entry into contract theory/mechanism design here: The government does not observe skill. Tax is based on income $z$, $T(z)$.

What happens if we had a tax $T(n)$ available?

Why did we not talk about this in the earlier derivations? Did we ignore the incentive compatibility constraints?
Elasticity of labor to taxes

Recall we derive elasticities on the linearized budget set. If marginal tax rate is $\tau$, labor supply is: $l = l(n(1 - \tau))$. Why the $n(1 - \tau)$? Why only $n(1 - \tau)$?

FOC of the agent for labor supply:

$$n(1 - \tau) = v'(l)$$

Totally differentiate this (key thing: skill is fixed!) to get elasticity of labor supply. Also equal to elasticity of income since skill fixed.

$$d(n(1 - \tau)) = v''(l)dl$$

$$\Rightarrow e = \frac{dl}{d(n(1 - \tau))} \frac{(1 - \tau)n}{l} = \frac{(1 - \tau)n}{lv''(l)} = \frac{v'(l)}{lv''(l)}$$

Is this compensated? uncompensated?
Direct Revelation Mechanism and Incentive Compatibility

We want to max social welfare and have exogenous revenue requirement (non transfer-related $E$).

We imagine a direct revelation mechanism. Every agent comes to government, reports a type $n'$. We assign allocations as a function of the report. $c(n')$, $z(n')$, $u(n')$. Why are we not assigning labor $l(n')$?

What are the constraints in this problem?

Feasibility (net resources sum to zero): $\int n c(n) dn \geq nl_n f(n) dn - E$.

Incentive compatibility:
Direct Revelation Mechanism and Incentive Compatibility

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What are the constraints in this problem?

Feasibility (net resources sum to zero): $\int c(n)f(n)dn \geq nl_nf(n)dn - E$.

Incentive compatibility:

$$c(n) - v \left( \frac{z(n)}{n} \right) \geq c(n') - v \left( \frac{z(n')}{n} \right) \forall n, n'$$

That’s a lot of constraints!
Envelope Theorem and First order Approach

Replace the infinity of constraints with agents' first-order condition. If we take derivative of utility wrt type $n$ at truth-telling

$$\frac{du_n}{dn} = \left( c'(n) - \frac{z'(n)}{n} v' \left( \frac{z(n)}{n} \right) \right) \frac{dn}{dn} + \frac{z(n)}{n^2} v' \left( \frac{z(n)}{n} \right)$$

What if report is optimally chosen?

Envelope condition:

$$\frac{du_n}{dn} = \frac{l_n v'(l_n)}{n}$$

Will replace infinity of constraints.

Is necessary, but what about sufficiency?
Full Optimization Program

\[ \max_{c_n, u_n, z_n} \int_n G(u_n) f(n) \, dn \quad \text{s.t.} \quad \int_n c_n f(n) \, dn \leq \int_n n l_n f(n) \, dn - E \]

and s.t. \( \frac{du_n}{dn} = \frac{l_n v'(l_n)}{n} \)

State variable: \( u_n \).

Control variables: \( l_n \), with \( c_n = u_n + v(l_n) \).

Why am I suddenly saying \( l_n \) is a control?

Use optimal control.
Hamiltonian and Optimal Control

The Hamiltonian is:

\[ H = \left[ G(u_n) + p \cdot (nl_n - u_n - v(l_n)) \right] f(n) + \phi(n) \cdot \frac{l_n v'(l_n)}{n} \]

\( p \): multiplier on the resource constraint.

\( \phi(n) \): multiplier on the envelope condition ("costate"). Depends on \( n \).

FOCs:

\[
\frac{\partial H}{\partial l_n} = p \cdot [n - v'(l_n)] f(n) + \frac{\phi(n)}{n} \cdot [v'(l_n) + l_n v''(l_n)] = 0
\]

\[
\frac{\partial H}{\partial u_n} = \left[ G'(u_n) - p \right] f(n) = -\frac{d\phi(n)}{dn}
\]

Transversality: \( \lim_{n \to \infty} \phi(n) = 0 \) and \( \phi(0) = 0 \).
Rearranging the FOCs

Take the integral of the FOC wrt $u_n$ to solve for $\phi(n)$:

$$-\phi(n) = \int_n^\infty [p - G'(u_m)] f(m) dm$$

Integrate this same FOC over the full space, using transversality conditions:

$$p = \int_0^\infty G'(u_n) f(m) dm$$

What does this say?

How can we make the tax rate appear? Use the agent’s FOC.

$$n - v'(l_n) = nT'(z_n)$$
Obtaining the Optimal Tax Formula

Recall that elasticity of income at $z_n$ is:

$$e(z_n) = \frac{(1-T'(z_n))n}{ln''(l)}$$

Rearranging the last term in the FOC for $l_n$:

$$\left[ v'(l_n) + l_n v''(l_n) \right] / n = [1 - T'(z_n)] \left[ 1 + 1/e(z_n) \right]$$

Let $g_m \equiv G'(u_m)/p$ be the marginal social welfare weight on type $m$.

Then, the FOC for $l_n$ becomes:

$$\frac{T'(z_n)}{1 - T'(z_n)} = \left( 1 + \frac{1}{e(z_n)} \right) \cdot \left( \int_n^\infty (1 - g_m) dF(m) \right) \cdot \frac{nf(n)}{nf(n)}$$

This is the Diamond (1998) formula.

What is different from the previously derived formula à la Saez (2001)?
Let’s go from types to observable income

How do we go from type distribution to income distribution?

Under linearized tax schedule, earnings are a function $z_n = nl(n(1 - \tau))$.

How do earnings vary with type?

$$\frac{dz_n}{dn} = l + (1 - \tau)n \frac{dl}{d(m(1 - \tau))} = ln \cdot (1 + e(z_n))$$

(intuition?)

Let $h(z)$ be the density of earnings, with CDF $H(z)$. The following relation must hold:

$$h(z_n) dz_n = f(n) dn$$

$$f(n) = h(z_n) ln(1 + e(z_n)) \Rightarrow ng(n) = zn h(z_n) (1 + e(z_n))$$

Let’s substitute income distributions for type distributions in the formula.
Optimal Tax Formula with No Income Effects

\[
\frac{T'(z_n)}{1 - T'(z_n)} = \left(1 + \frac{1}{e(z_n)}\right) \left(\frac{\int_{n}^{\infty} (1 - g_m) dF(m)}{nf(n)}\right) \quad \text{(primitives)}
\]

\[
= \frac{1}{e(z_n)} \left(\frac{1 - H(z_n)}{z_n h(z_n)}\right) \cdot (1 - G(z_n)) \quad \text{(incomes)}
\]

where:

\[
G(z_n) = \int_{n}^{\infty} g_m dF(m) = \int_{z_n}^{\infty} g_m dH(z_m)
\]

is the average marginal social welfare weight on individuals with income above \(z_n\) (change of variables to income distributions in last equality).

Rearrange, use definition of Pareto parameter \(\alpha(z) = (zh(z))/ (1 - H(z))\) to get same formula as before:

\[
T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e(z)}
\]
Recap:

“Mechanism design approach” requires you to specify *primitives* (utility function, uni-dimensional heterogeneity) as done in Mirrlees (1971).

“Sufficient stats approach” captures arbitrary heterogeneity conditional on \( z \) as long as well-behaved elasticities.

Yield same formula if can make the link between types and income distributions.

\[
\frac{T'(z_n)}{1 - T'(z_n)} = \left( 1 + \frac{1}{e(z_n)} \right) \left( \frac{\int_n^\infty (1 - g_m) dF(m)}{nf(n)} \right) \quad \text{(primitives)}
\]

\[
= \frac{1}{e(z_n)} \left( \frac{1 - H(z_n)}{z_n h(z_n)} \right) \cdot (1 - G(z_n)) \quad \text{(incomes)}
\]
NUMERICAL SIMULATIONS

$H(z)$ [and also $G(z)$] endogenous to $T(.)$. Calibration method (Saez Restud '01):

Specify utility function (e.g. constant elasticity):

$$u(c, z) = c - \frac{1}{1 + \frac{1}{\varepsilon}} \cdot \left( \frac{z}{n} \right)^{1 + \frac{1}{\varepsilon}}$$

Individual FOC $\Rightarrow z = n^{1+\varepsilon}(1 - T')^\varepsilon$

Calibrate the exogenous skill distribution $F(n)$ so that, using actual $T'(.)$, you recover empirical $H(z)$

Use Mirrlees '71 tax formula (expressed in terms of $F(n)$) to obtain the optimal tax rate schedule $T'$. 
NUMERICAL SIMULATIONS

\[
\frac{T'(z(n))}{1 - T'(z(n))} = \left(1 + \frac{1}{e}\right) \left(\frac{1}{nf(n)}\right) \int_{n}^{\infty} \left[1 - \frac{G'(u(m))}{\lambda}\right] f(m) \, dm,
\]

Iterative Fixed Point method: start with \( T'_0 \), compute \( z^0(n) \) using individual FOC, get \( T^0(0) \) using govt budget, compute \( u^0(n) \), get \( \lambda \) using \( \lambda = \int G'(u) f \), use formula to estimate \( T'_1 \), iterate till convergence.

Fast and effective method (Brewer-Saez-Shepard '10)
NUMERICAL SIMULATION RESULTS

\[ T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e(z)} \]

Take utility function with \( e \) constant

2) \( \alpha(z) = (zh(z))/(1 - H(z)) \) is inversely U-shaped empirically

3) \( 1 - G(z) \) increases with \( z \) from 0 to 1 \((\bar{g} = 0)\)

\( \Rightarrow \) Numerical optimal \( T'(z) \) is U-shaped with \( z \): reverse of the general results \( T' = 0 \) at top and bottom [Diamond AER'98 gives theoretical conditions to get U-shape]
FIGURE 5 – Optimal Tax Simulations

Source: Saez (2001), p. 224
Consider effect of small reform where marginal tax rates increased by $d\tau$ in $[z^*, z^* + dz^*]$.

What are the effects on tax receipts?

Mechanical effect net of welfare loss, $M$:

Every taxpayer with income $z$ above $z^*$ pays additional $d\tau dz^*$, valued at $(1 - g(z))d\tau dz^*$.

$$M = d\tau dz^* \int_{z^*}^{\infty} (1 - g(z))h(z)dz$$
BEHAVIORAL EFFECT PART 1: SUBSTITUTION

In $[z^*, z^* + dz^*]$, income changes by $dz$.

Marginal tax rate changes directly by $d\tau$, but also additionally indirectly by $dT'(z) = T''(z)dz$. Why? When is this not the case?

$$dz = -\varepsilon^c(z)z^* \frac{d\tau + dT'(z)}{1 - T'(z)} \Rightarrow dz = -\varepsilon^c(z)z^* \frac{d\tau}{1 - T'(z) + \varepsilon^c(z)z^* T''(z)}$$

Define the virtual density: density that would occur at $z$ if tax schedule replaced by linearized tax schedule. What is the linearized schedule $(\tau, R)$ such that income is $(1 - \tau)z + R$?

$$\frac{h^*(z)}{1 - T'(z)} = \frac{h(z)}{1 - T'(z) + \varepsilon^c(z)z^* T''(z)}$$
Overall elasticity/substitution effect is then:

\[ E = -\varepsilon(z)z^* \frac{T'(z)}{1 - T'(z)} h^*(z^*) d\tau dz^* \]

Can derive expression without taking into account endogenous (indirect) change in marginal tax rates if use the virtual density instead of true one.
INCOME EFFECT

Taxpayers with income above $z^*$ pay $-dR = d\tau dz^*$ additional taxes. Their change in income is:

$$dz = -\varepsilon(z)z \frac{T''(z)}{1 - T'} - \eta \frac{d\tau dz^*}{1 - T'(z)} \Rightarrow dz = -\eta \frac{d\tau dz^*}{1 - T'(z) + z\varepsilon(z) T''(z)}$$

Why?

Total income effect response:

$$I = d\tau dz^* \int_{z^*}^{\infty} -\eta(z) \frac{T'(z)}{1 - T'(z)} h^*(z) dz$$

At the optimum: $M + E + I = 0$. 
PUTTING THE EFFECTS TOGETHER

\[ \frac{T'(z)}{1 - T'(z)} = \frac{1}{\varepsilon_c(z)} \left( \frac{1 - H(z^*)}{z^* h(z^*)} \right) \]

\[ \times \left[ \int_{z^*}^{\infty} (1 - g(z)) \frac{h(z)}{1 - H(z^*)} \, dz + \int_{z^*}^{\infty} -\eta \frac{T'(z)}{1 - T'(z)} \frac{h^*(z)}{1 - H(z^*)} \, dz \right] \]


Change of variable from \( z \) to \( n \)?

Recall with a linear tax:

\[ \frac{\dot{z}_n}{z_n} = \frac{1 + \varepsilon_{(z_n)}}{n}. \]


\[ \frac{\dot{z}_n}{z_n} = \frac{1 + \varepsilon_{(z_n)}}{n} - \dot{z}_n \frac{T''(z_n)}{1 - T'(z_n)} \varepsilon_c(z(n)) \]
EXTENSIONS OF THE CORE INCOME TAXATION MODEL

1) Model includes only intensive earnings response. Extensive earnings responses [entrepreneurship decisions, migration decisions] ⇒ Formulas can be modified

2) Model does not include fiscal externalities: part of the response to $d\tau$ comes from income shifting which affects other taxes ⇒ Formulas can be modified

3) Model does not include classical externalities: (a) charitable contributions, (b) positive spillovers (trickle down) [top earners underpaid], (c) negative spillovers [top earners overpaid]

Classical general equilibrium effects on prices are NOT externalities and do not affect formulas [Diamond-Mirrlees AER ’71, Saez JpubE ’04]
EXTENSION 1: MIGRATION EFFECTS

Tax rates may affect migration (evidence on this next time).

Migration issues may be particularly important at the top end (brain drain).


Earnings $z$ are fixed, conditional on residence.

$P(c|z)$ is number of residents earning $z$ when disposable income is $c$, with $c = z - T(z)$.

Consider small tax reform $dT(z)$ for those earning $z$.

What is migration responding to? Marginal taxes?
ELASTICITY OF MIGRATION TO TAXES

Mechanical effect net of welfare is: \( M + W = (1 - g(z))P(c|z)dT \).

Why? Where is utility effect of changing country induced by taxes?

Migration responds to average taxes (or total taxes, since income fixed):

\[
\eta_m(z) = \frac{\partial P(c|z)}{\partial c} \frac{z - T(z)}{P(c|z)}
\]

Fiscal cost of raising taxes by \( dT(z) \) is: \( B = -\frac{T(z)}{z - T(z)} \cdot P(c|z) \cdot \eta_m \)

Optimal tax is where \( M + W + B = 0 \):

\[
\frac{T(z)}{z - T(z)} = \frac{1}{\eta_m(z)} \cdot (1 - g(z))
\]

What determines the elasticity \( \eta_m(z) \)?
MIGRATION EFFECTS IN THE STANDARD MODEL

$\eta_m(z)$ depends on size of jurisdiction: large for cities, zero worldwide $\Rightarrow$

1. Redistribution easier in large jurisdictions,
2. Tax coordination across countries increases ability to redistribute (big issue currently in EU),
3. visa system, cost of migration, ...

Top revenue maximizing tax rate formula (Brewer-Saez-Shepard '10):

$$\tau = \frac{1}{1 + a \cdot e + \bar{\eta}^m}$$

where $\bar{\eta}_m$ is the elasticity of top earners to disposable income.
EXTENSION 2: RENT SEEKING EFFECTS

Pay may not be equal to the marginal economic product for top income earners. Why? Overpaid or underpaid?


Actual output is \( y \), but individual only receives share \( \eta \) of actual output. To increase either productive effort or rent-seeking, effort is required.

\[
u^i(c, \eta, y) = c - h_i(y) - k_i(\eta)\]

Define bargained earnings: \( b = (\eta - 1)y \).

Average bargaining is \( E(b) \), extracted equally from everyone else (good assumption?) Means \( E(b) \) can be perfectly canceled by \( -T(0) \).
RENT SEEKING ELASTICITIES

Given tax, individual maximizes:

\[ u^i(c, y, \eta) = \eta \cdot y - T(\eta \cdot y) - h_i(y) - k_i(\eta) \]

What will \( y_i \) and \( \eta_i \) depend on?

Average reported income, productive income and bargained earnings in the top bracket:

\[ z(1 - \tau), \quad y(1 - \tau), \quad \eta(1 - \tau) \]

Total compensation elasticity \( e \): \( e = \frac{1 - \tau}{z} \frac{dz}{d(1 - \tau)} \) (what is it driven by?)

Real labor supply elasticity \( e_y \): \( e_y = \frac{1 - \tau}{y} \frac{dy}{d(1 - \tau)} \geq 0. \)

Thus the bargaining elasticity component \( e_b = \frac{db}{d(1 - \tau)} \frac{1 - \tau}{z} = s \cdot e \) with

\[ s = \frac{db}{dz} \frac{d(1 - \tau)}{d(1 - \tau)} \]

\( s \) and \( e_b \) positive if \( \eta > 1. \)
Suppose rent-seeking only at the top, \( E(b) = qb(1 - \tau) \) where \( q \) fraction of top earners.

Government maximizes tax revenues from top bracket earners:

\[
T = \tau[y(1 - \tau) + b(1 - \tau) - z^*]q - E(b)
\]

Why does \( E(b) \) enter?

\[
\tau^* = \frac{1 + a \cdot e_b}{1 + a \cdot e} = 1 - \frac{a(y/z)e_y}{1 + a \cdot e}
\]

How does \( \tau^* \) change with \( e, e_y, \) and \( e_b \)? When is \( \tau^* = 1 \) optimal?

Trickle up vs trickle down: what happens to \( \tau^* \) when top earners are overpaid? Underpaid?

How would you measure \( e_b \) (even \( b \) itself?)
REFERENCES (for lectures 2 and 3)


Laroque, G. “Indirect Taxation is Superfluous under Separability and Taste Homogeneity: A Simple Proof”, Economic Letters, Vol. 87, 2005, 141-144. (web)


