Two sets of alternatives for numerals

Main point: Modified (and especially superlative-modified) numerals pose a challenge to theories of numerals. The solution seems to require two sets of alternatives. While one of them seems to be the traditional Horn-set, it's not clear what the second one could be. We review two options and some challenges that come with them.

1 Preliminaries

* A numeral such as ‘three’ can be interpreted as ‘at least three’ or as ‘exactly three’.

(1) Examples that favor an ‘at least’ reading
   a. If you take three courses, you will pass. → If you take four courses, you will pass.
   b. Mary read three papers, if not more/#fewer.

(2) Examples that favor an ‘exactly’ reading
   a. Neither of them read three papers—John read two and Mary read four.
   b. (Cf.: Neither read many of the papers—# John read some and Mary read all.)

2 Horn-style alternatives: {1, 2, 3, …}

* Horn-style alternatives are a basic ingredient of ‘at least’ theories of numerals, which use them to derive the ‘exactly’ meaning as a scalar implicature:

(3) Mary read 3 papers.
   a. Semantic meaning: Mary read at least 3 papers.
   b. Alternatives: Mary read n papers.
   c. Stronger alternatives: Mary read 4 papers, Mary read 5 papers, …
   d. Scalar implicature: Mary read at least 3 papers and it is not the case that Mary read 4/5/… papers. ≡ Mary read exactly 3 papers. ✓
The same reasoning, however, leads to wrong predictions for modified numerals:

(4) Mary read more than 3 papers.
   a. **Semantic meaning:** Mary read more than 3 papers.
   b. **Alternatives:** Mary read more than \( n \) papers.
   c. **Stronger alternatives:** Mary read more than 4 papers, Mary read more than 5 papers, ...
   d. **Scalar implicature:** Mary read more than 3 papers and it is not the case that Mary read more than 4/5... papers. \( \equiv \) Mary read exactly 4 papers. \( \times \)

(5) Mary read at least 3 papers.
   a. **Semantic meaning:** Mary read at least 3 papers.
   b. **Alternatives:** Mary read at least \( n \) papers.
   c. **Stronger alternatives:** Mary read at least 4 papers, Mary read at least 5 papers, ...
   d. **Scalar implicature:** Mary read at least 3 papers and it is not the case that Mary read at least 4/5... papers. \( \equiv \) Mary read exactly 3 papers. \( \times \)

* Also, this reasoning is insufficient as it doesn't explain the following ignorance implicatures associated with superlative modifiers:

(6) a. This plane has 6 emergency exits. \( \Rightarrow \) Speaker knows how many there are.
   b. This plane has more than 6 emergency exits. \( \Rightarrow \) Speaker knows how many there are.
   c. This plane has at least 6 emergency exits. \( \Rightarrow \) Speaker doesn't know how many there are.

3 **Kennedy-style alternatives:** \{max = 3, max < 3,...\}

* Kennedy (2014) has a different proposal. Will it fare any better?

(7)
   a. (Normal) \[ \text{[three]} = \lambda P_{(d,t)} . \max\{n | P(n)\} = 3 \]
   b. (Typeshifted\(^1\)) \[ \text{[three]} = 3 \]
   c. (Comparative-modified) \[ \text{[more than three]} = \lambda P_{(d,t)} . \max\{n | P(n)\} > 3 \]
      \[ \text{[less than three]} = \lambda P_{(d,t)} . \max\{n | P(n)\} < 3 \]
   d. (Superlative-modified) \[ \text{[at most three]} = \lambda P_{(d,t)} . \max\{n | P(n)\} \leq 3 \]
      \[ \text{[at least three]} = \lambda P_{(d,t)} . \max\{n | P(n)\} \geq 3 \]

\(^1\)Using Partee (1987)'s BE and iota operations:

- \([\text{BE}] = \lambda Q_{(a,t)} . \lambda x_a . Q(\lambda y_a . y = x)\)
- \([\text{iota}] = \lambda P_{(a,t)} . tx_x[P(x)]\)
Kennedy defines bare and modified numerals in a way that eschews the wrong predictions in (4) and (5). In his theory, the modified numeral (just like the bare numeral) does not activate any alternatives, so no spurious alternatives are yielded:

(8) \[ \text{Mary read more than 3 papers} \]
\[ = \max\{n \mid \exists x[papers(x) \wedge \text{read}(x)(Mary) \wedge \#(x) = n]\} > 3 \checkmark \]

(9) \[ \text{Mary read at least 3 papers} \]
\[ = \max\{n \mid \exists x[papers(x) \wedge \text{read}(x)(Mary) \wedge \#(x) = n]\} \geq 3 \checkmark \]

Kennedy’s bare numerals have an ‘exact’ meaning by default, so he doesn’t need Horn-style alternatives to derive it. He argues, in fact, that Horn-style alternatives are not needed; rather, numerals have a different set of alternatives, namely, their modified variants. He uses this assumption to account for the observed ignorance implicatures of superlative-modified numerals.

(10) **Unmodified**: This airplane has six emergency exits.
   a. **Alternatives**: max > 6, max < 6, max ≥ 6, max ≤ 6
   b. The alternatives do not entail the utterance.
   c. No primary (ignorance) implicatures are derived.

(11) **Comparative-modified**: This airplane has more/less than six emergency exits.
   a. **Alternatives**: max = 6, max ≥ 6, max ≤ 6
   b. The alternatives do not entail the utterance.
   c. No primary (ignorance) implicatures are derived.

(12) **Superlative-modified, at least**: This airplane has at least six emergency exits.
   a. **Alternatives**: max = 6, max > 6, max < 6, max ≤ 6
   b. (max = 6) → (max ≥ 6),
      (max > 6) → (max ≥ 6)
   c. **Primary (ignorance) implicatures**: \{¬K(max = 6), ¬K(max > 6)\}

(13) **Superlative-modified, at most**: This airplane has at most six emergency exits.
   a. **Alternatives**: max = 6, max > 6, max < 6, max ≥ 6
   b. (max = 6) → (max ≤ 6),
      (max < 6) → (max ≤ 6)
   c. **Primary (ignorance) implicatures**: \{¬K(max = 6), ¬K(max < 6)\}

**Problem**: In this theory, superlative modifiers can only take the singular term (typeshifted) meaning of numerals as an argument, but superlative modifiers can combine with other kinds of things:
(14)  a. John saw at least Mary.  (DP)
b. Mary was at least satisfied.  (Adj)
c. He at most spanked the kid.  (VP)
d. At least some determiners aren’t determiners.  (Det)

(Krifka 1999)

⋆ To fix that, Kennedy might require Horn-alternatives also.

4 Kennedy- & Horn-style alternatives: \{max = 3, max < 3, \ldots\} \& \{1, 2, 3, \ldots\}

⋆ Kennedy’s meaning for superlative modifiers wasn’t flexible enough. Krifka (1999)’s might be:

(15)  a. \[[at least \alpha] F_K = \bigcup\{P | \langle P, \lambda F_K \rangle \in \alpha F_K A\}\]
b. \[[at most \alpha] F_K = \bigcup\{P | \langle \lambda F_K, P \rangle \in \alpha F_K A\}\]

⋆ This meaning crucially requires a ranking relation between \alpha and its alternatives. Krifka suggests that this ranking relation could be semantic, but also pragmatic. For example, it could be a pragmatic part relation on individuals:

(16)  At least John and Mary left.

a. Stronger alternatives: [John, Mary, and Sue], [John, Mary, Sue, and Bill], \ldots

b. Meaning constructed by at least: \lambda x. [people(x) \land left(x) \land John and Mary \leq_i x]

⋆ Back to numerals. Krifka’s meaning for numerals can only apply to arguments of a conjoinable type, so let’s assume that the numerals that combine with superlative modifiers are the non-typeshifted version, type \langle d t, t \rangle.

⋆ Krifka’s superlative modifiers perform the join operation over alternatives which stand in a particular ordering relation with respect to the assertion. Let’s assume this ordering refers to the value of max from Kennedy’s story:

(17)  \[[at most three_F] = \bigcup\{P | \langle P, \lambda F_{\langle d, t \rangle} \rangle \in \lambda F_{\langle d, t \rangle} A\}

= \bigcup\{P | \langle P, \lambda D_{\langle d, t \rangle} \cdot \max\{n | D(n)\} = 3 \rangle \in \{(\lambda D_{\langle d, t \rangle} \cdot \max\{n | D(n)\} = m, \lambda D_{\langle d, t \rangle} \cdot \max\{n | D(n)\} = m | m \geq_N n\}\}

= \bigcup\{\lambda D_{\langle d, t \rangle} \cdot \max\{n | D(n)\} = m | m \leq N 3\}

= \lambda D_{\langle d, t \rangle} \cdot \max\{n | D(n)\} \leq N 3

⋆ Conclusion: In this hybrid account numerals type \langle d t, t \rangle have Kennedy-style alternatives but numerals type d have Horn-style-alternatives. Thus, despite Kennedy’s claim, it looks like a necessary patch to his theory might need Horn-style alternatives also. Moreover, since Kennedy’s typeshifted meaning for numerals, type d, was always available, that means that Horn-style alternatives and their associated scalar implicature reasoning would be in constant competition with Kennedy’s exact meaning for numerals.
5 Mayr- & Horn-style alternatives: \( \{\langle,\rangle, \{\leq, \geq\}\}\) & \(\{1, 2, 3, \ldots\}\)

* If Kennedy presented an alternative to ‘at least’ theories to address (4), (5), Mayr (2013) however shows that, with the assumption of a second set of alternatives (for the modifiers themselves), the grammatical theory of implicatures can handle those examples also:

\[(18) \text{ At least three boys left.} \]

   a. \(O [C \ [\text{at least three boys left.}]]\)
   b. \(Alt[\text{At least three boys left}] = \{\text{at least 3 boys left, at least 4 boys left, \ldots, at most 3 boys left, at most 4 boys left, \ldots}\}\)
   c. \(O\) can only negate those alternatives which are innocently excludable.\(^2\)
   d. Let’s see which (if any) of the alternatives are innocently excludable:
      At least three boys left and …
      (i) \(\ldots \neg \text{at least 3 boys left.} \) \(\checkmark\), contradicts the prejacent
      (ii) \(\ldots \neg \text{at least 4 boys left.} \) \(\checkmark\), entails \text{at most 3 boys left}
      (iii) \(\ldots \neg \text{at least 5 boys left.} \) \(\checkmark\), entails \text{at most 4 boys left}
      …
      (iv) \(\ldots \neg \text{at most 3 boys left.} \) \(\checkmark\), entails \text{at least 4 boys left.}
      (v) \(\ldots \neg \text{at most 4 boys left.} \) \(\checkmark\), entails \text{at least 5 boys left}
      …
      (vi) None of the alternatives are innocently excludable, so we don’t get the incorrect scalar implicatures in (5) any more. \(\checkmark\)

* It seems that just Horn-style alternatives aren’t enough either.

* Note: it’s interesting how Mayr combines sets of modifiers with sets of Horn-style alternatives for numerals.

6 Conclusion and outlook

* Two different theories of numerals, yet both seem to require two sets of alternatives. While both seem to need Horn-style alternatives, which is the second set going to be: Kennedy-style or Mayr style alternatives?

* It is not clear how Mayr’s story in its current form could match Kennedy’s success w.r.t the ignorance implicatures of superlative-modified numerals (but not of comparative-modified or un-modified numerals). (Remember, it was to derive those implicatures that Kennedy came up with what we called Kennedy-style implicatures.) Also, it remains to be verified if/to what extent his story for superlative modifiers is compatible with the focus-sensivity and syntactic versatility of the superlative modifier.

\(^2\)He uses a version of the exhaustivity operator which negates only those alternatives in the set of alternatives \(C\) whose negation does not automatically require the truth of some other alternative in \(C\). For example, given a sentence such as \(A\) or \(B\) left, two possible alternatives are \(A\) left and \(B\) left. However, neither of these alternatives is innocently excludable because negating either one of them while asserting the prejacent would entail the other one: \(A\) or \(B\) left and \(it\ is\ not\ the\ case\ that\ A\ left\ entails\ B\ left.\)
On the other, Kennedy’s story also fails in a place where Mayr’s succeeds, that is, in deriving the following scalar implicature:

(19) Jack is required to read at least three books. \(\leadsto\) Jack is not required to read at least four books.

Kennedy’s story for this would correctly yield ignorance implicatures as well as one secondary implicatures, but he has no way to derive the implicature in (19). That is because he doesn’t have any mechanism to exploit the alternatives of the numeral separately from those of its modifier.

(20) Jack is required to read at least three books.
   a. \(\max\{n \mid [\exists x[\text{reg}(x)(\text{you}) \land \text{classes}(x) \land \#(x) = n]] \geq 3\}\)
   b. \(\Box[\max\{n \mid [\exists x[\text{reg}(x)(\text{you}) \land \text{classes}(x) \land \#(x) = n]] \geq 3\}\]

Mayr, on the other hand, shows that (19) has innocently excludable alternatives. Their negation would yield the desired implicature:

(21) Jack is required to read at least three books.
   a. \(\text{Alt}[\text{Jack is required to read at least three books}] = \{\text{Jack is required to read at least 3 books, Jack is required to read at least 4 books, \ldots, Jack is required to read at most 3 books, Jack is required to read at most 4 books, \ldots}\}\)
   b. Jack is required to read at least three books and \ldots
      (i) \(\neg\text{Jack is required to read at least 3 books. \(\times\), contradicts the prejacent}\)
      (ii) \(\neg\text{Jack is required to read at least 4 books. \(\checkmark\), implicature in (19)!}\)
      (Same for all numbers greater than 3.)

Conclusion: It’s not clear that we should pick either Kennedy- or Mayr-style alternatives. More generally, we need to rethink our notion of what Horn-sets can be like. Kennedy-style alternatives formed an only partially ordered set. Mayr-style alternatives formed non-monotonic sets. Quite different from the traditional notion of Horn-sets as sets ordered by strength.

References