Capital Account Controls and the Structure of the Banking Sector in a Small Open Economy

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Abstract:

This paper studies the welfare implications of different financial sector structures in small open economies (SOEs). There is a growing literature emphasizing the importance of bank net worth (borrowing) constraints and/or imperfect banking sector competition in matching empirical facts. However, little is known how the various sources of inefficiencies interact. I build a model of a SOE with a monopolistically competitive and concentrated financial sector, which faces a net worth constraint. There are two standard sources of inefficiency — pecuniary externalities, which lead to overinvestment, and a standard monopolistic underinvestment force. The optimal policy instruments include subsidies in certain periods and capital account controls in others, which is a good proxy for the behavior of emerging markets. For every country, there exists a financial sector with a particular banking sector concentration, for which the inefficiencies offset each other and no government intervention is required in some periods. Furthermore, this paper documents a novel theoretical result – the interaction between future binding bank net worth constraints and dynamic (future) underinvestment could lead to ex-ante overinvestment even in economies with a single monopolistic bank where there are no pecuniary externalities. This result implies that less competitive banking sectors — where banks have more to lose in the case of bankruptcy — need not be less prone to financial crises, as argued by the franchise value literature.

1 Introduction

Models that explicitly introduce a banking sector, facing some type of a borrowing or a bank net worth constraint, have gained in popularity in the new macro literature (for example, [Meh and Moran, 2010], [Gertler and Kiyotaki, 2010], [Gertler et al., 2012], [Adrian and Boyarchenko, 2012], [Maggiori, 2013], [Brunnermeier and Sannikov, 2013]). At the same time, many papers have emerged that include imperfectly competitive banking

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sectors (for example, [Gerali et al., 2010], [Hafstead and Smith, 2012], [Andres and Arce, 2012]). After the financial crises, it has become apparent that, in order to match the empirical evidence, it is of first order importance to introduce the financial sector into standard macroeconomic models in a realistic way. It is equally important to understand what are the inefficiencies that might arise due to the specific structure of the financial sector and whether policy makers can intervene in order to improve aggregate welfare. This paper attempts to address the latter.

While economists understand well by now how the presence of future binding net worth constraints/borrowing constraints can generate pecuniary externalities and how an imperfectly competitive banking sector would lead to underinvestment, little is known how those two sources of inefficiency interact. What does this environment imply for the optimality of existing policies such as capital account controls and subsidies on the borrowing rates of firms? These are the questions that I study in this paper.

I build a finite period model of a small open economy (SOE) with a banking sector, which is monopolistically competitive and concentrated, and faces a net worth borrowing constraint. Bankers borrow from foreigners at the risk free interest rate and, in turn, lend to domestic entrepreneurs/firms using a standard debt contract (SDC). Entrepreneurs are risk neutral and are the only consumers in the economy. The implicit assumption is that domestic firms can borrow only through the domestic banking sector due to its superior monitoring technology. Entrepreneurs have an access to a concave production technology and they default on their loans with some probability, at which point the bankers seize the assets of the firm. Every period and state of nature, bankers face a net worth borrowing constraint which forces them to finance at least a fraction of the loans they provide using their own equity. There is no bank default in this economy.

First, one can compare the behavior of more and less competitive banking sectors by varying the degree of substitution of loans, for a given number of banks. There are two forces in play. On the one hand, a less competitive banking sector wants to underinvest relative to a more competitive one due to a standard monopolistic effect. However, there is also an overinvestment force which is a novel theoretical result of this paper. The overinvestment force emerges from the interaction between the desire of a less competitive banking sector to underinvest in the future and the presence of a binding net worth constraint in the crisis state in the future. The intuition is the following. Due to its desire to underinvest in the future, a less competitive banking sector does not value an extra dollar of net worth in the crisis state, when the net worth constraint binds, as much as a more competitive banking sector. As a result, it does not perceive an extra dollar invested ex-ante, which depletes the net worth of the banking sector in the crisis state, to be as costly. If this overinvestment force dominates the ex-ante standard underinvestment force, a less competitive banking sector might end up
overinvesting relative to a more competitive one. This result is in contrast to the franchise value literature, which argues that it is always welfare improving to have a regulation that restricts the competitiveness of the banking sector, in addition to imposing a minimum bank capital requirement (see [Keeley, 1990], [Hellmann et al., 2000] and the Literature Review section of this paper).

Furthermore, one can compare the decentralized allocation to the constrained Central Planner’s allocation. There are two standard sources of inefficiency — pecuniary externalities and a monopolistic underinvestment force. The pecuniary externalities, which lead to overinvestment relative to the constrained Central Planner’s allocation, work through two different channels. First, bankers do not fully internalize the fact that the more they lend in period zero, the lower the marginal rate of return of the other bankers is, when the firm defaults. I call this channel a "bankruptcy" pecuniary externality and it is present even if the net worth constraint does not bind in the future. In addition, if the net worth constraint is binding in the crisis state in the future, the banker also does not internalize the fact that by lending more ex-ante, he decreases the return and the net worth of the other bankers during a crisis, which tightens the net worth constraints of the other bankers even further. I call this channel a "net worth constraint" pecuniary externality.

If the banking sector is imperfectly competitive, there is a standard monopolistic underinvestment force both in the current period and in future periods. The presence of monopolistic competition and binding net worth constraints in the crisis state in the future generate a third, novel, source of inefficiency and overinvestment in this model, which is separate from the pecuniary externalities. Under certain parametrization, even a single bank might end up overinvesting relative to the constrained Central Planner. The intuition is the following. The monopolistic bank wants to underinvest in the crisis state in the future and, as a result, does not value the marginal dollar of net worth in the crisis state as much as the Central Planner does. Therefore, the monopolistic bank is tempted to overinvest ex-ante. Whether the monopolistic bank will under-or-overinvest depends on how the standard current period underinvestment force compares to this novel overinvestment force.

Having understood the interaction between the different inefficiencies, I proceed to study how one can decentralize the constrained Central Planner’s allocation. I consider two instruments that have been used by a number of emerging economies — capital account controls on inflows in the form of a tax on the borrowing interest rate from foreigners and subsidies on firm borrowing interest rates. In addition, I allow the policy maker to provide a lump sum transfer/tax to firms in order to balance his budget and I also assume commitment on behalf of the policy maker. There is an infinite number of ways to decentralize the constrained Central Planner’s allocation using these instruments. Imposing the assumption that the policy maker uses only subsidies or capital account controls in every period and state of
nature, I consider one way to decentralize the equilibrium. Whether the policy maker utilizes subsidies or capital account controls depends crucially on the presence of uncertainty. The overinvestment forces are potentially present only if there is future uncertainty. Therefore, there might be a role for capital account controls only for economies which can end up in a crisis state in the future with a positive probability. If there is no future uncertainty and the economy is in a steady state, only the underinvestment force is present.2

One of the results that emerges is that there exists a country with an optimal number of banks (as a function of the degree of loan substitution). In that economy, the overinvestment force due to pecuniary externalities completely offsets the monopolistic underinvestment force. Therefore, it is possible that no ex-ante policy intervention is required, despite the presence of ex-ante inefficiencies. However, if the pecuniary externalities are the dominant force ex-ante, a capital account tax will be required to implement the constrained Central Planner’s allocation. This tax will be larger, the stronger the degree of substitution between loans is and the more banks the country has. If the underinvestment force dominates, the country should optimally impose a subsidy, which will be smaller, the larger the number of banks is and the higher the degree of substitution between loans is. Finally, it is interesting to note that even if a country has a single bank, which overinvests relative to the constrained Central Planner, due to the novel overinvestment force documented in this paper, one way to decentralize the constrained Central Planner’s allocation is by using subsidies in the current period and in the future, rather than by using capital account controls.3

Literature Review

There is substantial evidence that the banking sector is imperfectly competitive (due to regulation, market segmentation, product differentiation). [Claessens and Laeven, 2004], [Bikker and Spierdijk, 2008] and [Claessens, 2009] estimate the degree of imperfect competition of the financial sector using [Panzar and Rosse, 1987]’s methodology and show that it varies significantly across countries.4 Another important parameter in the model is banking sector concentration, which could be very different from the degree of monopolistic competition of the banking sector, given that in certain cases the threat of entry is sufficient to generate a fairly competitive banking industry. In fact, the correlation between banking sector concentration and the degree of banking sector competition appears to be slightly

2I consider an implementation where the policy maker always utilizes subsidies if the banking sector is imperfectly competitive and there is no future uncertainty (even if the net worth constraint is binding for both the Central Planner and the banker in the decentralized equilibrium and no policy instrument is required).

3It is important to note that these results represent only one way, among many, to implement the constrained Central Planner’s allocation.

4The measure captures to what extent the increase in input prices affects the marginal cost and total revenue of a given bank.
positive in the data (see Graph 1 in the Appendix). Therefore, it is important to have a model that can distinguish between the concentration of the banking sector and the degree of imperfect competition of the financial sector.

Graph 2 in the Appendix shows the banking sector concentration for a number of countries, which is measured as the assets held by the largest 3 banks as a fraction of total assets in the country. While there is significant heterogeneity across countries, banking sectors appear to be very concentrated. The assumption that most of the foreign borrowing is channeled through the banking sector also finds support in the data. In Graph 3 in the Appendix, which shows bank foreign debt as a fraction of total private foreign debt (excluding inter-firm borrowing), one can see that for many countries at least 50% of the private foreign borrowing is bank borrowing. For example, this is the case for Brazil, Hong Kong, Korea but also for many developed economies such as Greece, Austria, Germany, Sweden. Therefore, introducing formally the financial sector is important in order to understand how foreign inflows affect domestic investment and whether there is over or underinvestment relative to the constrained Central Planner’s allocation.

The types of policy instruments used by regulators to ensure financial sector stability are very different. They range from minimum bank capital requirements and capital account controls to directly regulating the competitiveness of the banking sector and subsidies for final borrowers. In this paper I focus on capital account controls in the form of a tax on foreign borrowing rates and also on subsidies on firm borrowing rates. More specifically, capital account controls that resemble a tax on foreign borrowing rates ("price" based capital account controls) have been implemented by Chile — from 1991 to 1998, Colombia — from 2007 to 2008, Thailand — from 2006 to 2008, Russia — from 2004 to 2006 and Brazil starting 2008, among others (see [Ostry et al., 2011]).

Government subsidies are ubiquitous and are often distributed via subsidized bank lending rates as modelled in this paper. For example, Brazil provides subsidized interest rates for corporate loans via the Brazil’s National Development Bank (BNDES). The subsidized credit amounts to about "27 percent of all productive credit". The subsidized rate at which the government lends to the BNDES is significantly lower than the nominal interest rate on government bonds. In 2008-2010, the former was 6 percent while the latter was around 12 percent. (for details and other sources see [Antunes et al., 2011]) The United States has the United States Small Business Administration (SBA) loan subsidy program, which guarantees loans to small businesses and provides an interest rate subsidy. For example, during

\[\text{\textcopyright 2011 The instruments used are either a direct tax or unremunerated reserve requirements (URRs), which work exactly like a tax.}\]

\[\text{\textcopyright 2011 \"Interest rates are negotiated between the borrower and the lender but are subject to SBA maximums, which are pegged to the prime rate, the LIBOR rate, or an optional peg rate." For details see <http://www.sba.gov/content/7a-terms-conditions>\]
the 2011 fiscal year, the subsidized new loans provided by the SBA amounted to 19.6 billion dollars (see [Dilger, 2011]). South Korea is another country that has relied on government subsidies on lending rates to enterprises during its industrialization period in the 70s and the 80s (see [Lee, 1996]).

There are a number of recent theoretical papers that explore how introducing an imperfectly competitive banking sector in standard macroeconomic models leads to a better data fit (see [Gerali et al., 2010], [Hafstead and Smith, 2012], [Andres and Arce, 2012]). There is also an older banking literature, which emphasizes imperfect banking sector competition (see [Klein, 1971] and [Freixas and Rochet, 2008] for a literature review). In the optimal bank regulation literature, [Hellmann et al., 2000] argue that it is optimal to restrict the competitiveness of the banking sector, in addition to imposing minimum bank capital requirements. It prevents banks from overinvesting and risk shifting since they want to prevent default in order to preserve their franchise value. [Keeley, 1990] tests the franchise value hypothesis. The natural experiment that he examines is the financial sector liberalization in the US in the mid 1960s and he confirms the link between competitiveness and higher bank default risk. The franchise value literature encouraged policy makers of many countries to introduce reforms that decreased the degree of banking sector competition (see [Boyd and DeNicolo, 2005]).

However, the more recent theoretical and empirical literature disputes the franchise value policy recommendations. For example, [Boyd and DeNicolo, 2005] show that higher banking sector concentration could potentially lead to more default by increasing risk shifting by the final firm. Weaker financial sector competition implies higher firm borrowing rates and increased incentive of firms to maximize the upside and, hence, to risk shift. This channel was missing in [Hellmann et al., 2000] since they considered a portfolio problem and did not model bankers and firms separately. On the empirical side, [Boyd and DeNicolo, 2005] show that there is no clear cut relationship between banking sector competition and financial sector stability. Using a Panzar and Rosse H-statistic measure of banking sector competition, which is a more precise measure than simply the banking sector concentration of a country, in a sample of 45 countries, [Schaeck et al., 2009] also find the opposite result — that more competitive banking sectors are less prone to systemic banking crises.

My paper contributes to the class of papers that introduces a new theoretical channel that pushes in the opposite direction of the key policy recommendation of the franchise value literature. Namely, I argue that the interaction between dynamic underinvestment and binding future net worth constraints imply that, for certain countries and states of nature, a more concentrated banking sector might end up overinvesting relative to a less concentrated one, leading to a larger loss of bank net worth in a future crisis and a more severe credit crunch.

Another strand of literature closely related to this paper are models where the financial
sector faces a net worth constraint or some type of borrowing constraint. One of the key assumptions of the model developed in this paper is that the financial sector faces a net worth constraint, which the Central Planner also has to take as given. The justification why such a net worth constraint will emerge in equilibrium is similar to [Holmstrom and Tirole, 1997]'s "skin in the game" argument where in order to mitigate potential moral hazard, lenders require that entrepreneurs invest their own net worth. There is a growing literature which models financial institutions as facing value at risk (VaR) constraints, which are constraints internally used by financial institutions. For example, [Adrian and Shin, 2011] microfound the VaR constraint as a way for lenders to place a limit on the leverage of a bank in order to prevent risk shifting, which becomes equivalent to preserving a fixed probability of default. [Adrian and Boyarchenko, 2012] model the VaR by assuming that bankers have to retain enough equity so that a certain fraction of losses is covered. Other papers that model the financial sector as facing VaR constraints are [Danielsson et al., 2011] and [Brunnermeier and Pedersen, 2009].

Another class of papers impose the constraint that the net present value or the net worth of the financiers has to exceed a certain value. For example, [Brunnermeier and Sannikov, 2013] impose the constraint that the banker’s net worth cannot exceed zero. [Gertler et al., 2012] and [Gertler and Kiyotaki, 2010] rely on a moral hazard story where the bankers have an access to a less efficient technology and, as a result, in order to prevent them from using it, lenders limit the amount of loans up to the point where the net present value the bankers receive has to be greater than or equal to what they would get if they use the less productive technology. [Maggiord, 2013] assumes that the net present value of the bank has to be greater than or equal to zero.

Even though in this paper I assume that the financiers face a net worth constraint following the literature described above, all of the analysis could have been done with a borrowing constraint in the spirit of [Kiyotaki and Moore, 1997]. The contribution of my paper to both the imperfect competition literature and the bank net worth constraint/borrowing constraint literature is to combine the two and examine the welfare implications that emerge.

Finally, my paper builds on the literature that justifies the imposition of capital account controls using pecuniary externalities. International finance papers which study pecuniary externalities include [Bianchi, 2011], [Caballero and Krishnamurthy, 2001], [Korinek, 2010], [Nikolov, 2011] and [Bianchi and Mendoza, 2012], among others. They build on the closed economy macroeconomic models with pecuniary externalities that lead to overinvestment pioneered by [Geanakoplos and Polemarchakis, 1986] and [Arnott et al., 1994]. The majority of the open economy models with pecuniary externalities do not explicitly introduce a banking sector and, as we saw in the data, a large fraction of the foreign borrowing is through the banking sector. Also in those models the borrowers are infinitesimally small while in the
data the banking sectors of all countries are very concentrated. The number of banks will affect the strength of the pecuniary externality and the degree of monopolistic competition. Therefore, it is an important parameter to consider.

2 Model

There is a single country which is a small open economy (SOE). There are three periods, \( t = 0, 1, 2 \) and there is uncertainty only in the middle period, \( t = 1 \). In \( t = 1 \), the economy can be in the high state (a boom) with a probability \( \pi_H \) and in the low state (a recession) with a probability \( \pi_L = 1 - \pi_H \). The shock is an aggregate shock. There are four types of agents — entrepreneurs/borrowers, bankers, foreign lenders and a policy maker. There are two goods — a capital good and a consumption good where the price of consumption is the numeraire good and is set equal to one.

There is a continuum of risk neutral entrepreneurs distributed uniformly on \([0, 1]\). They borrow from the banks using a short term standard debt contract (SDC). In some states of nature in \( t = 1 \), the entrepreneurs can default upon which their assets will be seized by the bankers. The entrepreneurs are the equity owners of the banks, but I assume that they cannot borrow from the banks they own equity in.\(^7\) Entrepreneurs have limited liability. If they default, they will be still allowed to borrow again and produce (no exclusion from debt markets for any amount of time).\(^8\) Entrepreneurs invest in \( t = 0, 1 \) and produce with a lag. The loans are imperfect substitutes and entrepreneurs can transform the consumption good into the capital good one-for-one. For simplicity, I assume that the entrepreneurs consume all the profits every period after producing and that the capital good depreciates one hundred percent after producing. These assumptions are not crucial for the final results but allow me to derive analytical results.

The banking sector is monopolistically competitive where the number of banks is finite and equal to \( n \). In the environment that I consider there will be no default by the banking sector. Banks maximize the dividends they pay to entrepreneurs. Bankers also face a net worth constraint, which limits the amount they can borrow from foreigners. I also assume that there is an infinite number of risk neutral foreign lenders that can lend money only

\(^7\)One can also think of this set up as having a representative family that splits into two agents in the beginning of period \( t = 0 \) (a banker and an entrepreneur who runs the firm) and they get back together at the end of period \( t = 2 \) and consume (see [Gertler and Kiyotaki, 2010]). Alternatively, I could have modelled bankers and entrepreneurs as agents that consume independently and the Central Planner places an exogenous weight on each agent. The intuition behind all the results will remain unchanged.

\(^8\)This assumption is not crucial and one can re-write the problem by modelling the entrepreneurs as overlapping generations instead or one can interpret the set up as if there is an entry of new entrepreneurs in case of aggregate default.
to the domestic banks and cannot directly lend to entrepreneurs. \(^9\) In expectation, foreign lenders receive the world risk free interest rate, \(R^f_t\).

### 2.1 The Problem of the Representative Entrepreneur

First, let us consider the problem of the entrepreneur. Every period the entrepreneur consumes, borrows from bankers and invests where the contract between the banker and the entrepreneur is a standard debt contract (SDC). In period \(t\), the banker offers a lending rate, \(R^l_t\), and the entrepreneur takes it as given and chooses how much to borrow, \(L_t\). The representative entrepreneur maximizes his utility given by

\[
\max_{C_t, K_t, L_t} \sum_{t=0}^{2} C_t
\]

subject to the period \(t\) budget constraint

\[
A_t K_t^{\alpha} - R^l_t L_{t-1} + L_t + D_t \geq C_t + K_t
\]

where the discount rate between periods is equal to one and \(C_t\) is the consumption of the entrepreneur. The entrepreneur has an access to a decreasing returns to scale production technology where period \(t\) output is given by \(A_t K_t^{\alpha}\). \(A_1 = A_L\) if the low state is realized and \(A_1 = A_H\) if the high state is realized where \(A_H > A_L\). \(A_2 = \bar{A}\) and is the same in both states of nature. \(K_t\) is period \(t\) investment which produces in \(t + 1\), \(D_t\) are the dividends paid by the banks which the entrepreneur takes as given. \(L_t\) is the amount of new aggregate loans, \(R^l_t L_{t-1}\) is the payment made on loans taken in \(t - 1\). The ex-post return on period \(t - 1\) loans is equal to \(R^l_t = \min\left\{ \frac{A_t K_t^{\alpha}}{L_{t-1}}, R_{t-1} \right\}\) where \(\hat{L}_{t-1} = \sum_{i=1}^{n} \frac{1}{n} L_{i,t-1}\) is a simple average. \(R^l_t = \frac{A_t K_t^{\alpha}}{L_{t-1}}\) is the ex-post return if the firm defaults\(^{10}\) and \(R^l_t = \hat{R}_{t-1}\) is the ex-post return if the firm does not default. I assume that in every period, the profits of the firm (if any) and also the dividends received from the banks are consumed, \(C_t = A_t K_t^{\alpha} - R^l_t L_{t-1} + D_t\), which implies \(L_t = K_t\). This assumption is made only as a simplification in order to derive intuitive

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\(^9\)One can endogenize this assumption by assuming that domestic banks have a better information regarding domestic investment projects relative to foreign investors or they have a better monitoring technology, for example.

\(^{10}\)The implicit assumption is that if the firm defaults on one bank, it defaults on all of them and that all banks receive a share of the output equal to the share of their loans out of total loans (since all the loans are equally productive).

Note that I assume that if the firm defaults, the bankers can seize only the output of the firms and not the dividends paid. (However, this assumption is irrelevant since dividends will be optimally paid only in the last period and, at that point, there will be no default since all the uncertainty is resolved in the middle period.)
analytical results. Assume that loans from different banks are imperfect substitutes which allows us to model the banking sector as monopolistically competitive. The CES aggregator over loans is given by

\[ L_t = \left[ \sum_{i=1}^{n} \frac{1}{n} (L_{i,t})^{\frac{\rho-1}{\rho}} \right]^\frac{\rho}{\rho-1} \]  

(3)

where \( L_{i,t} \) is the amount of loans the entrepreneur takes from bank \( i \) and \( \rho \in (1, \infty) \) is the elasticity of substitution between loans. If \( \rho \to \infty \), all the loans are perfect substitutes and if \( \rho \to 1 \) the functional form approaches Cobb Douglas which has an elasticity of substitution of one. The total expenditure on loans for each entrepreneur, if there is no default, is given by

\[ \bar{R}_t L_t = \sum_{i=1}^{n} \frac{1}{n} \bar{R}_{i,t} L_{i,t} \]  

(4)

The assumption that the entrepreneur consumes all the profits every period after producing and that capital depreciates one hundred percent makes the problem of the entrepreneur static. However, since I assume that in \( t = 1 \) there is no longer uncertainty and in \( t = 0 \) there is uncertainty I will write the \( t = 0 \) and the \( t = 1 \) problems separately to define notation and highlight the difference in the "uncertainty" versus the "no uncertainty" case.

I solve the model backwards. In \( t = 1 \), the uncertainty is resolved and since there will be no default in \( t = 1 \), then \( R_{2,t} = \bar{R}_1 \) and the entrepreneur maximizes

\[ \max_{L_{i,1}} \left[ \bar{A} (L_{1} (s_1))^{\alpha} - \bar{R}_{1} (s_1) L_{1} (s_1) + D_2 (s_1) \right] + A_1 L_0^{\alpha} - \bar{R}_{1} (s_1) L_0 + D_1 (s_1) \]

subject to equations 3 and 4. All the variables are a function of the period one state, \( s_1 \), which can be either high or low. Also \( \bar{A} \) is the period two TFP shock which is known in \( t = 1 \) and for simplicity \( \bar{A} \) is the same after either state of nature. One can think of \( \bar{A} \) as a steady state productivity. Section A.2 in the Appendix presents the details of the optimization problem. The first order condition with respect to \( L_{i,1} \) gives the standard demand for loans equations. The demand for aggregate loans is determined by banks equating the marginal cost of loans to the marginal benefit of loans (which here is simply the marginal benefit of capital since \( K_t = L_t \)).

\[ \alpha \bar{A} (L_{1})^{\alpha-1} = \bar{R}_{1} \]  

(5)

The first order condition with respect to \( L_{i,1} \) can be re-written as

\[ \frac{1}{\alpha} \bar{A} (L_{1})^{\alpha-1} = \bar{R}_{i,1} \]

\[ \bar{A} (L_{1})^{\alpha-1} = \bar{R}_{i,1} \]

11 Of course, the larger the net worth of the entrepreneur is, which he can reinvest, the lower the welfare loss is due to the fact that the banking sector is imperfectly competitive. However, as long as there is some borrowing in equilibrium, the qualitative results presented here remain.
\[ L_{i,1} = L_1 \left[ \frac{\tilde{R}_{i,1}}{\tilde{R}_1} \right]^{-\rho} \]

which determines the demand for bank specific loans and is standard in models with monopolistic competition. The interest rate on aggregate loans is given by

\[ \tilde{R}_1 = \left[ \sum_{i=1}^{n} \frac{1}{n} \left( \tilde{R}_{i,1} \right)^{(1-\rho)} \right]^{\frac{1}{1-\rho}} \]

Given that the problem is static, the period zero problem of the entrepreneur is given by

\[
\max_{L_{i,0}} \pi_H \left[ A_H (L_0)^{\alpha} - \tilde{R}_0 L_0 \right] + \pi_H D_1 (s_H) + \pi_L D_1 (s_L) + D_0
\]

where I assume for simplicity that the entrepreneur has no net worth in period zero and \( K_{-1} = 0 \). The model will be parametrized in such a way that there is always firm default in the low state. As a result, due to limited liability, entrepreneurs maximize only their profits in the non-default state since they lose their output in the case of default. Entrepreneurs will take period zero and period one dividend payments as given, which, in equilibrium, will be equal to zero. The first order conditions are very similar to the ones in period \( t = 1 \), with the only difference being that the aggregate demand for loans is given by \( \alpha A_H (L_0)^{\alpha-1} = \tilde{R}_0 \).

The equations for \( L_{i,0} \) and \( \tilde{R}_0 \) are identical but for the time subscript.

What is important to note is that the demand schedule for loans will be downward sloping and the imperfectly competitive banking sector will internalize that. This result, combined with the fact that if there is no default by the firm bankers can seize only a fraction of the output of the firm, will generate the standard underinvestment channel due to imperfect competition.\(^{12}\)

## 2.2 Banker \( i \)'s Optimization Problem

In this section I solve the optimization problem of banker \( i \). There are \( n \) banks and they provide loans that are imperfect substitutes. The banking sector is monopolistically competitive. It is important that this model allows us to separate the bank concentration effect, proxied by \( n \), from the imperfect competition effect, proxied by \( \rho \).

I solve the problem of banker \( i \) backwards. In period \( t = 1 \), banker \( i \) maximizes his net worth in \( t = 2 \), which is also equal to the dividends paid to the entrepreneurs, \( N_{i,2} = \)

\(^{12}\)It will be important that the fraction of the output that the banker will seize in equilibrium in the high state in \( t = 1 \) is exactly equal to the marginal product of capital.
Since all the uncertainty is resolved in $t = 1$, there will be no default in $t = 2$. Banker $i$ takes the first order conditions of the entrepreneur and the actions of the other bankers, $L_{j,1}$, as given, where $j \neq i$. Given $N_{i,1}$, the optimization problem of banker $i$ in $t = 1$ becomes

$$\max_{L_{i,1}} \tilde{R}_{i,1}^t (L_{i,1}) L_{i,1} - R_1^t [L_{i,1} - N_{i,1}]$$

subject to a net worth constrain which states that at least a fraction $\eta$ of the loans that bank $i$ issues have to be financed using the bank’s own net worth

$$N_{i,1} \geq \eta L_{i,1} \quad [\lambda_{i,1}] .$$

(8)

$\lambda_{i,1}$ is the Lagrangian of the borrowing constraint and it represents the marginal value of an extra dollar of net worth in period one as perceived by the banker. [Aghion et al., 1999] provide one way to microfound the functional form of the net worth constraint that I specify in equation 8. Instead of a net worth constraint, one could use a borrowing constraint as an alternative friction, which is used by the literature on pecuniary externalities. The borrowing constraint can be expressed using a similar functional form as equation 8 with the main difference being that $\eta$ would be endogenous and time varying. The rest of the analysis would be similar.

Notice that banker $i$ takes into account that his actions affect the return that he receives, $\tilde{R}_{i,1}^t (L_{i,1})$. The state variable is the period one net worth, $N_{i,1}$, which is a function of the realized state. If the firms default in $t = 1$, the period one net worth of banker $i$ is given by

$$N_{i,1} (s_L) = A_L (L_0)^{\alpha} \frac{L_{i,0}}{L_0} - R_0^t [L_{i,0} - N_{i,0}]$$

and if the firms do not default, the period one net worth of banker $i$ is

$$N_{i,1} (s_H) = \tilde{R}_{i,0} L_{i,0} - R_0^t [L_{i,0} - N_{i,0}]$$

**If the net worth constraint does not bind** ($\lambda_1 = 0$), and after imposing a symmetric equilibrium, the first order condition with respect to $L_{i,1}$ becomes

$$R_1^t = \gamma \tilde{R}_1^t$$

---

13Banker $i$ will optimally choose not to pay dividends prior to $t = 2$ since entrepreneurs are assumed to always consume them, which implies that the marginal value of dividends is one. The marginal value of the bank’s net worth will be always greater than or equal to one. Notice that all the results will go through if bankers and entrepreneurs are treated as separate agents and then the assumption that dividends have to be consumed when paid out can be relaxed.
where $R^f_1 \geq 1$ is the period one risk free world interest rate which is also the marginal cost (MC) of an extra dollar of loans provided by banker $i$. The mark-up is constant and is given by the following equation

$$\gamma = \frac{1}{1 - \frac{1}{\rho} (1 - \frac{1}{n}) - (1 - \alpha) \frac{1}{n}} \geq 1$$

(9)

The key result to notice is that if the net worth constraint is not binding, the first order condition is not a function of period zero variables and

$$L_1 = \left[ \frac{\alpha A}{\gamma R^f_1} \right]^{\frac{1}{1+\alpha}}$$

Let us consider the mark-up. Since $\left(1 - \frac{1}{\rho}\right) n > \left(1 - \alpha - \frac{1}{\rho}\right)$, it will be the case that $0 < \frac{1}{\gamma} \leq 1$. The mark-up is a function of the number of banks, $n$, the degree of substitution between loans, $\rho$, and the productivity of capital captured by $\alpha$. Notice that the larger $\rho$ is (the more competitive the banking sector is), the lower the mark up is $\frac{\partial \gamma}{\partial \rho} < 0$. Whether the mark-up decreases or increases as $n$ increases depends on whether $1 - \alpha > \frac{1}{\rho}$. If $1 - \alpha > \frac{1}{\rho}$, more banks implies lower mark-up $\frac{\partial \gamma}{\partial n} < 0$. Finally, the higher $\alpha$ is, the smaller the mark up is, $\frac{\partial \gamma}{\partial \alpha} < 0$. The latter result is intuitive since the larger $\alpha$ is, the larger the fraction of output that accrues to the banker is (which is clear from equation 5), and the smaller the incentive to underinvest. If $\alpha \to 1$ and if $n = 1$ (the banker fully internalizes his effect on output), then $\gamma = 1$. If both $\rho \to \infty$ and $n \to \infty$, the banking sector is perfectly competitive, in which case the equilibrium interest rate charged converges to the world risk free rate $\bar{R}^f_1 = R^f_1$ and $\gamma = 1$. If $n \to \infty$, the mark up converges to the standard monopolistic competition mark-up with a continuum of banks given by $\gamma = \frac{\rho}{\rho - 1}$. If $n = 1$, $\gamma = \frac{1}{\alpha}$ which coincides with the monopolistic case.

If the net worth constraint binds ($\lambda_1 > 0$), and after imposing a symmetric equilibrium, the amount of period one loans is determined by the net worth constraint

$$L_1 = \frac{1}{\eta} N_1$$

where

$$\lambda_1 = \frac{1}{\eta} \left[ \frac{1}{\gamma} \bar{R}^f_1 - R^f_1 \right]$$

and $\bar{R}^f_1 = \alpha A (L_1)^{\alpha-1}$. The marginal value of an extra dollar of bank net worth, $\lambda_1$, as perceived by the banker, is larger, the smaller the mark-up, $\gamma$, is. The banker will value his net worth more, the more competitive the economy is (the smaller the mark up is). The marginal value of an extra dollar of bank net worth will be important when I discuss how
the decentralized equilibrium compares to the constrained Central Planner’s allocation and also how the competitive decentralized equilibrium compares to the non-competitive one. In order to make the problem interesting, throughout the rest of the paper, I assume that the period one net worth constraint never binds in the high state in $t = 1$.

Next I solve for the optimal lending of banker $i$ at $t = 0$. Banker $i$ takes as given his optimal actions in both states of nature in $t = 1$. He also takes the actions of the other bankers in the economy as given. Banker $i$ maximizes his expected dividend payment in the last period

$$\max_{L_{i,0}} E_0 N_{i,2} + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}] = \max_{L_{i,0}} E_0 \left[ \bar{R}_{i,1} (L_{i,0}) - R_{1}^f \right] L_{i,1} (L_{i,0})$$

$$+ R_{1}^f \left( \pi_H \bar{R}_{i,0} (L_{i,0}) + (1 - \pi_H) A_L L_{0}^\alpha \frac{1}{L_{0}} - R_{0}^f \right) L_{i,0} + R_{1}^f R_{0}^f N_{i,0} + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}].$$

where $\bar{R}_{i,1} (L_{i,0})$ implies that $\bar{R}_{i,1}$ is a function of $L_{i,0}$ and so are $L_{i,1} (L_{i,0})$ and $\bar{R}_{i,0} (L_{i,0})$. One way to interpret the objective function of the bank in $t = 0$ is the following. The net worth in $t = 2$ is the sum of the expected profits in periods $t = 1$ and $t = 2$ plus the return on the starting net worth of banker $i$. Consider parametrization where the net worth constraint does not bind in $t = 1$ in the high state, $\lambda_{1} (s_H) = 0$. I proved that $L_{i,0}$ does not affect the optimal $L_{i,1}$ if the net worth constraint is not binding in $t = 1$, which will be the case in the high state in $t = 1$. I also assume that the economy starts in normal times where the net worth constraint does not bind in $t = 0$, $\lambda_{0} = 0$. Assuming no default by banker $i$ in the high state, after imposing a symmetric equilibrium, the first order condition with respect to $L_{i,0}$ can be summarized as (for details see Appendix, Section A.3)

$$MC (L_{0}) = -(1 - \pi_H) \lambda_{1} (s_L) \frac{\partial N_{i,1} (s_L)}{\partial L_{i,0}} = R_{1}^f E_0 \frac{\partial N_{i,1} (s_1)}{\partial L_{i,0}} = MB (L_{0}) \quad (10)$$

where

$$\frac{\partial N_{i,1} (s_L)}{\partial L_{i,0}} = A_L L_{0}^\alpha \left( 1 - \frac{1}{n} (1 - \alpha) \right) - R_{0}^f < 0 \quad (11)$$

$$\frac{\partial N_{i,1} (s_H)}{\partial L_{i,0}} = \alpha A_H L_{0}^\alpha \frac{1}{\gamma} - R_{0}^f > 0 \quad (12)$$

and if $\lambda_{1} (s_L) > 0$

$$\lambda_{1} (s_L) = \frac{1}{\eta} \frac{\partial N_{i,2} (s_L)}{\partial L_{i,1} (s_L)} = \frac{1}{\eta} \left[ \alpha \bar{A} (L_{1} (s_L))^{\alpha - 1} \frac{1}{\gamma} - R_{1}^f \right] > 0 \quad (13)$$

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14 The most interesting case will be when the constraint binds in the crisis state $\lambda_{1} (s_L) > 0$ as well, which will generate a recession and a credit crunch, but for now the equations presented are more general.
L_0 is determined by equating the expected marginal profit of an extra dollar of L_0, MB(L_0)\(^{15}\), to the marginal cost of an extra dollar of L_0, MC(L_0). If the banker lends one extra dollar in \(t = 0\) to the entrepreneur, the marginal benefit is the increase of expected period one profits, \(R^f_0 E_0 \frac{\partial N_i,1(s_1)}{\partial L_{i,0}}\). However, there is a cost associated with lending more in \(t = 0\) if the net worth constraint binds in the low state in \(t = 1\). An extra dollar lent in \(t = 0\) implies that in the low state, when the entrepreneur defaults, the net worth of the bank will decrease by \(-\frac{\partial N_i,1(s_L)}{\partial L_{i,0}}\). If the net worth constraint binds in the low state in \(t = 1\), the marginal value of an extra dollar of net worth in the low state is measured by the Lagrangian of the borrowing constraint in the low state \(\lambda_1(s_L)\). Hence, the marginal cost associated with lending one more dollar in \(t = 0\) is \(-(1-\pi_H)\lambda_1(s_L)\frac{\partial N_i,1(s_L)}{\partial L_{i,0}}\) which is the cost of making the borrowing constraint tighter. Notice that depending on the degree of monopolistic competition, \(\rho\), and the degree of banking sector concentration, \(n\), the equilibrium \(L_0\) will be different. Denote the decentralized equilibrium with a star.

**Definition 1** The non-competitive decentralized symmetric equilibrium is defined as a vector of quantities \(\{C_1^*(s_1), C_2^*(s_1), K_0^*, K_1^*(s_1), L_0^*, L_1^*(s_1)\}_{s_1 \in \{s_L, s_H\}}\) and prices \(\{\bar{R}^f_0, \bar{R}^f_1(s_1)\}_{s_1 \in \{s_L, s_H\}}\) such that

- The markets for loans, capital and the consumption good clear
- Banker \(i\) chooses \(L_{i,t}\) to maximize his expected net worth in \(t = 2\) taking into account the demand schedules for loans of the entrepreneurs, the net worth constraint and taking as given the actions of the other bankers \(L_{j,t}\) where \(j \neq i\)
- The representative entrepreneur chooses \(\{C_1^*(s_1), C_2^*(s_1), K_0^*, K_1^*(s_1), L_0^*, L_1^*(s_1)\}_{s_1 \in \{s_L, s_H\}}\), in order to maximize the profits of the firm taking as given the loan interest rates \(\{\bar{R}^f_0, \bar{R}^f_1(s_1)\}_{s_1 \in \{s_L, s_H\}}\)
- Foreign investors inelastically supply risk free loans to the domestic banking sector at the exogenous world interest rate \(R^f_t\)

Next, I prove existence and uniqueness and compare how the optimal investment varies with the degree of banking sector competition.

**Proposition 1** (i) If the parametrization is such that there is a crisis in the low state in \(t = 1\) and no crisis in the high state in \(t = 1\) and in \(t = 0\), \(\lambda_1(s_L) > 0, \lambda_1(s_H) = 0, \lambda_0 = 0\), the

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\(^{15}\)Note that the marginal benefit is net of the interest rate payment to foreigners.
equilibrium is unique and exists. (ii) Countries with a more competitive banking sector will borrow and invest more than countries with a less competitive banking sector, \( \frac{\partial L_{i,0}}{\partial \rho} > 0, \) if

\[
- (1 - \pi_H) \frac{\partial N_{i,1}(s_L) \partial \lambda_{i,1}(s_L)}{\partial L_{i,0}} < R_f^f \pi_H \frac{\partial^2 N_{i,1}(s_H)}{\partial L_{i,0} \partial \rho}
\]

(Assumption 1)

and \( \frac{\partial L_{i,0}}{\partial \rho} < 0 \) if Assumption 1 is not satisfied.

**Proof of Proposition 1:** See Appendix, Section A.6.

What is surprising about the result in Proposition 1 is that if Assumption 1 is not satisfied, countries with more competitive banking sectors (higher \( \rho \)) borrow and invest less (not more) than countries with less competitive banking sectors. The reason why one would expect less competitive banking sectors to invest less is that lower competition (smaller \( \rho \)) strengthens the standard underinvestment channel.

A less competitive banking sector would optimally want to underinvest in the current period \((t = 0)\) and the future period \((t = 1)\) relative to a more competitive banking sector. The current period underinvestment leads to the expected perceived period one profits to be lower for less competitive banking sectors, which is captured by the term \( \frac{\partial MB(\rho, L_{i,0})}{\partial \rho} > 0. \) However, the combination of future desire to underinvest and a binding net worth constraint in the crisis state in the future leads to an overinvestment force in \( t = 0 \) relative to the more competitive case. The intuition is the following. A less competitive banking sector wants to also underinvest in the future, in period one. As a result, an extra dollar in the crisis state in the future becomes less valuable for the less competitive banking sector, given that the perceived marginal value of an extra dollar of net worth is lower \( \frac{\partial \lambda_{i,1}(s_L)}{\partial \rho} > 0. \) Therefore, the perceived marginal cost of \( L_{i,0} \) is actually smaller, the less competitive the banking sector is, where an extra \( L_{i,0} \) is costly because it depletes the net worth of the bank in the crisis state \( \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} < 0. \) This latter overinvestment channel is captured by the term \( \frac{MC(\rho, L_{i,0})}{\partial \rho} > 0. \)

In summary, there are two forces in play when comparing a less competitive banking sector to a more competitive banking sector. The first force is a standard current period \((t = 0)\) underinvestment force. The second one is an overinvestment force due to the combination of the future underinvestment force \((t = 1)\) and a binding net worth constraint in the crisis state in the future. Therefore, the answer to the question whether a more or a less competitive banking sector experiences a larger investment boom ex-ante (prior to a crisis) will depend crucially on the interaction of the two effects. This result is in contrast to the franchise value literature, which argues that less competitive banking sectors are less prone to overinvestment since banks are concerned about preserving their franchise value. While
the model in this paper endogenously produces no bank default, the overinvestment force that I identify here (which is present even in the case of a single bank) will be a countervailing force to the franchise value argument.

2.3 Constrained Central Planner’s Problem

In order to determine the source of the inefficiencies in this economy, in this section I solve the constrained Central Planner’s (CP’s) problem. The CP faces the same constraints as the banker in the decentralized equilibrium. He has to take into account the same net worth constraints that the bankers face and also the first order conditions of the entrepreneurs.\textsuperscript{16} The CP chooses the amount of loans provided by every banker, taking into account that the equilibrium played is symmetric. There are two sources of inefficiency. The first source is pecuniary externalities, which will lead to overinvestment. The strength of the pecuniary externalities will vary with the number of banks, \( n \). The second inefficiency is due to monopolistic competition, which will lead to underinvestment. The strength of the underinvestment will vary with the degree of loan substitution, \( \rho \), and the number of banks, \( n \).

Solving the problem of the CP backwards, in \( t = 1 \), the CP maximizes the total welfare of the entrepreneurs, who also own the banks, subject to the banker’s net worth constraint. The problem simplifies to maximizing total output since the entrepreneurs, who are the only agents consuming in the economy, are risk neutral. (See Appendix, Section A.4 for detailed derivations of the CP’s problem.)

\[
\max_{L_1} \bar{A}L_1^\alpha - R_1^f [L_1 - N_1] + \lambda_1^{CP} [N_1 - \eta L_1]
\]

The first order condition with respect to \( L_1 \) is given by

\[
\alpha \bar{A}L_1^{\alpha-1} - R_1^f - \lambda_1^{CP} \eta = 0
\]

If the net worth constraint does not bind (\( \lambda_1^{CP} = 0 \)), the entrepreneur will invest up to the point where the marginal cost of an extra dollar of loans equals the marginal benefit of an extra dollar of loans from the Central Planner’s point of view, \( \alpha \bar{A}L_1^{\alpha-1} = R_1^f = \bar{R}_1^f \). If the constraint binds (\( \lambda_1^{CP} > 0 \)), then

\[
L_1 = \frac{1}{\eta} N_1
\]

where \( N_1 (s_L) = A_L L_0^\alpha - R_0^f (L_0 - N_0) \) and \( N_1 (s_H) = \alpha A_H L_0^\alpha - R_0^f (L_0 - N_0) \). The marginal value of an extra dollar of net worth when the constraint is binding in period one is given by

\textsuperscript{16}In this specific model, it does not make a difference whether the CP takes the first order conditions of the entrepreneurs as an additional constraint or not. However, this assumption is important for other models with a third agent, such as a consumer, in order to prevent transfer of resources from consumers to constrained bankers.
the Lagrangian $\lambda_1^{CP} = \frac{1}{\eta} \left( \alpha \tilde{A} L_1^{\alpha-1} - R^f_1 \right)$. The optimization problem in $t = 0$ is to maximize the sum of expected period one and period two output.

$$\max_{L_0} (1 - \pi_H) \left( \tilde{A} (L_1 (s_L))^\alpha - R^f_1 L_1 (s_L) \right) + \pi_H \left( \tilde{A} (L_1 (s_H))^\alpha - R^f_1 L_1 (s_H) \right) + R^f_1 \left( E_0 A_1 L_0^\alpha - R^f_0 L_0 \right) + R^f_1 R^f_0 N_0 + \lambda_0^{CP} [N_0 - \eta L_0]$$

Considering only parametrization where $\lambda_0^{CP} = 0$, $\lambda_1^{CP} (s_L) \geq 0$ and $\lambda_1^{CP} (s_H) = 0$, the first order condition with respect to $L_0$, is given by\footnote{The Appendix, Section A.4 I prove existence and uniqueness.}

$$MC^{CP} (L_0) = - (1 - \pi_H) \lambda_1^{CP} (s_L) \frac{\partial N_1 (s_L)}{\partial L_0} = R^f_1 \left( E_0 A_1 \alpha (L_0)^{\alpha-1} - R^f_0 \right) = MB^{CP} (L_0)$$

where $\frac{\partial N_1 (s_L)}{\partial L_0} = \left( \alpha A_L (L_0)^{\alpha-1} - R^f_0 \right) < 0$. If the bank net worth constraint in period one is binding, $\lambda_1^{CP} (s_L) > 0,$

$$\lambda_1^{CP} (s_L) = \left[ \alpha \tilde{A} (L_1 (s_L))^\alpha \right] \frac{1}{\eta} > 0$$

The way to interpret the first order condition is the following. An extra dollar invested in period zero will increase expected period one output, net of foreign debt payment, by $MB^{CP}$. However, there is an extra cost associated with an extra dollar invested in $t = 0$. In the low state in $t = 1$, an extra dollar invested in $t = 0$ will decrease the net worth of the banker by $- \frac{\partial N_1 (s_L)}{\partial L_0}$ and the value of an extra dollar of net worth in the low state as perceived by the CP is given by $\lambda_1^{CP} (s_L).$ Throughout the rest of the paper, the superscript $CP$ will denote the optimal allocation of the Central Planner.

3 Central Planner’s Allocation vs Decentralized Equilibrium

In this section I compare the CP’s allocation against the decentralized equilibrium. I show that there are two sources of inefficiency in this environment — pecuniary externalities, which lead to overinvestment relative to the constrained CP’s allocation, and imperfect competition of the banking sector, which leads to underinvestment relative to the constrained CP’s allocation.

To preview the results in this section, there will be two different channels through which the pecuniary externalities will generate overinvestment in this model. The intuition behind the first channel is that each banker does not fully internalize the fact that the more he invests in period zero, the more he increases the marginal return of all other bankers when
the representative entrepreneur defaults in period one. I will call this type of pecuniary externalities "bankruptcy" pecuniary externality. It will lead to overinvestment even if there is no binding net worth constraint in the future. In addition, if the banker’s net worth constraint is binding in the crisis state in the future, the pecuniary externalities will lead to overinvestment through a second channel. The more each banker invests in period zero, the lower the return and the net worth of all other bankers is in the crisis state, which will tighten the net worth constraints of all other bankers. I will call this second type of pecuniary externalities "net worth constraint" pecuniary externality. Since the Central Planner maximizes total output, he internalizes both of those externalities. The reason why I refer to these externalities as pecuniary externalities, is because one can think of the return each banker receives if the entrepreneur defaults as a price which, in equilibrium, is equal to the output of the entrepreneur divided by total loans. Since this price depends on the total output of the firm and the firm borrows from many different banks in order to invest (an assumption implicit in the monopolistic competition environment), the banker realizes that his actions only partially affect this price. Each banker perceives his effect on the rate of return during a crisis to be larger, the smaller \( n \) is. Therefore, a more concentrated banking sector implies weaker pecuniary externalities.

The second source of inefficiency in this model is due to the imperfect competition of the banking sector combined with the fact that the banker appropriates only part of the firm’s output. The banker maximizes only his own net worth/dividend payments and does not internalize the fact that the more he lends to the firm, the higher the profits of the entrepreneur are in the state of nature where the entrepreneur does not default.

Finally, I show that even if there are no pecuniary externalities, \( n = 1 \) (or they are weaker — \( n \) is small), a monopolistic bank might still overinvest relative to the CP as a result of the interaction between the desire to underinvest in the future in the crisis state and a binding net worth constraint during a crisis. The intuition why this is the case is the following. Given that the banker wants to underinvests in the future, he does not value an extra dollar of net worth in the crisis state as much as the CP does. Therefore, an extra dollar of loans provided in period zero, which decreases the net worth of the banker in a crisis, is perceived to be less costly. Whether the monopolistic bank ends up lending too much relative to the constrained CP depends on how this overinvestment force compares to the classic underinvestment force in period zero.

### 3.1 Bank Net Worth Constraint Never Binds

First, I examine the case where the net worth constraint does not bind either for the CP or the banker in the decentralized equilibrium in any period or state of nature.

**Proposition 2** If the net worth constraint does not bind for \( \forall t \) and in any state of nature
for either the CP or the banker in the decentralized equilibrium, the decentralized equilibrium exhibits underinvestment relative to the constrained CP’s allocation, $L^0_{CP} > L^*_0$, if

$$\pi_L A_L \left(\frac{1}{n} [\alpha - 1] + 1\right) + \pi_H A_H \frac{1}{\gamma} < \alpha E_0 A_1$$

(Assumption 2)

and overinvestment, $L^0_{CP} < L^*_0$, if Assumption 2 is violated. In the monopolistic case ($n = 1$), $L^0_{CP} > L^*_0$ and in the perfectly competitive case, ($n \to \infty$ and $\rho \to \infty$), $L^0_{CP} < L^*_0$, where $CE$ stands for competitive equilibrium.

**Proof of Proposition 2.** One can simplify the first order conditions of the CP and the banker when the net worth constraint never binds and write them as

$$MB(L_0) = R_1^f \left(\pi_L A_L L_0^{\alpha-1} \left(\frac{1}{n} [\alpha - 1] + 1\right) + \pi_H A_H L_0^{\alpha-1} \frac{1}{\gamma} - R_0^f\right) = 0$$

$$MB^{CP}(L_0) = R_1^f \left(E_0 A_1 \alpha (L_0)^{\alpha-1} - R_0^f\right) = 0$$

where $\gamma = \frac{1}{(1-\frac{1}{n})(1-\alpha)\frac{1}{\gamma}}$. Since both $MB'(L_0) < 0$ and $MB^{CP'}(L_0) < 0$ (For proof see Appendix, Proof of Proposition 1 and Section A.4), there will be underinvestment if $MB^{CP}(L_0) > MB(L_0)$, which will be true if Assumption 2 is satisfied. If $n = 1$, then $\gamma = \frac{1}{\alpha}$ and Assumption 2 will be always satisfied. If $n \to \infty$ and $\rho \to \infty$, Assumption 2 is violated.

If $n = 1$, the banker will underinvest relative to the CP as a result of a standard underinvestment channel where in the high state only a fraction of the output accrues to the entrepreneur. Since, the banker optimizes only his own profits and does not internalize the fact that higher investment increases the profits of the entrepreneur as well, he will underinvest relative to the constrained CP, who maximizes total output. It is surprising that in the perfectly competitive case, $n \to \infty$ and $\rho \to \infty$, the banker overinvests relative to the Central Planner even when the net worth constraint is not binding in any $t$ and state of nature. The intuition for this result is the following. When the entrepreneur defaults in the low state, bankers appropriate all of the output. In the perfectly competitive case, every banker takes the return received in the crisis state as given. As a result, every banker does not internalize the fact that the more he lends, the more he decreases the marginal rate of return of the other bankers (due to the concavity of the production technology of the firm).

\footnote{In terms of the actual math, due to the assumption about constant elasticity of substitution (CES) of loans, which is required in order to model monopolistic competition, every entrepreneur borrows from all bankers in order to invest. Therefore, the return in the crisis state is a function of aggregate variables which the bankers are too small to affect.}
when the entrepreneur defaults.\textsuperscript{19} This is why this model will exhibit pecuniary externalities which lead to overinvestment even if the net worth constraint does not bind. I call this type of pecuniary externalities "bankruptcy" pecuniary externality.

For any other combination of $n$ and $\rho$, whether there is over-or-underinvestment depends on whether Assumption 2 is satisfied, which determines whether the underinvestment force dominates the overinvestment force. Notice that the less competitive the banking sector is (small $\rho$), the more likely it is that Assumption 2 is satisfied since $\gamma'(\rho) < 0$ and hence the more likely it is that the economy exhibits underinvestment.

### 3.2 Binding Bank Net Worth Constraint In a Crisis

In this subsection I compare the decentralized equilibrium and the constrained Central Planner’s allocation assuming that the net worth constraint binds in the low state in $t = 1$ but does not bind in the high state in $t = 1$ and in $t = 0$. Essentially, this case maps to starting the economy in normal times where with some probability next period there will be a credit crunch crisis due to a binding bank net worth constraint, which would be the most interesting case to consider.

#### 3.2.1 Perfectly Competitive Case, $n \to \infty, \rho \to \infty$

In the perfectly competitive case, given the assumptions made above, I show that bankers always overinvest relative to the CP due to two sources of pecuniary externalities. The first one was described in Section 3.1 and is present even if the net worth constraint in the low state is not binding. The second source of overinvestment occurs due to the binding net worth constraint in the crisis state.

**Proposition 3** If $n \to \infty, \rho \to \infty$ and the net worth constraint binds only in the low state in $t = 1$ for both the CP and the banks in the decentralized equilibrium, then $L^C_P < L^C_E$.

**Proof of Proposition 3.** When $n \to \infty$ and $\rho \to \infty$, the only difference between the first order conditions of the CP and the banker in the competitive equilibrium is that

$$\left(\frac{\partial N(s_L)}{\partial L_0}\right)^{CP} < \left(\frac{\partial N(s_L)}{\partial L_0}\right)^{CE}.$$  

This inequality implies that $MB^{CP}(L_0) < MB^{CE}(L_0)$ and

\textsuperscript{19}In the perfectly competitive case, the banker also does not internalize the fact that he affects the return received by the other bankers in the high state where there is no default. However, in the high state, in equilibrium, the return the banker receives equals the marginal product of capital (and not the whole output as in the default state). Therefore, the overinvestment force is exactly offest by the underinvestment force due to the fact that the banker maximizes only a fraction of output in the high state while the CP’s objective function is to maximize the whole output in the high state. The intuition is similar as to why in a standard model with no default where the banker receives the marginal product of capital in every state of nature the CP’s allocation coincides with the CE.
MC^{CP} (L_0) > MC^{CE} (L_0) which combined with \( MC'' (L_0) > 0 \) and \( MB' (L_0) < 0 \) is sufficient to prove that for any parametrization that leads to the borrowing constraint binding for both the CP and the banker in the low state in \( t = 1 \), the banker will end up overinvesting relative to the CP.\(^{20}\)

On the one hand, the banker overinvests because he does not internalizes the fact that by investing too much in period zero he will decrease the marginal return the other bankers receive when the entrepreneur defaults (captured by \( MB^{CP} (L_0) < MB^{CE} (L_0) \)). This is the same source of overinvestment as in the case where the net worth constraint does not bind and is what I refer to as "bankruptcy" pecuniary externality.

The intuition behind the second source of the pecuniary externality is due to the binding net worth constraint in the crisis state and is captured by \( MC^{CP} (L_0) > MC^{CE} (L_0) \). Every banker does not internalize the fact that the more he invests in period zero, the lower the return of all other bankers is when the firm defaults. This leads to lower net worth and tighter net worth constraints of the other bankers. The CP internalizes this externality. I refer to this second type of pecuniary externality as "net worth constraint" pecuniary externalities.

3.2.2 Monopolistic Bank, \( n = 1 \)

It is informative to consider the monopolistic case, since \( n = 1 \) implies that the pecuniary externalities are turned off — the banker internalizes fully his impact on the return of the loans. The standard monopolistic force pushes towards underinvestment in both \( t = 0 \) and \( t = 1 \). However, the binding net worth constraint in the crisis state introduces a dynamic aspect to the problem of the monopolist. The interaction between the binding net worth constraint in the crisis state (low state in \( t = 1 \)) and the underinvestment force in the crisis state can lead to overinvestment in \( t = 0 \) relative to the CP’s allocation even when \( n = 1 \). The result is formally stated in Proposition 4 below.

**Proposition 4** If \( n = 1 \) and the net worth constraint binds only in the low state in \( t = 1 \) for both the CP and the monopolistic bank, then \( L_0^{CP} > L_0^* (n = 1) \) if

\[
\frac{R_1^f}{\eta} (1 - \alpha) \pi_H A_H (L_0^*)^\alpha - \frac{1}{\eta} (\alpha A_L (L_0^*)^{\alpha - 1} - R_0^f) [1 - \alpha] A (L_1^* (s_L))^{\alpha - 1} > 0
\]

where \( L_1^* = \frac{1}{\eta} \left( A_L (L_0^*)^\alpha - R_1^f [L_0 - N_0] \right) \). If Assumption 3 is violated, the monopolist over-invests relative to the Central Planner \( L_0^{CP} < L_0^* (n = 1) \).

**Proof of Proposition 4.** See Appendix, Section A.6. \(\blacksquare\)

\(^{20}\)Note that there will be still overinvestment even if the constraint does not bind in the future for the CP but binds in the crisis state for the banker from the CE.
The intuition why it is possible for the monopolist to overinvest relative to the CP is very similar as to why the monopolist might end up investing more in $t = 0$ than a perfectly competitive banking sector, which I discussed in Section 2.2. On the one hand, there is the standard underinvestment force which pushes the monopolist to want to underinvest relative to the CP in both periods zero and one. However, when the period one underinvestment force is combined with a binding net worth constraint in the crisis state, an overinvestment force emerges. The banker, unlike the CP, does not value an extra dollar of net worth in the crisis state as much as the CP since he wants to underinvest relative to the CP in the crisis state. Hence, he perceives the marginal cost of an extra dollar of $L_0$ to be smaller than the CP does, $MC(L_0(n = 1)) < MC^CP(L_0)$, which is why there is an overinvestment force. Whether the monopolist ends up overinvesting or not depends on the relative strength of the two forces. For example, as $\pi_H \to 1$, then the overinvestment force vanishes since the crisis state disappears and Assumption 3 is always satisfied. In contrast as $\eta \to 0$, which captures the degree of financial development (since lower $\eta$ implies that bankers can finance a larger fraction of loans using foreign loans rather than internal equity), the overinvestment force dominates. Therefore, in the limit, as it becomes very easy to finance domestic lending using bank debt, the overinvestment force dominates. Notice that these are only limiting results since $L^*$ is a function of $\eta$ and $\pi_H$ as well.

3.2.3 General Case

Having gained a better understanding of the externalities present in this model and whether they push towards over-or-underinvestment relative to the constrained CP’s allocation, next let’s consider the general case for any $n$ and $\rho$. In the general case, whether $L_0^CP < L_0^*$ or not will be determined by the relative strength of the overinvestment and the underinvestment forces. The overinvestment forces are due to the pecuniary externalities – both the "bankruptcy" and the "net worth constraint" pecuniary externality — and the combination of future desire to underinvest in the crisis state plus a binding net worth constraint in the crisis state. The underinvestment force is the standard imperfect competition underinvestment force in period zero.

**Proposition 5** For any $n$ and $\rho$ and if the net worth constraint binds only in the low state...
in $t = 1$ for both the CP and banker in the decentralized equilibrium, then $L^C_P > L^*_0$ if

$$R^f_{1} A_H (1 - 1) L^*_{0}^{\alpha - 1} - R^f_{1} (1 - \pi_H) A_L (1 - \alpha) \left( 1 - \frac{1}{n} \right) L^*_{0}^{\alpha - 1} +$$

underinvestment in $t = 0 \geq 0$

$$- (1 - \pi_H) \frac{1}{\eta} \left[ \alpha A (L^*_1 (s_L))^{\alpha - 1} - R^f_{1} (1 - \alpha) \left( 1 - \frac{1}{n} \right) A_L L^*_{0}^{\alpha - 1} \right]$$

"bankruptcy" pecuniary externality $\leq 0$

$$- (1 - \pi_H) \frac{1}{\eta} \left[ R^f_{0} - \left( 1 - (1 - \alpha) \frac{1}{n} \right) A_L L^*_{0}^{\alpha - 1} \right] \alpha A (L^*_1 (s_L))^{\alpha - 1} (1 - \frac{1}{n}) > 0$$

underinvestment in $t = 1 \leq 0$

where $L^*_1 = \frac{1}{\alpha} \left( A_L (L^*_0)^{\alpha} - R^f_{1} [L^*_0 - N_0] \right)$. If Assumption 4 is violated, then the banker over-invests relative to the Central Planner $L^C_P < L^*_0$.

**Proof of Proposition 5.** See Appendix, Section A.6.

If Assumption 4 is satisfied then the banker in the decentralized equilibrium underinvests relative to the CP, $L^C_P > L^*_0$, and if it’s not, he overinvests, $L^C_P < L^*_0$. The monopolistic competition pushes towards underinvestment in period $t = 0$. However, while the period zero underinvestment force makes it more likely for the banker to want to underinvest, the period one underinvestment force actually pushes towards overinvestment as is apparent in the formula for Assumption 4 presented above. The intuition is similar to the $n = 1$ case — the fact that bankers want to underinvest in $t = 1$ in the crisis state relative to the CP implies that they value an extra dollar of net worth by less than the CP. Hence an extra dollar of bank loans provided in period zero, which leads to lower net worth in the crisis state, is less costly from the perspective of the banker. From the formula for Assumption 4 one can also see the two pecuniary externalities forces both of which push towards overinvestment.

## 4 Decentralize the Constrained Central Planner’s Allocation

In this section I consider how the CP’s allocation can be decentralized. There are many different ways to do that. Two policy instruments that are often used in practice are loan subsidies to firms/entrepreneurs and also capital account controls which limit the inflow of funds into the country. The latter can be implemented using a tax on the return paid on foreign loans, which is the approach that I follow in this paper. I will assume that the only instruments available to the policy maker are subsidies on entrepreneur’s borrowing rates, $\tau_{i,t}^s$, a tax on the interest rate paid to foreigners, $\tau_{i,t}^{cc}$, and lump sum transfers (taxes if negative), $T_{i,t}$, to the bankers, where $i$ stands for banker $i$. To be more precise, when the firm is borrowing from bank $i$, the effective interest rate it faces is given by $(1 - \tau_{i,t}^s) \tilde{R}^f_{i,t}$ and when banker $i$ borrows from foreigners, the effective rate of return he has to pay is $(1 + \tau_{i,t}^{cc}) R^f_{i}$. I assume
commitment, which implies that all instruments, \( \{ \tau^{s}_{i,t}, \tau^{cc}_{i,t}, T_{i,t+1} \}_{t=0,1} \), are determined in the beginning of period \( t = 0 \), before entrepreneurs and bankers make any decisions. Hence, they take these instruments as given. The government will balance its budget every period, which will be achieved via the lump sum transfers to the bankers. In period \( t+1 \), if the entrepreneur does not default, he will receive from the government the effective subsidy on the loans he took in period \( t \). Similarly, the government will receive the tax from previous period loans of the bankers from the foreigners. The lump sum transfer to banker \( i \) in \( t+1 \); if there is no default by the representative entrepreneur, is given by

\[
T_{i,t+1} = \tau^{cc}_{i,t} R^{f}_{t} \left[ L_{i,t} - N_{i,t} \right] - \tau^{s}_{i,t} R^{l}_{i,t} L_{i,t}
\]

where \( L_{i,t} - N_{i,t} \) is the amount borrowed by banker \( i \) from foreigners. If the entrepreneur defaults, then the banker seizes all the output of the firm and the firm cannot even pay the subsidized interest rate payments. In that case

\[
T_{i,t+1} = \tau^{cc}_{i,t} R^{f}_{t} \left[ L_{i,t} - N_{i,t} \right]
\]

The subsidy has to be paid only conditional on the loan being repaid and, hence, will be paid only in the high state.\(^{21}\) For detailed derivation of the Ramsey Problem, see Section A.5 in the Appendix.

In this model, \( \tau^{cc}_{t} \) and \( \tau^{s}_{t} \) are not uniquely pinned down, which is intuitive since the banker will either underinvest or overinvest relative to the CP. As a result, only one of those instruments in every period and state of nature will be sufficient to replicate the constrained CP’s allocation. However, for every \( t \) and state of nature, there are many possible combinations of \( \tau^{cc}_{t} > 0 \) and \( \tau^{s}_{t} > 0 \) which implement the CP’s allocation. If one observes both subsidies and capital account controls in practice, it does not mean that the policies are necessarily sub-optimal.

One can make the argument that there are political and monetary costs to providing subsidies and collecting taxes. In this paper, I will not solve for the policy problem that would minimize those costs, but will consider one possible implementation, instead, that provides interesting intuition and meets the following criteria. First, at every point in time and in every state of nature, the policy maker can use only one instrument — either a subsidy or capital account controls. Also I assume that if there is no uncertainty in the future and the banking sector is imperfectly competitive, the policy maker always uses a subsidy. I would like to emphasize that this is only one way to implement the constrained CP’s allocation. Given the assumptions made, Proposition 6 specifies the optimal policy.

**Proposition 6** Assuming the policy maker can commit and the net worth constraint binds in the crisis state, the CP’s allocation can be decentralized using a lump sum transfer to entrepreneurs, \( T_{t} \), subsidy on entrepreneurs’ borrowing rates, \( \tau^{s}_{t} \geq 0 \), and a capital account control in the form of a tax on banker’s borrowing rates from foreigners, \( \tau^{cc}_{t} \geq 0 \). One possible implementation of the constrained Central Planner’s allocation is given by: \( \tau^{s}_{1} = 1 - \frac{1}{\tau^{cc}_{1}} \), \( \tau^{cc}_{1} = 0 \). If \( \tau^{cc}_{0} (\tau^{s}_{0} = 0) > 0 \), then \( \tau^{cc}_{0} = \tau^{cc}_{0} (\tau^{s}_{0}) \) and \( \tau^{s}_{0} = 0 \). If \( \tau^{cc}_{0} (\tau^{s}_{0} = 0) < 0 \), then \( \tau^{cc}_{0} = 0 \) and

\(^{21}\) The exact specification is not crucial for the results.
\(\tau^s_0 > 0\), where \(\tau^s_0\) is pinned down by \(\tilde{\tau}^{cc}_0 (\tau^s_0) = 0\) and

\[
\tilde{\tau}^{cc}_0 (\tau^s_0) = \Phi \left( R^f_1 \pi_H A_H \left( 1 + \frac{1}{\gamma (1 - \tau^s_0)} \right) + R^f_1 (1 - \pi_H) A_L (1 - \alpha) \left( 1 + \frac{1}{n} \right) \right) + \Phi (1 - \pi_H) \left[ \alpha \tilde{A} (L^\text{CP}_1 (s_L))^{\alpha - 1} - R^f_1 \right],
\]

\(\Phi = \frac{\left( L^\text{CP}_0 \right)^{\gamma - \alpha - 1} R^2_0 \left[ R^f_1 (1 - \pi_H) \tilde{A} (L^\text{CP}_1 (s_L))^{\alpha - 1} - R^f_1 \right]}{n} > 0\) and \(CP\) stands for the optimal allocation of the \(CP\).

**Proof of 6.** See Appendix, Section A.6. ■

In order to understand the optimal policy, it is worth discussing how to interpret the three period model, \(t = 0, 1, 2\), where all the uncertainty is resolved in \(t = 1\). As a reminder, the structure of the model allows me to study the following sequence of events:

1) In period \(t = 0\), agents face an uncertain period one output. For example, the presence of uncertainty can be justified by the discovery of a new technology in \(t = 0\) the returns of which are unknown.

2) In \(t = 1\), the uncertainty is realized and the economy can end up in a crisis state with probability \(1 - \pi_H\) if the technology was not very productive or in a non-crisis state with probability \(\pi_H\). If in the crisis state, the entrepreneur defaults and the bankers’ net worth is depleted to the point where their net worth constraints are binding. This environment proxies an economy with high aggregate default rates and a credit crunch.

3) In \(t = 2\), the economy converges to a steady state where there is no future uncertainty regarding the productivity of the technology and the probability of default in \(t = 2\) goes to zero.\(^{22}\) This latter assumption is fairly realistic when we think about the aggregate economy in normal times.

Let’s analyze the optimal policy backwards. In \(t = 1\) given that there is no future uncertainty and no future default or binding net worth constraints, all overinvestment forces are shut down. The only externality is the underinvestment force which is present only if the banking sector is imperfectly competitive and the optimal subsidy in period one is equal to \(\tau^s_1 = 1 - \frac{1}{\gamma}\). If the banking sector is perfectly competitive, then no intervention is required since \(\gamma = 1\).\(^{23}\)

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\(^{22}\) Also the definition of a steady state would imply no binding net worth constraints in \(t > 2\) if more periods were to be included.

\(^{23}\) Note that if the net worth constraint is binding for both the CP and the banker in \(t = 1\) in the crisis state no policy intervention would be required. This would be an alternative way to decentralize
The interesting question is what should be the optimal policy when there is a significant uncertainty regarding the productivity of the new technology and with some probability it can turn out to be unproductive, leading to large aggregate default and binding net worth constraints in the future. This is the environment that the policy maker and the agents face in $t = 0$. In $t = 0$, there will be overinvestment forces due to the pecuniary externalities and an underinvestment force if the banking sector is imperfectly competitive. (Notice that the third source of overinvestment will be no longer present in $t = 0$ since it was a result of the interaction between the future binding net worth constraint and the desire of bankers to underinvest in $t = 1$. However, by imposing a subsidy in the crisis state equal to $s_1 = 1$, the period one underinvestment force was shut down.) Whether the policy maker will use capital account controls or subsidies in period zero depends on whether the underinvestment or the overinvestment force dominates which will be determined by whether $\tilde{\tau}_0^{cc} (\tau_0^s) = 0$ is positive of negative.

First, notice that it is also possible that no regulation is required in $t = 0$ if the overinvestment and underinvestment forces exactly offset each other and $\tilde{\tau}_0^{cc} (\tau_0^s = 0) = 0$. This result is formally stated in the following Corollary 1.

**Corollary 1** For every country, where the net worth constraint binds in the crisis state for both the CP and the banker in the decentralized equilibrium, there exists $n^* (\rho)$, such that no period zero regulation is necessary, $\tau_0^{cc} = 0$ and $\tau_0^s = 0$.

The $n^*$, which one can solve for using the equation $\tilde{\tau}_0^{cc} (\tau_0^s = 0) = 0$, is given by

$$n^* (\rho) = 1 + \frac{R_1^f \pi_H A_H (1 - \alpha)}{\left( R_1^f + \alpha \bar{\rho} (L_1^{CP} (s_L))^{\alpha - 1} - R_1^f \right) \left( 1 - \pi_H \right) A_L (1 - \alpha) - R_1^f \pi_H A_H \frac{1}{\rho}}$$

where $L_1^{CP} = \frac{1}{\eta} \left( A_L (L_0^{CP})^\alpha - R_1^f [L_0^{CP} - N_0] \right)$. $L_0^{CP}$ and $L_1^{CP}$ are not a function of either $n$ or $\rho$ since the structure of the banking sector does not affect the CP’s problem. Also notice that the degree of monopolistic competition, $\gamma = \frac{1}{(1 - \frac{1}{\rho}(1 - \frac{1}{n}) - (1 - \alpha)\frac{1}{\rho})}$, is governed by two key parameters $- \rho$ and $n$. Therefore, even if the loans are perfect substitutes, $\rho \to \infty$, as long as $n < \infty$, the banking sector is still imperfectly competitive $- \gamma > 1$. The larger the degree of substitution between loans, the smaller is the number of banks for which no period zero regulation will be required, $n^* (\rho) < 0$. The intuition for this result is that larger $\rho$ weakens the underinvestment force while smaller $n$ strengthens the underinvestment force and weakens the pecuniary externality.

the constrained CP’s allocation. Introducing a subsidy in $t = 1$ in the low state if the banking sector is imperfectly competitive, even if not required, will change the period zero problem in an interesting way, which is why I choose to take this approach.

24If $\rho \to \infty$, then this environment maps to a model where the banking sector faces Cournot competition.
From equation 17, one can see that the smaller $n$ is, the more likely it is that in period zero the optimal policy is to have subsidy since smaller $n$ makes the underinvestment force stronger and also weakens the pecuniary externalities. Similarly, the smaller $\rho$ is, the stronger the underinvestment force is and the more likely it is that a subsidy will be required. In addition to comparative statics with respect to $\rho$ and $n$, one can consider the limiting cases of $\alpha \rightarrow 1$ and $\eta \rightarrow 0$. $\alpha \rightarrow 1$ affects the concavity of the production technology, which in turn affects the pecuniary externalities of this problem. The pecuniary externalities work through the fact that a single banker, if small, does not internalize the fact that the more he invests in $L_0$, the lower the marginal rate of return of the other bankers is in the crisis state. This leads to lower direct returns of other bankers — "bankruptcy" pecuniary externality — and if the net worth constraint binds in the crisis state, to tighter net worth constraints for the other bankers — "net worth constraint" pecuniary externality. If the production technology approaches a linear production technology, $\alpha \rightarrow 1$, then a single banker’s actions no longer affect the marginal return of the other bankers and the pecuniary externalities will disappear, i.e. $\tau^c_0 = 0$. If $\eta \rightarrow 0$, it is easier for bankers to lever and the "net worth constraint" pecuniary externality is stronger. As a result, $\tau^c_0 \rightarrow \infty$.

Next I provide the explicit formulas for subsidies and capital account taxes. Conditional on correcting for future underinvestment using a subsidy in $t = 1$, if a country has only a few banks and/or a small degree of substitution between loans, in $t = 0$, the policy maker should use a subsidy and not capital account controls. The formula for the optimal period zero subsidy conditional on $\tau^c_0 (s_0) < 0$ is given by

$$\tau^*_0 = 1 - \frac{\frac{1}{\gamma}R_1^f \pi_H A_H}{R_1^f \pi_H \alpha A_H - (1 - \pi_H) A_L (1 - \alpha) \left(1 - \frac{1}{n}\right) \left[R_1^f + \left[\alpha \bar{A} (L_1^{CP} (s_L))^{\alpha - 1} - R_1^f\right] \frac{1}{\gamma}\right]} \tag{18}$$

One can show that $\tau^*_0 (\rho) < 0$ and $\tau^*_0 (n) < 0$. Also if $n = 1$, then the optimal policy in period zero is a subsidy given by $\tau^*_0 = 1 - \alpha$ and also a subsidy in $t = 1$, $\tau^*_1 = 1 - \alpha$. What is interesting to note about this result is that even if the monopolist wants to overinvest, due to the interaction between future underinvestment and binding future net worth constraints, one set of optimal instruments to correct for this overinvestment are actually subsidies in period $t = 0$ and in period $t = 1$ rather than capital account controls.\(^{25}\)

In contrast, if the banking sector has a lot of banks and loans are highly substitutable, then capital account controls would be required in period zero conditional on optimal subsidies being implemented in $t = 1$ and the optimal capital account tax is given by

\(^{25}\) Notice that if $n = 1$ and there was ex-ante overinvestment and the policy maker had chosen to use no subsidy in $t = 1$ in the crisis state since the net worth constraint is binding, the period zero optimal policy would have been capital account controls. This would be an alternative way to implement the allocation to the case I examine in this paper.
\[
\tau_{0}^{cc} = -\Phi R^{f} \pi_{H} \alpha A_{H} \left( 1 - \frac{1}{\gamma} \right) + \left( R^{f} + \left[ \alpha \bar{A} \left( L_{1}^{CP} (s_{L}) \right) \alpha - R^{f} \right] \frac{1}{\eta} \right) \Phi (1 - \tau_{H}) A_{L} (1 - \alpha) \left( 1 - \frac{1}{n} \right)
\]

Conditional on a capital account controls being required, the tax is larger, the higher the degree of substitution between loans is \( \tau_{0}^{cc} (\rho) > 0 \) and the larger number of banks is \( \tau_{0}^{cc} (n) > 0 \).

5 Further Discussion

In practice, policy makers have an access to another instrument, in addition to the capital account controls, that can help them control overinvestment – minimum bank capital requirement. Minimum bank capital requirements require that banks finance at least a fraction of their loans using equity. Therefore, this policy instrument appears to be similar to the bank net worth constraint in this model, which both the bankers and the CP take as exogenous and given. However, there is one crucial difference. The bank net worth constraint is imposed exogenously by foreign lenders to prevent the banker and the CP from diverting part or all of the borrowed amount to be used for a project that the foreigner cannot seize, for example (see the discussion on net worth constraints in the literature review).\(^{26}\) In contrast, the minimum bank capital requirement is determined by the policy maker (CP) in order to correct for overinvestment. Therefore, those two constraints are very different.

One might ask the question why not simply use minimum bank capital requirements to correct for the overinvestment instead of imposing capital account controls. In the model I develop in this paper, equity is exogenous and fixed. However, if I were to allow bankers to raise costly equity, the minimum bank capital requirement will be no longer sufficient to replicate the constrained Central Planner’s allocation. Bankers will be tempted to raise too much costly equity in order to circumvent the minimum bank capital requirement and lend more than the socially optimal amount. In contrast, the capital account controls if implemented as a tax on foreign borrowing rates, as specified in this paper, will be sufficient to replicate the constrained CP’s allocation even if costly equity is introduced in the model. Therefore, it is not surprising that even though emerging markets impose minimum bank capital requirements, they also rely on capital account controls.

References


\(^{26}\) I don’t model explicitly the moral hazard due to the desire of the banker (and also the CP) to divert resources but the literature on net worth constraints has given different examples of how this can be done.


A Appendix

A.1 Graphs

On the y-axis, Graph 1 plots the H measure of banking sector competition calculated by [Claessens, 2009] using [Panzar and Rosse, 1987]’s methodology. If the H measure is equal to 1, the financial sector is perfectly competitive and the smaller the number is, the less competitive the financial sector is. On the x-axis, Graph 1 plots a measure of banking sector concentration calculated by the updated version of [Beck, Demirguc-Kunt and Levine, 2000]. The concentration measure is defined as the assets of the three largest banks as a share of assets of all commercial banks using BankScope data. (The Graph uses the numbers from the April 2013 version.)
**A.2 The Problem of the Entrepreneur**

I solve the problem of the entrepreneur backwards. In $t = 1$, all the uncertainty is resolved. The entrepreneur takes $R_1^t(s_1)$, $D_2(s_1)$ and $D_1(s_1)$ as given and optimizes
The first order condition with respect to $L_1(s_1)$ determines the demand for aggregate loans as a function of $\bar{R}_1(s_1)$

$$L_1(s_1) = \left[ \frac{\bar{R}_1(s_1)}{\alpha A} \right]^\frac{1}{\alpha - 1}$$  \hspace{1cm} (19)$$

Alternatively the problem can be re-written as (in order to solve for $L_{i,1}$)

$$\max_{L_{i,1}} \bar{A} \left( \left[ \sum_{i=1}^{n} \frac{1}{n} (L_{i,1})^{(\rho-1)/\rho} \right]^{\rho/\rho-1} \right)^{\alpha} \left( \sum_{i=1}^{n} \frac{1}{n} \bar{R}_{i,1} L_{i,1} \right)$$

The first order condition with respect to $L_{i,1}$ is given by

$$L_{i,1} = \left[ \frac{\bar{R}_{i,1}}{(L_1)^{\alpha-1+\frac{1}{\rho}} \alpha A} \right]^{-\rho} = L_1 \left[ \frac{\bar{R}_{i,1}}{\bar{R}_1} \right]^{-\rho}$$  \hspace{1cm} (20)$$

where the aggregate lending rate is $\bar{R}_1 = \left[ \sum_{i=1}^{n} \frac{1}{n} \left( \bar{R}_{i,1} \right)^{(1-\rho)} \right]^{\frac{1}{1-\rho}}$. In $t = 0$ the optimization problem simplifies to

$$\max_{L_{i,0}} \pi_H \left[ A_H \left( \left[ \sum_{i=1}^{n} \frac{1}{n} (L_{i,0})^{(\rho-1)/\rho} \right]^{\rho/\rho-1} \right)^{\alpha} \left( \sum_{i=1}^{n} \frac{1}{n} \bar{R}_{i,0} L_{i,0} \right) \right]$$

After taking first order conditions and re-arranging, the equilibrium system of period zero equations is

$$L_0 = \left[ \frac{\bar{R}_0}{\alpha A_H} \right]^\frac{1}{\alpha - 1}$$

$$L_{i,0} = \left[ \frac{\bar{R}_{i,0}}{(L_0)^{\alpha-1+\frac{1}{\rho}} \alpha A_H} \right]^{-\rho} = L_0 \left[ \frac{\bar{R}_{i,0}}{\bar{R}_0} \right]^{-\rho}$$  \hspace{1cm} (21)$$

$$\bar{R}_0 = \left[ \sum_{i=1}^{n} \frac{1}{n} \left( \bar{R}_{i,0} \right)^{(1-\rho)} \right]^{\frac{1}{1-\rho}}$$  \hspace{1cm} (22)$$

34
A.3 The Problem of Banker $i$

I solve the problem of banker $i$ backwards. Since all the uncertainty is resolved in the middle period $t = 1$, there is no default in $t = 2$. All the equations are a function of the state $s_1$ (for now the notation is suppressed). The banker maximizes

$$\max_{L_{i,1}} \tilde{R}_{i,1}^l (L_{i,1}) L_{i,1} - R_1^l [L_{i,1} - N_{i,1}] + \lambda_{i,1} [N_{i,1} - \eta L_{i,1}]$$

The first order condition with respect to $L_{i,1}$ is

$$\left[ \frac{\partial \tilde{R}_{i,1}^l (L_{i,1})}{\partial L_{i,1}} L_{i,1} + \tilde{R}_{i,1}^l - R_1^l \right] - \lambda_{i,1} \eta = 0 \quad (23)$$

where $\frac{\partial \tilde{R}_{i,1}^l (L_{i,1})}{\partial L_{i,1}}$ is given by totally differentiating the rewritten equation 20

$$\tilde{R}_{i,1}^l (L_{i,1}) = (L_{i,1})^{\frac{1}{\rho}} (L_1)^{\alpha - \frac{1}{\rho}} \alpha \tilde{A}$$

with respect to $L_{i,1}$ and taking into account the fact $\frac{\partial L_1}{\partial L_{i,t}} = \frac{1}{n} \left( \frac{L_1}{L_{i,t}} \right)^{\frac{1}{\rho}}$. $\frac{\partial \tilde{R}_{i,1}^l}{\partial L_{i,1}} = - \left[ \left( \frac{\tilde{R}_{i,1}^l}{R_1^l} \right)^{\rho} - ((\alpha - 1) \rho + 1) \frac{1}{n} \left( \frac{L_1}{L_{i,1}} \right)^{\frac{1}{\rho}} \right] \left( \frac{\tilde{R}_{i,1}^l}{L_1} \right) \frac{1}{\rho}$

If the net worth constraint does not bind in $t = 1$ ($\lambda_{i,1} = 0$), one can re-write equation 23 as (after plugging in equation 24)

$$L_{i,1} \left[ \left( \frac{\tilde{R}_{i,1}^l}{R_1^l} \right)^{-\rho} - \left( \rho (\alpha - 1) + 1 \right) \frac{1}{n} \left( \frac{L_1}{L_{i,1}} \right)^{\frac{1}{\rho}} \right] \frac{1}{\rho} \frac{\tilde{R}_{i,1}^l}{L_1} = \tilde{R}_{i,1}^l - R_1^l$$

Notice that if the net worth constraint is not binding, the first order condition is not a function of period zero variables. Since banks are symmetric, I consider the symmetric equilibrium which implies $\tilde{R}_{i,1}^l = \tilde{R}_1^l$ and $L_{i,1} = L_1$ and the first order condition simplifies to

$$\tilde{R}_1^l = \gamma R_1^l$$

where $\gamma = \frac{1}{1 - \frac{1}{n} \left( \frac{1 - \frac{1}{\rho} (1 - (\alpha - 1) \rho)}{1 - \alpha} \right)}$ is the mark-up.

If the net worth constraint binds in $t = 1$ ($\lambda_{i,1} > 0$), then the amount of loans in period one becomes

$$L_{i,1} = \frac{1}{\eta} N_{i,1}$$

where
\[
\lambda_{i,1} = \frac{1}{\eta} \left[ \bar{R}_{i,1}^f - R_1^f - L_{i,1} \left[ \left( \frac{\bar{R}_{i,1}^f}{\bar{R}_{i,1}^f} \right)^{-\rho} - (\rho (\alpha - 1) + 1) \frac{1}{n} \left( \frac{L_1}{L_{i,1}} \right)^\frac{1}{\rho} \right] \right] \]

\[
N_{i,1} (s_L) = A_L (L_0)^\alpha \frac{L_{i,0}}{L_0} - R_0^f [L_{i,0} - N_{i,0}]
\]

\[
N_{i,1} (s_H) = \bar{R}_{i,0} L_{i,0} - R_0^f [L_{i,0} - N_{i,0}]
\]

Next I solve the optimization problem of banker \(i\) in \(t = 0\). Assuming no default by banker \(i\) in \(t = 1\), which in equilibrium will be true, and that the net worth constraint binds in the low state in \(t = 1\) and does not bind in the high state in \(t = 1\), banker \(i\) maximizes the expected dividend payment in the last period (it is never optimal to pay dividends before \(t = 2\)).

\[
\max_{L_{i,0}} E_0 N_{i,2} + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}]
\]

\[
= \max_{R_{i,0}} \pi_H \left[ R_{i,1}^f (L_{i,0}, s_H) - R_1^f \right] L_{i,1} (s_H) + (1 - \pi_H) \left[ R_{i,1}^f (L_{i,0}, s_L) - R_1^f \right] L_{i,1} (s_L)
\]

\[
+ R_1^f \left( \pi_H \bar{R}_{i,0} (L_{i,0}) + (1 - \pi_H) A_L L_0^\alpha \frac{1}{L_0} - R_0^f \right) L_{i,0} + R_1^f R_0^f N_{i,0} + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}]
\]

Since in \(t = 1\) the problem is static in the states of nature where the net worth constraint is not binding and I consider parametrization where it is not binding in the high state in \(t = 1\), \(L_{i,0}\) will not affect \(L_{i,1} (s_H)\) and \(R_{i,1} (s_H)\). With that in mind, the first order condition with respect to \(L_{i,0}\) becomes

\[
(1 - \pi_H) \left[ \frac{\partial R_{i,1}^f (s_L)}{\partial L_{i,0}} L_{i,1} (s_L) + \left( \bar{R}_{i,1}^f (s_L) - R_1^f \right) \frac{\partial L_{i,1} (s_L)}{\partial L_{i,0}} \right] +
\]

\[
R_{i,1}^f \left( \pi_H \bar{R}_{i,0} + (1 - \pi_H) A_L L_0^\alpha \frac{1}{L_0} - R_0^f \right) +
\]

\[
R_{i,1}^f \left( \pi_H \frac{\partial \bar{R}_{i,0}}{\partial L_{i,0}} + (1 - \pi_H) A_L \left[ \alpha L_0^{\alpha - 1} \frac{\partial L_0}{\partial L_{i,0}} \frac{1}{L_0} - L_0^2 \frac{1}{n} \right] \right) L_{i,0} - \lambda_{i,0} \eta = 0
\]

where

\[
\frac{\partial \bar{R}_{i,1}^f (s_L)}{\partial L_{i,0}} = - \left[ \left( \bar{R}_{i,1}^f (s_L) \right)^\rho \left( \bar{R}_{i,1}^f (s_L) \right)^{(\alpha - 1) \rho + 1} \frac{1}{n} \left( \frac{L_1}{L_{i,1} (s_L)} \right)^\frac{1}{\rho} \right] \left( \frac{\bar{R}_{i,1}^f (s_L)}{L_1 (s_L)} \right) \frac{11 \partial N_{i,1} (s_L)}{\partial L_{i,0}}
\]

\[
\frac{\partial L_{i,1} (s_L)}{\partial L_{i,0}} = \frac{1}{\eta} \frac{\partial N_{i,1} (s_L)}{\partial L_{i,0}}
\]

\[
(25)
\]

36
\[ \frac{\partial \bar{R}_{i,0}^t}{\partial L_{i,0}} = - \left[ \left( \frac{\bar{R}_{i,0}^t}{R_0^t} \right)^\rho - ((\alpha - 1) \rho + 1) \frac{1}{n} \left( \frac{L_0}{L_{i,0}} \right)^{\frac{1}{\rho}} \right] \left( \frac{\bar{R}_{i,0}^t}{L_0^t} \right) \frac{1}{\rho} \] (26)

\[ \frac{\partial L_t}{\partial L_{i,t}} = \frac{1}{n} \left( \frac{L_t}{L_{i,t}} \right)^{\frac{1}{\rho}} \]

\[ \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} = A_L L_0^\alpha \frac{1}{L_0^t} + A_L \frac{1}{n} \left( \frac{L_0}{L_{i,0}} \right)^{\frac{1}{\rho}} \alpha \left( L_0 \right)^{\alpha - 1} \frac{L_{i,0}}{L_0} - A_L L_0^\alpha \frac{L_{i,0}}{L_0^2} \frac{1}{n} - R_0^f \]

\[ N_{i,1}(s_L) = A_L \left( L_0 \right)^\alpha \frac{L_{i,0}}{L_0} - R_0^f \left[ L_{i,0} - N_{i,0} \right] \]

\[ N_{i,1}(s_H) = \bar{R}_{i,0}^t L_{i,0} - R_0^f \left[ L_{i,0} - N_{i,0} \right] \]

Assume that we start from normal times where the borrowing constraint doesn’t bind in \( t = 0, \lambda_{i,0} = 0 \) and that the equilibrium is symmetric. One can re-write the first order condition with respect to \( L_{i,0} \) as

\[ MC \left( L_0 \right) = - (1 - \pi_H) \lambda_1 (s_L) \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} = R_1^t E_0 \frac{\partial N_{i,1}(s_1)}{\partial L_{i,0}} = MB \left( L_0 \right) \]

where

\[ \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} = A_L L_0^\alpha \left( 1 - \frac{1}{n} (1 - \alpha) \right) - R_0^f < 0 \]

\[ \frac{\partial N_{i,1}(s_H)}{\partial L_{i,0}} = \alpha A_H L_0^\alpha \frac{1}{\gamma} - R_0^f > 0 \]

and if \( \lambda_1 (s_L) > 0, \)

\[ \lambda_1 (s_L) = \frac{1}{\eta} \frac{\partial N_{i,2}(s_L)}{\partial L_{i,1}(s_L)} = \frac{1}{\eta} \left[ \frac{\alpha \bar{A} \left( L_1 (s_L) \right)^{\alpha - 1} \frac{1}{\gamma} - R_1^f}{} \right] > 0 \]

(28)

Notice that the bank will never choose an allocation that leads to bank bankruptcy in \( t = 1 \) because, given the assumptions made in this model, no bank loans would imply no investment and \( K_1 = L_1 = 0 \). As a result, if \( L_1 \to 0, \) then \( \lambda_1 (s_L) \to \infty \) and \( MC \left( L_0 \right) \to \infty \) and, therefore, banker \( i \) will never choose period zero allocation which will lead to bank default. If the entrepreneur has another source of income, then this result can be changed and one can generalize the model to include bank default. After plugging in all the equations for the case \( \lambda_1 (s_L) > 0, \)

\[ (1 - \pi_H) \frac{1}{\eta} \left[ \left( 1 - (1 - \alpha) \frac{1}{n} \right) A_L L_0^\alpha - R_0^f \right] \left[ \alpha \bar{A} \left( L_1 (s_L) \right)^{\alpha - 1} \frac{1}{\gamma} - R_1^f \right] + \]

\[ R_1^f \left( \pi_H \left( L_0 \right)^{\alpha - 1} \alpha A_H \frac{1}{\gamma} + (1 - \pi_H) A_L L_0^\alpha \left( 1 - (1 - \alpha) \frac{1}{n} \right) - R_0^f \right) = 0 \]

(29)
A.4 Constrained Central Planner’s Problem

The Central Planner (CP) chooses the amount of loans provided by every banker, taking into account that the equilibrium played is symmetric. The CP also takes into account the net worth constraint and the first order conditions of the entrepreneur. He internalizes his impact on the return of the bankers. The period one optimization problem of the CP is

\[
\max_{L_1} \bar{A}L_1^\alpha - R_1^f [L_1 - N_1] + \lambda_1^{CP} [N_1 - \eta L_1]
\]

The first order condition with respect to \( L_1 \) is given by

\[
\alpha \bar{A}L_1^{\alpha - 1} - R_1^f - \lambda_1^{CP} \eta = 0.
\]

If the net worth constraint does not bind (\( \lambda_1^{CP} = 0 \)), the bank will lend to the entrepreneur up to the point where the marginal cost equals the marginal benefit from the Central Planner’s point of view, \( \alpha \bar{A}L_1^{\alpha - 1} = R_1^f \). If the net worth constraint binds (\( \lambda_1^{CP} > 0 \)), then

\[
L_1 (s_L) = \frac{1}{\eta} N_1 (s_L)
\]

The optimization problem at \( t = 0 \) is given by

\[
\max_{L_0} (1 - \pi_H) \left( \bar{A} (L_1(s_L))^\alpha - \bar{R}_1^f L_1 (s_L) \right) \\
+ \pi_H \left( R_1^f (A_H (L_0))^\alpha - R_0^f (L_0) L_0 + \bar{A} (L_1(s_H))^\alpha - R_1^f (s_H) L_1 (s_H) \right) \\
+ \pi_H \left[ R_1^f (s_H) - R_1^f \right] L_1 (s_H) + (1 - \pi_H) \left[ R_1^f (s_L) - R_1^f \right] L_1 (s_L) \\
+ R_1^f \left( \pi_H R_0^f + (1 - \pi_H) A_L L_0^{\alpha - 1} - R_0^f \right) L_0 + R_1^f R_0^f N_0 + \lambda_0 [N_0 - \eta L_0]
\]

Re-writing the optimization problem, after plugging in for the first order conditions of the entrepreneur and simplifying,

\[
\max_{L_0} (1 - \pi_H) \left( \bar{A} (L_1(s_L))^\alpha - R_1^f L_1 (s_L) \right) \\
+ \pi_H \left( \bar{A} (L_1(s_H))^\alpha - R_1^f L_1 (s_H) \right) \\
+ R_1^f E_0 A_1 L_0^\alpha - R_1^f R_0^f L_0 + R_1^f R_0^f N_0 + \lambda_0 [N_0 - \eta L_0]
\]

Since \( L_1 (s_H) \) is not a function of \( L_0 \), the first order condition with respect to \( L_0 \) is given by

\[
(1 - \pi_H) \left[ \alpha \bar{A} (L_1(s_L))^{\alpha - 1} - R_1^f \right] \frac{\partial L_1 (s_L)}{\partial L_0} + R_1^f \left( E_0 A_1 \alpha (L_0)^{\alpha - 1} - R_0^f \right) - \lambda_0 \eta = 0
\]

where

\[
\frac{\partial L_1 (s_L)}{\partial L_0} = \frac{1}{\eta} \frac{\partial N_1 (s_L)}{\partial L_0} = \frac{1}{\eta} \left( \alpha A_L (L_0)^{\alpha - 1} - R_0^f \right)
\]
Combining all the equations, one gets

\[
MC^{CP} (L_0) = - (1 - \pi_H) \frac{1}{\eta} \left( \alpha A_L L_0^{\alpha - 1} - R_0^f \right) \left[ \alpha \bar{A} (L_1 (s_L))^{\alpha - 1} - R_1^f \right] 
\]

\[
= R_1^f \left( \alpha E_1 A_1 L_0^{\alpha - 1} - R_0^f \right) = MB^{CP} (L_0) 
\]  

(30)

Next I proof existence and uniqueness conditional on the equilibrium being such that \( \lambda_0^{CP} = 0, \lambda_1^{CP} (s_L) > 0 \) and \( \lambda_1^{CP} (s_H) = 0 \). First, I prove that \( MB^{CP} (L_0) = MC^{CP} (L_0) < 0 \).

\[
MB^{CP} (L_0) = R_1^f (\alpha - 1) \alpha E_0 A_1 L_0^{\alpha - 2} < 0
\]

\[
MC^{CP} (L_0) = (1 - \pi_H) (1 - \alpha) \left\{ \frac{\alpha A_L L_0^{\alpha - 1}}{\eta} \left[ \alpha \bar{A} (L_1 (s_L))^{\alpha - 1} - R_1^f \right] 
\right. 
\]

\[
+ \left[ \frac{1}{\eta} \left( \alpha A_L L_0^{\alpha - 1} - R_0^f \right) \right]^{2} \alpha \bar{A} (L_1 (s_L))^{\alpha - 2} \right\} > 0
\]

Since also \( \lim_{L_0 \to 0} [MB^{CP} (L_0) - MC^{CP} (L_0)] \to \infty \) and \( \lim_{L_0 \to \infty} [MB^{CP} (L_0) - MC^{CP} (L_0)] \to -\infty \), this is sufficient to prove existence and uniqueness.

### A.5 Ramsey Problem

First, I re-derive the problem of the entrepreneur given the presence of subsidies. As before, I solve the model backwards. In \( t = 1 \), all the uncertainty is resolved and the optimization problem of the entrepreneur after plugging in the budget constraint simplifies to

\[
\max_{L_1} \left[ \bar{A} (L_1 (s_1))^{\alpha} - \tilde{R}_1^f (s_1) L_1 (s_1) \right]
\]

where \( \tilde{R}_1^f (s_1) L_1 (s_1) = \sum_{i=1}^{n} \frac{1}{n} (1 - \tau_{i,1}^s) \tilde{R}_{i,1}^f L_{i,1} \). Notice that the subsidy is at the individual bank interest rate level and the aggregate interest rate, \( \tilde{R}_1^f \), is net of subsidies. The first order condition with respect to \( L_t \) determines the demand for aggregate loans as a function of \( L_1 (s_1) \)

\[
L_1 (s_1) = \left[ \frac{\tilde{R}_1^f (s_1)}{\alpha \bar{A}} \right]^{\frac{1}{\alpha - 1}}
\]

(32)

Alternatively the problem can be re-written as

\[
\max_{L_{i,1}} \bar{A} \left[ \left[ \sum_{i=1}^{n} \frac{1}{n} (L_{i,1})^{(\rho - 1)/\rho} \right]^{\frac{\rho}{\rho - 1}} \right]^\alpha - \sum_{i=1}^{n} \frac{1}{n} (1 - \tau_{i,1}^s) \tilde{R}_{i,1}^f L_{i,1}
\]
The first order condition with respect to $L_{i,t}$ is given by

$$\begin{align*}
L_{i,1} &= \left( 1 - \tau_{i,1}^s \right) \frac{\bar{R}_{i,1}^l}{\left( L_{1} \right)^{\alpha - 1 + \frac{1}{\rho} \alpha A}} \right)^{-\rho} = L_{1} \left[ \frac{\left( 1 - \tau_{i,1}^s \right) \bar{R}_{i,1}^l}{\bar{R}_{1}^l} \right]^{-\rho}
\end{align*}$$

Also the aggregate lending rate is

$$\bar{R}_{1}^l = \left[ \sum_{i=1}^{n} \frac{1}{n} \left( \left( 1 - \tau_{i,0}^s \right) \bar{R}_{i,0}^l \right) \right]^{\frac{1}{1-\rho}}$$

In $t = 0$, given that I consider parametrization where the firm defaults in the crisis state, the optimization problem is

$$\begin{align*}
\max_{L_{i,0}} \pi_H &= \left[ A_H \left( \sum_{i=1}^{n} \frac{1}{n} \left( L_{i,0} \right)^{(\rho-1)/\rho} \right) ^{\frac{\rho}{\rho-1}} \right] ^{\alpha} - \sum_{i=1}^{n} \frac{1}{n} \left( 1 - \tau_{i,0}^s \right) \bar{R}_{i,0}^l L_{i,0}
\end{align*}$$

After taking the first order conditions and re-arranging the equilibrium system of equations is

$$\begin{align*}
L_{0} &= \left( \frac{\bar{R}_{0}^l}{\alpha A_H} \right)^{\frac{1}{1-\rho}}
\tag{34}
\end{align*}$$

$$\begin{align*}
L_{i,0} &= \left[ \frac{\left( 1 - \tau_{i,0}^s \right) \bar{R}_{i,0}^l}{\left( L_{0} \right)^{\alpha - 1 + \frac{1}{\rho} \alpha A_H}} \right]^{-\rho} = L_{0} \left[ \frac{\left( 1 - \tau_{i,0}^s \right) \bar{R}_{i,0}^l}{\bar{R}_{0}^l} \right]^{-\rho}
\end{align*}$$

$$\begin{align*}
\bar{R}_{0}^l &= \left[ \sum_{i=1}^{n} \frac{1}{n} \left( \left( 1 - \tau_{i,0}^s \right) \bar{R}_{i,0}^l \right) \right]^{\frac{1}{1-\rho}}
\tag{35}
\end{align*}$$

Next I re-derive the banker’s problem taking into account the policy instruments. Since all the uncertainty is resolved in the middle period, $t = 1$, there is no firm default in $t = 2$. All the equations are a function of the state $s_1$ (for now the notation is suppressed). The banker maximizes his period two dividend payments/profits

$$\begin{align*}
\max_{L_{i,1}} \bar{R}_{i,1}^l \left( L_{i,1} \right) L_{i,1} - (1 + \tau_{i,1}^{cc}) R_{i,1}^l \left[ L_{i,1} - N_{i,1} \right] + T_{i,2} + \lambda_i \left[ N_{i,1} - \eta L_{i,1} \right]
\end{align*}$$

where $T_{i,2} = \tau_{i,1}^{cc} R_{i,1}^l \left[ L_{i,1} - N_{i,1} \right] - \tau_{i,1}^s \bar{R}_{i,1}^l L_{i,1}$ is predetermined in the beginning of period zero since I assumed that the policy maker is able to commit. Therefore, banker $i$ takes $T_{i,2}$ as given. The policy maker also anticipates the optimal actions of the bankers. The first order condition with respect to $L_{i,1}$ is

40
\[ \left[ \frac{\partial \tilde{R}_{i,1}^l (L_{i,1})}{\partial L_{i,1}} L_{i,1} + \tilde{R}_{i,1}^l - (1 + \tau_{i,1}^{cc}) R_1^f \right] - \lambda_{i,1} \eta = 0 \]

where \( \frac{\partial \tilde{R}_{i,1}^l (L_{i,1})}{\partial L_{i,1}} \) is given by totally differentiating the rewritten equation \( L_{i,1} = \left[ \frac{(1 - \tau_{i,1}^s) R_{i,1}^l}{(L_1)^{\alpha - 1} + \frac{1}{\alpha} \lambda} \right]^{-\rho} \) \( (1 - \tau_{i,1}^s) \tilde{R}_{i,1}^l = (L_{i,1})^{-\frac{1}{\rho}} (L_1)^{\alpha - 1 + \frac{1}{\rho}} \alpha \tilde{A} \)

with respect to \( L_{i,1} \) and taking into account the fact \( \frac{\partial L_t}{\partial L_{i,t}} = \frac{1}{n} \left( \frac{L_t}{L_{i,t}} \right)^\frac{1}{n} \).

If the net worth constraint does not bind in \( t = 1 \) (\( \lambda_{i,1} = 0 \)), then the first order condition with respect to \( L_{i,1} \) becomes

\[ \tilde{R}_{i,1}^l \left( 1 - \left[ 1 - \frac{1}{n} \left( \frac{L_1}{L_{i,1}} \right)^\frac{1}{\rho} ((\alpha - 1) \rho + 1) \right] \frac{1}{\rho} \right) - (1 + \tau_{i,1}^{cc}) R_1^f = 0 \]

If the net worth constraint is not binding, the first order condition is not a function of period zero variables. Since banks are symmetric, I consider the symmetric equilibrium which implies \( (1 - \tau_{i,1}^s) \tilde{R}_{i,1}^l = \tilde{R}_1^l \) and \( L_{i,1} = L_1 \). In a symmetric equilibrium

\[ \tilde{R}_1^l = \gamma (1 - \tau_{i,1}^s) (1 + \tau_{i,1}^{cc}) R_1^f \]

where \( \gamma = \frac{1}{(1 - \frac{1}{\rho} (1 - \frac{1}{\rho}) - (1 - \alpha) \frac{1}{\rho})} \) is the mark-up.

If the net worth constraint binds in \( t = 1 \) (\( \lambda_{i,1} > 0 \)), the amount of loans in period one becomes

\[ L_{i,1} = \frac{1}{\eta} N_{i,1} \]

where

\[ \lambda_{i,1} = \frac{1}{\eta} \left[ \tilde{R}_{i,1}^l \left( 1 - \left[ 1 - \frac{1}{n} \left( \frac{L_1}{L_{i,1}} \right)^\frac{1}{\rho} ((\alpha - 1) \rho + 1) \right] \frac{1}{\rho} \right) - (1 + \tau_{i,1}^{cc}) R_1^f \right] \]

\[ N_{i,1} (s_L) = A_L \left( L_0 \right)^{\alpha} \frac{L_{i,0}}{L_0} - (1 + \tau_{i,0}^{cc}) R_0^f [L_{i,0} - N_{i,0}] + T_{i,1} (s_L) \]

\[ N_{i,1} (s_H) = \tilde{R}_{i,0} L_{i,0} - (1 + \tau_{i,0}^{cc}) R_0^f [L_{i,0} - N_{i,0}] + T_{i,1} (s_H) \]
where $T_{i,1}(s_L) = \tau_{i,0}^c R_1^f [L_{i,0} - N_{i,0}]$ and $T_{i,1}(s_H) = \tau_{i,0}^c R_1^f [L_{i,0} - N_{i,0}] - \tau_{i,0}^s R_1^f L_{i,0}$.

In a symmetric equilibrium, $\bar{R}_{i,1}^f = \frac{\bar{R}_1^f}{(1-\tau_1^f)} = \frac{\alpha M L_0^{\alpha-1}}{(1-\tau_1^f)}$

$$L_1 = \frac{1}{\eta} N_1$$  \hspace{1cm} (36)

where $\lambda_1 = \frac{1}{\eta} \left[ \frac{\bar{R}_1^f}{(1-\tau_1^f)} - 1 - (1 + \tau_{i,0}^c) R_1^f \right]$.

Next, I solve the optimization problem of the banker in $t = 0$. Since, in equilibrium, there will be no default by banker $i$ in $t = 1$ and considering only parametrization where the net worth constraint binds in the low state in $t = 1$ and does not bind in the high state in $t = 1$, banker $i$ maximizes the expected dividend payment in the last period (notice that it is never optimal to pay dividends before $t = 2$).

$$\max_{L_{i,0}} E_0 N_{i,2} + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}]$$

$$= \max_{L_{i,0}} \pi_H \left[ \bar{R}_{i,1}^f (L_{i,0}; s_H) - (1 + \tau_{i,1}^c (s_H)) R_1^f \right] L_{i,1} (L_{i,0}; s_H)$$

$$+ (1 - \pi_H) \left[ \bar{R}_{i,1}^f (L_{i,0}; s_L) - (1 + \tau_{i,1}^c (s_L)) R_1^f \right] L_{i,1} (L_{i,0}; s_L)$$

$$+ \pi_H \left[ (1 + \tau_{i,1}^c (s_H)) R_1^f \left[ \bar{R}_{i,0} - (1 + \tau_{i,0}^c) R_0^f \right] L_{i,0}$$

$$+ (1 - \pi_H) \left[ (1 + \tau_{i,1}^c (s_L)) R_1^f \left[ A_L (L_0)^{\alpha} \frac{1}{L_0} - (1 + \tau_{i,0}^c) R_0^f \right] L_{i,0}$$

$$+ E_0 \left[ (1 + \tau_{i,0}^c (s_1)) R_1^f \left[ (1 + \tau_{i,1}^c) R_0^f N_{i,0} + T_{i,1} (s_1) \right] \right] + \lambda_{i,0} [N_{i,0} - \eta L_{i,0}]$$

Since in $t = 1$ the problem is static in the states of nature where the net worth constraint is not binding and I consider parametrization where it is not binding in the high state in $t = 1$, $L_{i,0}$ will not affect $L_{i,1} (s_H)$ and $R_{i,1} (s_H)$. With that in mind, the first order condition with respect to $L_{i,0}$ becomes

$$(1 - \pi_H) \left( \frac{\partial \bar{R}_{i,1}^f (s_L)}{\partial L_{i,0}} L_{i,1} (s_L) + \left( \bar{R}_{i,1}^f (s_L) - (1 + \tau_{i,1}^c (s_L)) R_1^f \right) \frac{\partial L_{i,1} (s_L)}{\partial L_{i,0}} \right)$$

$$+ \pi_H \left[ (1 + \tau_{i,1}^c (s_H)) R_1^f \left( \bar{R}_{i,0} + \frac{\partial \bar{R}_{i,0}}{\partial L_{i,0}} L_{i,0} \right) - E_0 \left[ (1 + \tau_{i,1}^c (s_1)) (1 + \tau_{i,0}^c) R_1^f R_0^f \right.$$  

$$+ (1 - \pi_H) \left[ (1 + \tau_{i,1}^c (s_L)) R_1^f \left[ A_L (L_0)^{\alpha} \frac{1}{L_0} + \left[ \alpha A_L (L_0)^{\alpha-1} \frac{1}{L_0} \partial L_0 \partial L_{i,0} - \frac{1}{n} A_L (L_0)^{\alpha} \frac{1}{L_0^2} \right] L_{i,0} \right] \right. \right]$$

$$- \lambda_{i,0} \eta = 0$$

where
A.6 Propositions

**Proposition 1:** (i) If parametrization is such that there is a crisis in the low state in \( t = 1 \) and no crisis in the high state in \( t = 1 \) and in \( t = 0 \), \( \lambda_1 (s_L) > 0, \lambda_1 (s_H) = 0, \lambda_0 = 0 \), the equilibrium is unique and exists. (ii) Countries with more competitive banking sector will borrow and invest more than countries with less competitive banking sector, \( \frac{\partial L_{i,0}}{\partial \rho} > 0 \), if

\[
\frac{\partial R_{i,1}^f (s_L)}{\partial L_{i,0}} = \frac{\partial R_{i,1}^f (s_L)}{\partial L_{i,1}} \frac{\partial L_{i,1}}{\partial L_{i,0}} = -\left[ \frac{L_1}{L_{i,1}} - \frac{1}{n} \left( \frac{L_1}{L_{i,1}} \right)^{\frac{1}{\rho}} \left( (\alpha - 1) \rho + 1 \right) \right] \frac{1}{\rho} \frac{L_1^{-1} R_{i,1}^f}{\eta} \frac{1}{\partial L_{i,0}} \frac{\partial N_{i,1} (s_L)}{\partial L_{i,0}}
\]

\[
\frac{\partial L_{i,1} (s_L)}{\partial L_{i,0}} = \frac{1}{\eta} \frac{\partial N_{i,1} (s_L)}{\partial L_{i,0}}
\]

\[
\frac{\partial R_{i,1}^f}{\partial L_{i,0}} = -\left[ \frac{L_0}{L_{i,0}} - \frac{1}{n} \left( \frac{L_0}{L_{i,0}} \right)^{\frac{1}{\rho}} \left( (\alpha - 1) \rho + 1 \right) \right] \frac{1}{\rho} L_0^{-1} R_{i,1}^f
\]

\[
\frac{\partial N_{i,1} (s_L)}{\partial L_{i,0}} = \frac{\partial L_{i,1} (s_L)}{\partial L_{i,0}} = A_L L_0^\alpha \frac{1}{\L_0} + A_L \frac{1}{n} \left( \frac{L_0}{L_{i,0}} \right)^{\frac{1}{\rho}} \alpha (L_0)^{\alpha - 1} \frac{L_{i,0}}{L_0} - A_L L_0^\alpha \frac{L_{i,0}}{\L_0^2} \frac{1}{n} - (1 + \tau_{i,0}^c) R_0^f
\]

\[
N_{i,1} (s_L) = A_L (L_0)^\alpha \frac{L_{i,0}}{L_0} - (1 + \tau_{i,0}^c) R_0^f [L_{i,0} - N_{i,0}]
\]

Assume that \( \lambda_{i,0} = 0 \). In a symmetric equilibrium \( L_t = L_{i,t} \) and \( (1 - \tau_t^s) \tilde{R}_{i,t} = \tilde{R}_{i,t}L_{i,0} - (1 + \tau_{i,0}^c) R_0^f [L_{i,0} - N_{i,0}] \)

\[
MC (L_0) = - (1 - \pi_H) \left( \frac{\alpha A (L_1 (s_L))^{\alpha - 1}}{\gamma (1 - \tau_1^c (s_L))} - (1 + \tau_1^c (s_L)) R_1^f \right) \frac{1}{\eta} \left( \left( 1 - \frac{1}{n} (1 - \alpha) \right) A_L L_0^{\alpha - 1} \right)
\]

\[
= \pi_H (1 + \tau_1^c (s_H)) R_1^f \frac{1}{\gamma} \frac{A_H (L_0)^{\alpha - 1}}{(1 - \tau_0^c (s_1))} - E_0 (1 + \tau_1^c (s_1)) (1 + \tau_0^c) R_1^f R_0^f
\]

\[
+ (1 - \pi_H) (1 + \tau_1^c (s_L)) R_1^f \left( 1 - (1 - \alpha) \frac{1}{n} \right) A_L (L_0)^{\alpha - 1} = MB (L_0)
\]
and \( \frac{\partial \lambda_2}{\partial \rho} < 0 \) if Assumption 1 is not satisfied.

**Proof of Proposition 1:** (i) In order to prove existence and uniqueness I will prove that \( MB'(L_0) - MC'(L_0) < 0 \). From equation 10

\[
\frac{\partial MB}{\partial L_0} - \frac{\partial MC}{\partial L_0} = R_1^f E_0 \frac{\partial N_1(s_1)}{\partial L_0 \partial L_0} + (1 - \pi_H) \left[ \lambda_1(s_L) \frac{\partial N_1(s_L)}{\partial L_0 \partial L_0} + \frac{\partial \lambda_1(s_L)}{\partial L_0} \frac{\partial N_1(s_L)}{\partial L_0} \right] < 0
\]

where from equations 13 and 12

\[
\frac{\partial N_1(s_L)}{\partial L_0 \partial L_0} = (\alpha - 1) A_L L_0^{\alpha - 2} \left( \frac{1}{\eta} [\alpha - 1] + 1 \right) < 0
\]

\[
\frac{\partial N_1(s_H)}{\partial L_0 \partial L_0} = (\alpha - 1) \alpha A_H L_0^{\alpha - 2} \frac{1}{\gamma} < 0
\]

\[
\frac{\partial N_1(s_L)}{\partial L_0} = A_L L_0^{-1} \left( \frac{1}{\eta} [\alpha - 1] + 1 \right) - R_0^f
\]

\[
\frac{\partial \lambda_1(s_L)}{\partial L_0} = \frac{1}{\eta} (\alpha - 1) \alpha \bar{A} (L_1(s_L))^{\alpha - 2} \frac{1}{\gamma} \frac{\partial N_1(s_L)}{\partial L_0}
\]

Notice that it will be the case that either \( \frac{\partial N_1(s_L)}{\partial L_0} < 0 \) and \( \frac{\partial \lambda_1(s_L)}{\partial L_0} > 0 \) or \( \frac{\partial N_1(s_L)}{\partial L_0} > 0 \) and \( \frac{\partial \lambda_1(s_L)}{\partial L_0} < 0 \), which implies that in either case \( \frac{\partial MB}{\partial L_0} - \frac{\partial MC}{\partial L_0} < 0 \). Combined with the fact that \( \lim_{L_0 \to 0} [MB(L_0) - MC(L_0)] \to \infty \) and \( \lim_{L_0 \to \infty} [MB(L_0) - MC(L_0)] \to -\infty \), this is sufficient to prove existence and uniqueness. Since \( MB > 0 \) if \( \lambda_1(s_L) > 0 \), it will have to be the case that if the equilibrium exists, also \( MC > 0 \) which will imply that, in equilibrium, \( \frac{\partial N_1(s_L)}{\partial L_0} < 0 \).

(ii) Totally differentiate \( MB(L_0) - MC(L_0) = 0 \) with respect to \( \rho \)

\[
MB(L_0) - MC(L_0) = R_1^f E_0 \frac{\partial N_{i,1}(s_1)}{\partial L_{i,0}} + (1 - \pi_H) \lambda_1(s_L) \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} = 0
\]

\[
\frac{\partial L_0^*}{\partial \rho} = -R_1^f \pi_H \frac{\partial^2 N_{i,1}(s_H;L_0)}{\partial L_{i,0} \partial \rho} - (1 - \pi_H) \frac{\partial N_{i,1}(s_L)}{\partial L_{i,0}} \frac{\partial \lambda_{i,1}(s_L;L_0)}{\partial \rho}
\]

where \( \frac{\partial (MB(L_0) - MC(L_0))}{\partial L_0} < 0 \) if \( \frac{\partial L_0^*}{\partial \rho} > 0 \) if \( (1 - \pi_H) \frac{\partial N_{i,1}(s_L)}{\partial L_0} \frac{\partial \lambda_{i,1}(s_L;L_0)}{\partial \rho} < R_1^f \pi_H \frac{\partial^2 N_{i,1}(s_H;L_0)}{\partial L_{i,0} \partial \rho} \)
and

\[
\frac{MB(\rho; L_0^*)}{\partial \rho} = R_1^f \pi_H \frac{\partial^2 N_{i,1}(s_H; L_0^*)}{\partial L_{i,0} \partial \rho} = -R_1^f \pi_H L_0^{\alpha_1 - 1} \alpha A_H \frac{1}{\gamma^2} \gamma'(\rho) > 0
\]

\[
\frac{MC(\rho; L_0^*)}{\partial \rho} = -(1 - \pi_H) \frac{\partial N_{i,1}(s_L)}{\partial L_0} \frac{\partial \lambda_{i,1}(s_L; L_0^*)}{\partial \rho} = (1 - \pi_H) \frac{1}{\eta} \left( \left(1 - (1 - \alpha) \frac{1}{n} \right) A_L L_0^{\alpha_1 - 1} - R_0^f \right) \alpha \bar{A} (L_1^* (s_L))^{\alpha_1 - 1} \frac{1}{\gamma^2} \gamma'(\rho) > 0
\]

where \( \gamma'(\rho) < 0 \).

**Proposition 4:** If \( n = 1 \) and the net worth constraint binds only in the low state in \( t = 1 \) for both the CP and the monopolistic bank, then \( L_0^{CP} > L_0^* (n = 1) \) if

\[
R_1^f \left( (1 - \alpha) \pi_H A_H \alpha L_0^{\alpha_1 - 1} + (1 - \pi_H) \frac{1}{\eta} \left( \alpha A_L (L_0^*)^{\alpha_1 - 1} - R_0^f \right) \right) \frac{1}{\eta} \left( \left(1 - (1 - \alpha) \frac{1}{n} \right) A_L L_0^{\alpha_1 - 1} - R_0^f \right) > 0
\]

(Assumption 3)

where \( L_1^* = \frac{1}{\eta} \left( A_L (L_0^*)^\alpha - R_1^f [L_0 - N_0] \right) \). If Assumption 3 is violated, then the monopolist overinvests relative to the Central Planner \( L_0^{CP} < L_0^* (n = 1) \).

**Proof of Proposition 4:** The proof is based on a local perturbation around the decentralized equilibrium \( L_0^* \) which is without loss of generality given that \( MB'(L_0) - MC'(L_0) < 0 \) and \( MB^{CP}(L_0) - MC^{CP}(L_0) < 0 \). Re-writing the first order conditions of the CP and the banker from the decentralized equilibrium, after imposing \( n = 1 \)

\[
MB(L_0) - MC(L_0) = R_1^f \left( \pi_H \alpha^2 A_H L_0^{\alpha_1 - 1} + (1 - \pi_H) A_L L_0^{\alpha_1 - 1} - R_0^f \right) + (1 - \pi_H) \frac{1}{\eta} \left[ \alpha \bar{A} (L_1 (s_L))^{\alpha_1 - 1} - R_1^f \right] \left( A_L L_0^{\alpha_1 - 1} - R_0^f \right)
\]

\[
MB^{CP}(L_0) - MC^{CP}(L_0) = R_1^f \left( E_0 A_1 \alpha (L_0)^{\alpha_1 - 1} - R_0^f \right) + (1 - \pi_H) \left[ \alpha \bar{A} (L_1 (s_L))^{\alpha_1 - 1} - R_1^f \right] \frac{1}{\eta} \left( \alpha A_L (L_0)^{\alpha_1 - 1} - R_0^f \right)
\]

Since \( MB'(L_0) - MC'(L_0) < 0 \) and \( MB^{CP}(L_0) - MC^{CP}(L_0) < 0 \), then \( L_0^{CP} > L_0^* (n = 1) \) if

\[
MB^{CP}(L_0^*) - MC^{CP}(L_0^*) - (MB(L_0^*) - MC(L_0^*)) = R_1^f \left( (1 - \alpha) \pi_H A_H \alpha L_0^{\alpha_1 - 1} + (1 - \pi_H) \frac{1}{\eta} \left( \alpha A_L (L_0^*)^{\alpha_1 - 1} - R_0^f \right) \right) \frac{1}{\eta} \left( \left(1 - (1 - \alpha) \frac{1}{n} \right) A_L L_0^{\alpha_1 - 1} - R_0^f \right) > 0
\]
which is true conditional on Assumption 3 being satisfied.

**Proposition 5:** For any \( n \) and \( \rho \) and if the net worth constraint binds only in the low state in \( t = 1 \) for both the CP and banker in the decentralized equilibrium, then \( L_0^{CP} > L_0^* \) if

\[
R_1^f \pi H A_H \left( 1 - \frac{1}{\gamma} \right) L_0^{a-1} - R_1^f (1 - \pi H) A_L (1 - \alpha) \left( 1 - \frac{1}{n} \right) L_0^{a-1} + \]

underinvestment in \( t = 0 \geq 0 \)


\[
= (1 - \pi H) \frac{1}{\eta} \left[ \alpha \tilde{A} (L_1^* (s_L))^{a-1} - R_1^f \right] (1 - \alpha) \left( 1 - \frac{1}{n} \right) A_L L_0^{a-1} \]

"bankruptcy" pecuniary externality \( \leq 0 \)


\[
- (1 - \pi H) \frac{1}{\eta} \left[ R_0^f - (1 - (1 - \alpha) \frac{1}{n}) A_L L_0^{a-1} \right] \alpha \tilde{A} (L_1^* (s_L))^{a-1} \left( 1 - \frac{1}{\gamma} \right) > 0 \quad (46)
\]

underinvestment in \( t = 1 \leq 0 \)

where \( L_1^* = \frac{1}{\eta} \left( A_L (L_1^*)^a - R_1^f [L_0 - N_0] \right) \). If Assumption 4 is violated, then the banker over-invests relative to the Central Planner \( L_0^{CP} < L_0^* \).

**Proof of Proposition 5:** The proof is based on a local perturbation around the decentralized equilibrium \( L_0^* \) which is without loss of generality given that \( MB' (L_0) - MC' (L_0) < 0 \) and \( MB'^{CP} (L_0) - MC'^{CP} (L_0) < 0 \). Re-writing the first order conditions of the banker from the decentralized equilibrium

\[
MB (L_0) - MC (L_0) = R_1^f \left( \pi H (L_0)^{a-1} \alpha A_H \frac{1}{\gamma} + (1 - \pi H) A_L L_0^{a-1} \left( \frac{1}{n} [\alpha - 1] + 1 \right) - R_0^f \right) \quad (47)
\]

\[
+ (1 - \pi H) \frac{1}{\eta} \left[ \left( \alpha - 1 \right) \frac{1}{n} + 1 \right] A_L L_0^{a-1} - R_0^f \left[ \alpha \tilde{A} (L_1^* (s_L))^{a-1} \frac{1}{\gamma} - R_1^f \right]
\]

Since \( MB' (L_0) - MC' (L_0) < 0 \) and \( MB'^{CP} (L_0) - MC'^{CP} (L_0) < 0 \), then \( L_0^{CP} > L_0^* \) if \( MB^{CP} (L_0^*) - MC^{CP} (L_0^*) - (MB (L_0^*) - MC (L_0^*)) > 0 \), which will be true if Assumption 4 is satisfied.

**Proposition 6:** Assuming the policy maker can commit and the net worth constraint binds in the crisis state, the CP’s allocation can be decentralized using a lump sum transfer to entrepreneurs, \( T_t \), subsidy on entrepreneurs’ borrowing rates, \( \tau_t^s \geq 0 \), and a capital account control in the form of a tax on banker’s borrowing rates from foreigners, \( \tau_t^{cc} \geq 0 \). One possible implementation of the constrained Central Planner’s allocation is given by: \( \tau_1^s = 1 - \frac{1}{\tilde{\gamma}} \), \( \tau_1^{cc} = 0 \). If \( \tilde{\tau}_0^{cc} (\tau_0^s = 0) > 0 \), then \( \tau_0^{cc} = \tilde{\tau}_0^{cc} (\tau_0^s) \) and \( \tau_0^s = 0 \). If \( \tilde{\tau}_0^{cc} (\tau_0^s = 0) < 0 \), then \( \tau_0^{cc} = 0 \) and
\[ \tau_0^s > 0, \text{ where } \tau_0^s \text{ is pinned down by } \tilde{\tau}_0^{cc}(\tau_0^s) = 0 \text{ and} \]

\[
\tilde{\tau}_0^{cc}(\tau_0^s) = -\Phi R_1^f \pi_H \alpha A_H \left(1 - \frac{1}{\gamma (1 - \tau_0^s)}\right) + \Phi R_1^f (1 - \pi_H) A_L (1 - \alpha) \left(1 - \frac{1}{n}\right) \tag{48}
\]

\[
+ \Phi (1 - \pi_H) \left[ \alpha \hat{A} \left( L_1^{CP} (s_L) \right)^{\alpha - 1} - R_1^f \right] \frac{1}{\eta} A_L (1 - \alpha) \left(1 - \frac{1}{n}\right)
\]

\[
\Phi = \left( L_0^{CP} \right)^{1 - \alpha} R_0^f \left[ R_1^f + (1 - \pi_H) \frac{1}{\alpha \hat{A} ( L_1^{CP} (s_L) )^{\alpha - 1} - R_1^f} \right] > 0 \text{ and } CP \text{ stands for the optimal allocation of the CP.}
\]

**Proof of Proposition 6:** In order to determine \( \tau_1^s \), compare the CP’s first order condition with respect to \( L_1 \), \( \tilde{R}_1^f = R_1^f \) to the first order condition of the banker with respect to \( L_1 \) from the Ramsey Problem, \( \tilde{R}_1^f = (1 - \tau_1^s) (1 + \tau_1^{cc}) \gamma R_1^f \). Since \( (1 - \tau_1^s) (1 + \tau_1^{cc}) \gamma = 1 \) and \( \tau_1^{cc} \geq 0, \tau_1^s \geq 0, \gamma \geq 1 \), then \( \tau_1^{cc} = 0 \) and \( \tau_1^s = 1 - \frac{1}{\gamma} \). In practice, the CP’s allocation can be decentralized in many different ways. Here, I derive the general formula for the optimal \( \tilde{\tau}_0^{cc}(\tau_0^s) \) as a function of \( \tau_0^s \in [0, 1] \). Combine the first order condition of the banker with respect to \( L_0 \) from the Ramsey Problem, equation 38, with the first order condition of the CP, equation 30. Taking into account the fact that \( \tau_1^{cc} = 0 \) and \( \tau_1^s = 1 - \frac{1}{\gamma} \), one can derive \( \tilde{\tau}_0^{cc}(\tau_0^s) \) specified in the proposition, where \( L_1^{CP} (s_L) = \frac{1}{\eta} N_1^{CP} (s_L) = \frac{1}{\eta} \left( A_L \left( L_0^{CP} \right)^{\alpha} - R_0^f \left[ L_0^{CP} - N_0 \right] \right) \).