Determiners and generalized quantifiers

1 *The*

- A *the*-phrase denotes an referential individual of type e. The definite determiner *the* is of type $\langle et, e\rangle$: it combines with a common noun of type $\langle e, t\rangle$ to return an referential individual of type e.[1]

  (2) a. $[\![\text{the cat}]\!] = \iota x_e[\mathsf{cat}(x)]$
  (The unique entity x such that x is a cat)

  b. $[\![\text{the}]\!] = \lambda P_{\langle e,t\rangle}.\iota x_e[P(x)]$
  ($[\![\text{the}]\!]$ applies to a predicate $P_{\langle e,t\rangle}$ and returns the unique entity x s.t. $P(x)$ is true.)

  c. $[\![\text{The cat snores}]\!] = [\![\text{snores}]\!]([\![\text{the cat}]\!])$
  $= (\lambda y_e.\mathsf{snores}(y))(\iota x_e[\mathsf{cat}(x)])$
  $= \mathsf{snores}(\iota x_e[\mathsf{cat}(x)])$

  The definite determiner *the* presupposes uniqueness.

  (3) a. [Pointing at one cat], the cat snores.
  b. [Pointing at two cats], # the cat snores.

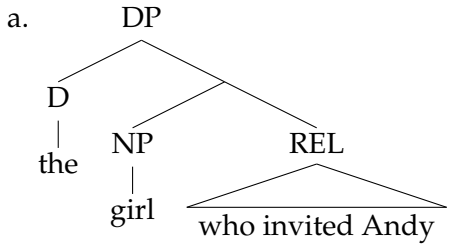
  This presupposition is introduced by the ι-operator:

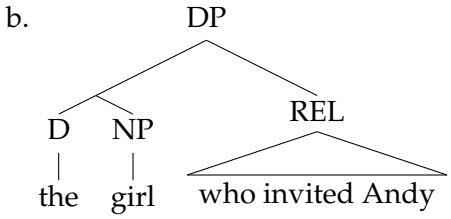
  (4) $\iota x_e[P(x)]$ is defined iff there exists exactly one x such that $P(x) = 1$.

- Adding relative clauses (REL)

  Discussion: Which of the following (simplified) trees correctly describes the structure of the definite description "the girl who invited Andy"? [In other words, does the relative clause "who invited Andy" modify "girl" or "the girl"?] Why?

  (5) the girl who invited Andy

  a.
  

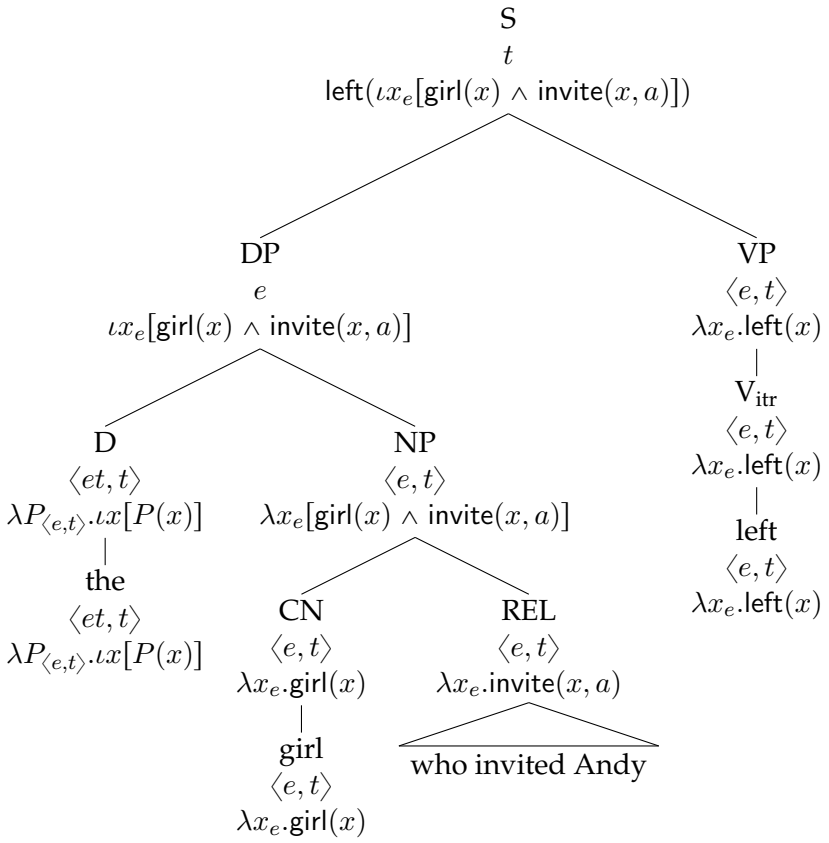
  b.
  

[1] The introduced lexicon of *the* is highly simplified. It doesn't explain why we can say "the boys", since it is nonsense to say "the unique boys". A more accurate lexical entry of *the* is as follows:

(1) $[\![\text{the}]\!] = \lambda P_{\langle e,t\rangle}.\exists x[P(x) \wedge \forall y[P(y) \rightarrow y \leqslant x]].$
$\iota x_e[P(x) \wedge \forall y[P(y) \rightarrow y \leqslant x]]$

- **Exercise:** Compose the following sentence:

  (6) The girl who invited Andy left.



2 Generalized quantifiers and quantificational determiners

- Recall: How would you translate the following English sentences in set-theoretical notations?

  (7) a. Some cat meows.
  b. Every cat meows.
  c. No cat meows.

- In set-theoretical notations, a quantificational determiners like *some* and *every* is defined as a relation between two sets of entities. Then, how should we define phrases like *some/every/no cat* and determiners like *some/every/no* in λ-notations?

2.1 Generalized quantifiers

- Quantificational DPs (e.g. *everything*, *something*, *nothing*, *every cat*, *some cat*, *no cat*) are not individuals:

  (8) *Law of Contradiction*
  a. Mary is coming and Mary is not coming. (Contradiction)
  b. Someone is coming and someone is not coming. (Not contradiction)

(9) *Law of Excluded middle*

a. Mary is coming or Mary is not coming. (Tautology)

b. Every is coming or everyone is not coming. (Not tautology)

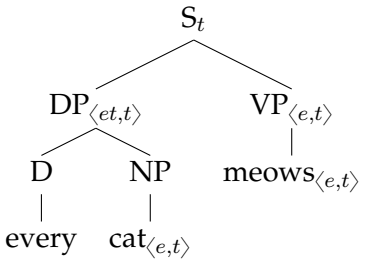
(10) *Only an e-type NP can normally license a singular discourse pronoun.*

a. John /the man/ a man walked in. He looked tired.

b. Every man /no man/ more than one man walked in. *He looked tired.

- We treat quantificational DPs as second-order functions of type $\langle et, t\rangle$, called **generalized quantifiers**. In (11), *moews* is an argument of *every cat*.

(11)



(12) a. $[\![\text{every cat}]\!] = \lambda P_{\langle e,t\rangle}.\forall x[\mathsf{cat}(x) \rightarrow P(x)]$

b. $[\![\text{every cat meows}]\!] = [\![\text{every cat}]\!]([\![\text{meows}]\!])$
$= (\lambda P_{\langle e,t\rangle}.\forall x[\mathsf{cat}(x) \rightarrow P(x)])(\lambda y_e.\mathsf{meows}(y))$
$= \forall x[\mathsf{cat}(x) \rightarrow \mathsf{meows}(x)])$

(13) a. $[\![\text{some cat}]\!] = \lambda P_{\langle e,t\rangle}.\exists x[\mathsf{cat}(x) \wedge P(x)]$

b. $[\![\text{some cat meows}]\!] = \exists x[\mathsf{cat}(x) \wedge \mathsf{meows}(x)]$

(14) a. $[\![\text{no cat}]\!] = \lambda P_{\langle e,t\rangle}.\neg\exists x[\mathsf{cat}(x) \wedge P(x)]$

b. $[\![\text{no cat meows}]\!] = \neg\exists x[\mathsf{cat}(x) \wedge \mathsf{meows}(x)]$

- Individuals (of type e) can also be shifted into generalized quantifiers via *type-lifting*.

(15) a. $[\![\text{Kitty}]\!] = j$

b. $\textsc{lift}([\![\text{Kitty}]\!]) = \lambda P_{\langle e,t\rangle}.P(j)$

c. $(\textsc{lift}([\![\text{Kitty}]\!]))([\![\text{meows}]\!]) = (\lambda P_{\langle e,t\rangle}.P(j))(\lambda x.\mathsf{meows}(x))$
$= (\lambda x.\mathsf{meows}(x))(j)$
$= \mathsf{meows}(j)$

2.2 Quantificational determiners

- The determiner *every* combines with a common noun of type $\langle e, t\rangle$ to return a generalized quantifier of type $\langle et, t\rangle$. Therefore, its type is quite complex: $\langle et, \langle et, t\rangle\rangle$.

(16) a. $[\![\text{every}]\!] = \lambda Q_{\langle e,t\rangle}\lambda P_{\langle e,t\rangle}.\forall x[Q(x) \rightarrow P(x)]$

b. $[\![\text{some}]\!] = \lambda Q_{\langle e,t\rangle}\lambda P_{\langle e,t\rangle}.\exists x[Q(x) \wedge P(x)]$

c. $[\![\text{no}]\!] = \lambda Q_{\langle e,t\rangle}\lambda P_{\langle e,t\rangle}.\neg\exists x[Q(x) \wedge P(x)]$