Multiple Exhaustifications and Multiple Scalar Items
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1. Background: the G-view of SIs

- Scalar Implicatures (SIs) were firstly considered as wholly pragmatic. (Grice 1975)
- Observing that SIs can be generated in embedding contexts, the *G(rammatical)-view of SIs* (Chierchia 2006; Fox 2007; Chierchia et al. 2012; a.o.) argues that SIs are compositionally derived from the lexicon of scalar items:
  - Scalar items are associated with a set of alternatives.
  - The alternatives grow point-wise until used by covert exhaustivity operator $O$ ($\approx$ only).
  - The $O$-operator affirms the prejacent and negates all the alternatives that are not entailed by the prejacent.

  $O(p) = p \land \forall q \in Alt(p)[p \notin q \rightarrow \neg q]$
  (the prejacent $p$ is true, and any alternative of $p$ not entailed by $p$ is false.)

  (2) Some of the students came. $\leadsto$ Not all of the students came.
    a. $Alt(\phi_{\text{some}}) = \{\phi_{\text{some}}, \phi_{\text{all}}\}$
    b. $O(\phi_{\text{some}}) = \phi_{\text{some}} \land \neg \phi_{\text{all}}$

  (3) Not all of the students came. $\leadsto$ Some of the students came.
    a. $Alt(\neg \phi_{\text{all}}) = \{\neg \phi_{\text{all}}, \neg \phi_{\text{some}}\}$
    b. $O(\neg \phi_{\text{all}}) = \neg \phi_{\text{all}} \land \neg \neg \phi_{\text{some}} = \neg \phi_{\text{all}} \land \phi_{\text{some}}$

  (4) If [\phi_{\text{OR}} you take salad OR dessert], you pay $20; but if you take both, there is a surcharge.
    $\leadsto$ If you take salad or dessert but not both, ...
    a. $Alt(\phi_{\text{OR}}) = \{\phi_{\text{OR}}, \phi_{\text{AND}}\}$
    b. $O(\phi_{\text{OR}}) = \phi_{\text{OR}} \land \neg \phi_{\text{AND}}$

- Target sentences of this talk:

  (5) a. John invited Andy or Billy or Cindy.
  b. Kai had some of the peas or the broccoli last night.
    or > some NP
  c. Some [students who missed some of the sessions] got an A. some [NP...some ...]

Goal: To capture the implicatures of sentences containing multiple scalar items.

Road map:

- Section 2: Sauerland’s (2012) puzzle
- Section 3: The O-Principle: $Alt(O\phi) = \{O\phi, \phi\}$
- Section 4: Predictions of the O-Principle

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1I thank Gennaro Chierchia, Uli Sauerland, and three anonymous reviewers of PLC 39 for helpful comments.
2. Sauerland’s (2012) puzzle

- Sauerland (2012): the G-view is overall preferable to the other approaches of SIs, including the pragmatic theory (Grice 1989), the lexical theory (Levinson 2000, Chierchia 2004), and their combination. However, the G-view predicts wrong interpretations for sentences with multiple scalar items.

(6) John invited Andy or Billy or Cindy.
   a.  nowrap;\( \rightarrow \) John invited only Andy, or only Billy, or only Cindy. \hspace{1cm} \text{Pic. 1}
   b. \( \not\rightarrow \) John invited only Andy, or only Billy, or only Cindy, or all of them. \hspace{1cm} \text{Pic. 2}

- The G-view predicts (7) to take an LF with multi-exhaustification like \( (7)_a \).

(7) John invited Andy or Billy or Cindy.
   a. \( O_1[O_2(A \lor_2 B) \lor_1 C] \)
   b. \( \mathcal{Alt}(O_2(A \lor B) \lor C) = \{O_2(A \lor B) \lor C, \overline{O_2(A \lor B) \land C}\} \)
   c. \( (7)_a \)
      \[ = [O_2(A \lor B) \lor C] \land \neg[O_2(A \lor B) \land C] \]
      \[ = [(A \lor_2 B) \lor C] \land \neg[(A \lor_2 B) \land C] \]
      \[ = A \lor_2 B \lor_2 C \]

- The G-view cannot exclude \( [A \land B \land C] \)

- The main proposal:

\[ \text{O-Principle: } \text{Alt}(O\phi) = \{O\phi, \phi\} \]

An O-operator activates an alternative that is equivalent to its prejacent:

The need of (8) has been discussed by Fox & Spector (2009) for the use of exhaustifications in DE contexts.

- (9b): the alternative set used by \(O_1\) contains two excludable alternatives (boxed).
- (9c): exercising \(O_1\) affirms the prejacent (= assertion + local SI) and negates both of the excludable alternatives, generating a global SI (underlined). The negation of the strengthened alternative collapses under the negation of the non-strengthened one; namely, \(\neg[O_2(A \lor B) \land C] \Rightarrow \neg[(A \lor B) \land C]\).

Therefore, the embedded or within the global SI isn’t strengthened, as we expected.

(9) John invited Andy or Billy or Cindy.

a. \(O_1[O_2(A \lor B) \lor C]\)

b. \(\text{Alt}(O_2(A \lor B) \lor C)\)

\[= \{O_2(A \lor B) \lor C, \ (A \lor B) \lor C, O_2(A \lor B) \land C \lor (A \lor B) \land C \} \]

c. (9c)

\[= [O_2(A \lor B) \lor C] \land \neg[O_2(A \lor B) \land C] \land \neg[(A \lor B) \land C] \]

\[= [(A \lor B) \lor C] \land \neg[(A \lor B) \land C] \]

- With the O-Principle, the G-view can exclude \([A \land B \land C]\).
- The heart of the O-principle is to generate the strongest SIs, especially the strongest global SI, in spirit of the Strongest Meaning Hypothesis.

(10) Strongest Meaning Hypothesis (Chierchia et al. 2012).

Let \(S = [S \ldots O(X) \ldots ]\) and \(S’ = [S’ \ldots X \ldots ]\).

Then everything else being equal, \(S’\) is preferred to \(S\) if \(S’\) is logically stronger than \(S\).
4. Predictions from the O-Principle

4.1. Overview

(11) Basic form:
\[ S = O_1\beta[O_2\alpha] \]
\[ = O_1[(\beta \ldots b \ldots) O_2 (\alpha \ldots a \ldots)] \]

- S has only two scopal elements, i.e., a and b, both are weak scalar items;
- \( \text{Alt}(b) = \{b, b^+\} \), where \( b^+ \) is a non-weaker alternative of b.
- \( \text{Alt}(\beta) = \{\beta, \beta^+\} \), where \( \beta^+ = \beta_{\{b^+\}} \), i.e., the result of replacing b in \( \beta \) with \( b^+ \).

Exercising \( O_1 \) yields two global SIs (underlined). But one of them is semantically weaker and hence collapses under the other one.

(12) \( O_1[\beta(O_2\alpha)] = \beta(\alpha_s) \land \neg\beta^+(\alpha) \land \neg\beta^+(O\alpha) = \begin{cases} \beta(\alpha_s) \land \neg\beta^+(\alpha) & \text{if } \neg\beta^+(\alpha) \Rightarrow \neg\beta^+(\alpha_s) \\ \beta(\alpha_s) \land \neg\beta^+(\alpha_s) & \text{otherwise} \end{cases} \)

Whether \( \neg\beta^+(\alpha_s) \) is entailed by \( \neg\beta^+(\alpha) \) or not depends on the monotonicity pattern of \( \beta^+(\alpha) \) w.r.t. \( \alpha \).

(13) \( O_1[\beta(O_2\alpha)] = \ldots = \begin{cases} \beta(\alpha_s) \land \neg\beta^+(\alpha) & \text{if } \neg\beta^+(\alpha) \Rightarrow \neg\beta^+(\alpha_s) \text{ i.e. } \beta^+(\alpha) \text{ is UE w.r.t. } \alpha \\ \beta(\alpha_s) \land \neg\beta^+(\alpha_s) & \text{otherwise i.e. } \beta^+(\alpha) \text{ is non-UE w.r.t. } \alpha \end{cases} \)

(\( \alpha \) takes a strengthened reading in the global SI iff \( \beta^+(\alpha) \) isn’t UE w.r.t. \( \alpha \).)

4.2. Pattern I: the Global SI doesn’t nest a local SI

**Prediction of UE:**

If \( \beta^+(\alpha) \) is UE w.r.t. \( \alpha \), then the proper global SI doesn’t nest a local SI.

- Example 1: *some* NP > or

(14) **Some** of the students will visit DC or Philadelphia during the break.

a. Assertion + local SI

Some students will visit DC or Phila but not both.

b. Global SI

Some students will visit neither DC nor Phila.

i. \( \leftarrow \) NEG [all the students will visit DC or Phila]

ii. \( \not\leftarrow \) NEG [all the students will visit DC or Phila but not both]

Empirically speaking, if the global SI were like (14b-ii), (14) would be incorrectly judged as true if all the students will visit one or both of the cities.

Check the Prediction of UE:

- \( \beta(\alpha) = [\text{SOME \ldots} (\alpha)]; \beta^+(\alpha) = [\text{ALL \ldots} (\alpha)]. \)

- [ALL \ldots (\alpha)] is UE w.r.t. \( \alpha \). Therefore the global SI of (14) to use the plain reading of or.
– Example 2: or > some NP

(15) Kai had [some of the peas] or the broccoli last night.
   a. \( \sim \neg [\text{Kai had the broccoli and some of the peas last night}] \)
   b. \( \sim \neg [\text{Kai had all of the peas last night}] \)

The alternative of the embedding or is and. The projection of and is UE w.r.t. the embedded some. Thus, the embedded some takes a plain reading in the global SI, as expected. (Cf. Sauerland 2004)

(16) a. \( O_1[O_2\phi_{\text{some}} \lor \phi_b] \)
    b. \( \text{Alt}(O_2\phi_{\text{some}} \lor \phi_b) = \{O_2\phi_{\text{some}} \lor \phi_b, \phi_{\text{some}} \lor \phi_b, O_2\phi_{\text{some}} \land \phi_b, \phi_{\text{some}} \land \phi_b\} \)
    c. \( (\text{16a}) \)
       \( = (O_2\phi_{\text{some}} \lor \phi_b) \land \neg (O_2\phi_{\text{some}} \land \phi_b) \land \neg (\phi_{\text{some}} \land \phi_b) \)
    i. If \( \phi_b = 1 \), then \( \phi_{\text{some}} = 0 \).
       (Kai had the broccoli, and he didn’t have any of the peas.)
    ii. If \( \phi_b = 0 \), then \( \phi_{\text{some}} = \neg \phi_{\text{all}} = 1 \).
       (Kai only had some and not all of the peas.)

4.3. Pattern II: the global SI nests a local SI

Prediction of Non-UE:

If \( \beta^+(\alpha) \) is not UE w.r.t. \( \alpha \), then the proper global SI nests a local SI.

– Example 3: some [NP...some ...]

(17) Some [students who missed some of the sessions] got an A.
   a. Assertion + local SI
      Some students who missed some but not all of the sessions got an A.
   b. Global SI
      Some student(s) who missed some but not all of the sessions didn’t get an A.
         i. \( \not\leftarrow \text{NEG} \) [All the students who missed any sessions got an A]
         ii. \( \leftarrow \text{NEG} \) [All the student who missed only some of the sessions got an A]

If the global SI were like (17b-), (17) would be judged as true under Scenario 2, where all the students who missed two sessions got an A.

Scenario 1: TRUE   Scenario 2: FALSE

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Check the Prediction of Non-UE

– \( \beta(\alpha) = [\text{SOME } (\alpha) (...)]; \beta^+(\alpha) = [\text{ALL } (\alpha) (...)]. \)
  – [ALL (\alpha) (...)] is DE wrt \( \alpha \). The global SI of (17) nests a local SI from the embedded some.
Example 4: some $[_{NP} \ldots \text{NumP} \ldots]$ 

(18) (Context: The government provides funding for applicants that have four or more children. The speaker complains that this policy isn’t performed strictly.)

“How applicants with three children got the funding.”

a. Assertion + local SI
Some applicants with exactly three children got the funding.

b. Global SI
Some applicants with exactly three children didn’t get the funding.

i. $\leftarrow$ NEG [all of the applicants with exactly three children got the funding]

ii. $\leftarrow$ NEG [all of the applicants with three or more children got the funding]

If the global SI were like (18b-ii), (18) would be incorrectly judged as true if all the applicants with exactly three children got the funding (although some applicants with 4+ children didn’t).

Check the Prediction of Non-UE:

$- \beta(\alpha) = \text{[SOME } (\alpha) \ldots \text{]; } \beta^{+}(\alpha) = \text{[ALL } (\alpha) \ldots \text{].}$

$- \text{[ALL } (\alpha) \ldots \text{]}$ is DE wrt $\alpha$. The global SI of (18) nests a local SI from the embedded numeral three, as we expected.

5. Conclusions

- Sentences with multiple scalar items take multiple exhaustifications, yielding a local SI and a global SI. Sauerland (2012) shows that the G-view cannot get the global SI right.

- Introducing Strongest Meaning Hypothesis into the derivational procedure of alternatives, I proposed an O-Principle to derive the correct global SIs.

- The O-Principle predicts that whether the global SI nests a local SI is determined by the monotonicity pattern of the matrix alternative w.r.t. the embedded scalar item.

References