# Productivity, Density, and Sorting

JOB MARKET PAPER

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**Abstract**

The strong, positive relationship between productivity and density has been used as evidence for the existence of agglomeration forces such as productivity spillovers. This paper presents an alternative hypothesis: In an economic geography model where firms choose locations to be close to their markets, sorting endogenously generates the density-productivity relationship in the absence of any pecuniary or non-pecuniary productivity spillovers. I introduce a new solution method for continuous space geography models that dramatically reduces the complexity of the equilibrium conditions and allows such model to generate more predictions than was previously possible in realistic geographies where firms and workers choose locations. Other cross-sectional relationships that have traditionally been used as evidence of agglomeration forces can also be derived in this environment where no such productivity spillovers exist. This geographic model of sorting breaks observational equivalence between firm sorting and agglomeration forces and allows for an indirect test of firm sorting. Under specific conditions, positive shocks to density can negatively affect average productivity through changes in the local composition of firms, inconsistent with models of agglomeration forces without sorting. Using restricted access establishment-level Census data, I document strong intra-city relationships between location and firm characteristics predicted by the model. I use the data to test for evidence of composition effects, instrumenting for the supply of new non-residential real estate construction using the geographic distribution of multi-city real estate developers, and find evidence of firm sorting.

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# 1 Introduction

Across and within cities, firms in dense locations – measured by population, establishment, or employment density – are more productive.[1]Figures 1a and 1b plot the relationship between total factor productivity[2] and establishment density for a sample of US manufacturing firms. Because transport costs are thought to be low, previous work has concluded that differences in access to local markets alone could not account for this persistent relationship. Because of this, the relationship has been used as evidence for the existence of productivity spillovers that work either through learning or inter-industry linkages.[3] These theories collectively posit a particular direction of causation: otherwise homogeneous firms become more productive when locating in denser areas.

This paper offers an alternative hypothesis: when firm mobility is added to a model with monopolistic competition and heterogeneous firms[4], sorting for market access generates productivity and density differences across locations, and endogenously generates the relationship between the two – even in the absence of pecuniary or non-pecuniary productivity spillovers and when underlying differences in demand across locations are small. In Section 2, I present a model in which locations differ by transport costs to and from markets at all other locations, affecting demand at each location. More productive entrepreneurs outbid others for locations with higher market potential, trading fixed costs of higher rents for higher variable profit. In turn, these firms attract more workers to those locations. This process generates equilibrium differences in market access, real estate prices, and firm productivity. Higher prices induce landowners to provide more density in areas where more productive firms locate. Firm sorting amplifies the same centripetal forces present in the new-economic geography literature. [5] Moreover, when firms sort on market access, the relationship between density and productivity arises endogenously.

The model is defined over a broad set of domains so that it may be taken to data from actual urban geographies with minimal analogy. This flexibility is similar to Allen & Arkolakis (2013), where trade takes place in a Ricardian framework with immobile,

[1]This stylized fact has been validated in multiple periods and across multiple continents.Ciccone & Hall (1996) were the first to differentiate between density and city size with respect to productivity. Recently work by Maré et al. (2006) and Combes et al. (2010)confirm the relationship using data from New Zealand and France, respectively.

[2]TFP measures at the establishment level for all responders to the Annual Survey of Manufacturers, from Petrin et al. (2011).

[3]See(Rosenthal & Strange, 2004).

[4]Melitz (2003)

[5] SeeFujita et al. (1999)

homogenous firms. My model adds a production setting with mobile, heterogeneous firms where the density of firms at each location is endogenously determined by real estate prices.

Differences between locations are a function of access to markets and variable costs, and therefore a function of the location decisions of all other firms and workers as well as of the space provision decisions of landowners. This complexity is endemic to new economic geography models, where it is often impossible to guarantee uniqueness or derive analytic relationships between key observables under realistic conditions.[6]

I introduce a novel solution method for this class of models, using location advantage as a sufficient statistic for the economic activity at a given location, an endogenously determined index,[7] and solving the model in two steps. First, I solve the firm, worker, and landowner decisions with respect to the index, and then use the general equilibrium conditions to solve for the mapping of index values onto locations. The first step reduces the dimensionality of these problems. The latter step recovers the complexity of the geographic interconnections and is necessary in order to understand the relative strength of the geographic centripetal and centrifugal forces or to describe the effects of geography-based policies. This second step does not deliver a closed-form solution and the equilibrium may not be unique, rendering it uninformative. However, the equilibrium conditions derived with respect to the index allow me to derive predictions based on observables that hold in any equilibrium. In Appendix 5, I show how this method can be applied to geography models with a broader set of assumptions, including location-specific and geographically determined spillovers.

Section 3 uses the model to derive predictions. I derive analytic relationships between observables (firm or location characteristics) and the location index, and use these in turn to derive analytic relationships of observables with respect to each other. In this way, the model delivers relationships between firm productivity, size, and profits, and firm density, employment or population density, and rents.

This paper also makes a theoretical and empirical contribution to a recent literature that proposes both the sorting[8] and selection[9] hypotheses to explain the closely related city-size-productivity relationship. In these models, firms are attracted to and sort into large cities in order to gain access to city-level agglomeration forces derived from city

[6]Fujita et al. (1999); Rossi-Hansberg (2005); Allen & Arkolakis (2013); Kyriakopoulou & Xepapadeas (2013)

[7]Davis & Dingel (2013) use a system of cities approach where distance from the city center is exogenously given as an index for location advantage. By contrast, here the index is endogenously determined across all locations, arising out of the effects of geographic frictions on the firms' production function.

[8]Gaubert (2014); Behrens et al. (2012); Maré & Graham (2009)

[9]Baldwin & Okubo (2006); Nocke (2006); Combes et al. (2012)

size. Gaubert (2014) assumes super-modularity directly between firm productivity and city size . In Behrens et al. (2010), productive firms take advantage of larger bases of local intermediates in larger cities. When, as in this literature, both forces are posited in tandem, the two become empirically indistinguishable. Any shock to the size of a city also affects the quality of firms sorting into the city. This fundamental identification problem, termed "observational equivalence" by Ellison and Glaeser (1997), prohibits the disentangling of firm and location characteristics when location decisions are endogenous.

The introduction of geography breaks the observational equivalence result. In my model, the sorting-induced density-productivity relationship occurs even in the absence of agglomeration forces. Firms sort on location advantage, which is based on networked proximity to other markets. Higher real estate prices in advantageous locations increase density. In equilibrium, advantageous locations are denser, but a location's density does not determine its advantage through agglomeration forces. Rather, density and firm quality at each location endogenously respond to geographic proximity.

This last theoretical innovation provides the key empirical advantage that enables me to distinguish firm sorting from agglomeration forces. Because density and firm quality both respond endogenously to location advantage, shocks to the sorting pattern of firms can affect the quality of firms at a location by changing the composition of firms the location, without affecting the underlying determinants of location advantage. While the model sometimes predicts positive relationships between positive shocks to density and average productivity, as predicted in an agglomeration force model, negative relationships emerge under specific circumstances. These negative relationships are not natural predictions of agglomeration force models. Introducing geography reframes the city-size-productivity literature in terms of the density-productivity relationship and provides this new mechanism underlying the sorting pattern of firms which can be tested. I test for and find evidence of composition effects that are consistent with a model where sorting and density are both endogenously responsive to location advantage.

Section 4 tests the predictions of the model using data collected from US Economic Censuses and Surveys between 1992 and 2007. The data show that firms in locations with higher establishment density have more sales and employees, are more productive, and pay higher rent and more rent per worker. Population density and productivity are positively related. These relationships hold within cities and across the US.

However, these predictions can also be derived from models of agglomeration forces. To begin testing for sorting, I first examine the location decisions of firms that expand and firms that relocate plants, using previous period productivity to predict new location density, and the subsequent effects of productivity five years forward. Although this strategy

can't rule out sorting on unobservables linked to density, I find evidence for inter- and intra-city sorting as well as mixed evidence for density effects.

Finally, I test the sorting hypothesis using the composition effects. To do this, I must isolate exogenous shocks to the supply of density. I use data from the Census of Finance and Insurance on the geography and construction expenditures of real estate development firms. Because commercial real estate development requires liquid collateral[10], such firms are exposed to real estate shocks in multiple cities, and price shocks in one city can both affect firm-level collateral and transfer resources away from projects in relatively lower-shocked cities. Both these channels appear to be in effect in the data.

I test two different composition effects. First, I isolate what the model predicts as the highest index locations in each city, ranking each tract in the city and dividing each city into percentiles. Construction of new non-residential real estate within the highest density-percentiles results in lower-productivity entrants. Second, I show that construction in a given tract lowers the productivity of entrants in relatively lower-rank tracts, but has no effect on higher-rank tracts.

These findings are consistent with the sorting hypothesis and inconsistent with the baseline agglomeration hypothesis, where higher density increases firm productivity. To be sure, these results cannot be interpreted as a rejection of agglomeration forces; models including both sorting and agglomeration forces may also predict the negative relationship I find. However, these results represent the first affirmation of the existence of intra-city firm sorting, and demonstrate the empirical flexibility of the indexing strategy.

My model has significant ramifications for a number of urban policies. As in other models of sorting, the model implies that significant mismeasurment of agglomeration forces may over or understate the benefits of placed-based urban policies, where municipalities subsidize the relocation of large, productive firms with the implicit assumption that incumbents gain pecuniary or non-pecuniary externalities. Moreover, the model has counter-intuitive predictions for the effects of relaxing zoning laws. In models with agglomeration forces, it is natural to think that allowing for the accumulation of more density will increase productivity locally. In this model, the opposite is often true, as new entrants can be of lower quality and reduce average productivity. This prediction stands in stark contrast to accepted wisdom.

Furthermore, my findings have important ramifications for our understanding of the gradient of land prices in cities. In models with agglomeration forces, firms pay for the productive amenities of cities, and a hedonic calculation would recover the full value of a location amenity. In this model, over and above differences in location quality, rents in

[10]Gyourko (2009)reports 1-to-1 leverage ratios.

central areas are bid up by more productive firms, and the overall rent gradient reflects both underlying differences in location quality, and differences in the underlying distribution in firm quality: if we want to understand why the centers of cities are more expensive than there peripheries, we must understand the sorting behavior of firms within cities.

# 2 Model

In this section, I outline the environment of the model, including assumptions on the geography and transportation costs in the domain. I set out the optimization problems for each of the three agents: workers, landowners, and entrepreneurs.

I then introduce the change of variable that will act as an index of location advantage. I separate the potential variable profit at each location into location-specific and firm-specific terms. The location-specific terms can be decomposed into prices and market size at all other locations, and the geographic relationship between locations. Together, these endogenously determined terms constitute all the geographic terms that affect variable profits, and the location index is the specific functional form for the effects of geography on variable profits. Effectively, a location's advantage in this model is its market access. In Appendix 5, I show how a broader set of production functions that include geographic productivity spillovers can be incorporated into this framework.

My solution method first solves the landowner space provision and firm location decisions with respect to this index. Because the terms comprising the index are geographic, their values cannot be determined in complicated spaces where closed-form solutions do not exist. However, the index is a sufficient statistic for all the effects of geography on firm worker and landowner decisions, so the mapping of firms, workers, and density to the index characterizes the full set of potential equilibria. To my knowledge, this is the first paper to employ such a method.

The full benefit of this solution method will be made more clear here and in the following section where I explore the model's predictions. Because the first order conditions of the agents are determined with respect to the index, they are free of geographic variables and hold under any equilibrium distribution of firms and workers. The predictions of the model will be derived from these first order conditions and therefore are also true in any equilibrium.

That said, to evaluate a particular equilibrium, it becomes necessary to reintroduce geographic variables. For instance, to evaluate the relative strengths of the geographic forces comprising location advantage, or to evaluate the effects of decreased trade costs due to transportation infrastructure development, the model must be solved for a particular equilibrium. To do this, I must map locations to index values.

The second step of the solution method is to solve for the mapping of location advantage to locations. I derive the equilibrium conditions for this mapping, which pins down both prices and market access in any particular geography. This system of nonlinear integral equations resembles the equilibrium conditions in Allen & Arkolakis (2013)as well as many other economic geography models. I show that under the particular assumptions of this model, an equilibrium must always exist. Uniqueness is guaranteed under particular conditions for transportation costs that are given in Appendix 3. Because these conditions may not be met, and because the imposition of uniqueness generates no further predictions in this context, I do not impose them.

## 2.1 Environment

The following subsection describes the geographic and economic environment of the model.

### 2.1.1 Geography

The goal of this model will be to be able to accept as a domain any realistic geography for which we have data. As such, the model is defined over $S$, any compact subset of Euclidean space $R^n$, $n \in \mathbb{N}$. All economic activities, production and consumption, take place at points $i \in S$. The flexibility of this domain allows for applications to real geographies, such as two-dimensional planes. The compact nature of the space guarantees the existence of a boundary, locations that are relatively distant. Given further assumptions on the transportation costs stated below, locations on the boundary will inevitably be economically remote, and thus less advantageous. This will be crucial in ensuring at least partial sorting.

Three kinds of agents take part in production and consumption in the space: landowners, workers, and entrepreneurs. The space is filled with a mass of immobile landowners, each endowed with a point $i \in S$. I refer to this as a landowner's location. All locations have landowners. Landowner locations are fixed. Workers and entrepreneurs choose their location (and their landowner). The requirement that economic activity is assigned a location constitutes the fundamental friction posed by space.

The space is further defined by a function governing transportation costs between all points in the space. Goods are sold from one point $i$ to another point $j$ with continuous, differentiable, and symmetric iceberg transport costs

$$1 < \tau(i, j) < \infty.$$

To help satisfy the existence of a sorting equilibrium, I impose symmetry and the tri-

angle inequality on $\tau(i,j)$.[11] No other functional form is placed on $\tau$. There's no direct comparison that can be made between transportation costs between $i$ and any two other locations $j_1$ and $j_2$. $\tau(i,j_1)$ and $\tau(i,j_2)$, even if we do know something about the euclidian distance between $i$ and the other points, e.g. if the vector $\vec{ij_1} > \vec{ij_2}$.

Locations differ by their relative proximity to other locations. On its own, this exogenous geography will not drive the location decisions of firms and workers. Rather, a location's advantage is determined by the equilibrium decision of workers, firms, and landowners. The joint actions of exogenous spatial characteristics and the endogenous economic potential available to agents at each location drive the sorting behavior of firms and the predictions of the model.

### 2.1.2 Production

A fixed set of firms use labor to produce differentiated goods. Firms are required to locate somewhere in order to produce. In addition, firms employ labor at their locations and pay location-specific wages. The total market of available labor is equal to some mass of $L$ workers. Differentiated goods are sold to consumers (workers, entrepreneurs, and landowners) across the entire space. Access to consumers, local production costs, and rents drive the location decisions of firms.

Landowners produce units of non-residential space and provide that space to a density of firms.

### 2.1.3 Consumption

All three agent types purchase and consume a CES aggregate of all the differentiated

[11] This is a technical assumption that will be necessary in order to ensure that points on the boundary are actually more distant from all other locations than nearby points, so that market access and prices both improve away from the boundary. A less strict condition is sufficient. For any point $k$ on the vector $\vec{ij}$, $\tau(i,j) > \tau(i,k)$ and $\tau(i,j) > \tau(k,j)$; intuitively for any journey between two points $i$ and $j$, transportation costs are lower for stops along the way. If travel between $i$ and $j$ always takes place on the euclidean vector $\vec{ij}$, this constraint intuitively means shorter trips facing the same geography must be less costly. In a world where shortest cost trips may include circuitous routes, this constraint may be unrealistic.

goods produced at all locations with an elasticity of substitution $\sigma > 1$.[12][13] The quantity of each good consumed will vary in equilibrium by location. Consumers will substitute towards goods produced locally, as such good face lower transportation costs, and goods produced by more productive firms, as the factory prices of such goods will be lower. [14]

#### 2.1.4 Timing

The model is static and all production, location, and consumption decisions are made simultaneously by all agents.

## 2.2 Setup

### 2.2.1 Worker location decision

A mass $L$ of homogenous workers provide one unit of labor inelastically. Workers consume at their location. Because my focus is on the commercial real estate market and the location decisions of firms, I will assume that workers do not participate in a real estate market.[15]

[12]Fixing landowners consumption at their location assumes balanced trade. This assumptions simplifies the notation significantly but does not drive the results. Instead, landowners and entrepreneurs can be assumed to be companies in which each worker owns stock. It is crucial, however, that some consumption remain localized. If all three agents could separate production and consumption choices entirely, then geographic frictions would no longer be a defining element of this model.

[13] Specifically, all three agents maximize their utility

$$U = \left[\int_{\omega\in\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution and $\omega \in \Omega$ is a good in the set of all available goods $\Omega$.

[14] The local price index at location $i$ is defined as

$$P(i)^{1-\sigma} = \int_{\omega\in\Omega} p(\omega, i)^{1-\sigma} d\omega$$

where $p(\omega, i)$ is the location-specific price of each good.

[15]An alternative would model workers as having an inelastic per capital housing demand at their location and the residential real estate market, segregated from the commercial real estate market, as a competitive fringe of builders with a constant marginal cost denominated in local labor. The worker equilibrium condition would remain similar to the one below, effectively multiplying the current location-specific wage by a constant equal to one plus the marginal cost of residential real estate. A more robust version would allow landowners to supply both residential and commercial real estate, and workers to commute between locations at a cost. As in Lucas & Rossi-Hansberg (2002) or Ahlfelt et. al.(2012), the relative density of commercial and residential activity would vary across locations. In this scenario, employment, but not firm

Because all goods will be sold to all locations, workers view locations as varying only by their respective price indexes. For any two locations, homogeneous workers must be indifferent between those two locations. In equilibrium, wages must exactly offset prices such that worker utility equalizes across space.

### 2.2.2 Firm location and pricing decisions

A set of entrepreneurs receive heterogeneous productivities and will each create a single firm to sell final goods to all agents at all locations.[16] Entrepreneurs face three decisions: (1) whether to produce (2) where to locate and (3) how to price their firm's good. Entrepreneurs simultaneously solve these three problems. An entrepreneur's location decision affects the variable and fixed costs of production they face as well as the demand they will face. The entrepreneur's optimal choice can be found by solving these three decisions in reverse: first determining the optimal price of the good at each potential location, then the optimal location given the pricing rule at each location, and finally whether to produce given the profits yielded by the optimal location.

Entrepreneurs draw a productivity $\psi$ from some distribution $G(\hat{\psi})$ where $\hat{\psi} = \psi^{\frac{1}{\sigma-1}}$.[17] The distribution need only be assumed to have an upper and lower bound, $0 < \psi_L < \psi_U < \infty$. The bounds on the distribution are technical assumptions that help ensure the existence of an equilibrium.

This firm-specific productivity lowers the marginal cost of production which is a function of a location-specific wage:

$$MC = \frac{w(i)}{\hat{\psi}}.$$

Note that in equilibrium, real wages will be identical across locations, thus the real marginal costs for a specific firm will not vary across locations: there are no location productivity advantages here.

Following the literature, at any location, the firm's price decision will be a constant markup over its marginal cost.[18] Through the wage, potential marginal costs, and therefore

density, is always positively correlated with firm productivity.

[16] I will use entrepreneurs and firms interchangeably throughout.

[17] Defining the distribution over a function of the productivity parameter rather than the productivity parameter itself simplifies the algebra.

[18] For a firm with a given productivity parameter $\psi$ at a given location $i$, the optimal factory price will be

the final goods' prices, will vary by location; the pricing decision can be folded into the location decision.

Because the wage and transportation costs to other markets differ across locations, firms face higher or lower demand at different locations. Consumers substitute towards local firms, whose goods pay lower transport costs, and firms at locations with lower marginal costs. Because of this variation in demand across locations, firms reap different levels of variable profit in different locations.

The firm pays a fixed cost of rent $\phi(i)$ (which is denominated in terms of units of consumption) to the landowner in order to rent space at location $i$. Rent does not depend on firm size.

Firms face a tradeoff between higher real variable profits and higher real rents. Entrepreneurs' utility is maximized when their real incomes, their firms' profits (in terms of price-index bundles of final goods), are maximized. The firm's maximization function can therefore be written as

$$i^* = argmax_{i \in S}\{\pi_f(i)\} = argmax_{i \in S}\left\{r(\psi, i)/\sigma - \phi(i)\right\}$$

where $r(\psi, i)$ is the revenue of a firm with productivity parameter $\psi$ at location $i$.

In order to examine the variable profit $r(\psi, i)/\sigma$, I first present the revenue, in units of consumption at $i$, from selling from point $i$ to point $j$:

$$r_j(\psi, i) = \frac{\left[\dfrac{w(i) \cdot \tau(i,j)}{P(j)\rho\hat{\psi}}\right]^{1-\sigma} R(j)}{P(i)} .$$

Revenue is a function of the markup, the firm's productivity $\psi$, the wage at $i$, as well as the price index at $j$, $P(j)$, the nominal size of the market at $j$, $R(j)$, the iceberg transportation costs between $i$ and $j$, $\tau(i,j)$, and the price index at $i$, $P(j)$. Summing over all markets $j \in S$, the expression for the variable profit at point $i$ :

$$r(\psi, i)/\sigma = \int_{j \in S} \frac{w(i)^{1-\sigma} \cdot \tau(i,j)^{1-\sigma}}{P(i)P(j)^{1-\sigma}\rho^{1-\sigma}\psi} \frac{R(j)}{\sigma} dj$$

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$$p(\psi, i) = \frac{w(i)}{\rho \cdot \hat{\psi}}$$

where $\rho = \dfrac{\sigma}{\sigma - 1}$ is one over the optimal markup.

The above equation yields the relationship between a firm's variable profit and its location. Rearranging terms, it is possible to separate the location-specific effects on variable profit from firm-specific productivity:

$$r(\psi, i)/\sigma = \psi \cdot \eta(i) \tag{1}$$

where $\eta(i)$ is the location-specific advantage term defined as

$$\eta(i) = \int_{j \in S} \frac{w(i)^{1-\sigma} \tau(i,j)^{1-\sigma} R(j)}{\sigma \rho^{1-\sigma} P(i) P(j)^{1-\sigma}} dj.$$

The value of $\eta(i)$ is dependent on the variable cost of production at $i$, $w(i)$, and the transportation cost-weighted proximity to markets $j$, both a function of the market size of $j$ and the price index at $j$. The market potential at $i$ can be expressed as:

$$Market\ Potential\ = \frac{\eta(i)P(i)}{w(i)^{1-\sigma}} = \int_{j \in S} \left[\frac{\tau(i,j)}{\rho P(j)}\right]^{1-\sigma} \frac{R(j)}{\sigma} dj.$$

This equation gives some intuition for the centripetal and centrifugal forces governing the model. The parameters of the utility function, the equilibrium distribution of firms and workers, and the exogenous geography (through $\tau$) jointly determine a location's market access. Locations that are relatively proximate to more productive firms and far from less productive firms will have lower price indexes, which decrease the effective market for a given firm. On the other hand, larger firms, their workers and landowners together form larger markets, and relative proximity to these markets increases market access via $R(j)$. Together with the variable costs at a given location $i$, these forces govern location $i$'s access to markets.

In addition to effects via market access, proximity to other firms affects location advantage at $i$ via $P(i)$, the price index at $i$. Proximity to productive firms reduces the price index at $i$, which reduces the wage $w(i)$, and thus marginal cost at $i$, and directly increases entrepreneur utility by increasing the total amount of consumption for a given level of profits.

Taken together, proximity to high-productivity firms increases affects advantage by (1) increasing the size of local markets, (2) stiffening competition, through lower price indexes in nearby locations, (3) decreasing marginal cost of production, and (4) increasing entrepreneur utility. The first two channels operate through market access, while the last two operate through effects on the price index at location $i$.

In equation (2), the ability to separate location- and firm-specific contribution to variable profit is a result of the particular assumptions of the production function. In particular, two features of equation (2) are crucial.

First, higher $\eta$ locations have higher variable profits for firms holding $\psi$ constant. All firms agree on which locations yield the highest variable profit. This ranking orders every location $i \in S$ according to $\eta(i)$. For any two locations $i$ and $j$, for any given level of firm productivity, marginal profits will be higher where $\eta$ is higher. The first proposition restates this:

> $Lemma\ 1$ : For any two locations $\eta_1$, $\eta_2 \in [\bar{\eta}, \underline{\eta}]$, if $\eta_1 > \eta_2$, then $r(\psi, \eta_1) > r(\psi, \eta_2)$ i.e., variable profits at $\eta_1$ are higher for all firms.

Lemma 1 follows directly from the production function and definition of $\eta$.

Second, $\hat{\psi}$ enters multiplicatively with $\eta$. As a consequence, difference in variable profit between any two locations is higher for more productive firms.

> $Lemma\ 2$ : For any $\eta_1$, $\eta_2 \in [\bar{\eta}, \underline{\eta}]$, if $\eta_1 > \eta_2$, and $\psi_1 > \psi_2$, then $r(\psi_1, \eta_1) - r(\psi_1, \eta_1) > r(\psi_2, \eta_2) - r(\psi_2, \eta_2)$.

Lemma 2 also follows directly from equation (2) and is a result of the super-modularity between location and productivity assumed in the production function. Locations that have higher equilibrium market access will be more sought after by more productive firms. As in the assignment literature,[19] this condition will, in equilibrium, lead firms to sort on productivity, with more productive firms capturing locations with greater market access.

The equilibrium parameter $\eta$ and the ability to rank any $i \in S$ according to $\eta$ will be central to the subsequent analysis. It is important at this stage to reiterate that the mapping of $\eta$ into locations is defined endogenously; it will be dependent on a particular arrangement of firms and workers in the space. As the analysis above demonstrates, the equilibrium decision of each firm depends on the decisions of all other firms. The complexity of this problem is enormous. Prices and market access at any given location $i$ will be impossible to pin down without knowing the prices and market access at every other location.

The introduction of the change of variable, $\eta$, greatly reduces the complexity of the task at hand. Without knowing the $\eta$-value of a particular location, the subsequent analysis will make claims on the economic activity at a given location $i$ conditional on the value of $\eta$ at $i$. After solving for the equilibrium behaviors of firms, landowner and workers with respect

[19] Costinot & Vogel (2009) use log-supermodularity to ensure matching.

to $\eta$, it will further be necessary to show the mapping $S \to [\underline{\eta}, \bar{\eta}]$ must exist. While this means predictions cannot be made regarding the economic activity at a specific location, I will subsequently show that the introduction of the endogenous variable allows for the derivation of analytic relationship between key economic variables in any equilibrium.

Finally, firms produce if their profits, conditional on their optimal location decision, are above zero:

$$r(\psi, \eta^*)/\sigma \geq \phi^*$$

where $\eta^*$ is the $\eta$ of the profit maximizing location for firm with productivity $\psi$, and $\phi^*$ is the rent at that optimal location.

### 2.2.3 Landowner supply decision

Each landowner is endowed with a location and must decide how much density to provide to firms at that location. Density is provided at increasing marginal construction cost according to an invertible, twice-differentiable cost function $c(h)$:

$$c'(h),\, c''(h) > 0,\ c(0) = 0$$

where $h$ is the density of firms at a particular location. Construction costs are denominated in baskets of final goods and no labor is required in construction.[20]

Landowners tradeoff the price-adjusted costs of providing density $c(h)$ against the price-adjusted rents $\phi$. In equilibrium, the landowner chooses rent $\phi$ and density $h$ to maximize profits

$$\pi_l = h\phi - c(h).$$

From the above equation, holding firm density, $h$ constant, higher rents increase profits unconditionally. The landowner's choice of $h$ is unconstrained. However, the choice of $\phi$ is constrained by the participation constraints of the firms: given each firms' outside

[20]Real construction costs differ from location to location due to differences in the price index. Landowners at higher $P(i)$ locations face higher unit costs of building, as real wages are higher at such locations. However, they also receive higher real rents (controlling for $\eta$ since $\phi(\eta)$ is denominated in units of final goods and therefore real rents are higher, all else equal, at locations with higher $P(i)$ as well. Therefore I write the cost function $c(h)$ as denominated in units of final goods. Thus, the landowners' provision decision is homogenous degree zero with respect to $P(i)$, and can be made entirely in terms of real units of goods. Denominating construction costs in final goods simplifies the algebra of the labor market clearing condition. Alternative specifications with costs denominated in labor are possible.

option, the landowners' pricing decision will affect the types of firms willing to locate at her location, and she must take this into account when setting rents.

To simplify this problem, note that for any given rent charged, the remaining choice of $h$ can be expressed as

$$\tilde{\pi}_l(\phi) = \max_h \{h\phi - c(h)\}$$

Given any rent $\phi$, optimal density will set marginal costs of density provision equal to marginal revenue, $\phi$.[21]

By the envelope theorem, the landowner's profits, conditional on optimally chosen density, are increasing in $\phi$

$$\tilde{\pi}_l'(\phi) = h > 0.^{22}$$

This allows the landowner's dual decision, choosing both firm density and rents at her location, to be expressed solely in terms of her choice of rent, as higher rents guarantee higher profits conditional on optimal density provision.

But recall that landowner profits holding $h$ constant are strictly increasing in $\phi$, and that this choice is constrained by the participation constraints of firms. Thus, the landowner's constrained optimization chooses highest rents possible, conditional on the willingness to pay of firms. A landowner's location is differentiated from others' by the location-specific parameter $\eta$. Each firm's willingness to pay for a space at location $\eta$ will be conditional on the firm's outside options, including the rent $\phi'$ at outside options $\eta' \in S \not\eta$, and variable profits at $\eta$ and locations $\eta'$. A firm of productivity $\psi$'s willingness to pay for space at location $\eta$ can be expressed as

$$WTP(\psi, \eta) = \min_{\eta' \in S\not\eta} \{\psi \cdot (\eta - \eta') + \phi'\}.$$

Intuitively, given a single outside option $\eta'$, a firm is willing to pay rent at $\eta$ equivalent to the difference in variable profit that firm would collect at location $\eta$ and what would be the variable profit at location $\eta'$, paying whatever rent is charged by the landowner at $\eta'$. The firm's willingness to pay for $\eta$ is therefore the minimum difference between profits at $\eta$ and profits at all other outside options $\eta' \in S \not\eta$, taking rents at those locations as given.

The landowner will choose the highest possible rent conditional on some firm type being willing to locate at her location. This is equivalent to choosing the firm with the maximum willingness to pay for her location.

[21] In the continuous space, the market for land is competitive, and marginal revenue equals rent. In a discretized version of the model, landowners in a finite set of points will each accommodate a mass of firms, and face a downward sloping demand curve due to firm heterogeneity. They therefore will act as quasi-monopolists in such a setup.

[22] The second order condition is satisfied by the assumption that $c''(h) > 0$.

$$\pi_l = max_\phi \{h\phi - c(h)\}$$

$$\text{s.t. } \phi \leq \max_\psi \{\min_{\eta' \in S_{\eta'}} \{\psi \cdot (\eta - \eta') + \phi'\}\}.$$

Notice that landowners differ only according to their location parameter $\eta$. In the equilibrium, the decisions of landowners and firms will jointly determine the matching function $\psi(\eta)$ between firm-types and the location index.

## 2.3 Equilibrium

In this section, I first introduce $\eta$ as a change of variable and define the rent function $\phi(\eta)$ and the matching function $\psi(\eta)$. I then show that, conditional on an equilibrium existing, sorting always exists. I derive the equilibrium conditions governing the functions $\psi(\eta)$, $h(\eta)$, and $\phi(\eta)$, and then prove that at least one equilibrium always exists by showing a solution for the mapping function of locations to indices $\eta(i)$ always exists. An equilibrium will be characterized by a function $\eta(i)$ that relates market access to locations in $S$, a mapping of firms to locations according to locations' market access $\psi(\eta)$, a rent curve $\phi(\eta)$, and a firm density function $h(\eta)$.

To find the functions $\psi(\eta)$, $h(\eta)$, $\phi(\eta)$, and $\eta(i)$, I derive six equilibrium conditions: (1) the firm spatial equilibrium, (2) the land development equilibrium at each location, (3) the real estate market clearing condition, (4) the labor market clearing condition, (5) the worker spatial equilibrium, and (6) the goods market clearing and balanced trade condition. The analysis herein will first assume an assignment $\eta(i)$, solve for $\phi(\eta)$, $h(\eta)$, and $h(\eta)$, and then return to solve for $\eta(i)$.

### 2.3.1 Change of variable

I begin by positing a mapping $\eta(i)$ which characterizes all occupied locations according to $\eta$. $\eta$ serves as index that transforms the space of all occupied locations[23] into a space $[\underline{\eta}, \bar{\eta}]$, with boundaries defined by the maximum and minimum values of $\eta$ in a given equilibrium. For each value of $\eta \in [\underline{\eta}, \bar{\eta}]$, a certain density of locations, expressed as $f(\eta)$, will share this value.

As an illustration, consider a two-dimensional circular geography with a unit radius, where transportation costs are linear with distance traveled, and all goods must travel through the center of the circle. In this geography, locations closer to the center always have lower transport costs to all other locations and therefore in any equilibrium, $\eta$ will be

[23] Given the assumptions of the model, in particular that $c(h)$ is continuous and $c(0) = 0$ , the equilibrium must always be regular and this transformation is always $S \rightarrow [\underline{\eta}, \bar{\eta}]$.

decreasing with distance to the center, with all locations on a fixed radius away from the center sharing the same index value $\eta$. For any radius $r$,

$$\eta(r) = (1-r)\bar{\eta} - r\underline{\eta}$$

with a density function $f(\eta)$ defined as

$$f(\eta) = 2\pi \frac{\bar{\eta} - \eta}{\bar{\eta} + \underline{\eta}}.$$

### 2.3.2 Rent gradient and firm sorting

Next, I introduce the rent function $\phi(\eta)$, characterizing the rents at each location according to the location index. Because the landowners' optimization problems differ only by $\eta$, landowners with identical $\eta$'s choose identical rents; $\eta$ is a sufficient statistic for $\phi$.

In order to characterize the rent function, I now introduce the firms' incentive compatibility constraints. In a spatial equilibrium, no entrepreneur can increase her implicit utility by changing locations. Deviations from optimal locations could potentially increase profits in one of two ways: an entrepreneur could move to a location with higher variable profits, or to a location with lower rent. Restating the firm's profit maximization implies neither of these options increase real profits for any entrepreneur.[24] Formally,

$$\forall \psi \in \Psi,\ \phi(\eta) - \phi(\eta_L) \leq \psi\left(\eta_L - \eta\right).\ \forall \eta_L < \eta \tag{2}$$

$$\forall \psi \in \Psi,\ \phi(\eta) - \phi(\eta_H) \geq \psi\left(\eta_H - \eta\right),\ \forall \eta_U > \eta \tag{3}$$

where $\eta$ is the location chosen by entrepreneur of productivity $\psi$, $\eta_L$ is an outside option with $\eta_L \leq \eta^*$, and $\eta_H$ is an outside option with $\eta_H \geq \eta^*$. Equations (2) and (3) are the incentive compatibility constraints for firms optimally locating in spaces with an index value of $\eta$.

It immediately follows from equations (2) and (3) that $\phi(\eta)$ is strictly increasing. If, instead, a local minimum existed at $\eta_1 \in (\underline{\eta}, \bar{\eta})$, all firms at locations to the left of $\eta_1$ could

[24] The firms' basic tradeoff between variable profit and rent is in terms of baskets of goods so that real profits and implicit utility are equivalent.

capture higher variable profit at locations with reduced rent. The firms' incentive compatibility constraints would then guarantee such locations would be unoccupied, contradicting the definition of all locations in $[\underline{\eta}, \bar{\eta}]$ as the set of occupied locations.

Next, I rewrite the entrepreneur's optimization problem in terms of $\eta$ and the profit function $\phi(\eta)$,

$$\pi_f = max_{\eta \in [\underline{\eta}, \bar{\eta}]} \{\psi \cdot \eta - \phi(\eta)\} \ ,$$

which yields the first order condition

$$\hat{\psi}(\eta) = \phi'(\eta) \ \forall \eta \in [\underline{\eta}, \bar{\eta}]. \tag{4}$$

Equation (4) defines the mapping of firms $\psi$ to locations $\eta$.[25] Appendix 2 verifies that given a matching function $\psi(\eta)$ and rent gradient $\phi'(\eta)$, landowners at every $\eta \in [\underline{\eta}, \bar{\eta}]$ choose rents such that equation (4) is satisfied. Because (4) is derived from the firm's optimization, and firm's optimization is the constraint on the landowner's price decision, it exactly satisfies the latter.

Finally, note that for firms to be at their optimum, the second order condition $\phi''(\eta) > 0$ must hold, which together with equation (4) implies that $\psi'(\eta) > 0$. This last result guarantees the positive assortative matching between firms and locations, and as expressed in the previous sub-section, is a product of the super-modularity of the production function. Proposition 1 formalizes this result

> $Proposition\ 1:$ **Firm sorting.** In any equilibrium, a strictly increasing function $\psi(\eta)$ exists; its inverse $\eta(\psi)$ is weakly positive. That is, in any equilibrium of rents, firm, and worker locations, for any $\eta_1 > \eta_2$, $\psi(\eta_1) > \psi(\eta_2)$. Furthermore, in any equilibrium of the above model, a non-degenerate distribution of $\eta(i)$

[25] This condition can also be derived from the incentive computability constraints using the Mirrlees conditions. Following Mirrlees (1976), we can further restrict the relevant outside options $\eta_H,\ \eta_L$ to be locations with the value of $\eta$ closest to $\eta^*$. Intuitively, if the firms have optimized, deviations from the optimum are increasingly detrimental, and therefore attention can be restricted to local deviations. Thus, for any choice set over which $\eta$ is continuous, equations (3) and (4) can be evaluated by taking the limit as $\eta_H \to \eta_L$. In the limit, both inequalities bind, becoming the mapping function in equation (4). In this way, the matching between firms and landowners is analogous to a mechanism design problem with a continuum of types with private information.

must exist for some locations in $S$. The only stable equilibria exhibit "strong" sorting, such that the one-to-one function $\psi(\eta)$ is continuously increasing.

Appendix 1 first proves the assumptions of the model prohibit a degenerate equilibrium distribution of $\eta(i)$, then that a location with a higher $\eta$ must be captured by firms with higher productivity parameter $\psi$.

The appendix further elaborates on the possible scope of exceptions to sorting. Any equilibrium must display "weak" sorting properties: the function $\psi(\eta)$ must be weakly increasing. When the function $f(\eta)$ is non-singular, sorting is strict. Singularities in $f(\eta)$ imply a positive mass of locations with a single value of $\eta$, and therefore a range of firms at that value of $\eta$. However, two firms of the same productivity may not be found at locations with different $\eta$'s. The appendix also shows that any weak sorting equilibrium is unstable.

The remaining analysis assumes a stable one-to-one matching of firms to location productivity. The appendix also notes small changes to the leading predictions of the model in the case of a weak sorting equilibrium, and shows the corollary to Proposition 1, that uniform density is never an equilibrium characteristic, and therefore agglomeration is a pervasive characteristic of this model.

### 2.3.3 Density gradient

Next, I rewrite the firm's profit maximization condition using the rent gradient

$$\tilde{\pi}_l = \max_h \{h \cdot \phi(\eta) - c(h)\}.$$

The first order condition of the above equation implicitly defined the density provision of landowners as a function of $\eta$ :

$$h(\eta) = c'^{-1}(\phi(\eta)). \tag{5}$$

Equation (5) sets the density of firms at a given location $i$, given the type of firm at location $i$, such that the cost of accommodating the marginal firm is equal to the willingness of that firm to pay for space at $i$. While the firm's optimization pins down the rent gradient, the landowner's optimization adjusts the density of firms at each point, thereby distributing the mass of firms in a space according to the index $\eta$. The density function $h(\eta)$ moves the

mass of firms from the space $\psi$, where the density is defined according to the distribution $g(\psi)$ to the index's space.

Because of the assumptions on the cost function, $h'(\eta) > 0$, and establishment density is increasing in $\eta$. In turn, landowners in more productive locations reap larger real profits, both because they attract more productive firms with higher willingness to pay, and because they optimally accommodate a higher density of firms.

### 2.3.4 Real estate market clearing condition

The previous two equilibrium conditions are derived conditional on a mapping of $\hat{\psi}$ into $\eta$, which moves the distribution of firms by productivity into a distribution of firms on the space of $\eta$. The landowners' optimal provision of density will feed, in equilibrium, back into the assignment of firms to locations, as more density at some locations shifts the mass of firms towards those locations. The matching of firms to locations and rents adjust accordingly so as to accommodate all firms which choose to produce.

Each $\psi$ has associated with it a specific density of firms, $g(\psi)$. The density function $h(\eta)$ describes the equilibrium density of firms at each location according to $\eta$. The function $h$ and $g$ are defined on different spaces and their distributions will not necessarily resemble one another.

However, the total mass of firms choosing to produce must equal the total mass of firms accommodated equilibrium, $H(\eta)$. Furthermore, the total mass of firms with at least a given productivity $\psi$ must be the mass of firms with at least $\psi$ in equilibrium. This intuition yields the following equilibrium constraint

$$\int_{\underline{\eta}}^{\eta_j} h(\eta) f(\eta) d\eta = \int_{\psi(\underline{\eta})}^{\psi(\eta_j)} g(\psi) d\psi$$

at any location $j$. Since this condition holds everywhere, differentiating at $\eta(j)$ we find

$$h(\eta) = \frac{g(\psi(\eta))\psi'(\eta)}{f(\eta)} \tag{6}$$

Equation (6) relates the mapping $\psi(\eta)$ to the firm density function $h(\eta)$ and the density function $f(\eta)$. For any given allocation of firms to locations, the initial density of firms of a specific productivity type must exist somewhere in the actual space.

### 2.3.5 Worker spatial equilibrium condition and wages

Workers must be indifferent between the bundle of goods they can consume at each location. For workers to be indifferent, wages at each location $i$ must exactly offset differences in the price index so workers can attain the same real wage $\lambda$ across locations.

$$w(i) = P(i) \cdot \lambda$$

Setting wages at a single location $j_1$ as the numeraire wage, the real wage can be expressed as $\lambda = \frac{1}{P(j_1)}$.

### 2.3.6 Labor market clearing condition

All labor $L$ must be used in production. For a given firm type $\psi$, the labor bill is equal to the amount of revenue each produces minus variable profits.

$$l(\eta) = \tfrac{\sigma \cdot \rho}{\lambda} \cdot \psi(\eta) \cdot \eta$$

Summing over all producing firms and dividing by wages, the total amount of labor must equal the local labor supply, $L$.

$$L = \int_{\underline{\hat{\psi}}}^{\hat{\psi}_U} \left[ \frac{\sigma \cdot \rho}{\lambda} \cdot \psi \cdot \eta(\psi) \cdot g(\psi(\eta)) \right] d\psi(\eta). \tag{7}$$

Where $\underline{\psi}$ is defined by $\underline{\psi} \equiv \phi(\underline{\eta})/\underline{\eta}$.

### 2.3.7 Solving the mapping $\psi(\eta)$

Equations (4), (5), (6), and (7) jointly determine, for any ordering $\eta$, the set of firms that produce, the arrangement of firms into locations, the matching of productivity to locations, the density of firms at each location, and the rent at each location. Putting equilibrium conditions in equations (4)-(7) together, I find the differential equation governing $\psi(\eta)$:

$$\psi(\eta) = c'' \left( \frac{g(\psi)\psi'(\eta)}{f(\eta)} \right) \cdot \left[ \frac{f(\eta)g(\psi)\psi''(\eta) + f(\eta)g'(\psi)\psi'(\eta)^2 - g(\psi)\psi'(\eta)f'(\eta)}{f(\eta)^2} \right]. \tag{8}$$

Note that the boundaries of the location productivity mapping function, $\bar{\eta}$, $\underline{\eta}$, as well as the function $f(\eta)$ are not yet defined.

In the space of $\eta$, the four equilibrium conditions provide the solution to an equilibrium where $\eta(i)$ is the mapping of locations to $\eta$. It remains to be shown that such an equilibrium exists. To do this, I introduce the last two equilibrium conditions, the worker spatial equilibrium the a balanced trade condition, and use them to solve for $\eta(i)$.

### 2.3.8 Balanced trade

Finally, because all three agents consume at their given or equilibrium locations, trade is balanced, i.e. total market for goods at each location must equal the total amount produced at the location. The local demand at $j$, $R(j)$, is therefore equal to the total revenue of all firms at location $j$. Recall each firm at location $i$ receive revenue

$$R(j) = P(j) \cdot r(\psi, j) \cdot h(\eta) = P(j) \cdot \psi \cdot \eta(j) \cdot \sigma \cdot h(\eta) \ .$$

### 2.3.9 Solving for the mapping $\eta(i)$ and the price index.

Equations (4) through (8) characterize any equilibrium in the space of $\eta$. While these equations are sufficient to derive predictions. However the general equilibrium requires further mapping of each physical location in $S$ to a value for the index $\eta$. I now use the balanced trade condition to solve for this mapping.

To solve for $\eta(i)$, I rewrite the equation for $\eta(i)$ imposing the balanced trade and worker spatial equilibrium conditions together with the mapping of firms to locations $\psi(\eta)$. This yields

$$\eta(i) = P(i)^{-\sigma} \int_{j \in S} \frac{\tau(i,j)^{1-\sigma}}{\rho^{1-\sigma} P(j)^{-\sigma}} \cdot \frac{g(\psi(\eta(j)))\psi'(\eta(j))}{(f(\eta))^2} \cdot (\psi(\eta(j)))^2 \cdot \eta(j) dj \tag{9}$$

while the price index can now be expressed as

$$P(i)^{1-\sigma} = \lambda \cdot \int_{j \in S} \left[ \frac{P(j)\tau(i,j)}{\rho\psi(\eta(j))} \right]^{1-\sigma} g(\psi) \cdot \psi^{'}(\eta(j)) \cdot dj \tag{10}$$

where $\lambda = \dfrac{1}{P(j_1)}$. Equations (9) and (10) constitute a system of nonlinear Hammerstein equations with a kernel of $\tau(i,j)^{1-\sigma}$ the solution to which determines the mappings $\eta(i)$ and $P(i)$.

Finally, note that for any equilibrium mapping of $\eta(i)$, $f(\eta)$ ensures the mass of points in $S$ is accounted for in $[\underline{\eta}, \bar{\eta}]$, such that

$$\int_{\underline{\eta}}^{\bar{\eta}} f(\eta) d\eta = \int_{j \in S} 1 \cdot dj.$$

$Proposition\ 2$ : An equilibrium exists and is described by equations (8)-(10).

$Proof$ : See appendix 3.

Appendix 3 proves the system exhibits at least one nontrivial solution and provides conditions for uniqueness. However, because the uniqueness conditions are not easily verifiable, and because they don't on their help provide predictions, I will not assume they are met through the remainder of the paper.

# 3 Predictions of the model

In this section, I use the model laid out in the previous section to derive predictions. In doing so, the full advantage of the location index is made plain. As in other geography models,[26] the interdependence of geographic decisions among many agents–in this case firms, workers, and landowners at each location–does not yield a closed-form solution and creates the potential for multiple equilibria. Because of these features, no predictions can be made for the relationship between geographic variables and location or firm characteristics. However, because the index functions as a sufficient statistic both for the location's endogenous characteristics and the characteristics of firms at the location, the model yields relationships between observable location and firm characteristics and the index that will hold in any equilibrium. I use these to derive predictions for key relationships between location characteristics and firm characteristics.

[26]See, for example, Allen & Arkolakis (2013) or Kyriakopoulou & Xepapadeas (2013).

These predictions could equally have been derived from geography-free models with agglomeration forces, and as a result they should not be considered tests of the sorting hypothesis or the geographic mechanisms of the model. Rather, they display the ability of the model, and the indexing method, to derive predictions in a geographic setting without solving the second-stage mapping between $\eta$, the location index containing all the geographic information, and locations $i$.

To test the sorting hypothesis, I then introduce the composition effects. These effects predict that shocks to density at a given location result in a change in productivity to firms at that location and at neighboring locations based on a change in the sorting pattern of firms.

I isolate two cases in which the direction of the change in productivity runs opposite the direction predicted by the agglomeration forces hypotheses. In particular, average productivity of firms at the most advantageous locations is reduced when those locations experience a positive shock to density. When density is positively shocked anywhere, the average productivity at less-advantageous neighboring locations is reduced, while the productivity at more advantageous neighboring locations increases. (This last effect accords with the agglomeration forces hypotheses). Because the first two predictions can't be derived from models where firms are ex-ante homogenous and density increases cause increases in firm productivity, they constitute a test of the sorting hypothesis.

## 3.1 Predictions for firm and location characteristics

In the set of equilibria derived in the previous section, the relationship between specific geographic locations and economic variables remains indeterminate. Underlying geography alone does not determine all subsequent economic decisions. Geographic determinism may be too restrictive a condition for any realistic model. In addition, there is no closed-form solution to the mapping of locations $i$ to the index $\eta$. These features of the model, common in geography models where networked geographic connections affect decisions, inhibits the derivation of predictions on observables.

Despite this, because the location index is a sufficient statistic for both the locations' characteristics and the characteristics of firms locating there, the indexing method yields predictions. Equations (4)-(10), generate relationships between observable location and firm characteristics and the location index. I show how predictions on key relationships between firm and location characteristics can be derived from these equations. These relationships are summarized in the following table

| | Location: | Rent | Est. den | Emp. den | Prices |
|---|---|---|---|---|---|
| | $\eta$ | $\phi(\eta)$ | $h(\eta)$ | $d(\eta)$ | $P$ |
| Firm: $\eta$ | | + | + | + | No |
| Productivity $\psi(\eta)$ | + | + | + | + | No |
| Size $l(\eta)$ | + | (+) | + | (+) | No |
| Profits $\pi(\eta)$ | + | (+) | + | (+) | No |

where $d(\eta)$ is employment density at locations with index value $\eta$, which has not been previously defined. Relationships in parentheses will not be explored in this paper. Note that there are no relationships between prices and other observable location and firm characteristics. Prices constitute one of the geographic components that form $\eta$, and can only be pinned down in a general equilibrium. Because predictions come from only those variables which can be expressed with respect to $\eta$, the model yields no predictions with respect to prices and other variables dependent on networked geographic interconnections.

Finally, note that models of agglomeration forces, where density positively affects productivity, predict the following relationships, with the exception of the positive relationship between density and profitability.[27] In particular, with the exception of predictions on firm profits, all the above predictions hold for the model derived in Appendix 5, where I re-derive $\eta$ using a combination of Marshallian agglomeration forces and pecuniary externalities.

### 3.1.1 The establishment density-productivity relationship

Using the equilibrium conditions derived in Section 2, I show the model's primary prediction: the productivity-density relationship. Unlike previous theories of firm sorting, this is a prediction, not an assumption of the model. The relationship arises because the most advantageous locations simultaneously attract the most productive firms and command the highest prices, inducing landowners to allow higher establishment density. Both landowners and firms react to the location's characteristic, and the establishment density-productivity relationship arises out of their correlated decisions.

To derive this relationship, I start by introducing the relationship between rents and location advantage and firm quality and location advantage, which will be used here and in the following predictions as well.

*Lemma* 3: In any sorting equilibrium, the mapping $\hat{\psi}(\eta)$ is increasing $\partial\hat{\psi}(\eta)/\partial\eta > 0$, and the rent gradient is convex $\partial^2\phi(\eta)/\partial\eta^2 > 0$. Firm density is increasing in

[27] In models where firms are ex-ante homogenous, and location effects create productivity differences, spatial equilibrium dictates that firms must pay fixed costs that offset the effect of higher productivity on profits.

$\eta$, $\partial h(\eta)/\partial\eta > 0$.

Lemma 3 follows directly from the first and second order conditions of the firm and landowner optimization problems. As ensured by super-modularity, more advantageous locations are matched to more productive firms. From equation (4), it is clear that this alone ensures the rent gradient is increasing and convex in $\eta$. Because, by equation (4), the rents are increasing in $\eta$, and landowners provide density such that the increasing marginal cost equals the rent, more advantageous locations must have higher density.

Next, I decompose the relationship between firm productivity and density into the relationship between productivity and location advantage, and density and location advantage.

$$\frac{d\psi(\eta)}{dh(\eta)} = \frac{d\psi(\eta)}{d\eta} \cdot \left(\frac{dh(\eta)}{d\eta}\right)^{-1}.$$

*Prediction* 1: Locations with higher firm density have more productive firms: $d\psi(\eta)/dh(\eta) > 0$.

The model's first prediction immediately follows from the above equation and Lemma 3.

### 3.1.2 The establishment density-employment relationship

The model predicts a positive relationship between establishment density and the size of firms. Intuitively, higher demand increases firm size through consumer substitution in two ways. First, more productive firms charge lower prices and capture larger shares of demand, irrespective of their location. Second, holding firm productivity constant, higher $\eta$ locations increase demand through a combination of lower local variable costs, and lower transportation costs to relatively more markets (higher local demand). Because firms sort, more productive firms grow larger due to a Matthew effect: the most productive firms, larger in their own right, purchase locations that push them to grow even more.[28]

Recall that the labor force hired by a firm at $\eta$ can be expressed as

$$l(\eta) = \tfrac{\sigma\cdot\rho}{\lambda} \cdot \psi(\eta) \cdot \eta.$$

Using the method set out in Section 3.1.1, the above equation leads to the second prediction:

[28]This in turn implies that part of the well-known productivity-scale relationship is a result of location-specific effects.

*Prediction* 2: Firms in denser locations will be larger.

Prediction 2 follows from the first order condition in the above equation and the sign of the relationships in Lemma 3. Intuitively, higher $\eta$ locations have both larger firms and higher firm density, and both effects positively affect the relationship between productivity and employment density.

### 3.1.3 The employment density-productivity relationship (population density/productivity relationship)

In the model, total employment density at a given location is equivalent to total population density at that location, as there is no commuting. Using the labor market clearing condition expressed with respect to $\eta$ and $\psi$ the equality

$$\int_{\underline{\eta}}^{\eta} d(\eta) \cdot f(\eta) d\eta = \int_{\underline{\psi}}^{\psi} \left[ \frac{\sigma \cdot \rho}{\lambda} \cdot \psi \cdot \eta(\psi) \cdot g(\psi(\eta)) \right] d\psi$$

must hold for each variable upper boundary $\psi(\eta)$ and $\eta$. Differentiating both sides and substituting equation (6) yields

$$d(\eta) = \frac{\sigma \cdot \rho}{\lambda} \cdot \psi \cdot \eta(\psi) \cdot h(\eta)$$

Prediction 3 follows from the above equation for employment density and Lemma 3.

*Prediction* 3: More productive firms locate in higher employment and higher employment density areas. $d\psi(\eta)/dd(\eta) > 0$

Intuitively, employment density is simply the firm size at a location times the density of such firms. This relationship would be positive even without the density response built into the model; if all landowners had a constant, inelastic supply of space, making the number of firms at each location constant, the fact that firms in more advantageous locations are larger ensures that firm productivity increases with employment density. Here, the additional density response only increases the size of the employment-location advantage elasticity.

### 3.1.4 Productivity and rent

In the model, firms pay for the advantage of higher demand inherent in higher-$\eta$ locations. In geography models with pecuniary externalities and homogenous firms, price differences would arise between locations without firm sorting, but productivity would be constant across locations. The introduction of heterogeneous productivity amplifies price differences and also ensures that more productive firms location in higher priced areas.

The relationship between rents and firm productivity can be expressed as

$$\frac{d\psi(\eta)}{d\phi(\eta)} = \psi(\eta) \cdot \frac{d\psi(\eta)}{d\eta}.$$

Intuitively, firms in locations that are more advantageous are more productive and locations that are more advantageous are more expensive. Prediction 4 immediately follows from the above equation and Lemma 3.

*Prediction* 4: More productive firms pay higher rents: $d\psi(\eta)/d\phi(\eta) > 0$.

### 3.1.5 The density-profitability relationship

While more productive firms pay higher fixed costs to operate in more productive locations, their overall profits are higher. Intuitively, for a location $i$ with a given $\eta$, more productive firms have higher sales at $i$, yet face the same fixed costs (and variable costs). Firms that are more productive than the firm assigned in equilibrium to $i$ would therefore have higher profits at $i$, yet because of the incentive compatibility constraints, their own location must be more profitable for them. Thus even though fixed costs are higher at high-$\eta$ locations, the more productive firms that locate there do so precisely because those higher fixed costs are not out-weighed by the variable profit gains they make, and their overall profits remain higher than their less productive counterparts at less costly locations. Finally, these locations are also denser, as higher prices induce landowners to increase the amount of density they provide.

The model yields the following, final major static prediction:

*Prediction* 5: Firms in denser locations are more profitable. $d\pi(\eta)/dh(\eta) > 0$.

*Proof* : First, note that $\pi'_f(\eta) > 0$, which is true by taking the first order condition of $\pi_f$ and substituting equation (4). With Lemma 3, this guarantees the result in *Prediction* 5.

See footnote 24 for an alternative explanation based on results from the mechanism design literature.

This prediction can only hold in models of firm sorting. Specifically, in models with agglomeration forces and homogenous firms, spatial equilibrium holds that price differences, wage differences, and location productivity must offset each other so that all firms are indifferent between locations. Firms pay for increased productivity with higher location-specific costs. If costs did not offset productive amenities, and profits differed across locations, firms in other locations could do better by relocating. In such models, differences in

profitability can only be maintained if entrepreneurs receive location-specific consumption amenities that are higher in unprofitable locations so that entrepreneurial utility equalizes across locations when profits do not.

## 3.2 Composition Effects

As with the existent sorting models, the predictions of the model thus far have failed to empirically distinguish the sorting hypothesis from models of agglomeration forces, as these alternatives can account for nearly all of the statics predictions in section (3.1). In effect, the observational equivalence result has up until now, continued to hold. In this section, I introduce the composition effect as a test capable of breaking the observational equivalence result and empirically distinguishing between the sorting and agglomeration forces hypotheses.

Under the sorting hypothesis, the productivity of firms at a given location is a function of the mapping $\psi(\eta)$, itself a function of the rent and density at all other locations Exogenous changes to the density of firms at a single location can affect the entire distribution. In the following subsection, I examine two specific instances where small, positive shocks to density generate changes to local productivity through composition effects. In the first case, the composition of firms in the center of the city is negatively affected by positive local changes in density. Second, I show how a shock to density anywhere generates negative effects on the productivity of firms in less advantageous locations and positive effects in more advantageous locations.

### 3.2.1 Composition changes in the urban core

Positive changes to the density in the urban core of the model, defined as the highest $\eta$ locations, reduces the average productivity of firms at those locations. Intuitively, the urban core, defined as the most advantageous set of locations in a given market, house the most productive firms. Positive changes in the density at the urban core must absorb firms that were formerly priced out of the core and less productive than the least productive firm there. Because the model has no density effects, the addition of more firms has no affect on the productivity of existing firms.

I define the average productivity between some cutoff $\eta_c$ and the most advantageous location $\bar{\eta}$

$$\tilde{\psi} = \frac{\int_{\psi(\eta_c)}^{\psi(\bar{\eta})} \psi g(\psi(\eta)) d\eta}{\int_{\psi(\eta_c)}^{\psi(\bar{\eta})} g(\psi(\eta)) d\eta}.$$

Equation (6), the real estate market clearing condition, ensures that the total density

of firms between productivity levels $\psi(\bar{\eta})$ and $\psi(\eta_c)$ is accounted for in the density of firms present between $\eta$ and $\eta_c$.

$$\int_{\psi(\eta_c)}^{\psi(\bar{\eta})} g(\psi(\eta))d\psi = \int_{\eta_c}^{\bar{\eta}} h(\eta)f(\eta)d\eta.$$

The real estate market clearing condition drives the composition effect. Because more real estate exists, given a positive shock, within the urban core, more firms must enter. For the condition to hold, the lower bound must move down so that the full mass of firm increases.

To model a shock to the supply density, for some location $\eta_1 \in [\eta_c, \bar{\eta}]$ I assume an idiosyncratic cost of development at that location, introducing a new parameter $\kappa \leq 1$ which affects construction costs for all locations $\eta \in [\eta_1, \bar{\eta}]$. Formerly, construction costs everywhere were identically defined according to the function $c(h(\eta))$. Now, the construction costs are redefined as

$$c_{new}(h(\eta)) \equiv c(h(\eta)) + \kappa(h) \cdot \epsilon.$$

with $\epsilon$ arbitrarily small and for a function $\kappa(\eta)$ defined as

$$\kappa(\eta) = \begin{cases} 0 & \eta < \eta_1 \\ a[h(\eta) - h(\eta_1)] & \eta \geq \eta_1 \end{cases}.$$

for some $a$. This functional form hypothesizes an arbitrarily small shock to density while preserving the smoothness of the functional gradients.

The cost of providing density above $\eta_1$ deviates from the otherwise symmetric cost across the rest of the space. A negative $a$, by equation (5) has a positive effect on density at $i$. Furthermore, the real estate market clearing condition now becomes

$$\int_{\psi_{new}(\eta_c)}^{\psi(\bar{\eta})} g(\psi(\eta))d\psi = \int_{\eta_c}^{\bar{\eta}} h_{new}(\eta)d\eta.$$

where $h_{new}(\eta)$ is the new density function and $\psi_{new}(\eta_c)$ is the new cutoff firm productivity. In particular $h_{new}(\eta) > h(\eta)$ when $a < 0$.

To accommodate the new density, the left-hand side of the condition must also increase, which is to say the total mass of firms between $\bar{\eta}$ and $\eta_c$ must expand. But this can only happen by lowering the lower bound, $\psi_{new}(\eta_c) < \psi(\eta_c)$. In order to accommodate more firms in the same space, new, less productive firms that were previously priced out must enter. Prediction 6 follows:

*Prediction* 6: Reductions in building costs in the urban core, defined as the most advantageous locations, by increasing density, decrease average productivity in those locations. $d\tilde{\psi}/da \leq 0$.

Note that the prediction holds weakly. As the sorting pattern changes so that less productive firms enter at each location, the quality of firms at those locations decreases marginally, reducing the density provided to them by the landowners. This attenuates the initial shock, however it cannot reverse the direction.

Note that without the index $\eta$, the composition effect only could not give clear predictions. Specifically, the urban core is defined with respect to $\eta$. Non-marginal changes in density have the potential to affect advantage of locations in unpredictable ways through geographic general equilibrium effects; increases in density at one location increase the size of local markets for nearby firms and affect price index at all locations. Without closed form solutions, it is impossible to predict the effects of density shocks as mediated through such geographic interconnections. The ability to define the urban core through the location index that can be held constant through marginal changes is crucial for this prediction.

The sign of this prediction obviously contrasts with models where firms are ex-ante identical and density causes increased productivity. In these models, with some exceptions, the productivity of firms at $\eta_1$ would increase, and the productivity of firms at nearby locations would either remain unchanged, or, through spillovers from firms at $\eta_1$, increase. This prediction may not hold in theories positing nonlinear relationship between density and productivity, where, at high levels of density, congestion forces caused decreases in productivity.

The direction of composition changes in the urban core is identified because the only margin for adjustment is the lower bound of firm productivity. The same shock to any other subset of locations $[\eta_{c1}, \eta_{c2}]$ would have two margins of adjustment, from both higher and lower ends of the productivity distribution. As such, the direction of any change would be ambiguous.

### 3.2.2 Composition changes to competing locations

Although the model has no clear predictions for the direction of composition changes when development costs are similarly shocked in such subsets of the space $[\eta_{c1}, \eta_{c2}]$, the model does have predictions for such a shock's effect on productivity at neighboring subsets $[\eta_{c0}, \eta_{c1}]$ and $[\eta_{c2}, \eta_{c3}]$, where $\eta_{c0} < \eta_{c1} < \eta_{c2} < \eta_{c3}$. Such shocks negatively affect the productivity of firms at less-advantageous neighboring locations $[\eta_{c0}, \eta_{c1}]$, and increase

productivity at more advantageous locations $[\eta_{c2}, \eta_{c3}]$.

Intuitively, the new firms between $[\eta_{c1}, \eta_{c2}]$ may come from either neighboring location, but will be marginal in either location. All firms in $[\eta_{c0}, \eta_{c1}]$ are priced out of the more advantageous locations. Those now entering those locations from below will be those on the margin, the most productive firms in $[\eta_{c0}, \eta_{c1}]$, who were closest to being willing to pay for $[\eta_{c1}, \eta_{c2}]$. The reverse is true for firms entering from above: the firms most easily enticed into the new space available at the less advantageous location must have been the most marginal, and therefore least productive in $[\eta_{c2}, \eta_{c3}]$. The removal of marginal firms from the upper and lower margins negatively and positively affect the quality of the average firms in $[\eta_{c0}, \eta_{c1}]$ and $[\eta_{c2}, \eta_{c3}]$, respectively.

Average firm productivity for this neighboring set of spaces is defined as

$$\tilde{\psi}_{NL} = \frac{\int_{\psi(\eta_{c0})}^{\psi(\eta_{c1})} \psi g(\psi(\eta)) d\eta}{\int_{\psi(\eta_{c0})}^{\psi(\eta_{c1})} g(\psi(\eta)) d\eta}.$$

If there is any adjustment on the lower margin of firms in $[\eta_{c1}, \eta_{c2}]$, $d\psi(\eta_{c1})/da \geq 0$. But this lower cutoff is also the upper cutoff of the neighboring subset $[\eta_{c0}, \eta_{c1}]$. In addition, shift in the matching function between $\eta_{c0}$ and $\eta_{c1}$ implies a weakly decreasing lower bound, $d\psi(\eta_{c0})/d\kappa \geq 0$, as even after adjustments in density provision in $[\eta_{c0}, \eta_{c1}]$, additional firms enter from below to compensate for firms leaving to $[\eta_{c1}, \eta_{c2}]$. Together, these two margins of adjustment move the average $\tilde{\psi}_N$ in the same downward direction.

> *Prediction* 7: Reductions in building costs between any locations $[\eta_{c1}, \eta_{c2}]$ reduces average productivity $\tilde{\psi}_{NL}$ in the set of less-advantageous neighboring locations $[\eta_{c0}, \eta_{c1}]$, $d\tilde{\psi}_{NL}/da \leq 0$. The reverse is true for more advantageous locations, $d\tilde{\psi}_{NU}/da \geq 0$.

Because only one or both margins of change may be in effect for the productivity boundaries of $[\eta_{c1}, \eta_{c2}]$, the signs of the predictions in Prediction 7 is weak.

In a model where proximity to density increases productivity, the additional density should positively affect productivity both at higher and lower density neighboring locations, holding the density at those locations constant. The differential impact on higher and lower indexed locations is unique to this model.

# 4 Evidence

In the following section, I use establishment-level data on US firms in order to test the predictions of the model. The model's static predictions on the relationships between loca-

tion and firm characteristics match the data. To my knowledge this is the most extensive documentation of these relationships at the tract level using US data. Again, these predictions are not exclusive to the model in this paper; most of these relationships are present in models of agglomeration forces, where density creates productivity differences. For this reason, the documentation of these relationships do not test the sorting hypothesis,

I then use the panel structure of the Census data to test for sorting among firms that move establishments and firms that expand into new markets. Confirming the literature (Gaubert, 2014), I find evidence for sorting at the city level. In addition, I find evidence for sorting within cities. I also use the panel nature of the data to test for density effects. When controlling for firm and industry effects, I find no evidence of density effects. While these results support the sorting hypothesis, they may be driven by sorting on unobservables unrelated to productivity.

Finally, I test for sorting using the composition effects established in section 3. I isolate shocks to construction expenditures – at the city center and at competing tracts – on the productivity of entrants. I instrument for supply shocks to density using inter-city real estate developer linkages. Positive density supply shocks at the city center and at more advantageous neighboring tracts have negative effects on the productivity of entrants. These results conform to the predictions of the sorting hypothesis and is inconsistent with models of agglomeration forces, where density increases the productivity of local firms.

## 4.1 Data

I use restricted access US Census Bureau data on all US establishments between 1992-2007. This includes yearly administrative data on employment and payroll from the Census' Longitudinal Business Database (LBD) and US Economic Census data from all economic Censuses from the years 1992, 1997, 2002, and 2007. In addition, I use IRS establishment-level data to supplement yearly sales data in the LBD and for micro-geographic data at the establishment level, including establishment address, zip code, Census Block and Census Tract. Of all establishments in the LBD in each year – numbering between 6.5 and 7.5 million – I am able to assign tract information to roughly 90%.[29] My sample excludes single-employee establishments. Reported sales in retail, wholesale, and non-tradable service sectors may be reflect differences in local price indexes. To mitigate this issue, I restrict my sample only to tradable sectors. My final sample is composed of between 4.5 and 5 million establishments per year. Column one of Table 1 reports summary

[29]Tract information is provided for between 30-60% of observations each year. Using address matching across multiple observations, I am able to assign an additional tract information to an additional 40% of firms. For 20%, I use zip codes to impute tract. A remaining 10% of establishments cannot be traced to specific tracts using the data available. A CES paper details the imputation process.

statistics for this sample.

The majority of my analysis will be at the tract level. I supplement this basic sample with public Decennial Population Census data on tract population, housing, and demographics, and more detailed data on a subset of establishments from Census Surveys, including the Annual Survey of Manufactures in order to confirm the robustness of my results.

Population and firm density are constructed using 2000 Population Census population and SSEL yearly count of active establishments over square miles of land in a tract, respectively. I construct output per worker as a measure of firm productivity. While output per worker is a measure of productivity commensurate with the model, it is realistic to assume the measure in the data is affected by capital levels and worker heterogeneity. To ensure my results are not driven by these forces that are unaccounted for in my model, I use value added per worker and gross margin (value added minus payroll), available for establishments responding to Census of Manufacturing, as well as total factor productivity, as calculated by Petrin, Reiter, and White (2012), where possible to confirm results. Columns two and three of Table 1 report summary statistics for these sub-samples, respectively.

Non-residential real estate data is taken from responses to rents at the establishment level from the Censuses of Manufacturers. Although rent does not vary by firm output or employment in the model, I compute rent per worker as an additional metric to ensure plant-size differences are not driving my results.

Finally, I isolate two further sub-samples of establishments: establishment relocations and firm expansions. The former group is identified using a methodology slightly more conservative than that of Lee (2008), isolating establishments that are part of multi-unit firms that shut down in one year and open in a new CBSA at least 50 miles away, in which the firm was not previously present, in the following year. The latter group isolates new establishments of multi-unit firms opened in CBSAs, at least 50 miles away, where there was previously no firm presence. Summary statistics for these two samples are provided in columns four and five of Table 1.

## 4.2 Evidence on static relationships

The following sub-section uses Census establishment-level data to test the predictions of the model. The cross-sectional relationships are broadly consistent with those predicted by the model.

Predictions 1-5 posit relationships between establishment level outcomes and tract-level variables. To test these relationships, I estimate the following equation

$$log(F_{it}) = \alpha_0 + \alpha_1 \cdot log(T_{tr,t}) + \alpha_2 \cdot X_{c,t} + \alpha_3 \cdot X_{i,t} + \epsilon_{it}$$

where $F_{it}$ is the establishment-level characteristic for establishment $i$ measured at time

$t$, $T_{tr,t}$ is the tract level characteristic for tract $tr$ , measured at time $t$, $X_{c,t}$ is a vector of city by industry by year fixed effects (used in the second specification only), and $X_{i,t}$ is a set of establishment variables at time $t$, including industry-year fixed effects using the full NAICS code,[30] age, and cubic polynomials of the establishment's latitude and longitude to account for potential spatial auto-correlation in the data.

Tables 2a and 2b reports relationships for each prediction. City-year fixed effects are added to even-numbered columns. Odd-numbered columns express the basic cross-sectional relationship, comparing tracts both within and across cities, while even columns display the relationship exploiting only within-city variation. All standard errors are clustered at the CBSA level.

Columns one and two report the elasticity between establishment sales per worker and establishment density. The relationship is consistent both across all locations and using within-city variation in column two, and is within the lower-end of the range in the literature, usually between 3% and 8%.[31] Columns three and four repeat the exercise with value added per worker as a measure of firm productivity. The elasticity is about 50% lower, but fairly constant whether examining between or within city variation. Columns five and six report the relationship using TFP and find a lower figure.

In Table 2b, Columns seven and eight test the firm size - establishment density relationship. The 5% elasticity is consistent both across all tracts and within cities only. Columns nine and ten find a consistent 20% elasticity between productivity and area commercial rents. Columns eleven and twelve and estimate an elasticity of the employment density-productivity relationship, excluding employment in the observed firm, of about 1%.

Finally, the model predicts that firms in denser locations will receive higher profits. Columns thirteen and fourteen find a positive relationship between log gross margin per employee, measured as value added minus payroll, divided by employment, and establishment density. Gross margin accounts for labor and intermediate inputs but does not account for capital stock. Because the data has poor measures of capital stock at the establishment level, accounting for depreciation without imputation is not possible. If depreciation rates or capital stocks vary significantly by tract density, then this relationship does not capture and is not driven by differences in profitability across density percentiles. This last measure must be taken purely as suggestive of the possibility that profits do indeed vary along with establishment size and worker productivity, across density percentiles.

The positive relationship between gross margin and density has an important secondary

[30] Because my sample spans the 2002 SIC / NAICS crossover, I use NAICS-year or SIC-year fixed effects for each code. Replicating the results for a range of years with just SIC or just NAICS codes does not affect the results

[31] See Rosenthal & Strange (2004) for a complete review of previous findings.

implication. Because the model and thus these empirics do not account for differences in labor force quality (such as education attainment), I risk mistaking worker productivity for firm productivity (see Combes et al. (2008) for a discussion thereof). If it was the case that worker productivity was driving the empirical relationships explored here, one would predict a negative or zero elasticity between margins, calculated as value added minus payroll, and density.

Figures 2a-2f report binned scatterplots where residuals of location and establishment characteristics from these regressions. The scatterplots show that these relationship are remarkably consistent across all density percentiles. Appendix figures replicate these non-linear plots using progressively fewer controls. Appendix Table 2 replicates Table 2 using the entire population of US establishments and finds similar results.

## 4.3 Evidence from movers

I use the above model to form predictions on the relocation decisions of plants and expansion decisions of multi-unit firms. The model does not directly make predictions about relocations or expansions: it is static and has no multi-unit plants. Nevertheless, observation of firm characteristics in one period allows me to measure such characteristics when the firm's establishments are divorced from their subsequent locations, making relocations and expansions useful descriptive evidence. I find evidence consistent with the sorting hypothesis, mixed evidence for city-size effects and no evidence for density effects. Above all, it should be stressed that these results do not show causal relationships, as they may be driven by selection on omitted variables unrelated to productivity.

I consider two distinct groups of movers: (1) multi-unit establishments that move at least one plant between cities at least fifty miles away, and (2) multiunit establishments that open new establishments in new cities at least fifty miles away. The data does not allow for the identification of relocations of single-unit firms or expansions of previously single-unit firms.

First, I consider multi-unit firms that relocate the production of a single establishment. I follow the literature (Lee, 2008) in identifying plant moves. I populate my sample by identifying only new plants with an identical 4-digit modal industry code as one or more plants within the same firm that closed in a separate city and state in the previous year. [32]This sample consists of roughly 0.01% of the total population of firms in each year. I estimate

[32]This is a more conservative definition than Lee 2008, who includes new plants within the same industry as existing plants that reduced output by at least 50% in the previous period. For this reason my sample is smaller than his.

$$log(h_{tr,e,t}) = \alpha_0 + \alpha_1 log(\psi_{i,t-1}) + \alpha_2 \cdot X_{e,t} + \alpha_3 X_{f,t-1} + \alpha_4 dist_{e,f} + \epsilon_{e,t}$$

where $h_{tr,e,t}$ is the establishment density of the tract where the relocated establishment $e$, as a part of firm $f$, exists in period $t$, $\psi_{f,t-1}$ is the firm's average productivity, calculated as output per worker at each establishment and weighted by establishment size, in period $t-1$, $X_{e,t-1}$ is a vector of establishment-specific variables including latitude and longitude cubes, industry-year fixed effects, and in some specifications will include industry by CBSA by year fixed effects, $X_{f,t-1}$ is a set of latitude and longitude cubes for firm centroid at $t-1$, and $dist_{e,f}$ is the distance between the firm centroid at $t-1$ and the establishment location at $t$.

Table 3 reports the results. Columns one and two report establishment movers while three and four repeat the exercise for firm expansions. As expected under sorting, output per worker in prior years positively predicts the density at the new location. When CBSA-year fixed effects are taken into account in even columns, the elasticity drops by between half to two thirds, although remains significant for plant relocations.

If firms did not sort on density, a zero or negative result would be expected driven by mean reversion. While suggestive of sorting, these results are limited by two identification problems. First, both sorting and agglomeration forces could be affecting my coefficients, and therefore their magnitude may not reflect the work of sorting alone. Second, the sorting hypothesis proposes sorting based on productivity. Instead, the observed sorting may be based on other firm unobservables that correlate with density. Because both the previous and current period measures of output could be the result of density-driven agglomeration forces, these measures may confound agglomeration forces for sorting on productivity where none exists. For that reason, they must be interpreted with caution.

Table 4 directly tests for agglomeration forces by examining how density at new locations affects productivity when conditioning on the firm's productivity in the previous period. I estimate

$$log(\psi_{e,t}) = \alpha_0 + \alpha_1 log(h_{tract,e,t}) + \alpha_2 log(\psi_{f,t-1}) + \alpha_3 \cdot X_{e,t} + \alpha_4 X_{f,t-1} + \alpha_5 dist_{e,f} + \epsilon_{e,t}$$

where $\psi_{e,t}$ is the output per worker of the new establishment at time $t$, and other variables are as before.

Columns one and four report a positive effect of density on establishment productivity. When city-year fixed effects are introduced in columns two and five, the point estimates fall and the effect becomes zero,. suggesting city-size, but not density effects, exist.

The last four regressions test use the multi-plant nature of these firms to control for firm-level productivity-year effects. Columns three and six introduce firm-industry-year

fixed effects. Here, I am comparing two relocations within the same firm-industry category in the same year. Columns four and eight add city-year fixed effects. I can reject the initial point estimates. Taken together, these regressions give weak evidence for city-size effects and no evidence for density effects.

It should be noted that these specifications do not account for differences in trends; firms which are increasingly productive may also sort into increasingly dense locations. Such effects would bias results positively. Again, these results must only be understood as suggestive.

Appendix Table 3 and Table 4 repeat the above exercise for all multi-unit movers across all industries. Though they find slightly more evidence for city-size effects and some evidence for density effects, they may be picking up price effects in non-tradable industries.

## 4.4 Composition Effect

Exogenous variation to the supply of density affects the quality of entrants, both in the urban core and at neighboring tracts. In the following subsection, I first propose a strategy for identifying exogenous variation to the supply of density, then test both composition effects. The OLS and IV results support the predictions of the model.

### 4.4.1 Instrumental Variable Approach

In order to isolate composition effects effect, I must first isolate exogenous, marginal changes to local construction costs. To do this, I use the inter-city linkages of real estate developers. Commercial real estate development is a leveraged industry (Gyourko, 2009). Developers such as real estate investment trusts, or REITs that also hold an lease real assets may be exposed to real estate shocks via effects on income from these assets. Changes in local real estate prices may therefore affect firms' propensity to supply space.

If such leaser-developers are active in more than one market, local shocks in one market can affect their ability to supply space in other markets. A firm with projects in Boston and Philadelphia, for example, may delay, sell, or scale-down a Boston project due to a negative shock to the market in Philadelphia that depletes the firms' assets. On the other hand, positive shocks to prices in Philadelphia may divert scarce resources away from Boston projects. I term the former effect an income effect and the latter a price effect.

Using the Census of Finance and Insurance, to which all lessors of real estate respond, I isolate over two hundred firms that operate in my period of observation in multiple cities and undertake new commercial real estate construction in at least one. I predict construction expenditures of a given firm's establishment using the construction expenditures and sales of single-unit leaser-developers operating in linked cities, that is, other cities where

the developers have an established presence, weighted by their previous-period payroll in each city. Both the income and price effects appear to be operational.

The first four columns of Table 5 show the relationship between predicted and actual construction expenditures for tract-level aggregates. Controlling for tract fixed effects in columns, the two are positively correlated and significant. Column two adds city and tract level controls. Columns three and four repeat the exercise using changes.

I use these predicted values as instruments for the level of construction expenditures at the tract and city levels in the following subsection in order to find the effects of these construction expenditures on the quality of entrants. The identifying assumption I make is that the level of sales and investment of single-unit establishments in linked cities affects the quality of entrants in another city only via construction expenditures of the linked firms. Several possible channels may violate this exclusions restriction and bias the results.

First, the exclusion restriction may be violated if linked cities are exposed to correlated real estate shocks. The model itself suggests the decision to enter each market is endogenous to a firm characteristics. If these endogenous links are formed between similar cities, and in particular if linked cities share characteristics that expose them to common price shocks, positive shocks in one market will appear as positive space supply shocks to the linked market. This will positively bias the sales instrument and negatively bias the expenditure instrument. In addition to correlated shocks, developers may specialize in similar types of markets, especially markets with particular similar trends. Similar trends in each market may appear and act as correlated shocks, biasing the instrument as above. Alternatively, developers may enter markets endogenously to hedge against idiosyncratic shocks. If endogenous links formed for hedging will negatively bias the sales instrument.

Columns five and six of Table 5 test for endogenous linkages by estimating the relationship between the tract-level aggregate predicted values for linked developers and city-level construction expenditures of non-linked developers. Appendix figures 1a and 1b show a binned scattered residual plot of these relationships. Relationships are positive but insignificant. Importantly, there is no evidence that developer linkages are formed to hedge idiosyncratic city-level shocks.

A third and fourth potential for bias enter as development may respond to income and price effects through other margins. The instrument relies on adjustments on the scope and timing of projects. Leaser-developers may select instead to adjust their selection of projects, choosing to delay the lowest-margin projects, or to alter the quality rather than scope or timing of a project. Lower quality sites or sites with lower-quality construction may attract lower quality firms or produce relatively less productive amenities. The former channel negatively biases results while the latter may be a positive bias.

Of these four channels, only site selection and market hedging negatively biases the sales instrument. Because there appears to be no evidence for market hedging, I only use the sales instrument.

### 4.4.2 Shock to supply of space in the urban core

Shocks to the supply of space in the urban core reduce the average productivity by accommodating the entrance of lower-quality firms.

In order to test this composition effect at the urban core using my sample, several ancillary assumptions must be made. First, for statistical power, I treat each of the 361 CBSAs reporting construction expenditures on commercial real estate as a distinct market, i.e., that no inter-city location decisions are made by firms. As stated in Section 3.2.1, the direction of the composition effect is only certain when the most-productive firm's location is fixed. If firms choose establishment locations within and between cities, more space in the city center many attract more productive firms from other, larger cities, attenuating any negative result.

Proposition 7 implies that the productivity of entrants relative to incumbents is decreasing in a shock to space provided in the urban core. Having isolated each city, the next step must be to identify the urban core, or the most advantageous locations in the city. Because location advantage, $\eta$, is a sufficient statistic for all economic activity, the model provides a natural proxy for $\eta$: establishment density. I rank each tract according to the establishments per square mile in each.

Next, the model is agnostic as to the cutoff threshold for the urban core. This provides a natural secondary test: a negative result must be robust to any arbitrary lower threshold. I break the city into density percentiles and test for an negative result in each. Specifically, I estimate

$$log(\psi_{p,e,t}) = \alpha_0 + \alpha_1 log(c_{p,t-1}) + \alpha_2 log(\psi_{p,i,t-1}) + \alpha_3 X_{e,t} + \alpha_4 X_{tract} + \epsilon_{pe,t}$$

where $\psi_{p,e,t}$ is the output per worker of entrants at time $t$ within the percentile cutoff threshold $p$, $\psi_{p,i,t-1}$ is the average productivity of incumbents within the cutoff at time $t-1$, $c_{p,t-1}$ is the dollar value of construction expenditures in the previous-period, $X_{e,t}$ is a vector of establishment controls, including, age, latitude and longitudinal cubes, and industry, and $X_{tract,t}$ is a vector of tract fixed effects, and where $p$ is, alternatingly, the 25th, 50th, 75th, 90th, and 95th percentile. While tract fixed effects are important for removing the baseline productivity differences between tracts, trends in productivity differences are removed by controlling directly for the lagged productivity of incumbents. Proposition 7 predicts $\alpha_1 < 0$, contrary to models where density causally improves productivity.

Because construction may respond to changes in demand, I instrument for supply shocks to construction using the predicted expenditures as described in Section 4.4.1, where the first stage estimates

$$log(\hat{c}_{p,t-1}) = \beta_0 + \beta_1 \cdot log(pred_{p,t-1}) + \epsilon_{p,t-1}.$$

Table 6 reports the results. Columns one, three, five, seven, and nine report the OLS results for the 25th, 50th, 75th, 90th, and 95th percentiles respectively. Coefficients are small and negative, and except for at the 75 percentile cutoff, significant.

Columns two, four, six, eight, and ten report IV results. When instrumenting for changes in supply, the effect of construction is negative, however not always significant. Although a positive effect cannot be excluded at every percentile threshold, the persistent, negative effects are consistent with the sorting hypothesis.

Although agglomeration forces cannot, on their own, explain the negative result at the 95th and 50th percentiles, this result does not refute their existence. Such forces, acting in conjunction with sorting, may bias the result in either direction. If agglomeration work directly through density, added space would amplify agglomeration forces. Alternatively, if agglomeration forces work through average productivity, the decreased productivity of entrants may amplify a negative sorting effect. A negative result therefore cannot act as a rebuttal of agglomeration forces but only as evidence of the existence of sorting forces.

Additionally, the test here implicitly assumes that agglomeration effects would take place within the first five years of a shock to density. If agglomeration forces take time to develop, perhaps through changes in the structure of the labor market as workers slowly adjust to the shock, the initial negative effects may fade over time and be replaced with the positive effects of agglomeration forces.

At their highest, the IV results predict about a three percent decrease in productivity of entrants for every doubling of construction activity. These results are not enormous but do suggest extremely powerful sorting forces. Congruent with the literature (Gyourko, 2009), I likely observe on average 20% of the newly-built office and manufacturing space in urban centers in this time period. While my time period reflects a period marked by booming commercial real estate construction, new construction adds to roughly 15%-20% vacancy rates in urban office space across this period. In addition, while most new entrants may be unaffected by changes to supply, the model predicts changes to productivity will occur through the productivity of marginal entrants. It is therefore possible that the observed coefficients are the result of the interaction of sorting forces with agglomeration forces.

Finally, if density and productivity are co-determined by unobserved variables, this result may be attained without sorting. For example, if local public goods improve productivity of firms and increase density while crowding reduces productivity of all firms, more

crowding would decrease the productivity of entrants in the absence of both sorting and agglomeration forces. While this specification cannot rule this possibility out, the following specification does.

### 4.4.3 Shock to supply of space in competing tracts

Shocks to landowner's ability to provide space in any particular location affects the productivity of entrants at neighboring locations differentially based on the relative quality of the shocked location. According to Proposition 10, a shock to density in one location systematically draws the least productive firms from more advantageous locations, improving average productivity at such locations, and the most productive firms from less advantageous locations reducing average productivity at such locations. To test Proposition 8, I use the ordering of tracts by density to estimate the following tract-level regressions

$$log(\tilde{\psi}_{NL,t}) = \alpha_0 + \alpha_1 \cdot log(cont_{tract,,t-1}) + \alpha_2 \cdot log(\tilde{\psi}_{NL,t-1}) + \alpha_3 X_{tract} + \alpha_4 \cdot X_t + \epsilon_{tract,t}$$

$$log(\tilde{\psi}_{NU,t}) = \alpha_0 + \alpha_1 \cdot log(cont_{tract,,t-1}) + \alpha_2 \cdot log(\tilde{\psi}_{NU,t-1}) + \alpha_3 X_{tract} + \alpha_4 \cdot X_t + \epsilon_{tract,t}$$

where $\tilde{\psi}_{NL,t}$ and $\tilde{\psi}_{NU,t}$ are the average productivity of entrants to the next-lowest and next-highest density tracts, respectively, $\tilde{\psi}_{NL,t-1}$ and $\tilde{\psi}_{NU,t-1}$ are the productivity of incumbents of the tracts at time $t-1$, $X_{tract}$ is a vector of tract-level variables including (and limited to at first) tract fixed effects and $X_t$ are year fixed-effects.

Columns one of Table 7 report the OLS results on less-advantageous locations. Positive shocks to construction shocks have negative effects on the productivity of entrants in the lower-ranked tract, consistent with the sorting hypothesis. Column two uses the same IV as described in Section 4.4.2. Consistent with the findings in the previous subsection, the IV delivers larger negative results. While the density percentiles in Section 4.4.2 likely group large swaths of connected tracts, neighbors in the tract ranking may or may not be physically adjacent. Column one's negative OLS result may be enabled by this divorce from common market conditions like localized demand shocks.

Column three and four replicate the exercise for the next-highest ranked tract. The OLS results are small and positive, consistent with the sorting hypothesis. The IV result in column four results in a noisy, slightly negative coefficient. Although I cannot rule out negative effects, the effects in columns one and three are statistically distinct and their difference is in the direction proposed by the model. Taken together, the results of columns one through four support the sorting hypothesis.

A competing explanation without sorting may be that positive shocks to density in one tract draw some volume of homogenous firms away from other tracts, and the decrease in density drives the negative relationship. Two pieces of evidence weight against this story.

First, because close-ranked tracts may be physically distant, this effect must generate a negative result despite diffusion of the effect of a single construction project on demand across the entire city. Moreover, this effect would operate equally on higher and lower ranked tracts, and would not account for the difference in the coefficients between columns one and three.

Nevertheless, I directly control for changes in establishment density in columns five through eight. Because density is an endogenous variable, these results should be taken lightly. However, the inclusion of observed establishment density directly does not significantly alter the results.

Although the magnitudes of the effects are in line with those found in Section 4.4.2, here I focus on localized effects and a much smaller percent of the overall U.S. geography. I am likely picking up more than 20% of new construction in these roughly one thousand tracts, and the coefficients speak to average effects on a far pool of number of entrants.

Using the model as guidance, the preceding two subsections have isolated positive shocks to density that result in effects on productivity that run counter to the predictions of agglomeration models. Shocks to space provided in the urban core tend to reduce the productivity of entrants relative to incumbents. Shocks to the amount of space provided in a particular location differentially affect other tracts based on their relative advantage. Taken together, these results confirm the suggestive evidence of intra-city firm sorting.

# 5 Conclusion

I have presented a model of heterogeneous firm location decisions where firms trade-off fixed costs of higher rents for increased variable profits, through decreased marginal costs and increased access to markets. Firm sorting and location advantage are co-determined: the sorting pattern interacts with differences in locations' exogenous geographic characteristics to generate endogenous differences in location advantage. Landowners in more advantageous locations are induced to provide more density. I show that in a monopolistic competition setting with heterogeneous firms where firm location decisions are endogenous, transportation costs to market alone are sufficient to induce the density-productivity relationship. The model both fits the cross-sectional relationships in the data and can be tested against an alternative hypothesis where agglomeration forces alone drive the density-productivity relationship.

The presence of geography often creates tractability issues: it prevents analytic solutions, creates the potential for multiple equilibria, and precludes clear predictions. Faced with this tradeoff, those studying the economic effects of space have largely divorced them-

selves of the space's theoretical concerns. I introduce a change of variable, indexing the continuous space geography by endogenous location advantage. Location characteristics which are tied directly to geography (in this model, prices and demand) cannot be analytically pinned down, but the remaining location and firm characteristics can. This strategy allows for geographic analysis which neither ignores the role of geography nor succumbs to its complexity. The model is not geographically deterministic but does provide analytic predictions on observables that match the data well. Both these cross-sectional relationships and evidence from establishment moves and firm expansions are consistent with the firm sorting hypothesis.

However, this body of evidence fails to causally distinguish between sorting and agglomeration effects, as does the previous literature on the subject. To that end, the introduction of geography pays dividends. In the model, location advantage is a geographic concept. The composition at firms at each location affect neither the marginal cost of producing at that location, which is defined by the price index, nor the market access of that location, which is defined by a location's geographic proximity to endogenously determined goods markets. In contrast to the literature on inter-city firm sorting, the composition of firms at each location therefore does not affect the sorting behavior of firms. This allows me to test how shocks to certain locations affect firm productivities at other locations in unexpected ways by affecting the pattern of sorting without affecting underlying location fundamentals.

By testing for these composition effects, I find evidence for the existence of intra-city firm sorting. Examining both the effects of shocks to density in city centers on the productivity of entrants there, as well as the effects of shocks to density at any location on the composition of firms at neighboring locations, I find effects consistent with the sorting hypothesis but inconsistent with models which use agglomeration forces to explain the density-productivity relationship. This constitutes the first reduced-form evidence in favor of the firm sorting hypothesis. To isolate exogenous variation in the supply of space, I develop an instrumental variable based on the inter-city linkages of lessor-developers. Predicting construction expenditures at each location based on the sales and expenditures of single-unit lessor-developers in linked cities, I find predicted expenditures confirm the OLS results and the sorting hypothesis.

While the evidence now weighs in favor of the existence of firm sorting within cities, it does not refute the existence of agglomeration forces such as productivity spillovers. Second, the existent evidence against firm sorting at the inter-city level should be reevaluated within the context of within-city sorting. Combes et al (2012), for example, do not examine whether intra-city heterogeneity can explain the apparent lack of pattern in the productiv-

ity cutoffs of firms or the thicker right tail of firm productivity among larger cities. It may well be that sorting within cities replicates the patterns observed in the data.

The flexibility of the location index approach extends beyond its use here. While the model presented in this paper takes a strong stand on the nature of location advantage, the tools I present can be applied more broadly in models where multiple sorting geographic forces are at work. Appendix A5 shows how the strategy can be used to incorporate sorting and agglomeration effects without affecting the static predictions of the model. In future work, the index approach may be used as expressed there in order to quantify the relative strengths of these forces and their interactive effects.

This paper and this new approach bridge two distinct approaches to spatial frictions. Economic geography, which attempts to account for the effects of spatial frictions in a world where locations are networked by their geographic interconnections, and urban economics, where locations are essentially islands that trade freely with each other (or not at all in certain sectors). Spatial equilibria in urban economics are divorced from actual concerns about proximity, yet equilibrium conditions deliver clear, testable predictions. In economic geography, intractability inhibits models from delivering such clear predictions. This new approach to proximity seeks a middle ground, where geography matters but a broad set of predictions can still be made.

The mechanisms explored in this paper and the empirical results suggest significant new channels through which urban policies may affect urban growth. Restrictions on urban development through zoning regulations may act to change the composition of firm productivities. Zoning regulation, by increasing competition for space, may push out less productive firms.

Two major caveats must be expressed. First, the composition effect operates on the margin. Major changes to supply restrictions affect other general equilibrium channels and can easily overturn the toughened sorting mechanism. Second, this result is dependent on the functional form assumptions of the model. In particular, there are no productivity spillovers in the baseline model. As explored in Appendix A5, the inclusion of productivity spillovers could overturn this result. Nevertheless, the sorting channel should be considered as potentially existent cause of productivity effects of marginal policy changes in zoning restrictions.

In the model, transportation costs are the decisive factor governing market access and therefore the differential density across locations. All roads lead to Rome. However, in this model, decreased transportation costs can also increase market access in the periphery, and push production into the periphery. This double-edged sword, as hypothesized in other geography models (Krugman, 1991), reflects the dichotomy between the opening

of the Erie Canal as the impetus for the explosion of density in New York City, and the construction of the suburban highways as a precursor to New York's mid-century urban decline. All roads also lead away from Rome. However, in this model, the flattening of the rent curve is attenuated by the changing composition of firms: as only the most productive firms remain, the cost of the center remains higher than would be predicted in a model of homogenous firm quality.

Finally, although this paper did not seek to estimate the strength of market access, its predictions reconcile two literatures that have previously done so. Papers in the geography literature such as Ahlfelt et al. (2012) have estimated large impacts of changes in market access on the distribution of economic activity. Papers in urban economics that attempt to measure the relative strength of market access such as Ellison & Glaeser (1994)have been more skeptical of the continued importance of location to markets. The results here offer a possible solution: while real differences in market access may be small, they may interact with real estate market elasticities and the sorting behavior of heterogenous firms to create large differences in the distribution of economic activity.

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# Figures and Tables

**Table 1: Summary Statistics**

| | Observations | Mean | Std. Dev. |
|---|---|---|---|
| | Panel 1: Cross-section of establishments | | |
| *Sales* | 5,351,354 | 5,530,992 | 1,022,721 |
| *Employment* | 5,351,354 | 23.64 | 129.94 |
| *Payroll* | 5,351,354 | 854,970 | 7,710,470 |
| *Value Added* | 784,364 | 7,080,749 | 75,009,740 |
| | Panel 2 :Relocated Establishments | | |
| *Sales* | 65,146 | 6,129,400 | 6,126,009 |
| *Employment* | 65,146 | 33.96 | 160.69 |
| *Payroll* | 65,146 | 1,302,700 | 7,549,590 |
| | Panel 3: Firm Expansions | | |
| *Sales* | 179,956 | 6,668,400 | 5,041,589 |
| *Employment* | 179,956 | 33.215 | 105.305 |
| *Payroll* | 179,956 | 1,314,547 | 6,434,390 |
| | Panel 4: All New Establishments | | |
| *Sales* | 1,145,664 | 2,566,148 | 4,356,515 |
| *Employment* | 1,145,664 | 16.99742 | 110.6184 |
| *Payroll* | 1,145,664 | 553,311 | 4,708,828 |

Note. Sample one includes manufacturing or business services establishments responding to Economic Censuses between 1992 and 2007, excluding sole proprietorships, for which geographic data is available or could be imputed from address records. Value added is taken from the subset of firms responding to the Census of Manufactures only. Sample two includes manufacturing or business services establishments responding to Economic Censuses between 1992 and 2007, excluding sole proprietorships and single-unit plants, for which geographic data is available or could be imputed from address records and which are categorized as relocated establishments. Relocations are defined as establishments opened between census years in the same industry (4-digit SIC or NAICS) as an "origin" plant within the same firm that closed the prior year. Relocations within CBSAs or between CBSAs but less than 50 miles apart are excluded. Sample three includes established analogously categorized as firm expansions, defined as new establishments that are part of pre-existing multi-unit firms, opening in a CBSA at least 50 miles away from any other establishments of the same firm. Establishment density is computed as the number of establishments per square mile of land at the tract level. Sample four is the subset of sample one that are new establishments, defined as establishments less than 5 years old at the time of response to the Economic Census.

## Table 2a: Productivity vs Establishment Density

| VARIABLES | Log output per worker | | Log VA per worker | | Log TFP | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| *Log establishment density* | 0.050*** | 0.043*** | 0.026*** | 0.024*** | 0.003*** | 0.0021*** |
| | (0.002) | (0.003) | (0.002) | (0.002) | (0.0006) | (0.0006) |
| *CBSA/Year FEs* | No | Yes | No | Yes | No | Yes |
| Constant | 7.46 | 7.1 | 4.7 | -3.89 | 3.46 | 1.55 |
| Observations | 5,351,354 | 5,351,354 | 784,364 | 784,364 | 569,032 | 569,032 |
| R-squared | 0.39 | 0.49 | 0.29 | 0.52 | 0.47 | 0.48 |

*** p<0.01, ** p<0.05, * p<0.1

Note. Samples include manufacturing or business services establishments excluding sole proprietorships for which geographic data is available or could be imputed from address records. Sample in columns one and two are all such respondents to Economic Censuses between 1992 and 2007. Sample in columns three and four are manufacturing firms in that year range. VA is value added, reported in the Census of Manufacturers and constructed using output minus value of inputs. Sample in columns five and six are all such respondents to the Annual Survey of Manufacturing between 1993 and 2010. TFP measures are taken from Petrin, White, and Reiter (2011). Establishment density is computed as the number of establishments per square mile of land at the tract level. All regressions are at the establishment-year level and control for age of establishment, cubic polynomials for establishment latitude and longitude, industry-year fixed effects for the establishment's full SIC or NAICS code, and tract-level demographic controls including average age, gender, racial composition, education composition, and income levels. All standard errors clustered at the CBSA level.

## Table 2b: Other Cross-sectional Relationships

| VARIABLES | Log employment | | Log output per worker | | Log output per worker | | Log gross margin PW | |
|---|---|---|---|---|---|---|---|---|
| | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) |
| *Log establishment density* | 0.051*** | 0.057*** | | | | | 0.023*** | 0.023*** |
| | (0.001) | (0.001) | | | | | (0.002) | (0.002) |
| *Log avg. rent per worker* | | | 0.010*** | 0.065*** | | | | |
| | | | (0.005) | (0.005) | | | | |
| *Log employment density* | | | | | 0.011*** | 0.008*** | | |
| | | | | | (0.001) | (0.001) | | |
| *CBSA/Year FEs* | No | Yes | No | Yes | No | Yes | No | Yes |
| Constant | -5.04 | -5.66 | 2.8 | -3.2 | 9.74 | 7.1 | 3.66 | -3.55 |
| Observations | 5,351,354 | 5,351,354 | 5,351,354 | 5,351,354 | 5,351,354 | 5,351,354 | 784,364 | 784,364 |
| R-squared | 0.27 | 0.4 | 0.41 | 0.49 | 0.38 | 0.49 | 0.28 | 0.51 |

*** p<0.01, ** p<0.05, * p<0.1

Note. Samples include manufacturing or business services establishments responding to Economic Censuses between 1992 and 2007, excluding sole proprietorships, for which geographic data is available or could be imputed from address records. Establishment density is computed as the number of establishments per square mile of land at the tract level. Average rent per worker is the tract-level average of respondents to questions on rent in the Census of Manufacturers. Employment density is the employees per square mile of land area, excluding those belonging to the observational establishment. Gross margin per worker is value added minus payroll, divided by the number of employees. All regressions are at the establishment-year level and control for age of establishment, cubic polynomials for establishment latitude and longitude, industry-year fixed effects for the establishment's full SIC or NAICS code, and tract-level demographic controls including average age, gender, racial composition, education composition, and income levels. All standard errors clustered at the CBSA level.

## Table 3: Sorting on Productivity

| VARIABLES | Log establishment density | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | Establishment relocations | | Firm expansions | |
| *Prior-year log output per worker* | 0.028*** | 0.004 | 0.086*** | 0.047*** |
| | (0.007) | (0.005) | (0.014) | (0.011) |
| *CBSA FEs* | No | Yes | No | Yes |
| Constant | 22.44 | 86.97 | 15.55 | 64.28 |
| Observations | 65,146 | 65,146 | 179,956 | 179,956 |
| R-squared | 0.58 | 0.68 | 0.6 | 0.7 |

*** p<0.01, ** p<0.05, * p<0.1

Note. Samples include manufacturing or business services establishments responding to Economic Censuses between 1992 and 2007, excluding sole proprietorships and single-unit plants, for which geographic data is available or could be imputed from address records. Sample in columns one and two use establishment relocations, defined as establishments opened between census years in the same industry (4-digit SIC or NAICS) as an "origin" plant within the same firm that closed the prior year. Relocations within CBSAs or between CBSAs but less than 50 miles apart are excluded. Sample in columns three and four use firm expansions, defined as new establishments that are part of pre-existing multi-unit firms, opening in a CBSA at least 50 miles away from any other establishments of the same firm. Establishment density is computed as the number of establishments per square mile of land at the tract level. All regressions are at the establishment-year level and control for age of establishment, industry-year fixed effects for the establishment's full SIC or NAICS code, cubic polynomials for both the establishment latitude and longitude and the latitude and longitude of the closed plant or firm's geographic center, respectively, and the distance between the old and new plants or the new establishment and firm's geographic center, respectively. All standard errors clustered at the CBSA level.

## Table 4: Effects of Density

| VARIABLES | Log output per worker, 5 years forward | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| | Establishment Movers | | | | Firm Expansions | | | |
| *Log est. density, destination* | 0.0070** | 0.0016 | 0.0035 | -0.0004 | 0.0046** | -0.0030 | 0.0008 | -0.0027 |
| | (0.0031) | (0.0033) | (0.0032) | (0.0036) | (0.0024) | (0.0025) | (0.0020) | (0.0024) |
| Firm-level log output pw | 0.2806*** | 0.2735*** | . | . | 0.1407*** | 0.290*** | . | . |
| | (0.0104) | (0.0106) | . | . | (0.0131) | (0.012) | . | . |
| *CBSA fixed effects* | No | Yes | No | Yes | No | Yes | No | Yes |
| *Origin lat/lon cubes* | Yes | Yes | No | No | Yes | Yes | No | No |
| *Industry, year fixed effects* | Yes | Yes | No | No | Yes | Yes | No | No |
| *Firm, industry, year fixed eff* | No | No | Yes | Yes | No | No | Yes | Yes |
| Constant | 0.6 | 7.52 | 14.5 | 30 | 2.98 | 6.84 | 5.5 | -1.3 |
| Observations | 40,064 | 40,064 | 40,064 | 40,064 | 129,120 | 129,120 | 129,120 | 129,120 |
| R-squared | 0.93 | 0.94 | 0.94 | 0.95 | 0.93 | 0.93 | 0.95 | 0.96 |

*** p<0.01, ** p<0.05, * p<0.1

Note. Samples include manufacturing or business services establishments responding to Economic Censuses between 1992 and 2007, excluding sole proprietorships and single-unit plants, for which geographic data is available or could be imputed from address records. Sample in columns through four use establishment relocations, defined as establishments opened between census years in the same industry (4-digit SIC or NAICS) as an "origin" plant within the same firm that closed the prior year. Relocations within CBSAs or between CBSAs but less than 50 miles apart are excluded. Sample in columns five through eight use firm expansions, defined as new establishments that are part of pre-existing multi-unit firms, opening in a CBSA at least 50 miles away from any other establishments of the same firm. Establishment density is computed as the number of establishments per square mile of land at the tract level the year of observed relocation or expansion. Prior census year log output per worker is the average output per worker of all other plants in the year of relocation or expansion. Log output per worker, five years forward is the productivity measure of the new establishment taken five years after the relocation or expansion. All regressions are at the establishment-year level and control for age of establishment, industry-year fixed effects for the establishment's full SIC or NAICS code, cubic polynomials for the establishment latitude and longitude, dummies for the closure of the new plant 5 years after moving, and the distance between the old and new plants or the new establishment and firm's geographic center, respectively. All standard errors clustered at the CBSA level.

## Table 5: Predicted Construction Expenditures

| VARIABLES | Log construction expenditures, tract level | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | Levels | | Changes | |
| *Log predicted sales, tract level* | 0.105*** | 0.104*** | | |
| | (0.034) | (0.035) | | |
| *Change in log predicted sales, tract level* | | | 0.086** | 0.084*** |
| | | | (0.026) | (0.026) |
| *CBSA size* | No | Yes | No | Yes |
| *CBSA/Year FE* | Yes | Yes | No | No |
| Constant | -0.21 | -420 | -0.08 | -2.58 |
| Observations | 4,088 | 4,088 | 4,088 | 4,088 |
| R-squared | 0.22 | 0.22 | 0.01 | 0.02 |

*** p<0.01, ** p<0.05, * p<0.1

Note. Sample includes all tracts for which Real Estate Investment Trusts respond to questions in the Census of Finance and Insurance respondents on expenditures on new construction on commercial real estate, which have establishments reporting sales at least one other CBSA, and for which geographic information was present or could be imputed from address files. Log construction expenditures at the tract level is the log of the sum of the dollar value of all construction expenditures on commercial real estate, including office and manufacturing space, reported. Columns one and two report this measure in levels and columns three and four report changes between census years at the tract level. Log predicted sales are the predictions based on current-year sales of single-unit firms responding to the same census questions in CBSAs where the firm responding in the observed tract has other establishments present, weighted by the percent of sales at the firm level in that CBSA in the previous year. Changes report log differences for this predicted value. All regressions are at the tract level and include controls for year fixed effects. All standard errors clustered at the CBSA level.

## Table 6: Composition Effect At Urban Core

| VARIABLES | Log output per worker | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 25th percentile | | 50th percentile | | 75th percentile | | 90th percentile | | 95th percentile | |
| | OLS | IV | OLS | IV | OLS | IV | OLS | IV | OLS | IV |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| *Log const. exp.* | -0.007** | -0.004 | -0.006* | -0.003 | -0.001 | -0.029** | -0.002** | 0.011 | -0.005** | -0.007 |
| | (0.003) | (0.004) | (0.003) | (0.004) | (0.001) | (0.014) | (0.011) | (0.012) | (0.002) | (0.015) |
| *Angrist-Pischke value* | | 6.56 | | 8.56 | | 16.38 | | 14.92 | | 21.42 |
| Observations | 1,145,664 | 1,145,664 | 883,331 | 883,331 | 452,092 | 452,092 | 150,050 | 150,050 | 61,401 | 61,401 |
| R-squared | 0.34 | 0.38 | 0.35 | 0.37 | 0.33 | 0.38 | 0.46 | 0.38 | 0.39 | 0.35 |

*** p<0.01, ** p<0.05, * p<0.1

Note. Samples include all new entrants, defined as establishments less than 5 years old at the time of the Economic Census, in manufacturing or business services industries responding to Economic Censuses between 1992 and 2007 for which geographic data is available or could be imputed from address records, in the listed density percentile or above. Density percentiles are constructed by ranking tracts in each CBSA according to the establishments per square mile of land in the tract. Log construction expenditures is the tract-level sum of reported construction expenditures on new office or manufacturing space, in thousands of dollars, of REITs in the tract, or the instrumented value (see text for IV strategy and procedure). All regressions control for CBSA fixed effects, year fixed effects, age latitude longitude cubic functions, and full industry code fixed effect of establishments, prior-period average log output per worker of incumbent establishments. All standard errors are clustered at the CBSA level.

## Table 7: Composition Effect At Neighboring Tracts

| VARIABLES | Log output per worker | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Lower-ranked neighbor | | Higher-ranked neighbor | | Lower-ranked neighbor | | Higher-ranked neighbor | |
| | OLS | IV | OLS | IV | OLS | IV | OLS | IV |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| *Log const. expend.* | -0.048*** | -0.060*** | 0.001 | -0.004 | -0.027*** | -0.071*** | 0.001 | -0.014 |
| | (0.012) | (0.030) | (0.011) | (0.04) | (0.008) | (0.030) | (0.010) | (0.05) |
| *Neighbor density* | No | No | No | No | Yes | Yes | Yes | Yes |
| *Angrist-Pischke value* | | 19.01 | | 31.74 | | 19.91 | | 8.69 |
| Observations | 4,088 | 4,088 | 4,088 | 4,088 | 4,088 | 4,088 | 4,088 | 4,088 |
| R-squared | 0.45 | 0.38 | 0.45 | 0.36 | 0.48 | 0.38 | 0.46 | 0.36 |

*** p<0.01, ** p<0.05, * p<0.1

Note. Samples include all new entrants, defined as establishments less than 5 years old at the time of the Economic Census, in manufacturing or business services industries responding to Economic Censuses between 1992 and 2007 for which geographic data is available or could be imputed from address records, in tracts neighboring tracts for which there are REITs with positive construction expenditures on new commercial real estate. Neighboring tracts are defined first by ranking all tracts within the CBSA according to the number of establishments per square mile of land area. Neighboring tracts are those with ranks directly above or below tracts with REIT activity, and for which the relative ranks did not switch between any of the four census years. Note that these tracts are not necessarily geographically proximate. All standard errors are clustered at the CBSA level

Figure 1a: Productivity vs Density[33]

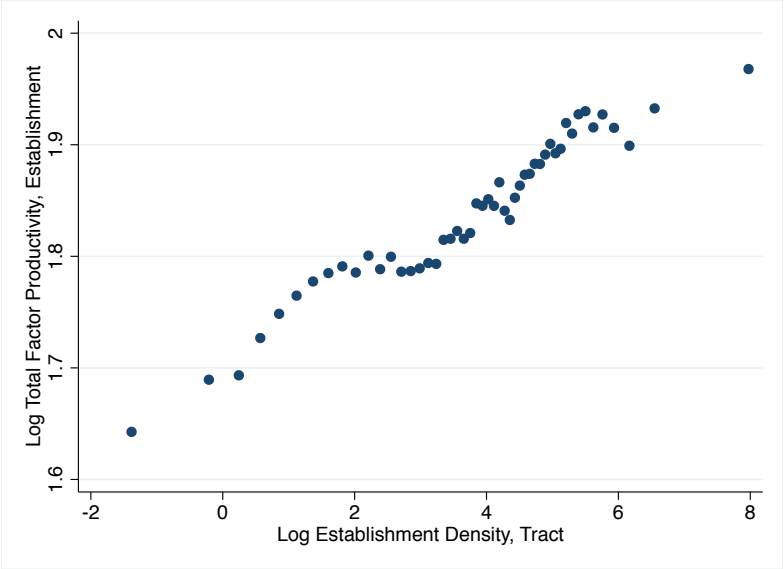
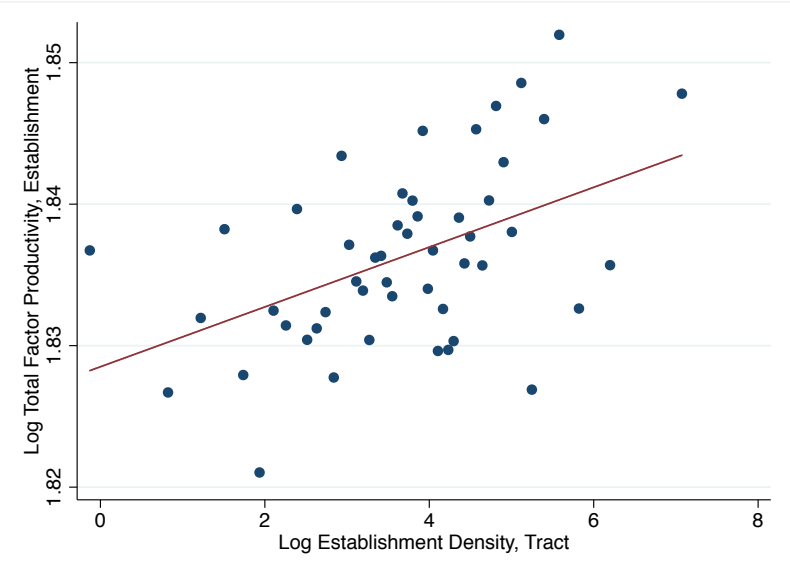



Figure 2a: Productivity vs Density, Industry and CBSA fixed effects[34]




[33]Note. Sample is all establishments responding to Annual Survey of Manufacturers between 1993 and 2010 at the establishment-year level. Productivity measures are TFP measures from Petrin et al. (2011). Establishment density is number of establishments per square mile of land area in the tract. Figure is a binned scatter plot of residuals, accounting only for year fixed effects, plant age, and cubic polynomials in plant latitude and longitude.

[34]Note. Sample is all establishments responding to Annual Survey of Manufacturers between 1993 and 2010 at the establishment-year level. Productivity measures are TFP measures from Petrin et al. (2011). Establishment density is number of establishments per square mile of land area in the tract. Figure is a binned scatter plot of residuals, accounting only for industry by CBSA by year fixed effects, plant age, and cubic polynomials in plant latitude and longitude as well as tract level demographics including median income, age, gender racial and education composition.

Figure 2a: Log output per worker vs log establishment density

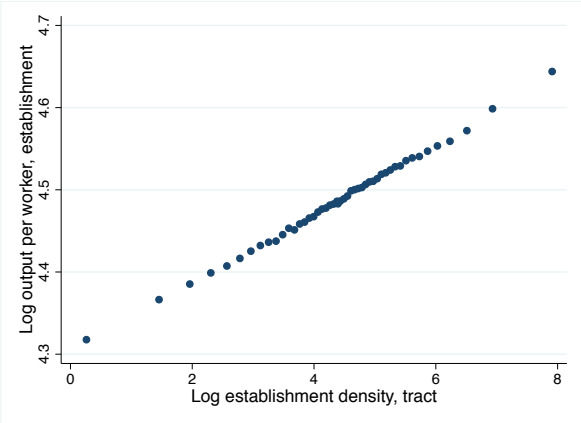



Figure 2b: Log value added per worker vs log establishment density

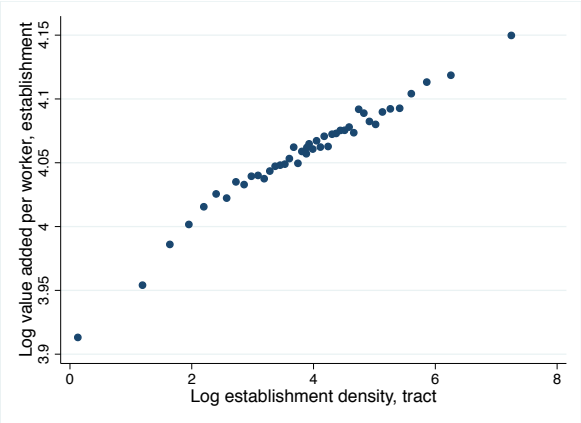



Figure 2c:Log establishment size vs log establishment density

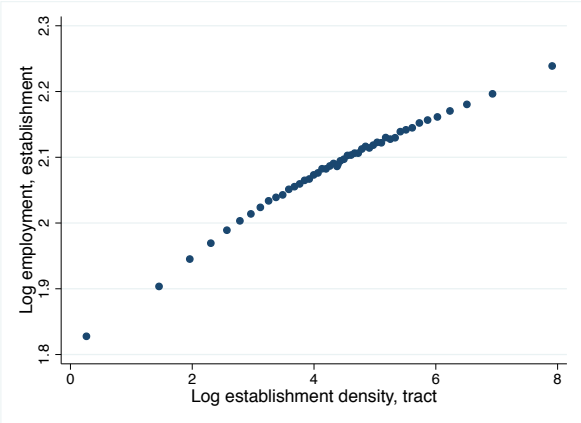



Figure 2d: Log output per worker vs log rent per worker

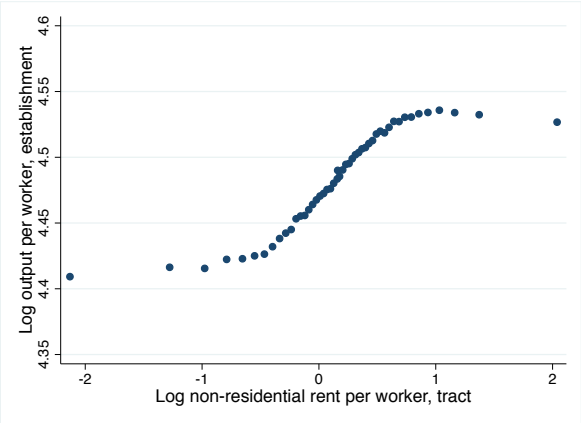



Figure 2e: Log output per worker vs log population density

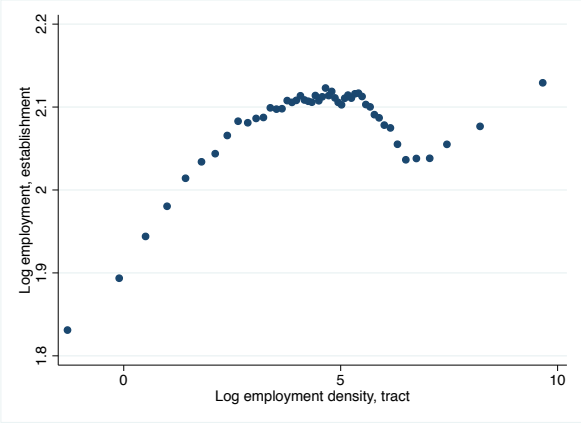
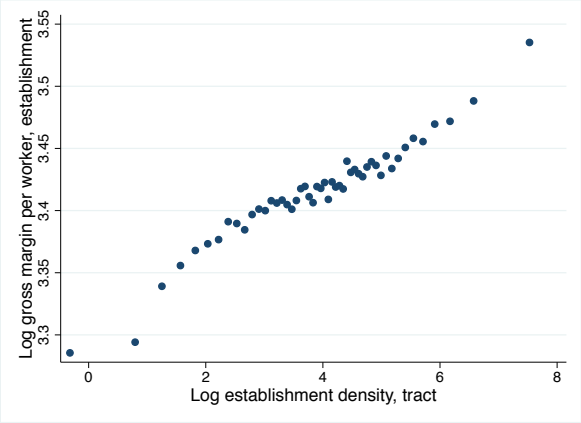



Figure 2f: Log gross margin per worker vs log establishment density




Note. All figures are binned scattered residuals of tract level variables on establishment level variables, controlling for establishment age, industry- and CBSA-year fixed effects, as well as cubic polynomials in establishment latitude and longitude.

# Appendix

## A.1 Proof of sorting equilibrium.

The proof will proceed as follows. First, conditional on there being multiple values of $\eta$ for different locations, I show that higher $\eta$ locations must in equilibrium attract higher $\psi$ firms. I then show that differences in $\eta$ must exist by ruling out cases in which no differences exist in the space $S$.

### A1.1 Sorting conditional on non-degenerate distribution

Assume the opposite, that there exists two locations $i$ and $j$ where $i$ has a higher location specific productivity, so $\eta(i) > \eta(j)$, but firms at $i$ have lower productivity $\psi_2$ than firms at $j$, with productivity $\psi_1$, so $\psi_1 > \psi_2$. Rents must be such that firms in neither location wish to move. Rent is a fixed cost. It is incurred by firms of different productivities in the same way, whereas, by Proposition 2, higher $\eta$ has differential effects on firms according to their productivity. Lower rent at $j$ must compensate the higher productivity firm at $j$ for decreased variable profit. $\phi(i) - \phi(j) \geq \frac{(\rho\psi_1)^{\sigma-1}}{\sigma}(\eta(i) - \eta(j))$, but since $\psi_1 > \psi_2$, this implies $\phi(i) - \phi(j) > \frac{(\rho\psi_2)^{\sigma-1}}{\sigma}(\eta(i) - \eta(j))$. But this violates the second incentive compatibility constraint of the firm (See equation (4)) at $j$, $\phi(j) - \phi(i) \geq \frac{(\rho\psi_2)^{\sigma-1}}{\sigma}(\eta(i) - \eta(j))$. Therefore the firm at $j$ could not have optimally chosen $j$ in equilibrium.

### A1.2 Proof of impossibility of complete non-sorting equilibrium

Next, I show that an equilibrium in which no location differs by $\eta(i)$, i.e. that $\eta(i)$ is constant for any $i \in S^n$, can be ruled out. The proof evaluates changes in the value of $\eta(i)$ at a convex boundary of the space $S$. Because at the boundary $\eta(i)$ cannot be constant, non-sorting equilibria can be ruled out.

Again I proceed by contraction, assuming for every $i \in S^n$ $\eta(i) = \eta(\not{i})$. Then we can write the equation for $\eta(i)$, according to equation (10), as

$$\eta(i) = \int_{j \in S} P(i)^{-\sigma}(1 - \xi(i)) \left[\frac{\tau(i,j)}{\rho P(j)}\right]^{1-\sigma} h(j)\psi(j)^{\sigma} \cdot \eta(j)dj.$$

Pulling out the constant value of $\eta$ and simplifying

$$\text{(A1.1)}\ P(i)^{\sigma} = \int (1-\xi(i)) \left[\frac{\tau(i,j)}{\rho}\right]^{1-\sigma} P(j)^{1-\sigma} h(j)\psi(j)dj.$$

Differentiating with respect to $i$ and substituting in using equation (12)

$$\text{(A1.2)}\ \sigma P(i)^{-\sigma-1}\nabla_i P(i) \cdot \int_{j\in S} \left[\frac{\tau(i,j)}{\rho P(j)}\right]^{1-\sigma} h(j)\psi(j)dj$$

$$= P(i)^{-\sigma} \cdot (1-\sigma) \int_{j\in S} \tau(i,j)^{-\sigma}\nabla_i\tau(i,j)P(j)^{\sigma-1}\rho^{\sigma-1}h(j)\psi(j)dj$$

$$\nabla_i P(i) \cdot \int_{j\in S} \sigma P(i)^{-1}\tau(i,j)^{1-\sigma}P(j)^{\sigma-1}\rho^{\sigma-1}h(j)\psi(j)dj$$

$$= (1-\sigma) \int_{j\in S} \tau(i,j)^{-\sigma}\nabla_i\tau(i,j)P(j)^{\sigma-1}\rho^{\sigma-1}h(j)\psi(j)dj$$

where,

$$\nabla_i P(i) = \int_{j\in S^n} \left[\frac{\tau(i,j)}{\rho}\right]^{1-\sigma} P(j)^{1-\sigma}\psi(j)^{\sigma-1}h(j)dj^{\frac{\sigma}{1-\sigma}} \cdot \int_{j\in S^n} \tau(i,j)^{-\sigma}\nabla_i\tau(i,j)\rho^{\sigma-1}P(j)^{1-\sigma}\psi(j)^{\sigma-1}h(j)dj.$$

Equation (A1.2) can be rewritten as

$$\text{(A1.3)}\ \int_{j\in S^n} \tau(i,j)^{-\sigma}\nabla_i\tau(i,j)\left(P(j)^{1-\sigma}\psi(j)^{\sigma-1}\cdot z(i)\cdot h(j) + (\sigma-1)P(j)^{\sigma-1}h(j)\psi(j)\right)dj = 0$$

where

$$z(i) = \int_{j\in S^n} \left[\frac{\tau(i,j)}{\rho}\right]^{1-\sigma} P(j)^{1-\sigma}\psi(j)^{\sigma-1}h(j)dj^{\frac{\sigma}{1-\sigma}} \cdot \int_{j\in S} \sigma P(i)^{-1}\tau(i,j)^{1-\sigma}P(j)^{\sigma-1}\rho^{\sigma-1}h(j)\psi(j)dj$$

is strictly positive over $i \in S^n$.

Expressing this condition by grouping locations for which $\nabla\tau(i,j)$ is positive, $J_2 \in S^n$, and negative, $J_1 \in S^n$,

$$\int_{J_1} \tau(i,j)^{-\sigma} \nabla_i \tau(i,j) \left(P(j)^{1-\sigma}\psi(j)^{\sigma-1} \cdot z(i) \cdot h(j) + (\sigma-1)P(j)^{\sigma-1}h(j)\psi(j)\right) dj =$$
$$\int_{J_2} \tau(i,j)^{-\sigma} \nabla_i \tau(i,j) \left(P(j)^{1-\sigma}\psi(j)^{\sigma-1} \cdot z(i) \cdot h(j) + (\sigma-1)P(j)^{\sigma-1}h(j)\psi(j)\right) dj$$

But in the limit as $i$ approaches the edge of the space, the definition of $\tau(i,j)$ reduces the set of $J_2$ to zero. While the right hand side of the equation must go to zero, every term on the left hand side is by definition positive, making the sum itself positive. This condition is therefore a contradiction when evaluated at a convex boundary of the space.

### A1.3 Partial sorting equilibria

At this point, it may still be possible for some group of locations $j$ to have equivalent location productivity potential $\bar{\eta} = \eta(j)$. In this case, some subset of firms are made indifferent between all locations $j$, landowners charge $\bar{\phi} = \phi(j)$ constant rents, and density is constant $\bar{h} = h(j)$ across all locations $j$. Marginal costs of production $w(j)$ may differ across these locations as may the price index $P(j)$ and therefore market access $\dfrac{\eta(j)P(j)}{w(j)^{1-\sigma}}$.

Although complete non-sorting has been ruled out, this partial sorting equilibrium necessitates a many-to-one match of some set of firm types to locations. In this case, the mapping $\psi(\eta)$ is discontinuous and the rent curve $\phi(\eta)$ is kinked at $\bar{\eta}$.

## A2. Landowner density and firm choice decisions

A firm's willingness to pay for a space $i$ depends on its variable profit at $i$ vs that at any other location $j$ as well as the rents that firm would face at $j$. In particular, a firm with productivity parameter $\psi$ will be willing to pay rent for space $i$ according to the function:

$$\text{(A2.1)}\ WTP(\psi, i) = \min_j \left\{\psi^{\sigma-1}\left(\eta(i) - \eta(j)\right) + \phi(j)\right\}$$

where $\phi(j)$ is the rent faced by firms at location $j$ and $\eta$ is the location-specific productivity parameter. More productive firms derive higher profits from any location, but the rent schedule derived from the IC constraints (equations (3) and (4)) makes it such that firms are unwilling to locate at less productive locations, because the rent savings are outweighed by the productivity loss, and unwilling to locate at more productive locations, because the higher rents must at least offset increased profits.

The IC constraints previously derived ensure rents are such that no firm wishes to move, but this does not immediately imply that the firm with the highest willingness to pay for a particular space is the one assigned to that space. The latter condition would ensure landowners find it optimal to provide density to the firm type matched to their location.

The following argument shows that these IC constraints do ensure just that: assuming a matching function of firms to locations and a schedule of rents supporting the firms incentives to locate, the firm with the highest willingness to pay for a given space is the firm matched to that location.

I first choose to compare the willingness to pay for space $i$ of firm $\psi(\eta(i))$with any firm $\psi < \psi(\eta(i))$.

Choosing specifically the location $k$ to which it is assigned, the willingness to pay of such a firm is less than or equal to the difference between its variable profit at $i$ and $k$ plus rent at $k$:

$$\text{(A2.2) } WTP(\psi, i) = \min_j \left\{\psi^{\sigma-1}\left(\eta(i) - \eta(j)\right) + \phi(j)\right\} \leq \psi^{\sigma-1}\left(\eta(i) - \eta(k)\right) + \phi(k)$$

However, rearranging the terms in the upper IC constraint for this firm, rent at this location, which is the willingness to pay for $i$ of the firm $\psi(\eta(i))$, or $\phi(i)$ must be greater than the right hand side of the above equation:

$$WTP(\psi, i) = \min_j \left\{\psi^{\sigma-1}\left(\eta(i) - \eta(j)\right) + \phi(j)\right\} \leq \psi^{\sigma-1}\left(\eta(i) - \eta(k)\right) + \phi(k) \leq \phi(i).$$

An identical argument (omitted) is made for firms above $\psi$, using the lower IC constraint. I therefore conclude that the firm with the highest willingness to pay for location $i$ must be the firm matched to $i$ using the function $\psi(\eta(i))$ in an equilibrium where rents support firm location incentives.

Alternatively, we can write the equilibrium landowner maximization problem

$$\text{(A2.3) } \phi(i) = \max_\psi \left\{\min_j \left\{\psi\left(\eta(i) - \eta(j)\right) + \phi(j)\right\}\right\}$$

Again dropping the minimization, i.e. reducing each firm's decision to one between their assigned location and $i$, and substituting the equilibrium condition for rents (recall)

$$\psi(\eta) = \left[\left(\frac{\partial\phi(\eta)}{\partial\eta}\right)\right]^{\frac{1}{\sigma-1}}$$

we can rewrite the landowners problem as

$$\text{(A2.4)}\ \phi(i) = \max_{\psi} \left\{ \psi(\eta)^{\sigma-1} \left(\eta(i) - \eta(j)\right) + \int_{\underline{\eta}}^{\eta(j)} \psi(\eta)^{\sigma-1} d\eta \right\}$$

Finally, using the implicit function theorem (since $\eta(j)$ changes with $\psi$) and a change of variables:

$$\phi(i) = \max_{\psi} \left\{ \psi(\eta)^{\sigma-1} \left(\eta(i) - \eta(\psi(j))\right) + \int_{\psi(\underline{\eta})}^{\psi(\eta(j))} \psi(\eta)^{\sigma-1} \left(\frac{\partial \psi}{\partial \eta}\right)^{-1} d\psi \right\}$$

we can evaluate the first order condition:

$$0 = (\sigma - 1)\psi(\eta)^{\sigma-2} \left(\eta(i) - \eta(\psi(j))\right) - \psi(\eta)^{\sigma-1} \left(\frac{\partial \psi}{\partial \eta}\right)^{-1} + \psi(\eta)^{\sigma-1} \left(\frac{\partial \psi}{\partial \eta}\right)^{-1}$$

which is solved where $\eta(\psi(i)) = \eta(\psi(j))$ or by choosing the firm that has already been assigned to that location.

## A3. Proof of Existence and conditions for uniqueness

### A3.1 Existence

First, equations (10)-(12) can be written as a system of nonlinear Hammerstein equations of the second kind.

Equation (11) in particular can be rewritten as

$$\text{(A3.1)}\ a(i) = \int \tau(i,j)^{1-\sigma} F\left(a(j), b(j)\right) dj$$

where the functions $a(j) = \dfrac{\eta(j)}{P(j)^{-\sigma}}$ and $b(j) = P(j)^{1-\sigma}$ are defined by an integral equation with a kernel of $\tau(i,j)^{1-\sigma}$ and a nonlinear function

$$F(a(j), b(j)) =$$
$$\rho^{\sigma-1} \cdot g(\psi(a(j) \cdot b(j)^{\frac{-\sigma}{1-\sigma}}))\psi'(a(j) \cdot b(j)^{\frac{-\sigma}{1-\sigma}}) \cdot \psi(a(j) \cdot b(j)^{\frac{-\sigma}{1-\sigma}})^{2-2\sigma} \cdot a(j) \cdot b(j)^{\frac{1}{\sigma-1}},$$

In turn, the function $b(j)$ can be used to similarly rewrite equation (12) as

$$\text{(A3.2) } b(i) = \int \tau(i,j)^{1-\sigma} G\left(a(j), b(j)\right) dj$$

where the function $b(j)$ is defined over an integral equation with a kernel of $\tau(i,j)^{1-\sigma}$ and a nonlinear function

$$G\left(a(j), b(j)\right) = b(j) \cdot \rho^{\sigma-1} \psi(a(j) \cdot b(j)^{\frac{-\sigma}{1-\sigma}})^{\sigma-1} g(\psi) \cdot \psi'(a(j) \cdot b(j)^{\frac{-\sigma}{1-\sigma}})$$

Together, equations (A3.1) and (A3.2) constitute a system of nonlinear Hammerstein equations of the second kind. Agarwal et al. (2008)show the existence for such a system on four conditions they refer to as (C1)-(C4). I will show that conditions (C1)-(C4) are satisfied in the system described by (A3.1) and (A3.2), and therefore that an equilibrium exists.

First, by assumption, the kernel of this system of equations $\tau(i,j)^{1-\sigma}$ is continuous and non-negative. This satisfies condition (C1). Note that $F\left(a(j), b(j)\right)$ and $G\left(a(j), b(j)\right)$ accept only non-negative arguments and must be non-negative everywhere, and are continuous and closed for any non-negative elements $a(j),\ b(j)$, since, by equation (10), $\psi(\eta(i))$ and $\psi'(\eta(i))$ are continuous functions. This satisfies condition (C2). The distribution of firm productivities is bounded by some maximum $\bar{\psi}$ and $g(\bar{\psi})$, maximum density for some firm type, is finite. Notice that because the rent at each location is finite, the derivative function $\psi'(\eta)$ must be finite everywhere and we can define $\bar{\psi}'$ as its maximum.

Next, define

$$d(i) = \sup \int_S \tau(i,j) dj$$

and

$$w_{F1}(a(j)) = a(j),\ w_{F2}(b(j)) = b(j)^{\frac{1}{\sigma-1}},\ w_{G1}(a(j)) = 1,\ w_{G2}(b(j)) = b(j)\ q = \rho^{\sigma-1} \cdot \bar{\psi} \cdot \bar{\psi}' \cdot g(\bar{\psi})$$

Then, by construction, $G\left(a(j), b(j)\right)$ and $F\left(a(j), b(j)\right)$ are both less than $q \cdot w_{F1}(a(j)) \cdot w_{F2}(b(j))$ and $q \cdot w_{G1}(a(j)) \cdot w_{G2}(b(j))$, respectively, which are finite. This satisfies conditions (C3).

Finally, note that

$$\alpha > dq\alpha^{\frac{2\sigma-1}{\sigma-1}}$$

for

$$\alpha < (dq)^{\frac{1-\sigma}{2\sigma-1}},$$

fulfilling (C4). This completing the sufficient conditions for existence of a nontrivial solution to the system.

### A3.2 Uniqueness

In the general case, the equilibrium of this model will not be unique with respect to the mapping of firms to locations. However, two forms of restrictions on the kernel, or the trade costs, and therefore restrictions on the geography underlying the model, admit a single equilibrium.

Golomb (1935)shows that the system of equations in (A3.1) and (A3.2) has a unique solution if the following Lipschitz condition is satisfied:

$$\text{(A3.3)}\ \left(F(a_1,b_1)-F(a_2,b_2)\right)^2 \leq k_1^2\left(\left((a_1-a_2)^2+\left((b_1-b_2)^2\right)\right.\right.,$$

$$\left(G(a_1,b_1)-G(a_2,b_2)\right)^2 \leq k_2^2\left(\left((a_1-a_2)^2+\left((b_1-b_2)^2\right)\right.\right.,$$

for any possible $a_1,\ b_1,\ a_2,\ b_2$, and some $k_1,\ k_2$ such that

$$k_1^2+k_2^2<\lambda$$

where $\lambda$ is the smallest eigenvector from the kernel defined by

$$K(i,j)=\int_{S^n}\left(\tau(i,r)\tau(j,r)\right)^{1-\sigma}dr$$

### A3.3 Stability

The equilibrium is point-wise locally stable if no small group of entrepreneurs or workers can increase their welfare by moving to a different location and no group of landowners can increase profits by changing the amount of density they provide. I show that any equilibrium with a one-to-one, continuous matching $\psi(\eta)$ is stable.

First, no group $\epsilon$ of landowners $i \in \epsilon$ can improve profits by adjusting the density of firms at their locations by some fixed amount $\gamma$. Adjusting density downward lead (by

Appendix 2) to a decrease in profits and to a decrease in $\eta(i)$. Adjusting density upwards for each landowner reduces profits by a fixed amount through increased marginal cost for each landowner, while potentially increasing $\eta(i)$ via higher local demand. As $\epsilon$ becomes small, the effect of local changes due to $\epsilon\gamma$ on $\eta(i)$ go to zero, since $\eta(i)$ is defined with respect to all points $j \in S^n$. For any change $\gamma$ in density, there is an $\epsilon$ small enough such that $\triangle\eta(i) < c(h(i)+\gamma)-c(h(i))$. Since this is true for any change in density $\gamma$, no arbitrarily small group of landowners can increase profits by deviating in their density provision.

Next, no small group of firms and workers can improve their profits by moving to another location. Firms moving to a new location pay higher rents for the increase in density but may benefit from better market access, as higher demand for their goods from other firms and workers in their deviating group drive up variable profits at the new location. Following Allen and Arkolakis (2013), this cannot be the case when $\frac{d\pi(i)}{dh(i)} < 0$ which is the case if $\frac{d\eta(i)}{dh(i)} < -\frac{d\phi(i)}{dh(i)}$. Intuitively, because $\eta(i)$ is defined with respect to the entire space, smaller groups of firms have an increasingly smaller effect on $\eta(i)$, while the negative effect of density on profits via the direct impact on $\phi(i)$ is constant.

Worker's real wages are always equalized across locations, so no independent move by workers can improve their utility.

## A4. Conditions for a mono-centric equilibrium

Urban models and some parts of the new economic geography literature (Fujita Krugman Venables 1999) have traditionally assumed a single, central business district. No such organization of space is assumed in this model. Although the model may yield a single center of economic activity, other equilibria may have multiple "centers". Because cities often have a single business center and because the literature has so often assumed geographies with exogenous centers, in this appendix I examine conditions under which economic activity must necessarily be mono-centric in the space.

The space $S$ is mono-centric if there are no troughs of economic activity. As shown in sections 2 and 3 above, $\eta(i)$ is a sufficient statistic for economic activity at any location. So conditions that exclude local minima in $\eta(i)$ are sufficient conditions to guarantee that any equilibria is mono-centric. For the remainder of this appendix, I refer to local maxima and minima when I discuss the curvature of the function $\eta(i)$ in the space $S$.

To exclude local minima in $\eta(i)$ is to exclude any equilibria for which, for some $i \in S$, $\nabla\eta(i) = 0$ and $\nabla^2\eta(i) < 0$. To simplify the computation we rewrite

$$\text{(A4.1)}\ \eta(i) = P(i)^{-\sigma}a(i)$$

where $a(i) = \int_{j \in S} \tau(i,j)^{1-\sigma} z(j) dj$ and $P(i) = \int_{j \in S} \tau(i,j)^{1-\sigma} y(j) dj$.
The first order condition is therefore

$$\text{(A4.2)} \ \frac{\partial \left(P(i)^{-\sigma}\right)}{\partial i} = -\frac{\partial \left(a(i)\right)}{\partial i} \frac{P(i)^{-\sigma}}{a(i)}.$$

Next, I evaluate the second derivative of $\eta(i)$. At a local minima, this must be positive.

$$\text{(A4.3)} \ \frac{\partial^2 \eta}{\partial i^2} = \frac{\partial^2 \left(P(i)^{-\sigma}\right)}{\partial i^2} a(i) + 2 \frac{\partial \left(P(i)^{-\sigma}\right)}{\partial i} \frac{\partial \left(a(i)\right)}{\partial i} + \frac{\partial^2 \left(a(i)\right)}{\partial i^2} P(i)^{-\sigma}$$

With further substitution of the first order condition, equation (A4.3) can be rewritten written as

$$\text{(A4.4)} \ \frac{\partial^2 \eta}{\partial i^2} = \frac{\partial^2 \left(P(i)^{-\sigma}\right)}{\partial i^2} a(i) - 2 \frac{P(i)^{-\sigma}}{a(i)} \left( \frac{\partial \left(a(i)\right)}{\partial i} \right)^2 + \frac{\partial^2 \left(a(i)\right)}{\partial i^2} P(i)^{-\sigma}$$

or

$$\frac{\partial^2 \eta}{\partial i^2} =$$
$$(1-\sigma) \int_{j \in S} \left( (-\sigma) \tau(i,j)^{-\sigma-1} \nabla \tau(i,j) + \tau(i,j)^{-\sigma} \nabla^2 \tau(i,j) \right) y(j) dj \cdot a(i) - 2 \frac{P(i)^{-\sigma}}{a(i)} \left( \frac{\partial \left(a(i)\right)}{\partial i} \right)^2$$
$$+ (1-\sigma) \int_{j \in S} \left( (-\sigma) \tau(i,j)^{-\sigma-1} \nabla \tau(i,j) + \tau(i,j)^{-\sigma} \nabla^2 \tau(i,j) \right) z(j) dj \cdot P(i)^{-\sigma}$$

which is greater than zero if

$$-\sigma \tau(i,j)^{-\sigma-1} \nabla \tau(i,j) + \tau(i,j)^{-\sigma} \nabla^2 \tau(i,j) > 0$$

or

$$\text{(A4.5)} \ \nabla^2 \tau(i,j) > \sigma \frac{\nabla \tau(i,j)}{\tau(i,j)}$$

When transportation costs are sufficiently convex relative to the elasticity of substitution, no local minima are possible. Intuitively, space in local minima is worse than spaces in any direction. Moving in any direction brings firms closer to their own local demand

but further from centers of demand on the other side of the local minima. Convex transportation costs and substitutability of goods jointly make the such tradeoffs of proximate markets for further markets too dear. The result is that distributions with local centers of activity and local valleys cannot be supported.

It should be noted that this is a sufficient but not a necessary condition for single-peaked equilibria. If this condition is violated, the first term is negative while the second is positive. Depending on the functions $z(j)$, $y(j)$, $a(j)$, and $P(i)$, the second derivative may still be negative everywhere, in some places, or nowhere.

Using the flexible functional form for transportation costs $\tau(i,j) = (1 + \|i - j\|)^d$, where transport costs depend on the distance between points $i$ and $j$ and the parameter $d$, which reflects the extent of convexity in the transportation cost, the condition in (A4.5) is

$$d > \frac{1 + \sqrt{1 + 4\sigma}}{2}$$

## A5. Incorporating productivity spillovers

The framework laid out in Section 2 can be expanded to incorporate a flexible form of productivity spillovers. In this section I present a model where locations differ by an endogenous location productivity amenity. The index strategy combined with functional form assumptions regarding the relationship between exogenous and endogenous firm productivity allow productivity spillovers to be incorporated into location-specific productivity such that the equilibrium conditions of Section 2 hold with only slight modifications.

As before, firms sell differentiated goods at a markup over marginal cost to all locations $j \in S^n$. However, firm productivity, now denoted by $\tilde{\psi}$ is now endogenously defined by the firm's location. Firm variable profit at location $i$ can be expressed as

$$r(\psi, i)/\sigma = \int_{S^n} \frac{P(i)^{-\sigma} \cdot (1 + \xi(i))^{1-\sigma} \cdot \tau(i,j)^{1-\sigma}}{P(j)^{1-\sigma} \rho^{1-\sigma} \psi(i)^{1-\sigma}} \frac{R(j)}{\sigma} dj$$

where firm productivity is a function of exogenous firm productivity $\psi$ and location-specific productivity spillovers

$$\psi(i) = f(\psi, s(i))$$

The location-specific productive amenities $s$ is a function of the density, productivity, and distance of other firms.

$$s(i) = f(H, \Psi, D)$$

where $D$ is a the (exogenous) distance function between locations, $\Psi$ is the (endogenous) mapping of firm productivities to (all) locations, and $H$ is the (endogenous) function governing densities at all points $j \in S$.

A sufficient condition for isomorphism between this model and the model in Section 2 is for firm productivity and location productivity spillovers to be multiplicatively separable:

$$\psi(i) = \psi \cdot s(i).$$

Note that this model conforms to the standard agglomeration model when $\psi = 1$ for every firm. When both $\psi$ and $s(i)$ are variable, more productive firms will experience larger effects from the same value of $s(i)$, which is an feature of other models in the literature (Gaubert 2013) and suggested by empirical evidence (Combs et al 2012).

Under the above assumption, firm variable profits at $i$ can be expressed as

$$r(\psi, i)/\sigma = \psi \cdot \tilde{\eta}(i)$$

where $\tilde{\eta(i)} = s(i) \cdot \eta(i)$. Landowner, firm, and worker optimal decisions follow as before, now as a function of $\tilde{\eta}$ rather than $\eta$.

Note that $s(i)$ is determined endogenously. Because $\eta$ is endogenous, this does not affect equations (7)-(10). However, the mapping of location productivity $\tilde{\eta}(i)$ to locations is now altered:

$$\tilde{\eta}(i) = s(i) \int_{j \in S} P(i)^{-\sigma}(1 - \xi(i)) \left[\frac{\tau(i,j)}{\rho P(j)}\right]^{1-\sigma} h(j)\psi(j)^{\sigma} \cdot \eta(j) dj.$$

In order to solve for an equilibrium, a solution must be provided for $\tilde{\eta}$ as well as $s(i)$ in conjunction with the unaltered equation for the price index. Following the proof in A3.1, a non-trivial solution to a system of three integral equations of the second kind, provided a specific functional form for $s(i)$ is determined, can be shown to exist. As with $\eta(i)$, restrictions on the value of spillovers will be necessary in order to ensure conditions (C1)-(C4) are met.

## A.6 Appendix Figures and Tables

**Appendix Table A.1: Summary Statistics, Tradables and Non-tradables**

| | Observations | Mean | Std. Dev. |
|---|---|---|---|
| | All establishments, cross-section | | |
| *Sales* | 14,780,463 | 5,477,300 | 6,585,840 |
| *Employment* | 14,780,463 | 21.83 | 114.51 |
| *Payroll* | 14,780,463 | 732,940 | 703,132 |
| *Value Added* | 1,372,309 | 5,677,300 | 6,148,140 |

Note. Sample includes all establishments responding to Economic Censuses between 1992 and 2007, excluding sole proprietorships, for which geographic data is available or could be imputed from address records. Value added is taken from the subset of firms responding to the Census of Manufactures only.

## Appendix Table A.2: Cross-sectional relationships, Tradables and Non-tradables

| VARIABLES | Log output per worker | | Log VA per worker | | Log employment | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| *Log establishment density* | 0.049*** | 0.033*** | 0.090*** | 0.057*** | 0.044*** | 0.044*** |
| | (0.004) | (0.003) | (0.003) | (0.003) | (0.003) | (0.002) |
| *CBSA/Year FEs* | No | Yes | No | Yes | No | Yes |
| Constant | 7.97 | 4.45 | 9.32 | 6.28 | 3.46 | 1.55 |
| Observations | 14,780,463 | 14,780,463 | 1,372,309 | 1,372,309 | 14,780,463 | 14,780,463 |
| R-squared | 0.46 | 0.47 | 0.46 | 0.47 | 0.47 | 0.48 |
| | Log output per worker | | Log output per worker | | Log gross margin PW | |
| | (9) | (10) | (11) | (12) | (13) | (14) |
| *Log avg. rent per worker* | 0.090*** | 0.057*** | | | | |
| | (0.003) | (0.003) | | | | |
| *Log employment density* | | | 0.032*** | 0.035*** | | |
| | | | (0.003) | (0.003) | | |
| *Log establishment density* | | | | | 0.011* | 0.014*** |
| | | | | | (0.006) | (0.005) |
| *CBSA/Year FEs* | No | Yes | No | Yes | No | Yes |
| Constant | 9.32 | 6.28 | 9.06 | 6.33 | -8.47 | -19.13 |
| Observations | 14,780,463 | 14,780,463 | 5,351,354 | 5,351,354 | 1,372,309 | 1,372,309 |
| R-squared | 0.46 | 0.47 | 0.46 | 0.47 | 0.32 | 0.33 |

*** p<0.01, ** p<0.05, * p<0.1

Note. Samples include manufacturing or business services establishments responding to Economic Censuses between 1992 and 2007, excluding sole proprietorships, for which geographic data is available or could be imputed from address records. Establishment density is computed as the number of establishments per square mile of land at the tract level. Average rent per worker is the tract-level average of respondents to questions on rent in the Census of Manufacturers. Employment density is the employees per square mile of land area, excluding those belonging to the observational establishment. Gross margin per worker is value added minus payroll, divided by the number of employees. All regressions are at the establishment-year level and control for age of establishment, cubic polynomials for establishment latitude and longitude, industry-year fixed effects for the establishment's full SIC or NAICS code, and tract-level demographic controls including average age, gender, racial composition, education composition, and income levels. All standard errors clustered at the CBSA level.

## Appendix Table A.3: Sorting on Productivity, all Movers

| VARIABLES | Log establishment density | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | Establishment relocations | | Firm expansions | |
| *Prior-year log output per worker* | 0.095*** | 0.050*** | 0.032*** | 0.012*** |
| | (0.005) | (0.007) | (0.006) | (0.005) |
| *CBSA FEs* | No | Yes | No | Yes |
| Constant | 1.37 | 10.03 | 12.39 | 47.66 |
| Observations | 280,126 | 280,126 | 557,544 | 557,544 |
| R-squared | 0.52 | 0.58 | 0.14 | 0.28 |

*** p<0.01, ** p<0.05, * p<0.1

Note. Samples include all establishments responding to Economic Censuses between 1992 and 2007, excluding sole proprietorships and single-unit plants, for which geographic data is available or could be imputed from address records. Sample in columns one and two use establishment relocations, defined as establishments opened between census years in the same industry (4-digit SIC or NAICS) as an "origin" plant within the same firm that closed the prior year. Relocations within CBSAs or between CBSAs but less than 50 miles apart are excluded. Sample in columns three and four use firm expansions, defined as new establishments that are part of pre-existing multi-unit firms, opening in a CBSA at least 50 miles away from any other establishments of the same firm. Establishment density is computed as the number of establishments per square mile of land at the tract level. All regressions are at the establishment-year level and control for age of establishment, industry-year fixed effects for the establishment's full SIC or NAICS code, cubic polynomials for both the establishment latitude and longitude and the latitude and longitude of the closed plant or firm's geographic center, respectively, and the distance between the old and new plants or the new establishment and firm's geographic center, respectively. All standard errors clustered at the CBSA level.

## Appendix Table A.4: Effects of Density on Movers, Tradable and Non-tradables

| VARIABLES | Log output per worker, 5 years forward | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| | Establishment relocations | | | | Firm expansions | | | |
| *Log est. density, destination* | 0.010*** | 0.008*** | 0.010*** | 0.008*** | 0.010*** | 0.003* | 0.004*** | 0.001 |
| | (0.001) | (0.002) | (0.002) | (0.001) | (0.002) | (0.002) | (0.001) | (0.001) |
| *Firm-level log output PW* | 0.294*** | 0.290*** | . | . | 0.211 | 0.208 | . | . |
| | (0.013) | (0.012) | . | . | (0.006) | (0.006) | . | . |
| *CBSA fixed effects* | No | Yes | No | No | No | Yes | No | Yes |
| *Origin lat/lon cubes* | Yes | Yes | No | No | Yes | Yes | No | No |
| *Industry, year fixed effects* | Yes | Yes | No | No | Yes | Yes | No | No |
| *Firm, industry, year fixed eff* | No | No | Yes | Yes | No | No | Yes | Yes |
| Constant | 3.95 | 10.72 | 7.42 | 9.09 | 8.23 | 14.1 | 6.01 | 9.28 |
| Observations | 173,692 | 173,692 | 173,692 | 173,692 | 370,526 | 370,526 | 370,526 | 370,526 |
| R-squared | 0.94 | 0.94 | 0.96 | 0.96 | 0.93 | 0.93 | 0.96 | 0.96 |

*** p<0.01, ** p<0.05, * p<0.1

Note. Samples includes all establishments responding to Economic Censuses between 1992 and 2007, excluding sole proprietorships and single-unit plants, for which geographic data is available or could be imputed from address records. Sample in columns one through four use establishment relocations, defined as establishments opened between census years in the same industry (4-digit SIC or NAICS) as an "origin" plant within the same firm that closed the prior year. Relocations within CBSAs or between CBSAs but less than 50 miles apart are excluded. Sample in columns five through eight use firm expansions, defined as new establishments that are part of pre-existing multi-unit firms, opening in a CBSA at least 50 miles away from any other establishments of the same firm. Establishment density is computed as the number of establishments per square mile of land at the tract level the year of observed relocation or expansion. Prior census year log output per worker is the average output per worker of all other plants in the year of relocation or expansion. Log output per worker, five years forward is the productivity measure of the new establishment taken five years after the relocation or expansion. All regressions are at the establishment-year level and control for age of establishment, industry-year fixed effects for the establishment's full SIC or NAICS code, cubic polynomials for the establishment latitude and longitude, dummies for the closure of the new plant 5 years after moving, and the distance between the old and new plants or the new establishment and firm's geographic center, respectively. All standard errors clustered at the CBSA level.

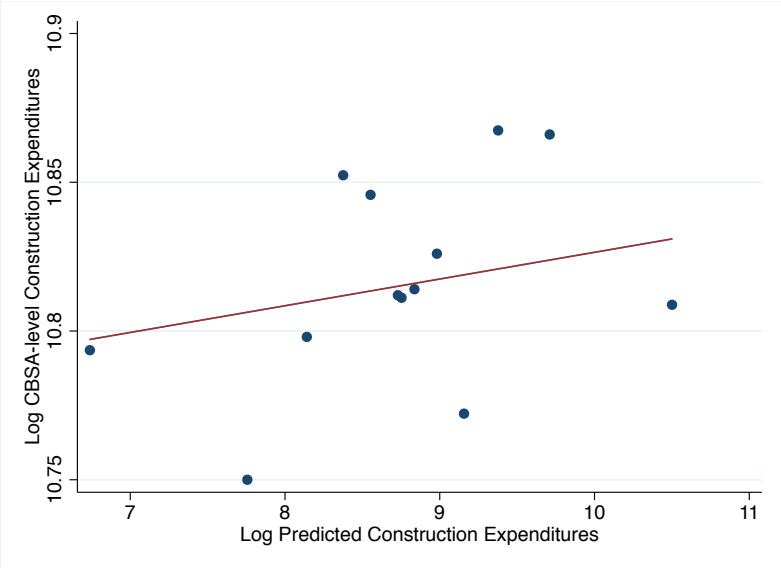
## Appendix Table A.5: Predicted and CBSA-level Construction Expenditures

| VARIABLES | Log construction exp. of unlinked firms, CBSA | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | Levels | | Changes | |
| *Log predicted sales, tract* | 0.009 | 0.004 | | |
| | (0.020) | (0.019) | | |
| *Change in log predicted sales, CBSA level* | | | 0.004 | 0.006 |
| | | | (0.015) | (0.015) |
| *CBSA size* | No | Yes | No | Yes |
| *CBSA/Year FE* | Yes | Yes | No | No |
| Constant | 1.03 | -604 | 0 | -22.25 |
| Observations | 4,088 | 4,088 | 4,088 | 4,088 |
| R-squared | 0.81 | 0.84 | 0.03 | 0.07 |

*** p<0.01, ** p<0.05, * p<0.1

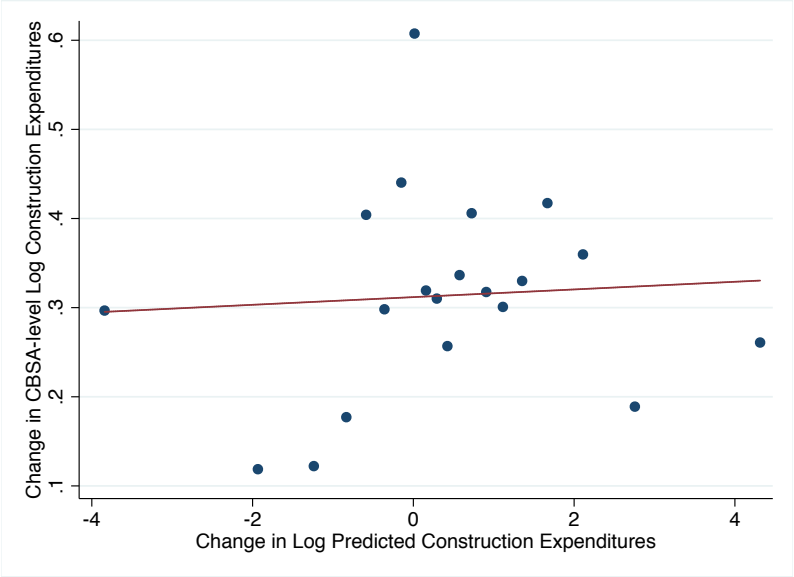
Note. Sample includes all tracts for which Real Estate Investment Trusts respond to questions in the Census of Finance and Insurance respondents on expenditures on new construction on commercial real estate, which have establishments reporting sales at least one other CBSA, and for which geographic information was present or could be imputed from address files. Log construction expenditures at the CBSA level is the log of the sum of the dollar value of all construction expenditure, including office and manufacturing space, reported by all single-unit (i.e., unlinked) REITs in the CBSA. Columns one and two report this measure in levels and columns three and four report changes between census years at the tract level. Log predicted sales are the predictions based on current-year sales of single-unit firms responding to the same census questions in CBSAs where the multi-unit REIT responding in the observed tract has other establishments present, weighted by the percent of sales at the firm level in that CBSA in the previous year. Changes report log differences for this predicted value. All regressions are at the tract level and include controls for year fixed effects. All standard errors clustered at the CBSA level.

Appendix Figure A.1: Predicted construction expenditures vs CBSA-level sums[35]




[35]Note. Figure is binned scattered of residuals of regression in Column 2 of Appendix Table A.5. Sample includes all tracts for which Real Estate Investment Trusts respond to questions in the Census of Finance and Insurance respondents on expenditures on new construction on commercial real estate, which have establishments reporting sales at least one other CBSA, and for which geographic information was present or could be imputed from address files. Log construction expenditures at the CBSA level is the log of the sum of the dollar value of all construction expenditure, including office and manufacturing space, reported by all single-unit (i.e., unlinked) developers in the CBSA. Log predicted sales are the predictions based on current-year sales of single-unit firms responding to the same census questions in CBSAs where the multi-unit developers responding in the observed tract has other establishments present, weighted by the percent of sales at the firm level in that CBSA in the previous year.

Appendix Figure A.2: Predicted construction expenditures vs CBSA-level sums, changes[36]




[36]Note. Figure is binned scattered of residuals of regression in Column 4 of Appendix Table A.5. Sample includes all tracts for which Real Estate Investment Trusts respond to questions in the Census of Finance and Insurance respondents on expenditures on new construction on commercial real estate, which have establishments reporting sales at least one other CBSA, and for which geographic information was present or could be imputed from address files. Change in log construction expenditures at the CBSA level is the change between census years of the log of the sum of the dollar value of all construction expenditure, including office and manufacturing space, reported by all single-unit (i.e., unlinked) developers in the CBSA. Change in log predicted sales are the predictions based on current and previous census-year sales of single-unit firms responding to the same census questions in CBSAs where the multi-unit developers responding in the observed tract has other establishments present, weighted by the percent of sales at the firm level in that CBSA in the previous year.

# Appendix Figures A.3a-A.6c[37]

A B C

Figures A.3a-c: Log output per worker vs log establishment density

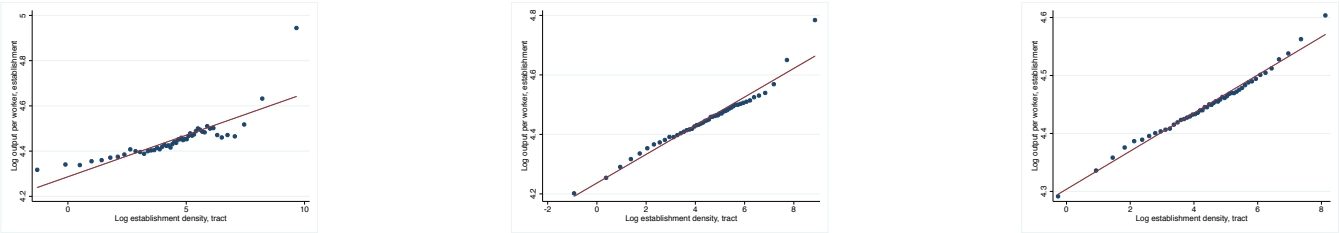



Figure A.4a-c: Log value added PW vs log establishment density

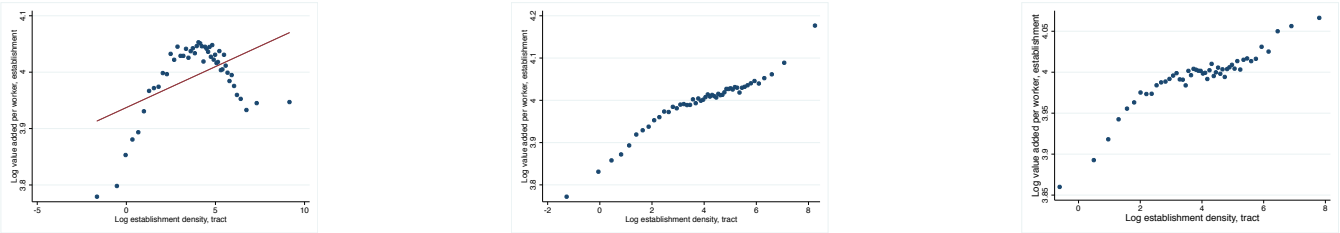



Figure A5.a-c: Log employment vs log establishment density

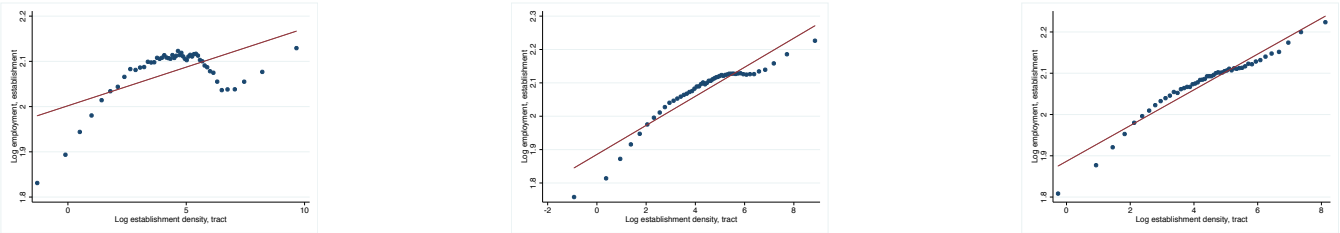
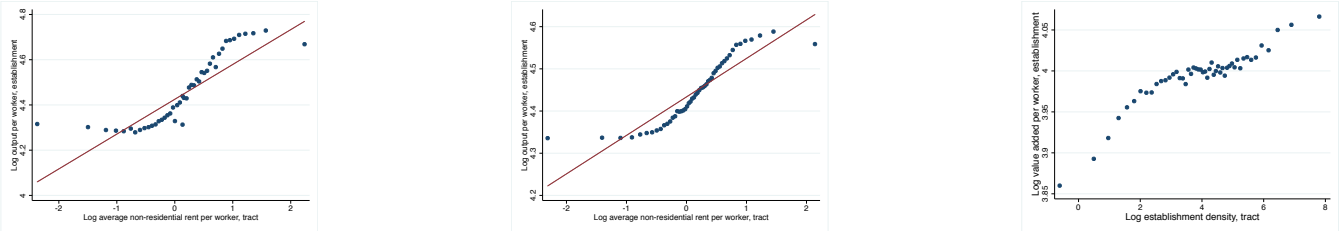



Figure A6.a-c: Log output PW vs log average non-residential rent PW




[37]Note. Figures are binned scattered residuals of establishment and tract-level variables. Figures in column A plot the raw relationship, controlling only for year fixed-effects. Column B controls for establishment-level variables including industry-year fixed effects, establishment age, and cubic polynomials in attitude and longitude. Figures in Column C control add CBSA-year fixed effects. Variables are as defined in Figures 2a-f.

# Appendix Figures A.3a-A.6c[38]

A B C

Figure A7.a-c: Intermediate good intensity vs log establishment density

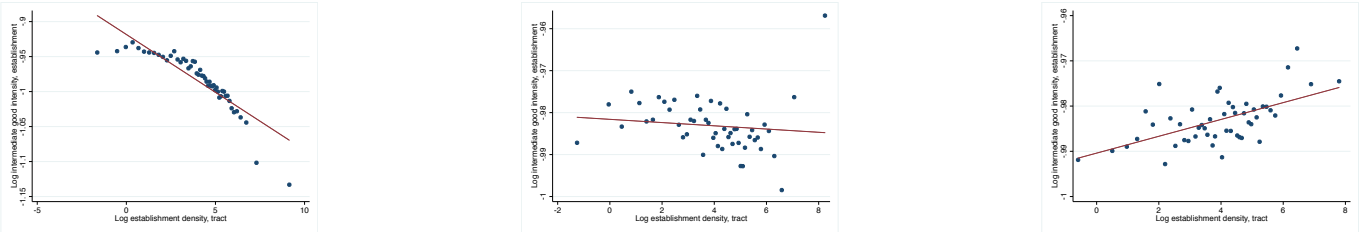
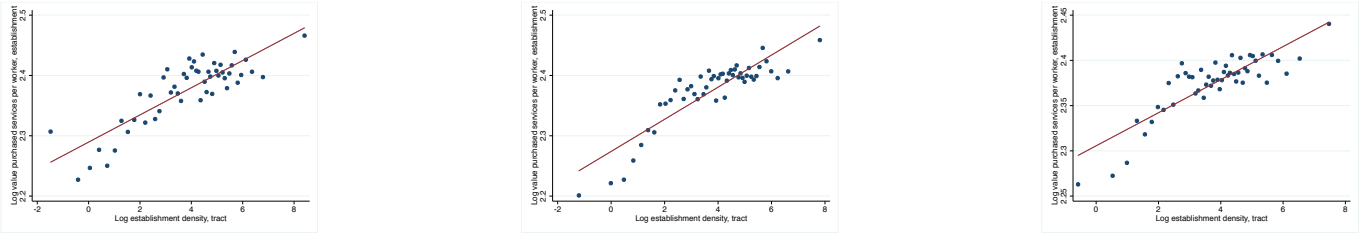



Figure A8.a-c: Log purchased services PW vs log establishment density




[38]Note. Figures are binned scattered residuals of establishment and tract-level variables. Figures in column A plot the raw relationship, controlling only for year fixed-effects. Column B controls for establishment-level variables including industry-year fixed effects, establishment age, and cubic polynomials in latitude and longitude. Figures in Column C control add CBSA-year fixed effects. Log intermediate input usage is calculated as total value sales minus value added minus divided by employment. Log value purchased services per worker is calculated as total purchased services, as reported in the Census of Manufactures, divided by employment.