Financing Entrepreneurial Production:  
Security Design with Flexible Information Acquisition

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Abstract

We propose a new theory of the use of debt and non-debt securities in financing entrepreneurial production, positing that the investor can acquire costly information on the entrepreneur’s project before making the financing decision. We show that debt is optimal when information is not valuable for production, while the combination of debt and equity is optimal when information is valuable. These predictions are consistent with the empirical facts regarding the finance of entrepreneurial businesses. Flexible information acquisition allows us to characterize the payoff structures of optimal securities without imposing usual assumptions on feasible securities or belief distributions.

Keywords: security design, debt, combination of debt and equity, flexible information acquisition.

JEL: D82, D86, G24, G32, L26

1 Introduction

Why are projects with different natures usually financed by different types of securities? Specifically, both debt and non-debt securities are commonly viewed in different real-world corporate finance contexts, but it is less clear why debt is optimal for financing some projects while non-debt securities are optimal for others. This paper provides a new answer to this question, under a single and natural premise that the investor can acquire costly information on the entrepreneur’s project before making the financing decision.

The literature of security design usually postulates that an entrepreneur with a project but without financial resources proposes specific contracts to an investor to get finance. The entrepreneur is often modeled as an expert who is more informed about the project. However, this common approach misses a crucial point: in reality some investors are better able than the entrepreneur to acquire information and thus to assess a project’s uncertain market prospects, drawing upon their industry experience. For instance, start-ups seek venture capital, and most venture capitalists are themselves former founders of successful start-ups, so they may be better able to determine whether new technologies match the market. Tirole (2006) points out that one shortcoming of the classical corporate finance literature is that it overlooks this informational advantage of investors. Our paper addresses this concern by uncovering the interaction between entrepreneurs’ security design and investors’ endogenous information acquisition and screening. It enables us to provide a theory of debt and non-debt securities within a single framework, and in particular, to show under what conditions debt or non-debt securities will be optimal. These results are consistent with the empirical evidence regarding the finance of entrepreneurial businesses.

In our model, an entrepreneur has the potential to produce but has no money to start the project, and thus the investor’s endogenous information advantage over the entrepreneur leads to a new informational friction. Specifically, in our model, the investor can flexibly (defined later) acquire costly information about the project’s uncertain cash flow before making the financing decision. Only when the investor believes that the project is good enough, will she finance the project. Hence, the entrepreneur’s real production depends on the investor’s information acquisition, but these two are conducted separately, constituting the friction at the heart of our model.

1In our model, screening refers to the decision of whether or not to finance the project after acquiring information. As our model does not feature entrepreneurs’ private information, our notion of screening is however different from the notion of separating (different types of entrepreneurs) commonly used in the literature.

2We do not attempt to deny that entrepreneurs in reality may have private information about their technologies, which has been discussed extensively in the previous literature. Rather, we highlight the overlooked fact that investors may acquire information and become more informed about the potential match between new technologies and the market.
Our model predicts standard debt and the combination of debt and equity as optimal securities in different circumstances. When the project’s ex-ante market prospects are already good and clear or the screening cost is high, the optimal security is debt, which does not induce information acquisition. This prediction is consistent with the evidence that conventional start-ups and mature private businesses rely heavily on plain-vanilla debt finance from investors who are not good at screening, such as relatives, friends, and banks (see Berger and Udell, 1998, Kerr and Nanda, 2009, Robb and Robinson, 2014). The intuition is clear: since the benefit of screening does not justify its cost, the entrepreneur finds it optimal to avoid costly information acquisition by using debt, the least information-sensitive security. The investor thus makes the investment decision based on her prior. This intuition for the optimality of debt is different from the conventional wisdom, as our mechanism does not feature adverse selection or signaling.

In contrast, when the project’s ex-ante market prospects are obscure or the cost of screening is low, the optimal security is the combination of debt and equity that induces the investor to acquire information. Regarding cash flow rights only, this is equivalent to participating convertible preferred stock. This prediction fits well with the empirical facts (Sahlman, 1990, Gompers, 1999). In particular, convertible preferred stocks have been used in almost all the contracts between entrepreneurs and venture investors, and nearly half of them are participating, as documented in Kaplan and Stromberg (2003). Participating convertible preferred stock is popular in particular for the early rounds of investment (Kaplan and Stromberg, 2003), when the friction is more severe.

The optimality of the combination of debt and equity is subtle. First, the entrepreneur wants to induce the investor to screen only if the investor screens in a potentially good project and screens out bad ones. That is, any project with a higher ex-post cash flow should have a better chance to be financed ex-ante. Only when the investor’s payoff is high in good states while low in bad states, the investor has the right incentive to distinguish between these different states by developing such a screening rule, because she only wants to invest when the likelihood of high payment is high. Consequently, the entrepreneur can maximally benefit from this by ensuring that her own payoff is also high when the investor’s is. Therefore, an equity component with payments that are strictly increasing in the project’s cash flow is offered, encouraging the investor to acquire adequate information to distinguish between any different states. In addition, the investor’s information after screening may still be imperfect, albeit perhaps with a better

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3The formal mathematical definitions of debt, equity, and the combination of debt and equity in our framework are given in Sections 3.1 and 3.2. In defining them, we focus on the qualitative aspects of their cash flow rights but ignore the aspects of control rights. Specifically, debt means the security pays all the cash flow in low states but a constant face value in high states, while equity means the security payoff and its residual are both strictly increasing in the fundamental. Consistent with the reality, debt is also more senior than equity in our framework.

4Notable results regarding debt as the least information-sensitive security to mitigate adverse selection include Myers and Majluf (1984), Gorton and Pennacchi (1990) and DeMarzo and Duffie (1999).

5Our model features continuous state, but we use the notions of good and bad at times to help develop intuitions.
posterior. In other words, the investor might still end up financing a bad project after screening. Thus, downside protection is necessary to prevent the investor from rejecting the project without any information acquisition. This justifies the debt component. These intuitions also suggest that straight or leveraged equity alone is not optimal for financing entrepreneurial production, consistent with reality (Kaplan and Stromberg, 2003, Lerner, Leamon and Hardymon, 2012).

One methodological contribution of our work is to characterize the payment structure, or in visual terms, the “shape”, of the optimal securities without either distributional assumptions or restrictions on the feasible security space. A new concept, flexible information acquisition, helps achieve this goal. For instance, debt, with its flatter shape, is less likely than equity to prompt screening. Moreover, in screening, a debt holder tends to pay attention to states with low cash flows, as the payments are constant over states with high cash flows so there is no point in differentiating the latter. In contrast, levered equity holders tend to pay attention to states with high cash flows, as they benefits from the upside payments. An arbitrary security determines the investor’s incentives for screening and attention allocation in this state-contingent way, and these in turn affect the entrepreneur’s incentives in designing the security. The traditional approach of exogenous information asymmetry does not capture these incentives. Recent models of endogenous information acquisition do not capture such flexibility of incentives adequately, since they only consider the amount or precision of information (see Veldkamp, 2011, for a review). Our approach of flexible information acquisition, following Yang (2013, 2015), is based on the literature of rational inattention (Sims, 2003, Woodford, 2008), but has a different focus. It captures not only how much but also what kind of information the investor acquires through state-contingent attention allocation. In our setting, when screening is desirable, the optimal security encourages the investor to allocate adequate attention to all states so as to effectively distinguish potentially good from bad projects, and thus delivers the highest possible ex-ante profit to the entrepreneur. This mechanism helps generate the exact shape of different optimal securities.

Our parsimonious framework accommodates a variety of theoretical corporate finance contexts and real-world scenarios of financing entrepreneurial production. On the one hand, we view the investor as a screening expert, which is natural but often overlooked. As the cost of screening

6The acknowledgement of investors’ screening dates back to Knight (1921) and Schumpeter (1942). Apart from extensive anecdotal evidence (see Kaplan and Lerner, 2010, Da Rin, Hellmann and Puri, 2011, for reviews), recent empirical literature (Chemmanur, Krishnan and Nandy, 2012, Kerr, Lerner and Schoar, 2014) has also identified direct screening by various types of investors. Theoretical developments in this direction have been surveyed in Bond, Edmans and Goldstein (2012). However, most of those papers focus on the role of competitive financial markets in soliciting or aggregating the information of investors or speculators (for instance, Boot and Thakor, 1993, Fulghieri and Lukin, 2001, Axelson, 2007, Garmaise, 2007, Hennessy, 2013, on security design) rather than the role of screening by individual investors. In reality, most firms are private and do not have easy access to a competitive financial market. A burgeoning security design literature highlights individual investors’ endogenous information advantage directly (Dang, Gorton and Holmstrom, 2011, Yang, 2013), but these models are built to capture the asset-backed securities market as an exchange economy and not fit for our setting of financing entrepreneurial production.
pertains both to the project’s nature and to the investor’s information expertise, it also allows us to cover various investors, including family and friends, banks, and venture capitalists. On the other hand, we highlight two particular aspects of the entrepreneur, capturing the nature of private businesses that account for most firms. First, the entrepreneur is financially constrained. Second, her human capital is inalienable, which means the investor cannot take over the project and the entrepreneur has bargaining power in designing the security. These settings fit the notion of entrepreneur-led financing proposed by Admati and Pfleiderer (1994) and the idea in Rajan (2012) that entrepreneurs’ human capital is important in the early stages of firms’ life cycles. These assumptions are also completely benign; even relaxing them does not affect our main results. This paper, to the best of our knowledge, is the first to investigate the interplay between security design and an individual investor’s screening in a corporate finance production setting.

Related Literature. In addition to the security design literature that identifies debt as the most information-insensitive form of finance (as mentioned in footnote 6), this paper is related to a series of theoretical papers that predict that non-debt securities (including equity and convertibles) can be optimal in various circumstances with asymmetric information (see Brennan and Kraus, 1987, Constantinides and Grundy, 1989, Stein, 1992, Nachman and Noe, 1994, Chemmanur and Fulghieri, 1997, Inderst and Mueller, 2006, Chakraborty and Yilmaz, 2011, Chakraborty, Gervais and Yilmaz, 2011, Fulghieri, Garcia and Hackbarth, 2013). Even closer to the present paper are Boot and Thakor (1993), Manove, Padilla and Pagano (2001), Fulghieri and Lukin (2001), Axelson (2007), Garmaise (2007), and Hennessy (2013), all of which highlight the competitive financial markets’ role in soliciting or aggregating investors’ private information. These papers, however, do not consider an individual investor’s screening directly. Also different from these papers, our model allows for state-contingent decisions of information acquisition, which reflects the fact that the investor is able to acquire information in a very detailed, careful manner. This methodology helps to model arbitrary feasible securities over continuous states with arbitrary distributions and information structures, allowing us to characterize the matching between different projects and different optimal securities in an exhaustive way.

Our model also contributes to the venture contract design literature by highlighting screening. Security design is one focus of modern research in entrepreneurial finance and innovation, but the literature mostly focuses on control rights (Berglof, 1994, Hellmann, 1998, Kirilenko, 2001), monitoring (Ravid and Spiegel, 1997, Schmidt, 2003, Casamatta, 2003, Hellmann, 2006), and refinancing and staging (Admati and Pfleiderer, 1994, Bergemann and Hege, 1998, Cornelli and Yosha, 2003, Repullo and Suarez, 2004) and tends to ignore screening. Further, most of these models only focus on one class of optimal security. In contrast, our model unifies debt and non-

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7The results of optimal securities continue to hold if the project is transferrable or if the entrepreneur does not have full bargaining power in designing the security. See subsection 3.3 and appendix A.2.

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debt-like securities in a general framework and provides a consistent mapping of their optimality to different real-world circumstances.

A new strand of literature on the real effects of rating agencies (see Kashyap and Kovrijnykh, 2013, Opp, Opp and Harris, 2013) is also relevant. On behalf of investors, the rating agency screens the firm, which does not know its own type. Information acquisition may improve social surplus through ratings and the resulting investment decisions. Unlike this literature, we study how different shapes of securities interact with the incentives to allocate attention in acquiring information and therefore the equilibrium financing choice.

The remaining of this paper proceeds as follows. Section 2 specifies the economy. Section 3 characterizes the optimal securities. Section 4 further characterizes under what conditions debt or non-debt securities will be optimal. Section 5 performs comparative statics on the optimal securities. Section 6 concludes.

2 The Model

We present a model focusing on the interplay between security design and flexible screening. We highlight one friction: the dependence of real production on information acquisition and the former’s simultaneous separation from the latter.

2.1 Financing Entrepreneurial Production

Consider a production economy with two dates, \( t = 0, 1 \), and a single consumption good. There are two agents: an entrepreneur lacking financial resources and a deep-pocket investor, both risk-neutral. Their utility function is the sum of consumptions over the two dates: \( u = c_0 + c_1 \), where \( c_t \) denotes an agent’s consumption at date \( t \). In what follows we use subscripts \( E \) and \( I \) to indicate the entrepreneur and the investor, respectively.

The financing process of the entrepreneur’s risky project is as follows. To initiate the project at date 0, the underlying technology requires an investment of \( k > 0 \). If financed, the project generates a non-negative verifiable random cash flow \( \theta \) at date 1. The project cannot be initiated partially. Hence, the entrepreneur has to raise \( k \), by selling a security to the investor at date 0. The payment of a security at date 1 is a mapping \( s : \mathbb{R}_+ \to \mathbb{R}_+ \) such that \( s(\theta) \in [0, \theta] \) for any \( \theta \). We focus only on the cash flow of projects and securities rather than the control rights.

The security design and information acquisition both happen at date 0. The agents have a common prior \( \Pi \) on the potential project’s future cash flow \( \theta \), and neither party has any private information ex-ante.\(^8\) The entrepreneur designs the security, and then proposes a take-it-or-leave-it offer to

\(^8\)We can interpret this setting as that the entrepreneur may still have some private information about the future cash flow, but she does not have any effective ways to signal that to the investor. Signaling has been extensively
the investor at price $k$. Facing the offer, the investor acquires information about $\theta$ in the manner of rational inattention (Sims, 2003, Woodford, 2008, Yang, 2013, 2015), updates beliefs on $\theta$, and then decides whether to accept the offer. The information acquired is measured by reduction of entropy. The information cost per unit reduction of entropy is $\mu$, defined as the cost of screening. We elaborate this information acquisition process in more detail in subsection 2.2.

The assumptions implicit in the setting reflect the key features of financing entrepreneurial production, in particular the role of screening. First, the entrepreneur cannot undertake the project except by external finance. This is consistent with the empirical evidence that entrepreneurs and private firms are often financially constrained (Evans and Jovanovic, 1989, Holtz-Eakin, Joulfaian and Rosen, 1994). Even in mature firms, managers may seek outside finance where the internal capital market does not work well for risky projects (Stein, 1997, Scharfstein and Stein, 2000). Second, the investor can acquire information about the cash flow and thus screen the project through her financing decision. This point not only accounts for the empirical evidence but also sets this model apart from most of the previous security design literature, which features the entrepreneur’s exogenous information advantage. These two points together lead to the dependence and separation of real production and information acquisition, which is the key friction in our model.

It is worth noting which aspects of finance in the production economy are abstracted away, and how much they affect our analysis. First, to focus on screening, we set aside moral hazard. To ignore moral hazard is common in the security design literature, especially when hidden information is important (see DeMarzo and Duffie, 1999, for a justification). Second, we do not focus on the bargaining process and the allocation of control rights. We assume that the entrepreneur’s human capital is inalienable, so that direct project transfer is impossible and the entrepreneur has the bargaining power to design the security. In subsection 3.3 we formally demonstrate that even if the project is transferrable, it is not optimal to transfer the project at any fixed price. Moreover, in appendix A.2 we discuss a general allocation of bargaining power between the two agents and we show that our main results are unaffected unless the investor’s bargaining power is too strong. Third, consistent with the security design literature, we do not allow for partial financing or endogenous investment scale choice. Since our theory can admit discussed in the literature and already well understood, so we leave it aside.

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9As common in the corporate finance literature (see Tirole, 2006 for an overview), entrepreneurs are typically viewed as financially constrained. Alternatively, the entrepreneur may have her own capital but cannot acquire information, so that she wants to hire an information expert to improve the investment decision. This alternative situation boils down to a consulting problem. A large literature on the delegation of experimentation (for example, Manso, 2011) considers such consulting problems in corporate finance, which is beyond the scope of this paper.

10This notion of entrepreneur-led financing is also common in the literature (Brennan and Kraus, 1987, Constantinides and Grundy, 1989, Admati and Pfleiderer, 1994). Together with the differentiation argument in Rajan (2012), this assumption also broadly corresponds to the earlier incomplete contract literature, which suggests that ownership should go to the entrepreneur when firms are young (Aghion and Tirole, 1994).
any prior distribution, a fixed investment requirement in fact enables us to capture projects with differing natures in an exhaustive sense. Fourth, we do not model the staging of finance, and we accordingly interpret the cash flow $\theta$ as already incorporating the consequences of investors’ exiting. Hence, each round of investment may be mapped to our model separately with a different prior. Last, we do not model competition among investors. The last two points pertain to the structure of financial markets, which is tangential to the friction we consider.

2.2 Flexible Information Acquisition

We model the investor’s screening by flexible information acquisition (Yang, 2015). This captures the nature of screening and allows us to work with arbitrary securities over continuous states and without distributional assumptions. Fundamentally, the entrepreneur can design the security’s payoff structure arbitrarily, which may induce arbitrary incentive of attention allocation by the investor in screening the project. This therefore calls for an equally flexible account of screening to capture the interaction between the shape of the securities and the incentives to allocate attention. To map to the reality, flexible information acquisition also captures the fact that the investor can acquire information in a very detailed and careful manner.

The essence of flexible information acquisition is that it captures not only how much but also which aspects of information an agent acquires. Consider an agent who chooses a binary action $a \in \{0, 1\}$ and receives a payoff $u(a, \theta)$, where $\theta \in \mathbb{R}_+$ is the fundamental, distributed according to a continuous probability measure $\Pi$ over $\mathbb{R}_+$. Before making a decision, the agent may acquire information through a set of binary-signal information structures, each signal corresponding to one optimal action. Specifically, she may choose a measurable function $m : \mathbb{R}_+ \rightarrow [0, 1]$, the probability of observing signal 1 if the true state is $\theta$, and acquire binary signals $x \in \{0, 1\}$ parameterized by $m(\theta)$; $m(\theta)$ is chosen to ensure that the agent’s optimal action is 1 (or 0) when observing 1 (or 0). By choosing different functional forms of $m(\theta)$, the agent can make the signal correlate with the fundamental in any arbitrary way. Intuitively, for instance, if the agent’s payoff is sensitive to fluctuations of the state within some range $A \subset \mathbb{R}_+$, she would pay more attention to this range by making $m(\theta)$ co-vary more with $\theta$ in $A$. This gives us a desirable account to model an agent’s incentive to acquire different aspects of information.

The conditional probability $m(\cdot)$ embodies a natural interpretation of screening. In our setting of financing entrepreneurial production, conditional on a cash flow $\theta$, $m(\theta)$ is the probability of the project’s being screened in and thus getting financed. It is state-contingent, capturing the investor’s incentive to allocate attention in screening a project. In particular, the absolute value

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11 In general, an agent can choose any information structure. But an agent always prefers binary-signal information structures in binary decision problems. See Woodford (2008) and Yang (2015).

12 Technically, this allows agents to choose signals drawn from any conditional distribution of the fundamental.
of the first order derivative $|dm(\theta)/d\theta|$ represents the screening intensity: when it is larger, the investor differentiates the states around $\theta$ better. Thus, in what follows we call $m(\cdot)$ a screening rule.

We then characterize the cost of information acquisition. As in Woodford (2008) and Yang (2015), the amount of information conveyed by a screening rule $m(\cdot)$ is defined as the expected reduction of uncertainty through observation of the signal, where the uncertainty associated with a distribution is measured by Shannon’s entropy $H(\cdot)$. This reduction from agents’ prior entropy to expected posterior entropy can be calculated as:

$$I(m) = \mathbb{E}[g(m(\theta))] - g(\mathbb{E}[m(\theta)])
,$$

where $g(x) = x \cdot \ln x + (1 - x) \cdot \ln (1 - x)$, and the expectation operator $\mathbb{E}(-)$ is with respect to $\theta$ under the probability measure $\Pi$.\(^{13}\) Denote by $M = \{m \in L(R_+, \Pi) : \theta \in R_+, m(\theta) \in [0, 1]\}$ the set of binary-signal information structures, and $c : M \to R_+$ the cost of information. The cost is assumed to be proportional to the associated mutual information:

$$c(m) = \mu \cdot I(m)
,$$

where $\mu > 0$ is the cost of information acquisition per unit of reduction of entropy.\(^{14,15}\)

Built upon flexible information acquisition, the agent’s problem is to choose a functional form of $m(\theta)$ to maximize expected payoff less information cost. We characterize the optimal screening rule $m(\theta)$ in the following proposition. We denote $\Delta u(\theta) = u(1, \theta) - u(0, \theta)$, which is the payoff gain of taking action 1 over action 0. We also assume that $\Pr[\Delta u(\theta) \neq 0] > 0$ to exclude the trivial case where the agent is always indifferent between the two actions. The proof is in Yang (2013) (see also Woodford, 2008, for an earlier treatment).

**Proposition 1.** Given $u$, $\Pi$, and $\mu > 0$, let $m^*(\theta) \in M$ be an optimal screening rule and

$$\bar{\pi}^* = \mathbb{E}[m^*(\theta)]$$

\(^{13}\)Formally, we have

$$I(m) = H(\Pi) - \int_x H(\Pi(\cdot|x))\Pi_x dx
,$$

where $\Pi$ denotes the prior, $x$ the signal received, $\Pi(\cdot|x)$ the posterior distribution, and $\Pi_x$ the marginal probability of signal $x$. Under binary-signal structure, standard calculation yields the result above.

\(^{14}\)Although the cost $c(m)$ is linear in mutual information $I(m)$, it does not mean it is linear in information acquisition. Essentially, mutual information $I(m)$ is a non-linear functional of the screening rule $m(\cdot)$ and the prior $\Pi$, micro-founded by the information theory.

\(^{15}\)The functional form of the information cost, following the literature of rational inattention, is not crucial in driving our qualitative results. See Woodford (2012) and Yang (2015) for discussions on related properties of this cost function.
be the corresponding unconditional probability of taking action 1. Then,
i) the optimal screening rule is unique;
ii) there are three cases for the optimal screening rule:
\[ \bar{\pi}^* = 1, \ \text{i.e.,} \ \text{Prob}[m^* (\theta) = 1] = 1 \ \text{if and only if} \]
\[ E \left[ \exp \left( -\mu^{-1} \cdot \Delta u(\theta) \right) \right] \leq 1; \quad (2.1) \]
\[ \bar{\pi}^* = 0, \ \text{i.e.,} \ \text{Prob}[m^* (\theta) = 0] = 1 \ \text{if and only if} \]
\[ E \left[ \exp \left( \mu^{-1} \cdot \Delta u(\theta) \right) \right] \leq 1; \]
\[ 0 < \bar{\pi}^* < 1 \text{ and } \text{Prob}[0 < m^* (\theta) < 1] = 1 \ \text{if and only if} \]
\[ E \left[ \exp \left( \mu^{-1} \cdot \Delta u(\theta) \right) \right] > 1 \text{ and } E \left[ \exp \left( -\mu^{-1} \cdot \Delta u(\theta) \right) \right] > 1; \quad (2.2) \]
in this case, the optimal screening rule \( m^*(\theta) \) is determined by the equation
\[ \Delta u(\theta) = \mu \cdot \left( g'(m^*(\theta)) - g'(\bar{\pi}^*) \right) \quad (2.3) \]
for all \( \theta \in \mathbb{R}_+ \), where
\[ g'(x) = \ln \left( \frac{x}{1-x} \right). \]

Proposition 1 fully characterizes the agent’s possible optimal decisions of information acquisition. Case a) and Case b) correspond to the scenarios of optimal action 1 or 0. These two cases do not involve information acquisition. They correspond to the scenarios in which the prior is extreme or the cost of information acquisition is sufficiently high. But case c), the more interesting one, involves information acquisition. In particular, the optimal screening rule \( m^*(\theta) \) is not constant in this case, and neither action 1 nor 0 is optimal ex-ante. This case corresponds to the scenario where the prior is not extreme, or the cost of information acquisition is sufficiently low. In Case c) where information acquisition is involved, the agent equates the marginal benefit of information with its marginal cost, as indicated by condition (2.3). So doing, the agent chooses the shape of \( m^*(\theta) \) according to the shape of payoff gain \( \Delta u(\theta) \) and her prior \( \Pi \). In the next section we will see that the shape of \( m^*(\theta) \) is crucial in characterizing the way in which the investor screens a project.

\[ ^{16} \text{See Woodford (2008), Yang (2013, 2015) for more examples on this decision problem.} \]
3 Security Design

Now let us consider the entrepreneur’s security design problem. Denote the optimal security of the entrepreneur by $s^*(\theta)$. The entrepreneur and the investor play a dynamic Bayesian game. Concretely, the entrepreneur designs the security, and then the investor screens the project given the security designed. Hence, we apply Proposition 1 to the investor’s information acquisition problem, given the security, and then solve backwards for the entrepreneur’s optimal security. To distinguish from the general decision problem in Section 2.2, we denote the investor’s optimal screening rule as $m_s(\theta)$, given the security $s(\theta)$; hence the investor’s optimal screening rule is now denoted by $m_s^*(\theta)$.

We formally define the equilibrium as follows.

**Definition 1.** Given $u$, $\Pi$, $k$ and $\mu > 0$, the sequential equilibrium is defined as a combination of the entrepreneur’s optimal security $s^*(\theta)$ and the investor’s optimal screening rule $m_s(\theta)$ for any generic security $s(\theta)$, such that

i) the investor optimally acquires information given any generic security $s(\theta)$: $m_s(\theta)$ is prescribed by Proposition 1, and

ii) the entrepreneur designs the optimal security:

$$s^*(\theta) \in \arg \max_{0 \leq s(\theta) \leq \theta} E[m_s(\theta) \cdot (\theta - s(\theta))] .$$

According to Proposition 1, there are three possible investor behaviors, given the entrepreneur’s optimal security. First, the investor may optimally choose not to acquire information and simply accept the security as proposed. This implies that the project is certainly financed. Second, the investor may optimally acquire some information, induced by the proposed security, and then accept the entrepreneur’s optimal security with a positive probability. In this case, the project is financed with a probability that is positive but less than one. Third, the investor may simply reject the security without acquiring information, which implies that the project is certainly not financed. All the three cases can be accommodated by the equilibrium definition. This third case, however, represents the outside option of the entrepreneur, who can always offer nothing to the investor and drop the project. Accordingly, we focus on the first two cases. The following lemma helps distinguish the first two types of equilibrium from the third.

**Lemma 1.** The project can be financed with a positive probability if and only if

$$\mathbb{E} \left[ \exp(\mu^{-1} \cdot (\theta - k)) \right] > 1 .$$

\[3.1\]

\[17\] The specification of belief for the investor at any generic information set after information acquisition is implied by Proposition 1, provided the definition of $m_s(\theta)$. 

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Lemma 1 is an intuitive investment criterion. It implies that the project is more likely to get financed if the prior of the cash flow is better, if the initial investment $k$ is smaller, or if the cost of screening $\mu$ is lower. When condition (3.1) is violated, the investor will reject the proposed security, whatever it is.

Condition (3.1) appears different from the ex-ante NPV criterion, which suggests that a project should be financed for sure when $E[\theta] - k > 0$. In our model with screening, by Jensen’s inequality, condition (3.1) implies that any project with positive ex-ante NPV will be financed with a positive probability. Moreover, some projects with negative ex-ante NPV may also be financed with a positive probability. This is consistent with our idea that real production depends on information acquisition. With the potential of screening, the ex-ante NPV criterion based on a fixed prior is generalized to a new information-adjusted one to admit belief updating.

The following Corollary 1 implies that the entrepreneur will never propose to concede the entire cash flow to the investor if the project is financed. This corollary is straightforward but worth stressing, in that it helps illustrate the key friction by showing that the interests of the entrepreneur and the investor are not perfectly aligned.

**Corollary 1.** When the project can be financed with a positive probability, $s^*(\theta) = \theta$ is not an optimal security.

In what follows, we assume that condition (3.1) is satisfied, and characterize the entrepreneur’s optimal security, focusing on the first two types of equilibria with a positive screening cost $\mu > 0$. As we will see, the entrepreneur’s optimal security differs between the two cases, which implies that the investor screens the project differently. We further show that to transfer the project at a given price is always sub-optimal, which also justifies the security design approach. Finally, for additional intuitions, we consider two limiting cases, one with infinite and one with zero screening cost.

### 3.1 Optimal Security without Inducing Information Acquisition

In this subsection, we consider the case in which the entrepreneur’s optimal security is accepted by the investor without information acquisition. In other words, the entrepreneur finds screening not worthwhile and wants to design a security to deter it. Concretely, this means $Pr[m^*_s(\theta) = 1] = 1$. We first consider the investor’s problem of screening, given the entrepreneur’s security, then characterize the optimal security.

Given a security $s(\theta)$, the investor’s payoff gain from accepting rather than rejecting the security is

$$\Delta u_I(\theta) = u_I(1, \theta) - u_I(0, \theta) = s(\theta) - k.$$  \hspace{1cm} (3.2)
According to Proposition 1 and conditions (2.1) and (3.2), any security \( s(\theta) \) that is accepted by the investor without information acquisition must satisfy

\[
\mathbb{E} \left[ \exp \left( -\mu^{-1} \cdot (s(\theta) - k) \right) \right] \leq 1. \tag{3.3}
\]

If the left-hand side of inequality (3.3) is strictly less than one, the entrepreneur could lower \( s(\theta) \) to some extent to increase her expected payoff gain, without affecting the investor’s incentives. Hence, condition (3.3) always holds as an equality in equilibrium.

By backward induction, the entrepreneur’s problem is to choose a security \( s(\theta) \) to maximize her expected payoff

\[
u_E(s(\cdot)) = \mathbb{E} [\theta - s(\theta)]
\]

subject to the investor’s information acquisition constraint

\[
\mathbb{E} \left[ \exp \left( -\mu^{-1} \cdot (s(\theta) - k) \right) \right] = 1,
\]

and the feasibility condition \( 0 \leq s(\theta) \leq \theta \).

In this case, the entrepreneur’s optimal security is a debt. We characterize this optimal security by the following proposition, along with its graphical illustration in Figure 1. It is easy to see that the face value of the debt is unique in this case.

**Proposition 2.** If the entrepreneur’s optimal security \( s^*(\theta) \) induces the investor to accept the security without acquiring information, then it takes the form of a debt:

\[
s^*(\theta) = \min (\theta, D^*)
\]

where the unique face value \( D^* \) is determined in equilibrium.

It is intuitive that debt is the optimal means of finance when the entrepreneur finds it optimal not to induce information acquisition. Since screening is not worthwhile, and the entrepreneur wants to design a security to deter it, debt is the least information-sensitive form that provides the entrepreneur’s desired expected payoff. From another perspective, the optimal security enables the investor to break even between acquiring and not acquiring information. Specifically, it is the flat part of debt that mitigates the investor’s incentive to acquire information to the extent at which she just gives up acquiring information, while delivering the highest possible expected

---

\( ^{18} \)With this feasibility condition, the entrepreneur’s individual rationality constraint \( \mathbb{E} [\theta - s(\theta)] \geq 0 \) is automatically satisfied, which is also true for the later case with information acquisition. This comes from the fact that the entrepreneur has no endowment. It also implies that the entrepreneur always prefers to undertake the project, which is consistent with real-world practices. However, it is not correct to interpret this as that the entrepreneur would like to contract with any investor, as we do not model the competition among different investors.
payoff to the entrepreneur. This implies that the optimal security should be as flat as possible when the limited liability constraint is not binding, which leads to debt.

Interestingly, although in this case the investor only provides material investment (rather than acquire costly information), the expected payment $\mathbb{E}[s^*(\theta)]$ exceeds the investment requirement $k$. The extra payment exceeding $k$ works as a premium to make the investor comfortable with accepting the offer with certainty, as otherwise, without screening she might worry about financing a potentially bad project.

On the other hand, however, the prediction here implies that there is always a limit to the amount of optimal leverage, even without resorting to costs of financial distress which is typical in trade-off theories of debt. In our framework, intuitively, this derives from the separation of real production and information acquisition. As long as the face value is high enough to prevent costly and unnecessary information acquisition, the entrepreneur wants to retain as much as possible, inducing a cap on the optimal face value of debt.

The optimality of debt here accounts for the real-world scenarios in which new projects are financed by fixed-income securities. When a project’s market prospects are good and thus not much extra information is needed, it is optimal to deter or mitigate investor’s costly information acquisition by resorting to a debt security, which is the least information-sensitive. Interestingly, the rationale for debt in our model does not feature adverse selection, but rather a cost-benefit trade-off of screening. Empirical evidence suggests that many conventional businesses and less revolutionary start-ups relying heavily on plain vanilla debt finance from investors who are not good at screening, such as relatives, friends, and traditional banks (for example, Berger and Udell, 1998, Kerr and Nanda, 2009, Robb and Robinson, 2014), as opposed to more sophisticated financial contracts with venture capital or buyout funds.

The optimality of debt described here is also conceptually different from that in Yang (2013),
who considers security design with flexible information acquisition in a comparable exchange economy. In that model, a seller has an asset in place and proposes a security to a more patient buyer to raise liquidity. The buyer can acquire information about the asset’s cash flow before purchasing. In that model, debt is optimal because it offers the greatest mitigation of the buyer’s adverse selection and hence helps achieve a higher selling price. In the present production economy, however, the investment requirement is fixed, and the optimality of debt derives from the aforementioned cost-benefit analysis of screening (i.e., information acquisition).

3.2 Optimal Security Inducing Information Acquisition

Here we characterize the entrepreneur’s optimal security that does induce the investor to acquire information and to accept the security with positive probability but not certainty. In other words, the entrepreneur finds screening desirable in this case and designs a security to incentivize it. According to Proposition 1, this means \( \text{Prob}[0 < m_s(\theta) < 1] = 1 \).

Again, according to Proposition 1 and conditions (2.2) and (3.2), any generic security \( s(\theta) \) that induces the investor to acquire information must satisfy

\[
E \left[ \exp \left( \mu^{-1} \left( s(\theta) - k \right) \right) \right] > 1
\] (3.4)

and

\[
E \left[ \exp \left( -\mu^{-1} \left( s(\theta) - k \right) \right) \right] > 1 \, ,
\] (3.5)

Given such a security \( s(\theta) \), Proposition 1 and condition (2.3) also prescribe that the investor’s optimal screening rule \( m_s(\theta) \) is uniquely characterized by

\[
s(\theta) - k = \mu \cdot (g'(m_s(\theta)) - g'(\pi_s)) \, ,
\] (3.6)

where

\[
\pi_s = E [m_s(\theta)]
\]

is the investor’s unconditional probability of accepting the security. In what follows, we denote by \( \pi^*_s \) the unconditional probability induced by the entrepreneur’s optimal security \( s^*(\theta) \).

We derive the entrepreneur’s optimal security backwards. Taking account of investor’s response \( m_s(\theta) \), the entrepreneur chooses a security \( s(\theta) \) to maximize

\[
u_E(s(\cdot)) = E [m_s(\theta) \cdot (\theta - s(\theta))] \] (3.7)
subject to (3.4), (3.5), (3.6), and the feasibility condition $0 \leq s(\theta) \leq \theta$.\textsuperscript{19}

We first offer an intuitive roadmap to investigate the optimal security and the associated screening rule, highlighting their key properties. Then we follow with a formal proposition to characterize the optimal security. The detailed derivation is in Appendix A.1.\textsuperscript{20}

First, the investor’s optimal screening rule $m^*_s(\theta)$, induced by the optimal security $s^*(\theta)$, must increase in $\theta$. When the entrepreneur finds it optimal to induce information acquisition, screening by the investor benefits the entrepreneur. Effective screening makes sense only if the investor screens in a potentially good project and screens out bad ones; otherwise it lowers the total social surplus. Under flexible information acquisition, this implies that $m^*_s(\theta)$ should be more likely to generate a good signal and to result in a successful finance when the cash flow $\theta$ is higher, while more likely to generate a bad signal and a rejection when $\theta$ is lower. Therefore, $m^*_s(\theta)$ should be increasing in $\theta$. As we will see, the monotonicity of $m^*_s(\theta)$ and the shape of $s^*(\theta)$ are closely interrelated.

To induce an increasing optimal screening rule $m^*_s(\theta)$, the optimal security $s^*(\theta)$ must be increasing in $\theta$ as well, according to the first order condition of information acquisition (3.6). Intuitively, this monotonicity reflects the dependence of real production on information acquisition: the entrepreneur is willing to compensate the investor more in the event of higher cash flow to encourage effective screening. Unlike the classical security design literature, which often restricts the feasible set to non-decreasing securities, our model uncovers the intrinsic force that drives the pervasiveness of increasing securities in reality.

We also argue that the non-negative constraint $s(\theta) \geq 0$ is not binding for the optimal security $s^*(\theta)$ for any $\theta > 0$. Suppose $s^*(\tilde{\theta}) = 0$ for some $\tilde{\theta} > 0$. Since $s^*(\theta)$ is increasing in $\theta$, for all $0 \leq \theta \leq \tilde{\theta}$ we must have $s^*(\theta) = 0$. This violates the foregoing argument that $s^*(\theta)$ must be increasing in $\theta$. Intuitively, zero payment in states with low cash flows gives the investor too little incentive to acquire information, which is not optimal for the entrepreneur. The security with zero payment in states with low cash flows looks closest to levered common stock, which is the least commonly used security between entrepreneurs and investors in practice (Kaplan and Stromberg, 2003, Kaplan and Lerner, 2010, Lerner, Leamon and Hardymon, 2012).

For closer examination of the optimal security, a perturbation argument on the security design problem gives the entrepreneur’s first order condition. Specifically, denote by $r^*(\theta)$ the marginal contribution to the entrepreneur’s expected payoff $u_E(s(\cdot))$ of any feasible perturbation to the optimal security $s^*(\theta)$.\textsuperscript{21} As $s^*(\theta) > 0$ for any $\theta > 0$, it is intuitive to show that for any $\theta > 0$:

\textsuperscript{19}Again, the entrepreneur’s individual rationality constraint $\mathbb{E}[m_0(\theta) \cdot (\theta - s(\theta))] \geq 0$ is automatically satisfied.

\textsuperscript{20}To facilitate understanding, the intuitive investigation of the optimal security is not organized in the same order as the derivation goes in the Appendix, but all the claims in the main text are guaranteed by the formal proofs.

\textsuperscript{21}Formally, $r^*(\theta)$ is the Frechet derivative, the functional derivative used in the calculus of variations, of $u_E(s(\cdot))$ at $s^*(\theta)$. It is analogical to the commonly used derivative of a real-valued function of a single real variable but
\[
\begin{cases}
    r^*(\theta) = 0 & \text{if } 0 < s^*(\theta) < \theta, \\
    \geq 0 & \text{if } s^*(\theta) = \theta,
\end{cases}
\]

which is further shown to be equivalent to

\[
(1 - m^*_s(\theta)) \cdot (\theta - s^*(\theta) + w^*) \begin{cases}
    = \mu & \text{if } 0 < s^*(\theta) < \theta, \\
    \geq \mu & \text{if } s^*(\theta) = \theta,
\end{cases}
\]

where \(w^*\) is a constant determined in equilibrium.

We argue that the optimal security \(s^*(\theta)\) follows the 45° line in states with low cash flows and then increases in \(\theta\) with some smaller slope in states with high cash flows. That is, the residual of the optimal security, \(\theta - s^*(\theta)\), also increases in \(\theta\) in states with high cash flows. According to the entrepreneur’s first order condition (3.8) and the monotonicity of \(m^*_s(\theta)\), if \(s^*(\hat{\theta}) = \hat{\theta}\) for some \(\hat{\theta} > 0\), it must be \(s^*(\theta) = \theta\) for any \(0 < \theta < \hat{\theta}\). Similarly, if \(s^*(\hat{\theta}) < \hat{\theta}\) for some \(\hat{\theta} > 0\), then for any \(\theta > \hat{\theta}\) it must be \(s^*(\theta) < \theta\), again by condition (3.8) and the monotonicity of \(m^*_s(\theta)\). In addition, Corollary 1 rules out \(s^*(\theta) = \theta\) for all \(\theta > 0\) as an optimal security. Thus, since \(s^*(\theta)\) is increasing in \(\theta\), the limited liability constraint can only be binding in states with low cash flows. Importantly, given condition (3.8) and, again the monotonicity of \(m^*_s(\theta)\), when the limited liability constraint is not binding in states with high cash flows, not only \(s^*(\theta)\) but also \(\theta - s^*(\theta)\) are increasing in \(\theta\). In other words, \(s^*(\theta)\) is dual monotone when it deviates from the 45° line in states with high cash flows.

The shape of the optimal security \(s^*(\theta)\) reflects the friction of the economy. Recall that the monotonicity of \(s^*(\theta)\) reflects the dependence of real production on information. The monotonicity of \(\theta - s^*(\theta)\), however, reflects their separation: the entrepreneur wants to retain as much as possible while incentivizing the investor to screen the project. Specifically, the area between \(s^*(\theta)\) and the 45° line not only captures the entrepreneur’s retained benefit, but also reflects the degree to which the allocation of cash flow is inefficient when screening is desirable. This is intuitive: dependence implies that the investor should get all the cash flow, but separation precludes proposing such a deal, as shown in Corollary 1. The competition of the two forces is alleviated in a most efficient way: rewarding the investor more but also retaining more in better states. In this sense, again, our

\[\text{generalized to accommodate functions on Banach spaces.}\]

\[\text{This argument can be seen by contradiction. Suppose } s^*(\theta) < \theta \text{ when } 0 < \theta < \hat{\theta}. \text{ By the monotonicity of } m^*_s(\theta), \text{ we know that } 1 - m^*_s(\theta) > 1 - m^*_s(\hat{\theta}). \text{ We also know that } \theta - s^*(\theta) > \hat{\theta} - s^*(\hat{\theta}) = 0. \text{ Thus, we have } (1 - m^*_s(\theta))(\theta - s^*(\theta) + w^*) > (1 - m^*_s(\hat{\theta}))(\hat{\theta} - s^*(\hat{\theta}) + w^*) \geq \mu, \text{ the last inequality following the second row of condition (3.8) because } s^*(\hat{\theta}) = \hat{\theta}. \text{ But this in turn implies that } (1 - m^*_s(\theta))(\theta - s^*(\theta) + w^*) > \mu, \text{ which violates the first row of condition (3.8) because } s^*(\theta) < \theta \text{ requires } (1 - m^*_s(\theta))(\theta - s^*(\theta) + w^*) = \mu, \text{ a contradiction.}\]

\[\text{In the formal proofs we further show that the limited liability constraint must be binding for some states } (0, \hat{\theta}) \text{ with } \hat{\theta} > 0.\]
prediction of dual monotonicity derives endogenously from the friction of the economy, whereas in the previous literature it is commonly posited by assumptions.

Formally, the following proposition characterizes the optimal security $s^*(\theta)$ that induces the investor to acquire information.

**Proposition 3.** If the entrepreneur’s optimal security $s^*(\theta)$ induces the investor to acquire information, then it takes the following form:

$$s^*(\theta) = \begin{cases} \theta & \text{if } 0 \leq \theta \leq \hat{\theta} \\ \hat{s}(\theta) & \text{if } \theta > \hat{\theta} \end{cases},$$

where $\hat{\theta}$ is determined in equilibrium and the unconstrained part $\hat{s}(\theta)$ satisfies:

i) $\hat{\theta} < \hat{s}(\theta) < \theta$;

ii) $0 < d\hat{s}(\theta)/d\theta < 1$.\(^{24}\)

Finally, the corresponding optimal screening rule satisfies $dm_s^*(\theta)/d\theta > 0$.

![Figure 2: The Unique Optimal Security with Information Acquisition](image)

Proposition 3 offers a clear prediction on the entrepreneur’s optimal security when screening is desirable. The form of this security most closely resembles participating convertible preferred stock, with $d\hat{s}(\theta)/d\theta$ as the conversion ratio, which grants holders the right to receive both the face value and their equity participation as if it was converted, in the real-world event of a public offering or sale. The payoff structure shown in Figure 2 may be also interpreted as debt

\(^{24}\)In Appendix A.1, we provide the implicit function that determines $d\hat{s}(\theta)/d\theta$ and further show that $d^2\hat{s}(\theta)/d\theta^2 < 0$, which implies that the unconstrained part is concave, as illustrated in Figure 2. Specifically, we interpret this as a state-contingent conversion ratio, with which the entrepreneur retains more shares in better states. Kaplan and Stromberg (2003) have documented the frequent use of contingent contracts in venture finance and private equity buyouts, which is consistent with the state-contingent conversion ratio described here.
plus equity (common stock),\textsuperscript{25} or participating convertible debt.\textsuperscript{26} This prediction is consistent with the empirical evidence of venture contracts documented in Kaplan and Stromberg (2003), who find that 94.4\% of all financing contracts are convertible preferred stock,\textsuperscript{27} among which 40.8\% are participating, and the participating feature is especially frequent in the earlier rounds of investment. Kaplan and Stromberg (2003) suggest that participating is preferable even to straight convertible preferred stock for screening purpose. Our model implies the same: straight convertible preferred stock is better than debt but still not optimal, because its flat payoffs in intermediate states do not provide enough incentive for the investor to differentiate these states. Our prediction also fits in line with earlier evidence (Sahlman, 1990, Bergemann and Hege, 1998, Gompers, 1999) on the popularity of participating convertible preferred stock and the combination of debt and equity in financing young firms and new projects. For brevity, in what follows we refer to the optimal security in this case as convertible preferred stock or the combination of debt and equity, and use the two terms interchangeably.

Flexible information acquisition plays an important role in predicting the shape of convertible preferred stock. When screening is desirable, on the one hand, a globally increasing security incentivizes the investor to pay sufficient attention to all states so as to discriminate between potentially good and bad projects. On the other hand, a higher conversion ratio $d\hat{s}(\theta)/d\theta$ induces the investor to screen the underlying project more intensively, at a higher cost to the entrepreneur.\textsuperscript{28} Hence, the entrepreneur weighs the benefit of screening against its cost by choosing the optimal conversion ratios on a state-contingent basis to ensure the highest possible ex-ante profit.

Our prediction of the multiple of convertible preferred stock, defined as the ratio of the face value $\hat{\theta}$ to the investor’s initial investment $k$, is also consistent with the empirical evidence (Kaplan and Stromberg, 2003, Lerner, Leamon and Hardymon, 2012). The multiple is a key characteristic of convertible preferred stock, sometimes taken as analogous to investment returns.

**Corollary 2.** The multiple of convertible preferred stock, as the optimal security $s^*(\theta)$ inducing information acquisition, is greater than one, that is, $\hat{\theta} > k$.

Like the debt case, this property again derives from the fact that the entrepreneur should offer

\textsuperscript{25}The package of redeemable preferred stock (can be viewed as a form of debt) and common stock is used as equivalent to participating convertible preferred stock in practice. But the package is not popular, since it is harder to assign reasonable value to each component of the package. See Lerner, Leamon and Hardymon (2012) for more detailed discussion on this point. Also see Stein (1992) and Cornelli and Yosha (2003) for theoretical expositions on the advantage of convertible securities over the combination of debt and equity when characteristics other than the cash flow rights are taken into account.

\textsuperscript{26}Compared to equity (common stock), debt and preferred stock are identical in our model, as the model only features two tranches and no dividends.

\textsuperscript{27}If we include convertible debt and the combination of debt and equity, this number increases to 98.1\%.

\textsuperscript{28}In the Appendix A.1, we formally show that the screening intensity $|d\tilde{s}(\theta)/d\theta|$ in the converting region is increasing in $d\tilde{s}(\theta)/d\theta$. 

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the investor a premium for accepting the offer with positive probability. Even after screening, the investor’s information is still imperfect. Thus, this premium makes the investor comfortable with not rejecting the offer, because even with information acquisition the investor may still end up financing a potentially bad project.

Finally, comparison of the production with an exchange economy helps show why our model can predict both debt and non-debt securities. In a production economy (a primary financial market), costly information contributes to the output, whereas in an exchange economy (a secondary financial market) it only helps reallocate existing resources. Specifically, in an exchange economy as modeled in Dang, Gorton and Holmstrom (2011) and Yang (2013), information is socially wasteful for the following two reasons. First, the information acquisition leads to endogenous adverse selection and results in illiquidity. Second, information is costly per se. As a result, to discourage information acquisition is desirable. In the present paper, however, the entrepreneur and the investor jointly tap the project’s cash flow if the investor accepts the proposed security, not if the security is rejected. Thus, the present model features a production economy in which the social surplus may depend positively on costly information. In this case, adverse selection is no longer the focus. Instead, the entrepreneur may want to design a security that encourages the investor to acquire information favorable to the entrepreneur. That is, debt may no longer be optimal when information acquisition is desirable.29

3.3 Project Transfer

This subsection considers the potential for transferring the project at a fixed non-negative price and shows that it is not optimal even though agents are risk neutral. It can be seen as tantamount a scenario in which the entrepreneur works for the investor for a fixed wage, without moral hazard on the entrepreneur’s side. We show that even if the project is transferrable, the entrepreneur still finds it not optimal.

The key to understand this idea is to posit project transfer as one of the feasible securities, as in Figure 3, and show that this security is not optimal. When the entrepreneur proposes a project transfer to the investor at a fixed price \( p \geq 0 \), it is equivalent to proposing a security \( s(\theta) = \theta - p \) without the non-negative constraint \( s(\theta) \geq 0 \). To see why, notice that if the investor accepts the offer of transfer and undertakes the project, she gets the entire cash flow \( \theta \) and pay \( p \) as an upfront cost. This interpretation allows us to analyze project transfer in our security design framework.

29This contrast is reminiscent of Hirshleifer (1971), which distinguishes between information value in an exchange and in a production economy. Earlier mechanism design literature on information gathering also hints at this difference, suggesting that the contribution of information provision on liquidity would differ accordingly (Cremer and Khalil, 1992, Cremer, Khalil and Rochet, 1998a,b).
To see why the equivalent security \( s(\theta) = \theta - p \) is feasible but not optimal, observe that the non-negative constraint \( s(\theta) \geq 0 \) is not binding in either case of security design, as shown in Section 3.1 and Section 3.2. Hence, we may also consider a larger set of feasible securities, which is still restricted by the limited liability constraint \( s(\theta) \leq \theta \) but allows negative payoffs to the investor. As debt and the combination of debt and equity are still the only two optimal securities in this generalized problem, and \( s(\theta) = \theta - p \) (project transfer) is feasible, we conclude that project transfer is not optimal to the entrepreneur at any transfer price \( p \). Intuitively, project transfer is not optimal because it does not follow the least costly way to compensate the investor, whether or not information acquisition is induced.

**Proposition 4.** Transferring a project at a fixed price \( p \geq 0 \) is always sub-optimal for the entrepreneur.

The timeline and sequence of moves in the economy is important for Proposition 4. Consider an alternative sequence in which the investor acquires information only after the transfer. In this economy friction is no longer present, because real production and information acquisition are both performed by the investor after the transfer. With this timeline, project transfer is optimal to the entrepreneur through setting a price that is equivalent to the expected profit of the investor. In this case, the entrepreneur’s bargaining power is too strong in the sense that she can prevent the investor from acquiring information when proposing the transfer deal, which essentially removes the friction from the economy. In practice, however, it is common (and reasonable) for investors to have the option of acquiring information about the project before the transfer. This justifies our sequence of moves and suggests that transfer is not optimal when screening is inevitable.
4 Optimal Securities in Different Circumstances

A natural question is: in our production economy, when is debt optimal, and when is the combination of debt and equity (convertible preferred stock) optimal? Having characterized the optimal securities with and without inducing screening, we take them together and determine the optimal security given the characteristics of the production economy. This implies that the entrepreneur chooses different optimal securities and thus different capital structures in order to finance projects with differing natures. We focus on cases where the project can be financed with a positive probability and thus security design makes sense, that is, when condition (3.1) is satisfied.

4.1 NPV Dimension

We first investigate how the optimal security varies when the ex-ante NPV of the project is different.

**Proposition 5.** Consider the ex-ante NPV (i.e., \( \mathbb{E}[\theta] - k \)) of the project:

i). if \( \mathbb{E}[\theta] - k \leq 0 \), the optimal security \( s^*(\theta) \) is convertible preferred stock; and

ii). if \( \mathbb{E}[\theta] - k > 0 \), \( s^*(\theta) \) may be either convertible preferred stock or debt.

Intuitively, a negative NPV project can only be financed by convertible preferred stock: only through screening could the investor’s belief be updated. This is consistent with the conventional wisdom that a negative NPV project can never be financed by debt with a given, fixed belief.

Interestingly, convertible preferred stock may be optimal for financing both negative- and positive-NPV projects, but the underlying mechanisms of screening are subtly different. In both cases, the dependence of real production on information acquisition is strong.

When the project has a zero or negative NPV, convertible preferred stock is used to encourage the investor to screen in a potentially good project. The investor will never finance the project without screening it, because it incurs an expected loss even if the entrepreneur promises the entire cash flow. Thus, if it is to be financed, the only way is to use convertible preferred stock to encourage screening. This implies that the dependence of real production on information acquisition is strong due to the relatively poor prior, and thus the friction is accordingly severe. When the investor acquires information, she may expect either a good signal, which leads to a deal, or a bad signal, which results in a rejection, but the ex-ante probability of financing the project becomes positive since a potentially good project can be screened in. Hence, the entrepreneur is better off by proposing convertible preferred stock.

In contrast, when the project has a positive NPV, convertible preferred stock may still be used, but now the aim is to encourage the investor to screen out a potentially bad project. Here, the
dependence of real production on information acquisition is still strong due to a relatively mediocre prior. In the status quo where the investor is unable to screen the project, the entrepreneur can finance the positive-NPV project certainly (i.e., with probability one) by proposing debt with a sufficiently high face value. However, when the investor can acquire information, such certain financing (with probability one) could be too expensive, because it leaves too little for the entrepreneur. Instead, the entrepreneur could retain more by offering convertible preferred stock, less generous, and invite the investor to screen the project. Although this results in less than certain financing for the project, the entrepreneur’s total expected profit could be higher, since a potentially bad project may be screened out, which justifies convertible preferred stock as optimal.

Finally, debt may be optimal for some positive-NPV projects. When the prior is sufficiently good, the dependence of real production on information acquisition is weak, and thus the benefit from screening does not justify the cost. In this case, it is optimal for the entrepreneur to propose debt to deter costly screening while still retaining enough profit.

4.2 The Efficiency Dimension

We then go deeper to investigate how the optimal security varies when the extent of the informational friction in the economy changes. This helps reveal the mechanism of this model at a more fundamental level.

To understand how the optimal security evolves with the severity of friction, we consider a frictionless centralized economy in which real production and information acquisition are aligned. We define a new efficiency dimension with help of this centralized economy. If and only if the friction in the decentralized economy is not severe in the sense that an optimal security can achieve efficiency, the optimal security is debt and screening is not induced in equilibrium. If and only if the friction is severe in the sense that an optimal security cannot achieve efficiency, the optimal security is convertible preferred stock and screening is induced. This dichotomy again highlights the close interconnection of the shape of the optimal security, the role of screening, and the extent of friction in the production economy.

First let us define the expected social surplus and the efficiency dimension. In the decentralized economy, the expected surplus is the difference between the expected profit of the project and the cost of screening, both of which are functions of the screening rule. Thus, an optimal security achieves efficiency if the induced optimal screening rule maximizes expected social surplus in equilibrium.

**Definition 2.** An optimal security in the decentralized economy achieves efficiency if and only
if the associated optimal screening rule \( m^*_s(\theta) \) satisfies:

\[
m^*_s(\theta) \in \arg \max_{0 \leq m(\theta) \leq 1} \mathbb{E}[m(\theta) \cdot (\theta - k)] - \mu \cdot I(m(\theta)).
\] (4.9)

To facilitate discussion, we characterize a frictionless centralized economy to help benchmark the friction in the corresponding decentralized economy. In the centralized economy, \( u, \Theta, \Pi, k \) and \( \mu \) are given as the same. However, we assume that the entrepreneur has sufficient initial wealth and can also screen the project. Thus, real production still depends on information acquisition, but the two are aligned. In this economy, security design is irrelevant. The entrepreneur’s problem is to decide whether to undertake the project directly, to screen it, or to abandon it. The entrepreneur’s payoff gain from undertaking the project rather than abandoning it is

\[
\Delta u_I(\theta) = u_I(1, \theta) - u_I(0, \theta) = \theta - k.
\]

We denote an arbitrary screening rule in the centralized economy by \( m_c(\theta) \) and the optimal screening rule by \( m^*_c(\theta) \). Thus, the entrepreneur’s problem in the centralized economy is

\[
\max_{0 \leq m_c(\theta) \leq 1} \mathbb{E}[m_c(\theta) \cdot (\theta - k)] - \mu \cdot I(m_c(\theta)).
\] (4.10)

By construction, the entrepreneur’s objective (4.10) in the centralized economy is exactly the expected social surplus in the decentralized economy. Hence, we can examine the centralized economy to analyze the efficiency of equilibria in the corresponding decentralized economy.

Since the optimal screening rules are unique for both the centralized and the decentralized economy, efficiency is achieved if and only if information is acquired in the same manner in both.

**Lemma 2.** An optimal security in the decentralized economy achieves efficiency if and only if the associated optimal screening rule \( m^*_s(\theta) \) satisfies

\[
\text{Prob}[m^*_s(\theta) = m^*_c(\theta)] = 1,
\]

where \( m^*_c(\theta) \) is the optimal screening rule in the corresponding centralized economy.

The efficiency concept in Lemma 2 demonstrates the role of screening in the production economy and provides a natural measure of the severity of friction in the decentralized economy. Fundamentally, we may view the expected social surplus (4.10) or (4.9) in our production economy as a production function, with information characterized by the screening rule \( m_c(\theta) \) or \( m(\theta) \) as the sole input. This again fits with the idea that real production depends on information acquisition. Consequently, efficiency is achieved if and only if the optimal security in the decentralized economy
delivers the same equilibrium allocation of input, information acquisition, as the centralized economy. If the optimal security does this, friction in the decentralized economy is not severe, as it can be effectively removed by optimal security design. Otherwise, friction is severe, in that it cannot be completely removed even by an optimal security.

By the efficiency concept in Definition 2 and Lemma 2, we can characterize the optimal securities over the efficiency dimension, as follows.

**Proposition 6. In the decentralized production economy:**

i) the optimal security $s^*(\theta)$ is debt if and only if friction in the decentralized economy is not severe, i.e., an optimal security achieves efficiency; and

ii) $s^*(\theta)$ is convertible preferred stock if and only if the friction is severe, i.e., an optimal security cannot achieve efficiency.

These results are important because they answer the question why different projects should be financed by different types of securities by pointing to its origin: the friction between real production and information acquisition. In the decentralized economy, real production is performed by the entrepreneur while information acquisition by the investor. This separation is always present, unchanged despite the different exogenous characteristics of the economy. Hence, the severity of the friction is reflected in the extent to which real production depends on information acquisition. If the friction is severe, the dependence is strong, which makes screening worthwhile and makes convertible preferred stock optimal. Similarly, if the friction is not severe, the dependence is weak, screening does not justify its cost, and debt is optimal.

Our predictions help unify the empirical evidence. They are particularly suited for entrepreneurial businesses. Debt financing is popular for conventional projects and for investors who have less expertise in screening, i.e., when the informational friction is not severe. Instead, financing with convertible preferred stock (or the combination of debt and equity) is common for innovative projects, especially in the early rounds, which benefit more from screening, that is, when the friction is severe.

5 Comparative Statics of the Optimal Security

For additional intuitions, we look at numerical comparative statics on the shape of the optimal securities with respect to two empirical dimensions: the profitability of the project and its uncertainty.\footnote{We are not aware of any analytical comparative statics pertaining to functionals. An analytical comparative statics requires a total order, which is not applicable for our security space. Even for some ordered characteristics of the optimal security, for instance, the face value, analytical comparative statics are not achievable. Thus, we rely on numerical results to deliver intuitions and leave analytical work to future research. Numerical analysis in our framework is tractable but already fairly technical. The algorithm and codes are available upon request.} When the environment varies, the role of screening changes, and the way in which
the entrepreneur incentivizes screening changes accordingly, producing different shapes of optimal securities.

5.1 Project Profitability

First, we consider the effects of variations in the project’s profitability on the shape of the optimal security $s^*(\theta)$, holding constant the project’s market prospects (i.e., the prior distribution of the cash flow $\theta$), and the cost of screening, $\mu$. Thus, a decrease in the investment requirement $k$ implies that the project is more profitable ex-ante.

We show the results in Figure 4. The investment $k$ takes three increasing values: 0.4, 0.475, and 0.525. When $k = 0.4$, the optimal security is debt; for two other projects with larger $k$, one with positive and one with negative ex-ante NPV, it is convertible preferred stock. In particular, the face value $\hat{\theta}$ and the conversion ratios $d\hat{s}(\theta)/d\theta$ of the convertible preferred stock are both increasing in $k$. For the prior of the cash flow $\theta$, we take a normal distribution with mean 0.5 and standard deviation 0.125, and then truncate and normalize this distribution to the interval $[0, 1]$.

The screening cost $\mu$ is fixed at 0.2.

![Figure 4: Change of Investment with $E[\theta] = 0.5, \mu = 0.2$](image)

The comparative statics with respect to the profitability of the project serve as a detailed illustration of Proposition 5. When the project is sufficiently profitable ex-ante ($k = 0.4$), the friction is not severe and the project will be financed by debt without inducing screening. When the project looks mediocre in terms of its profitability but still has a positive ex-ante NPV ($k = 0.475$), the friction becomes severe, and information acquisition becomes worthwhile to screen bad projects out, so that convertible preferred stock becomes optimal. When the project is not profitable in the sense that its NPV is negative ($k = 0.525$), the friction is more severe, and the only way for the entrepreneur to obtain financing is to propose convertible preferred stock and expect a potentially good project to be screened in. For this type of project, in particular,
screening is more valuable, and hence the entrepreneur is willing to compensate the investor more generously to induce more effective screening, as seen in Figure 4.

5.2 Project Uncertainty

We then consider how varying the degree of the project’s uncertainty affects the optimal security $s^*(\theta)$. Concretely, we consider different prior distributions of the cash flow $\theta$ with the same mean, ranked by second order stochastic dominance.\(^\text{31}\) We also hold constant the investment requirement $k$ and the cost of screening $\mu$. Note that, the effect of varying uncertainty cannot be accounted for by any argument involving risks, because both the entrepreneur and the investor are risk-neutral. Instead, we still focus on friction and the role of screening to explain these effects.

Interestingly, the comparative statics with respect to uncertainty depend on the sign of the project’s ex-ante NPV. As implied by Proposition 5, the role of screening differs when these signs differ. This further leads to different patterns of comparative statics when the degree of uncertainty varies.

First, we consider projects with positive ex-ante NPV and increasing uncertainty. The results are shown in Figure 5, the left panel illustrating the priors of the cash flow $\theta$ and the right panel the evolution of the optimal security. When the project is the least uncertain, the optimal security is debt. For more uncertain projects convertible preferred stock becomes optimal, while the face value $\hat{\theta}$ and the conversion ratios $\hat{d} s(\theta)/d\theta$ are both increasing in uncertainty. For the priors, we take normal distributions with mean 0.5 and standard deviations 0.125 and 0.25, and then truncate and normalize them to the interval $[0, 1]$. We also construct a third distribution, in which the project is so uncertain that the cash flow has a greater probability of taking extreme values in $[0, 1]$.

The comparative statics in this case demonstrate how varying uncertainty affects the screening-out of bad projects, given positive ex-ante NPV. When the project is least uncertain, it is least likely to be bad, which implies that screening-out is least relevant and debt financing is accordingly optimal. When uncertainty increases, the project is more likely to be bad, and screening-out becomes more valuable. Hence, the entrepreneur finds it optimal to propose a more generous convertible preferred stock to induce screening-out.

Next we consider projects with negative ex-ante NPV, focusing on those that may be financed with a positive probability due to screening-in through convertible preferred stock. The results are shown in Figure 6: both the face value $\hat{\theta}$ and the conversion ratio $d\hat{s}(\theta)/d\theta$ of the convertible preferred stock are decreasing in uncertainty. The priors are generated as we did in Figure 5. The

\(^{31}\)There are other ways to measure the project’s uncertainty. For comparative statics, our idea is to find a partial order of uncertainty over the space of distributions, while to keep the project’s ex-ante NPV constant. Thus, second order stochastic dominance is a natural choice.
investment is $k = 0.525$ and the cost of screening is $\mu = 0.2$.

The comparative statics in this case are also intuitive, according to the role of screening-in. Given negative ex-ante NPV, the investor screens in potentially good projects. In contrast to the positive-NPV case, here the increase in uncertainty means that the ex-ante negative-NPV project is more likely to be good. Thus, to acquire costly information to screen in a potentially good project becomes less necessary. Therefore, the entrepreneur wants to propose a less generous convertible preferred stock for less costly screening. Not surprisingly, the resulting convertible preferred stock moves away from the $45^\circ$ line when the project is more uncertain.
6 Conclusion

This paper posits a new type of informational friction to investigate security design. Real production depends on information acquisition, but these two functions are performed separately by entrepreneur and investor. We predict that debt, which does not induce screening, is optimal when the dependence is weak and the friction is therefore not severe, whereas the combination of debt and equity (participating convertible preferred stock), which does induce screening, is optimal when the dependence is strong and the friction accordingly severe. These predictions are supported by the empirical evidence.

This paper contributes to the security design literature in several respects, as well as to the broader corporate finance and contract design literature. By flexible information acquisition, we can work with arbitrary securities over continuous states while dispensing with usual distributional assumptions, thus offering an exhaustive examination at the question why projects with different natures should be financed by different types of securities.
A Derivation, Extensions, and Proofs

A.1 Derivation of Convertible Preferred Stock as the Optimal Security

This appendix derives the optimal security $s^*(\theta)$ when it induces information acquisition. We proceed by two steps.

First, we solve for an “unconstrained” optimal security without the feasibility condition $0 \leq s(\theta) \leq \theta$. We denote the solution by $\hat{s}(\theta)$. We also denote the corresponding screening rule by $\hat{m}_s(\theta)$. The unconstrained optimal security recovers the unconstrained part $\hat{s}(\theta)$ of the eventual optimal security in Proposition 3. After that, we resume the feasibility condition and characterize the optimal security $s^*(\theta)$.

**Lemma 3.** In an equilibrium with information acquisition, the unconstrained optimal security $\hat{s}(\theta)$ and its corresponding screening rule $\hat{m}_s(\theta)$ are determined by

$$\hat{s}(\theta) - k = \mu \cdot \left( g'(\hat{m}_s(\theta)) - g'\left(\pi^*_s\right)\right), \tag{A.1}$$

where

$$\pi^*_s = \mathbb{E}[m^*_s(\theta)],$$

and

$$(1 - \hat{m}_s(\theta)) \cdot (\theta - \hat{s}(\theta) + w^*) = \mu, \tag{A.2}$$

where

$$w^* = \mathbb{E} \left[ (\theta - s^*(\theta)) \frac{g''(\pi^*_s)}{g''(m^*_s(\theta))} \right] \left( 1 - \mathbb{E} \left[ \frac{g''(\pi^*_s)}{g''(m^*_s(\theta))} \right] \right)^{-1},$$

in which $\pi^*_s$ and $w^*$ are two constants determined in equilibrium, and $s^*(\theta)$ and $m^*_s(\theta)$ are the solutions of the original constrained problem.

Lemma 3 exhibits the relationship between the unconstrained optimal security $\hat{s}(\theta)$ and the corresponding screening rule $\hat{m}_s(\theta)$. Condition (A.1) specifies how the investor responds to the unconstrained optimal security by adjusting her screening rule. On the other hand, condition (A.2) is derived from the entrepreneur’s optimization problem. It indicates the entrepreneur’s optimal choices of payments across states, given the investor’s screening rule. In equilibrium, $\hat{s}(\theta)$ and $\hat{m}_s(\theta)$ are jointly determined.

Although it is not tractable to fully solve the system of equations (A.1) and (A.2), we are able to deliver important analytical characteristics of the unconstrained optimal security $\hat{s}(\theta)$ and the corresponding screening rule $\hat{m}_s(\theta)$.

**Lemma 4.** In an equilibrium with information acquisition, the unconstrained optimal security $\hat{s}(\theta)$...
and the corresponding screening rule \( \hat{m}_s(\theta) \) satisfy

\[
\frac{\partial \hat{m}_s(\theta)}{\partial \theta} = \mu^{-1} \cdot \hat{m}_s(\theta) \cdot (1 - \hat{m}_s(\theta))^2 > 0 ,
\]

(A.3)

and

\[
\frac{\partial \hat{s}(\theta)}{\partial \theta} = 1 - \hat{m}_s(\theta) \in (0,1).
\]

We have several interesting observations from Lemma 4. First, condition (A.3) implies that the unconstrained optimal screening rule \( \hat{m}_s(\theta) \) is strictly increasing. Second, condition (A.4) implies that the unconstrained optimal security \( \hat{s}(\theta) \) is also strictly increasing. These are because, according to Proposition 1, we have \( \text{Prob}[0 < \hat{m}_s(\theta) < 1] = 1 \) in this case, and thus the right hand sides of (A.3) and (A.4) are positive. It follows immediately that the residual of the unconstrained optimal security, \( \theta - \hat{s}(\theta) \), is also strictly increasing. Last, the unconstrained optimal security \( \hat{s}(\theta) \) is strictly concave. This is because conditions (A.3) and (A.4) imply that

\[
\frac{\partial^2 \hat{s}(\theta)}{\partial \theta^2} = -\mu^{-1} \cdot \hat{m}_s(\theta) \cdot (1 - \hat{m}_s(\theta))^2 < 0.
\]

Therefore, the unconstrained optimal security \( \hat{s}(\theta) \) is an increasing concave function of \( \theta \).

Given Lemma 4, it is instructive to have the following lemma to illustrate the possible relative positions between the unconstrained optimal security and the feasibility constraints.

**Lemma 5.** Three possible relative positions between the unconstrained optimal security \( \hat{s}(\theta) \) and the feasibility constraints \( 0 \leq s(\theta) \leq \theta \) may occur in equilibrium, in the \( \theta \sim s \) space:

i) \( \hat{s}(\theta) \) intersects with the 45\(^\circ\) line \( s = \theta \) at \( (\hat{\theta}, \hat{\theta}) \), \( \hat{\theta} > 0 \), and does not intersect with the horizontal axis \( s = 0 \);

ii) \( \hat{s}(\theta) \) goes through the origin \( (0,0) \), and does not intersect with either the 45\(^\circ\) line \( s = \theta \) or the horizontal axis \( s = 0 \) for any \( \theta \neq 0 \);

iii) \( \hat{s}(\theta) \) intersects with the horizontal axis \( s = 0 \) at \( (\tilde{\theta}, 0) \), \( \tilde{\theta} > 0 \), and does not intersect with the 45\(^\circ\) line \( s = \theta \).

In the three different cases, the actual optimal security \( s^*(\theta) \) will be constrained by the feasibility condition in different ways. For example, \( s^*(\theta) \) will be constrained by the 45\(^\circ\) line \( s = \theta \) in Case i) while by the horizontal axis \( s = 0 \) in Case iii). By imposing the feasibility conditions, we have the following characterization for \( s^*(\theta) \):

**Lemma 6.** In an equilibrium with information acquisition, the corresponding optimal security
\( s^*(\theta) \) satisfies

\[
s^*(\theta) = \begin{cases} 
\theta & \text{if } \hat{s}(\theta) > \theta \\
\hat{s}(\theta) & \text{if } 0 \leq \hat{s}(\theta) \leq \theta \\
0 & \text{if } \hat{s}(\theta) < 0
\end{cases},
\]

where \( \hat{s}(\theta) \) is the corresponding unconstrained optimal security.

Lemma 6 is helpful because it tells us how to construct an optimal security \( s^*(\theta) \) from its corresponding unconstrained optimal security \( \hat{s}(\theta) \). Concretely, \( s^*(\theta) \) will follow \( \hat{s}(\theta) \) when the latter is within the feasible region \( 0 \leq s \leq \theta \). When \( \hat{s}(\theta) \) goes out of the feasible region, the resulting optimal security will follow one of the feasibility constraints that is binding.

We apply Lemma 6 to the three cases of the unconstrained optimal security \( \hat{s}(\theta) \) described in Lemma 5. This gives the three potential cases of the optimal security \( s^*(\theta) \), respectively.

**Lemma 7.** In an equilibrium with information acquisition, the optimal security \( s^*(\theta) \) may take one of the following three forms:

i) when the corresponding unconstrained optimal security \( \hat{s}(\theta) \) intersects with the 45° line \( s = \theta \) at \( (\hat{\theta}, \hat{\theta}) \), \( \hat{\theta} > 0 \), we have

\[
s^*(\theta) = \begin{cases} 
\theta & \text{if } 0 \leq \theta < \hat{\theta} \\
\hat{s}(\theta) & \text{if } \theta \geq \hat{\theta}
\end{cases};
\]

ii) when the corresponding unconstrained optimal security \( \hat{s}(\theta) \) goes through the origin \( (0,0) \), we have \( s^*(\theta) = \hat{s}(\theta) \) for \( \theta \in \mathbb{R}^+ \);

iii) when the corresponding unconstrained optimal security \( \hat{s}(\theta) \) intersects with the horizontal axis \( s = 0 \) at \( (\tilde{\theta}, 0) \), \( \tilde{\theta} > 0 \), we have

\[
s^*(\theta) = \begin{cases} 
0 & \text{if } 0 \leq \theta < \tilde{\theta} \\
\hat{s}(\theta) & \text{if } \theta \geq \tilde{\theta}
\end{cases}.
\]

The optimal security \( s^*(\theta) \) takes different shapes in the three potential cases. In Case i), \( s^*(\theta) \) follows a debt in states with low cash flows but increases in states with high cash flows. In Case iii), \( s^*(\theta) \) has zero payment in states with low cash flows, while is an increasing function in states with high cash flows. Case ii) lies in between as a knife-edge case.

We proceed by determining whether these three potential cases are valid solutions to the entrepreneur’s problem in an equilibrium with information acquisition. Interestingly, not all the three cases can occur in equilibrium.

**Lemma 8.** If the entrepreneur’s optimal security \( s^*(\theta) \) induces the investor to acquire information in equilibrium, then it must follow Case i) in Lemma 7, which corresponds to a participating convertible preferred stock with a face value \( \hat{\theta} > 0 \).
Together with the lemmas already established, Lemma 8 immediately leads to Proposition 3. Intuitively, Case ii) and Case iii) in Lemma 7 cannot sustain an equilibrium with information acquisition because the investor is underpaid. Recall that the investor provides two types of inputs. The first is the investment required to initiate the project, and the second is the costly information to screen the project. As a result, the entrepreneur wants to make sure that the investor is sufficiently compensated for both inputs to be willing to accept the security. This argument is further strengthened by Corollary 2, which suggested that $\hat{\theta}$ should be larger than the investment requirement $k$.

A.2 General Allocation of Bargaining Power

This appendix extends our baseline model to a more general setting that allows for the arbitrary allocation of bargaining power between the entrepreneur and the investor. It demonstrates that our framework and qualitative results are robust to the allocation of bargain power.

Without loss of generality, let the entrepreneur’s bargaining power in security design be $1 - \alpha$ and the investor’s $\alpha$. Suppose a third party in the economy knows $\alpha$, designs the security and proposes it to the investor. The investor acquires information according to the security and decides whether or not to accept this offer. The third party’s objective function is an average of the entrepreneur’s and the investor’s utilities, weighted according to the bargaining power of each. When $\alpha = 0$, this reduces to our baseline model. The derivations for the results are the same as in the baseline model.

In this setting, the third-party’s objective function, that is, the payoff gain, is

$$u_T(s(\theta)) = \alpha \cdot (\mathbb{E}[(s(\theta) - k) \cdot m(\theta)] - \mu \cdot I(m)) + (1 - \alpha) \cdot \mathbb{E}[(\theta - s(\theta)) \cdot m(\theta)].$$

We can show that, with information acquisition, the equation that governs information acquisition is still the same as condition (3.6):

$$s(\theta) - k = \mu \cdot (g'(m_s(\theta)) - g' (\pi_s)),$$

while the equation that characterizes the optimality of the unconstrained optimal security becomes

$$r(\theta) = (2\alpha - 1) \cdot m(\theta) + (1 - \alpha) \cdot \mu^{-1} \cdot m(\theta) \cdot (1 - m(\theta)) \cdot (\theta - s(\theta) + w).$$

The following two propositions characterize the optimal security in the general setting. \[32\]

\[32\]The proofs for the extended model follow those for the benchmark model closely, so we do not repeat them in the appendix.
Proposition 7. When $0 \leq \alpha < 1/2$ and information acquisition happens in equilibrium, the unconstrained optimal security $\hat{s}(\theta)$ and the corresponding screening rule $\hat{m}_s(\theta)$ satisfy

$$\frac{d\hat{s}(\theta)}{d\theta} = \frac{1 - \hat{m}_s(\theta)}{1 - \frac{\alpha}{1-\alpha}\hat{m}_s(\theta)} \in (0, 1)$$

and

$$\frac{d\hat{m}_s(\theta)}{d\theta} = \frac{\mu^{-1} \cdot \hat{m}_s(\theta) \cdot (1 - \hat{m}_s(\theta))^2}{1 - \frac{\alpha}{1-\alpha}\hat{m}_s(\theta)} > 0.$$}

Also, all the results from Lemma 5 to Lemma 8 and from Proposition 1 to Proposition 6 still hold.

Proposition 8. When $1/2 \leq \alpha \leq 1$, the optimal security features $s^*(\theta) = \theta$.

Our generalized results show that when the investor has some bargaining power, but not too much, all the qualitative results remain unchanged. But if the investor’s bargaining power is strong, the optimal security is most favorable to the investor: a complete takeover. This is intuitive considering the situation from the standpoint of friction. If the entrepreneur dominates, she will still play a considerable role in real production, which depends on the investor’s information acquisition. Thus, the presence of friction still calls for a meaningful security design that follows our interaction between the shape of the securities and the incentive for screening. On the contrary, if the investor dominates, she may take over the project and effectively eliminate the friction. In this case, real production and information acquisition are joined and security design becomes less relevant. This corresponds to the empirical fact that buyouts and takeovers are common for mature companies, where the role of entrepreneurs and founders is no longer inalienable, a point also highlighted in Rajan (2012) and Lerner, Leamon and Hardymon (2012).

A.3 Proofs

This appendix provides all proofs omitted above.

Proof of Lemma 1. We first prove the “only if” part. Suppose that

$$\mathbb{E} \left[ \exp(\mu^{-1}(\theta - k)) \right] \leq 1.$$

According to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the investor will still reject the offer without acquiring information. Since $s(\theta) \leq \theta$, the project cannot be initiated in this case.

Then we prove the “if” part. Let $t \in (0, 1)$. Since $\mathbb{E} \left[ \exp(\mu^{-1}(t \cdot \theta - k)) \right]$ is continuous in $t$, there exists $t < 1$ such that

$$\mathbb{E} \left[ \exp(\mu^{-1}(t \cdot \theta - k)) \right] > 1.$$
Hence, according to Proposition 1, the security \( s_t(\theta) = t \cdot \theta \) would be accepted by the investor with a positive probability. Moreover, let \( m_t(\theta) \) be the corresponding screening rule. As \( s_t(\theta) \) would be accepted with a positive probability, \( m_t(\theta) \) cannot be always zero. Hence, the entrepreneur’s expected payoff is \( \mathbb{E}[(1 - t) \cdot \theta \cdot m_t(\theta)] \), which is strictly positive.

Note that the security \( s_t(\theta) \) is a feasible security. Hence, the optimal security \( s^*(\theta) \) will also be accepted with a positive probability and deliver a positive expected payoff to the entrepreneur. This concludes the proof.

Proof of Corollary 1. The proof is straightforward following the above proof of Lemma 1. Proposing \( s^*(\theta) = \theta \) gives the entrepreneur a zero payoff, while proposing \( s_t(\theta) = t \cdot \theta \) constructed in the proof of Lemma 1 gives her a strictly positive expected payoff. This suggests that \( s^*(\theta) = \theta \) is not optimal.

Proof of Proposition 2. The Lagrangian of the entrepreneur’s problem is

\[
\mathcal{L} = \mathbb{E} \left[ \theta - s(\theta) + \lambda \cdot \left( 1 - \exp \left( \mu^{-1} \cdot (k - s(\theta)) \right) \right) + \eta_1(\theta) \cdot s(\theta) + \eta_2(\theta) \cdot (\theta - s(\theta)) \right],
\]

where \( \lambda, \eta_1(\theta) \) and \( \eta_2(\theta) \) are multipliers.

The first order condition is

\[
\frac{d\mathcal{L}}{ds(\theta)} = -1 + \lambda \cdot \mu^{-1} \cdot \exp \left( \mu^{-1} \cdot (k - s(\theta)) \right) + \eta_1(\theta) - \eta_2(\theta) = 0.
\]

We first consider a special case that is helpful for us to solve the optimal security. If \( 0 < s(\theta) < \theta \), the two feasibility conditions are not binding. Thus \( \eta_1(\theta) = \eta_2(\theta) = 0 \), and the first order condition is simplified as

\[
-1 + \lambda \cdot \mu^{-1} \cdot \exp \left( \mu^{-1} \cdot (k - s(\theta)) \right) = 0.
\]

By rearrangement, we get

\[
s(\theta) = k - \mu \cdot \ln(\lambda^{-1} \cdot \mu). \quad (A.6)
\]

We denote the right hand side of \( (A.6) \), which is irrelevant of \( \theta \), as \( D^* \). By definition, we have \( D^* > 0 \). Also, it is straightforward to have

\[
-1 + \lambda \cdot \mu^{-1} \cdot \exp \left( \mu^{-1} \cdot (k - D^*) \right) = 0. \quad (A.7)
\]

In what follows, we characterize the optimal solution \( s^*(\theta) \) for different regions of \( \theta \).

First, we consider the region of \( \theta > D^* \). We show that \( 0 < s^*(\theta) < \theta \) in this region by
contradiction.

If \( s^*(\theta) = \theta > D^* \), we have \( \eta_1(\theta) = 0 \) and \( \eta_2(\theta) \geq 0 \). From the first order condition (A.5) we obtain
\[
-1 + \lambda \cdot \mu^{-1} \cdot \exp \left( \mu^{-1} \cdot (k - \theta) \right) = \eta_2(\theta) \geq 0. \tag{A.8}
\]

On the other hand, as \( \theta > D^* \), we have
\[
-1 + \lambda \cdot \mu^{-1} \cdot \exp \left( \mu^{-1} \cdot (k - D^*) \right) > -1 + \lambda \cdot \mu^{-1} \cdot \exp \left( \mu^{-1} \cdot (k - \theta) \right) . \tag{A.9}
\]

Conditions (A.7), (A.8), and (A.9) construct a contradiction. So we must have \( s^*(\theta) < \theta \) if \( \theta > D^* \).

Similarly, if \( s^*(\theta) = 0 \), we have \( \eta_1(\theta) \geq 0 \) and \( \eta_2(\theta) = 0 \). Again from the first order condition (A.5) we obtain
\[
-1 + \lambda \cdot \mu^{-1} \cdot \exp \left( \mu^{-1} \cdot k \right) = -\eta_1(\theta) \leq 0 . \tag{A.10}
\]

On the other hand, as \( D^* > 0 \), we have
\[
-1 + \lambda \cdot \mu^{-1} \cdot \exp \left( \mu^{-1} \cdot (k - D^*) \right) < -1 + \lambda \cdot \mu^{-1} \cdot \exp \left( \mu^{-1} \cdot k \right) . \tag{A.11}
\]

Conditions (A.7), (A.10), and (A.11) construct another contradiction. So we must have \( s^*(\theta) > 0 \) if \( \theta > D^* \).

Therefore, we have shown that \( 0 < s^*(\theta) < \theta \) for \( \theta > D^* \). From the discussion above for this specific case, we conclude that \( s^*(\theta) = D^* \) for \( \theta > D^* \).

We then consider the region of \( \theta < D^* \). We show that \( s^*(\theta) = \theta \) in this region.

Since \( s^*(\theta) \leq \theta < D^* \), we have
\[
-1 + \lambda \cdot \mu^{-1} \cdot \exp \left( \mu^{-1} \cdot (k - s^*(\theta)) \right) > -1 + \lambda \cdot \mu^{-1} \cdot \exp \left( \mu^{-1} \cdot (k - D^*) \right) . \tag{A.12}
\]

From condition (A.7), the right hand side of this inequality (A.12) is zero. Together with the first order condition (A.5), the inequality (A.12) implies that \( \eta_1(\theta) = 0 \) and \( \eta_2(\theta) > 0 \). Therefore, we have \( s^*(\theta) = \theta \) in this region.

Also, from the first order condition (A.5) and condition (A.7), it is obvious that \( s^*(D^*) = D^* \).

To sum up, the entrepreneur’s optimal security without inducing the investor to acquire information features a debt with face value \( D^* \) determined by condition (A.6).

We need to check that there exists \( D^* > 0 \) and the corresponding multiplier \( \lambda > 0 \) such that
\[
\mathbb{E} \left[ \exp \left( -\mu^{-1} \cdot (\min(\theta, D^*) - k) \right) \right] = 1 , \tag{A.13}
\]
where \( D^* \) is determined by condition (A.6).

Consider the left hand side of condition (A.13). Clearly, it is continuous and monotonically decreasing in \( D^* \). When \( D^* \) is sufficiently large, the left hand side of (A.13) approaches \( \mathbb{E} \left[ \exp \left( -\mu^{-1} \cdot (\theta - k) \right) \right] \), a number less than one, which is guaranteed by condition (3.3) as well as the feasibility condition \( s(\theta) \leq \theta \). On the other hand, when \( D^* = 0 \), it approaches \( \exp \left( \mu^{-1} \cdot k \right) \), which is strictly greater than one. Hence, there exists \( D^* > 0 \) such that condition (A.13) holds.

Moreover, from condition (A.6), we also know that \( D^* \) is continuous and monotonically increasing in \( \lambda \). When \( \lambda \) approaches zero, \( D^* \) approaches negative infinity, while when \( \lambda \) approaches positive infinity, \( D^* \) approaches positive infinity as well. Hence, for any \( D^* > 0 \) there exists a corresponding multiplier \( \lambda > 0 \).

Last, suppose \( D^* \leq k \). It is easy to see that this debt would be rejected by the investor due to Proposition 1, a contradiction.

Finally, by condition (3.3) again, since the optimal security \( s^*(\theta) \) satisfies
\[
\mathbb{E} \left[ \exp \left( -\mu^{-1} \cdot (s^*(\theta) - k) \right) \right] = 1,
\]
Jensen’s inequality implies that \( \mathbb{E}[s^*(\theta)] > k \) given \( \mu > 0 \). This concludes the proof. \( \square \)

**Proof of Lemma 3.** We derive the entrepreneur’s optimal security \( s^*(\theta) \) and the corresponding unconstrained optimal security \( \hat{s}(\theta) \) through variational methods. Specifically, we characterize how the entrepreneur’s expected payoff responds to the perturbation of her optimal security.

Let \( s(\theta) = s^*(\theta) + \alpha \cdot \varepsilon(\theta) \) be an arbitrary perturbation of the optimal security \( s^*(\theta) \). Note that the investor’s optimal screening rule \( m_s(\theta) \) appears in the entrepreneur’s expected payoff \( u_E(s(\cdot)) \), according to condition (3.7), and it is implicitly determined by the proposed security \( s(\theta) \) through the functional equation (3.6). Hence, we need to first characterize how \( m_s(\theta) \) varies with respect to the perturbation of \( s^*(\theta) \). Taking derivative with respect to \( \alpha \) at \( \alpha = 0 \) for both sides of (3.6) leads to
\[
\mu^{-1} \varepsilon(\theta) = g''(m_s^*(\theta)) \cdot \frac{\partial m_s(\theta)}{\partial \alpha} \bigg|_{\alpha=0} - g''(\pi_s^*) \cdot \mathbb{E} \left[ \frac{\partial m_s(\theta)}{\partial \alpha} \right]_{\alpha=0}.
\]

Take expectation of both sides and we get
\[
\mathbb{E} \left[ \frac{\partial m_s(\theta)}{\partial \alpha} \right]_{\alpha=0} = \mu^{-1} \cdot \left( 1 - \mathbb{E} \left[ (g''(m_s^*(\theta)))^{-1} \right] \cdot g''(\pi_s^*) \right)^{-1} \cdot \mathbb{E} \left[ (g''(m_s^*(\theta)))^{-1} \varepsilon(\theta) \right].
\]
Combining the above two equations, for any perturbation \( s(\theta) = s^*(\theta) + \alpha \cdot \varepsilon(\theta) \), the investor’s
screening rule $m_s(\cdot)$ is characterized by
\[
\frac{\partial m_s(\theta)}{\partial \alpha} \bigg|_{\alpha=0} = \mu^{-1} \cdot (g''(m^*_s(\theta)))^{-1} \varepsilon(\theta) \\
+ \mu^{-1} \cdot (g''(m^*_s(\theta)))^{-1} \cdot \mathbb{E} \left[ (g''(m^*_s(\theta)))^{-1} \varepsilon(\theta) \right] \\
\frac{(g''(\pi^*_s))^{-1} - \mathbb{E} [(g''(m^*_s(\theta)))^{-1}]}{\cdot (g''(m^*_s(\theta)))^{-1}}. \tag{A.14}
\]

We interpret condition (A.14). The first term of the right hand side of (A.14) is the investor’s local response to $\varepsilon(\theta)$. It is of the same sign as the perturbation $\varepsilon(\theta)$. When the payment of the security increases at state $\theta$, the investor is more likely to accept the security at this state. The second term measures the investor’s average response to perturbation $\varepsilon(\theta)$ over all states. It is straightforward to verify that the denominator of the second term is positive due to Jensen’s inequality. As a result, if the perturbation increases the investor’s payment on average over all states, she is more likely to accept the security as well.

Now we can calculate the variation of the entrepreneur’s expected payoff $u_E(s(\cdot))$, according to condition (3.7). Taking derivative of $u_E(s(\cdot))$ with respect to $\alpha$ at $\alpha = 0$ leads to
\[
\frac{\partial u_E(s(\cdot))}{\partial \alpha} \bigg|_{\alpha=0} = \mathbb{E} \left[ \frac{\partial m_s(\theta)}{\partial \alpha} \bigg|_{\alpha=0} (\theta - s(\theta)) \right] - \mathbb{E} \left[ m^*_s(\theta) \cdot \varepsilon(\theta) \right]. \tag{A.15}
\]

Substitute (A.14) into (A.15) and we get
\[
\frac{\partial u_E(s(\cdot))}{\partial \alpha} \bigg|_{\alpha=0} = \mathbb{E} [r(\theta) \cdot \varepsilon(\theta)], \tag{A.16}
\]
where
\[
r(\theta) = -m^*_s(\theta) + \mu^{-1} \cdot (g''(m^*_s(\theta)))^{-1} \cdot (\theta - s^*(\theta) + w^*) \tag{A.17}
\]
and
\[
w^* = \mathbb{E} \left[ (\theta - s^*(\theta)) \frac{g''(\pi^*_s)}{g''(m^*_s(\theta))} \right] \left( 1 - \mathbb{E} \left[ \frac{g''(\pi^*_s)}{g''(m^*_s(\theta))} \right] \right)^{-1}.
\]

Note that $w^*$ is a constant that does not depend on $\theta$ and will be endogenously determined in the equilibrium. Besides, $r(\theta)$ is the Frechet derivative of the entrepreneur’s expected payoff $u_E(s(\cdot))$ at $s^*(\theta)$, which measures the marginal contribution of any perturbation to the entrepreneur’s expected payoff when the security is optimal. Specifically, the first term of (A.17) is the direct contribution of perturbing $s^*(\theta)$ disregard the variation of $m^*_s(\theta)$, and the second term measures the indirect contribution through the variation of $m^*_s(\theta)$. This Frechet derivative $r(\theta)$ plays an important role in shaping the entrepreneur’s optimal security.

To further characterize the optimal security, we discuss the Frechet derivative $r(\theta)$ in detail. Recall that the optimal security would be restricted by the feasibility condition $0 \leq s^*(\theta) \leq \theta$. 

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Let
\[ A_0 = \{ \theta \in \Theta : \theta \neq 0, s^* (\theta) = 0 \} , \]
\[ A_1 = \{ \theta \in \Theta : \theta \neq 0, 0 < s^* (\theta) < \theta \} , \]
\[ A_2 = \{ \theta \in \Theta : \theta \neq 0, s^* (\theta) = \theta \} . \]

Clearly, \( \{ A_0, A_1, A_2 \} \) is a partition of \( \Theta \setminus \{ 0 \} \). Since \( s^* (\theta) \) is the optimal security, we have
\[ \left. \frac{\partial u_E (s(\cdot))}{\partial \alpha} \right|_{\alpha=0} \leq 0 \]
for any feasible perturbation \( \varepsilon (\theta) \). Therefore, condition (A.16) implies
\[ r (\theta) \begin{cases} \leq 0 & \text{if } \theta \in A_0 \\ = 0 & \text{if } \theta \in A_1 \\ \geq 0 & \text{if } \theta \in A_2 \end{cases} . \tag{A.18} \]

According to Proposition 1, when the optimal security \( s^* (\theta) \) induces the investor to acquire information, we have \( 0 < m^*_s (\theta) < 1 \) for all \( \theta \in \Theta \). Hence, condition (A.18) can be rearranged as
\[ \frac{r (\theta)}{m^*_s (\theta)} = -1 + \mu^{-1} \cdot (1 - m^*_s (\theta)) \cdot (\theta - s^* (\theta) + w^*) \begin{cases} \leq 0 & \text{if } \theta \in A_0 \\ = 0 & \text{if } \theta \in A_1 \\ \geq 0 & \text{if } \theta \in A_2 \end{cases} . \tag{A.19} \]

Recall condition (3.6), given the optimal security \( s^* (\theta) \), the investor’s optimal screening rule \( m^*_s (\theta) \) is
\[ s^* (\theta) - k = \mu \cdot \left( g' (m^*_s (\theta)) - g' (\pi^*_s) \right) , \tag{A.20} \]
where
\[ \pi^*_s = \mathbb{E} [m^*_s (\theta)] \]
is the investor’s unconditional probability of accepting the optimal security \( s^* (\theta) \). Conditions (A.19) and (A.20) as a system of functional equations jointly determine the optimal security \( s^* (\theta) \) when it induces the investor’s information acquisition.

Finally, when we focus on the unconstrained optimal security \( \hat{s}(\theta) \), note that is would not be restricted by the feasibility condition. Hence, the corresponding Frechet derivative \( r(\theta) \) would be always zero at the optimum. On the other hand, the investor’s optimal screening rule would not

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\(^{33}\)A perturbation \( \varepsilon (\theta) \) is feasible with respect to \( s^* (\theta) \) if there exists \( \alpha > 0 \) such that for any \( \theta \in \Theta \), \( s^* (\theta) + \alpha \cdot \varepsilon (\theta) \in [0, \theta] \).
be affected. As a result, conditions (A.20) and (A.19) become

\[ \hat{s}(\theta) - k = \mu \cdot (g'(\hat{m}_s(\theta)) - g'(\pi_s^*)) , \]

where

\[ \bar{p}_s^* = E[m^*_s(\theta)] , \]

and

\[ (1 - \hat{m}_s(\theta)) \cdot (\theta - \hat{s}(\theta) + w^*) = \mu , \]

where

\[ w^* = E \left[ (\theta - s^*(\theta)) \frac{g''(\pi_s^*)}{g''(m^*_s(\theta))} \right] \left( 1 - E \left[ \frac{g''(\pi_s^*)}{g''(m^*_s(\theta))} \right] \right)^{-1} , \]

in which \( \overline{p}_s^* \) and \( w^* \) are two constants that do not depend on \( \theta \). This concludes the proof.

**Proof of Lemma 4.** From Lemma 3, \( (\hat{s}(\theta), \hat{m}_s(\theta)) \) satisfies the two equations (A.1) and (A.2). By condition (A.2), we get

\[ \hat{m}_s(\theta) = 1 - \frac{\mu}{\theta - \hat{s}(\theta) + w^*} . \quad (A.21) \]

Substituting (A.21) into (A.1) leads to

\[ \mu^{-1} (\hat{s}(\theta) - k) = g' \left( \frac{\mu}{\theta - \hat{s}(\theta) + w^*} \right) - g'(\pi_s^*) . \]

Taking derivatives of both sides of the above equation with respect to \( \theta \) leads to

\[ \mu^{-1} \cdot \frac{d\hat{s}(\theta)}{d\theta} = g''(\hat{m}_s(\theta)) \cdot \frac{d\hat{m}_s(\theta)}{d\theta} \]

\[ = g''(\hat{m}_s(\theta)) \cdot \frac{\mu \cdot \left( 1 - \frac{\hat{s}(\theta)}{\theta - \hat{s}(\theta) + w^*} \right)}{(\theta - \hat{s}(\theta) + w^*)^2} \]

\[ = \frac{1 - \frac{\hat{s}(\theta)}{\theta - \hat{s}(\theta) + w^*} - \mu}{\theta - \hat{s}(\theta) + w^*} , \]

where we use

\[ g''(x) = \frac{1}{x(1-x)} \]

while deriving the third equality. Rearrange the above equation, and we get

\[ \frac{d\hat{s}(\theta)}{d\theta} = \frac{\mu}{\theta - \hat{s}(\theta) + w^*} \]

\[ = 1 - \hat{m}_s(\theta) , \]

where the last equality follows (A.21).
Again, taking derivatives of both sides of the above equation with respect to \( \theta \) leads to

\[
\mu^{-1} \cdot \frac{d\hat{s}(\theta)}{d\theta} = g''(\hat{m}_s(\theta)) \cdot \frac{d\hat{m}_s(\theta)}{d\theta} = \frac{1}{\hat{m}_s(\theta) (1 - \hat{m}_s(\theta))} \cdot \frac{d\hat{m}_s(\theta)}{d\theta}.
\]

Hence

\[
\frac{d\hat{m}_s(\theta)}{d\theta} = \mu^{-1} \cdot \hat{m}_s(\theta) \cdot (1 - \hat{m}_s(\theta)) \cdot \frac{d\hat{s}(\theta)}{d\theta} = \mu^{-1} \cdot \hat{m}_s(\theta) \cdot (1 - \hat{m}_s(\theta))^2.
\]

This completes the proof. \( \square \)

**Proof of Lemma 5.** From Lemma 4, it is easy to see that the slope of \( \hat{s}(\theta) \) is always less than one. Hence, Lemma 5 is straightforward. \( \square \)

**Proof of Lemma 6.** We proceed by discussing three cases.

Case 1: We show that \( \hat{s}(\theta) > \theta \) implies \( s^*(\theta) = \theta \).

Suppose \( s^*(\theta) < \theta \). Then we have \( s^*(\theta) < \hat{s}(\theta) \). Since both \( (s^*(\theta), m^*_s(\theta)) \) and \( (\hat{s}(\theta), \hat{m}_s(\theta)) \) satisfy condition (3.6), we must have \( m^*_s(\theta) < \hat{m}_s(\theta) \). Therefore,

\[
\frac{r(\theta)}{m^*_s(\theta)} = -1 + \mu^{-1} \cdot (1 - m^*_s(\theta)) \cdot (\theta - s^*(\theta) + w^*)
\]

\[
> -1 + \mu^{-1} \cdot (1 - \hat{m}_s(\theta)) \cdot (\theta - \hat{s}(\theta) + w^*)
\]

\[
= 0,
\]

which implies \( s^*(\theta) = \theta \), a contradiction.

Note that, the logic for the inequality above is as follows. Since \( (\hat{s}(\theta), \hat{m}_s(\theta)) \) satisfies condition (A.2), we must have \( \theta - \hat{\theta} + w^* > 0 \). Hence, \( \hat{s}(\theta) > s^*(\theta) \) implies that

\[
\theta - s^*(\theta) + w^* > \theta - \hat{s}(\theta) + w^* > 0.
\]

Also by noting that

\[
1 - m^*_s(\theta) > 1 - \hat{m}_s(\theta) > 0,
\]

we get the inequality above.

Hence, we have \( s^*(\theta) = \theta \) in this case.

Case 2: We show that \( \hat{s}(\theta) < 0 \) implies \( s^*(\theta) = 0 \).

Suppose \( s^*(\theta) > 0 \). Then we have \( s^*(\theta) > \hat{s}(\theta) \). By similar argument we know that \( m^*_s(\theta) > 40 \).
\( \hat{m}_s(\theta) \). Therefore,

\[
\frac{r(\theta)}{m^*_s(\theta)} = -1 + \mu^{-1} \cdot (1 - \hat{m}_s(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\
< -1 + \mu^{-1} \cdot (1 - \hat{m}_s(\theta)) \cdot (\theta - \hat{s}(\theta) + w^*) \\
= 0 ,
\]

which implies \( s^*(\theta) = 0 \). This is a contradiction. Hence, we have \( s^*(\theta) = 0 \) in this case.

Case 3: We show that \( 0 \leq \hat{s}(\theta) \leq \theta \) implies \( s^*(\theta) = \hat{s}(\theta) \).

Suppose \( \hat{s}(\theta) < s^*(\theta) \). Then similar argument suggests \( r(\theta)/m^*_s(\theta) < 0 \), which implies \( s^*(\theta) = 0 < \hat{s}(\theta) \). This is a contradiction.

Similarly, suppose \( s^*(\theta) < \hat{s}(\theta) \). Similar argument suggests that \( r(\theta)/m^*_s(\theta) > 0 \), which implies \( s^*(\theta) = \theta > \hat{s}(\theta) \). This is, again, a contradiction. Hence, we have \( s^*(\theta) = \hat{s}(\theta) \) in this case.

This concludes the proof. \( \square \)

**Proof of Lemma 7.** Apply Lemma 5 to Lemma 6, then Lemma 7 is straightforward. \( \square \)

**Proof of Lemma 8.** We prove by contradiction. Suppose that the last two cases in Lemma 7 can occur in equilibrium. Hence, there exists a \( \tilde{\theta} \geq 0 \), such that \( s^*(\theta) = 0 \) when \( 0 \leq \theta \leq \tilde{\theta} \) and \( s^*(\theta) = \hat{s}(\theta) \) when \( \theta > \tilde{\theta} \).

Note that, \( s^*(\theta) \) is strictly increasing when \( \theta > \tilde{\theta} \). Also, since we focus on the equilibrium with information acquisition, there must exist a \( \theta' \) such that \( s^*(\theta') > k \); otherwise the optimal security would be rejected without information acquisition. Therefore, there exists a \( \theta' > \tilde{\theta} \) such that \( s^*(\theta') = \hat{s}(\theta') = k \). Recall condition (A.1), we have

\[
m^*_s(\theta') = \bar{\pi}^*_s .
\]

Moreover, notice that we have \( s^*(\theta') \in (0, \theta') \), we have

\[
0 = r(\theta') = -m^*_s(\theta') + \mu^{-1} \cdot m^*_s(\theta') \cdot (1 - m^*_s(\theta')) \cdot (\theta' - s^*(\theta') + w^*) \\
= -\bar{\pi}^*_s + \mu^{-1} \cdot \bar{\pi}^*_s \cdot (1 - \bar{\pi}^*_s) \cdot (\theta' - k + w^*) \\
= \mu^{-1} \cdot \bar{\pi}^*_s \cdot (1 - \bar{\pi}^*_s) \cdot (\theta' - k) + \mathbb{E}[r(\theta)] ,
\]
where

\[
\mathbb{E}[r(\theta)] = -\bar{\pi}_s^* + \mu^{-1}\left(\mathbb{E}\left[\frac{(\theta - s(\theta)) g''(\bar{\pi}_s^*)}{g''(m(\theta))}\right] / g''(\bar{\pi}_s^*) + w^* \mathbb{E}\left[\frac{1}{g''(m(\theta))}\right]\right)
\]

\[
= -\bar{\pi}_s^* + \mu^{-1}\left(w^* \cdot \left(1 - \mathbb{E}\left[\frac{g''(\bar{\pi}_s^*)}{g''(m(\theta))}\right] / g''(\bar{\pi}_s^*) + w^* \mathbb{E}\left[\frac{1}{g''(m(\theta))}\right]\right)\right)
\]

\[
= -\bar{\pi}_s^* + \mu^{-1} \cdot \frac{w^*}{g''(\bar{\pi}_s^*)}
\]

\[
= -\bar{\pi}_s^* + \mu^{-1} \cdot \pi_s^* \cdot (1 - \bar{\pi}_s^*) \cdot w^* .
\]

We can express the expectation term \(\mathbb{E}[r(\theta)]\) in another way. Note that, for any \(\theta \in [0, \tilde{\theta}]\), by definition we have

\[
r(\theta) = -m_s^*(\theta) + \mu^{-1} \cdot m_s^*(\theta) \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*)
\]

\[
= -\hat{m}_s(\bar{\theta}) + \mu^{-1} \cdot \hat{m}_s(\bar{\theta}) \cdot (1 - \hat{m}_s(\bar{\theta})) \cdot (\theta - 0 - \tilde{\theta} + \bar{\theta} + w^*)
\]

\[
= r(\tilde{\theta}) - \mu^{-1} \cdot \hat{m}_s(\bar{\theta}) \cdot (1 - \hat{m}_s(\bar{\theta})) \cdot (\tilde{\theta} - \theta)
\]

\[
= -\mu^{-1} \cdot \hat{m}_s(\bar{\theta}) \cdot (1 - \hat{m}_s(\bar{\theta})) \cdot (\tilde{\theta} - \theta) .
\]

Also, as \(s^*(\theta) = \hat{s}(\theta)\) for any \(\theta > \tilde{\theta}\), we have \(r(\theta) = 0\) for all \(\theta > \tilde{\theta}\). Hence,

\[
\mathbb{E}[r(\theta)] = -\mu^{-1} \cdot \hat{m}_s(\bar{\theta}) \cdot (1 - \hat{m}_s(\bar{\theta})) \int_0^{\tilde{\theta}} (\tilde{\theta} - \theta) d\Pi(\theta) .
\]

Therefore, we have

\[
\mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k) = -\mathbb{E}[r(\theta)]
\]

\[
= \mu^{-1} \cdot \hat{m}_s(\bar{\theta}) \cdot (1 - \hat{m}_s(\bar{\theta})) \int_0^{\tilde{\theta}} (\tilde{\theta} - \theta) d\Pi(\theta) .
\]

Now we take the tangent line of \(s^*(\theta)\) at \(\theta = \tilde{\theta}\). The tangent line intersects \(s = k\) at \(\tilde{\theta}'\), which is given by

\[
\frac{k}{\tilde{\theta}' - \tilde{\theta}} = \frac{d\hat{s}^*(\theta)}{d\theta} \bigg|_{\tilde{\theta}} = 1 - \hat{m}_s(\tilde{\theta}) .
\]

Hence, we have

\[
\tilde{\theta}' = \tilde{\theta} + \frac{k}{1 - \hat{m}_s(\tilde{\theta})} .
\]

Also, note that we have shown that for any \(\theta \geq \tilde{\theta}\), we have

\[
\frac{ds^*(\theta)}{d\theta} = \frac{d\hat{s}(\theta)}{d\theta} = 1 - \hat{m}_s(\theta) = 1 - m_s^*(\theta) .
\]
Hence,
\[
\frac{d^2 s^*(\theta)}{d\theta^2} = -\mu^{-1} \cdot m_s^*(\theta) \cdot (1 - m_s^*(\theta))^2 < 0.
\]

Therefore, \( s^*(\theta) \) is strictly concave for \( \theta \geq \tilde{\theta} \), and we also have \( \tilde{\theta}' < \theta' \). Consequently, by condition (A.24) and then conditions (A.22) and (A.23), we have

\[
\tilde{\pi}_s^* \cdot (1 - \tilde{\pi}_s^*) \cdot \left( \tilde{\theta} + \frac{\tilde{m}_s(\tilde{\theta})}{1 - \tilde{m}_s(\tilde{\theta})} \cdot k \right) = \tilde{\pi}_s^* \cdot (1 - \tilde{\pi}_s^*) \cdot (\tilde{\theta}' - k) < \tilde{\pi}_s^* \cdot (1 - \tilde{\pi}_s^*) \cdot (\tilde{\theta}' - k) = \tilde{m}_s(\tilde{\theta}) \cdot (1 - \tilde{m}_s(\tilde{\theta})) \int_0^{\tilde{\theta}} (\tilde{\theta} - \theta) d\Pi(\theta).
\]

On the other hand, by Jensen’s inequality, we know that

\[
\tilde{\pi}_s^* \cdot (1 - \tilde{\pi}_s^*) > \mathbb{E} \left[ m_s^*(\theta) \cdot (1 - m_s^*(\theta)) \right].
\]

Therefore, we have

\[
\tilde{m}_s(\tilde{\theta}) \cdot (1 - \tilde{m}_s(\tilde{\theta})) \int_0^{\tilde{\theta}} (\tilde{\theta} - \theta) d\Pi(\theta) > \tilde{\pi}_s^* \cdot (1 - \tilde{\pi}_s^*) \cdot \left( \tilde{\theta} + \frac{\tilde{m}_s(\tilde{\theta})}{1 - \tilde{m}_s(\tilde{\theta})} \cdot k \right) > \mathbb{E} \left[ m_s^*(\theta) \cdot (1 - m_s^*(\theta)) \right] \cdot \left( \tilde{\theta} + \frac{\tilde{m}_s(\tilde{\theta})}{1 - \tilde{m}_s(\tilde{\theta})} \cdot k \right).
\]

Expand the expectation term above and rearrange, we get

\[
\tilde{m}_s(\tilde{\theta}) \cdot (1 - \tilde{m}_s(\tilde{\theta})) \int_0^{\tilde{\theta}} (\tilde{\theta} - \theta) d\Pi(\theta) < \tilde{m}_s(\tilde{\theta}) \cdot (1 - \tilde{m}_s(\tilde{\theta})) \int_0^{\tilde{\theta}} (-\theta) d\Pi(\theta) \leq 0.
\]

Nevertheless, the left hand side of the above inequality should be positive, which is a contradiction. This concludes the proof.

**Proof of Proposition 4.** We first consider the case with a positive transfer price \( p > 0 \). Suppose the corresponding security \( s(\theta) = \theta - p \) is optimal in a generalized security design problem without the non-negative constraint. However, this security can be accommodated by neither Proposition 2 or Proposition 3, which exclusively characterize the optimal security in the generalized security design problem, a contradiction.

By Corollary 1, we also know that the security \( s(\theta) = \theta \) that represents transfer with a zero
price is not optimal. This concludes the proof.

Proof of Corollary 2. First, note that $s^*(\theta)$ is strictly increasing and continuous. Also, note that there exists a $\theta''$ such that $s^*(\theta'') > k$; otherwise, the offer will be rejected without information acquisition.

Therefore, there exists a unique $\theta'$ such that $s^*(\theta') = k$, which ensures that $m^*_s(\theta') = \pi^*_s$, and

$$r(\theta') = -\pi^*_s + \mu^{-1} \cdot \pi^*_s \cdot (1 - \pi^*_s) \cdot (\theta' - s^*(\theta') + w^*)$$

$$= \mu^{-1} \cdot \pi^*_s \cdot (1 - \pi^*_s) \cdot (\theta' - s^*(\theta')) + \mathbb{E}[r(\theta)].$$

Note that $\mathbb{E}[r(\theta)] > 0$ and $\theta' - s^*(\theta') \geq 0$, we have $\theta' < \hat{\theta}$. As $\theta' = s^*(\theta') = k$, it follows that $\hat{\theta} > \theta' = k$. This concludes the proof.

Proof of Proposition 5. When we have $\mathbb{E}[\theta] \leq k$ and

$$\mathbb{E}\left[\exp\left(\mu^{-1}(t \cdot \theta - k)\right)\right] > 1,$$

according to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the security would induce the investor to acquire information and accept it with a positive (but less than one) probability. The only optimal security for this case is convertible preferred stock. This concludes the proof.

Proof of Lemma 2. The “if” part is straightforward, following the definition of efficiency. The “only if” part is ensured by the fact that the optimal screening rule is always unique given an arbitrary security, established in Proposition 1.

Proof of Proposition 6. We state a useful lemma to begin. It allow us to focus on the first two types of equilibria for welfare analysis.

Lemma 9. A project is initiated with a positive probability in the decentralized economy if and only if it is initiated with a positive probability in the corresponding centralized economy.

Proof of Lemma 9. With the objective function (4.10) in the centralized economy, the entrepreneur’s optimal screening rule $m^*_c(\theta)$ is characterized by Proposition 1. Specifically, the investor will initiate the project without information acquisition, i.e., $\text{Prob}[m^*_c(\theta) = 1] = 1$ if and only if

$$\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] \leq 1,$$
will skip the project without information acquisition, i.e., \( \text{Prob}[m_c^*(\theta) = 0] = 1 \) if and only if
\[
\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] \leq 1,
\]
and will initiate the project with probability \( 0 < \bar{\pi}_c^* < 1 \), \( \bar{\pi}_c^* = \mathbb{E}[m_c^*(\theta)] \), if and only if
\[
\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] > 1 \quad \text{and} \quad \mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1,
\]
in which \( m_c^*(\theta) \) is determined by
\[
\theta - k = \mu \cdot (g'(m_c^*(\theta)) - g'(\bar{\pi}_c^*)).
\]

It is straightforward to observe that, the project is initiated with a positive probability in the frictionless centralized economy if and only if
\[
\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1.
\] (A.25)

Note that, condition (A.25) is the same as condition (3.1) in Lemma 1 that gives the investment criterion in the corresponding decentralized economy. This concludes the proof. \( \square \)

We continue the proof of Proposition 6. By Lemma 9, once we prove the "only if" parts of both cases of debt and convertible preferred stock, the "if" parts will get proved simultaneously.

First, consider the case when \( s^*(\theta) \) is debt. In this case, we have \( \text{Prob}[m_c^*(\theta) = 1] = 1 \), and
\[
\mathbb{E}[\exp\left(-\mu^{-1} \cdot (s^*(\theta) - k)\right)] \leq 1,
\]
both from Proposition 1. Since \( s^*(\theta) < \theta \) when \( \theta > D^* \), it follows that
\[
\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)] \leq 1,
\]
which implies that \( \text{Prob}[m_c^*(\theta) = 1] = 1 \), also by Proposition 1. Hence, we know that
\[
\text{Prob}[m_c^*(\theta) = m_c^*(\theta)] = 1,
\]
which suggests that \( s^*(\theta) \), as debt, achieves efficiency, according to Lemma 2.

Second, consider the case when \( s^*(\theta) \) is convertible preferred stock that induces information acquisition. In this case, we have \( \text{Prob}[0 < m_c^*(\theta) < 1] = 1 \), and
\[
\mathbb{E}[\exp\left(-\mu^{-1} \cdot (s^*(\theta) - k)\right)] > 1,
\]
again both from Proposition 1. Since $s^*(\theta) < \theta$ when $\theta > \hat{\theta}$, the relationship between $\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))]$ and 1 is ambiguous. If

$$
\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] \leq 1,
$$

we have $\text{Prob}[m^*_s(\theta) = 1] = 1$, and information acquisition is not induced in the centralized economy. It follows that

$$
\text{Prob}[m^*_s(\theta) = m^*_c(\theta)] \neq 1.
$$

Otherwise, if

$$
\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] > 1,
$$
suppose we also have $\text{Prob}[m^*_s(\theta) = m^*_c(\theta)] = 1$, then according to condition (A.20), we have

$$
\text{Prob}[s^*(\theta) = \theta] = 1,
$$

which violates Corollary 1. A contradiction. As a result, from Lemma 2, we know that $s^*(\theta)$, as convertible preferred stock, cannot achieve efficiency. This concludes the proof.

References


