Appendix to
A Multivariate Model of Strategic Asset Allocation

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Appendix A: Derivation of Main Equations in Text

We first summarize three results on matrix algebra that will be convenient in deriving the expressions given in the text.

**Result 1.**

\[
\begin{align*}
  z_{t+1}' &= (\Phi_0 + \Phi_1 z_t + v_{t+1})' - E_t (z_{t+1}'_{t+1}) \\
  &= \Phi_0 \Phi_0' + \Phi_1 z_t \Phi_0' + v_{t+1} \Phi_0' + \Phi_0 z_t' \Phi_1' + \Phi_1 z_t' \Phi_1' \\
  &\quad + v_{t+1} z_t' \Phi_1' + \Phi_0 v_{t+1}' + \Phi_1 z_t v_{t+1}' - E_t (z_{t+1}'_{t+1}) \\
  &= v_{t+1} \Phi_0' + v_{t+1} z_t' \Phi_1' + \Phi_0 v_{t+1}' + \Phi_1 z_t v_{t+1}' + v_{t+1} v_{t+1}' - \Sigma_v.
\end{align*}
\]

**Result 2.**

\[
\begin{align*}
  r_{i,t+1} - E_t (r_{i,t+1}) &= x^{(i-1)}_{t+1} + r_{1,t+1} - E_t (x^{(i-1)}_{t+1} + r_{1,t+1}) \\
  &= v^{(i)}_{t+1} + v^{(1)}_{t+1}
\end{align*}
\]

where \(x^{(i-1)}_{t+1}\) denotes the \((i-1)\)th element of the excess return vector \(x_{t+1}\) and likewise with \(v_{t+1}\.\)

**Result 3.** Any quadratic form \(z_{t+1}'Mz_{t+1}\) admits the following vector form representation:

\[
\begin{align*}
  z_{t+1}'Mz_{t+1} &= \text{vec}(M)' \text{vec}(z_{t+1}'_{t+1})
\end{align*}
\]

where \(\text{vec}(\cdot)\) is the vectorization operator.

**Result 4.** (Muirhead, 1982, pp.518)

\[
\begin{align*}
  \text{Var}_t (\text{vec}(v_{t+1}'_{t+1})) &= \left( I_m^2 + \sum_{i,j}^m \left( Q_{ij} \otimes Q_{ij}' \right) \right) (\Sigma_v \otimes \Sigma_v),
\end{align*}
\]

where \(Q_{ij}\) is a \(m \times m\) zero matrix except for the \((i,j)\)th element which is equal to 1, and \(\otimes\) is the kronecker product operator.

Unconditional distribution of the state vector \(z_t\).

The linearity of the VAR system (4) implies that the state vector \(z_t\) inherits the normality of the shocks \(v_{t+1}\). It has unconditional mean \(\mu_z\) and variance-covariance matrix \(\Sigma_{zz}\) given by

\[
\begin{align*}
  \mu_z &= (I_m - \Phi_1)^{-1} \Phi_0, \\
  \text{vec}(\Sigma_{zz}) &= (I_m^2 - \Phi_1 \otimes \Phi_1)^{-1} \text{vec}(\Sigma_v).
\end{align*}
\]
Derivation of Equation (10)

The log return on the portfolio \( r_{p,t+1} \) is a discrete-time approximation to its continuous-time counterpart. We begin by specifying the return processes for the short-term instrument \( B_t \) and other risky assets \( P_t \) in continuous time:

\[
\frac{dB_t}{B_t} = \mu_b \, dt + \sigma_b \, dW_t,
\]

\[
\frac{dP_t}{P_t} = \mu_t \, dt + \sigma \, dW_t,
\]

where \( \mu_b \) and \( \mu_t \) are the drifts, \( \sigma_b \) and \( \sigma \) are the diffusion, and \( W_t \) is a \( m \)-dimensional standard Brownian motion.\(^{15} \) We allow the drifts to depend on other state variables, but for notational simplicity, we suppress this dependency and simply use the time subscript. Moreover, note that the same \( W_t \) appears in the two equations.

We can obtain the log return on each asset using Ito’s Lemma:

\[
d \log B_t = \left( \frac{dB_t}{B_t} \right) - \frac{1}{2} \left( \sigma_b \sigma_b' \right) dt,
\]

\[
d \log P_{i,t} = \left( \frac{dP_{i,t}}{P_{i,t}} \right) - \frac{1}{2} \left( \sigma_i \sigma_i' \right) dt,
\]

where \( \sigma_i \) is the \( i \)th row of the diffusion matrix \( \sigma \), and \( i = 1, \ldots, n - 1 \).

Let \( V_t \) be the value of the portfolio at time \( t \). We will use \( d \log V_t \) to approximate \( r_{p,t+1} \). By Ito’s Lemma,

\[
d \log V_t = \left( \frac{dV_t}{V_t} \right) - \frac{1}{2} \left( \frac{dV_t}{V_t} \right)^2.
\]

We will now derive these two terms in order:

\[
\frac{dV_t}{V_t} = \alpha_t \left( \frac{dP_t}{P_t} \right) + (1 - \alpha_t \iota) \frac{dB_t}{B_t}
\]

\[
= \alpha_t \left( d \log P_t + \frac{1}{2} [\sigma_i \sigma_i'] dt \right) + (1 - \alpha_t \iota) \left( d \log B_t + \frac{1}{2} (\sigma_b \sigma_b') dt \right)
\]

\[
= \alpha_t \left( d \log P_t - d \log B_t \cdot \iota \right) + d \log B_t
\]

\[
+ \frac{1}{2} \alpha_t \left( [\sigma_i \sigma_i'] - \sigma_b \sigma_b' \cdot \iota \right) dt + \frac{1}{2} \sigma_b \sigma_b' dt,
\]

where \( \iota \) is a \( n \times 1 \) vector of ones and the bracket \([ \cdot ]\) denotes a vector with \( \sigma_i \sigma_i' \) the \( i \)th entry. Next,

\[
\left( \frac{dV_t}{V_t} \right)^2 = \alpha_t \left( d \log P_t - d \log B_t \cdot \iota \right) \left( d \log P_t - d \log B_t \cdot \iota \right)' \alpha_t + \left( d \log B_t \right)^2
\]

\[
+ 2 \alpha_t \left( d \log P_t - d \log B_t \cdot \iota \right) \left( d \log B_t \right) + o \left( dt \right),
\]

\(^{15}\)The dimensions of \( \mu_b, \mu, \sigma_b, \sigma \) are \( 1 \times 1, (n - 1) \times 1, 1 \times m, (n - 1) \times m \), respectively.
where the \( o(dt) \) terms vanish because they involve either \((dt)^2\) or \((dt)(dW_t)\).

Now, from equation (31)–(33) and ignoring \( dt \) terms,
\[
d\log P_t - d\log B_t \cdot \mu = (\sigma - \mu \cdot \sigma_b) dW_t.
\]

Thus,
\[
(d\log P_t - d\log B_t \cdot \mu) (d\log P_t - d\log B_t \cdot \mu)' = (\sigma - \mu \cdot \sigma_b)(\sigma - \mu \cdot \sigma_b)',
\]
\[
(d\log P_t - d\log B_t \cdot \mu)(d\log B_t) = (\sigma - \mu \cdot \sigma_1) \cdot \sigma_b'.
\]

Collecting these results and using our notation for excess returns: \( x_{t+1} = d\log P_t - d\log B_t \cdot \mu, \) \( r_{1,t+1} = d\log (B_t) \) and \( dt = 1, \)
\[
r_{p,t+1} = d\log V_t
\]
\[
= \alpha_i' x_{t+1} + r_{1,t+1} + \frac{1}{2} \alpha_i' ([\sigma_i \sigma_i'] - \sigma_b \sigma_b' \cdot \mu)
\]
\[
- \frac{1}{2} [\alpha_i' (\sigma - \mu \cdot \sigma_b)(\sigma - \mu \cdot \sigma_b)'+ \alpha_i' (\sigma - \mu \cdot \sigma_b) \sigma_b']
\]

Using the notation in the VAR system with the Cholesky decomposition for \( \Sigma = GG' \), \( \sigma_i - \sigma_b \) is equal to the \( i \)th row of \( G, G_i \). Hence,
\[
(\sigma - \mu \cdot \sigma_b)(\sigma - \mu \cdot \sigma_b)' = G_{2,2}G_{2,2}' = \Sigma_{xx},
\]
\[
\sigma_b \sigma_b' = G_1 G_1' = \sigma_1^2,
\]
\[
\sigma_i \sigma_i' = G_i G_i' + \sigma_b G_i' + \sigma_b G_i' + \sigma_b \sigma_b',
\]
\[
[\sigma_i \sigma_i'] = \sigma_2^2 + 2 \sigma_{1x} + \sigma_1^2 \mu,
\]
\[
(\sigma - \mu \cdot \sigma_b) \sigma_b' = G_{2,2}G_1' = \sigma_{1x},
\]

where \( G_{2n} \) denotes the submatrix formed by taking the 2nd to \( n \)th rows of \( G \).

With these terms, the return on the portfolio is
\[
r_{p,t+1} = \alpha_i' x_{t+1} + r_{1,t+1} + \frac{1}{2} \alpha_i' (\sigma_x^2 + 2 \sigma_{1x}) - \frac{1}{2} \alpha_i' \Sigma_{xx} \alpha_i - \alpha_i' \sigma_{1x},
\]
\[
= \alpha_i' x_{t+1} + r_{1,t+1} + \frac{1}{2} \alpha_i' (\sigma_x^2 - \Sigma_{xx} \alpha_i).
\]

**Solving for the Optimal Portfolio Rule.**

Subtracting the log Euler equation (12) with \( i = 1 \) from (12), we obtain
\[
E_t (r_{i,t+1} - r_{1,t+1}) + \frac{1}{2} \text{Var}_t (r_{i,t+1} - r_{1,t+1}) = \text{Cov}_t \left( \frac{\theta}{\psi} \Delta c_{t+1} + (1 - \theta) r_{p,t+1}, r_{i,t+1} \right) - \text{Cov}_t \left( \frac{\theta}{\psi} \Delta c_{t+1} + (1 - \theta) r_{p,t+1}, r_{1,t+1} \right)
\]
\[
- \frac{1}{2} \left( \text{Var}_t (r_{i,t+1}) - \text{Var}_t (r_{1,t+1}) - \text{Var}_t (r_{i,t+1} - r_{1,t+1}) \right).
\]
Using the budget constraint (11) and the trivial identity $\Delta c_{t+1} = (c_{t+1} - w_{t+1}) - (c_t - w_t) + \Delta w_{t+1},$

$$\frac{\theta}{\psi} \Delta c_{t+1} + (1 - \theta) r_{p,t+1} = \frac{\theta}{\psi} (c_{t+1} - w_{t+1}) + \gamma r_{p,t+1} + \text{time terms and constants.}$$

Thus, equation (36) can be written as

$$E_t (r_{i,t+1} - r_{1,t+1}) + \frac{1}{2} \text{Var}_t (r_{i,t+1} - r_{1,t+1}) = \frac{\theta}{\psi} [\sigma_{i,c-w,t} - \sigma_{1,c-w,t}] + \gamma [\sigma_{i,p,t} - \sigma_{1,p,t}]$$

$$- \frac{1}{2} \text{Var}_t (r_{i,t+1}) - \text{Var}_t (r_{1,t+1}) + \text{Var}_t (r_{i,t+1} - r_{1,t+1}).$$

We will derive these terms now.

Using the equation for log return on the portfolio and ignoring time $t$ terms and constants,

$$\sigma_{i,p,t} = \text{Cov}_t (\alpha_t' x_{t+1} + r_{1,t+1}, r_{i,t+1})$$

$$= \alpha_t' (\Sigma_{x,t}^{(i)} + \sigma_{1x}) + \sigma_t^{(i)} + \sigma_{1}^2;$$

$$\sigma_{1,p,t} = \text{Cov}_t (\alpha_t' x_{t+1} + r_{1,t+1}, r_{1,t+1})$$

$$= \alpha_t' \sigma_{1x} + \sigma_1^2.$$

To evaluate the conditional covariances $\sigma_{i,c-w,t}$ and $\sigma_{1,c-w,t}$, we use the conjectured policy rule for the consumption-wealth ratio.

$$\sigma_{i,c-w,t} = \text{Cov}_t (c_{t+1} - w_{t+1} - E_t (c_{t+1} - w_{t+1}), r_{i,t+1} - E_t (r_{i,t+1}))$$

$$= \text{Cov}_t \left( B_1' v_{t+1} + (\Phi_0 + \Phi_1 z_t + v_{t+1})' B_2 (\Phi_0 + \Phi_1 z_t + v_{t+1}, v_{t+1}^2 + v_{t+1} v_{t+1}^2) \right)$$

$$= \text{Cov}_t \left( B_1' v_{t+1} + \Phi_0' B_2 z_t + z_t' \Phi_1 B_2 v_{t+1} + v_{t+1}^2 B_2 \Phi_0 + v_{t+1}^2 B_2 \Phi_1 z_t, v_{t+1}^2 + v_{t+1} v_{t+1}^2 \right)$$

$$= B_1' \left( \Sigma_{v,t}^{(i)} + \Sigma_{v,t}^{(i)} \right) + \Phi_0' B_2 \left( \Sigma_{v,t}^{(i)} + \Sigma_{v,t}^{(i)} \right) + \left( \Sigma_{v,t}^{(i)} + \Sigma_{v,t}^{(i)} \right)' B_2 \Phi_0$$

$$+ z_t' \Phi_1 B_2 \left( \Sigma_{v,t}^{(i)} + \Sigma_{v,t}^{(i)} \right) + \left( \Sigma_{v,t}^{(i)} + \Sigma_{v,t}^{(i)} \right)' B_2 \Phi_1 z_t,$$

where the second equality follows from using Result 1 and 2, and $\Sigma_{v,t}^{(i)}$ denotes the $i$th column of $\Sigma_v$. Note that $B_2$ is not necessarily symmetric, so that we cannot combine any of the terms in the expression above. Similarly, for the return on the short-term bond we have

$$\sigma_{1,c-w,t} = B_1' \Sigma_{v,t}^{(i)} + \Phi_0' B_2 \Sigma_{v,t}^{(i)} + \left( \Sigma_{v,t}^{(i)} \right)' B_2 \Phi_0 + z_t' \Phi_1 B_2 \Sigma_{v,t}^{(i)} + \left( \Sigma_{v,t}^{(i)} \right)' B_2 \Phi_1 z_t.$$

Therefore

$$\sigma_{i,c-w,t} - \sigma_{1,c-w,t} = B_1' \Sigma_{v,t}^{(i)} + (\Phi_0' B_2 + z_t' \Phi_1 B_2) \Sigma_{v,t}^{(i)} + \left( \Sigma_{v,t}^{(i)} \right)' (B_2 \Phi_0 + B_2 \Phi_1 z_t)$$

$$= B_1' \Sigma_{v,t}^{(i)} + (\Phi_0' + z_t' \Phi_1)' (B_2 + B_2) \Sigma_{v,t}^{(i)}$$

$$= \left( \Sigma_{v,t}^{(i)} \right)' B_1 + \left( \Sigma_{v,t}^{(i)} \right)' (B_2 + B_2) (\Phi_0 + \Phi_1 z_t)$$

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Combining the two equations (24) and (38), we obtain a linearized budget constraint (11) into this equation and taking expectations we obtain

where

\[ A \chi \text{ term of state variables.} \]

Solving for the Optimal Consumption Rule.

Solving for the Optimal Consumption Rule.

We derive first equation (26). To derive this equation, note that log consumption growth verifies the following trivial identity: \[ \Delta c_{t+1} = (c_{t+1} - w_{t+1}) - (c_t - w_t) + \Delta w_{t+1}. \] Substituting the log-linearized budget constraint (11) into this equation and taking expectations we obtain

\[ E_t(\Delta c_{t+1}) = E_t(c_{t+1} - w_{t+1}) - (c_t - w_t) + E_t(\Delta w_{t+1}) \]

\[ = E_t(c_{t+1} - w_{t+1}) - (c_t - w_t) + E_t(r_{p,t+1}) + \left(1 - \frac{1}{\rho}\right)(c_t - w_t) + k. \]

Combining the two equations (24) and (38), we obtain a difference equation in \( c_t - w_t \), given in (26).

Next we show that both the expected log return on the wealth portfolio \( E_t r_{p,t+1} \) and the variance term \( \chi_{p,t} \) in equation (24) for expected log consumption growth are quadratic functions of the vector of state variables.

Taking conditional expectations of equation (10) and substituting the portfolio policy rule \( \alpha_t = A_0 + A_1 z_t \),

\[ E_t(r_{p,t+1}) = \alpha'_t E_t(x_{t+1}) + E_t(r_{1,t+1}) + \frac{1}{2} \alpha'_t (\sigma_x - \Sigma_{xx} \alpha_t) \]

\[ = (A'_0 + z'_t A'_1)H_x(\Phi_0 + \Phi_1 z_t) + H_1(\Phi_0 + \Phi_1 z_t) \]

\[ + \frac{1}{2} (A'_0 + z'_t A'_1) \sigma_x - \frac{1}{2} (A'_0 + z'_t A'_1) \Sigma_{xx} (A_0 + A_1 z_t) \]

\[ = \Gamma_0 + \Gamma_1 z_t + \Gamma_2 \text{vec}(z'_t), \]

where

\[ \Gamma_0 = A'_0 H_x \Phi_0 + H_1 \Phi_0 + \frac{1}{2} A'_0 \sigma_x^2 - \frac{1}{2} A'_0 \Sigma_{xx} A_0, \]

\[ \Gamma_1 = \Phi'_0 H'_x A_1 + A'_0 H_x \Phi_1 + H_1 \Phi_1 + \frac{1}{2} \sigma_x A_1 - A'_0 \Sigma_{xx} A_1, \]

\[ \Gamma_2 = \text{vec}(A'_1 H_x \Phi_1)' - \frac{1}{2} \text{vec}(A'_1 \Sigma_{xx} A_1)', \]
and \( H_1 \) and \( H_x \) are selection matrices that select the short-term real interest rate and the vector of excess returns from the full state vector.

We now evaluate the variance term
\[
\chi_{p,t} = \frac{1}{2} \left( \frac{\theta}{\psi} \right) \text{Var}_t (\Delta c_{t+1} - \psi r_{p,t+1}).
\]

Using the trivial identity for \( \Delta c_{t+1} \) and the budget constraint (11), substituting the conjecture for the consumption rule
\[
c_t - w_t = b_0 + B'_1 z_t + z'_t B z_t = b_0 + B'_1 z_t + \text{vec} (B_2)' \text{vec} (z_t z'_t)
\]
and \( \alpha_t = A_0 + A_1 z_t, \) and ignoring time \( t \) terms and constants, we can write the argument of the variance as:
\[
\Delta c_{t+1} - \psi r_{p,t+1} = [B'_1 + \Phi'_0 (B_2 + B'_2) + (1 - \psi) A'_0 H_x + (1 - \psi) H_1] v_{t+1}
\]
\[
+ z'_t [\Phi'_1 (B_2 + B'_2) + (1 - \psi) A'_1 H_x] v_{t+1}
\]
\[
+ \text{vec} (B_2)' \text{vec} (v_{t+1} v'_{t+1})
\]
\[
= [\Pi_1 + z'_t \Pi_2] v_{t+1} + \text{vec} (B_2)' \text{vec} (v_{t+1} v'_{t+1}),
\]
where
\[
\Pi_1 \equiv B'_1 + \Phi'_0 (B_2 + B'_2) + (1 - \psi) A'_0 H_x + (1 - \psi) H_1,
\]
\[
\Pi_2 \equiv \Phi'_1 (B_2 + B'_2) + (1 - \psi) A'_1 H_x.
\]

Since \( v_{t+1} \) is conditionally normally distributed, all third moments are zero. Thus,
\[
\text{Var}_t (\Delta c_{t+1} - \psi r_{p,t+1})
\]
\[
= \Pi_1 \Sigma_v \Pi_1' + [2 \Pi_1 \Sigma_v \Pi_2] z_t + \text{vec} (\Pi_2 \Sigma_v \Pi_2') \text{vec} (z_t z'_t)
\]
\[
+ \text{vec} (B_2)' \text{Var}_t \left( \text{vec} (v_{t+1} v'_{t+1}) \right) \text{vec} (B_2),
\]
and \( \text{Var}_t (\text{vec} (v_{t+1} v'_{t+1})) \) is given by the expression in Result 4 above. Putting these pieces together, we have
\[
\chi_{p,t} = V_0 + V_1 z_t + V_2 \text{vec} (z_t z'_t),
\]
where
\[
V_0 = \frac{\theta}{2 \psi} \left[ \Pi_1 \Sigma_v \Pi_1' + \text{vec} (B_2)' \text{Var}_t (\text{vec} (v_{t+1} v'_{t+1})) \text{vec} (B_2) \right],
\]
\[
V_1 = \frac{\theta}{2 \psi} [2 \Pi_1 \Sigma_v \Pi_2],
\]
\[
V_2 = \frac{\theta}{2 \psi} \left[ \text{vec} (\Pi_2 \Sigma_v \Pi_2') \right].
\]
We can now solve for the coefficients of the optimal consumption rule. Simple substitution of the expressions for \(E_t r_{p,t+1}\) and \(\chi_{p,t}\), and the expression for the conditional expectation of \((c_t + w_{t+1})\) into the RHS of (26) yields
\[
c_t - w_t = \Xi_0 + \Xi_1 z_t + \Xi_2 \text{vec}(z'_t),
\]
where
\[
\Xi_0 = \rho [-\psi \log \delta + k - V_0 + (1 - \psi) \Gamma_0 + b_0 + B'_1 \Phi_0 \\
+ \text{vec}(B_2) \text{vec}(\Phi_0 \Phi'_0) + \text{vec}(B_2) \text{vec}(\Sigma)] ,
\Xi_1 = \rho [-V_1 + (1 - \psi) \Gamma_1 + B'_1 \Phi_1 + 2 \Phi'_0 (B_2 + B_2) \Phi_1] ,
\Xi_2 = \rho [-V_2 + (1 - \psi) \Gamma_2 + \text{vec}(\Phi'_1 B_2 \Phi_1)' ] .
\]
Equation (39) confirms our initial conjecture on the form of the consumption-wealth ratio. Notice that \(\Xi_0, \Xi_1, \Xi_2\) depend on \(b_0, B_1, B_2\). Therefore, for the solution to be consistent, \(\{b_0, B_1, B_2\}\) must solve the following set of equations:
\[
\begin{align*}
b_0 &= \Xi_0, \\
B_1 &= \Xi'_1, \\
\text{vec}(B_2) &= \Xi'_2.
\end{align*}
\]
The resulting set of values for \(b_0, B_1\) and \(\text{vec}(B_2)\) determines the optimal consumption rule.

**Verification that the optimal portfolio rule is independent of \(\psi\) given \(\rho\).**

From equations (22) and (23) in text, and equation (37) in the Appendix, we can write \(A_0\) and \(A_1\) as:
\[
\begin{align*}
A_0 &= \left( \frac{1}{\gamma} \right) \Sigma^{-1}_{xx} \left( H_x \Phi_0 + \frac{1}{2} \sigma^2_x + (1 - \gamma) \sigma_{1x} \right) + \left( 1 - \frac{1}{\gamma} \right) \Sigma^{-1}_{xx} \frac{-A_0}{1 - \psi} \\
&= \left( \frac{1}{\gamma} \right) \Sigma^{-1}_{xx} \left( H_x \Phi_0 + \frac{1}{2} \sigma^2_x + (1 - \gamma) \Sigma \sigma_{1x} \right) - \left( 1 - \frac{1}{\gamma} \right) \Sigma^{-1}_{xx} \left[ (\Sigma \Phi'_x)' \frac{B_1}{1 - \psi} + (\Sigma \Phi'_x)' \frac{B_2 + B_2}{1 - \psi} \Phi_0 \right] \\
A_1 &= \left( \frac{1}{\gamma} \right) \Sigma^{-1}_{xx} H_x \Phi_1 + \left( 1 - \frac{1}{\gamma} \right) \Sigma^{-1}_{xx} \frac{-A_1}{1 - \psi} \\
&= \frac{1}{\gamma} \Sigma^{-1}_{xx} (H_x \Phi_1) - \left( 1 - \frac{1}{\gamma} \right) \Sigma^{-1}_{xx} \left[ (\Sigma \Phi'_x)' \frac{B_2 + B_2}{1 - \psi} \Phi_1 \right] .
\end{align*}
\]
Thus, showing that the optimal portfolio rule is independent of \(\psi\) given \(\rho\) is equivalent to showing that \(B_1 = B_1/(1 - \psi)\) and \(B_2 = B_2/(1 - \psi)\) are independent of \(\psi\) given \(\rho\).

First, consider \(B_2\). From (40), we have
\[
(1 - \psi) \text{vec}(B_2) = \rho [-V'_2 + (1 - \psi) \Gamma'_2 + (1 - \psi) \text{vec}(\Phi'_1 B_2 \Phi_1)] .
\]
Using the definition of $\mathbf{V}_2$, we have

$$-\mathbf{V}_2' = \frac{1 - \gamma}{2(1 - \psi)} \text{vec} \left[ (\mathbf{\Phi}'_1 (\mathbf{B}_2 + \mathbf{B}_2') + (1 - \psi) \mathbf{A}'_1 \mathbf{H}_x) \Sigma_\nu (\mathbf{\Phi}'_1 (\mathbf{B}_2 + \mathbf{B}_2') + (1 - \psi) \mathbf{A}'_1 \mathbf{H}_x)' \right]$$

$$= \frac{1 - \gamma}{2(1 - \psi)} \text{vec} \left[ (1 - \psi)^2 (\mathbf{\Phi}'_1 (\mathbf{B}_2 + \mathbf{B}_2') + \mathbf{A}'_1 \mathbf{H}_x) \Sigma_\nu (\mathbf{\Phi}'_1 (\mathbf{B}_2 + \mathbf{B}_2') + \mathbf{A}'_1 \mathbf{H}_x)' \right]$$

$$= \frac{1 - \gamma}{2(1 - \psi)} \text{vec} \left[ (\mathbf{\Phi}'_1 (\mathbf{B}_2 + \mathbf{B}_2') + \mathbf{A}'_1 \mathbf{H}_x) \Sigma_\nu (\mathbf{\Phi}'_1 (\mathbf{B}_2 + \mathbf{B}_2') + \mathbf{A}'_1 \mathbf{H}_x)' \right]$$

$$\equiv (1 - \psi)\mathbf{\nabla}_2',$$

which is independent of $\psi$, since $\mathbf{A}_1$ does not depend on $\psi$, given $\mathbf{B}_2$.

Similarly, using the definition of $\Gamma_2$, we have:

$$(1 - \psi)\Gamma_2' = (1 - \psi) \left[ \text{vec} (\mathbf{A}'_1 \mathbf{H}_x \mathbf{\Phi}_1) - \frac{1}{2} \text{vec} (\mathbf{A}'_1 \Sigma_{xx} \mathbf{A}_1) \right],$$

which is also independent of $\psi$, since $\mathbf{A}_1$ does not depend on $\psi$, given $\mathbf{B}_2$.

Thus, (41) reduces to

$$\text{vec} (\mathbf{B}_2) = \rho \left[ \mathbf{\nabla}_2' + \Gamma_2' + \text{vec} (\mathbf{\Phi}'_1 \mathbf{B}_2 \mathbf{\Phi}_1) \right].$$

This is a quadratic equation in $\mathbf{B}_2$, whose coefficients do not depend on $\psi$, except for $\rho$. The loglinearization coefficient $\rho = 1 - \exp(E_x[c_t - w_t])$ does depend on $\psi$ indirectly, through the dependence of $E_x[c_t - w_t]$ on $b_0$, $\mathbf{B}_1$ and $\mathbf{B}_2$, which are functions of $\psi$. Consequently, the solution for $\mathbf{B}_2$ will also be independent of $\psi$ given $\rho$.

Using the same logic, we can show that $\mathbf{B}_1$ is independent of $\psi$. From (40) we have

$$(1 - \psi)\mathbf{B}_1 = \rho [ -\mathbf{V}_1' + (1 - \psi) \Gamma_1' + (1 - \psi) \mathbf{\Phi}'_1 \mathbf{B}_1 + (1 - \psi) \mathbf{\Phi}'_1 (\mathbf{B}_2 + \mathbf{B}_2') \mathbf{\Phi}_1].$$

(42)

Now,

$$-\mathbf{V}_1' = \frac{1 - \gamma}{(1 - \psi)} \left[ (1 - \psi) \mathbf{\Pi} \Sigma_\nu (\mathbf{\Phi}'_1 (\mathbf{B}_2 + \mathbf{B}_2') + \mathbf{A}'_1 \mathbf{H}_x)' \right]$$

$$= (1 - \gamma) (1 - \psi) \left[ \mathbf{\Pi} \Sigma_\nu (\mathbf{\Phi}'_1 (\mathbf{B}_2 + \mathbf{B}_2') + \mathbf{A}'_1 \mathbf{H}_x)' \right]$$

$$\equiv (1 - \psi)\mathbf{\nabla}_1'$$

where

$$\mathbf{\Pi}_1 = \mathbf{\Pi}_1' + \mathbf{\Phi}'_0 (\mathbf{B}_2 + \mathbf{B}_2') + \mathbf{A}'_0 \mathbf{H}_x + \mathbf{H}_1.$$  

Both $\mathbf{\nabla}_1$ and $\mathbf{\Pi}_1$ are also independent of $\psi$, since $\mathbf{A}_0$ and $\mathbf{A}_1$ do not depend on $\psi$, given $\mathbf{B}_1$ and $\mathbf{B}_2$.

Also, $\Gamma_1$ is only a function of $\mathbf{B}_1$ and $\mathbf{B}_2$ via its dependence on $\mathbf{A}_0$ and $\mathbf{A}_1$, not of $\psi$. Therefore, (42) becomes

$$\mathbf{B}_1 = \rho \left[ \mathbf{\nabla}_1' + \mathbf{\Phi}'_1 \mathbf{B}_1 + \mathbf{\Phi}'_1 (\mathbf{B}_2 + \mathbf{B}_2') \mathbf{\Phi}_1 \right].$$

8
which again implies that the solution for $B_1$ does not depend on $\psi$ given $\rho$. This completes our proof.

**Value function when $\psi = 1$.**

First, note that we have just proved that $B_1$ and $B_2$ do not depend on $\psi$ given $\rho$. We now derive an expression for the value function when $\psi = 1$. The value function is given by

$$V_t = (1 - \delta) - \frac{\psi}{1 - \psi} \left( \frac{C_t}{W_t} \right)^{1 - \psi}$$

$$= \exp \left\{ - \frac{\psi}{1 - \psi} \log (1 - \delta) + \frac{b_0}{1 - \psi} + \frac{B_1'}{1 - \psi} z_t + \frac{\text{vec} (B_2')}{1 - \psi} \text{vec} (z_t^t) \right\}$$

$$= \exp \left\{ B_0 + B_1' z_t + \text{vec} (B_2') \text{vec} (z_t^t) \right\},$$

where $B_1'$, and $\text{vec} B_2$ are independent of $\psi$ given $\rho$, but $B_0$ does depend on $\psi$.

We now find the limiting expression for $B_0$ when $\psi = 1$:

$$B_0 = - \frac{\psi}{1 - \psi} \log (1 - \delta) + \frac{b_0}{1 - \psi}$$

$$= - \frac{\psi}{1 - \psi} \log (1 - \delta)$$

$$+ \frac{\rho}{1 - \rho} \left[ - \frac{\psi}{1 - \psi} \log \delta + \frac{k}{1 - \psi} - \frac{V_0}{1 - \psi} + \Gamma_0 + \frac{B_1'}{1 - \psi} \Phi_0 + \frac{\text{vec} (B_2')}{1 - \psi} \text{vec} (\Phi_0 \Phi_0^t) + \frac{\text{vec} (B_2')}{1 - \psi} \text{vec} (\Sigma_v) \right]$$

$$= \frac{1}{1 - \psi} \frac{\rho}{1 - \rho} \left[ - \frac{1 - \rho}{\rho} \psi \log (1 - \delta) - \psi \log \delta + k \right]$$

$$+ \frac{\rho}{1 - \rho} \left[ - \frac{V_0}{1 - \psi} + \Gamma_0 + \frac{B_1'}{1 - \psi} \Phi_0 + \frac{\text{vec} (B_2')}{1 - \psi} \text{vec} (\Phi_0 \Phi_0^t) + \frac{\text{vec} (B_2')}{1 - \psi} \text{vec} (\Sigma_v) \right].$$

Substituting $k = \log \rho + ((1 - \rho)/\rho) \log (1 - \rho)$ into the first term of the last equality, and noting that $\rho = \delta$ when $\psi = 1$, we have

$$\frac{1}{1 - \psi} \frac{\rho}{1 - \rho} \left[ - \frac{1 - \rho}{\rho} \psi \log (1 - \delta) - \psi \log \delta + k \right]$$

$$= \frac{1}{1 - \psi} \frac{\rho}{1 - \rho} \left[ - \frac{1 - \rho}{\rho} \psi \log (1 - \delta) - \psi \log \delta + \log \rho + \frac{1 - \rho}{\rho} \log (1 - \rho) \right]$$

$$= \frac{1}{1 - \psi} \frac{\rho}{1 - \rho} \left[ \frac{1 - \rho}{\rho} (1 - \psi) \log (1 - \delta) + (1 - \psi) \log \delta \right]$$

$$= \log (1 - \delta) + \frac{\delta}{1 - \delta} \log \delta,$$

which is independent of $\psi$. 

9
From previous results, we know that all the terms in the second term of the last equality are independent of $\psi$ given $\rho$, except for $V_0/(1-\psi)$. We now verify that this term is also independent of $\psi$ given $\rho$:

$$
V_0 = \frac{V_0}{1-\psi}
= -\frac{1 - \gamma}{2} \left[ \frac{\Pi_1 \Sigma_v \Pi'_1}{(1-\psi)^2} + \frac{\text{vec}(B_2')}{1-\psi} \frac{\text{Var}_t(\cdot)}{1-\psi} \right]
= -\frac{1 - \gamma}{2} \left[ \frac{\Pi_1 \Sigma_v \Pi'_1}{(1-\psi)^2} + \text{vec}(B_2') \frac{\text{Var}_t(\cdot)}{1-\psi} \text{vec}(B_2) \right],
$$

where the second term is independent of $\psi$ given $\rho$, and the first one is also verifiably independent of $\psi$:

$$
\frac{\Pi_1 \Sigma_v \Pi'_1}{(1-\psi)^2} = \left( \frac{B_1'}{1-\psi} + A'_0 H_x + H_1 + \phi'_0 \frac{B_2 + B_2'}{1-\psi} \right) \Sigma_v \left( \frac{B_1'}{1-\psi} + \phi'_0 \frac{B_2 + B_2'}{1-\psi} \right)
= (B_1' + A'_0 H_x + H_1 + \phi'_0 (B_2 + B_2')) \Sigma_v (B_1 + \phi'_0 + B_2 + B_2'),
$$

which does not depend on $\psi$ given $\rho$.

Thus when $\psi = 1$ we have:

$$
B_0 = \log (1-\delta) + \frac{\delta}{1-\delta} \log \delta
+ \frac{\delta}{1-\delta} \left[ -V_0 + \Gamma_0 + B_1' \phi_0 + \text{vec}(B_2) \text{vec}(\phi_0') + \text{vec}(B_2') \text{vec}(\Sigma_v) \right].
$$

Now, since $\rho$ is independent of $\psi$ in the special case $\psi = 1$—because $\rho = \delta$—, we conclude that $B_0$, $B_1$ and $B_2$ are independent of $\psi$ when $\psi = 1$.

**Derivation of $E[V_t]$.**

From the previous section, we learn how to obtain the coefficient matrices $B_0$, $B_1$ and $B_2$. Now we want to evaluate explicitly

$$
E(V_t) = E \left[ \exp \left( B_0 + B_1' z_t + B_2' z_t \right) \right]
$$

where $z_t$ has a multivariate normal distribution

$$
z_t \sim \mathcal{N}(\mu_z, \Sigma_{zz}),
$$

with

$$
\mu_z = (I_m - \Phi_1)^{-1} \Phi_0,
\text{vec}(\Sigma_{zz}) = (I_{m^2} - \Phi_1 \otimes \Phi_1)^{-1} \text{vec}(\Sigma_v).
$$
First, consider a change of variable. Define

\[ h_t = z_t - \mu_z. \]

Then,

\[ B_0 + B'_1 z_t + z'_t B_2 z_t = C_0 + C_1 h_t + h'_t C_2 h_t, \]

where

\[ C_0 \equiv B_0 + B'_1 \mu_z + \mu'_z B_2 \mu_z, \]  \hspace{1cm} (43)
\[ C_1 \equiv B'_1 + 2 \mu'_z B_2, \]  \hspace{1cm} (44)
\[ C_2 \equiv B_2. \]  \hspace{1cm} (45)

To calculate \( E(V_t) \),

\[
E(V_t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp \left( B_0 + B'_1 z_t + z'_t B_2 z_t \right) \\
\times \left( \frac{1}{\sqrt{2\pi}} \left| \Sigma_{zz} \right|^{-\frac{1}{2}} \exp \left( -\frac{1}{2} (z_t - \mu_z)' \Sigma^{-1}_{zz} (z_t - \mu_z) \right) \right) dz_t \cdots dz_{mt}
\]

\[
= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp \left( C_0 + C_1 h_t + h'_t C_2 h_t \right) \\
\times \left( \frac{1}{\sqrt{2\pi}} \left| \Sigma_{zz} \right|^{-\frac{1}{2}} \exp \left( -\frac{1}{2} h'_t \Sigma^{-1}_{zz} h_t \right) \right) dh_t \cdots dh_{mt}
\]

\[
= \frac{1}{\sqrt{2\pi}} \left| \Sigma_{zz} \right|^{-\frac{1}{2}} \cdot \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp \left( C_0 + C_1 h_t - \frac{1}{2} h'_t \left( \Sigma^{-1}_{zz} - 2C_2 \right) h_t \right) dh_t \cdots dh_{mt}
\]

\[
= \frac{1}{\sqrt{2\pi}} \left| \Sigma_{zz} \right|^{-\frac{1}{2}} \cdot \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp \left( C_0 + C_1 h_t - \frac{1}{2} h'_t \left( \Sigma^{-1}_{zz} - 2C_2 \right) h_t \right) dh_t \cdots dh_{mt}
\]

\[
= \frac{1}{\sqrt{2\pi}} \left| \Sigma_{zz} \right|^{-\frac{1}{2}} \cdot \tilde{E} \left( \exp \left( C_0 + C_1 h_t \right) \right),
\]

where the expectation \( \tilde{E} \) is taken as if \( h_t \) is normally distributed with mean zero and covariance matrix \( \left( \Sigma^{-1}_{zz} - 2C_2 \right)^{-1} \). Thus, we immediately have

\[
E(V_t) = \frac{\left| \Sigma_{zz} \right|^{-\frac{1}{2}}}{\left| \left( \Sigma_{zz}^{-1} - 2C_2 \right) \right|^\frac{1}{2}} \cdot \exp \left( C_0 + \frac{1}{2} C_1 \left( \Sigma_{zz}^{-1} - 2C_2 \right)^{-1} C_1^t \right). \]  \hspace{1cm} (46)
Appendix B: Numerical Procedure

Equations (21) and (40) show that the coefficients $\{A_0, A_1\}$, $\{b_0, B_1, \text{vec}(B_2)\}$ in the optimal policy rules are functions of the underlying parameters. When there is one state variable as in Campbell and Viceira (1999), solving explicitly for these coefficients is manageable. However, with multiple state variables, such an exercise is practically impossible. Therefore, we employ a simple numerical procedure to find these coefficients instead.

To find the coefficients of the optimal portfolio rule for each value of $\gamma$, we use the fact that they are independent of $\psi$ given $\rho$. Thus, for each $\gamma$, we fix a value for $\rho$, choose an arbitrary value for $\psi$, and start with some initial values for $\{B_1, \text{vec}(B_2)\}$—denote these by $\{B_1^{(1)}, \text{vec}(B_2)^{(1)}\}$. Through equation (21), this implies a set of values for $\{A_0, A_1\}$—denote these by $\{A_0^{(1)}, A_1^{(1)}\}$.

With $\rho$, $\{A_0^{(1)}, A_1^{(1)}\}$, $\{B_1^{(1)}, \text{vec}(B_2)^{(1)}\}$, we can compute the coefficients $\{\Xi_1, \Xi_2\}$ in the $c - w$ difference equation (39). By equating these coefficients with the $\{B_1, \text{vec}(B_2)\}$ in the conjectured policy function, we have a new set of values for $\{B_1, \text{vec}(B_2)\}$—call them $\{B_1^{(2)}, \text{vec}(B_2)^{(2)}\}$. Since the initial values are arbitrary, $\{B_1^{(2)}, \text{vec}(B_2)^{(2)}\}$ will be different from $\{B_1^{(1)}, \text{vec}(B_2)^{(1)}\}$ in general. Thus, we recompute $\{A_0, A_1\}$ using $\rho$, and $\{B_1^{(2)}, \text{vec}(B_2)^{(2)}\}$ to get $\{A_0^{(2)}, A_1^{(2)}\}$. We obtain then a new set of values for $\{\Xi_1, \Xi_2\}$. We continue until values of $\{B_1, \text{vec}(B_2)\}$, and hence $\{A_0, A_1\}$, converge.

The convergence criterion for $\{B_1, \text{vec}(B_2)\}$ is rather stringent. We first calculate the maximum of the squared deviations of all elements from 2 consecutive iterations. We then require for parameter convergence that the sum of 20 such consecutive maxima be less than 0.00001.

Note once again that coefficients $\{A_0, A_1\}$ are the same for all values of $\psi$ given the log-linearization parameter $\rho$. In the special case $\psi = 1$, we also have that $\rho = \delta$, so that the solution is exact, and choosing a value for $\rho$ is equivalent to choosing a value for $\delta$. When $\psi$ is not equal to one, we implement a recursive procedure similar to the one described in Campbell and Viceira (1999, 2001). Given an initial value of $\rho$, we compute coefficients $\{A_0, A_1\}$, $\{b_0, B_1, \text{vec}(B_2)\}$ using the procedure described above. From $\{b_0, B_1, \text{vec}(B_2)\}$, we can compute $E_t[c_t - w_t]$, and a new value of $\rho$. We iterate until convergence.
C Appendix C: Construction of Hypothetical Real Bonds

Recall that the first element of our VAR system is the ex post real bill return. Therefore, the ex ante log real bill return at time $t + 1$ is the first element of $E_t(z_{t+1}) = \Phi_0 + \Phi_1 z_t$. In other words, the log real yield at time $t$ is given by

$$\hat{y}_{1t} = H_1 \cdot E_t(z_{t+1}) \equiv H_1 \cdot \hat{z}_{t,t+1},$$

where $H_1 \equiv (1, 0, ..., 0)$ and $\hat{z}_{t,t+1} \equiv E_t(z_{t+1})$.

The next step is to assume that the log expectations hypothesis holds for the real term structure; that is,

$$y_{n,t} = \frac{1}{n} \sum_{i=0}^{n-1} E_t(y_{1,t+i}),$$

where $y_{n,t}$ is the log yield on a real bond with maturity $n$. Note that we have implicitly assume that inflation risk premium is zero. An estimate of $y_{n,t}$ can be easily constructed as follows:

$$\hat{y}_{n,t} = \frac{1}{n} \sum_{i=0}^{n-1} \hat{y}_{1,t+i} = \frac{1}{n} \sum_{i=0}^{n-1} H_1 \cdot \hat{z}_{t,t+i+1}.$$

To compute $\hat{z}_{t,t+i+1}$, we can iterate the VAR(1) system forward to get

$$\hat{z}_{t,t+k} = \left( \sum_{j=0}^{k-1} \Phi_1^j \right) \Phi_0 + \Phi_1^k z_t.$$

Using this result, log yield can be expressed as a function of current state variables:

$$\hat{y}_{n,t} = \frac{1}{n} H_1 \sum_{i=1}^{n} \hat{z}_{t,t+i}$$

$$= \frac{1}{n} H_1 \sum_{i=1}^{n} \left[ \left( \sum_{j=0}^{i-1} \Phi_1^j \right) \Phi_0 + \Phi_1^i z_t \right]$$

$$= \frac{1}{n} H_1 (Q_c + Q_n z_t)$$

where

$$Q_n \equiv \sum_{i=1}^{n} \Phi_1^i = \Phi_1 (I_m - \Phi_1)^{-1} (I_m - \Phi_1^n),$$

$$Q_c \equiv (I_m - \Phi_1)^{-1} (I_m - Q_n) \Phi_0,$$
and $I_m$ is the identity matrix, $m = \dim(z_t)$.

Finally, the 1-period return on a hypothetical real $n$-period bond is calculated as

$$r_{n,t+1} = n \hat{y}_{n,t} - (n - 1) \hat{y}_{n-1,t+1} \approx n \hat{y}_{n,t} - (n - 1) \hat{y}_{n,t+1}$$

And the excess return on the hypothetical real $n$-period bond is

$$r_{n,t+1} - r_{1,t+1} = (n \hat{y}_{n,t} - (n - 1) \hat{y}_{n,t+1}) - \hat{y}_{t+1} = \left\{ H_1 (Q_c + Q_n z_t) - \frac{n-1}{n} H_1 (Q_c + Q_n z_{t+1}) \right\} - H_1 (\Phi_0 + \Phi_1 z_t).$$

The next step is to construct a real perpetuity from these zero-coupon bonds. Campbell, Lo and MacKinlay (1997) show how to use a loglinearization framework to construct real perpetuity returns. Specifically, their equations (10.1.16) and (10.1.17) show that the log yield on a real perpetuity or “consol” $y_{c,\infty,t}$ is given by

$$y_{c,\infty,t} = (1 - \rho_c) \sum_{i=0}^{\infty} \rho_c^i r_{c,\infty,t+i+1},$$

where $r_{c,\infty,t+i}$ is the one-period log return on a perpetuity at time $t+i$ and $\rho_c = 1 - \exp(E[-p_{c,t}])$, where $p_{c,t}$ is the log “cum-dividend” price of the perpetuity including its current coupon payout.

Taking conditional expectations at time $t$ and imposing the expectations hypothesis,

$$y_{c,\infty,t} = (1 - \rho_c) \sum_{i=0}^{\infty} \rho_c^i H_1 \tilde{z}_{t,i+1} = H_1 (1 - \rho_c) \left( \sum_{i=0}^{\infty} \rho_c^i \Phi_1^j \right) \Phi_0 + H_1 (1 - \rho_c) \left( \sum_{i=0}^{\infty} \rho_c^i \Phi_1^{i+1} \right) z_t.$$

It is straightforward to show that

$$\sum_{i=0}^{\infty} \rho_c^i \Phi_1^j = \frac{1}{1 - \rho_c} (I_m - \rho_c \Phi_1)^{-1},$$

$$\sum_{i=0}^{\infty} \rho_c^i \Phi_1^{i+1} = (I_m - \rho_c \Phi_1)^{-1} \Phi_1.$$

Thus, the log yield can be expressed as function of the VAR parameters, current state variables and the loglinearization constant $\rho_c$:

$$y_{c,\infty,t} = H_1 (I_m - \rho_c \Phi_1)^{-1} \Phi_0 + H_1 (1 - \rho_c) (I_m - \rho_c \Phi_1)^{-1} \Phi_1 z_t.$$
D Appendix D: Tables
**TABLE A**

Variability of Asset Demands

A: Quarterly Sample (1952Q2 - 1999Q4)

\[ \gamma = 5, \psi = 1, \rho = 0.92^{1/4} \]

<table>
<thead>
<tr>
<th></th>
<th>( \text{Var}(\alpha^m)/\text{Var}(\alpha) ) (%)</th>
<th>( \text{Var}(\alpha^h)/\text{Var}(\alpha) ) (%)</th>
<th>( \text{Cov}(\alpha^m, \alpha^h)/\text{Var}(\alpha) ) (%)</th>
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Percentage of Total Variation Explained By:

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<th></th>
<th>( rtb_t )</th>
<th>( xr_t )</th>
<th>( xb_t )</th>
<th>( y_t )</th>
<th>( (d - p)_t )</th>
<th>( spr_t )</th>
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**Note:** \( rtb_t \) = ex post real T-Bill rate, \( xr_t \) = excess stock return, \( xb_t \) = excess bond return, \( (d - p)_t \) = log dividend-price ratio, \( rb_t \) = relative bill rate, \( spr_t \) = yield spread. The bond is a 5-year nominal bond in the monthly dataset and a 20-year in the annual dataset.
TABLE A (Ctd.)

Variability of Asset Demands

B: Annual Sample (1890-1998)

\[ \gamma = 5, \psi = 1, \rho = 0.92^{1/4} \]

<table>
<thead>
<tr>
<th></th>
<th>( \text{Var}(\alpha^m)/\text{Var}(\alpha) ) (%)</th>
<th>( \text{Var}(\alpha^h)/\text{Var}(\alpha) ) (%)</th>
<th>( \text{Cov}(\alpha^m, \alpha^h)/\text{Var}(\alpha) ) (%)</th>
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Percentage of Total Variation Explained By:

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<th>( y_t )</th>
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<td>-0.76</td>
<td>-1.31</td>
<td>0.82</td>
<td>0.07</td>
<td>1.35</td>
<td>0.00</td>
</tr>
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<td></td>
<td>-13.90</td>
<td>14.86</td>
<td>-30.49</td>
<td>0.62</td>
<td>-3.57</td>
<td>90.88</td>
</tr>
</tbody>
</table>

Note: \( rtb_t \) = ex post real T-Bill rate, \( xr_t \) = excess stock return, \( xb_t \) = excess bond return, \( (d - p)_t \) = log dividend-price ratio, \( rb_t \) = relative bill rate, \( spr_t \) = yield spread. The bond is a 5-year nominal bond in the monthly dataset and a 20-year in the annual dataset.
## TABLE B

VAR Estimation Results
Nominal Bills, Stocks and Real Consol Bond


<table>
<thead>
<tr>
<th>Variable</th>
<th>(rtb_t)</th>
<th>(xr_t)</th>
<th>(xrcb_t)</th>
<th>(y_t)</th>
<th>((d - p)_t)</th>
<th>(spr_t)</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent</td>
<td>((t))</td>
<td>((t))</td>
<td>((t))</td>
<td>((t))</td>
<td>((t))</td>
<td>((t))</td>
<td>((p))</td>
</tr>
</tbody>
</table>

**VAR Estimation Results**

<table>
<thead>
<tr>
<th>(rtb_{t+1})</th>
<th>0.435</th>
<th>0.005</th>
<th>0.015</th>
<th>0.270</th>
<th>-0.001</th>
<th>0.428</th>
<th>0.338</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t)</td>
<td>(6.154)</td>
<td>(0.775)</td>
<td>(0.359)</td>
<td>(3.478)</td>
<td>(-1.173)</td>
<td>(2.261)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(xr_{t+1})</td>
<td>1.866</td>
<td>0.079</td>
<td>0.919</td>
<td>-2.341</td>
<td>0.050</td>
<td>0.201</td>
<td>0.084</td>
</tr>
<tr>
<td>(t)</td>
<td>(1.559)</td>
<td>(0.953)</td>
<td>(1.580)</td>
<td>(-2.627)</td>
<td>(2.404)</td>
<td>(0.077)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>(xrcb_{t+1})</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>(t)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(y_{t+1})</td>
<td>-0.001</td>
<td>0.004</td>
<td>-0.000</td>
<td>0.948</td>
<td>0.000</td>
<td>0.120</td>
<td>0.868</td>
</tr>
<tr>
<td>(t)</td>
<td>(-0.024)</td>
<td>(1.505)</td>
<td>(-0.009)</td>
<td>(18.671)</td>
<td>(0.002)</td>
<td>(1.151)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>((d - p)_{t+1})</td>
<td>-1.972</td>
<td>-0.072</td>
<td>-0.819</td>
<td>1.640</td>
<td>0.959</td>
<td>-0.955</td>
<td>0.932</td>
</tr>
<tr>
<td>(t)</td>
<td>(-1.615)</td>
<td>(-0.803)</td>
<td>(-1.333)</td>
<td>(1.765)</td>
<td>(44.168)</td>
<td>(-0.355)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(spr_{t+1})</td>
<td>0.009</td>
<td>-0.000</td>
<td>0.009</td>
<td>0.026</td>
<td>-0.000</td>
<td>0.743</td>
<td>0.540</td>
</tr>
<tr>
<td>(t)</td>
<td>(0.304)</td>
<td>(-0.045)</td>
<td>(0.586)</td>
<td>(0.815)</td>
<td>(-0.232)</td>
<td>(10.996)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

**Cross-Correlation of Residuals**

<table>
<thead>
<tr>
<th></th>
<th>(rtb)</th>
<th>(xr)</th>
<th>(xrcb)</th>
<th>(y)</th>
<th>((d - p))</th>
<th>(spr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(rtb)</td>
<td>0.551</td>
<td>0.228</td>
<td>-0.465</td>
<td>-0.390</td>
<td>-0.228</td>
<td>0.183</td>
</tr>
<tr>
<td>(xr)</td>
<td>-</td>
<td>7.764</td>
<td>-0.351</td>
<td>-0.164</td>
<td>-0.981</td>
<td>0.023</td>
</tr>
<tr>
<td>(xrcb)</td>
<td>-</td>
<td>-</td>
<td>1.236</td>
<td>-0.408</td>
<td>0.333</td>
<td>0.111</td>
</tr>
<tr>
<td>(y)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.256</td>
<td>0.196</td>
<td>-0.776</td>
</tr>
<tr>
<td>((d - p))</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.946</td>
<td>-0.056</td>
</tr>
<tr>
<td>(spr)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.172</td>
<td>-</td>
</tr>
</tbody>
</table>

**Note:** \(rtb_t\) = ex post real T-Bill rate, \(xr_t\) = excess stock return, \(xrcb_t\) = excess real consol bond return, \((d - p)_t\) = log dividend-price ratio, \(y_t\) = nominal T-bill yield, \(spr_t\) = yield spread. The bond is a 5-year nominal bond in the quarterly dataset and a 20-year for the annual dataset.
TABLE B (Ctd.)
VAR Estimation Results
Nominal Bills, Stocks and Real Consol Bond

B: Annual Sample (1890 - 1998)

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>(rtb_t)</th>
<th>(x_{rt1})</th>
<th>(xrcb_{rt})</th>
<th>(y_t)</th>
<th>((d - p)_t)</th>
<th>(spr_t)</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(rtb_{t+1})</td>
<td>0.309</td>
<td>-0.056</td>
<td>0.000</td>
<td>0.604</td>
<td>-0.009</td>
<td>-0.548</td>
<td>0.235</td>
</tr>
<tr>
<td></td>
<td>(2.299)</td>
<td>(-1.467)</td>
<td>(0.004)</td>
<td>(2.546)</td>
<td>(-0.320)</td>
<td>(-1.081)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(x_{rt1})</td>
<td>-0.096</td>
<td>0.078</td>
<td>-0.211</td>
<td>0.052</td>
<td>0.136</td>
<td>1.682</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td>(-0.270)</td>
<td>(0.650)</td>
<td>(-1.100)</td>
<td>(0.080)</td>
<td>(2.385)</td>
<td>(1.217)</td>
<td>(0.314)</td>
</tr>
<tr>
<td>(xrcb_{t+1})</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(1.000)</td>
</tr>
<tr>
<td>(y_{t+1})</td>
<td>-0.058</td>
<td>-0.013</td>
<td>-0.017</td>
<td>0.897</td>
<td>-0.006</td>
<td>0.098</td>
<td>0.774</td>
</tr>
<tr>
<td></td>
<td>(-2.142)</td>
<td>(-0.270)</td>
<td>(-1.100)</td>
<td>(0.080)</td>
<td>(-1.343)</td>
<td>(0.956)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>((d - p)_{t+1})</td>
<td>-0.362</td>
<td>-0.136</td>
<td>0.192</td>
<td>-0.928</td>
<td>0.826</td>
<td>-1.505</td>
<td>0.719</td>
</tr>
<tr>
<td></td>
<td>(-1.202)</td>
<td>(-1.310)</td>
<td>(1.047)</td>
<td>(-1.498)</td>
<td>(12.505)</td>
<td>(-1.113)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(spr_{t+1})</td>
<td>0.030</td>
<td>0.003</td>
<td>0.011</td>
<td>0.093</td>
<td>0.004</td>
<td>0.767</td>
<td>0.541</td>
</tr>
<tr>
<td></td>
<td>(1.565)</td>
<td>(0.500)</td>
<td>(0.962)</td>
<td>(1.672)</td>
<td>(1.245)</td>
<td>(9.785)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

Cross-Correlation of Residuals

<table>
<thead>
<tr>
<th>(rtb)</th>
<th>(xr)</th>
<th>(xrcb)</th>
<th>(y)</th>
<th>((d - p))</th>
<th>(spr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(rtb)</td>
<td>7.619</td>
<td>-0.169</td>
<td>-0.839</td>
<td>0.126</td>
<td>0.109</td>
</tr>
<tr>
<td>(xr)</td>
<td>-</td>
<td>17.429</td>
<td>0.166</td>
<td>-0.149</td>
<td>-0.721</td>
</tr>
<tr>
<td>(xrcb)</td>
<td>-</td>
<td>-</td>
<td>12.416</td>
<td>-0.579</td>
<td>-0.026</td>
</tr>
<tr>
<td>(y)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.235</td>
<td>0.205</td>
</tr>
<tr>
<td>((d - p))</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>16.104</td>
<td>-0.185</td>
</tr>
<tr>
<td>(spr)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.977</td>
</tr>
</tbody>
</table>

Note: \(rtb_t\) = ex post real T-Bill rate, \(xr_t\) = excess stock return, \(xrcb_t\) = excess real consol bond return, \((d - p)_t\) = log dividend-price ratio, \(y_t\) = nominal T-bill yield, \(spr_t\) = yield spread. The bond is a 5-year nominal bond in the quarterly dataset and a 20-year for the annual dataset.
### TABLE C

**VAR Estimation Results**
Nominal Bills, Stocks, Real Consol Bond, and Nominal Bond


<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>$rtb_t$</th>
<th>$xr_t$</th>
<th>$xnb_t$</th>
<th>$xrcb_t$</th>
<th>$y_t$</th>
<th>$(d-p)_t$</th>
<th>$spr_t$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$rtb_{t+1}$</td>
<td>0.593</td>
<td>0.012</td>
<td>-0.058</td>
<td>0.128</td>
<td>0.236</td>
<td>-0.001</td>
<td>0.409</td>
<td>0.352</td>
</tr>
<tr>
<td></td>
<td>(5.344)</td>
<td>(1.984)</td>
<td>(-1.929)</td>
<td>(1.967)</td>
<td>(3.129)</td>
<td>(-0.721)</td>
<td>(2.178)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$xr_{t+1}$</td>
<td>0.962</td>
<td>0.038</td>
<td>0.329</td>
<td>0.276</td>
<td>-2.148</td>
<td>0.047</td>
<td>0.310</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td>(0.558)</td>
<td>(0.394)</td>
<td>(0.729)</td>
<td>(0.261)</td>
<td>(-2.286)</td>
<td>(2.277)</td>
<td>(0.119)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>$xnb_{t+1}$</td>
<td>0.375</td>
<td>-0.038</td>
<td>-0.189</td>
<td>0.280</td>
<td>0.314</td>
<td>0.003</td>
<td>2.968</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>(0.727)</td>
<td>(-1.473)</td>
<td>(-1.128)</td>
<td>(0.866)</td>
<td>(0.731)</td>
<td>(0.441)</td>
<td>(2.700)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>$xrcb_{t+1}$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(1.000)</td>
<td>(1.000)</td>
</tr>
<tr>
<td>$y_{t+1}$</td>
<td>-0.042</td>
<td>0.002</td>
<td>0.015</td>
<td>-0.029</td>
<td>0.957</td>
<td>-0.000</td>
<td>0.125</td>
<td>0.869</td>
</tr>
<tr>
<td></td>
<td>(-0.769)</td>
<td>(0.816)</td>
<td>(0.778)</td>
<td>(-0.935)</td>
<td>(19.692)</td>
<td>(-0.260)</td>
<td>(1.204)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$(d-p)_{t+1}$</td>
<td>-0.980</td>
<td>-0.027</td>
<td>-0.362</td>
<td>-0.112</td>
<td>1.428</td>
<td>0.962</td>
<td>-1.074</td>
<td>0.932</td>
</tr>
<tr>
<td></td>
<td>(-0.539)</td>
<td>(-0.262)</td>
<td>(-0.757)</td>
<td>(-0.102)</td>
<td>(1.457)</td>
<td>(44.524)</td>
<td>(-0.401)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$spr_{t+1}$</td>
<td>0.019</td>
<td>0.000</td>
<td>-0.004</td>
<td>0.017</td>
<td>0.024</td>
<td>-0.000</td>
<td>0.741</td>
<td>0.540</td>
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<tr>
<td></td>
<td>(0.506)</td>
<td>(0.175)</td>
<td>(-0.298)</td>
<td>(0.802)</td>
<td>(0.779)</td>
<td>(-0.138)</td>
<td>(10.975)</td>
<td>(0.000)</td>
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</table>

**Cross-Correlation of Residuals**

<table>
<thead>
<tr>
<th></th>
<th>$rtb_t$</th>
<th>$xr_t$</th>
<th>$xnb_t$</th>
<th>$xrcb_t$</th>
<th>$y_t$</th>
<th>$(d-p)_t$</th>
<th>$spr_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$rtb_t$</td>
<td>0.545</td>
<td>0.239</td>
<td>0.389</td>
<td>-0.462</td>
<td>-0.384</td>
<td>-0.240</td>
<td>0.181</td>
</tr>
<tr>
<td>$xr_t$</td>
<td>-</td>
<td>7.751</td>
<td>0.228</td>
<td>-0.355</td>
<td>-0.170</td>
<td>-0.981</td>
<td>0.025</td>
</tr>
<tr>
<td>$xnb_t$</td>
<td>-</td>
<td>-</td>
<td>2.670</td>
<td>0.543</td>
<td>-0.764</td>
<td>-0.245</td>
<td>0.195</td>
</tr>
<tr>
<td>$xrcb_t$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.236</td>
<td>-0.414</td>
<td>0.337</td>
<td>0.112</td>
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<td>$y_t$</td>
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<td>-</td>
<td>-</td>
<td>0.255</td>
<td>0.202</td>
<td>-0.776</td>
</tr>
<tr>
<td>$(d-p)_t$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.932</td>
<td>-0.058</td>
</tr>
<tr>
<td>$spr_t$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.172</td>
</tr>
</tbody>
</table>

**Note:**
- $rtb_t$ = ex post real T-Bill rate,
- $xr_t$ = excess stock return,
- $xrcb_t$ = excess real consol bond return,
- $(d-p)_t$ = log dividend-price ratio,
- $y_t$ = nominal T-bill yield,
- $xnb_t$ = excess nominal long bond return,
- $spr_t$ = yield spread. The bond is a 5-year nominal bond in the quarterly dataset and a 20-year for the annual dataset.
### TABLE C (Ctd.)

**VAR Estimation Results**

Nominal Bills, Stocks, Real Consol Bond, and Nominal Bond

#### B: Annual Sample (1890 - 1998)

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>( rtb_t )</th>
<th>( xr_t )</th>
<th>( xnb_t )</th>
<th>( xrcb_t )</th>
<th>( y_t )</th>
<th>( (d-p)_t )</th>
<th>( spr_t )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>(t)</td>
<td>(t)</td>
<td>(t)</td>
<td>(t)</td>
<td>(t)</td>
<td>(t)</td>
<td>(t)</td>
<td>(p)</td>
</tr>
</tbody>
</table>

#### VAR Estimation Results

- \( rtb_{t+1} \)
  - 0.305 (-2.258)
  - 0.052 (0.902)
  - 0.122 (2.380)
  - 0.002 (-0.147)
  - 0.700 (-1.177)
  - 0.004 (0.000)
  - -0.781 (0.059)
  - 0.240 (0.059)

- \( xr_{t+1} \)
  - -0.093 (-0.262)
  - 0.074 (0.616)
  - -0.098 (-0.332)
  - -0.212 (-1.114)
  - -0.025 (-0.036)
  - 0.132 (2.371)
  - 1.869 (1.251)
  - 0.059 (0.305)

- \( xnb_{t+1} \)
  - 0.223 (1.890)
  - 0.106 (2.954)
  - -0.196 (-1.485)
  - 0.023 (0.274)
  - -0.117 (-0.333)
  - 0.012 (5.118)
  - 2.566 (0.393)
  - 0.393 (0.000)

- \( xrcb_{t+1} \)
  - 0.000 (0.000)
  - 0.000 (0.000)
  - 0.000 (0.000)
  - 0.000 (0.000)
  - 0.000 (0.000)
  - 0.000 (1.000)
  - 0.000 (0.000)
  - 0.000 (0.000)

- \( y_{t+1} \)
  - -0.059 (-2.393)
  - -0.012 (-1.746)
  - 0.036 (1.280)
  - -0.017 (1.037)
  - 0.925 (12.568)
  - -0.005 (1.115)
  - 0.029 (0.249)
  - 0.779 (0.000)

- \( (d-p)_{t+1} \)
  - -0.373 (-1.260)
  - -0.124 (-1.174)
  - 0.363 (1.163)
  - 0.197 (1.103)
  - -0.642 (-0.992)
  - 0.840 (13.412)
  - -2.198 (-1.459)
  - 0.723 (0.000)

- \( spr_{t+1} \)
  - 0.030 (1.593)
  - 0.002 (0.415)
  - -0.013 (-0.649)
  - 0.011 (0.953)
  - 0.083 (1.557)
  - 0.004 (1.142)
  - 0.791 (8.432)
  - 0.543 (0.000)

#### Cross-Correlation of Residuals

<table>
<thead>
<tr>
<th></th>
<th>( rtb )</th>
<th>( xr )</th>
<th>( xnb )</th>
<th>( xrcb )</th>
<th>( y )</th>
<th>( (d-p) )</th>
<th>( spr )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( rtb )</td>
<td>7.592</td>
<td>-0.168</td>
<td>-0.020</td>
<td>-0.842</td>
<td>0.115</td>
<td>0.101</td>
<td>-0.155</td>
</tr>
<tr>
<td>( xr )</td>
<td></td>
<td>17.422</td>
<td>-0.017</td>
<td>0.166</td>
<td>-0.146</td>
<td>-0.723</td>
<td>0.195</td>
</tr>
<tr>
<td>( xnb )</td>
<td></td>
<td></td>
<td>5.099</td>
<td>0.257</td>
<td>-0.651</td>
<td>-0.059</td>
<td>0.262</td>
</tr>
<tr>
<td>( xrcb )</td>
<td></td>
<td></td>
<td></td>
<td>12.416</td>
<td>-0.585</td>
<td>-0.026</td>
<td>0.618</td>
</tr>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.221</td>
<td>0.191</td>
<td>-0.894</td>
</tr>
<tr>
<td>( (d-p) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15.996</td>
<td>-0.179</td>
</tr>
<tr>
<td>( spr )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.975</td>
</tr>
</tbody>
</table>

**Note:** \( rtb_t = \) ex post real T-Bill rate, \( xr_t = \) excess stock return, \( xrcb_t = \) excess real consol bond return, \( (d-p)_t = \) log dividend-price ratio, \( y_t = \) nominal T-bill yield, \( xnb_t = \) excess nominal long bond return, \( spr_t = \) yield spread. The bond is a 5-year nominal bond in the quarterly dataset and a 20-year for the annual dataset.
### TABLE D
Mean Asset Demands with Hypothetical Real Bonds
(Annual Sample: 1890 - 1998)

#### A: Nominal Bills, Stocks, and Real Consol Bond

<table>
<thead>
<tr>
<th>State Variables:</th>
<th>Constant</th>
<th>Full VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma = 1, \psi = 1, \rho = 0.92 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td>200.45</td>
<td>226.91</td>
</tr>
<tr>
<td>Real Consol Bond</td>
<td>-54.43</td>
<td>-64.46</td>
</tr>
<tr>
<td>Cash</td>
<td>-46.02</td>
<td>-62.45</td>
</tr>
<tr>
<td>( \gamma = 2, \psi = 1, \rho = 0.92 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td>100.58</td>
<td>140.21</td>
</tr>
<tr>
<td>Real Consol Bond</td>
<td>-1.02</td>
<td>-10.05</td>
</tr>
<tr>
<td>Cash</td>
<td>0.43</td>
<td>-30.16</td>
</tr>
<tr>
<td>( \gamma = 5, \psi = 1, \rho = 0.92 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td>40.67</td>
<td>65.69</td>
</tr>
<tr>
<td>Real Consol Bond</td>
<td>31.03</td>
<td>44.67</td>
</tr>
<tr>
<td>Cash</td>
<td>28.30</td>
<td>-10.36</td>
</tr>
<tr>
<td>( \gamma = 20, \psi = 1, \rho = 0.92 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td>10.71</td>
<td>17.40</td>
</tr>
<tr>
<td>Real Consol Bond</td>
<td>47.06</td>
<td>82.94</td>
</tr>
<tr>
<td>Cash</td>
<td>42.24</td>
<td>-0.34</td>
</tr>
<tr>
<td>( \gamma = 2000, \psi = 1, \rho = 0.92 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td>0.82</td>
<td>-0.84</td>
</tr>
<tr>
<td>Real Consol Bond</td>
<td>52.34</td>
<td>97.86</td>
</tr>
<tr>
<td>Cash</td>
<td>46.84</td>
<td>2.98</td>
</tr>
</tbody>
</table>

**Note:** "Constant" column reports mean asset demands when the VAR system only has a constant in each regression, corresponding to the case in which risk premia are constant and realized returns on all assets, including the short-term real interest rate, are i.i.d. "Full VAR" column reports mean asset demands when the VAR system includes all state variables. The nominal bond is a 5-year nominal bond in the quarterly dataset and a 20-year in the annual dataset.
### TABLE D (ctd.)

**Mean Asset Demands with Hypothetical Real Bonds**  
*(Annual Sample: 1890 - 1998)*

#### B: Nominal Bills, Stocks, Real Consol Bond and Nominal Bond

<table>
<thead>
<tr>
<th>State Variables:</th>
<th>Constant</th>
<th>Full VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 1, \psi = 1, \rho = 0.92$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td>198.39</td>
<td>232.42</td>
</tr>
<tr>
<td>Real Consol Bond</td>
<td>-68.22</td>
<td>-97.56</td>
</tr>
<tr>
<td>Nominal Bond</td>
<td>143.96</td>
<td>301.23</td>
</tr>
<tr>
<td>Cash</td>
<td>-174.13</td>
<td>-336.09</td>
</tr>
</tbody>
</table>

| $\gamma = 2, \psi = 1, \rho = 0.92$ |          |          |
| Stocks           | 99.61    | 137.51   |
| Real Consol Bond | -7.54    | -30.05   |
| Nominal Bond     | 68.06    | 163.36   |
| Cash             | -60.13   | -170.81  |

| $\gamma = 5, \psi = 1, \rho = 0.92$ |          |          |
| Stocks           | 40.34    | 60.01    |
| Real Consol Bond | 28.87    | 39.15    |
| Nominal Bond     | 22.51    | 64.02    |
| Cash             | 8.27     | -63.18   |

| $\gamma = 20, \psi = 1, \rho = 0.92$ |          |          |
| Stocks           | 10.71    | 15.95    |
| Real Consol Bond | 47.08    | 81.05    |
| Nominal Bond     | -0.26    | 19.78    |
| Cash             | 42.47    | -16.79   |

| $\gamma = 2000, \psi = 1, \rho = 0.92$ |          |          |
| Stocks           | 0.93     | 0.61     |
| Real Consol Bond | 53.09    | 95.75    |
| Nominal Bond     | -7.77    | 7.58     |
| Cash             | 53.75    | -3.94    |

**Note:** “Constant” column reports mean asset demands when the VAR system only has a constant in each regression, corresponding to the case in which risk premia are constant and realized returns on all assets, including the short-term real interest rate, are i.i.d. “Full VAR” column reports mean asset demands when the VAR system includes all state variables. The nominal bond is a 5-year nominal bond in the quarterly dataset and a 20-year in the annual dataset.
<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$E[V_t]$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal Bills and Stocks</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.164</td>
</tr>
<tr>
<td>2</td>
<td>0.088</td>
</tr>
<tr>
<td>5</td>
<td>0.050</td>
</tr>
<tr>
<td>20</td>
<td>0.015</td>
</tr>
<tr>
<td>2000</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Nominal Bills, Stocks, and Nominal Bond</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>14.911</td>
</tr>
<tr>
<td>2</td>
<td>0.446</td>
</tr>
<tr>
<td>5</td>
<td>0.086</td>
</tr>
<tr>
<td>20</td>
<td>0.018</td>
</tr>
<tr>
<td>2000</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Nominal Bills, Stocks, and Real Consol Bond</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.175</td>
</tr>
<tr>
<td>2</td>
<td>0.090</td>
</tr>
<tr>
<td>5</td>
<td>0.053</td>
</tr>
<tr>
<td>20</td>
<td>0.038</td>
</tr>
<tr>
<td>2000</td>
<td>0.014</td>
</tr>
<tr>
<td><strong>Nominal Bills, Stocks, Nominal Bond, and Real Consol Bond</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>37.37</td>
</tr>
<tr>
<td>2</td>
<td>0.526</td>
</tr>
<tr>
<td>5</td>
<td>0.095</td>
</tr>
<tr>
<td>20</td>
<td>0.023</td>
</tr>
<tr>
<td>2000</td>
<td>0.014</td>
</tr>
</tbody>
</table>