Firm Dynamics and Growth Measurement in France*

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Abstract

In this paper we use the same methodology as Aghion et al. (2017a) to compute missing growth estimates from creative destruction in France. We find that from 2004 to 2015, about 0.5 percentage point of real output growth per year is missed by the statistical office, which is about the same as what was found in the U.S. We look at how missing growth varies across French sectors and regions, and we look at the underlying establishment and firm dynamics. In particular we show that the similar missing growth estimates between France and the U.S. hide noticeable differences in the market share of entrant and exiting plants between the two countries.

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1 Introduction

In 1938, economist Alvin Hansen explained in his Presidential Address before the American Economic Association that in his opinion, the United States faced inexorable weak growth in the long term which he denoted “secular stagnation” (Hansen, 1939). The nation was just emerging from the Great Depression, and Hansen did not anticipate another World War that would stimulate a rebound in public spending and thereby of aggregate demand. Since then, the world has experienced remarkable growth but also another major crisis in 2007. This “Great Recession” led some economists to revive the expression “secular stagnation” to characterize a situation that they assimilated to the one described by Hansen in 1938 (see Summers, 2014 and Teulings and Baldwin, 2014 for an overview).

In particular for Gordon (2012) the risk of secular stagnation reflects a supply problem, and the age of great innovations is past. Gordon uses the metaphor of a fruit tree to describe the evolution of productivity for the past 150 years: in the same way as low-hanging fruits are easier to catch and more juicy than high-hanging fruits, the second industrial revolution—that of electricity and chemistry—produced a higher productivity wave than the third revolution—the ICT wave. This is illustrated in Figure 1 below, taken from Bergeaud et al. (2016). A similar argument is made by Bloom et al. (2017) who push the view that the secular decline in productivity growth has to do with the fact that in any sector new ideas get harder and harder to find over time.

Schumpeterian economists are more optimistic about the future. A first argument (e.g., see Aghion, 2016) is that the ICT revolution has radically and durably improved IT-producing technology; meanwhile globalization (which was concomitant with the ICT revolution) has substantially increased the potential returns on innovation—hence generating a scale effect—as well as the potential downside of not innovating—hence inducing a competition effect.

A second argument, is that innovation may not be properly reflected in actual measures
Notes: This figure plots the trend of TFP growth rate of the U.S. and the Euro Area since 1890. Trends have been obtained through a HP filter with a penalty parameter calibrated to filter less than 30-year-long cycles. Euro Area is the aggregate of France, Germany, Italy, Spain, Netherlands, Finland, Portugal, and Belgium. See Bergeaud et al. (2016) for more details and sources.

of productivity growth. Already in 1996, the Boskin Commission (Boskin et al., 1996) would describe how bias could arise in the measure of inflation from the direct quality adjustment done when incumbents upgrade their products. This report was widely discussed in France at the time (see, e.g., Lequiller, 1997) and a main conclusion was that the bias associated with incumbent innovation should be significantly lower in France.\footnote{One reason for this, is that in France the weights of the various products in the CPI are readjusted every year which helps reduce the “substitution bias” emphasized by the Boskin Commission. And the French Statistical Office (INSEE) plans to rely more systematically on high frequency scanner data which should reduce this bias even further at least for non-durable goods (see Léonard et al., 2017).}
In this paper, however, we focus on a different source of bias associated with creative destruction, i.e., with quality improvements by new producers who replace incumbent producers. The following Figure 2 helps motivate the idea that creative destruction could partly explain missing growth. Figure 2 depicts over time for U.S. manufacturing industries the relationship between the level of creative destruction and the correlation between TFP growth and the intensity of innovation, as measured by the number of patents on the other hand. More precisely we did the following: each year over the period from 1993 to 2008, we computed the rank-rank correlation between TFP growth and the number of patents per employee, for 26 manufacturing industries. TFP growth has been computed using the NBER-CES manufacturing industry database and the number of patents have been taken directly from the USPTO and correspond to granted patent filed by U.S. companies and inventors. We then measured the level of creative destruction in each year and for each sector by half the sum of job creation and job destruction rates taken from the Quarterly Workforce Indicators series from the Census. Figure 2 shows that the correlation between TFP growth and patenting, is lower in year when creative destruction is higher.²

Why should creative destruction make it harder to fully measure the contribution of innovation to productivity growth? Aghion et al. (2017a), henceforth ABBKL, argue that in sectors where new products replace old ones, the statistical office does not correctly assess how much of the increase in monetary value from the sector is due to inflation versus real productivity growth. The standard procedure in such cases is to assume that the quality-adjusted inflation rate is the same as for other items in the same category that the statistical office can follow over time, i.e., products that are not subject to creative destruction. This procedure is referred to as “imputation” in the U.S.

²Note that a similar figure could be obtained by plotting the average level of creative destruction for each sector over the period against the rank-rank correlation between patents and TFP growth for each sector (instead of doing it for each year).
ABBKL develop a methodology to quantify the bias that arises from relying on imputation to measure U.S. productivity growth in cases of creative destruction. Using the Schumpeterian growth paradigm, ABBKL provide explicit expressions for missing growth from creative destruction and estimate this missing growth to lie between 0.4 and 0.8 percentage point on average per year over the past thirty years in the aggregate U.S. economy. This corresponds to about one fourth to one third of “true” productivity growth that has been missed. Furthermore, ABBKL find no evidence for a clear time trend in this “missing growth” in the U.S.

This growth mismeasurement from creative destruction and imputation is not specific to the U.S., however. In this paper we use the same methodology as in ABBKL to compute missing growth from creative destruction and imputation in France. We find missing growth

Notes: Rank-Rank correlation between TFP growth rate and number of patents per employees has been calculated each year among a set of 26 manufacturing industries using Spearman’s formula. Patents correspond to granted application filed by U.S. companies or inventors and are distributed by the year of application. Creative destruction has been computed as half the sum of job creation and job destruction flow. Data are for 1993–2008.

\[ \text{Notes:} \text{ Rank-Rank correlation between TFP growth rate and number of patents per employees has been calculated each year among a set of 26 manufacturing industries using Spearman’s formula. Patents correspond to granted application filed by U.S. companies or inventors and are distributed by the year of application. Creative destruction has been computed as half the sum of job creation and job destruction flow. Data are for 1993–2008.} \]

For details on PPI and CPI in Europe, see Eurostat (2012); OECD (2002); Ahnert and Kenny (2004) and the ILO and IMF guidelines.
estimates that are remarkably similar between France and the U.S. That missing growth from creative destruction should not be too different between the two countries, is hinted at by Guédès (2004) which looks at the 1998–2003 period. During this period, the monthly rate of item substitutions in the CPI ranges between 4.1 and 4.5% and the average monthly frequency of “non-comparable” substitutions (those from which it is not possible to find a replacement item of comparable quality) ranges between 2.5 and 3.1% in the French CPI. These numbers are quantitatively very similar to their counterparts in the U.S. as reported by ABBKL (e.g., roughly 50% of substitutions are judged to be comparable and substitution happens at similar monthly frequency, see Aghion et al., 2017a, Online Appendix A).

But what is more surprising is that we obtain similar estimates of missing growth from creative destruction in France and the U.S. despite the fact that, as we shall see below, the underlying plant dynamics are markedly different between the two countries.

After thirty years (from 1945 until 1975) over which France was growing faster than the U.S. in terms of per capita GDP capita (3.8% annual growth on average in France vs. 2.1% for the U.S. when the 1945–1950 period is excluded), convergence stopped in the mid 70s; and from 1995 onwards the U.S. economy has grown faster than the French economy (1.4% annual growth in the U.S. vs. 0.9% in France) as shown in Figure 3 below. What happens to this comparison between France and the U.S. when we factor in missing growth from creative destruction in the two countries? Is the gap between U.S. and French GDP growth increased or reduced when adding missing growth? These are among the questions we address in this paper.\(^4\)

\(^4\)And both the BLS and the INSEE rely mostly to imputation to deal with non-comparable substitutions: “In France, we generally estimate the evolution of prices by the evolution of the average price observed among products that are followed and are considered to be in the same variety. The remaining changes are considered to be a quality effect.” (translated from Guédès, 2004).

\(^5\)Some existing literature has documented the state of firm dynamics in France: Picart (2006) shows that a small share of firms are responsible for most of job creations (see also Cette et al., 2017); Picart (2008b) considers the low dynamism of French SMEs and Bacheré (2017) shows that firms with a number of employees between 250 and 5,000 has been creating most jobs from 2009 to 2015. We contribute to this literature by
The remaining part of the paper is organized as follows. Section 2 outlines the theoretical framework and the methodology in ABBKL to compute missing growth from creative destruction for the whole economy. Section 3 presents the data and descriptive statistics and derives the missing growth estimates for France. Section 4 looks at the extent to which the comparison between missing growth estimates in France and the U.S. is mirrored by the comparison between the firm and plant dynamics in the two countries. Finally, Section 5 concludes.

2 The ABBKL Theoretical Setup

In this section we present a simplified version of the formal setup developed by ABBKL to compute aggregate missing growth from creative destruction.

showing new evidence on creation, destruction and growth of French establishment in comparison with the U.S.
2.1 Basic Setup

Time is discrete and in each period the economy is populated by a one-period lived representative consumer who consume a final good. The final good is produced using a CES production technology\(^6\)

\[
Y = \left( \int_0^N \left[ q_\omega y_\omega \right]^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{1}{\sigma-1}}, \tag{1}
\]

where \( y_\omega \) denotes the quantity and \( q_\omega \) the quality of intermediate input \( \omega \) currently used to produce the final good, \( N \) denotes the number of intermediate varieties currently available for producing the final good, and \( \sigma > 1 \) denotes the elasticity of substitution between intermediate inputs.

Each intermediate input \( y_\omega \) is in turn produced one-for-one with labor according to

\[
y_\omega = l_\omega, \tag{2}
\]

where \( l_\omega \) is the amount of labor used to produce intermediate good \( \omega \).

The final good sector is assumed to be competitive, so that each intermediate good is paid its marginal productivity in producing the final good, whereas intermediate producers are monopolistic.

We assume that intermediate producer \( \omega \) charge in equilibrium a price which is a constant markup factor \( \mu > 1 \) times marginal cost, i.e., we have\(^7\)

\[
p_\omega = \mu W, \tag{3}
\]

for all intermediate products \( \omega \).

\(^6\)We remove time subscripts for notational simplicity.

\(^7\)This constant markup factor may be determined by a competitive fringe or may equal the optimal markup in which case we have \( \mu = \frac{\sigma}{\sigma-1} \).
We now introduce (exogenous) innovation that increase the quality of the intermediates products. We consider two types of innovations in this setup.

- **Creative destruction by new entrant (d)**

  At each point in time in any intermediate sector \( \omega \), with exogenous probability \( \lambda_d \in [0, 1) \) a new entrant replaces the incumbent firm in that sector. This induces a quality improvement of the intermediate product by a factor

  \[
  \gamma_d = q_{\omega,t+1}/q_{\omega,t} > 1.
  \]

- **Own improvement by incumbent (i)**

  With exogenous probability \( \lambda_i \in [0, 1) \) a surviving incumbent improves the quality of her intermediate product \( \omega \) by a factor

  \[
  \gamma_i = q_{\omega,t+1}/q_{\omega,t} > 1.
  \]

The main difference between the two type of innovations is that the producer is replaced by a new producer in the case of creative destruction whereas the same producer stays in the market in cases of incumbent own innovation. Furthermore, as we will argue below, the two types of innovations have implications for how the quality adjustment of the new product is done by the statistical office.

AABKL allow for a third type of innovation, namely variety expansion which increases \( N \) over time as a result of new entry. For simplicity, here we shall abstract from this other potential source of growth and take \( N \) to be constant.
2.2 Missing Growth

2.2.1 Missing growth as mismeasured inflation

Let \( M = YP \) denote aggregate \textit{nominal} output (or aggregate expenditure on the final good), where \( P \) is the price index, and \( Y \) denotes real aggregate final output \( Y \). By definition, gross real output growth between \( t \) and \( t + 1 \) satisfies

\[
\frac{Y_{t+1}}{Y_t} = \frac{M_{t+1}}{M_t} \frac{P_t}{P_{t+1}},
\]

where \( \frac{P_t}{P_{t+1}} \) is the inverse of the true (quality-adjusted) inflation rate. We shall assume that nominal output growth \( \frac{M_{t+1}}{M_t} \) is perfectly well measured, in which case growth mismeasurement is entirely due to mismeasured (quality-adjusted) inflation.

More formally, if \( \frac{\hat{P}_t}{\hat{P}_{t+1}} \) denotes measured inverse inflation, then measured real output growth is equal to

\[
\frac{\hat{Y}_{t+1}}{\hat{Y}_t} = \frac{M_{t+1}}{M_t} \frac{\hat{P}_t}{\hat{P}_{t+1}},
\]

so that the rate of "missing growth" at date \( t \) can be expressed as

\[
MG_t = \ln \left( \frac{Y_{t+1}}{Y_t} \right) - \ln \left( \frac{\hat{Y}_{t+1}}{\hat{Y}_t} \right) = \ln \left( \frac{\hat{P}_{t+1}}{\hat{P}_t} \right) - \ln \left( \frac{P_{t+1}}{P_t} \right).
\]  \hspace{1cm} (6)

Thus there will be positive missing growth whenever inflation is being overstated.
2.2.2 True and measured inflation rates

Let us first compute the true inflation rate. First, one can show (see the Appendix A) that the equilibrium aggregate price index is given by

\[ P = \mu W \left( \int_0^N q_\omega^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}. \]  

(7)

Using the above expression for the price index, we can compute the true inflation rate as a function of the frequency and size of the various types of innovations. Namely (see ABBKL for a proof), the true inflation rate is given by:

\[ \frac{P_{t+1}}{P_t} = \frac{W_{t+1}}{W_t} \left[ 1 + \lambda_d \left( \gamma_d^{\sigma-1} - 1 \right) + (1 - \lambda_d) \lambda_i \left( \gamma_i^{1-\sigma} - 1 \right) \right]^{\frac{1}{1-\sigma}}. \]  

(8)

The term \( \lambda_d \left( \gamma_d^{\sigma-1} - 1 \right) \) captures the creative destruction part of the price change; and the term \( (1 - \lambda_d) \lambda_i \left( \gamma_i^{\sigma-1} - 1 \right) \) captures the incumbent’s own quality-improving innovation part of the price change.

We now turn our attention to the measured inflation rate. Assume that the statistical office perfectly observes nominal values such as the nominal output growth \( \frac{M_{t+1}}{M_t} \) and the nominal wage growth \( \frac{W_{t+1}}{W_t} \). Hence, the entire missing growth arises from not properly measuring quality-adjusted price changes of intermediate inputs. Let us also assume that the statistical office knows the production structure including the value of the elasticity of substitution between intermediate inputs \( \sigma \). Yet, the statistical office must find ways to compute inflation rates when new items replace existing items. Here, our main assumption is that the statistical office uses the average price change on all products with a surviving producer to compute the inflation rate from innovation for the overall economy: that is, it computes the aggregate quality-adjusted price growth for the entire economy as being equal to the average price growth over all products that are not subject to creative destruction (i.e., products that are unchanged or products that
are subject to incumbent own innovation). This is in line with what we said in the introduction regarding how statistical agencies deal with product substitution in the CPI.

Under these assumptions, the measured inflation rate is given by:

\[
\frac{\hat{P}_{t+1}}{P_t} = \frac{W_{t+1}}{W_t} \left[ 1 + \lambda_i \left( \gamma_i^{\sigma} - 1 \right) \right]^{\frac{1}{1-\sigma}}.
\] (9)

### 2.2.3 Equilibrium missing growth

Recall that missing growth between \( t \) and \( t + 1 \) is equal to the log-difference between the measured and the true inflation rate (see (6)). Under the above assumptions about innovation processes and the procedure of the statistical office, it is straightforward to show that missing growth is equal to:

\[
MG = \frac{1}{\sigma - 1} \ln \left( 1 + \frac{\lambda_d \left[ \gamma_d^{\sigma} - 1 \right] - \lambda_i \left( \gamma_i^{\sigma} - 1 \right)}{1 + \lambda_i \left( \gamma_i^{\sigma} - 1 \right)} \right).
\] (10)

There is missing growth if the products with a surviving producer are not representative for the quality improvement of products subject to incumbent own innovation. Missing growth may arise since not all the products of surviving producers are subject to incumbent own innovation (\( \lambda_i < 1 \)) and since the step sizes between the two type of innovations may differ (\( \gamma_i \neq \gamma_d \)).

### 2.3 Estimating Missing Growth Using Market Shares

One could use equation (10) to indirectly infer missing growth by first estimating the frequencies and size of the two types of innovations, i.e., \( \lambda_d, \lambda_i, \gamma_d, \gamma_i \). García-Mácia et al. (2016) and

\[^8\text{ABBKL assume that the statistical office perfectly observes frequency, } \lambda_i \text{ and step size, } \gamma_i, \text{ of incumbent own innovation. This amounts to assuming both, that in the case of incumbent own innovation the statistical office implements direct adjustments (e.g., using hedonics or expert judgment), and that these adjustments involve no mistake.}\]
ABBKL use indirect inference to estimate these parameters that relies on detailed information on firms’ employment by age, job creation, job destruction, etc.

However, ABBKL propose an alternative and simpler method to estimate missing growth too: this alternative method uses information on the market shares of entrant establishments (plants), of surviving plants that stay in the market, and of exiters throughout the time period we consider. This method is quite attractive as it allows us to abstract from the details of the innovation process, in particular we do not have to compute the arrival rates and step size of the various types of innovations. Yet, to develop this approach we need to assume that establishments produce a constant number of products so that creative destruction (and brand new varieties) come from entering establishments. In this paper as in ABBKL, we shall proxy market shares of plants using employment, but we will also explore alternative measures like total payroll. Appendix A (as well as ABBKL) gives the technical details about how to match market share of plants to the missing growth expression presented in equation (10).

In a nutshell, let \( L_t \) denote total employment (or payroll) at date \( t \), \( X_t \) denote the employment of continuers at date \( t \), i.e. of the set of plants operating in both periods \( t \) and \( t + 1 \), and \( E_t \) denote the employment of exiting plants at date \( t \). We have by definition

\[
L_t = X_t + E_t.
\]

Similarly, if \( L_{t+1} \) denotes total employment at date \( t + 1 \), \( X_{t+1} \) denote the employment or payroll of continuers from date \( t \) at date \( t + 1 \), and \( F_{t+1} \) denotes the employment or payroll of new entrants (i.e., of new entering plants) at date \( t + 1 \), we have:

\[
L_{t+1} = X_{t+1} + F_{t+1}.
\]

Then, under the assumption of a constant number of products per plant, ABBKL show that
aggregate missing growth from imputation between periods $t$ and $t+1$, can be simply expressed as a function of the growth in market share of continuers between $t$ and $t+1$, namely:\footnote{The proof is reproduced in Appendix A.}

$$MG_{t+1} = \frac{1}{\sigma - 1} \left[ \ln \left( \frac{X_t}{L_t} \right) - \ln \left( \frac{X_{t+1}}{L_{t+1}} \right) \right], \quad (11)$$

where $\frac{X_t}{L_t}$ is the market share of continuers at date $t$ and $\frac{X_{t+1}}{L_{t+1}}$ is the market share of those same continuers at date $t + 1$.

Thus true growth exceeds measured growth (i.e., missing growth is positive) whenever the market share of continuing incumbents shrinks over time. The imputation done by the statistical office is based on information of the continuers. So intuitively, the difference between true growth and measured growth is equal to the difference between true growth and continuers’ average productivity growth. This relative productivity growth cannot directly be observed, but the dynamics in market share reflects information about it. The market share of continuers shrinks between $t$ and $t + 1$ precisely when the average productivity of continuers grows more slowly than the average productivity of the overall economy. Together with an estimate for the elasticity of substitution, $\sigma$, data on market share dynamics can be used to back out the underlying difference in productivity growth.

For our analysis in Section 4 it will be useful to reexpress the above missing growth expression as $MG_{t+1} = \frac{1}{\sigma - 1} \left[ \ln \left( 1 - \frac{E_t}{L_t} \right) - \ln \left( 1 - \frac{F_{t+1}}{L_{t+1}} \right) \right]$. For $\frac{E_t}{L_t}$ and $\frac{F_{t+1}}{L_{t+1}}$ small, we can approximate $MG_{t+1}$ as

$$MG_{t+1} \approx \frac{1}{\sigma - 1} \left[ \frac{F_{t+1}}{L_{t+1}} - \frac{E_t}{L_t} \right]. \quad (12)$$
3 Missing Growth in France

In this section, we implement the approach developed in ABBKL and presented above on French plant data to derive aggregate missing growth estimates for France.

3.1 Data Source

Our data are based on administrative sources and cover all French establishments (plants) from 1993 to 2015. Our main source is the CLAP (“Connaissance Locale de l’Appareil Productif”) dataset from 2003 which we augment with information taken from matched employer employees data, namely the DADS (“Déclaration Annuelle des Donnees Sociales”) dataset to back date to the year 1993. CLAP is constructed using various administrative sources (social security, business registry...) and provides firm-level and plant-level information on employment and wage remuneration for the various types of activities in commercial and non-commercial sectors of the French economy as long as it involves a labor income. CLAP also reports the date of creation of the plant. This is arguably the most reliable and broadest source of information at the establishment level in France but is unfortunately “only” available since 2003. Prior to that year, we rely to aggregate matched employer-employees data at the plant level from the DADS. While this other database is very accurate when it comes to worker level information, it is less complete than CLAP for plant level information. In particular, DADS only reports employment in total headcount at the end of the year.\footnote{Note that the U.S. Census’s Longitudinal Business Database (LBD) also measures employment by headcount.} For this reason, our baseline results will consider the period 2004–2015.

We restrict attention to establishments from non-farm business sectors for consistency with the U.S. Census’s Longitudinal Business Database (LBD). For each establishment in our datasets and each year, we have information on the precise location of the establishment, its
date of registration, the size of its workforce (i.e., employment by the plant), its total payroll and the firm’s value added. We do not consider individual firms,\textsuperscript{11} except when they involve some kind of labor income and the results are therefore unaffected by the numerous changes in regulation and incentives since 2008 associated with the introduction of the “auto-entrepreneur” status (see, e.g., Aghion et al., 2017b for more details on individual firms). Our last data cleaning operation is to delete from our sample plants with specific juridical category such as bailiffs, legal entity under public law subject to commercial law (transport activity), legal person and body subject to administrative law (water supply, university refectory) and mutual funds.

We also drop from the sample plants that reports a workforce of zero both in terms of full time equivalents and in terms of total headcount. Table 1 shows the number of plants in our final sample each year, along with plants’ average employment size.\textsuperscript{12}

### 3.2 Measuring the Market Share Growth of Continuers

From this database, we can infer \(E_t\) and \(F_{t+1}\) as well as \(L_t\) and \(L_{t+1}\) from information on plants’ employment shares.\textsuperscript{13} More precisely, let \(B\) denote the first period of operation and \(D\) denote the last year of operation of a plant. Then, let \(L(t, B \leq b, D \geq d)\) denote the total employment or payroll in period \(t\) of plants who were born before or in period \(b\) and die in period \(d\) or after.

\textsuperscript{11} Individual firms (“entreprises individuelles”) are firms that are owned by individuals that bear all the legal responsibilities associated with the business. Individual firms typically includes craftsmanship. Such firms can have employees but they remain typically small for fiscal reasons.

\textsuperscript{12} On average 61\% of French establishments in the non-farm business sector have an employment equals to zero over the period 2003–2015. This high proportion is mainly attributable to non salaried individuals owning their companies.

\textsuperscript{13} We will also explore alternative proxies for the market share of plants based on payroll and value added.
Using previous notations, we therefore have:

\[
E_t = L(t, B \leq t, D = t);
\]

\[
F_{t+1} = L(t + 1, B = t + 1, D \geq t + 1);
\]

\[
L_t = L(t, B \leq t, D > t);
\]  \hspace{1cm} (13)

Which are sufficient to estimate missing growth through equation (12). How do we measure these quantities? A natural way is to map $t$ to the data year, $B$ to the first year the plant appears in the dataset and $D$ to the last year the plant appears in the dataset. This would
implicitly assume that entry and exit in our data correspond to entry and exit in the market. However, in practice entering the database does not necessarily mean fully entering the market, e.g., because it may take time for firms to accumulate customers and market share and recruit workers. Also, some establishments may appear in the database even during the development phase of their products. Hence, the mapping between the model and the data is likely to be more accurate if we consider a plant to be an entrant a few years after the firm has appeared in the database.

This in turn calls for mapping $B$ into a year in the dataset plus $k$ years of lag, where $k > 0$. Concretely, as in ABBKL we remove from the database all plants that are less than $k$ year old. An entrant is therefore defined as a firm of $k$ years old. Another assumption is that when a plant stops its activity, it immediately exits the dataset. This is not always true in practice and some plants can survive in the data for many years with 0 employment. In that case, we consider $D$ to be the last year with positive employment for the plant. Because of the truncation of our data, we have to assume that these plant do not reenter after showing 0 employment. Reentry is a relatively rare event in French data, just like in the U.S. LBD, and whenever this happens, we delete the establishment from the database.\footnote{More generally, entry and exit in the database do not necessarily correspond to actual creation and destruction of plants. Alternative reasons include relocation and acquisition which both generate a change in the establishment identifier. This issue is inherent to all establishment data where an establishment is usually defined as the combination of a firm identifier and an address. Thus over the 2003–2004 period, more than 60% of entries in our database correspond to actual creations. Note however that each relocation and acquisition simply adds one entry and one exit. Thus from equation (12) this does not affect our missing growth estimate, provided that the employment growth of affected plants is not systematically above or below average.}

### 3.3 Results

Figure 4 shows the evolution of mean employment growth as a function of the plant’s age, based on our full data sample. We see that employment growth is extraordinarily high for young plants but stabilizes rapidly after about 5 years. This in turn justifies to focus on the
market share dynamics of mature plants and by setting $k$ equal to 5 years to compute missing growth, as ABBKL do when computing missing growth for the U.S.

FIGURE 4: AVERAGE EMPLOYMENT GROWTH BY AGE

Notes: Employment growth is computed by establishments age groups, each year, using full-time equivalent employment. Results are then averaged over all years from 2004 to 2015.

Table 2 shows the missing growth estimates using the employment share of continuing plants for our baseline period 2004–2015. We consider this period as our baseline since over that period we can directly use the CLAP database which we consider to be more accurate. We set $\sigma = 4$ in addition to taking $k = 5$. This choice of the elasticity of substitution is consistent with ABBKL and in line with the median value across producers within a same product category from Hottman et al. (2016). To measure the market share, we here use employment at the plant level which we measure first using a full time equivalent count (column 1) and then by using total headcount at the end of the year (column 2). On average over the baseline period, yearly missing growth is around 0.5 percentage points which is in the same order of magnitude as what ABBKL find for the U.S. For a more precise comparison between France and the U.S., we refer the reader to the second row which focuses on the 2006–2013 period as in ABBKL,
and on the second column which uses headcount as in ABBKL. In this case, missing growth in France is equal to 0.60 which is remarkably close to the corresponding missing growth estimate for the U.S. over the same period, namely 0.74 (see Aghion et al., 2017a, Table 1).

In the other columns of Table 2, we measure as a robustness check the market share using different proxies. Column 3 uses total payroll which yields larger missing growth estimates than using employment, a result that ABBKL also reports (see Aghion et al., 2017a, Table 2). Then, in column 4, we estimate the value added of the plant splitting the firm’s total value added\(^{15}\) between that of each of its establishments weighted by their employment size. For the vast majority of single-establishment firms (94% of firms on average, over 2003–2015), this is equivalent to considering the exact value added of the plant. For the remaining firms, this uses the assumption of a constant level of productivity across all its plants. Finally, in column 5, we measure the intensive margin of employment by hours worked which we compute using the DADS dataset.\(^{16}\) The resulting estimates of missing growth are very close to what we find in our baseline results presented in the first column of Table 2.

In light of the results presented in Table 2, we can confidently assume that our estimate of missing growth lies around 0.5pp per year on average from 2004 to 2015. Over this period, measured TFP growth in France was on average 1% per year\(^{17}\) which means that missing growth represented about a third of total “true” growth. Next, in Tables 3 and 4, we explore the sensitivity of our baseline estimate to the choice of parameters \(k\) and \(\sigma\). We let the values of \(k\) vary from 3 to 7 and the values of \(\sigma\) vary from 3 to 5. While the effect of an increase in \(\sigma\) is clearly predictable from equation (11), the effect of an increase in \(k\) is somewhat harder

\(^{15}\)Value added at the firm level has been computed using the INSEE firm level balance sheet dataset: FICUS/FARE.

\(^{16}\)In theory, this should be equivalent to using full-time equivalent. The difference is that full-time equivalent weight employment by working time only up to a given number of hours worked (35 hours per week), above which its weight is assumed to be 1.

\(^{17}\)This estimation use TFP data computed by the Bank of France to which we expressed a labor augmenting terms. Note that because of methodological differences in the TFP calculation this number is not one to one comparable to the one in ABBKL which is based on the mutifactor TFP series calculated by the BLS.
to predict. Yet we see from Table 3 that missing growth increases slightly with $k$ and reaches 0.66 percentage points per year on average from 2004 to 2015 when $k$ is set to 7.

**TABLE 2: Missing growth at the plant level**

<table>
<thead>
<tr>
<th></th>
<th>FTE</th>
<th>Headcount</th>
<th>Payroll</th>
<th>Value Added</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004–2015</td>
<td>0.46</td>
<td>0.59</td>
<td>0.70</td>
<td>0.49</td>
<td>0.48</td>
</tr>
<tr>
<td>2006–2013</td>
<td>0.42</td>
<td>0.60</td>
<td>0.61</td>
<td>0.58</td>
<td>0.39</td>
</tr>
</tbody>
</table>

*Notes: Entries are percentage points per year. $\sigma = 4$, $k = 5$. Data for total hours worked are not available in 2015 so column 5 stops in 2014.*

Next, we extend our results by moving back in time, using information drawn from the DADS, as explained previously. The DADS is matched employer employee dataset and is not originally dedicated to be a register of all the plants in France. Therefore we cannot consider the estimations over the period before 2004 to be as accurate as those for the 2004–2015 period. In any case, Table 5 presents average yearly missing growth in percentage point for the whole 1994–2015 period. We find an average yearly missing growth estimate of 0.46 using end of year headcount as the measure for market share.

For the sake of comparison with ABBKL, we also average yearly missing growth estimates over the 1996–2005 time period: we find an average yearly missing growth estimate of 0.38pp, compared to 0.55 in the U.S. (see Aghion et al., 2017a, Table 1). Over this period, yearly TFP growth is roughly equal to 2% on average in France which suggests that missing growth represents about a fifth of total “true” productivity growth.

So far, we have computed missing growth for the whole non-farm business economy. We now take a look at missing growth at the sectoral level. We thus split the whole economy into 10 broad industry groups and compute missing growth within each of these groups. Table 6
TABLE 3: Missing growth, at different values of $k$

<table>
<thead>
<tr>
<th></th>
<th>$k = 3$</th>
<th>$k = 5$</th>
<th>$k = 7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004–2015</td>
<td>0.23</td>
<td>0.46</td>
<td>0.66</td>
</tr>
<tr>
<td>2006–2013</td>
<td>0.24</td>
<td>0.42</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Notes: Entries are percentage points per year. $\sigma = 4$ and market share is measured using full-time equivalents.

TABLE 4: Missing growth, at different values of $\sigma$

<table>
<thead>
<tr>
<th></th>
<th>$\sigma = 5$</th>
<th>$\sigma = 4$</th>
<th>$\sigma = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004–2015</td>
<td>0.34</td>
<td>0.46</td>
<td>0.68</td>
</tr>
<tr>
<td>2006–2013</td>
<td>0.32</td>
<td>0.42</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Notes: Entries are percentage points per year. $k = 5$ and market share is measured using full-time equivalents.

reports the results for missing growth on average from 2004 to 2012 in column 1. In particular, Table 6 shows that missing growth figures are lower for manufacturing than for other sectors, as this was also the case in the U.S. (see Aghion et al., 2017a, Table 5). It also shows that our results for the whole non-farm business economy is not driven by one particular sector. Next, we compute an estimate of the average level of creative destruction in each of these sectors over the 2004–2012 period. Here creative destruction is measured as the sum of entry and exit rates of plants divided by 2. We see that missing growth tends to be higher in sectors with more creative destruction.

We next look how missing growth is geographically distributed across French regions. Indeed, CLAP reports the location of each plants and it is therefore straightforward to compute

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18The reason we stop in 2012 is because of a change in the sectoral classification that occurred in 2008. The CLAP database continued to report the previous classification but only up to 2012. It would be possible to use a crosswalk to update the results up to 2015 but this would also add some noise.
TABLE 5: Missing growth, long run

<table>
<thead>
<tr>
<th>Period</th>
<th>Missing Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994–2015</td>
<td>0.46</td>
</tr>
<tr>
<td>1996–2005</td>
<td>0.38</td>
</tr>
<tr>
<td>2006–2013</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Notes: Entries are percentage points per year. $\sigma = 4$, $k = 5$ and market share is measured using total headcount. The period subdivision is reduced to 1996-2005 and 2006-2013 for the sake of comparison with U.S. estimates from Aghion et al., 2017a, Table 1.

missing growth estimates locally. Figures 5(a) and 5(b) show how missing growth is geographically distributed across French regions and French “departements”.\(^{19}\) We find that missing growth is higher in the Cote-D’Azur region, in the northern Atlantic coast and around Toulouse and Lyon. Interestingly, these are regions where creative destruction is higher (the correlation between creative destruction and missing growth at the “departement” level is reported in Figure 6). In addition, these are regions in which measured productivity growth is higher than in the rest of France, which suggests that territorial inequality in terms of economic development is even worse than what existing growth measures suggest. One notable exception is the city of Paris which show a very small amount of missing growth (despite a high level of creative destruction). This is due to the fact that Paris experiences a high entry rate of plants but also a high exit rate, so that the market share of continuers remains rather stable over time.

Finally, in Table 7 we compare missing growth figures using plants’ employment shares with missing growth estimates using firms’ employment shares (in each case we consider both, full-time employment equivalents and headcounts on December 31 to measure employment). We

\(^{19}\)There are 22 regions in mainland France in the period we consider, and 96 départements.
TABLE 6: MISSING GROWTH, BY INDUSTRY

<table>
<thead>
<tr>
<th>Industry</th>
<th>Missing Growth</th>
<th>Creative Destruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extractive industry</td>
<td>0.10</td>
<td>7.8</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.04</td>
<td>9.3</td>
</tr>
<tr>
<td>Construction</td>
<td>0.40</td>
<td>12.2</td>
</tr>
<tr>
<td>Retail</td>
<td>0.78</td>
<td>10.9</td>
</tr>
<tr>
<td>Hotels, restaurants</td>
<td>0.75</td>
<td>11.0</td>
</tr>
<tr>
<td>Logistic &amp; Communication</td>
<td>0.36</td>
<td>11.8</td>
</tr>
<tr>
<td>Finance</td>
<td>0.66</td>
<td>10.9</td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.71</td>
<td>14.7</td>
</tr>
<tr>
<td>Health</td>
<td>0.18</td>
<td>9.7</td>
</tr>
<tr>
<td>Social and personal services</td>
<td>0.74</td>
<td>11.7</td>
</tr>
</tbody>
</table>

Notes: Missing growth is given in percentage points per year and is measured using $k = 5$, $\sigma = 4$ and market share is measured using full-time equivalents. The period considered is 2004–2012 due to change in the definition of sectors. Over this period, the missing growth for the whole non-farm business sector is 0.48 percentage point. Creative Destruction is the average of entry and exit rate of plants.

FIGURE 5: MISSING GROWTH BY GEOGRAPHICAL AREAS

Notes: Missing growth is computed in percentage point per year. $\sigma = 4$ and $k = 5$. Market share is measured using full-time equivalent employment. The period considered is 2004–2015.
FIGURE 6: CORRELATION BETWEEN MISSING GROWTH AND CREATIVE DESTRUCTION

Notes: Missing growth has been computed at the departement level and corresponds to value presented in Figure 5(b) while creative destruction is defined as half the sum of entry and exit rates of plants. Both are taken as average over the period 2004–2015. Paris is “departement” number 75. A complete list of “departements” and their corresponding numbers can be found from the INSEE.

see that missing growth estimates using firms’ employment shares are lower than when using plants’ employment shares, as also found in ABBKL when looking at U.S. firms and plants.

TABLE 7: MISSING GROWTH, FIRMS VS. PLANTS

<table>
<thead>
<tr>
<th></th>
<th>Firms</th>
<th></th>
<th>Plants</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FTE</td>
<td>Headcount</td>
<td>FTE</td>
<td>Headcount</td>
</tr>
<tr>
<td>2004–2015</td>
<td>0.17</td>
<td>0.10</td>
<td>0.46</td>
<td>0.59</td>
</tr>
<tr>
<td>2006–2013</td>
<td>0.20</td>
<td>0.11</td>
<td>0.42</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Notes: Entries are percentage points per year. $\sigma = 4$ and $k = 5$. 

25
4 The Underlying Plant Dynamics in France vs. the U.S.

In the previous section we showed that missing growth in France is sizable, at about 0.5 percentage point per year on average, and of comparable magnitude as missing growth in the U.S. Recall that when restricting attention to $\sigma = 4$ and $k = 5$, the missing growth estimate we found for France based on headcount employment was equal to 0.60 on average between 2006 and 2013. Using the same methodology, ABBKL found a missing growth estimate only slightly higher and equal to 0.74. At the same time average yearly measured growth was lower in France than in the U.S. over that period (and this was also true for the longer 1996–2013 time period) so that missing growth represents a higher share of total GDP growth in France than in the U.S. Yet it is remarkable that missing growth in the two countries be so close to each other. Does this mean that the underlying plant dynamics is similar across the two countries? In fact we will see in this section that the answer is no: namely, both the market share $F_{t+1} L_{t+1}$ of future entrants and the market share $E_{t} L_{t}$ of past exiters differ markedly between France and the U.S., but these differences end up canceling each other out, thereby leading to similar missing growth estimates in the two countries (see equation (12)).

4.1 The Age-Size Nexus

Figure 7 depicts the employment share of plants by age on average over the period 2003–2015, respectively for France (in blue) and the U.S. (in red). Data for the U.S. have been directly drawn from the Census’s Business Dynamics Statistics (BDS) and are based on the work of John Haltiwanger, Javier Miranda and Ron Jarmin, among others (see, e.g., Haltiwanger et al., 2013). We see that young plants account for a higher share of employment in France, especially in the case of plants younger than 5 years. On the other hand, older plants account for a higher share of employment in the U.S. This figure pools all establishments over all years and aggregate them by age groups. Figure 7 clearly shows that past a certain age the average plant
size grows faster with age in the U.S. than in France, and that the U.S. economy has larger old plants than France.

FIGURE 7: EMPLOYMENT SHARE OF PLANTS BY AGE

![Employment Share of Plants by Age](image)

**Notes:** Employment share by age group of establishments is computed each year using total headcount. Results are then averaged over the period 2003–2015.

Figure 8 provides additional supporting evidence of these findings by reproducing the same kind of exercise as in Hsieh and Klenow (2014). More specifically, we plot the average size for establishment at different ages in the cross-section. We see that up to age 21–25, the life cycle of establishments is rather similar between the two countries. However, it differs quite dramatically for firms that are older than 26 years. A more dynamic way of showing the link between size and age can be found in Appendix B, Figure B2 where we only consider establishments born in 1993 which we follow over time up to year 2015.

The fact that average plants’ employment grows faster with age in the U.S. compared to

---

20 Note that the analysis in Figure 1 in Hsieh and Klenow (2014) is done on manufacturing plants only.
21 Figure B1 in Appendix B reproduces the same graph for France but extending up to 40 year old. We see that there is also a discontinuity for the older establishments, but these represents a small part of the total economy, especially compared to the U.S.
Notes: Average Employment by age group of establishments is computed each year using total headcount and normalized to 1 for the youngest age group (similar to Hsieh and Klenow (2014, Figure 1). Results are then averaged over all years from 2003 to 2015.

France, is likely to reflect both: (i) that promising businesses have better investment and financing opportunities in the U.S., and (ii) that the selection process towards plants with high growth potential operates more efficiently in the U.S. than in France. All in all, this suggests that misallocation should be larger in France than in the U.S. (see Hsieh and Klenow, 2009). This in turn could be due to a number of factors that have been widely documented: these include the higher degree of market frictions in France (as reflected in the OECD Indicators of Product Market Regulation), firm size regulation in France, (Garicano et al., 2016), labor market adjustments (Ridder and Berg, 2003; Picart, 2008a), entry regulation (Bertrand and Kramarz, 2002) and even corporate real-estate frictions (Bergeaud and Ray, 2017). All these frictions hinder firms and establishments dynamics in France.
4.2 Entry, Exit, and Missing Growth

The evidence shown above suggests that the market share of entrants is higher in France than in the U.S. However, missing growth figures are comparable between the two countries, which in turn suggests that the market share of exiters should also be higher in France.

In Figure 9 we computed the average exit rate for all years between 2003 and 2015 at different age bins. First, we see that French plants exit more often at all age. See also Figure B3 in the Appendix B, which considers all firms that were born in 1993 and compute the exit rate of survivors every year in our sample period until 2014.

![FIGURE 9: Exit rate of Plants by Age](image)

**Notes:** Exit rate by age group of establishments is computed each year using total headcount. Results are taken indifferently all years from 2003 to 2015.

Next, one can compare the market share of exiters between France and the US over the period 2006–2013. We find the following results

\[
\left( \frac{E_t}{L_t} \right)_{FR} = 0.065 > \left( \frac{E_t}{L_t} \right)_{US} \approx 0.033.
\]
What about entry rates? We can show that employment share $\frac{F_{t+1}}{L_{t+1}}$ of very young firms is indeed higher in France than in the U.S. More precisely, on average over the period 2006–2013 we have

$$\left( \frac{F_{t+1}}{L_{t+1}} \right)_{FR} = 0.082 > \left( \frac{F_{t+1}}{L_{t+1}} \right)_{US} \approx 0.051,$$

where the U.S. numbers are computed using the Census’s BDS. Recall that entry is defined as an establishment reaching the age of 5. Thus overall, both the employment rate of new entrants and that of exiters are higher in France than in the U.S., but the difference $F_{t+1} - E_t$ which by (12) translates into missing growth estimates is similar between France and the U.S.

This similarity in missing growth estimates means that the relative market share of continuers decreases at a relatively similar pace in the two countries. Yet the employment share of entrants and exiters is larger in France than in the U.S.

Note that these comparisons between France and the U.S. involve the relative *market share* of entrants vs. exiters. It does not mean that entry and exit rates are necessarily larger in France than in the U.S. In fact, entry and exit rates of plants in the two countries are of the same order of magnitude, as seen in Figures 10(a) and 10(b).
FIGURE 10: ENTRY/EXIT RATE IN FRANCE AND THE U.S.

(a) France

(b) United States

Notes: Entry rate is defined as the number of new establishments at t divided by the stock of establishments at t. Exit rate is defined as the number of establishments that disappeared at t divided by the stock of establishments at t. Net Entry Rate is the difference between the two.

5 Conclusion

In this paper we used plant-level information to compute missing growth estimates for France. We first found that in absolute terms missing growth from imputation is slightly lower in France than in the U.S.: this implies that, if anything, the growth differential between the U.S. and France over the past fifteen years has been (slightly) underestimated by the statistics. Second, while in the U.S. between a third and a fourth of true productivity growth was missed according to ABBKL, in France missing growth from imputation is closer to one third of total true productivity growth between 2004 and 2015. Third, we found that missing growth is higher in French sectors or regions with higher rates of creative destruction and with higher measured growth rates, suggesting that geographical economic dynamism differences could have been underestimated in the case of France. Fourth, we found that the similarity between French
and U.S. in terms of missing growth hides differences in the market shares of new entrants and exiters between those two countries: both the employment share of new entrants and that of exiters are higher in France than in the U.S., but the differences in these employment shares between the two countries (almost) cancel each other out when computing missing growth estimates.

The analysis in this paper suggests several avenues for future work. This paper includes replication of the model developed in ABBKL which could be easily extended to other countries. Preliminary results in the UK and Japan show that missing growth seem to lie within the same range in these countries, and similar computations are ongoing in Italy and Sweden. Another extension would be to look at how the French Statistical Office could improve its measurement of productivity growth. Finally, our results have implications for growth, fiscal and labor market policies and their impact on growth. For example minimum wage policy uses estimates of the yearly level of inflation. Should the fact that inflation is overestimated, lead us to revisit minimum wage policy?
References


A Theoretical Appendix

In this Appendix we explain how ABBKL derive the expression for missing growth as a function of the growth in the market share of continuers over time.

First, using the fact that, in each period, the final good sector maximizes current final output $Y$ with respect to $\{y_\omega\}_{\omega=0}^N$ subject to $M = \int_0^N y_\omega p_\omega d\omega$, where $M$ is current aggregate nominal expenditure, we can easily show that in equilibrium the demand for an intermediate product $y_\omega$ sold at price $p_\omega$ by the final good sector, is given by

$$y_\omega = q_\omega^{\sigma-1} \left[ \frac{P}{p_\omega} \right]^{\frac{\sigma}{1-\sigma}} \frac{M}{P},$$  \hspace{1cm} (A1)

where the aggregate price index is in turn given by:\textsuperscript{22}

$$P = \left( \int_0^N \left[ \frac{p_\omega}{q_\omega} \right]^{\frac{1-\sigma}{\sigma}} d\omega \right)^{\frac{1}{1-\sigma}}.$$ \hspace{1cm} (A2)

Together with the equilibrium price setting of intermediate input producer we then yields (7).

\textsuperscript{22}The first-order condition when maximizing $Y = \left( \int_0^N [q_\omega y_\omega]^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$ with respect to $\{y_\omega\}_{\omega=0}^N$ subject to the budget constraint $M = \int_0^N y_\omega p_\omega d\omega$, can be written as

$$\lambda p_\omega = q_\omega^{\sigma-1} y_\omega^{-\frac{1}{\sigma}} \left( \int_0^N [q_\omega y_\omega']^{\frac{\sigma-1}{\sigma}} d\omega' \right)^{\frac{\sigma}{\sigma-1}}, \forall \omega,$$

where $\lambda$ is the Lagrange multiplier attached to the budget constraint. Integrating over all $\omega$’s yields

$$\lambda = Y/M = P^{-1}.$$  

This, together with the above first-order condition, yield (A1). Furthermore, note that (A1) implies

$$p_\omega y_\omega = q_\omega^{\sigma-1} P^\sigma p_\omega^{1-\sigma} (M/P).$$

Integrating over all $\omega$’s then gives (A2).
Next, we define the market share of any intermediate product $\omega$ as follows

$$s_\omega \equiv \frac{p_\omega y_\omega}{M},$$

where $M = \int_0^N p_\omega y_\omega d\omega = PY$. This, together with (A1), yields:

$$s_\omega = \left(\frac{q_\omega P}{p_\omega}\right)^{\sigma-1}.$$

Given this expression for the market share we have

$$\frac{X_{t+1}}{L_{t+1}} = \int_{I_{t+1}} \left(\frac{q_{\omega,t+1}P_{t+1}}{p_{\omega,t+1}}\right)^{\sigma-1} d\omega,$$

where $I_{t+1}$ denotes the set of continuers at date $t + 1$, i.e., all the varieties produced by the same incumbent producer in both periods $t$ and $t + 1$.

The aggregate market share of those same continuers at date $t$ is equal to:

$$\frac{X_t}{L_t} = \int_{I_{t+1}} \left(\frac{q_{\omega,t}P_t}{p_{\omega,t}}\right)^{\sigma-1} d\omega.$$

But then, using the fact that $p_{\omega,t+1} = \mu W_{t+1}$ and $p_{\omega,t} = \mu W_t$ we have:

$$\frac{X_{t+1}}{L_{t+1}} / \frac{X_t}{L_t} = \left(\frac{W_{t+1}/P_{t+1}}{W_t/P_t}\right)^{1-\sigma} \frac{\int_{I_{t+1}} q_{\omega,t+1}^{\sigma-1} d\omega}{\int_{I_{t+1}} q_{\omega,t}^{\sigma-1} d\omega}.$$

By the law of large numbers a fraction $(1 - \lambda_i)$ of continuers are not innovating so that for these plants: $q_{\omega,t+1} = q_{\omega,t}$; and a fraction $\lambda_i$ of continuers are innovating so that for these plants:
\( q_{\omega, t+1} = \gamma_i q_{\omega, t} \). This in turn yields,

\[
\frac{\int_{t+1} q_{\omega, t+1} d\omega}{\int_{t+1} q_{\omega, t} d\omega} = 1 - \lambda_i + \lambda_i \gamma_i^{\sigma-1},
\]

which implies that:

\[
\frac{X_{t+1}}{L_{t+1}} \frac{X_t}{L_t} = \left( \frac{W_{t+1}/P_{t+1}}{W_t/P_t} \right)^{1-\sigma} (1 - \lambda_i + \lambda_i \gamma_i^{\sigma-1})
\]

This equation allows us to express the true inflation rate \( P_{t+1}/P_t \) as a function of the growth in market share of continuers \( \frac{X_{t+1}}{L_{t+1}} \frac{X_t}{L_t} \).

Next, recall that the measured inflation rate satisfies:

\[
\frac{\bar{P}_t}{P_{t+1}} = \frac{W_t}{W_{t+1}} \left[ 1 - \lambda_i + \lambda_i \gamma_i^{\sigma-1} \right]^{\frac{1}{\sigma-1}}.
\]

Putting the last two equations together leads to:

\[
MG_{t+1} = \frac{1}{\sigma-1} \ln \left( \frac{X_t}{L_t} \frac{X_{t+1}}{L_{t+1}} \right).
\]
B Additional Figures

FIGURE B1: AVERAGE SIZE OF PLANTS BY AGE — CROSS SECTION

Notes: Average Employment by age group of establishments has been computed each year using total headcount and normalized to 1 for the youngest age group, as in Hsieh and Klenow (2014, Figure 1). Results are then averaged over the period 2011–2015.
FIGURE B2: EMPLOYMENT SHARE OF PLANTS BY AGE

Notes: This Figure restricts on establishments born in 1993 and compute their average employment using total headcount each year until 2014 when they are aged 21.

FIGURE B3: PROBABILITY OF EXIT OF PLANTS BY AGE

Notes: This Figure restricts on establishments born in 1993 and compute the exit rate each year until 2014 when they are aged 21. Exit rate is defined as the ratio of exiters at t, divided by the total number of firms born in 1993 that are still alive at t. At age 21, 8.16% of plants born in 1993 were still active.