Good Rents versus Bad Rents: R&D Misallocation and Growth

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March 2022

PRELIMINARY VERSION

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Abstract

Firm price-cost markups may reflect (a) higher step sizes from quality innovations that confer significant knowledge spillovers onto other firms, and/or (b) higher process efficiency than competing firms. We write down an endogenous growth model in which, compared with the *laissez-faire* equilibrium, the social planner would generally like to reallocate research resources towards high markup firms in case (a) so as to enhance knowledge spillovers but not in case (b). We then exploit unit price variation across high versus low markup firms in French manufacturing to assess the relative strength of these two forces. Viewed through the lens of our model, the French data imply that large firms typically display high process efficiency and low step size. The policy implication is that, to reach the social optimum, French research subsidies should favor only those high markup firms with "good" rents.

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The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System, the Banque de France, or the Eurosystem. This work is supported by a public grant overseen by the French National Research Agency (ANR) as part of the "Investissements d'Avenir" program (reference: ANR-10-EQPX-17 - Centre d'accès securisé aux données – CASD), and has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 786587). Philippe Aghion also acknowledges the support of EUR grant ANR-17-EURE-0001.

1 Introduction

A perennial debate weighs the static distortions created by price-cost markups against the dynamic incentives that such markups provide for private sector innovation. Classic examples include Dasgupta and Stiglitz (1980) and Tirole (1988). More recently, the debate has evolved to include cross-firm heterogeneity in markups and innovation effort. Evidence for heterogeneous markups includes Edmond, Midrigan, and Xu (2015, 2018), De Loecker and Eeckhout (2018), Haltiwanger, Kulick, and Syverson (2018), De Loecker, Eeckhout, and Unger (2020), Baqaee and Farhi (2020), and Autor, Dorn, Katz, Patterson, and Van Reenen (2020).[1] Among reasons for heterogeneous research intensity are firm differences in process efficiency (Cavenaile, Celik, and Tian, 2021; Voronina, 2022), research productivity (De Ridder, 2019), or ability to implement innovations (Akcigit, Celik, and Greenwood, 2016; Ma, 2021).

Less attention has been paid to the possibility that the *source* of markups may differ across firms — with implications for the optimal allocation of research efforts across firms. Markups may differ because firms differ in the step size of their quality-improving innovations, such as in Klette and Kortum (2004) or Akcigit and Kerr (2018). Quality innovation can confer significant knowledge spillovers onto other firms, who can build on their innovations. Alternatively, markups may vary across firms because firms differ in their process efficiency (which arguably spills over to other firms less easily) or in their ability to circumvent regulations, obtain permits, or impose entry barriers on potential competitors (which bear no obvious knowledge externality).[2]

[1] A number of papers provide evidence for heterogeneous markups in the form of incomplete exchange rate pass-through. See Gopinath and Rigobon (2008), Gopinath, Itskhoki, and Rigobon (2010), Fitzgerald and Haller (2014), and Amiti, Itskhoki, and Konings (2019).

[2] One paper which documents heterogeneity in the source of markups is Akcigit, Baslandze, and Lotti (2018). Using firm-level evidence from Italy, these authors contrast small firms (whose rents stem from innovation) with market leaders (whose rents rely importantly on political connections). Only the former source of rents bestows knowledge spillovers on future

In this paper, we analyze the optimal allocation of R&D in an economy with multiple sources of markup heterogeneity across firms, and we contrast it with the market equilibrium under *laissez-faire*.[3] In the model we develop, firms differ both in their quality advantage over other firms and in their degree of proprietary process efficiency. Firms can improve quality through innovating, whereas the level of proprietary process efficiency for each firm is given once for all. Both high quality steps and high process efficiency enable a firm to charge above-average markups, yet only high quality steps bestow knowledge spillovers on subsequent innovators (firms with better proprietary process efficiency do not).[4]

Our analysis sheds light on whether the allocation of research under *laissez-faire* is excessively or insufficiently tilted towards high-rent firms, depending upon whether the main source of markup heterogeneity across firms is quality steps versus proprietary process efficiency. The planner wishes to undo the static misallocation of production labor created by markup dispersion, but also endeavors to allocate research labor optimally. In particular, the planner wants to shift innovation effort toward high quality step firms and, as a byproduct, away from high process efficiency firms.

We use data on French manufacturing from 2012 to 2019 to infer the extent to which firms differ in their quality step sizes and process efficiency. According to our theory, high quality steps allow firms to raise their prices to charge high markups. And high process efficiency allows firms to charge high markups by not passing through their lower marginal cost into their prices.

innovators and generates growth externalities in their environment.

[3]To focus on the allocation of research across firms, we fix total research labor in this economy. That is, we set aside the question of whether the market devotes too little labor to research versus production.

[4]On the question of whether firms differ in their knowledge spillovers, the literature has drawn an important distinction between product innovation and process innovation. Product innovation appears easier for competing firms to reverse engineer and build upon. And process innovation appears easier for firms to keep secret from their competitors. This has historically tilted patenting toward products and away from processes — see the survey by Cohen, Levin, Schmalensee, and Willig (1989). Relatedly, Moser (2005, 2013) argues that the fraction of innovations that are patented varies over time and across industries depending on how much firms can keep their ideas secret if they do not patent them.

Thus, among firms with higher markups, high-price firms take big quality steps and low-price firms enjoy higher process efficiency. We can use this same methodology to address the extent to which large firms charge higher markups because of higher process efficiency, and whether (some) small firms charge higher markups because of their innovativeness on quality.

We find that French manufacturing firms do indeed differ in their price-cost markups, as measured by the ratio of their revenues to their total costs. And firms differ in their unit prices conditional on these markups, suggesting heterogeneity in their quality steps versus process efficiency. When we infer the process efficiency of firms by dividing their markups by their price levels, we find that firm differences in process efficiency are quite persistent, consistent with our model.

A key feature of the French data that we identify is that employment is lower at high-price firms and higher in high process efficiency firms within industries. Viewed through the prism of our model, this negative elasticity of employment with respect to price and positive elasticity with respect to process efficiency requires: first, a negative correlation between step size and process efficiency across firms; and second, more variation in process efficiency than quality step sizes across firms. As a result, large firms must typically have high process efficiency and low step size. Thus, according to our model and inference on the French data, research subsidies should not favor larger French firms.

This paper relates to several strands of literature: First, to the literature on competition, R&D, and growth such as Dasgupta and Stiglitz (1980), Aghion, Harris, and Vickers (1997), Aghion, Harris, Howitt, and Vickers (2001), and Aghion, Bloom, Blundell, Griffith, and Howitt (2005). Acemoglu and Akcigit (2012) use a step-by-step innovation model with leaders and followers to analyze the growth and welfare implications of various patent protection policies. They conclude that patents should provide stronger protection to firms with bigger technological leads over their rivals. There is only one source of markups in their model, however, namely quality. We contribute to this literature by considering multiple sources of markup heterogeneity at once, and by proposing an empirical method to infer the primary source of markup heterogeneity in an economy.

Second, our paper overlaps with the recent literature on whether market power and markup dispersion are inhibiting growth. Examples include Akcigit and Ates (2019), De Ridder (2019), Aghion, Bergeaud, Boppart, Klenow, and Li (2021), and Liu, Mian, and Sufi (2022). Our analysis is distinct in encompassing two sources of markup heterogeneity — process efficiency and quality steps — which allows us to characterize the socially optimal R&D allocation across firms with different markup sources and to contrast it with the *laissez-faire* equilibrium.

Third and finally, our effort connects to an older industrial organization literature looking at competition, regulation, and R&D allocation. This literature emphasizes the trade off between the regulatory distortions under monopoly and the duplication of R&D and/or entry costs under competition. For examples see Tirole (1988), Laffont and Tirole (1993), and Armstrong, Cowan, and Vickers (1994).

The rest of the paper proceeds as follows: Section 2 lays out our endogenous growth model of multiproduct firms with differing quality innovation step sizes and process efficiency levels. In Section 3 we calibrate the key model parameters based on the moments we document in the French data. Section 4 concludes.

2 Theory

We start by laying out the theoretical setting. We pursue by characterizing the planner's solution and then by specifying a decentralized equilibrium. We next solve and compare the allocation of the planner to the decentralized outcome. We suppress time indices whenever it should not lead to confusion.

2.1 Setup

Household and preferences There is representative household with the following preferences over a final output good

$$\sum_{t=0}^{\infty} \beta^t \log(C_t). \tag{1}$$

Furthermore, the household can supply each period L units of production labor and Z units of R&D labor without generating disutility.

Final good production Final output, Y is a Cobb-Douglas bundle of a unit interval of intermediate goods which come at qualities $q(i)$s:

$$Y = \exp\left(\int_0^1 \log\left[q(i)y(i)\right] di\right).$$

The final output good can be used for two purposes: it can be consumed, C, or—as will be explained below—used to cover production overhead cost, O.

Intermediate input production There is a "large" number of J firms which can produce the intermediate goods $i \in [0, 1]$. Each firm produces at a line-specific quality level $q(i, j)$ and a firm-specific "process efficiency" $\varphi(j)$. More specifically, a firm j can produce at their respective quality level $q(i, j)$ with constant labor productivity $\varphi(j)$, i.e.,

$$y(i, j) = \varphi(j) \cdot l(i, j), \tag{2}$$

where $l(i, j)$ denotes production labor used in line i by firm j.

The quality levels at which a firm produces change endogenously over time as a result of R&D activity. Each firm has access to a linear R&D technology with heterogeneous step sizes. That is, if $x \cdot \psi_z$ units of research labor are used by a particular firm j, this firm innovates in x randomly drawn lines. In a selected line the currently highest existing quality across firms is taken and increased by a factor $\gamma(j)$. The innovating firm j can then produce at this higher quality from the next period onward. The initial distribution of highest quality levels across firms is exogenously given. As the quality innovations are building on each other they therefore imply a positive spillovers on future innovators.

Overall the J firms in our model differ exogenously in two dimensions their level of process efficiency $\varphi(j)$ and their innovation stepsize $\gamma(j)$. In the following we assume both dimension taking on two potential values (high H and low L). We further assume $\gamma_L > \varphi_H/\varphi_L$.[5] Given the binary heterogeneity in both γ and φ there are four type of firms: $k = \{HH, HL, LH, LL\}$. Here HL denotes a firms with a high process efficiency and a low innovation stepsize and so on.

On top of the linear production cost firms use resources on fixed production cost called "overhead". I.e., in order to be active in a period a firm that has the highest quality level in $n(j)$ lines needs to spend $\frac{1}{2}\psi_o\, n(j)^2 Y$ units of final output on overhead.

In the following we assume that all firms of the same type k start out with the same number of line, $n_{k,0}$, in which they have the highest quality $q_0(i)$ across all firms. With the convex overhead cost schedule this ensures that it is optimal for the planner to always keep homogeneity within type and the planner's problem can be characterized in terms of four representative type of firms.[6] We assume that ϕ_k denotes the fraction of firms of type k.

Aggregates and resource constraints Aggregate resources used on overhead are then given by $O = \sum_{j=1}^{J} \frac{1}{2}\psi_o\, n(j)^2 Y$ and the economy's resource constraint

[5] This will ensure that it is always optimal to produce by the highest quality firm in each line. This condition also ensures production in the highest quality firm (irrespective of its type) in the decentralized economy we study below.

[6] The same will also hold true in the decentralized equilibrium that we study. Along a balanced growth path, which we will study below, homogeneity within type is automatically fulfilled and this assumption does not put any additional restrictions.

reads

$$Y = C + O. \tag{3}$$

Next, we have the production and R&D labor resource constraints which are

$$Z = \psi_z \sum_{j=1}^{J} x(j) = \psi_z \sum_k J\phi_k x_k, \tag{4}$$

(where the second equality again exploits homogeneity within types) and

$$L = \sum_{j=1}^{J} \int_0^1 l(i,j)\, di. \tag{5}$$

Finally, we have an accounting equation that says that the total number of products of highest quality must sum to 1 across firms

$$\sum_k S_k = 1 \text{ where } S_k \equiv J\phi_k n_k. \tag{6}$$

2.2 Planner's problem

With homogeneity within the four type of firms $k = \{HH, HL, LH, LL\}$, the planner's problem can be characterized as follows:

$$\max_{\{C_t, Q_{t+1}, n_{k,t+1}, x_{k,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(C_t), \tag{7}$$

where $k = \{HH, HL, LH, LL\}$, subject to

$$C_t = Q_t \exp\left(\sum_k J\phi_k n_{k,t} \log(\varphi_k)\right)\left(1 - \psi_o \sum_k \frac{1}{2} J\phi_k n_{k,t}^2\right) L, \tag{8}$$

$$Z = \psi_z \sum_k J\phi_k x_{k,t}, \tag{9}$$

$$Q_{t+1} = Q_t \exp\left(\sum_k J\phi_k x_{k,t} \log\left(\gamma_k\right)\right), \tag{10}$$

$$n_{k,t+1} = n_{k,t}\left(1 - \sum_{k'} J\phi_{k'}x_{k',t}\right) + x_{k,t},\ \forall k, \tag{11}$$

and a given $Q_0 = \exp\left(\int_0^1 \log\left(q_0(i)\right) di\right)$, $n_{HH,0}$, $n_{LH,0}$, $n_{HL,0}$, and $n_{LL,0}$ and some non-negativity constraints

$$n_{k,t+1} \geq 0,\ x_{k,t} \geq 0,\ \forall k, t. \tag{12}$$

Equation (8) captures the resource constraint, i.e., that consumption equals output minus overhead. Here we already have exploited the fact that it is always optimal to set $l(i) = L$ due to the Cobb-Douglas technology. Furthermore, we exploited that it is always optimal to produce by the highest quality firm in a given line.[7] Output can then be written as the product of the geometric mean quality, $Q_t \equiv \exp\left(\int_0^1 \log\left(q_t(i)\right) di\right)$, the geometric average of process efficiency, $\exp\left(\sum_k J\phi_k n_{k,t} \log\left(\varphi_k\right)\right)$, and L. The term $\left(1 - \psi_o \sum_k \frac{1}{2} J\phi_k (n_{k,t})^2\right)$ captures output net of overhead. Equation (9) captures the constraint on researcher labor. Finally, (10) captures the law of motion of the average quality level and (11) gives the law of motion of the number of highest quality lines by each type of firm.

2.3 Decentralized economy

In the decentralized economy we assume competitive markets with the exception of intermediate good production. So final output production can be characterized as the behavior of a representative firm solving

$$\max_{\{y(i)\}_{i=0}^1} P\exp\left(\int_0^1 \log\left(q(i)y(i)\right)di\right) - \int_0^1 p(i)y(i)\, di. \tag{13}$$

In the following, we normalize the price of the final output $P = \exp\left(\int_0^1 \log\left(p(i)/q(i)\right) di\right)$ to one in all periods.

The representative household supplies inelastically L units of production

[7] This follows from the restriction $\gamma_L > \varphi_H/\varphi_L$ we made above.

labor and Z units of research labor to the labor market and solves

$$\max_{\{C_t, A_{t+1}\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(C_t), \tag{14}$$

subject to $A_{t+1} = A_t(1 - r_t) + w_t L + w_{z,t} Z - C_t$, $\forall t$ and a standard no-Ponzi game condition. Here A denotes wealth, r the interest rate and w and w_z the wage rates of production and research labor, respectively.

Intermediate input production The J intermediate input producers own patents to produce at particular qualities in given lines. The distribution of qualities across lines increases endogenously due to innovations. In each line the different firms that produce at different quality levels then compete à la Bertrand. We solve at this point already for the equilibrium pricing decision under Bertrand competition at intermediate input level. We will then arrive at an expression for period profits of a firm and this allows us to directly focus on the dynamic firm problem in isolation.

As we assumed $\gamma_L > \varphi_H / \varphi_L$, the firms that has the highest quality patent in a line is the "leading" firm, i.e., the firm with the lowest quality-adjusted marginal cost. Due to the Cobb-Douglas structure, under Bertrand competition, it is always optimal for this leading firm j in a given line i to set its quality-adjusted price equal to the quality-adjusted marginal cost of the second-best quality producer j', i.e., $\frac{p(i,j(i),j'(i))}{q(i,j(i))} = \frac{w}{q(i,j'(i))\varphi(j')}$. This price setting implies the following markup factor over marginal cost charged by producer $j(i)$ in a line i

$$\mu(i, j(i)) = \gamma_j \frac{\varphi(j(i))}{\varphi(j'(i))}. \tag{15}$$

The markup is equal to the step size (which depends on the identity of the producing firm) times the ratio of process efficiency of the producing firm relative to the process efficiency of the second-best firm. So the process efficiency of the second-best firm, which is either high or low, does influence markups (and therefore profits from a given line). Due to the Cobb-Douglas structure of final output production, sales in each product line are given by Y and independent of the quality level and prices. Hence, operating profits of a producing firm in a given line (before overhead cost) are given $Y(1-1/\mu(i,j(i)))$.

Total period profit of firm j then depend on the number of lines in which the firm has the highest quality patent, $n(j)$, and the share of these lines in which they face a high productivity second-best firm $h(j)$. These two variables $n(j)$ and $h(j)$ are the two individual state variables in the dynamic firm problem. So total profits after overhead expressed relative to output Y are given by

$$\pi\left(j,n,h\right)=nh\left(1-\gamma_j\frac{\varphi_H}{\varphi_j}\right)+n(1-h)\left(1-\gamma_j\frac{\varphi_L}{\varphi_j}\right)-\frac{1}{2}\psi_o n^2. \tag{16}$$

When we again assume homogeneity within type, (16) can be expressed as four type-specific profit functions $\pi_k(n,h)$ for $k = \{HH,HL,LH,LL\}$ that depend on the individual state variables $n(j)$ and $h(j)$. The profit functions are type-specific as they depend on φ_j and γ_j of the producing firm j (see (16)). The dynamic firm problem can then be expressed as follows:

$$V_{k,0}=\max_{\{x_t,n_{t+1},h_{t+1}\}_{t=0}^{\infty}}\sum_{t=0}^{\infty}Y_t\left[\pi_k(n_t,h_t)-x_t\psi_z\frac{w_{z,t}}{Y_t}\right]\prod_{s=0}^{t}\left(\frac{1}{1+r_s}\right)$$

subject to

$$n_{t+1}=n_t(1-Z/\psi_z)+x_t,\ \forall t, \tag{17}$$

$$h_{t+1}n_{t+1}=h_tn_t(1-Z/\psi_z)+S_tx_t,\ \forall t, \tag{18}$$

for a given n_0 and h_0 and some non-negativity constraints $x_t\geq 0$, $n_{t+1}\geq 0$. That is, each firm is choosing the R&D activity such to maximize the net present value of firm profits. The constraints (17) give the dynamics of number of lines in which the firm produces. There the firms take the aggregate rate of creative destruction Z/ψ_z as exogenously given. The second set of constraints (18) gives the change in the share of line in which the firm faces a high process efficiency second-best firm. In this process the firm take into account that the newly innovated lines are drawn from the total pool of line in which a fraction S is currently served by high process efficiency firms.

Market clearing and aggregates We have the labor markets clearing conditions (4) and (5) and the asset market clearing condition

$$\sum_k J\phi_k V_k = A.$$

Finally, we have the accounting equation (6) and an accounting equation giving the aggregate share of lines served by high process efficiency firms

$$S_t = S_{HH,t} + S_{HL,t} = J\left(\phi_{HH} n_{HH,t} + \phi_{HL} n_{HL,t}\right). \tag{19}$$

Equilibrium definition A decentralized equilibrium is then defined as a sequence of quantities and prices that jointly solve the final producer problem, the intermediate producer problems, the household problem and is in line with market clearing and all the aggregate constraints.

2.4 Definition of a BGP

We define a balanced growth path (BGP) in the standard way, i.e., as a path along which all quantities grow at constant rates. In the following we focus on an interior such BGP in which growth is strictly positive and all the type of firms are active. Under some parameter restriction such an interior BGP exists and is unique (both in the planner's problem as well as for the decentralized equilibrium). We next characterize the BGP in the planner's solution and then move on to the decentralized equilibrium and compare the two. We will denote variables along the BGP by upper bar, i.e., $\bar{r}$ will denote the interest rate along the decentralized BGP. Along a BGP, in both the planner's solution as well as the decentralized equilibrium, the distribution of the number of lines across firms will be stationary and output, consumption, the geometric mean of quality, $Q_t = \exp\left(\int_0^1 \log\left(q_t(i)\right) di\right)$ and total resources used for overhead grow at some endogenous rate $\bar{g}$.

2.5 Welfare along the BGP

As consumption grows along the BGP at a constant endogenous rate $\bar{g}$ welfare in (1) can be rewritten as

$$\frac{1}{1-\beta}\left(\log(\bar{C}_0) + \frac{\beta}{1-\beta}\log(1+\bar{g})\right), \tag{20}$$

where $\bar{C}_0 \equiv C_t(1+\bar{g})^{-t}$ is the detrended consumption level along the BGP. That is, welfare can be written as a weighted sum of the logarithm of the consumption *level* and *growth*, where the relative weight put on the logarithm of the gross growth rate is $\frac{\beta}{1-\beta}$. As output and innovations are both produced from distinct type of labor which are in fixed exogenous supply (L and Z), discrepancies along the BGP in the consumption level $\bar{C}_0$ and the growth rate $\bar{g}$ between the decentralized equilibrium and planner's solution solely arise from differences in the allocation of the fixed L and Z resources across the heterogeneous firms. In this sense our model focuses entirely on misallocation, both statically and in terms of R&D resources and shuts down potential distortions on the amount of resources devoted to R&D.

In the decentralized equilibrium as well as the planner's solution the detrended consumption level along the BGP can be written as the following product

$$\bar{C}_0 = (1-\bar{o}) \cdot Q_0 \cdot \Phi \cdot \mathcal{M} \cdot L, \tag{21}$$

where $\bar{o} \equiv O/Y = \psi_o \sum_k \frac{1}{2J\phi_k}\bar{S}_k^2$ is the fraction of output used for overhead, $Q_0 = \exp\left(\int_0^1 \log\left(q_0(i)\right) di\right)$ is the initial geometric mean quality level, $\Phi = \exp\left(\sum_k \bar{S}_k \log\left(\varphi_k\right)\right) = \varphi_L \Delta^{\bar{S}}$ is the geometric average of process efficiency, and $\mathcal{M}$ captures potential misallocation of labor across lines. The last term $\mathcal{M}$ is equal to one for the social planner, whereas it is smaller than one due to markup dispersion across lines in the decentralized equilibrium. In the decentralized equilibrium this allocative efficiency term is the ratio of geometric relative to the arithmetic average of the inverse markups across lines, or formally

$$\mathcal{M} = \frac{\exp\left(\int_0^1 \log \frac{1}{\mu(i,j(i))}\, di\right)}{\int_0^1 \frac{1}{\mu(i,j(i))}\, di} \leq 1. \tag{22}$$

The terms $\bar{o}$ and the aggregate process efficiency level Φ are both functions of the distribution of lines provided by the different type of firms $\bar{S}_k$. Overhead resources $\bar{o}$ are minimized if all firms are of equal size $n_j = 1/J, \forall j$ irrespective of their type (which implies $\bar{S}_k = \phi_k, \forall k$). In contrast, aggregate process efficiency Φ is maximized if the high process efficient firms serve the whole market, i.e., $\bar{S} = 1$. As the decentralized equilibrium and the planner's solution will give rise to a different market share distribution along the BGP (and there is markup dispersion in the decentralized equilibrium) the detrended consumption level $\bar{C}_0$ will differ between the two solution concepts.

Similarly to the level, also the growth rate can be expressed as a function of the market share distribution along the BGP. We have

$$1 + \bar{g} = \exp\left(\frac{Z}{\psi_z}\left[\sum_k J\phi_k \bar{n}_k \log\left(\gamma_k\right)\right]\right) = \left(\bar{\gamma}\right)^{\frac{Z}{\psi_z}}. \tag{23}$$

where $\bar{\gamma} \equiv \Pi_k \gamma_k^{\bar{S}_k}$, and $\bar{S}_k \equiv J\phi_k \bar{n}_k$. Here, $\bar{\gamma}$ is the geometric mean of the step size γ_k weighted by the share of lines $\bar{S}_k$ served by type k firms. This equation again holds true for both the decentralized equilibrium as well as the planner's solution. Growth rate is higher when more products are produced by firms with higher step size. The intuition for this result is the following: As total research labor is fixed and all firms have the same linear R&D technology the rate of creative destruction is equal to $\frac{Z}{\psi_z}$. Now along a balanced growth path—in order for the firm size distribution to be stationary—all the types of firms have to innovate at a rate that is proportional to their market share $\bar{S}_k$. The growth rate is then simply given by the market share-weighted geometric average of the step size raised to the rate of creative destruction $\frac{Z}{\psi_z}$.

This shows how the main ingredients into welfare—the detrended consumption level plus the growth rate—are both a function of the market share distribution across firms. In order to produce, the quality improvements have to be developed in house and this creates an interesting trade-off. The planner weighs off the level and growth effect as highlighted in (20). In contrast, the market share distribution along the decentralized BGP is determined by the relative profitability (capability to charge markups) across firms which will generally lead to inefficiencies. We will next characterized the optimal market share distribution chosen by the planner along the BGP and then contrast it with the decentralized equilibrium.

2.6 Characterizing the BGP of the planner's solution

How does the planner determine the market shares $\bar{S}_k = J\phi_k\bar{n}_k$? As shown in the Online Appendix A, the optimal differences between any two $\bar{n}_k$ and $\bar{n}_{k'}$ satisfy

$$\bar{n}_k - \bar{n}_{k'} = \frac{1-\bar{o}}{\psi_o}\left[\log\left(\frac{\varphi_k}{\varphi_{k'}}\right) + \left(1 + \frac{Z/\psi_z}{1/\beta - 1}\right)\log\left(\frac{\gamma_k}{\gamma_{k'}}\right)\right], \tag{24}$$

where $\bar{o} = O/Y$ is again the output share of total overhead costs along the BGP. This result says that the planner chooses a higher long-run market share for firms with higher process efficiency φ and for firms with higher step size γ. The reason for this is that increasing the number of products in high φ firms increases the aggregate level of process efficiency Φ in production, whereas increasing the number of products in high γ firms has on top of a similar level effect also a positive effect on the long-run growth rate $\bar{g}$.[8] As a consequence, the relative weight the planner puts on differences in step sizes versus process efficiency is increasing in the consumer's patience (β) or in the amount of available research labor (Z/ψ_z). The differences in the optimal long-run

[8]Remember that choosing a high long-run market share for one firms also implies that this firm uses a larger share of the R&D labor.

number of products across firm are moderated by the scalar in front of the overhead cost curve. If the overhead cost curve shifts down (as ψ_o decreases) the optimal long-run differences in firm size are magnified.

The level of $\bar{n}_k$ is determined by combining (24) with the accounting equation (6) on the aggregate number of products. Namely, as we derive in the Online Appendix, the optimal share of products produced by type-k firms is given by

$$\bar{S}_k = \phi_k \left(1 + \nu_k \frac{\sqrt{1 + (\frac{2J}{\psi_o} - 1) \sum_k \phi_k \nu_k^2} - 1}{\sum_k \phi_k \nu_k^2} \right), \tag{25}$$

where

$$\nu_k \equiv \log \left(\frac{\varphi_k}{\prod_{k'} \varphi_{k'}^{\phi_{k'}}} \right) + \left(1 + \frac{Z/\psi_z}{1/\beta - 1} \right) \log \left(\frac{\gamma_k}{\prod_{k'} \gamma_{k'}^{\phi_{k'}}} \right)$$

summarizes the technology level of type-k firms relative to the geometric mean across firms. Since the differences in γ have additional implications for the long-run growth rate, the term in front of $\log(\gamma_k)$ exceeds 1 and the planner places more weight on step size differences.[9]

As we noted in the setup of the planner's problem, the planner finds it optimal to allocate the same amount of labor to each product line. Hence, $\bar{S}_k$ is also the share of labor allocate to type-k firms or $L_k/L = \bar{S}_k$. Furthermore, since the rate of innovation in each firm $\bar{x}_k/\bar{n}_k$ is equal to the aggregate rate of creative destruction and all firms have the same R&D efficiency, $\bar{S}_k$ is also the share of R&D labor allocated to type-k firms.

[9] In order to guarantee an interior BGP we assume that the parameters satisfy the following condition

$$0 < \frac{\sqrt{1 + (\frac{2J}{\psi_o} - 1) \sum_k \phi_k \nu_k^2} - 1}{\frac{J}{\psi_o} \sum_k \phi_k \nu_k^2} < 1. \tag{26}$$

This is a necessary for the solution given by (25) to generate $\bar{o} \in (0, 1)$. When this condition is satisfied, we can show that the share of products $\bar{S}_k$ approaches the share of firms ϕ_k when the overhead cost schedule shifts up (higher ψ_o). The convex overhead cost is the reason why the planner does not allocate all products to a particular type of firm with the highest combination of step size and process efficiency.

2.7 Characterizing the BGP of the decentralized equilibrium

Using the first-order conditions of the firm's problem, we show in Online Appendix A that the number of products produced by a type-k firm along an interior BGP satisfies

$$\bar{n}_k = \frac{1}{J} + \frac{\bar{S}(\Delta - 1) + 1}{\gamma_L \psi_o}\omega_k, \tag{27}$$

where $\bar{S}$ is the share of products produced by firms with high process efficiency, Δ is the process efficiency gap φ_H/φ_L and $\omega_k \equiv \left(\sum_{k'} \phi_{k'} \frac{\gamma_L\varphi_L}{\gamma_{k'}\varphi_{k'}}\right) - \frac{\gamma_L\varphi_L}{\gamma_k\varphi_k}$ is the markup advantage of a type-k firm vis-à-vis an average firm. Firms with higher markups, either due to higher step size or higher process efficiency, have higher than average share of products. This results in the product shares in the decentralized equilibrium

$$\bar{S}_k = \phi_k \left(1 + \omega_k \frac{(\Delta - 1)(\phi_{HH} + \phi_{HL}) + 1}{\gamma_L \frac{\psi_o}{J} - (\Delta - 1)(\phi_{HH}\omega_{HH} + \phi_{HL}\omega_{HL})}\right), \tag{28}$$

which differs from the expression for the planner's product shares. In contrast to the planner's solution, the relative market shares are just pinned down by the relative markups and are independent of the relative weight β consumers put on growth and level effects. As an increase in the minimum step size γ_L shifts—for a given Δ and Γ—the markup distribution to the right, γ_L has an influence on the decentralized market shares along the BGP. Such an effect is not in the planner's solution, where the optimal market shares only depends on relative step size and relative process efficiency Γ and Δ.

Furthermore, the relative labor share of two firms j and k facing the same share of high process efficiency competitors $\bar{S}$ is given by

$$\frac{\lambda_j(\bar{S})}{\lambda_k(\bar{S})} = \frac{\gamma_k\varphi_k}{\gamma_j\varphi_j}, \tag{29}$$

as all firms are facing along the BGP the same share of high process efficiency competitors $\bar{S}$. Since production wages are the same across firm and $\bar{S}_k$ is equal to the sales share of type-k firms, the relative employment of type k firms given by

$$\frac{L_k}{L_{k'}} = \frac{\lambda_k(\bar{S})\bar{S}_k}{\lambda_{k'}(\bar{S})\bar{S}_{k'}} = \frac{\gamma_{k'}\varphi_{k'}}{\gamma_j\varphi_k}\frac{\bar{S}_k}{\bar{S}_{k'}}. \tag{30}$$

This expression says that firms with high markups (high $\gamma_k\varphi_k$) have employment shares that are lower than their sales and product shares. This is a contrast to the planner's solution, where employment and market share coincide for any given firm. On top of this effect there is also heterogeneity in labor allocated to the different production lines *within* a firm along the competitive BGP. This is because the markup across production lines within a firm differs by a factor Δ depending on whether the second-best firm is of high or low process efficiency. The amount of labor a firm devotes to line i where it faces a high efficiency competitor relative to the amount of labor it devotes to line i' where it faces a low efficiency competitor is therefore given by

$$l(i,j) = \Delta\, l(i',j). \tag{31}$$

This gives rise to an additional static efficiency loss as the planner would equalize the amount of production labor across all lines within a firm. The heteorogeneity in labor allocation across lines lowers the level of detrended consumption relative to the planner's allocation and shows up as $\mathcal{M} < 1$ in our welfare decomposition (20). The allocative efficiency term becomes along the BGP

$$\mathcal{M} = \frac{\Delta^{\bar{S}}}{\Delta\bar{S} + 1 - \bar{S}} \frac{\exp\left(-\sum_k \bar{S}_k \log(\varphi_k\gamma_k)\right)}{\sum_k \bar{S}_k \frac{1}{\varphi_k\gamma_k}} < 1.$$

Finally, as in the planner's problem, the share of research labor allocated to type-k firms is the same as the share of products produced by type-k firms because the arrival rate of new products matches the aggregate rate of creative destruction $\bar{z} = Z/\psi_z$ for all firms.

2.8 Planner's solution versus the decentralized equilibrium

In general, the planner's allocation is different from the decentralized allocation. First, conditional on a given number of product lines in which a firm is the highest quality producer, the planner and decentralized equilibrium differ in the allocation of employment across product lines. Equation (30) implies that the planner always want more employment in the firms with higher $\gamma_k \varphi_k$ than the decentralized equilibrium, conditional on the product shares. Both step size and process efficiency gaps generate markup and labor share heterogeneity across firms of different $\gamma_k \varphi_k$. On top of that the type of second-best firms will additional generate variations in production labor across lines within firms in the decentralized equilibrium. Along the BGP each firm has a share $\bar{S}$ of lines in which they face a high type second-best in in such lines production labor is higher compared to the remaining $1 - \bar{S}$ lines by a factor of Δ. In contrast, the planner wants to allocate the same amount of labor to each line.

Second, the decentralized equilibrium in general will deviate from the planner's product shares leading to static and/or dynamic misallocation. We will use three extreme cases to illustrate this.

First, let us consider the case where step sizes are the same across firms $\gamma_k = \gamma$. In this case, there is no dynamic misallocation in the sense that the growth rate is $1 + \bar{g} = \gamma^{\frac{Z}{\psi_z}}$ in both the decentralized equilibrium and the planner's problem. However, the planner's share of products allocated to the high process efficiency firms ($\bar{S}_H^P$) can differ from the share in the decentralized equilibrium ($\bar{S}_H^D$). So this is an extreme case where efficiency boils done to only static efficiency. We show in Online Appendix A that

$$\frac{\bar{S}_H^P - \phi_H}{\bar{S}_H^D - \phi_H} = \frac{(\frac{2J}{\psi_o} - 1)\,(\log \Delta)\,\frac{\frac{\psi_o}{J}\frac{\gamma\Delta}{\Delta-1} - (\Delta-1)\phi_H(1-\phi_H)}{(\Delta-1)\phi_H+1}}{\sqrt{1 + (\frac{2J}{\psi_o} - 1)\,(\log \Delta)^2\,(1-\phi_H)\phi_H} + 1}, \tag{32}$$

which implies that $\bar{S}_H^P - \bar{S}_H^D$ increases with the common step size γ. In the decentralized equilibrium, the gap in profit share between the high and low types shrinks with γ and $\bar{S}_H^D$ approaches ϕ_H as γ goes to infinity. The step size does not affect $\bar{S}_H^P$.

Hence, when firms have the same step size but different process efficiency levels, the long-run growth rate is the same in the planner's solution and the decentralized equilibrium but the level of consumption can be lower in the decentralized equilibrium due to static misallocation (differences in employment and product shares). These differences result in differences in the share of overhead costs $\bar{o}$, aggregate process efficiency $\Phi = \Delta^{\bar{S}_H}$ and allocative efficiency (markup dispersion) $\mathcal{M}$ in decomposition (20).

Another polar case is where all firms have the same process efficiency $\varphi_k = \varphi$. This is equivalent to setting $\Delta = 1$. Define ϕ_H as the share of firms with γ_H and $\bar{S}_H$ the share of products they produce. For both the planner and decentralized equilibrium, the geometric-mean of the step sizes is given by

$$\bar{\gamma} = \Gamma^{\bar{S}_H} \gamma_L.$$

As a consequence, the growth rate increases with the share of products produced by firms with the high step size $\bar{S}_H$.

We show in Online Appendix A that

$$\frac{\bar{S}_H^P - \phi_H}{\bar{S}_H^D - \phi_H} = \frac{(\frac{2J}{\psi_o} - 1)\left(\frac{1/\beta - 1 + Z/\psi_z}{1/\beta - 1} \log \Gamma\right) \frac{\gamma_H}{\Gamma - 1} \frac{\psi_o}{J}}{\sqrt{1 + (\frac{2J}{\psi_o} - 1)\left(\frac{1/\beta - 1 + Z/\psi_z}{1/\beta - 1} \log \Gamma\right)^2 (1 - \phi_H)\phi_H} + 1} \tag{33}$$

which implies that $\bar{S}_H^P - \bar{S}_H^D$ increases in γ_H holding fixed Γ. In the decentralized equilibrium, profit shares approaches 1 as γ_H and γ_L increase. Hence, the gap in profit share between the high and low types shrinks and $\bar{S}_H^D$ approaches ϕ_H when γ_H increases while Γ stays constants. However, the planner only cares about Γ and $\bar{S}_H^P$ does not change when γ_H increases while Γ stays constant. Therefore, when firms differ in step sizes, growth rates in the decentralized equilibrium will in general deviate from the growth rate in the planner's problem.

A final special case is the one were the product $\varphi_k \cdot \gamma_k$ is the same across all firms. This is the case when $\Delta = \Gamma$ and process efficiency and the step size are perfectly negatively correlated, i.e., $\phi_{HH} = \phi_{LL} = 0$. In this case, as markups are equalized across firms the number of lines per firm along the decentralized BGP is equalized or $\bar{n}^D_{LH} = \bar{n}^D_{HL} = 1/J$ (see (27) and note that ω_k is equal to zero for both groups k). Interestingly, this is an allocation that minimizes overhead cost $\bar{o}$. However, the decentralized equilibrium does not take into account the dynamic positive externality generated by the large step size firms. The planner would indeed along the BGP choose a larger market share of the large step size firms LH and a lower market share for the high process efficiency firms HL. Formally, we have

$$\bar{n}^P_{LH} - \bar{n}^P_{HL} = \frac{1-\bar{o}}{\psi_o} \frac{Z/\psi_z}{1/\beta - 1} \log(\Gamma) > 0. \tag{34}$$

Hence, the planner would indeed sacrifice some static process efficiency and increase the overhead cost $\bar{o}$ to instead exploit the larger growth potential of the higher step size firms. The extent to which this is done depends on the scalar in front to the overhead cost curve ψ_o, the growth potential of the economy determined by the available R&D labor Z/ψ_z and the discount rate β.

3 Calibration

In this section, we calibrate 10 parameters—$\psi_o, Z/\psi_z, \beta, \gamma_L, \gamma_H, \Delta, \{\phi_{k,k'}\}$—in the BGP of the decentralized equilibrium to fit 1) variation in markups, price and productivity across firms, 2) variation in product prices within firms, 3) dispersion and skewness of the sales-share distribution, 4) aggregate markup, productivity growth rate and interest rate.[10] We will also address measurement

[10]We only need 9 moments for 10 parameters because we have the restriction $\sum_k \phi_k = 1$.

errors that may contaminate measures of firm-level markups, prices and productivity.

We will first provide a heuristic description of how the aforementioned moments discipline the parameters in the model. Then we describe how we address measurement errors in the data. After describing the overall strategy of the calibration, we will summarize how we construct the target moments from firm-level and product-level data. Finally, we will show the calibration results and use the calibrated parameter values to compare allocations under the decentralized equilibrium with the planner's allocation.

3.1 Intuition for calibration

Here, we will provide some intuition for how certain moments are informative of particular parameters.[11] First, recall that in the decentralized equilibrium, the price of good i is given by

$$p(i, j(i), j'(i)) = \frac{w \cdot \gamma(j(i))}{\varphi(j'(i))},$$

where $j(i)$ and $j'(i)$ index respectively the producing and the second-best firm. We can calculate firm j's price index as the sales-weighted average of the prices of all products produced by the firm. In the Cobb-Douglas case, this coincides with the unweighted average of product prices as the sales shares are the same across products in that case. Along a BGP firms innovate upon a randomly drawn line from a stationary pool of types of producing firms. Hence, along the BGP, the share of products produced by type k firms ($\bar{S}_k$) is the same as the share of products of any firm where the second-best producer is of type k.

[11] It should be noted however that the parameters are calibrated jointly. The model is nonlinear in parameters and hence the effect of a parameter on outcomes in general depends on the value of other parameters.

Using these two properties of the model, the "average" price level for a firm j is

$$p_{j,t} = \bar{w}_0(1+\bar{g})^t \cdot \gamma_j \sum_{k'} \frac{\bar{S}_{k'}}{\varphi_{k'}} \propto \gamma_j. \tag{35}$$

Equation (35) says that along the BGP, cross-firm price variation is entirely driven by cross-firm differences in the step size of innovations γ_j. The heterogeneity in process efficiency does not explain any of the differences in the price level across firms as such productivity differences just affect the markups but leave prices unaffected due to Bertrand competition. This gives us the following prediction about the dispersion of the prices *across* firms

$$Var_j(\log(p_j)) = Var_j(\log(\gamma_j)) = (\phi_{HH} + \phi_{LH})(\phi_{LL} + \phi_{HL})(\log \Gamma)^2, \tag{36}$$

where Var_j is the variance operator using the distribution of firm types. Given the distribution parameters, the dispersion of prices across firms is increasing in the gap in step sizes Γ.

Next, we define firm-level TFPR as firm-level revenue over firm-level costs and firm-level TFPQ as TFPR divided by the firm-level price index we described earlier. Since we do not have physical capital in our model, TFPR is proportional to firm markup which is the inverse of firm-level labor share. Recall that the labor share of a firm j along the BGP is given by

$$\lambda_j = \frac{\bar{S}\varphi_H + (1-\bar{S})\varphi_L}{\gamma_j \varphi_j} \tag{37}$$

where $\bar{S} = \bar{S}_{HH} + \bar{S}_{HL}$ is the sales-share of high process efficiency firms. Hence, cross-firm differences in TFPR are driven both by differences in the step sizes of innovation and by process efficiency heterogeneity i.e.

$$\text{TFPR}_j \propto \gamma_j \varphi_j. \tag{38}$$

As a consequence, TFPQ of a firm—defined as its TFPR divided by firm price level—is proportional to its process efficiency:

$$\text{TFPQ}_j \equiv \frac{\text{TFPR}_j}{p_j} \propto \varphi_j. \tag{39}$$

Therefore, dispersion in firm-level TFPQ is given by

$$Var_j(\log(\text{TFPQ}_j)) = Var_j(\log(\varphi_j)) = (\phi_{HH} + \phi_{HL})(\phi_{LL} + \phi_{LH})(\log \Delta)^2 \tag{40}$$

and is informative of the gap in process efficiency Δ. All else equal, higher Δ implies higher TFPQ dispersion.

Furthermore, given the dispersion in TFPQ and prices, dispersion in TFPR is informative about the covariance of step sizes and process efficiency across firms as we have

$$\begin{aligned} & \frac{Var_j(\log(\text{TFPR}_j)) - Var_j(\log(\gamma_j)) - Var_j(\log(\varphi_j))}{2} \\ = \ & Cov_j(\log(\gamma_j), \log(\varphi_j)) \\ = \ & (\phi_{HH}\phi_{LL} - \phi_{HL}\phi_{LH})(\log \Gamma)(\log \Delta). \end{aligned} \tag{41}$$

Given the gap in step sizes and process efficiency, the covariance increases with $\phi_{HH}\phi_{LL} - \phi_{HL}\phi_{LH}$. For example, the covariance is negative if the distribution of types has more weight on high step size and low process efficiency firms than high step size and high process efficiency firms ($\phi_{HL}\phi_{LH} > \phi_{HH}\phi_{LL}$).

Given Δ, price dispersion across lines *within* a firm is determined by the share of lines where the firm faces a high process efficiency second-best producer. Since along the BGP this share is equal to $\bar{S}$ for all firms, the *within* firm price dispersion weighted by the sales is given by

$$Var_i(\log(p(i, j(i)))) = \bar{S}(1 - \bar{S})(\log \Delta)^2. \tag{42}$$

Note that this is different from the dispersion of prices across firms.

Furthermore, we have

$$\frac{Var_i(\log(p(i,j(i))))}{Var_j(\log(\text{TFPQ}_j))} = \frac{\bar{S}(1-\bar{S})}{(\phi_{HH}+\phi_{HL})(1-\phi_{HH}-\phi_{HL})}.$$

From (28), we can see that the above ratio approaches 1 when ψ_o/J increases and each firm's sales share approaches $1/J$. Hence, the value of within firm price dispersion relative to across firm TFPQ dispersion is helpful for pinning down the overhead cost parameter.

On the other hand, the aggregate cost-weighted markup is informative about the step size γ_L given relative step sizes Γ and process efficiency Δ. Aggregate markup along the BGP satisfies

$$\frac{\bar{Y}}{wL} = \frac{1}{\bar{S}\Delta + 1 - \bar{S}} \frac{\gamma_L}{\sum_k \bar{S}_k \frac{\gamma_L}{\gamma_k} \frac{\varphi_L}{\varphi_k}}. \tag{43}$$

Hence an increase in γ_L shifts—for given relative step sizes Γ and relative process efficiency Δ—the entire markup distribution to the right.

Finally, as shown in (28), the sales share distributions across firms depend on the underlying distribution of types. Hence, we use the dispersion and skewness (median relative to mean) of the sales share distribution together with the covariance of TFPQ and firm level prices as three moments for calibrating the value of ϕ_{HH}, ϕ_{HL} and ϕ_{LH}. The value of ϕ_{LL} is determined by one minus the sum of these three values.

3.2 Accounting for measurement errors

The previous section described our general strategy for disciplining the parameters in our model. Before carrying out the calibration, we need to also lay out a strategy for dealing with measurement errors in the data as studies found large measurement errors in firm-level measured TFPR and prices (see for example Bils, Klenow, and Ruane, 2021). Here we lay out a strategy that addresses classical multiplicative measurement errors commonly used in the literature.

Let $\hat{v}$ denote the measured value of variable v. Suppose measured price, TFPR and TFPQ for a firm j is assumed to be related to the true price, TFPR and TFPQ as follows

$$\begin{aligned}
\ln \widehat{p}_j &= \ln p_j + \epsilon_j^p && (44)\\
\ln \widehat{\text{TFPR}}_j &= \ln \text{TFPR}_j + \epsilon_j^{\text{TFPR}} && (45)\\
\ln \widehat{\text{TFPQ}}_j &\equiv \ln \frac{\widehat{\text{TFPR}}_j}{\widehat{p}_j} = \ln \text{TFPQ}_j + \epsilon_j^{\text{TFPR}} - \epsilon_j^p && (46)
\end{aligned}$$

where ϵ_j^{TFPR} and ϵ_j^p are independent of each other, p and TFPR. Note that we construct measured TFPQ by dividing measured TFPR by measured price.

The dispersion in measured price, TFPR and TFPQ across firms are given by

$$\begin{aligned}
Var_j(\ln \widehat{p}_j) &= Var_j(\ln \gamma_j) + Var_j(\epsilon_j^p)\\
Var_j(\ln \widehat{\text{TFPR}}_j) &= Var_j(\ln \gamma_j + \ln \varphi_j) + Var_j(\epsilon_j^{\text{TFPR}})\\
Var_j(\ln \widehat{\text{TFPQ}}_j) &= Var_j(\ln \varphi_j) + Var_j(\epsilon_j^{\text{TFPR}}) + Var_j(\epsilon_j^p)
\end{aligned}$$

while

$$\frac{Var_j(\ln \widehat{\text{TFPR}}_j) - Var_j(\ln \widehat{p}_j) - Var_j(\ln \widehat{\text{TFPQ}}_j)}{2} = Cov_j(\ln \gamma_j, \ln \varphi_j) - Var_j(\epsilon_j^p).$$

Therefore, the dispersion in measured price and TFPQ across firms overstates the true dispersion in step sizes and process efficiency. Also, the gap between the dispersion in measured TFPR and the dispersion in measured prices and TFPQ understates the true covariance between step sizes and process efficiency when there are large measurement errors in prices. Hence, we need to know the degree of measurement errors $Var_j(\epsilon_j^p)$ and $Var_j(\epsilon_j^{\text{TFPR}})$ to correctly infer the parameters in the model.

How do we gauge the extent of measurement errors? For the firms in our sample, we have a measure of labor input that is from a separate source and is not used to construct TFPR.[12] We construct a firm's labor input $\widehat{l_j}$ using this measure. Suppose $\widehat{l_j}$ deviates from the true labor input of a firm l_j by a classical multiplicative error

$$\ln \widehat{l_j} \quad = \quad \ln l_j + \epsilon_j^l, \tag{47}$$

where measurement error ϵ_j^l is independent of the measurement errors in prices and TFPR as well as step sizes and process efficiency. Given parameters, the model implies a relationship between a firm's employment , a firm's price and TFPQ through the relationship between employment share, step sizes and process efficiency. Measurement errors attenuate this relationship towards zero. Therefore, we can project measured employment from the independent source onto measured prices and TFPQ to generate additional moments to pin down the degree of measurement errors in prices and TFPQ.

More precisely, an OLS regression of $\ln \widehat{l_j}$ on $\ln \widehat{p_j}$ and a constant yields the slope coefficient

$$\hat{\beta}_{l,p} = \frac{Cov_j(\ln l_j, \ln \gamma_j)}{Var_j(\ln \gamma_j) + Var_j(\epsilon_j^p)}$$

while regressing on $\ln \widehat{\text{TFPQ}}_j$ and a constant yields the slope coefficient

$$\hat{\beta}_{l,\text{TFPQ}} = \frac{Cov_j(\ln l_j, \ln \varphi_j)}{Var_j(\ln \varphi_j) + Var_j(\epsilon_j^{\text{TFPR}}) + Var_j(\epsilon_j^p)}.$$

Given model parameters, both coefficients approach zero as measurement errors in prices and TFPR increase. Hence, we will account for measurement error by adding these coefficients to our calibration targets.

[12] We construct this measure using the matched employer-employee data (DADS) which we aggregate at the firm level, while TFPR is constructed using only information from the firm's balance sheet using FARE. See the Online Appendix B for more detail.

3.3 Data

Having laid out the general calibration strategy, let us next describe how we construct calculate the calibration targets from firm-level and price-level data. Our main source of data is the balance sheet and income statements of all firms that are subject to the standard corporate tax scheme in France (FARE). From this data, we construct firm-level measures of nominal value added (VA_j), the total wage bill (W_j), and total net tangible and intangible asset value (K_j). We augment these data using the matched employer-employee dataset (DADS), which contains detailed information on the wage structure of each employee in France, including the total number of hours worked within the year. We use the DADS data to construct a measure of total hours worked (H_j), which we use to address measurement error. More details about the dataset and cleaning procedures are given in the Online Appendix B.

Merging FARE and DADS leaves out some firms, namely those without any paid employees. We also focus on the manufacturing sector. At the end, our dataset is an unbalanced panel of 95,219 unique firms observed over the period 2012–2019 (403,492 observations).[13]

To construct a measure of firm-level TFPR, we first compute industry-specific cost share

$$\alpha_{s,t} = \frac{\sum_{s(j,t)=s} rK_{j,t}}{\sum_{s(j,t)=s} rK_{j,t} + W_{j,t}} \quad \in [0,1]$$

where $s(j,t)$ is the firm j industry during year t. $\alpha_{s,t}$ is therefore the average value of the cost share of capital in a given industry in year t (we consider a breakdown of the manufacturing sector into 21 2-digit-NACE industries). We

[13] We use the "consolidated" version of FARE which aggregate legal units (or "siren") into enterprise groups ("entreprises profilés"). This procedure mostly concerns the largest firms and allows a better consideration of their balance sheets. Indeed, the largest groups are often made up of numerous legal entities, each of them corresponding to an entry in the unconsolidated version of FARE. For this reason, we have to restrict the sample to 2012–2019.

take $r = 0.1$ as a benchmark. Then the TFPR of a firm j in year t is computed as

$$\widehat{\text{TFPR}}_{j,t} = \frac{VA_{j,t}}{K_{j,t}^{\alpha_{s(j,t),t}} W_{j,t}^{1-\alpha_{s(j,t),t}}}$$

Measuring average unit price

To estimate the average unit price for the calculation of firm-level TFPQ, we use the "Enquête Annuelle de Production" (EAP). The EAP is a survey of manufacturing firm which covers all firms with more than 20 employees and which splits sales $P_j Y_j$ at the product level $p(i,j)y(i,j)$.[14]. The database also informs us about the quantity $y(i,j)$ of each product i sold by firm j.

Therefore, for each firm j and each product i produced during year t we observe both firm j's quantity $y_t(i,j)$ of product i sold during year t and the total revenue $p_t(i,j)y_t(i,j)$. We construct unit price $\widehat{p_t(i,j)}$ by dividing product sales by product quantity sold i.e. $(p_t(i,j)y_t(i,j))/y_t(i,j)$ and estimate firm-level price $\widehat{P}_{j,t}$ as the sales weighted average of product unit prices

$$\widehat{P}_{j,t} = \prod_{i=1}^{N_j} \widehat{p_t(i,j)}^{\omega_{i,j,t}}, \tag{48}$$

where $N_{j,t}$ is the number products sold by the firm and $\omega_{i,j,t}$ is the weight of product i in firm j's production in year t, defined as

$$\omega_{i,j,t} = \frac{p_t(i,j)y_t(i,j)}{\sum_{i'=1}^{N_{j,t}} p_t(i',j)y_t(i',j)}.$$

To account for the fact that prices are aggregated across products with potentially different units, we standardize the unit price $\widehat{p(i,j)}$ prior to computing the value of $\widehat{P}_j$ in (48) by dividing it by the average value of unit price observed over all products belonging to the same product groups. We

[14]Our product groups correspond to the classification used by the PRODCOM survey conducted by Eurostat. See Online Appendix B for more details. There are about 4,000 different product codes.

construct 1,100 groups product groups using the same first 6 digit of the product codes.

We then construct TFPQ at the firm level as:

$$\widehat{\text{TFPQ}}_{j,t} = \frac{\widehat{\text{TFPR}}_{j,t}}{\widehat{P}_{j,t}}.$$

Our final dataset, with information on unit price, TFPR and TFPQ, includes 32,641 firms with on average about 50,000 product level observations per year.

3.4 Baseline calibration

Table 1 displays the data moments that we use to calibrate the parameters of the model. We will briefly describe how we calculate the moments and refer the reader to Online Appendix C for details. We calculate the target for dispersion in measured price, TFPQ and TFPR by first calculating firm-level price, TFPQ and TFPR as described in the previous section and then calculating their dispersion across firms within industry-year groups. We calculate aggregate dispersion in each year by averaging industry dispersion with industry cost-share weights.

Next, we calculate within firm price dispersion for firms with two or more products in a given year. As prices may differ systematically across product attributes we classify products by product groups and standardize each product price by dividing the price by the average unit price of the corresponding product group. For each firm in a year, we then calculate the dispersion of standardized prices across its products. To arrive at our final measure, we calculate an average of firm-level dispersion in each year by weighing each firm by its value-added share and then average across years.

To calculate moments of the sales share distribution, for each industry-year, we calculate the sales share of the firms in the industry and calculate the standard deviation and median sales share. We then standardize the industry-year moments by the mean sales-share, which is by definition the inverse of the number of firms in the industry. We calculate the calibration target by averaging across industries using industry value-added shares.

The next two moments are slope coefficients obtained when we regress log firm employment within an industry year on log of firm-level prices and log of firm-level TFPQ, controlling for industry-year fixed effects.

We obtain the remaining target values from external sources. For average markups, we use the estimate from Hall (2018) over 1988–2015 for U.S. manufacturing. We take average annual rate of manufacturing productivity growth from EU-KLEMS over 1995–2017 and the real interest rate for U.S. from 1996 to 2016 as estimated by Farhi and François (2018).

Table 2 display the calibrated parameter values. When fitting the parameters, we give more weight to the regression coefficients of labor input on firm price and TFPQ (moments 7 and 8) in Table 1 because these are the most informative about the correlation between step sizes and process efficiency across firms. Since in the model price and TFPQ are proportional to process efficiency and step size respectively, these coefficients ask for parameters such that on average firm size increases with firm process efficiency but declines with firm step size in the model. However, all else equal, firm employment increases with both process efficiency and step size. Hence, the model needs a negative correlation between step size and process efficiency ($\phi_{HH}\phi_{LL} < \phi_{LH}\phi_{H}$) to fit the empirical relationship between size, price and TFPQ. It also needs the step size gap Γ to be smaller than the process efficiency gap Δ so that large firms have high process efficiency. We find that we need $\Gamma = 1.02$ and $\Delta = 1.07$.

We also find that $\phi_{HL} < \phi_{LH}$ so that the median firm has low process efficiency and high step size. This allows the model to generate median sales share below the mean as we see in the data. Given Γ, the dispersion in price increases when $\phi_{HL} + \phi_{LL}$ approaches half. Therefore, the dispersion of price in the data pins down ϕ_{LL} to 0.109.

The calibrated level of γ_L is 1.22, close to the target on aggregate markup level. We need the discount factor β to be 0.977 to fit the interest rate target and R&D costs ψ_z/Z to be 7.542 to fit the productivity growth rate. There is large dispersion in firm sales share in the data. The model picks ψ_o/J = 0.0875 to fit this dispersion. However, with only four types of firms, the model is not able to fit the size of the dispersion observed in the data. Finally, as a by-product of our calibration, we find that measurement error in prices is about 3% of the observed dispersion in firm-level prices while measurement error in TFPR is 3/4 of the dispersion in measured firm-level TFPR.

Table 1: Baseline calibration

Targets	Data	Model
1. Dispersion in firm-level prices, $Var_j(\log \hat{p})$	0.672	0.022
2. Dispersion in firm-level TFPQ, $Var_j(\log \widehat{\text{TFPQ}})$	0.837	0.139
3. Dispersion in firm-level TFPR, $Var_j(\log \widehat{\text{TFPR}})$	0.154	0.116
4. Within firm dispersion in product prices, $Var_{j(i)}(\log \widehat{p(i,j)})$	1.017	0.001
5. Dispersion in firm sales shares (StDev/Mean)	5.304	0.205
6. Skewness in firm sales shares (Median/Mean)	0.126	0.916
7. Project firm employment share on firm price, $\hat{\beta}_{\bar{E},p}$	-0.030	-0.030
8. Project firm employment share on firm TFPQ, $\hat{\beta}_{\bar{E},\text{TFPQ}}$	0.032	0.032
9. Markup level	1.22	1.23
10. Productivity growth rate (ppt/year)	2.83	2.82
11. Interest rate (ppt/year)	5.2	5.2

Source: 1 to 8: authors' calculations from DADS, EAP and FARE, French manufacturing, 2012–2019. 9: Hall (2018), U.S. manufacturing, 1988–2015. 10: EU-KLEMS, French manufacturing, TFP growth in labor-augmenting form, 1995–2017. 11: Farhi and François (2018), U.S. all economy, 1996–2016.

3.5 Welfare decomposition

We next evaluate the decomposition in equation (20) at the calibrated parameters to compare welfare in the decentralized equilibrium with social optimum. Table 3 displays the components in equation (20) and Table 4 compares the allocation of products by firm types. The first row of Table 3 displays the distance of the decentralized economy relative to the first best in

Table 2: Baseline calibrated parameters

	Model parameters	Value
ψ_o/J	overhead cost	0.0875
γ_L	lowest step size	1.22
γ_H	highest step size	1.24
$\Gamma = \gamma_H/\gamma_L$	Step size gap	1.018
Δ	Process efficiency gap	1.069
ϕ_{HH}	Share of firms with high process efficiency and high step size	0.001
ϕ_{HL}	Share of firms with high process efficiency and low step size	0.223
ϕ_{LH}	Share of firms with low process efficiency and high step size	0.667
ϕ_{LL}	Share of firms with low process efficiency and low step size	0.109
β	discount factor	0.977
ψ_z/Z	R&D cost relative to R&D labor	7.542
	Measurement error parameters	**Value**
$\frac{Var_j(\epsilon^P)}{Var_j(\ln \widehat{P}_j)}$	Measurement error in price relative to observed price	0.033
$\frac{Var_j(\epsilon^{\text{TFPR}})}{Var_j(\ln \widehat{\text{TFPR}}_j)}$	Measurement error in TFPR relative to observed TFPR	0.75

consumption equivalence form. Namely, we calculate the percent increase ξ in the consumption level $\bar{C}_0$ in the decentralized equilibrium such that welfare is the same as the planner's allocation or formally

$$\log(1+\xi) = \log\left(\frac{\bar{C}_0^P}{\bar{C}_0^D}\right) + \frac{\beta}{1-\beta}\log\left(\frac{1+\bar{g}^P}{1+\bar{g}^D}\right).$$

Table 3: Welfare comparison

Welfare loss in consumption equivalence terms ξ	0.656%
Relative growth $(1+\bar{g}^P)/(1+\bar{g}^D)$	1.00045
Relative consumption level $\bar{C}_0^P/\bar{C}_0^D$	0.987
Relative $(1-\bar{o}^P)/(1-\bar{o}^D)$	0.995
Relative process efficiency Φ^P/Φ^D	0.991
Relative allocative efficiency $\mathcal{M}^P/\mathcal{M}^D$	1.0008

Overall, the welfare loss is 0.656% in consumption equivalence term. The planner chooses a higher growth rate (2.88 vs 2.83 ppt) than in the decentralized equilibrium as it is optimal to allocate more products to high step size firms as shown in Table 4. The growth rate difference is small because the calibrated step size gap is small. On the other hand, the consumption level $\bar{C}_0$ is lower in the planner's solution as the planner sacrifices some resources on overhead and allocates a smaller market shares to high process efficiency firms in order to exploit the growth potential. The higher overhead cost share of the planner arises because it is optimal to have more products produced by each high step size firms. In Table 4, the planner increases the number of products produced by each high step size firm. Finally, the planner has slightly higher allocative efficiency. This term is small because $\gamma_k \varphi_k$ do not vary much due to the negative correlation between step sizes and process efficiency and as a consequence there is not so much markup dispersion.

Table 4: Product share (in ppt) and firm size, planner vs. decentralized

	$\bar{S}_{HH}$	$\bar{S}_{HL}$	$\bar{S}_{LH}$	$\bar{S}_{LL}$	High proc. eff share	High step size share
Planner	0.20	17.4	81.8	0.6	17.6	82.0
Decentralized	0.15	30.7	60.9	8.3	30.9	61.1
	$\bar{n}_{HH}$	$\bar{n}_{HL}$	$\bar{n}_{LH}$	$\bar{n}_{LL}$		
Planner	1.95	0.78	1.23	0.06		
Decentralized	1.52	1.38	0.91	0.76		

The first panel displays the share of products produced by firms of each type. "High proc. eff share" = $\bar{S}_{HH} + \bar{S}_{HL}$ and "High step size share" = $\bar{S}_{HH} + \bar{S}_{LH}$. The $\bar{n}_k$ are the number of products produced by each firm type relative to the mean $1/J$.

4 Conclusion

In this paper, we characterized the optimal research allocation in an economy where markup heterogeneity may be due to both differences in the step size of quality innovations and to differences in process efficiency across firms. To the extent that, unlike process efficiency, quality innovations confer knowledge spillovers onto other firms, we find the social planner will tilt innovation effort toward high quality step firms to enhance knowledge spillovers and thereby growth. At the same time, the planner will seek to undo the static misallocation of production labor created by markup dispersion.

We used data on French manufacturing firms from 2012 to 2019 to calibrate our model, and inferred a negative correlation between the step size of innovations and process efficiency across firms, and more variation in process efficiency than in quality step sizes across firms. This, in turn, implied that larger firms in France typically possess higher process efficiency but lower step sizes for their quality innovation. A corollary is that research subsidies should not favor larger firms in the context of the French economy. Interestingly, it turns out that the French research subsidy system does exactly the opposite — see Aghion, Antonin, and Bunel (2021, chapter 12).

There are additional sources of firm heterogeneity that we did not model that can also affect R&D misallocation. Examples include firm differences in research efficiency such as Luttmer (2011), about which the planner may have less information as in Akcigit et al. (2021). This could generate size differences that are unrelated to markup differences.

Our paper features a representative consumer, but could be extended to feature heterogeneity of firm ownership. In this way our theory and empirics could be extended to connect to a growing literature on income and wealth inequality such as Aghion, Akcigit, Bergeaud, Blundell, and Hémous (2019), Boar and Midrigan (2019), Piketty (2018); Piketty and Saez (2003), and Song, Price, Guvenen, Bloom, and Von Wachter (2019).

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