Gov 2006: Formal Political Theory II 2013
Harvard University
Notes on Garfinkel-Skaperdas (2007): Economics of Conflict: An Overview

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1 Technologies of Conflict

The main idea is that agents can invest in appropriating power: they produce 'guns' instead of 'butter'. Let G_1 be the gun choice of the first agent, G_2 that of the second agent (there are only two of them).

We need some kind of probability that appropriation given G_1 and G_2 indeed works out for agent 1. This is what the power function is for:

$$p(G_1, G_2) = \frac{f(G_1)}{f(G_1) + f(G_2)} \tag{1}$$

with $f(\dot{})$ being non-negative and increasing (and assume $p(G_1, G_2) = \frac{1}{2}$ if $f(G_1)+f(G_2) = 0$).

In particular, let us work with the 'power form' or 'ratio form':

$$p(G_1, G_2) = \frac{G_1^m}{G_1^m + G_2^m} \tag{2}$$

with $m > 0$ and $m \leq 1$. m is the effectiveness of guns.

Another form is the 'logit specification' where $f(G_i) = e^{kG_i}$ for $k > 0$:

$$p(G_1, G_2) = \frac{e^{kG_1}}{e^{kG_1} + e^{kG_2}} = \frac{1}{1 + e^{k(G_2 - G_1)}}, \tag{3}$$

note that in this specification if we add a constant C to both parties' guns, p remains the same. This is less used, but it has a nice interpretation: if performance in a battle is stochastic ($G_i + \epsilon_i$ where ϵ_i is the error term) and the error term follows an extreme value distribution then the outcome of the battle will follow the logit specification.

For equation **??** and the special case of $m = 1$ we can derive that it is the winning probability when the performance of a party is $\theta_i G_i$, where the error term θ_i follows an exponential distribution.

We can also introduce one party to be favored in the conflict by choosing $\phi \in (0, 1)$:

$$p(G_1, G_2) = \frac{\phi f(G_1)}{\phi f(G_1) + (1 - \phi) f(G_2)}. \tag{4}$$

2 Competing for a Resource

Next, we need to decide on the utility function of agents. What is the trade-off? Butter versus guns: there is a $2R$ resource that can be taken over with guns with probability $p(G_i, G_j)$ by agent i. Butter is whatever is not consumed as guns:

$$V_i(G_1, G_2) = p_i(G_1, G_2) 2R - G_i. \tag{5}$$

What is the optimal gun choice?

$$\frac{\partial V_i(G_1, G_2)}{\partial G_i} = \frac{\partial p_i(G_1, G_2) 2R}{\partial G_i} - 1. \tag{6}$$

Using the ratio form:

$$\frac{m G_i^{m-1} G_j^m}{(G_1^m + G_2^m)^2} 2R = 1. \tag{7}$$

(SOC?). Therefore we have two equations:

$$\frac{m G_1^{m-1} G_2^m}{(G_1^m + G_2^m)^2} 2R = 1, \tag{8}$$

and

$$\frac{m G_2^{m-1} G_1^m}{(G_1^m + G_2^m)^2} 2R = 1, \tag{9}$$

Divide them by each other to get: $G_1 = G_2$. The Nash equilibrium is going to be symmetric. Plugging back in yields:

$$m(G_1^{2m-1}) 2R = 4 G_1^{2m}, \tag{10}$$

or

$$G_1 = G_2 = \frac{mR}{2} \tag{11}$$

Thus $p^* = \frac{1}{2}$ and

$$V_i(G_1, G_2) = \frac{1}{2} 2R - \frac{mR}{2} = (1 - \frac{m}{2}) R. \tag{12}$$

(Comparative statics?). Note that if taking over was not possible (or could be credibly contracted against, agents' combined utilities would be higher - a bit like a prisoner's dilemma). In the realist school of international relations the key assumptions is that of anarchy though: no authority exists that can credibly enforce contracts. The school (which is more like a loose group of similar approaches) argues that no-one answer's countries 911 calls...

3 Guns versus Butter

Now we introduce productivity as well as different resources for agents (R_i and R_j). Assume that butter production is such that:

$$R_i = G_i + \frac{X_i}{\beta_i}, \tag{13}$$

where just think of X_i as the butter that a country likes to have. So far we have been assuming $\beta_i = 1$. Note that β_i is a measure of productivity, the higher it is the more X_i can be produced out of any resource. Let country 1 be more productive: $\beta_1 \geq \beta_2$.

Let us recalculate our Nash Equilibrium with this new setup. Now the maximand is:

$$V_i(G_1, G_2) = p_i(G_1, G_2) \sum_{j=1}^{2} \beta_j (R_j - G_j). \tag{14}$$

It is interesting to think about how you would express the cost of war now in this setup. What is the optimal gun choice now? FOC:

$$\frac{\partial V_i(G_1, G_2)}{\partial G_i} = \frac{\partial p_i(G_1, G_2)}{\partial G_i} (\sum_{j=1}^{2} \beta_j (R_j - G_j)) - \beta_i p_i(G_1, G_2) = 0. \tag{15}$$

(SOC?).

Next consider the special case $\beta_1 = \beta_2$. Then we get (by dividing the two FOC's by each other):

$$G_1^* = G_2^* = \frac{m}{m+1} \tilde{R}, \tag{16}$$

and utilities:

$$V_1^* = V_2^* = \frac{1}{m+1} \tilde{R}, \tag{17}$$

where $\tilde{R} = \frac{R_1 + R_2}{2}$ (Comparative Statics?) Note that the size of the resource allocated to i does not enter into utility, only the average size. This is called the 'paradox of power' (the poorer side devotes relatively more to appropriation) by Jack Hirshleifer (1991). But why is this? It is actually not all that surprising given the all-or-nothing nature of war. What if there was a cost of transfer or if war destroyed some of the resource?

Even more interestingly, in the general case ($\beta_i, \beta_j \geq 0$) we can similarly derive:

$$\frac{G_1^*}{G_2^*} = \left(\frac{\beta_2^*}{\beta_1^*} \right)^{\frac{1}{m+1}} \tag{18}$$

Note that who ever has the higher productivity will end up with fewer guns, so their utility will be lower! (Is this because of specialization? In some sense both agents are maximizing the same thing, aren't they?)

4 Settlement in the Shadow of Conflict

We now introduce bargaining, that was the key idea of James Fearon (1995) (and actually Thomas Schelling (1960) earlier on). The idea is that conflict is costly ex post, so why not agree on the same distribution of resources without actually fighting? Fearon theorized that with unitary actors the three reasons could be (1) informational asymmetry with incentives to misrepresent information; (2) a commitment problem and (3) issue indivisibility.

In these setups we assume the following structure of the game: (1) parties choose guns; (2) they negotiate (anyone can make any transfer); (3) any party can start a war. We solve by backward induction.

What is the disincentive to war? It could be costliness: that $1-\phi$ fraction ($\phi \in (0,1)$) of the total resource gets destroyed. Then there will be at least one division of the common resource whose choice strictly dominates fighting (which one?).

Another way to get war to be strictly dominated is to introduce risk aversion: assume $U(\cdot)$ is strictly concave. Then the outcome of war is the expected utility:

$$E[V_i(G_1,G_2)] = p_i(G_1,G_2)U(\sum_{j=1}^{2}\beta_j(R_j - G_j)) + (1-p_i(G_1,G_2))U(0). \tag{19}$$

But then there is the settlement where each actor receives a share of the resource equal to its potential might yielding:

$$V_i(G_1,G_2) = U(p_i(G_1,G_2)\sum_{j=1}^{2}\beta_j(R_j - G_j)) + (1-p_i(G_1,G_2))U(0). \tag{20}$$

By Jensen's inequality (**??**)>(**??**), so war is strictly dominated.

Another way to make war endogenously inefficient ex post is to assume that whoever owns the resource originally can put it to better use (but could this not go the other way around?). Assume, for instance, that conflict is over $T_1 + T_2$ where T_i is the holding of territory by agent i. Then assume that to produce butter we need not just X_i (butter earlier) but some T_i too:

$$V_i = B_i = F(T_i, X_i), \tag{21}$$

where the production technology $F(\cdot)$ is strictly concave and increasing. We also assume complementarity: $F_{T_i X_i} > 0$, so that one extra unit of either input enhances the other inputs productivity. Then fix X_i at some level. Under this setup again consider dividing up the resource based on power probabilities. This would yield a utility:

$$V_i = B_i = F(p_i(G_1,G_2)T, X_i), \tag{22}$$

while in war the expected utility is

$$E[V_i] = E[B_i] = p_i(G_1,G_2)F(T,X_i) + (1-p_i(G_1,G_2))F(0,X_i), \tag{23}$$

so again by Jensen's inequality (??)>(??). We have used the concavity of the production technology but should technology be concave? (Also as a technicality do we need the complements assumption, or just that the Hessian of $F(\cdot)$ is negative definite, so that the cross-partials are not too negative.)

Next, let us solve the model, knowing that peace will endure. How does this affect the choice of guns?

Let us go back to the special $\beta_1 = \beta_2 = 1$ case. By backward induction, we calculate the utilities in case of war first. This utility is simply:

$$V_i^w(G_1, G_2) = p_i(G_1, G_2)\phi(2\tilde{R} - G_1 - G_2), \tag{24}$$

where recall that $\tilde{R} = \frac{R_1+R_2}{2}$. Under the settlement with share δ_i going to i the utility is:

$$V_i^s(G_1, G_2) = \delta_i(G_1, G_2)(2\tilde{R} - G_1 - G_2). \tag{25}$$

Observe that if agents expected a war to occur they would still choose:

$$G_i^w = \frac{m}{m+1}2\tilde{R}, \tag{26}$$

since the multiplicative ϕ term disappears in the FOC's, leading to payoffs (butter) from war:

$$V_i^w = \frac{\phi}{m+1}2\tilde{R}, \tag{27}$$

while,

$$V_i^s = \frac{1}{m+1}2\tilde{R}. \tag{28}$$

Now what if the division of the resource occurs according to winning probabilities? Then $\delta_i(G_1, G_2) = p_i(G_1, G_2)$. The case is identical to the benchmark one (again since ϕ drops out). In this simple structure the actual lack of fighting does not influence gun choices at all. (And this is usually the way the resource is thought to be divided up in bargaining.)

Next what if the surplus from avoiding war gets divided up, say equally? This is the Nash-bargaining solution. We assume:

$$V_1^s - V_1^w = V_2^s - V_2^w. \tag{29}$$

Substituting in yields:

$$(\delta_1(G_1, G_2) - \phi p_1(G_1, G_2))(2\tilde{R} - G_1 - G_2) = (1 - \delta_1(G_1, G_2) - \phi + \phi p_1(G_1, G_2))(2\tilde{R} - G_1 - G_2), \tag{30}$$

which can be rearranged to:

$$\delta_i(G_1, G_2) = \phi p_i(G_1, G_2) + (1-\phi)\frac{1}{2}, \tag{31}$$

which is a weighted average of relative power and a fair distribution. Interestingly, the more destructive war is (the lower is ϕ) the less it matters for the division of the resource. This is because conflict is a worse threat in this case as it destroys so much that getting a lot of the little left yields little bargaining power.

Now we use backward induction again. Agents knowing they will divide up the resource according to (**??**), they simultaneously choose guns. The FOC's are (using the chain rule and (**??**)):

$$\frac{mG_1^{m-1}G_2^m}{(G_1^m+G_2^m)^2}\phi(2\tilde{R}-G_1-G_2)=\delta_1(G_1,G_2), \tag{32}$$

and

$$\frac{mG_2^{m-1}G_1^m}{(G_1^m+G_2^m)^2}\phi(2\tilde{R}-G_1-G_2)=1-\delta_1(G_1,G_2), \tag{33}$$

Substituting and dividing through yields:

$$G_1^*=G_2^*=\frac{\phi m}{\phi m+1}\tilde{R}. \tag{34}$$

Comparative statics: $\phi<1$ so the gun choice in this case is lower than under war (or if the surplus is divided based on winning probabilities). This is because guns are less effective at getting at the bargaining surplus. The utilities now are also higher:

$$V_i^*=\frac{1}{\phi m+1}\tilde{R}. \tag{35}$$

The more destructive war is, the higher is the utility. This is because war is off the equilibrium path but wasting resources on guns can occur if it can be used as a credible enough threat to appropriate the other actor's resources.

5 Why Fight?

Garfinkel and Skaperdas then discuss informational asymmetries and commitment problems as causes of ex post inefficient wars. In particular, they build a model of the commitment problem. Fearon (1995) argues for three sources of the commitment problem: first-strike advantage, preventive war, and strategic territory. All of these can be thought economically as involving incomplete contracting. Nowadays, the commitment problem is probably the most accepted cause of inefficient wars, and resonates well with the traditional starting assumption of anarchy of the realist school of international relations (but is more precise).

Garfinkel and Skaperdas choose to model the commitment problem in a different light from Fearon, given that they have incorporated the first-stage choice of guns. If there are two periods and the adversary is eliminated by the second period, no wasteful gun production needs to take place then.

To find the subgame perfect equilibrium we go to the terminal nodes of the game. If there is no war in the first period then the game at time $t=2$ will just mimic the one-period model. (Would an infinite horizon change the picture? Recall that with a simple prisoner's dilemma the choice between a finite and an infinite horizon makes a big difference: in a finitely repeated version of the game, mutual defection is still the only equilibrium, while for sufficiently forward-looking players the folk theorem holds in the infinite version. The commitment problem is actually not susceptible to the same phenomenon, why not? Because the adversary is eliminated in a war forever, therefore no punishment (like grim trigger) is possible.)

In deriving the second-period choices, the paper changes the basic form of the game, but to make it more comparable to the previous models, let us use the same setup as before. Then under the split-the-surplus role second-period gun choices have been found to be:

$$G_{12}^* = G_{22}^* = \frac{\phi m}{\phi m + 1}\tilde{R}. \tag{36}$$

and

$$V_{i2}^* = \frac{1}{\phi m + 1}\tilde{R}. \tag{37}$$

Now go back to the war decision in the first period. If there is war then no guns need to be purchased in the next period and instead if the war is won utility will be: $\phi 2\tilde{R}$, where ϕ comes in because of the destructiveness of the first-period war.

Let $\lambda \in (0, 1]$ be the discount factor. The higher this discount factor is, the more patient players are. Then player i's utility is:

$$U_i = V_{i1} + \lambda V_{i2}. \tag{38}$$

When there is war then the expected utility is:

$$E[U_i^w] = p_i(G_{11}, G_{21})\phi(2\tilde{R} - G_1 - G_2)(1 + \lambda), \tag{39}$$

while if there is settlement then utility is:

$$E[U_i^s] = \delta_i(G_{11}, G_{21})(2\tilde{R} - G_1 - G_2) + \lambda\frac{1}{\phi m + 1}\tilde{R}. \tag{40}$$

For war not to occur in the first period we need that: $U_i^s \geq U_i^w$ for both players. (How would bargaining over δ change this?) For instance in the symmetric case $p = \frac{1}{2}$ we have $\delta = \frac{1}{2}$ too so settlement is preferred when: