Section 10

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1. Externalities, Coase Theorem, Coasian Bargaining, social optimum, efficient outcomes, property rights, transaction costs
2. Cournot Competition, Monopoly, Oligopoly

1 Externality and Coasian Bargaining

- Coase Theorem: With well-defined property rights, complete contracts and no transaction costs, agents can always negotiate to reach an efficient outcome.
- We will explore this topic through an easy example
- JoJo Pa and Donald Coats are neighbors. Don writes for a living, and works at home. JoJo Pa plays the piccolo professionally, and practices at home quite often. The two occupy adjacent apartments with thin walls. Although most building residents complain vehemently of the noise they hear from their neighbors, the thin walls are a blessing for Don, as he loves to listen in to JoJo's beautiful piccolo playing. If only he practiced more!

  Luckily, their apartments feature large balconies, which border each other and are separated by a metal screen. This screen initially separates the balconies into identical pieces of 10 square meters each, but the screen can be moved as the occupants of the two apartments see fit. Thus, let $(b_J, m) = (10, 0)$ and $(b_D, m) = (10, 0)$ represent the initial endowments of balcony space and daily hours of music that JoJo and Don claim

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by virtue of their apartment ownership, respectively. Assume that we can represent their utility functions as follows:

$$\begin{aligned} U_J(b_J, m) &= b_J + 2m - m^2 \\ U_D(b_D, m) &= b_D + 2m \end{aligned}$$

1. If JoJo maximizes her utility in isolation, how much music will she play?
   Solutions:

   The first-order condition to JoJo's maximization problem,

$$\begin{aligned} \max_m \quad U_J(b_J, m) &= b_J + 2m - m^2 \\ s.t. \quad b_J &= 10, \end{aligned}$$

   is simply $2 - 2m = 0$, so JoJo plays 1 hour of music.

2. Identify all Pareto optimal allocations of b and m that give JoJo and Don at least as much utility as they achieve when JoJo optimizes in isolation. Assume that moving the screen is costless (ie. no transaction costs).
   Solutions:

   At $m = 1$, $U_J = 11$ and $U_D = 12$, so Pareto superior allocations have to beat those levels. Pareto optimal allocations solve the social maximization problem

$$\begin{aligned} \max_{b_J, b_D, m} \quad U_J(b_J, m) + U_D(b_D, m) &= b_J + 2m - m^2 + b_D + 2m \\ s.t. \quad b_J + b_D &= 20, \end{aligned}$$

   whose first-order condition is $4 - 2m = 0$, so JoJo will now play 2 hours of music. This is because the marginal social benefit of playing more music (a constant 4) is greater than the marginal private benefit of JoJo (a constant 2).

   Social welfare is then $U_J + U_D = 24 > 11 + 12$. In addition we need $b_J + 2m - m^2 =$

$b_j > 11$ and $b_D + 2m = b_D + 4 > 12$, so Pareto optimal allocations take the form

$$\begin{aligned}(b_J, m) &= (x, 2)\\ (b_D, m) &= (20 - x, 2)\\ x &\epsilon [11, 12].\end{aligned}$$

3. If JoJo and Don bump into each other in the elevator and discuss the possibility of trading balcony space for music, what will determine which of the Pareto optimal allocations you identified above they actually settle on?

   Solutions:

   Trading can create an additional unit of utility, which could potentially be shared between the two players in an infinite number of ways (ie., with any fraction between 0 and 1 going to JoJo and the difference to Don). The particular outcome that results will depend on the structure of the negotiation (bargaining), which can be modeled using game theory.

What if instead of enjoying music, Don is annoyed by it? Let $U_D(b_D, m) = b_D - 2m$. What are the socially optimal outcomes?

Solutions:

At $m = 1$, $U_J = 11$ and $U_D = 9$, so Pareto superior allocations have to beat those levels. Pareto optimal allocations solve the social maximization problem

$$\begin{aligned}\max_{b_J, b_D, m} \quad U_J(b_J, m) + U_D(b_D, m) &= b_J + 2m - m^2 + b_D - m\\ s.t. \qquad b_J + b_D &= 20,\end{aligned}$$

whose first-order condition is $1 - 2m = 0$, so JoJo will now play 0.5 hours of music. This is less than the original 1 because the marginal social benefit of playing more music is below the net marginal private benefit (a different way of saying this is that the net marginal social cost is higher than the net marginal private cost).

Social welfare now is $U_J + U_D = 20.25 > 11 + 9$. In addition, we need $b_J + 2m - m^2 =$

$b_j + 0.75 > 11$ and $b_D - m = b_D - 0.5 > 9$, so Pareto optimal allocations take the form

$$\begin{aligned}(b_J, m) &= (x, 2)\\ (b_D, m) &= (20 - x, 2)\\ x &\epsilon [10.25, 10.5].\end{aligned}$$

Go back to $U_D(b_D, m) = b_D + 2m$. Now assume that moving the screen is costly, because the apartments have floor-to-ceiling windows looking out on the balcony. To maintain privacy, plants will have to be purchased to line the interior side of the window along whatever space is transferred. To capture this, let the cost of transferring Δ units of balcony space be $\alpha\Delta$, paid by the receiver of space, where $0 < \alpha < 1$.

1. Since JoJo is the one playing the music, assume that she can make a take-it-or-leave-it offer to Don (this means JoJo has all the 'bargaining power' in this game). What space transfer and level of music will her offer specify?

   Solutions:

   With JoJo having the power to make a take-it-or-leave-it offer, she solves the optimization problem

$$\begin{aligned}\max_{\Delta,m} \quad & U_J(10 + (1-\alpha)\Delta, m)\\ s.t. \quad & U_D(10 - \Delta, m) \geq 12,\end{aligned}$$

   where we assume that Don will accept the deal at indifference. Having all bargaining power means that all the surplus from negotiation is taken by JoJo and Don is kept at his previous utility level. The constraint $U_D(10 - \Delta, m) \geq 12$, is a participation constraint, that tells us how much Don should be required to pay/receive to make it individually rational for him to agree to the deal.

   Plugging the arguments into the utility functions, we can rewrite the optimization problem as

$$\begin{aligned}\max_{\Delta,m} \quad & 10 + (1-\alpha)\Delta + 2m - m^2\\ s.t. \quad & 10 - \Delta + 2m = 12.\end{aligned}$$

Since the objective function is increasing in $\triangle$, JoJo will set $\triangle$ as high as possible, i.e. $\triangle = 2m - 2$ from the individual rationality constraint. Thus the problem can be reduced to

$$\max_{m} \quad 10 + (1-\alpha)(2m-2) + 2m - m^2.$$

The first-order condition is therefore $2(1-\alpha) + 2 - 2m = 0$, so $m^* = 2 - \alpha < 2$ and $\triangle^* = 2 - 2\alpha$.

2. Explain why the optimal level of music now is different from the level chosen when moving the screen is costless.

   Solutions:

   Note that a transaction cost of zero results in a Pareto optimal outcome, because JoJo is able to fully internalize the positive externality of her music. On the other hand, a transaction cost of 100% destroys all gains from trade, and she doesn't internalize any of it. In this setup, ability to internalize the externality decreases 1-for-1 with the transaction cost, a result of the fact that we have assumed utility is quasi-linear in balcony space.

2 Monopoly and Oligopoly vs Perfect Competition

- Under monopoly, a single firm with market power optimizes its own decision without being affected by rivals
- Under oligopoly, the behavior of multiple firms can endogenously influence market price and demand. Agents thus have to worry about strategic interaction and what other firms will do. Under duopoly we have two firms
- Under perfect competition, firms are price takers. Equilibrium price is exogenous and no single agent can take an action significant enough to be able to influence market outcomes
- Cournot competition with two firms: firms choose quantities Q_1 and Q_2 with price being equal to $p(Q_1 + Q_2)$
- Let demand be linear: $p = a - b(Q_1 + Q_2)$

- We explore the Nash Equilibrium of the game: each agent (firm) knows what the other firm is doing and gives a best response to that

- Firm one solves:

$$\max_{Q_1} pQ_1 - c_1(Q_1)$$

- Assume linear costs: $c(Q_1) = c_1 Q_1$ and $c(Q_2) = c_2 Q_2$ for simplicity

- Then FOC for firm 1 (using $p = a - b(Q_1 + Q_2)$)

$$(a - c_1) - 2bQ_1^* - bQ_2 = 0$$

- Thus the best response of firm 1 to firm 2's Q_2 is:

$$Q_1^*(Q_2) = \frac{a - c_1 - bQ_2}{2b},$$

  as long as this quantity is non-negative

- Solving firm 2's problem yields a similar result because of the similarity:

$$Q_2^*(Q_1) = \frac{a - c_2 - bQ_1}{2b},$$

- In Nash Equilibrium we have both of these equations holding, or more simply: $Q_1^*(Q_2^*(Q_1^*)) = Q_1^*$:

$$Q_1^* = \frac{a - c_1 - b\frac{a - c_2 - bQ_1^*}{2b}}{2b}$$

- This leads to $Q_1^* = \frac{a - 2c_1 + c_2}{3b}$ and similarly: $Q_2^* = \frac{a - 2c_2 + c_1}{3b}$ and $Q^* = \frac{2a - c_1 - c_2}{3b}$

- $p^* = a - bQ^* = \frac{a + c_1 + c_2}{3}$

- Profits: $\pi_1 = (p^* - c_1)Q_1^* = \frac{(a - 2c_2 + c_1)^2}{9b}$ and $\pi_2 = \frac{(a - 2c_1 + c_2)^2}{9b}$

- Note that under monopoly ($Q_2 = 0$) we would get: $Q_1^* = \frac{a - c_1}{2b}$, if $c_1 = c_2$, the duopolic firms produce less each but market supply is greater than under monopoly ($2\frac{a-c}{3b} > \frac{a-c}{2b}$) and thus price is lower under duopoly than under monopoly ($\frac{a+2c}{3} < \frac{a+c}{2}$) and firms profits are lower under duopoly ($\frac{(a-c)^2}{9b} < \frac{(a-c)^2}{4b}$) as profits are started to be competed away