Ambiguity and Extremism in Elections

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Abstract

We analyze a model in which voters are uncertain about the policy preferences of candidates. Two forces affect the probability of electoral success: proximity to the median voter and campaign contributions. First, we show how campaign contributions affect elections. Then we show how the candidates may wish to announce a *range* of policy preferences, rather than a single point. This strategic ambiguity balances voter beliefs about the appeal of candidates both to the median voter and to the campaign contributors. If primaries precede a general election, they add another incentive for ambiguity, because in the primaries, the candidates do not want to reveal too much information, to maintain some freedom of movement in the policy space for the general election. Ambiguity has an option value.

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1 Introduction

The standard two-candidate model of electoral competition has two implications: convergence to the ideal policy of the median voter and unambiguous policy platforms. Convergence to the median is complete in a Downsian (Downs 1957) model in which the two candidates care only about winning elections. It is partial in a "partisan" model in which the candidates care about policy per se (Wittman 1983, Calvert 1985) and can make a commitment to their electoral platforms[1]. Partial convergence means that the two candidates propose policies (much) closer to each other than their ideal ones. As for ambiguity, in an important early contribution Shepsle (1972) shows that, with risk-averse voters, the candidates have an interest in being as clear as possible in announcing their welfare-maximizing policy. Any uncertainty about their policy platforms would affect the parties negatively in the eyes of the voters.

The observed patterns of real world elections in two-party systems seem rather far from the implications of these basic models. No commentators of recent American presidential elections would argue that the two parties have moved closer to each other consistently. Clinton was a moderate democrat but quite often extreme groups are quite influential in both parties, and presidential candidates have taken increasingly polarized positions. McCarty, Poole and Rosenthal (2006) present a swathe of evidence on the evolution of polarization in the United States and its large recent increase. Speaking of convergence in American politics, today, is at best highly debatable. In the recent (2007) French presidential election, the two candidates took positions very far from each other and showed no interest in converging, not even in words. The same applies to recent Spanish and Italian legislative elections of 2008. Even less plausible is the implication that candidates are unambiguous in their pre-electoral policy statements. Just the opposite: most candidates are very careful not to take clear positions on key issues and also often change their positions when useful and depending on the audience. The ambiguity of pre-electoral speeches and the rhetorical contortions to avoid

[1] See Alesina and Rosenthal (1995) and the references cited therein for a review of these models.

taking a clear-cut position have reached levels that are often borderline comical.[2]

We develop a model that generates ambiguous policy platforms that can be even more extreme than the candidates' ideal policies. More precisely, in equilibrium, the candidates may offer the voters an ideological interval within which they position their platforms, but they purposely do not reveal which policy they stand for within that interval. The policy interval proposed by the candidates may be more extreme than their ideal policy.

The model combines four elements: first, partisan preferences of candidates (namely, candidates have ideal policies that they would like to implement); second, uncertainty about the true ideal policies of the candidates; third, uncertainty about the distribution of voters' preferences and specifically about the position of the median voter; fourth, campaign contributions that, holding everything else constant, affect the probability of victory of the two candidates. Ambiguity of platforms emerges in equilibrium as a result of the candidates balancing two forces: the need to converge towards the median and the need to raise contributions that may flow from groups with interests (or ideologies) positioned at the extreme of the ideological spectrum.

For the sake of argument, we can think of contributions as "money," but they an also take the form of unpaid time of activists, political strikes organized by unions (mainly in a non-US context) and the like. Extremism arises when the effect of campaign contributions from extreme groups dominates the gain in probability of winning from convergence to the median voter. This is why ambiguity may be useful. In an attempt to gain contributions without losing too many voters in the middle of the political spectrum, the candidates may choose to be less than precise about their policy stands, even though all contributors and voters are aware of this incentive. Obviously, ex ante uncertainty about the true ideal policies of the candidates is crucial for this result on ambiguity of platforms. Note that, like in an arms race or in advertising, in equilibrium campaign contributions may not affect the probability

[2] One particularly amusing example was when one Republican presidential candidate in the 2008 race, Mitt Romney, changed his position on abortion a few times ("every even year," as John Mc Cain pointed out) and the last time he did it, he cited the issue of "cloning" (sic!) as his justification for his change of mind on abortion.

of victory of the two candidates, even though they would affect their choice of policy.

We can parametrize the extent of ambiguity (i.e., the size of the ideological interval offered by candidates) and the degree of convergence as a function of parameters that can be easily interpreted, such as the distribution of voter preferences, the amount of uncertainty about the position of the median voter, the marginal cost of campaign contributions, the distance of parties' ideal policies, and these policies' (a)symmetry relative to the median voter's ideal policy. We also show how the evolution of certain parameters (for instance, the importance and role of contributions) may affect the equilibrium, even holding constant the *ex ante* (i.e., before-contributions) distribution of voters' preferences.

The basic version of the model applies generically to any two-party electoral context. We then propose an extension of it that incorporates primaries, a more specifically American electoral feature. The primary system adds another dimension of ambiguity. During the primaries, the candidates seek to win the nomination of their party before proceeding to a general election. The ideal platform to win the two elections may not be the same, and both would depend on the result of the primary of the other party: namely, a candidate may be farther from the median of his/her party than his opponent in the primaries but more likely to beat one (but possibly not the other) candidate of the opposing party. We show that, even without the additional effect of campaign contributions, the primary game adds a dimension of ambiguity. In the primaries, candidates have an incentive not to reveal their true ideologies. Obviously, the primary system, together with campaign contributions (both for the primary race and the presidential race), would compound the ambiguity effect on platforms.

Many before us have noted the inability of simple "traditional" models to explain the richness of diverse observations offered by electoral contests. The issue of (lack of) convergence has been attributed to the inability of making commitments to moderate pre-electoral platforms (Alesina 1988), but this model is not compatible with parties taking positions even more extreme than their ideal policies. Alesina and Rosenthal (2000) discuss extremism of presidential candidates when facing an adverse Congress, an issue which we do not address here. Glaeser, Ponzetto and Shapiro (2005) derive the adoption of extreme policies from the incentive that parties have to increase voter turnout. Extremists vote only if the policies proposed to them are not too middle-of-the-road. Campante (2007) presents a model in which campaign contributions influence how much parties choose to redistribute and presents evidence consistent with the fact that more polarization in the population leads to more contributions that polarize parties' policies. McCarty, Poole and Rosenthal (2006) discuss the importance of contributors to the increased polarization of American politics and relate both of them to the increased inequality. They show that both contributions from Political Action Committees (PAC's) and soft money from individuals are highly ideological. In particular, soft money comes from extremists, and its importance has risen recently.[3]

Models of ambiguity in politics are more rare. Shepsle (1972) investigates in a Downsian model under what condition of risk aversion or not there would be ambiguous platforms. Alesina and Cukierman (1990), stud instead partisan model in which an incumbent has an interest in introducing noise so as not to allow the voters to learn his ideal policy by perfectly observing his policies while in office. In that model, however, only the incumbent can be ambiguous: there is no strategic game in platforms.[4] Aragonès and Postlewaite (2002) analyze a model in which candidates compete to win an election and in which there is a fixed set of policy alternatives. They provide conditions under which there exists a pure strategy equilibrium in which there is ambiguity–in the sense that, in equilibrium, voters do not know with probability 1 which policy a candidate will implement if she wins the election. Callendar and Wilkie (2007)[5] consider a model in which candidates may misrepresent their policy preferences (i.e., lie) but have heterogeneous costs of so doing. Candidates make promises that may differ from their intentions. Thus, their model is a signalling model with

[3] See these authors for a more complete review of the political science literature on this issue.

[4] Several papers discuss the choice of policy (especially monetary policy) in situations where the public does not know the type of policymakers, but these are not models of electoral competition. See Persson and Tabellini (2002) for a survey.

[5] See also Kartik and McAfee (2007).

differential costs and a continuum of types. They characterize the symmetric perfect Bayesian equilibria that survive the universal divinity refinement of Banks and Sobel (1987). In such equilibria, a positive measure of candidates do not locate at the median voter–i.e., there is policy divergence. The effects of ambiguity and divergence analyzed in our paper are quite different from those just reviewed.

The paper is organized as follows: The first part of section 2 presents the basic model without uncertainty about candidates' ideal policies but with contribution. This part of the model delivers contributions driven extremism. The second part of section 2 derives ambiguity of candidates' platforms, that is, we illustrate the possibility that candidates choose a (possibly large) interval in the issue line rather than a single point. In section 3, we provide another setting in which there will be ambiguity: when policy positions are announced before primary elections. Section 4 discusses extensions, applications and limitations of the model and concludes. All the proofs are in the appendix.

2 The Model

2.1 The Basic Setup

Consider two candidates who have to locate on the ideological (policy) space defined by the unit interval $[0,1]$. The "left-wing" candidate (player X) has a policy bliss point $x_b \in [0,1/2]$, and the "right-wing" candidate (player Y) has a policy bliss point $y_b \in [1/2,1]$. We do not model entry of third candidates. Simultaneously and non-cooperatively, player X chooses location x, and player Y chooses location y. For now, we take x and y to be real numbers (i.e., points) on the unit interval, but below we will allow them to be sub-intervals. Denoting p as the probability that candidate X wins the election, we assume that the preferences of player X can be represented by the utility function

$$U_X = -p\left(x - x_b\right)^2 - \left(1-p\right)\left(y - x_b\right)^2.$$

Similarly, for player Y ,we have

$$U_Y = -(1-p)\left(y-y_b\right)^2 - p\left(x-y_b\right)^2.$$

Electoral outcomes–encapsulated by the probability p–are determined by two forces. The median voter's ideal policy is a random variable ε with expected value of 1/2, distributed uniformly on the interval $\left[\bar{\varepsilon}, \underline{\varepsilon}\right]$, and we parameterize the magnitude of the shock by assuming that $\bar{\varepsilon} - \underline{\varepsilon} = 1/\beta$. The probability that candidate X wins the elections is then:

$$p = \frac{\frac{x+y}{2} - \left(\frac{1}{2} - \frac{1}{2\beta}\right)}{1/\beta}.$$

We have chosen specific functional forms in order to obtain closed form solutions as much as possible and for an easier interpretation, but as we discuss below, the results do not depend qualitatively on these specific parameterization, at least until noted below. By taking the first order conditions, which are explicitly described in the appendix, one can show that:

Proposition 1 *There exists a unique interior Nash equilibrium in candidate locations. Also, in the unique interior Nash equilibrium: (a) if $x_b = 1 - y_b$ then $x^* = 1 - y^*$ (b) $x^* > x_b$ and $y^* < y_b$, (c) if $x_b < 1 - y_b$ then $x^* < 1 - y^*$and (d) if $x_b > 1 - y_b$ then $x^* > 1 - y^*$*

The proof is well known; see, for instance, Alesina and Rosenthal (1995). Point (a) shows that, if the candidates' ideal policies are symmetric around the expected median, the chosen policies are also symmetric. Point (b) implies partial convergence: both parties offer policies that are closer to each other than their ideal points. Point (c) shows that, if a party is farther from the expected median than the other one in ideal policies, it will be farther in policies and it will have a lower probability of winning. As an illustration of this result, consider the case where $x_b = 1/5, y_b = 3/4$. In this case, $x^* = 0.358$, and $y^* = 0.619$.

It is also interesting to investigate the role of the degree of uncertainty captured by the inverse of the parameter β. It is easy to prove the following:

Proposition 2 *If $\beta \to \infty$ then $x^* = y^* = 1/2$ and $p = 1/2$. If $\beta \to 0$ then $x^* = x_b$ $y^* = y_b$ and $p = 1/2$. If $x_b = 1 - y_b$ a reduction in β increases polarization, i.e. x^* goes down and y^* goes up.*

The proof is obvious. If there is no uncertainty, the two parties converge to the median, since any asymmetry around the median would make one party a sure winner. If $\beta \to 0$, the support of the distribution of the median voter goes to infinity. Any pair of positions offered by the two parties does not influence the probability of electoral outcome that remains at 1/2; therefore, the two parties may as well adopt the bliss point as their platforms.[6] If the candidate locations around the media are symmetric, then the probability of victory in equilibrium is 1/2, but $\partial p/\partial x = \beta/2$. Thus, the marginal benefit to party X of converging toward the median (if X is below the median) is increasing in β : therefore, less uncertainty pushes the party closer to the median. The implication is clear: one should observe more polarization in systems where the position of the median is harder to predict.[7]

Also, for given policies chosen by the two parties, an increase in uncertainty (reduction of β) favors the party closer to the expected median. This can be easily seen by noting that

$$\partial p/\partial\beta = \frac{x+y}{2} - \frac{1}{2}.$$

This means that a reduction of uncertainty favors party X when its platform is closer to 1/2 the expected median.

Summarizing: the critical result here (in addition to existence, of course) is the partial convergence effect. The parties move closer to each other than their ideal policies. It would make no sense, in fact, for a candidate to announce a policy more extreme than his ideal: by moving closer to his ideal, he would increase his probability of winning and would also

[6] This can be easily verified by applying L'Hôpital's Rule to take the limit of the probability of victory of X as $\beta \longrightarrow 0$.

[7] One could think of an interesting and more realistic extension to a multidimensional setting. A lowering of predicability of the position of the median for given party platforms (vector of policy proposals) may arise from an increase in the dimensionality of the relevant policy space. In other words, it may become more difficult to predict how an election would turn for given platforms if the voters differ on a host of issues.

propose a policy closer to his ideal. Also note that, in the model which we use, we did not include a preference for holding office per se. The candidates care only about the policy outcome, and they want to win so that they can implement the desired policy. If they also had an incentive to win per se, the amount of convergence would increase, since the candidates would be more willing to trade off the policy location for an increase in the probability of winning by converging to the median.[8] In other words, the ideological space that lies on the right (left) of the right (left) wing party is completely irrelevant: no policies will be ever proposed in that space. If the two candidates are relatively close in ideology, a large part of the ideologic spectrum is never travelled.

2.2 Contributions

Consider the problem of two contributors (one for the right and one for the left) who make contributions after the candidates locate in the ideological line[9]. The contributors may be a group, but we assume that they act as a single agent, we do not explore issues related to the internal organization of lobbies, free riding and the like. For simplicity, the bliss points of the contributors are at the extreme of the expected political spectrum, i.e. they have bliss points of 0 and 1. Nothing of relevance would change if the contributors were more extreme than the two parties' bliss points but strictly in the interior of the expected ideological space. Note that we have to talk about the "expected" ideological space because its extreme, as well as the median, is perturbed by the shock. The contributors have measure zero as voters.[10]

[8] In this case the expected utility of, say, candidate X would be written as follows

$$U_X = -p((x - x_b)^2 + h) - (1 - p)(y - x_b)^2 .$$

where $h > 0$ represents the benefits of holding office *per se*. Candidate Y's utility function would-be symmetric. Party See Alesina and Rosenthal (1995) for further discussion.

[9] The assumption of non-simultaneous moves is largely for technical reasons. It removes the need to analyze a four-player game, rather than two two-player games that both turn out to be supermodular. We conjecture that our results extend to that setting, but the four-player game is not supermodular, and hence we cannot apply our argument to that setting. There are also interesting dynamic aspects in this contribution game like bandwagon effects that are not the focus of the present paper.

[10] This technical assumption is needed for the following reason. Imagine a negative realization of the shock to the median voter such that the latter is equal to $1/2 - \epsilon$. This implies that the realization of the distribution of voters is from $-\epsilon$ to $1 - \epsilon$. We continue to take the right-wing contributor as an agent with

The view that contributors are relatively extreme is commonly held; see, for instance, the recent discussion in McCarty, Poole and Rosenthal (2007). One explanation is intensity of preferences: extreme groups are especially far in preferences from middle-of-the-road policies, and they have a stronger incentive to move policies away from the middle. Individuals at the extreme have more to lose in terms of utility by their candidate losing given concavity of preferences; therefore, they have, *ceteris paribus,* a stronger incentive to contribute.[11] For simplicity, we can think of contributions as money, but they could also be time and free labor contributed by volunteers and party activists. For instance, the right-wing party closer to the wealthiest may get more monetary contributions, while the left-wing parties may get more free labor from activists, union members, etc.

The contributors decide how much to give taking the other contribution as given–that is, we consider Nash equilibria of the contribution game. Let us define c_x and c_y as the contributions received by parties X and Y, respectively. They affect the position of the median voter: we capture the idea that money spent for campaign activities switches undecided voters in the middle of the political spectrum toward one of the two. It is important to keep in mind that, in our model, contributors act in their own interests and not necessarily purely in the candidate's interest. In equilibrium, the left-wing contributor gives only to the left-wing party and vice versa. In fact, if the left-wing interest group gave money to the right-wing party, the latter would use it to move the median voter to the right, and this clearly cannot be in the interest of the left-wing interest group. Note that we assume that interest groups cannot affect the position of the parties directly, because, for instance, the parties cannot deliver what they promise to the interest groups.[12]

When the left-wing and right-wing contributors give amounts c_x and c_y in contributions, the expected median voter becomes $1/2 - c_x + c_y$. Thus, the probability that party X wins

bliss point at 1, but we ignore him as a voter.

[11] Obviously, we could have both extreme contributors and moderate ones, but as long as the former contribute more, the assumptions of the model would capture that.

[12] See Grossman and Helpman (2004) for models on interest groups' contribution and for discussions of various issues of commitment.

for given policies x and y is given by:

$$p(\cdot) \equiv \Pr(X\ wins) = \Pr(\varepsilon < (x+y)/2 - c_x + c_y) = \frac{\frac{x+y}{2} + c_x - c_y - \left(\frac{1}{2} - \frac{1}{2\beta}\right)}{1/\beta}$$

This expression shows that a higher c increases the probability of an X victory for a given policy. With no shocks, party X could win for sure, even with a policy farther to the original median of $1/2$ than policy y so that $x + y < 1$. However, the policies x and y are a function of c_x.and c_y. Therefore, the contributors affect the policy outcomes in two ways, by changing the expected median voter ideal policy and therefore also changing the probability of electoral outcomes for given policies and the policies chosen by the two parties.

Also, note that:

$$\partial \Pr(Xwins) \partial c_x = \beta.$$

If $\beta \longrightarrow 0$, as we saw before, the uncertainty is so large that $p = 1/2$ for any policy chosen by the two parties, and the effect of contribution on the expected median goes to zero. Therefore (as we will see more formally below), no contributions are paid. The marginal effects of contributions is increasing in the precision with which the median voter is known: with a relatively small range of variation in the possible position of the median voter, contributions have stronger marginal effects. This seems reasonable: the more one knows about the median voter (or the "swing voters," in common parlance), the more campaign advertising can be targeted correctly and effectively. Needless to say, opposite advertising can counterbalance. In fact, the specification of the contributions from the right-wing group with bliss point equal to 1 is symmetric.

Let us now consider the maximization problem of the left wing contributor with a bliss point of zero.

$$\max_{c_x}\left\{\begin{array}{c} p\left[x(c_x,c_y),y(c_x,c_y),c_x,c_y)\right]U(x(c_x,c_y),0) \\ +(1-p\left[x(c_x,c_y),y(c_x,c_y),c_x,c_y)\right])U(y(c_x,c_y),0)-H_x(c_x) \end{array}\right\}$$

In this expression, $H_x(c_x)$ represents the cost of contributions, which we assume to be convex with $H_x(0)=0$, $H_x'(0)=0$, $H_x'(\infty)=\infty$. After rearrangement, the first order condition can be written as follows, and those of the right-wing contributor are symmetric:

$$\beta\left[(U(x(c_x,c_y),0)-U(y(c_x,c_y),0)\right]+p(\cdot)\left[\partial U/\partial x\partial c_x-\partial U/\partial y\partial c_x\right]=\partial H/\partial c_x.$$

The objective function and first order condition of the right-wing contributor (with a bliss point of 1) is symmetric. The first term captures the effect of a change probability of election for given policies due to the change in expected median generated by campaign contributions, recalling that $\partial p/\partial c_x=\beta$. The second term captures the changes in policy of the two parties induced by campaign contributions for given probabilities.

2.3 Extremism and contributions

In this brief section, we show two results. The first is that the presence of campaign contributions polarizes the political system. The second is that if one of the two contributors is "stronger"–say it has more resources to spend the political equilibrium–it not surprisingly "tilts" in its direction, and we show precisely in what way.

Let us define $x_c^*(\ y_c^*)$ as the two equilibrium policy chosen by the parties, in order to distinguish them from $x^*(y^*)$, the equilibrium policies without contributions. The polarizing effect of contributions can be stated as follows:

Proposition 3 *If $H_x(c_x)=H_y(c_y)$ and $x_b=1-y_b$ then $x_c^*=1-\ y_c^*, x_c^*<x^*, y_c^*>y^*, c_x^*=c_y^*$ and $p=1/2$.*

This proposition shows that the policies adopted by the two candidates are more extreme that those chosen without contributions. Note also that contributions in this case are a pure waste, since they wash out and the probability of electoral outcomes in unchanged. In fact, with risk-averse voters, contributions decrease aggregate welfare because they increase the polarization of policies and therefore the ex ante uncertainty about the ex post realized policy. Needless to say, one should not infer from this result any implication about the optimality of laws that regulate or restrict campaign contributions. The model is not rich enough in this dimension to analyze the issue; however, the polarizing effect has to be taken into account in any policy discussion about contributions.

We now show that, if one contributor is "stronger" than the other, the equilibrium is biased in his favor. An obvious way to model strength is to have lower costs of contributions.

Proposition 4 *Suppose that H_x and H_y are α_x-convex and α_y-convex*[13] *respectively with $\alpha_x > \alpha_y$ with $k_x > k_y$, then $c_x^* < c_y^*$.*

2.4 Ambiguity of platforms

We now introduce uncertainty concerning the true beliefs (i.e., the bliss points) of the two candidates. For simplicity, we assume that there are only two types of candidates; extending this to a continuum of types is discussed informally below; a formal treatment is left for future research.

Thus, there are two potential types of each candidate: candidate X has bliss point x_L with probability q_x and bliss point x_R with probability $1 - q_x$ such that $1/2 \geq x_R > x_L \geq 0$. Similarly, candidate Y has bliss point y_R with probability q_y and bliss point y_L with probability $1 - q_y$ such that $1/2 \leq y_L < y_R \leq 1$.We now allow the two candidates to choose not simply a *point* but an *interval* in the policy space. Obviously, in the previous model, with no uncertainty about party ideal policies, they had no reason to do so. Let the choice of interval for candidate x_i of type i be $\left[\underline{x_i}, \bar{x}_i\right]$ and for candidate y_i be $\left[\underline{y_i}, \bar{y}_i\right]$. We assume that, if elected, the two candidates are free to choose any policies within their announced intervals but may not choose policies outside those intervals, which is the logical

[13] A function f is α-convex if $f'' \geq \alpha$ everywhere.

extension to the assumption that we used thus far that candidates are committed to their policy platforms.

This is a dynamic game of incomplete information, and our solution concept is Perfect Bayesian Equilibrium. The out-of-equilibrium beliefs of the voters are as follows: If candidate X does not follow the equilibrium strategy, then she is believed to be candidate type x_L if she deviates by expanding the range to the left and type x_R if she expands the range to the right. Similarly, if candidate Y does not follow the equilibrium strategy, then she is believed to be of type y_R if the expands the range to the right and type y_L if she expands the range to the left.[14] If a candidate narrows the range, then she is believed to be the type of candidate whose preferred policy is within the range. If both types expand beyond the range on both sides, then voters glean no information (i.e., their posterior is their prior).

Our first result shows that, in the setting we have considered, there exists no equilibrium: either pooling or separating. This "negative" result is important to generate intuition.

Proposition 5 *Assume $\beta = 2$. Then for any y_L, y_R, x_L, x_R there does not exist a Perfect Bayesian Equilibrium in which either $\underline{x}_L = x_L = \underline{x}_R$, $\bar{x}_L = x_R = \bar{x}_R$ and $\underline{y}_L = y_L = \underline{y}_R$, $\bar{y}_L = y_R = \bar{y}_R$, nor does there exist a separating equilibrium.*

The intuition is as follows: Fix player Y's strategy. Because of the uniform distribution assumption, there is always one type of player X that has a higher probability of defeating player Y in the separating equilibrium. If the contributions effect is large (i.e., H_x is such that contribution costs are low), then this is player x_L; if it is small, then it is player x_R. Because the players get to implement any policy in their ranges ex post, the probabilities of victory are the only relevant consideration. So a separating equilibrium cannot exist, because one player always wants to pool and can mimic the other player[15]. Similarly, a

[14] In the appendix, we show that this result holds with an alternative specification of out-of-equilibrium beliefs where candidates who deviate are assumed to be the extreme type. Specifically, if candidate X does not follow the equilibrium strategy, then she is believed to be candidate type x_L, and if candidate Y does not follow the equilibrium strategy, then she is believed to be of type y_R.

[15] There is a first-order gain from doing so and a second-order loss from the change in interval and the consequent effect on policy choice.

pooling equilibrium cannot exist, because one player always wants to separate.

The uniform distribution is responsible for this non-existence. In order to gain intuition, note that the candidates face a trade-off between the "median voter effect" and the "campaign contributions effect." That is, by moving towards the median for given contributions, they increase their chances of winning, and by moving towards the extreme, they get more contributions by moving the median. An equilibrium with ambiguity would require that somehow the two incentives balance out. Consider the left-wing candidate, X. An equilibrium with ambiguity requires that the right-wing and left-wing types choose the same interval. Thus, for the right-wing type, the cost of separating is to lose too many contributions, while for the left-wing type, the cost is to lose too many votes by revealing his distance from the median voter. With linearity in the probability of elections in contributions, one of the two effects always dominates. This means that the probability of candidate X (for example) winning is monotonic in x. Whether it is increasing or decreasing depends on H_x versus H_y, i.e., the relative magnitude of the two effects. Thus, a pooling equilibrium cannot exist.

Instead of the uniform distribution, then, suppose that the aggregate shock to voter preferences is given by a variant of the beta distribution[16] with CDF $1/2 - (1 - q^{1/2})^2$. For $\bar{\varepsilon} - \underline{\varepsilon} = 1/2$, the probability that candidate X wins is then given by

$$\Pr(X \text{ wins}) = \frac{1}{2} - \left(1 - \beta\left(\frac{x+y}{2} + c_x - c_y - \left(\frac{1}{2} - \frac{1}{2\beta}\right)\right)^{1/2}\right)^2.$$

We then have

Proposition 6 *Assume that the shock follows the above beta distribution. Fix candidate Y's strategy. Then there exist y_L, y_R, x_L, x_R and $\alpha \geq 0$ such that there exists a Perfect Bayesian Equilibrium in which $\underline{x}_L = x_L = \underline{x}_R$, $\bar{x}_L = x_R = \bar{x}_R$.*

The intuition is as follows: Given the ability to choose any policy within the announced

[16] This is sometimes referred to as the generalized Kumuraswamy distribution.

range ex post, the only difference in payoffs between the pooling equilibrium and a deviation from it is the probability of winning the election. Under the above distributional assumption, the probability of winning is non-monotonic in location–it has an inverted U-shape. Thus, if x_L is sufficiently left of the peak of the hump and x_R is sufficiently right of it, then both types benefit from being believed to be an intermediate type. Translating this into fundamentals, the non-mononiticity that gives rise to the pooling equilibrium arises when the median voter effect (which is largest at $x = 1/2$ and smallest at $x = 0$) decreases slowly from 1/2 and the more rapidly and when symmetrically the campaign contributions effect (which is largest at $x = 0$ and smallest at $x = 1/2$) decreases slowly from 0 and the more rapidly.

We should also point out that the result is somewhat stronger than it appears as stated. Although we say there exist y_L, y_R, x_L, x_R, we have chosen particular parameters of the distribution[17]. For different values of y_L, y_R, x_L, x_R, one can choose different parameters and sustain the pooling equilibrium. Moreover, we have fixed candidate Y's strategy and considered the deviations by x_L and x_R. It is also possible to construct examples in which all types y_L, y_R, x_L, x_R pool, using different parameters for the distribution of shocks to the median voter.

It is also important to note that, with this specification of shocks, contributions have a convex effect on election probabilities (e.g., $d^2 \Pr(X \text{ wins})/dc_x^2 > 0$). This is a sufficient condition for the ambiguity result, but we conjecture that it may be a necessary condition as well. Economically, this implies that there are increasing returns to contributions. Note that we are talking about total contributions[18]: it is indeed likely that small total contributions would do very little, since they may be fixed costs in setting up campaign headquarters, buying TV time, etc. Beyond a certain level, decreasing returns to campaign contribution may settle in.

[17] The general CDF is $1/2 - (1 - q^a)^b$.

[18] Empirically, a lot of campaign contributions are individually small, and in fact, in the US there are limits to the size of individual contributions. See Campante (2007) for further discussion.

3 Ambiguity in Primaries

Up to this point, our model could apply to any two-party election. In the case of the United States, the Presidential general election is preceded by primaries in which the positioning of candidates is just as important, and the candidates face a two-stage game in which they have to choose a position (or range of positions) that wins them both the nomination and the general election. This system adds another dimension that creates incentives to be ambiguous. Consider, for instance, the primary of the left-wing party, in which left-leaning voters participate. The candidates would like to convince the voters that they are relatively left-wing to win the primary; on the other hand, they do not want to reveal themselves to be extreme lest they lose the general election. Voters are rational, and in the primaries, they vote taking into account the consequences of their choice in the general election, but a more extreme leftist in the primaries would prefer a more leftist candidate, because he trades off the probability of electability in exchange for a more extreme policy in a different way than a more moderate voter. In this section we analyze this setting. For the sake of simplicity, we return to the case of no contributions, but since contributions add another force in favor of ambiguity, then to the extent that we obtain ambiguity in primaries even without them, their role would be to add another source of it.

Consider two candidates in party X who are competing for their party's nomination and who denote their bliss point as x_L and $1/2 \geq x_R \geq x_L$. They are uncertain who party Y will nominate, and for simplicity, suppose that they both have a uniform prior on $[1/2, 1]$ about the bliss point of their opponent–and hence their policy choice if they win the general election. This assumption is made for simplicity and in order to allow us to focus on the strategic primary game within one part only. Uncertainty regarding the own and opponent party primaries is more appealing when, as in the 2008 election, neither the sitting president nor the vice president is running for reelection.

As above, a strategy for each of the candidates in party X is a policy range $\left[\underline{x}_i, \bar{x}_i\right]$, where i indexes the candidate. Again, prior beliefs are that candidate X has bliss point x_L with probability q_x and bliss point x_R with probability $1 - q_x$. This implies that the voters know that there are two positions competing in the primaries, one more leftist than the other, but they do not know which candidate is which. This informational structure is the simplest possible to analyze, but the qualitative nature of the result would generalize to more complex cases. Out of equilibrium, beliefs are formed as above. The candidate from party X who wins the primary may announce policy again at the general election phase. However, voters are aware of the announcements during the primary. *Inter alia*, this implies that if a separating equilibrium occurred in the primary phase, then the candidate is revealed, and no announcement at the general election can counter those beliefs because of their inability to commit to policy choices. If, however, a pooling equilibrium were to occur at the primary phase, the candidates would not reveal their true identities and could continue to be ambiguous in the general election if they wish.

At the primary election phase, candidates face a trade-off between clarity and flexibility. If they play a strategy that reveals their type (in equilibrium) in the primary, then they have no "room to move" in the general election, because they cannot commit to policies. One player may generically have an advantage *in the primary election* by being believed to be more left-wing. But, for instance, when the primary candidate bliss points are symmetric ($x_L = 1/2 - x_R$), then their payoffs, given the pooling or separating primary equilibrium, are equal. Thus, the option value considerations from the general election dominate and make the pooling equilibrium more attractive for both "left-leaning" candidates. Moreover, since the payoffs are continuous in the bliss points, there exists a neighborhood of "close to symmetric" bliss points under which ambiguity is beneficial. We now establish this formally.

Again, we seek to sustain a pooling equilibrium in which $\underline{x}_L = x_L = \underline{x}_R$ and $\bar{x}_L = x_R = \bar{x}_R$. The payoff to player X_L in such an equilibrium is

$$U_{X_L} = -p_L \pi_L (x - x_L)^2 - (1 - p_L)(1 - \pi_R)(y - x_L)^2 - (1 - p_L)\pi_R (x' - x_L)^2,$$

where p_L is the probability that player X_L wins the primary, π_L is the probability that she wins the general, π_R is the probability that player X_R wins the general, and x' is the policy choice of player X_R in that case. Again noting that, conditional on winning, player X_L implements her most preferred policy and so does player X_R, we can simplify the payoff as follows:

$$U_{X_L} = -(1-p_L)(1-\pi_R)(y-x_L)^2 - (1-p_L)\pi_R(x_R-x_L)^2.$$

To establish that the pooling equilibrium exists, we must show that neither player X_L nor player X_R has a profitable deviation in that equilibrium.

Proposition 7 *There exist x_L and x_R such that there exists a Perfect Bayesian Equilibrium of the primary game in which $\underline{x}_L = x_L = \underline{x}_R$, $\bar{x}_L = x_R = \bar{x}_R$ and $\underline{y}_L = y_L = \underline{y}_R$, $\bar{y}_L = y_R = \bar{y}_R$.*

The intuition for this result is as follows: At the primary stage, neither player X_L nor X_R knows which candidate from party Y they will face in the general election if they win the primary. In a pooling equilibrium, they retain the option of changing their policy announcement in the general election to anywhere within the range $[\underline{x}, \bar{x}]$. In a separating equilibrium, they are known to have bliss points x_L and x_R, respectively. Because they cannot commit to enact a different policy, voters know that this is what they will choose if they win. This is equivalent to constraining their strategy in the general to be equal to that in the setting of Proposition 1. In a pooling equilibrium, both players X_L and X_R retain the option value of tailoring their announcements: i.e., maintaining the beliefs from the pooling equilibrium at the primary stage or changing announcements and thus being believed to be their true type. This option value can be large enough to swamp the negative effect of pooling the in the primary for either player, and thus it is possible that both prefer the pooling equilibrium and do not wish to deviate from it. In fact, when the primary candidates' bliss points are symmetric (around some point on $[0, 1/2]$), the probability of winning the primary is the *same* in both the pooling primary equilibrium and the separating primary equilibrium. Thus, the only incremental consideration is the general election, and the option value considerations are dominant and favor the pooling equilibrium at the primary stage.

It is worth noting that, if party Y had a known candidate at the time of the party X primary, there could not be ambiguity in primaries in party X. However, if we reintroduce contributions into the model, there would again be a force toward ambiguity in the party X primary.

4 Conclusions and Extensions

According to traditional models, in a two-party electoral contest, the party platforms should be clear, unambiguous and close to each other. In reality, however, the two parties often do not converge, and they make ambiguous policy promises, attempting to cover a vast ideological space.

This paper provides a model that is consistent with both observations. What drives extremism and ambiguity is the parties' need to trade-off two forces: the gains in votes obtained by converging to the middle and the benefit of campaign contributions that influence voters' behavior. Contributions often accrue to the parties if they move away from the middle ground and towards extreme groups that feel especially strongly about certain issues. Ambiguous polices allow the parties to attract contributions without committing to extreme policies that would alienate middle-of-the-road voters. Obviously, rational voters would recognize this incentive, but with some ex ante uncertainty about the true beliefs of the candidates, the latter may maintain in equilibrium a certain amount of ambiguity in their platforms. Primaries add another dimension of ambiguity. The candidates in the primaries do not want to reveal their types, given the uncertainty about who they will face in the general elections and the need to win both the primaries and the general election. In this setting, ambiguity at the primary phase provides option value for the general election phase.

Several extensions seem worth exploring in future research. First, the fact that the contributors are at the extremes of the policy spectrum is a simplification meant to capture the fact that groups distant from the median have a strong incentive to pull parties away from the middle. This is related to intensity of preferences. If no contributions were made, the parties would converge, so those at the extreme have to pull them away in some other way than with their votes. The point is even clearer in a multidimensional setting. In fact, in that case, the groups that are extreme and feel strongly about one particular issue will contribute precisely on that one. So for instance, the gun lobby, which feels strongly about that issue, will pull a party toward extreme positions on gun control, gay rights activist will do the same, focusing on their preferred issue and not caring about other ones, and so on.[19] Multidimensional voting models present a significant increase in analytical complexity relative to unidimensional ones. Although we have chosen not to formally explore formally this avenue, this is an excellent area for future research. One result that would be quite different in multidimensional voting models is the one about "wasted" campaign contributions. In a unidimensional voting model, campaign contributions pull the parties in opposite directions and may counterbalance each other, But in a multidimensional voting model, only the groups who feel very strongly about one issue may contribute, i.e., the gun lobby may contribute to have no gun control, but those who prefer gun control may not be organized to counteract.

Second, another simplification is our assumption that there are only two possible types of candidates. The obvious generalization is to a continuum of candidates, but the basic intuition of our result on ambiguity of platforms would seem to generalize under appropriate regularity conditions.

Third, we have proven existence of equilibrium but not uniqueness. There may be other equilibria. Normally, the possibility of many different equilibria is seen as a limitation of a model. However, the complexity and the variety of outcomes that one observes in real-world

[19] See Campante (2007) and McCarty Poole and Rosenthal (2007) for recent analysis on the polarizing effects of soft money.

electoral competitions suggest that, depending on the particular combination of factors and forces at play, different equilibria may indeed materialize in different elections. Perhaps this is why electoral campaigns are entertaining: because they are difficult to predict. Certainly, models that predict that there would always be full or almost-full convergence with very precise policy platforms may have unique equilibria, but they do not seem to capture much of real-life electoral dynamics. Of course, certain refinements of sequential equilibrium may narrow the equilibrium set.

Fourth, the model with primaries could be extended to analyze the view often mentioned by commentators that a long and detailed voting record (in the Senate, for instance) is "baggage" that negatively affects the electoral chances of a Presidential candidate, while newcomers may have an advantage. Our model is consistent with this observation: it is more difficult for somebody with a long and detailed voting record to be ambiguous in his Presidential campaign and in the primaries in order to appeal to a broad range of contributors and voters. A newcomer has more room to be ambiguous in his platform; on the other hand, contributors may trust him/her less. That is an interesting trade-off that could be analyzed with our framework. In other words, the degree to which the candidate can be ambiguous in the primaries (i.e., a contest before the general election) is in turn affected by the candidate's "baggage" due to ambiguity (or lack thereof) in his/her previous set of votes.

Finally, like any model with two candidates, one always disregards the issue of entry of third parties. In principle, potential entry may create yet another incentive for ambiguity: by occupying a larger range of positions, an existing candidate could make entry more difficult, but this conjecture would of course need to be established more precisely and formally.

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5 Appendix

5.1 Proposition 1 First Order Conditions

The first-order condition for candidate X is

$$\frac{(x-x_b)^2\beta-(y-x_b)^2\beta}{2}+2(x-x_b)\beta\left(\frac{x+y}{2}-\frac{1}{2}+\frac{1}{2\beta}\right)=0.$$

Similarly, for candidate Y, we have

$$\frac{(x-y_b)^2\beta-(y-y_b)^2\beta}{2}+2(y-y_b)\beta\left(\frac{x+y}{2}-\frac{1}{2}+\frac{1}{2\beta}\right)=0.$$

5.2 Omitted Proofs

Proof of Proposition 3. The game is a supermodular game since

$$\frac{d^2U_x}{dxdy} = \frac{d^2U_y}{dxdy} = \beta\left(y - x\right) \geq 0,$$

since $y > x$ by construction and $\beta \geq 0$. The comparative static that $x_c^* < x^*, y_c^* > y^*$ follows from Topkis (1998) Theorem 4.2.2. The conclusion that $x_c^* = 1-\ y_c^*$ and $c_x^* = c_y^*$ follows from the fact that this is a two-player constant sum game. That $p = 1/2$ follows from substituting into the formula for p. ■

Proof of Proposition 4. Given the candidate locations, the contribution game is supermodular since

$$\frac{d^2U_x}{dc_xdc_y} = \frac{d^2U_y}{dc_xdc_y} = 0 \geq 0$$

and the desired comparative static follows from Topkis (1998) Theorem 4.2.2. ■

Proof of Proposition 5. Fix the strategy of other types and consider a deviation by type x_L. In the pooling equilibrium her payoff is

$$\begin{aligned} U_X^P &= -p^P\left(x_L - x_L\right)^2 - \left(1 - p^P\right)\left(q_y y_L + (1-q_y)y_R - x_L\right)^2 \\ &= \left(p^P - 1\right)\left(q_y y_L + (1-q_y)y_R - x_L\right)^2, \end{aligned}$$

where p^P is the probability that she wins in the pooling equilibrium. If she deviates in a way which removes her bliss point from the interval then *a fortiori* she is worse off than deviating in a way that keeps her bliss point in the interval. It is therefore sufficient to consider only the latter deviation. Suppose that player x_L deviates by expanding her interval on the left. Denote the probability that player x_L wins under the deviation as $p_{x_L}^S$. Supposing that player x_R deviates by expanding her range to the right this probability is denoted $p_{x_R}^S$. The differences in payoffs are then

$$\begin{aligned}
\Delta_{x_L} &= \left(p^P - p^S_{x_L}\right)\left(\frac{1}{2}\left(y_L + y_R\right) - x_L\right)^2, \\
\Delta_{x_R} &= \left(p^P - p^S_{x_R}\right)\left(\frac{1}{2}\left(y_L + y_R\right) - x_R\right)^2.
\end{aligned}$$

Note that $\left(p^P - p^S_{x_L}\right) + \left(p^P - p^S_{x_R}\right) = 0$, and hence if $\Delta_{x_L} > 0$ then $\Delta_{x_R} < 0$ and if $\Delta_{x_R} > 0$ then $\Delta_{x_L} < 0$. Thus if one player prefers the separating equilibrium then the other does not and hence the separating equilibrium does not exist. Similarly, the pooling equilibrium cannot exist because one player always prefers to deviate. Now consider the deviation which involves narrowing the range. Again, each type is still able to choose their preferred policy ex post, and the change in beliefs is the same as above so that the benefit from this deviation is the same as above. Hence neither the separating nor the pooling equilibrium exists. ■

Proof of Proposition 6. It suffices to provide an example in which such an equilibrium exists. To that end, let $\beta = 2$ and consider candidate X. The probability of winning in the pooling equilibrium is

$$p^P = \frac{1}{2} - \left(1 - 2\left(\frac{\frac{1}{2}x_L + \frac{1}{2}x_R + y^*}{2} + c_x - c_y - \frac{1}{4}\right)^{1/2}\right)^2$$

The payoff to player x_L in the pooling equilibrium is

$$\left(p^P - 1\right)\left(y^* - x_L\right)^2,$$

and similarly for player x_R. The probability that player x_L wins if she deviates is

$$p^S_{x_L} = \frac{1}{2} - \left(1 - 2\left(\frac{x_L + y^*}{2} + c_x - c_y - \frac{1}{4}\right)^{1/2}\right)^2,$$

and her payoff is

$$\left(p^S_{x_L} - 1\right)\left(y^* - x_L\right)^2.$$

Again, the analogous expression for player x_R is apparent. Let $x_L = 0.1, x_R = 0.4$ and fix y^* at 0.75. Also, choose H_x and H_y such that $c_x = c_y$. Then calculating the difference in payoffs for the players in the pooling equilibrium compared to deviating we find $\Delta x_L = 0.179$ and $\Delta x_E = 0.002$. Thus neither has a profitable deviation. ■

Proof of Proposition 7. Again, we need only provide an example: so consider $x_L = 0.2, x_R = 0.3$ and $\beta = 2$. This implies that in both the pooling and separating equilibrium $p_L = 1/2$. Thus we can write the expected utility of player X_L in the separating equilibrium as

$$\begin{aligned} EU^S_{X_L} &= -\frac{1}{2}\int\left(1-\pi_R\left(x,y\right)\right)\left(y-x_L\right)^2 f\left(y\right)dy - \frac{1}{2}\int \pi_R\left(x,y\right)\left(x_R-x_L\right)^2 f\left(y\right)dy \\ &= \frac{\left(x_L\right)^3}{2} - \frac{9\left(x_L\right)^2}{8} + \frac{5x_L}{6} - \frac{13}{64} - \frac{1}{8}(1+4x_L)\left(x_L-x_R\right)^2. \end{aligned}$$

Similarly, for player X_R we have

$$\begin{aligned} EU^S_{X_R} &= -\frac{1}{2}\int\left(1-\pi_R\left(x,y\right)\right)\left(y-x_L\right)^2 f\left(y\right)dy - \frac{1}{2}\int \pi_L\left(x_L-x_R\right)^2 f\left(y\right)dy \\ &= \frac{\left(x_R\right)^3}{2} - \frac{9\left(x_R\right)^2}{8} + \frac{5x_R}{6} - \frac{13}{64} - \frac{1}{8}(1+4x_R)\left(x_L-x_R\right)^2. \end{aligned}$$

The difference between the pooling and separating equilibrium payoffs is that in the pooling equilibrium the winner of the primary has the option to reveal herself as her true type. Because in the general election the winner of the primary can always mimic her announcement, the payoff to either player in the pooling equilibrium is at least as great in the separating equilibrium. Since the probability of winning the primary is the same in either the pooling or separating equilibrium, neither player wishes to deviate from pooling. ■