Loss Aversion in Politics
Supplementary Material
Online Appendix

Alberto Alesina[*] Francesco Passarelli[†]

February 2017

Contents

[*]Harvard, NBER, and Igier Bocconi.
E-mail: aalesina@harvard.edu
Web: http://scholar.harvard.edu/alesina/home
[†]University of Turin and Bocconi.
E-mail: francesco.passarelli@unibocconi.it
Web: https://cardiogroup.academia.edu/FrancescoPassarelli

1. A Parametric Model

This Section contains a parametric version of the model in the main text of the paper. It computes closed form solutions and numerical simulations. They are presented with references to the main results in the paper.

Each type t has benefit from policy p, according to the function $B : \mathbb{R}^2 \to \mathbb{R}_+$ and cost according to the function $C : \mathbb{R}^2 \to \mathbb{R}_+$. Let the benefit function be $B(t,p) = pt$ and let the cost function be $C(t,p) = p^2$. These two functions satisfy assumptions A1-A3.

1.1 Without Loss Aversion

The utility of a type t voter from policy p is

$$V(t,p) = pt - p^2$$

V is concave: the global maximum is characterized by the following first order condition. Voter i sets her desired policy p_i according to the following rule:

$$p_i = \frac{1}{2}t_i$$

1.2 With Loss Aversion

With loss aversion, policies both above and below the status quo involve an additional welfare loss.

$$V(t_i, p \mid p^S) = \begin{cases} pt - p^2 - \lambda(p^S - p)t & \text{if} \quad p \geq p^S \\ pt - p^2 - \lambda(p^2 - p^{S\,2}) & \text{if} \quad p < p^S \end{cases}$$

Voter i's ideal policy p_i solves the first order condition. The choice rule is as follows:

$$p_i = \begin{cases} \frac{1+\lambda}{2} t_i & \text{for} \quad t_i < \check{t} = \frac{2}{1+\lambda} p^S \\ p^S & \text{for} \quad \check{t} \leq t_i \leq \hat{t} \\ \frac{1}{2(1+\lambda)} t_i & \text{for} \quad t_i \geq \hat{t} = 2(1+\lambda)p^S \end{cases}$$

Median Voter Equilibrium - Proposition 1

i) *The policy outcome is the status quo if the median voter is an intermediate type; i.e.* $t_m \in \left[\check{t}, \hat{t}\right]$.

Let m be the median voter. If the type of the median voter t_m is in $\left[\frac{2}{1+\lambda}p^S,\ 2(1+\lambda)p^S\right]$ then the policy preferred by the median voter p_m is p^S. If $t_m \notin \left(\frac{2}{1+\lambda}p^S,\ 2(1+\lambda)p^S\right)$, the policy adopted is

$$p_m = \begin{cases} \frac{1+\lambda}{2} t_m & \text{for} \quad t_m < \frac{2}{1+\lambda} p^S \\ \frac{1}{2(1+\lambda)} t_m & \text{for} \quad t_m > 2(1+\lambda)p^S \end{cases}$$

ii) *If* $t_m \in \left[\check{t}, \hat{t}\right]$, *a shock affecting the preferences of the median will lead to a policy change only if it is sufficiently large. The size of the "inertia" interval* $\left[\check{t}, \hat{t}\right]$ *increases in the loss aversion parameter* λ.
If $t_m \in \left(\frac{2}{1+\lambda}p^S, 2(1+\lambda)p^S\right)$ a marginal change in the median voter location does not lead to any change in the policy adopted:

$$\frac{\partial p_m}{\partial t_m} = 0$$

Notice that $\lambda = 0 \iff \hat{t} = \tilde{t}$. The "inertia" interval is a singleton if and only if the voter is not loss averse. Moreover, the larger the loss aversion the bigger the inertia interval:

$$\frac{\partial \check{t}}{\partial \lambda} = -\frac{2}{(1+\lambda)^2}p^S \leq 0 \quad \text{and} \quad \frac{\partial \hat{t}}{\partial \lambda} = 2p^S \geq 0$$

Sufficiently large shocks are needed to produce change.

iii) *Voters' ideal policies are less dispersed with loss aversion than without it. If* $t_m \notin \left[\check{t}, \hat{t}\right]$, *a policy change occurs, but it is smaller than with no loss aversion.*
If $t_m < \frac{2}{1+\lambda}p^S$, the bliss point of the median voter without loss aversion is $\frac{1+\lambda}{2}t_m > \frac{1}{2}t_m$, where the RHS of this inequality is her bliss point with no loss aversion. If $t_m > 2(1+\lambda)p^S$, the bliss point is $\frac{1}{2(1+\lambda)}t_m \leq \frac{1}{2}t_m$. Thus under loss aversion the policy is closer to the status quo.

iv) *Suppose a) the status quo is low and the majority decides for a higher policy; or suppose b) the status quo is high and the same majority decides to change it for a lower policy. In the first case the majority chooses a lower policy, compared to the second case.*
Consider a given t_m outside of the "inertia" interval. The policy outcome "indirectly" depends on the position of the status quo. If the status quo is *high* and such that $t_m < \frac{2}{1+\lambda}p^S$, the policy is $\frac{1+\lambda}{2}t_m$. If it is low and such that $t_m > 2(1+\lambda)p^S$, the policy is $\frac{1}{2(1+\lambda)}t_m < \frac{1+\lambda}{2}t_m$. This result is shown in Figure 1.

v) *Once a new policy has been approved and becomes the status quo, more than the strict majority of people do not want to return to the previous status quo.*
Suppose a change in the position of the median occurs, say the median is now such that $t_m > \hat{t}^1 = 2(1+\lambda)p^{S1}$. The new policy is $p^1 = p_m = \frac{1}{2(1+\lambda)}t_m$. Such a policy becomes the new status quo p^{S2} and the upper bound of the inertia interval is now $\hat{t}^2 = 2(1+\lambda)p^{S2} = t_m$. Wile p_m was once voted by the simple majority of voters, it is now preferred to any of the policies lower than p_m with more than the majority of votes. In particular, if F is the distribution of types, the share of voters voting for $p_m - e$, e being arbitrarily small, is strictly greater than $1 - F(\check{t}^2)$. The latter is larger than $1 - F(\hat{t}^2) = 0.5$.

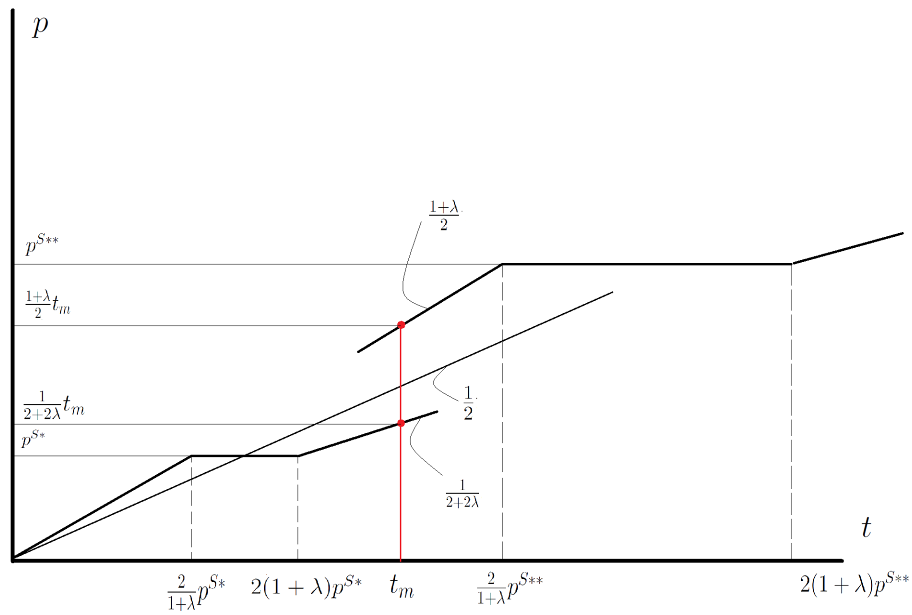


Figure 1: Entrenchment

1.3 Old and Young Voters

Individuals live two periods. The birth rate is b. At each period both young and old decide the policy. The fraction of young people in the population is $\sigma = \frac{1+b}{2+b}$. Given a status quo p^S, a policy p will pass if and only if more than half of the population prefers it to the status quo, i.e.

$$p > p^S \iff (\frac{1}{2+b})F(\hat{t}^o) + \frac{1+b}{2+b}F(\hat{t}^y) < 0.5$$

$$p < p^S \iff (\frac{1}{2+b})\left[1 - F(\check{t}^o)\right] + \frac{1+b}{2+b}\left[1 - F(\check{t}^y)\right] < 0.5$$

A young i sets her preferred policy according to the rule

$$p_i = \begin{cases} \frac{1+\frac{\lambda}{2}}{2}t_i & \text{for} \quad t_i < \check{t}^y = \frac{2}{1+\frac{\lambda}{2}}p^S \\ p^S & \text{for} \quad \check{t}^y \leq t_i \leq \hat{t}^y \\ \frac{1}{2(1+\frac{\lambda}{2})}t_i & \text{for} \quad t_i \geq \hat{t}^y = 2(1+\frac{\lambda}{2})p^S \end{cases}$$

while an old individual i sets her preferred policy according to the rule

$$p_i = \begin{cases} \frac{1+\lambda}{2}t_i & \text{for} \quad t_i < \check{t}^o = \frac{2}{1+\lambda}p^S \\ p^S & \text{for} \quad \check{t}^o \leq t_i \leq \hat{t}^o \\ \frac{1}{2(1+\lambda)}t_i & \text{for} \quad t_i \geq \hat{t}^o = 2(1+\lambda)p^S \end{cases}$$

Chance and Size of Reforms - Proposition 2

i) *The share of people who want the status quo is always larger amongst the old generation than the young one.*
Notice that $\check{t}^y = \frac{4}{2+\lambda}p^S > \frac{2}{1+\lambda}p^S = \check{t}^o$ and $\hat{t}^y = 2(1+\frac{\lambda}{2})p^S < 2(1+\lambda)p^S = \hat{t}^o$. Thus the "inertia" interval is smaller in young generations:

$$\left[\check{t}^y, \hat{t}^y\right] \subseteq \left[\check{t}^o, \hat{t}^o\right]$$

By monotonicity of a probability measure: $1-F(\check{t}^y) < 1-F(\check{t}^o)$ and $F(\hat{t}^y) < F(\hat{t}^o)$. Thus a smaller share of the young generation is in favor of the status quo compared to the old generation. Notice that, for fixed t_i, by the moderation effect

$$\frac{\partial p_i}{\partial \lambda}\begin{cases} \geq 0 & \text{for} \quad t_i < \check{t} \\ = 0 & \text{for} \quad \check{t} \leq t_i \leq \hat{t} \\ \leq 0 & \text{for} \quad t_i \geq \hat{t} \end{cases}$$

More simply bliss points are closer to the status quo in the old generation compared to the young one: $\frac{1+\frac{\lambda}{2}}{2}t_i < \frac{1+\lambda}{2}t_i$ and $\frac{1}{2(1+\frac{\lambda}{2})}t_i > \frac{1}{2(1+\lambda)}t_i$. So, every majority in the old generation will want a policy closer to the status quo than the same majority among the youth.

iii) *The lower the birth rate, b, the larger the set of parameter values for which the status quo does not change.*
Assume, as an illustration, F is uniform on $[0,2]$:

$$p > p^S \iff (1+\lambda)p^S - \frac{1+b}{2+b}\frac{\lambda}{2}p^S < 0.5$$

$$p < p^S \iff 1 - \frac{1}{1+\lambda}p^S + \frac{1+b}{2+b}\left[\frac{1}{1+\lambda}p^S - \frac{1}{1+\frac{\lambda}{2}}p^S\right] < 0.5$$

Consider the converse of the first condition: $(1+\lambda)p^S - \frac{1+b}{2+b}\frac{\lambda}{2}p^S \geq 0.5$ and assume $p^S < 0.5$. For fixed b, the higher λ the larger the left hand side. Moreover, the lower b, the lower λ can be in order to satisfy that inequality. We can see this by rearranging the inequality. We get:

$$\lambda < [\frac{1}{2p^S} - 1][\frac{2(2+b)}{3+b}] = \bar{\lambda}$$

Notice that

$$\frac{d\bar{\lambda}}{db} = \frac{4}{b^2}[\frac{1}{2p^S} - 1] > 0$$

The inequality holds since $p^S < 0.5$. As a result the upper bound $\bar{\lambda}$ of the set of values that λ can take to satisfy the inequality increases in the rate b.

Equilibrium Reform - Proposition 2 continued

Assume at time k policy $p^k \neq p^S$ is feasible, WLOG $p^k > p^S$. Let $t^y(p^k)$ be the type of young voter whose bliss point is p^k and $t^o(p^k)$ be the old voter whose bliss point is p^k. Specifically:

$$\begin{aligned} t^y(p^k) = & \ 2(1+\tfrac{\lambda}{2})p^k \\ t^o(p^k) = & \ 2(1+\lambda)p^k \end{aligned}$$

Observe that $t^y(p^k) < t^o(p^k)$. If p^k is feasible then

$$(\frac{1}{2+b})F(t^o(p^k))+\frac{1+b}{2+b}F(t^y(p^k)) = (\frac{1}{2+b})F(2(1+\lambda)p^k)+\frac{1+b}{2+b}F(2(1+\frac{\lambda}{2})p^k) = 0.5 \tag{1}$$

Since we assumed that F is uniform $[0,2]$, equation 1 becomes:

$$\left[1+\lambda(1-\frac{1+b}{2(2+b)})\right]p^k = 0.5$$

iv) *Assume a constituency for a reform exists in period k. The reform is smaller in absolute value if the birth rate is lower.*

When the birth rate b increases the share of the young generation increases. For the equation to hold p^k must increase:

$$\frac{\partial p^k}{\partial b} = \frac{\lambda}{\left[2(2+b)+\lambda(3+b)\right]^2} > 0$$

With a higher birth rate policies that are more distant from the status quo become feasible. A lower birth rate will move the feasible equilibrium reform closer to the status quo.

Ripe Times for Reforms - Proposition 3

Let e be the life expectancy growth rate. Young and old voters' *perceived* loss aversion parameters are then

$$\lambda^y = L^y(e,k) = \frac{\lambda}{2+2ek} \quad \text{and} \quad \lambda^o = L^o(e,k) = \frac{\lambda}{1+2e(k-1)}$$

The date k^* at which time is ripe for overruling the status quo with a higher policy is the integer immediately subsequent to $\tilde{k}$ where such a $\tilde{k}$ solves the following equation:

$$(\frac{1}{2+b})(1+\frac{\lambda}{1+2e(\tilde{k}-1)}p^S) + \frac{1+b}{2+b}(1+\frac{\lambda}{2+2e\tilde{k}})p^S = 0.5$$

$\tilde{k}$ is then the time when voters who want to change represent fifty per cent of the population:

i) *If life expectancy grows, there exists a "ripe time" k^* such that a change in the status quo becomes politically feasible.*
To show that $\tilde{k}$ exists, let $p^S = \frac{1}{3}$, $\lambda = 1$, $b = 0$ and $e < \frac{1}{4}$ (life expectancy grows but not too fast, otherwise the ripe time could be less than 1). The equation above becomes

$$-4e^2k^2 + (4e^2 - 2e)k + 1 = 0$$

The only feasible solution to the quadratic equation is

$$\tilde{k} = \frac{1}{2} - \frac{1}{4e} + \sqrt{\frac{1}{2} - \frac{1}{4e} + \frac{1}{16e}}$$

Thus $\tilde{k}$ exists.

ii) *k^* is decreasing in life expectancy growth rate, e.*
Since we assumed $e < \frac{1}{4}$, the derivative with respect to e is negative:

$$\frac{d\tilde{k}}{de} = \frac{1}{4} + \frac{1}{2\sqrt{\frac{1}{2} - \frac{1}{4e} + \frac{1}{16e}}}(\frac{1}{4} - \frac{1}{16e^2}) < 0$$

1.4 Policy Motivated Parties

Consider two candidates/parties with policy preferences. The left-wing one prefers $\bar{l} = 0$ and the right-wing one prefers $\bar{r} = 1$. They have strictly concave utility functions on the policy space.
Assume the utility of the right party is

$$U(p) = p^{\frac{1}{4}}$$

for $p \in [0, 1]$, while the left party has utility

$$U(1 - p) = (1 - p)^{\frac{1}{4}}$$

Both right and left-wing party's utility functions are continuous increasing on $p < 0$ and decreasing on $p > 1$. So the candidates have no incentive to propose platforms outside $[0, 1]$. A party wins when the median voter prefers his platform to the competitor's one. Candidate left proposes policy x, while right proposes policy y.
The two parties do not know the exact position of the median voter. We assume they share a subjective uniform probability distribution of the median voter with expected value $t_m = 1.6$, support $[-0.4, 3.6]$.
Given candidates' policies (x, y) and status quo p^S, the probability that party left wins is

$$P(T(x, y, p^S) > t_m + \varepsilon) = \frac{T(x, y, p^S) + 0.4}{4}$$

where $T(x, y, p^S)$ is the type of the voter indifferent between policy x, and y. When policies are $x < p^S < y$. This means that

$$V(T, x\,|p^S) = xT - x^2 - \lambda(p^S - x)T = yT - y^2 - \lambda(y^2 - p^{S2}) = V(T, y\,|p^S)$$

and reduces to

$$T(x, y, p^S) = \frac{x^2 - y^2 - \lambda(y^2 - p^{S2})}{x - y - \lambda(p^S - x)}$$

Without loss aversion ($\lambda = 0$), it becomes

$$T(x, y) = x + y$$

We have

$$T_x(x, y, p^S) = \frac{2x\left[x - y - \lambda(p^S - x)\right] - (1 + \lambda)\left[x^2 - y^2 - \lambda(y^2 - p^{S2})\right]}{\left[x - y - \lambda(p^S - x)\right]^2} > 0$$

$$T_y(x, y, p^S) = \frac{(1 + \lambda)\, y^2 - 2\,(1 + \lambda)^2\, xy + 2\,(1 + \lambda)\,\lambda p^S y + \lambda p^{S2} + x^2}{\left((1 + \lambda)\, x - y - \lambda p^S\right)^2} > 0$$

$$T_{p^S}(x, y, p^S) = \frac{2\lambda p^S[x - y - \lambda(p^S - x)] + \lambda[x^2 - y^2 - \lambda(y^2 - p^{S2})]}{\left((1 + \lambda)\, x - y - \lambda p^S\right)^2} < 0$$

Without loss aversion, the partial derivatives of the indifferent type would simply be:

$$T_x(x, y) = T_y(x, y) = 1 \quad \text{and} \quad T_{p^S}(x, y) = 0$$

With loss aversion, the equilibrium (x^*, y^*) must satisfy the two FOCs:

$$-\frac{1}{4}\frac{T + 0.4}{4}(1 - x)^{-\frac{3}{4}} + \frac{T_x}{4}(1 - x)^{\frac{1}{4}} - \frac{T_x}{4}(1 - y)^{\frac{1}{4}} = 0$$

$$\frac{1}{4}(1 - \frac{T + 0.4}{4})y^{-\frac{3}{4}} - \frac{T_y}{4}y^{\frac{1}{4}} + \frac{T_y}{4}x^{\frac{1}{4}} = 0$$

Such a system of nonlinear equations needs not have an analytical solution. Hence, by a numerical computation performed by Matlab we can study the equilibrium in a neighborhood of the status quo. By setting $\lambda = 0.5$, $p^S = 0.5$ we obtain $(x^*, y^*) \approx (0.007,\ 0.700)$.

Convergence - Proposition 4

Loss aversion leads the two candidates to propose closer platforms than without loss aversion.

By computing the equilibrium for a grid of values of λ around 0.5 we get that

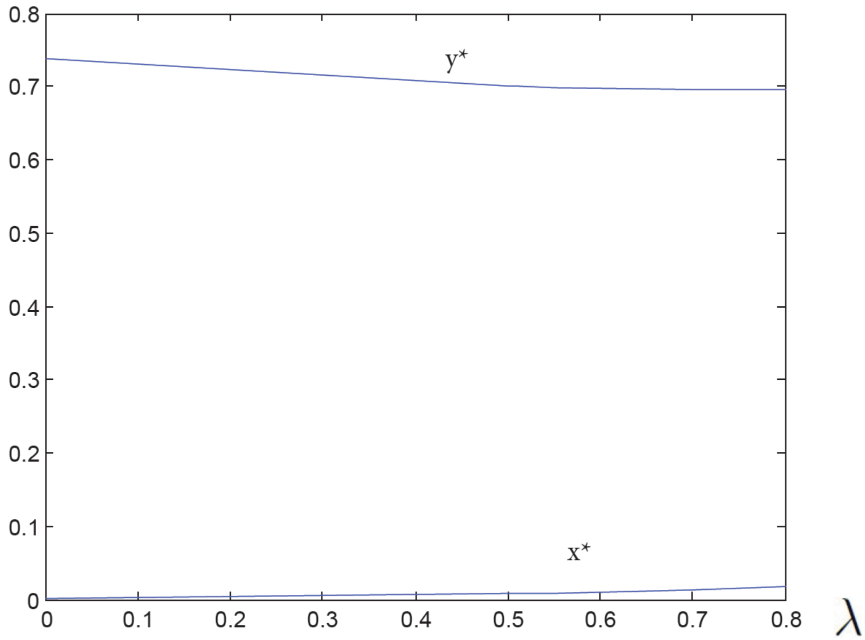


Figure 2: Higher convergence

the distance between the two equilibrium platforms is smaller when loss aversion is higher,

$$\frac{\partial\left(y^{*}-x^{*}\right)}{\partial\lambda}<0$$

This result is shown in Figure 2.

Dynamic Status Quo Bias - Proposition 5

i) *If both candidates' utility functions are sufficiently steep and concave, then equilibrium platforms x^* and y^* positively depend on the status quo.*

ii) *If in addition the loss aversion parameter λ is sufficiently large, the expected policy outcome is positively affected by the status quo.*

By either computing the value of the derivatives $\frac{\partial x^*}{\partial p^S}$ and $\frac{\partial y^*}{\partial p^S}$or directly solving for the value of (x^*, y^*) in a grid of values of p^S around 0.5, we obtain that two equilibrium platforms and the expected policy increase in the status quo (see numerical computations in Figure 3)

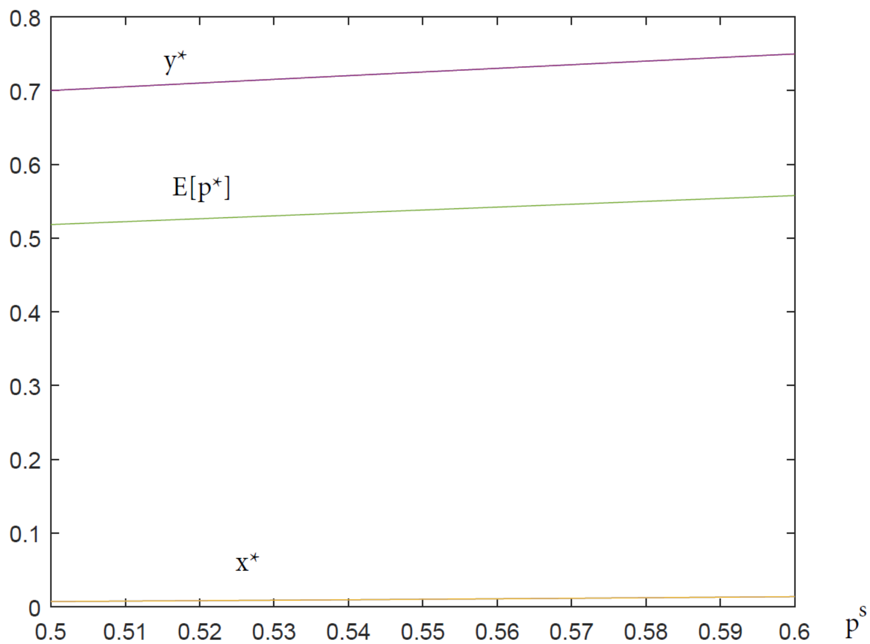


Figure 3: Dynamic status quo bias

2. An example: Fiscal Policy

2.1. Public Good provision

Here we apply the framework described in the main paper to a basic Meltzer and Richard model with public good provision.[1] The policy consists in the provision of a non-excludable public good financed by a proportional income tax. Agents enjoy utility from consumption of a private good (c_i) and the public good (g) that we measure here in per capita terms. Instead of a public good, we could have a lump sum redistribution; the results are identical.

Let the utility function be quasi-linear in c_i, and concave and increasing in g:

$$u(c_i, g) = c_i + H(g)$$

($H' > 0$, $H'' < 0$). Individuals are heterogeneous in income: let y_i be the income of individual i, and denote by $\bar{y}$ the average income. Denote with m the individual with the median income. The government budget is balanced and the prices of c

[1] The model of this section is a stylized version of Meltzer and Richard (1981) as presented by Persson and Tabellini (2000, pp. 48-50).

and g are normalized to 1. Indirect utility of voter i is then:

$$V(y_i, g) = y_i + H(g) - \frac{y_i}{\bar{y}} g$$

Her most preferred level of g is:

$$g_i = H'^{-1}\left(\frac{y_i}{\bar{y}}\right)$$

Policy preference functions are single peaked and the bliss points negatively depend on individual incomes: richer individuals want a smaller government because the private cost of one unit of public good $\left(\frac{y_i}{\bar{y}}\right)$ is higher for them. The equilibrium is the median voter's most preferred policy (g_m). The normative implication is that the majority rule, or Downsian electoral competition, implements the social optimum only if the median voter's income equals the average income. If instead the income distribution is skewed toward the right (i.e. $y_m < \bar{y}$), the voting outcome is overspending and overtaxation.

Loss aversion

Let g^S be the status quo amount of public good. Lower public good provision or additional taxes are both a loss, while more public good or tax reductions are a gain. Under loss aversion indirect utility is:

$$V(g, y_i \mid g^S) = \begin{cases} V(y_i) - \lambda \frac{y_i}{\bar{y}}(g - g^S) & \text{if} \quad g \geq g^S \\ V(y_i, g) - \lambda \left[H(g^S) - H(g)\right] & \text{if} \quad g < g^S \end{cases}$$

The bliss point, i.e., the most preferred amount of g is:[2]

$$g_i = \begin{cases} H'^{-1}\left(\frac{y_i(1+\lambda)}{\bar{y}}\right) & \text{if} \quad y_i < \check{y} \\ g^S & \text{if} \quad \check{y} \leq y_i \leq \hat{y} \\ H'^{-1}\left(\frac{y_i}{\bar{y}(1+\lambda)}\right) & \text{if} \quad y_i > \hat{y} \end{cases}$$

Suppose median income declines by a small amount compared to the mean, i.e., inequality increases, at least according to this measure. In the standard model that would always imply a change in policy: higher taxes and more public good. In the model with loss aversion, instead, an increase in income inequality may lead to no changes in taxation as long as the change in inequality does not push the

[2] Where $\check{y} \equiv \frac{1}{(1+\lambda)} H'(g^S)\bar{y}$, and $\hat{y} \equiv (1+\lambda)\, H'(g^S)\bar{y}$. If $y_i < \check{y}$ then $H'^{-1}\left(\frac{y_i(1+\lambda)}{\bar{y}}\right) > g^S$ and if $y_i > \hat{y}$ then $H'^{-1}\left(\frac{y_i(1+\lambda)}{\bar{y}}\right) < g^S$. Also bserve that both $\check{y}$ and $\hat{y}$ negatively depend on the status quo, g^S (cf. Proposition 1 in the main text).

parameter values outside the range in which the status quo prevails. This result helps in rationalizing for instance why a large increase in income inequality in the US and in other countries has not been accompanied by an immediate increase in redistributive policies.

In addition, with loss aversion, the marginal cost of more public good is higher and the marginal benefit of less public good is lower. Therefore, compared to the standard Meltzer and Richard model, the rich increase their demand for public good and the poor reduce theirs.[3] The level of disagreement about the size of government is lower in a loss averse society. This result may help rationalize why the poor would not impose an expropriatory level of taxation and the rich might accept a certain moderate level of taxes. In other words the *moderation effect* helps making sense of non extreme forms of taxation in democracies.

2.2. Lump Sum Transfers

As in Meltzer and Richard (1981), the policy consists in a lump-sum transfer financed by a proportional income tax. Individuals are heterogeneous in labor productivity (x_i). The distribution of x_i is common knowledge, and its average is normalized to one: $\bar{x} = 1$. Individuals are risk neutral and draw utility from consumption and disutility from labor. Their utility is:

$$v_i = c_i - U(l_i)$$

where c is consumption, l is labor, and $U(\cdot)$ is an increasing and convex function with $U(0) = 0$. Labor is the only factor of production. The government can levy a linear income tax τ and provide a non-negative lump sum transfer r. The budget constraint of individual i is:

$$c_i = x_i\, l_i(1 - \tau) + r$$

The balanced public budget constraint is (the population size is one):

$$\tau\, L = r$$

where L is total labor supply (and income). Individual labor choice is:

$$l_i^* \in \arg\max_{l_i} \quad x_i\, l_i(1 - \tau) + r - U(l_i)$$

[3] An alternative and not mutually exclusive argument for why the poor may not want to aggressively expropriate the rich is the *Prospect for Upward Mobility* (POUM) hypothesis by Bénabou and Ok (2001).

The individual optimality condition,

$$x_i\,(1-\tau) - U'(l_i) = 0$$

yields individual labor supply:

$$l_i^* = U'^{-1}(x_i(1-\tau))$$

Since $U'^{-1}(\cdot)$ is an increasing function, individual (and total) labor supply increases in productivity and decreases in taxes.

Using the government budget constraint, the individual policy preference function (recall that $\bar{x} = 1$) is:

$$V_i(\tau) = l_i^* x_i(1-\tau) + \tau L^*(\tau) - U(l_i^*)$$

where $L^*(\tau)$ is the equilibrium total labor supply function. Recall that $y_i = l_i^* x_i$ and $\bar{y} = L^*(\tau)$. Then,

$$V_i(\tau) = y_i(1-\tau) + \tau\bar{y} - U\left(\frac{y_i}{x_i}\right)$$

Applying the envelope theorem, and maximizing, yields the optimality condition (Meltzer and Richard, 1981, eq. (13), p. 920), which pins down the individuals' bliss points:

$$\bar{y} + \tau\frac{\partial\bar{y}}{\partial\tau} - y_i \leq 0$$

Bliss points are interior only for individuals whose labor productivity is lower than the average, $x_i < 1 \rightarrow y_i < \bar{y}$. All the other types want zero tax.

Let us now apply loss aversion. We assume that an individual brackets taxes and transfers separately: i.e., she perceives the monetary amount of taxes that she pays separately with respect to the amount of transfers that she receives. This is consistent with the extensive literature and familiar observation that people treat out-of-pocket expenses (taxes, in this case) differently than opportunity costs (transfers, in this case). Alternatively one can assume that the individual perceives separately the two dimensions of her labor supply choice: leisure and money. This would imply a slightly different, but qualitatively equivalent model of loss aversion. Let τ^S be the status quo tax rate. By loss aversion, the indirect utility is computed relative to the status quo, and losses are overweighed.

Thus for individuals who enjoy positive net transfers, it becomes:[4]

$$V(\tau, y_i \mid \tau^S) = \begin{cases} \begin{array}{l} \left[y_i(1-\tau) - y_i^S(1-\tau^S)\right](1+\lambda) + \tau\bar{y} - \tau^S\bar{y}^S \\ -U(\frac{y_i}{x_i}) + U(\frac{y_i^S}{x_i}) \end{array} & \text{if} \quad \tau \geq \tau^S \\ \begin{array}{l} y_i(1-\tau) - y_i(1-\tau^S) + \left[\tau\bar{y} - \tau^S\bar{y}^S\right](1+\lambda) \\ -\left[U(\frac{y_i}{x_i}) - U(\frac{y_i^S}{x_i})\right](1+\lambda) \end{array} & \text{if} \quad \tau < \tau^S \end{cases}$$

[4] Of course we constraint τ and τ^S to be lower than the tax rate that maximizes the Laffer curve.

where y_i^S and $\bar{y}^S$ are the status quo individual and total income, respectively. The first line says that, by loss aversion, the voter overweighs the reduction of personal income (a loss) when she decides for a tax increase. The second line says that she overweighs the loss of transfers, and the increased disutility from labor, if the tax rate decreases. By the envelope theorem, the optimality condition is then

$$\frac{\partial V}{\partial \tau}(\tau, y_i \mid \tau^S) = \begin{cases} \bar{y} + \tau \frac{\partial \bar{y}}{\partial \tau} - y_i(1+\lambda) \gtreqless 0 & \text{if} \quad \tau \geq \tau^S \\ \left[\bar{y} + \tau \frac{\partial \bar{y}}{\partial \tau}\right](1+\lambda) - y_i \gtreqless 0 & \text{if} \quad \tau < \tau^S \end{cases}$$

Starting from this point, all results of the Fiscal Policy model presented above also hold in this model of lump sum redistribution.

3. Projection Bias
Subsection 4.2.2

We define a period as the length of time in which a certain outcome becomes the status quo. Individual i lives n periods. We are at the beginning of period 1, and future periods are indexed by k ($k = 2, .., n$), with no discounting for future utility. Each voter chooses her ideal policy in each period assuming that at period k, the ideal policy of period $k-1$, p^{k-1}, becomes the new status quo. Thus, p^{k-1} is a policy variable in period $k-1$, while in period k it is a predetermined state variable. In period 1, the (exogenous) status quo policy is p^0.

In order to account for a projection bias, we follow Loewenstein et al. (2003) in assuming that at the beginning of her residual life voter i's predicted lifetime utility is a weighted average between two lifetime utility functions with different reference points: the current status quo, p^0, and the one-period lagged status quo, p^{k-1},

$$\tilde{V}(t_i, p^1, .., p^n \mid p^0) = V(t_i, p^1 \mid p^0) + (1-\alpha)\sum_{k=2}^{n} V(t_i, p^k \mid p^{k-1}) + \alpha \sum_{k=2}^{n} V(t_i, p^k \mid p^0)$$

α parametrizes projection bias ($0 \leq \alpha \leq 1$): if $\alpha = 1$, then i perceives her preferences in period k will not change as a result of a change in the status quo, p^{k-1}; when $\alpha = 0$, she has no projection bias. With this formulation, a voter thinks ex ante that with probability $(1-\alpha)$ she will never become accustomed to the new policy, and with probability α she will completely accustom to the new policy after one single period.

Proposition 1 below states that voter i's perceived loss aversion is $\frac{\lambda(1+\alpha(n-1))}{n}$. Intuitively, ex ante the voter thinks that with probability α her preferences will not change, thus she will bear the cost of change for n periods, and her perceived loss aversion is λ. With probability $(1-\alpha)$ the cost is gone after one single period and her perceived loss aversion is $\frac{\lambda}{n}$.

Proposition 1 *(Projection Bias)*
In the presence of a projection bias parametrized by α, a voter living for n periods sets the policy at period 1 as if her loss aversion parameter were $\frac{\lambda(1+\alpha(n-1))}{n}$.

Proof. Let us proceed backward. In period n, any individual i chooses her policy in order to maximize her perceived residual lifetime utility $\tilde{V}(t_i, p^{n-1}, p^n \mid p^0)$:

$$p_i^n \in \arg\max_{p^n} \begin{cases} V(t_i, p^n) - \lambda(1-\alpha)\left[C(t_i, p^n) - C(t_i, p^{n-1})\right] & \text{if} \quad p^n \geq p^{n-1} \\ \qquad -\lambda\alpha\left[C(t_i, p^n) - C(t_i, p^0)\right] & \text{and} \quad p^n \geq p^0 \\ \\ V(t_i, p^n) - \lambda(1-\alpha)\left[B(t_i, p^{n-1}) - B(t_i, p^n)\right] & \text{if} \quad p^n < p^{n-1} \\ \qquad -\lambda\alpha\left[B(t_i, p^0) - B(t_i, p^n)\right] & \text{and} \quad p^n < p^0 \end{cases}$$

In period n the ideal policy p_i^n is a function of the state variables p^{n-1} and p^0: $p_i^n = T(p^{n-1}, p^0)$.
At time $n-1$ the ideal policy p_i^{n-1} is such that:

$$p_i^{n-1} \in \arg\max_{p^{n-1}} \left\{ \tilde{V}(t_i, p^{n-2}, p^{n-1}, T(p^{n-1}, p^0) \mid p^0) + \tilde{V}(t_i, p^{n-1}, T(p^{n-1}, p^0) \mid p^0) \right\}$$

The ideal policy of period $n-1$ coincides with the voter's plan to keep this policy unchanged: $p^{n-1} = p_i^n$. Voting for any alternative policy $p^{n-1} \neq p_i^n$ would result in a loss of utility in period $n-1$. Applying this argument backward, the best policies of periods $2, \ldots, n$ are such that: $p^1 = p_i^2 = \cdots = p_i^n$.
We can now see how the voter sets her most preferred policy p^1. Lifetime perceived utility at period 1 is $\tilde{V}(t_i, p^1, .., p^n \mid p^0)$ given by

$$V(t_i, p^1 \mid p^0) + (1-\alpha)\sum_{k=2}^{n} V(t_i, T(p^k, p^{k-1}) \mid T(p^{k-1}, p^{k-2})) + \alpha \sum_{k=2}^{n} V(t_i, T(p^k, p^{k-1}) \mid p^0)$$

Observe that, for $K > 2$, $T(p^k, p^{k-1}) = T(p^{k-1}, p^{k-2})$. This implies that in the second term of the above expression, we have $V(t_i, T(p^k, p^{k-1}) \mid T(p^{k-1}, p^{k-2})) = V(t_i, p^1)$. Thus with probability $(1-\alpha)$, experienced utility for the next $(n-1)$ periods starting from period 2 will be equal to utility with no loss aversion. Further observe that $p_i^k = p^1$ implies $T(p^k, p^{k-1}) = p^1$ for any $k \geq 2$. Therefore we can rewrite $\tilde{V}(t_i, p^1, .., p^n \mid p^0)$ as $V(t_i, p^1 \mid p^0) + (1-\alpha)(n-1)V(t_i, p^1) + \alpha(n-1)V(t_i, p^1 \mid p^0)$.
After some algebraic manipulation, we obtain:

$$\begin{cases} nB(t_i, p^1) - nC(t_i, p^1) - \lambda\left[C(t_i, p^1) - C(t_m, p^0)\right] \\ -\lambda\alpha(n-1)\left[C(t_i, p^1)) - C(t_i, p^0)\right] & \text{if} \quad p^1 \geq p^0 \\ \\ nB(t_i, p^1) - nC(t_i, p^1) - \lambda\left[B(t_i, p^0) - B(t_i, p^1)\right] \\ -\lambda\alpha(n-1)\left[B(t_i, p^0) - B(t_i, p^1))\right] & \text{if} \quad p^1 < p^0 \end{cases}$$

Maximizing this function w.r.t. p^1, yields voter i's ideal policy plan

$$p_i^1 \text{ solves } \begin{cases} B_p(t_i, p^1) - (1 + \frac{\lambda(1+\alpha(n-1)}{n})C_p(t_i, p^1) = 0 & \text{if } p^1 > p^0 \\ p^1 = p^0 & \text{if } p^1 = p^0 \\ (1 + \frac{\lambda(1+\alpha(n-1))}{n})B_p(t_i, p^1) - C_p(t_i, p^1) = 0 & \text{if } p^1 < p^0 \end{cases}$$

and $p_i^2 = ... = p_i^n = p_i^1$

This proves that the median sets the policy at the first period *as if* her perceived loss aversion were $\frac{\lambda(1+\alpha(n-1))}{n}$. ■

4. Electoral Competition with Forward-Looking Voters
Subsection 6.1.1

Assume all voters have a two-period planning horizon. In each period voters choose the platform that maximizes their expected residual lifetime utility. Specifically, in period 2 voter i's indirect utility is

$$V(t_i, p^2 \mid p^{*1}) = \begin{cases} V(t_i, p) - \lambda \left[C(t_i, p^2) - C(t_i, p^1)\right] & \text{if} \quad p^2 \geq p^1 \\ V(t_i, p) - \lambda \left[B(t_i, p^1) - B(t_i, p^2)\right] & \text{if} \quad p^2 < p^1 \end{cases}$$

Voters choose their ideal policy, p_i^2, exactly as in the static model. Since their residual life in one period, their perceived loss aversion is λ. Note that in period 2 the policy p^1 is predetermined and is taken as given. It is the realization of the competition in period 1; thus $p^1 \in \{x^{*1}, y^{*1}\}$.

In period 1 individuals choose how to vote according to the following intertemporal utility function

$$V(t_i, p^1 \mid p^0) + V(t_i, p^2 \mid G(p^1)) \tag{2}$$

The second term incorporates the rational expectation of the future reference point, $G(p^1) = \mathbf{E}(X^1(p^1), Y^1(p^1))$, where $x^{*1} = X^1(p^1)$ and $y^{*1} = Y^1(p^1)$ are the two equilibrium platforms in period 1 and the expectation operator $\mathbf{E}$ is computed according the equilibrium probability $P(X^1(p^1), Y^1(p^1))$.

As in the model of Section 4, the following lemma says each voter perceives a lower degree of loss aversion in period 1.

Lemma 1 *In period 1, all voters choose the policy based on a "perceived" loss aversion parameter $\tilde{\lambda} < \lambda$. In period 2, they choose the policy based on λ.*

Proof. Lemma 1
The second statement of the lemma is trivial because their residual life is one period (cf. also Lemma **??**). As for the first statement, let $G(p^1) \equiv P(X^1(p^1), Y^1(p^1)) \cdot X^1(p^1) + [1 - P(X^1(p^1), Y^1(p^1))] \cdot Y^1(p^1)$ be the expected equilibrium policy in period 2 if policy p^1 is implemented in period 1. The two equilibrium platforms in the second period, x_2^* and y_2^*, depend on the outcome of the electoral competition in period 1: $x_2^* = X^1(p^1)$ and $y_2^* = Y^1(p^1)$. The first derivative $G_p(p^1)$ is positive since a higher policy in period 1 leads to a higher expected policy outcome in period 2 (cf. Proposition 5-*ii)*). Both candidates propose a higher platform and $P(X^1(p^1), Y^1(p^1))$ decreases at the margin. Furthermore, the argument in the proof of Proposition 1 also applies here: no voter has incentive to choose p^1 such that $p^1 > p^0$ and $G(p^1) < p^1$, or such that $p^1 < p^0$ and $G(p^1) > p^1$. Thus we can re-write

voter i's intertemporal utility as follows:

- if $G(p^1) \geq p^1 \geq p^0$ then intertemporal utility is

$$V(t_i, p^1) + V(t_i, G(p^1)) - \lambda\left[C(t_i, p^1) - C(t_i, p^0)\right] - \lambda\left[C(t_i, G(p^1)) - C(t_i, p^1)\right]$$

- if $G(p^1) \leq p^1 < p^0$ then intertemporal utility is

$$V(t_i, p^1) + V(t_i, G(p^1)) - \lambda\left[B(t_i, p^0) - B(t_i, p^1)\right] - \lambda\left[B(t_i, p^1) - B(t_i, G(p^1))\right]$$

Taking the derivative wrt p^1 of the above expressions we have, respectively,

$$\begin{array}{l} B_p(t_i, p^1) - C_p(t_i, p^1)(1 + G_p(p^1)) - \lambda C_p(t_i, p^1) G_p(p^1) \\ B_p(t_i, p^1) - C_p(t_i, p^1)(1 + G_p(p^1)) + \lambda B_p(t_i, p^1) G_p(p^1) \end{array}$$

Let $\tilde{\lambda} \equiv \lambda \frac{G_p(p^1)}{(1+G_p(p^1))} < \lambda$. The optimality condition is

$$\begin{cases} B_p(t_i, p^1) - (1 + \tilde{\lambda}) C_p(t_i, p^1) \gtreqless 0 & \text{if} \quad G(p^1) \geq p^1 \geq p^0 \\ (1 + \tilde{\lambda}) B_p(t_i, p^1) - C_p(t_i, p^1) \gtreqless 0 & \text{if} \quad G(p^1) \leq p^1 < p^0 \end{cases}$$

In period 1 voter i chooses the policy based on a perceived loss aversion parameter, $\tilde{\lambda}$, that is lower than λ. ■

Lower perceived loss aversion in period 1 yields lower moderation in policy preferences. The following proposition says how this affects electoral competition in the two periods.

Proposition 2 (Dynamic status quo bias with forward-looking voters)
Compared to the case of single-period planning horizon, if voters have a two-period planning horizon then,
i) parties propose less convergent platforms in period 1;
ii) parties propose more extreme platforms in period 2.

Proof. By Lemma 1 above, in period 1 voters are less subject to loss aversion, because they anticipate the effect of their current choice on the policy equilibrium of period 2. Due to lower loss aversion, their preferences are more dispersed. Thus the two parties propose less moderate platforms in period 1. In period 2 voters' residual life is one period, thus their perceived loss aversion is λ. Electoral competition takes place as in the second period of the dynamic model of Section 4 of the main paper. The only difference is in the state variable, the status quo, which can be either a more right-wing (left-wing) policy if the winner in period 1 was the right-wing (left-wing) candidate. Thus, independently of the winner in period 1, the status quo of period 2 is less a moderate policy. ■

Summing up, the risk of a "political drift" towards one or the other extreme of the political space is larger when voters have a two-period planning horizon.

5. Political Cycles in Old and Young Societies
Subsection 6.1.2

In subsection 6.1.2 of the main paper we claim that political cycles should be smaller but more persistent in ageing societies than in younger ones. One should observe that, say, after a victory of the left-wing candidate in period 1 the chance to "return" to a given right-wing policy in the second period is smaller in the ageing society than in the young one.

Let the initial status quo in period 1 be the same in both societies. Call it p^0. Take the case in period 1 the left-wing candidate won the election.[5] Let $p^{1o} = x^{1o*}$ be the policy implemented in period 1 by the old society, and let $p^{1y} = x^{1y*}$ be the policy implemented in period 1 by the young society. By Proposition 4 in the main text, the former policy is closer to p^0 than the latter because the ageing society perceives a higher degree of loss aversion. Thus candidates converge more in the ageing society. Therefore, $p^{1o} > p^{1y}$. In the second period, the status quo is a more right-wing policy in the old society compared to the young one. We want to prove that, starting from these two different status quos, a right-wing victory in period 2 that brings back the policy to the right is less likely in the old society than in the young one. To make this comparison we need the right-wing platforms in the second period to be the same in both societies: $y^{2o*} = y^{2y*}$. Let P^{2o} and P^{2y} the probability of a left-wing victory in the second period. We have to show that, holding y^2 constant, $P^{2o} > P^{2y}$. This inequality originates from higher perceived loss aversion in ageing society. Thus we can prove our claim by showing that in equilibrium the probability of a left-wing victory in the second period is increasing in loss aversion parameter: $\partial P^2/\partial\lambda > 0$. This is what we do below.

By (10) in the main paper, P^2 positively depends on the type of the cutoff voter in the second period, $t_{ind}^{LA} = T^{LA}(x^{*2}, y^{*2}, p^1, \lambda)$. Thus we have to show that $\partial t_{ind}^{LA}/\partial\lambda > 0$. Since $p^1 = x^{1*}$, we can write $t_{ind}^{LA} = T^{LA}(x^{*2}, y^{*2}, x^{1*}, \lambda)$. Differentiating t_{ind}^{LA} wrt λ yields

$$\frac{\partial t_{ind}^{LA}}{\partial\lambda} = T_{x^1}^{LA}\frac{\partial x^{1*}}{\partial\lambda} + T_{\lambda}^{LA} + T_{x^2}^{LA}\frac{\partial x^{2*}}{\partial\lambda} + T_{y}^{LA}\frac{\partial y^{2*}}{\partial\lambda} + T_{x^2}^{LA}\frac{\partial x^{2*}}{\partial x^{1*}}\frac{\partial x^{1*}}{\partial\lambda} + T_{y}^{LA}\frac{\partial y^{2*}}{\partial x^{1*}}\frac{\partial x^{1*}}{\partial\lambda}$$

The first term in the RHS is negative since $\frac{\partial x^{1*}}{\partial\lambda} > 0$ by Proposition 4, and $T_{x^1}^{LA} < 0$ is the partial derivative of the cutoff type wrt the status quo of period 2. The latter is negative by (47) in the main text. The second term can be either or positive, however it is negligible if there is enough symmetry in the cutoff's voter utility function, which we assume. Specifically, $B(t_{ind}^{LA}, x^{1*}) - B(t_{ind}^{LA}, x^{2*}) \simeq C(t_{ind}^{LA}, x^{2*}) - B(t_{ind}^{LA}, x^{1*})$. The loss of benefits due to choosing the right-wing policy in the second

[5] The proof for the case the right-wing candidate won follows the same steps.

period is similar to the increase in cost due to choosing the left-wing policy. The third term is positive since, as shown in the proof of Proposition 4, $T^{LA}_{x^2} > 0$, and $\frac{\partial x^{2*}}{\partial \lambda} > 0$. The fourth and the sixth terms are zero, because we are holding y^2 constant. The fifth term is positive. Thus a sufficient condition for $\frac{\partial t^{LA}_{ind}}{\partial \lambda} > 0$ is that $T^{LA}_{x^1}\frac{\partial x^{1*}}{\partial \lambda} + T^{LA}_{x^2}\frac{\partial x^{2*}}{\partial \lambda} > 0$. Note that $\frac{\partial x^{1*}}{\partial \lambda}$ and $\frac{\partial x^{2*}}{\partial \lambda}$ represent the (positive) marginal effect of loss aversion on the equilibrium platforms of the two periods. If we assume this effect is the same, the sufficient condition becomes $T^{LA}_{x^1} + T^{LA}_{x^2} > 0$. By (47), $T^{LA}_{x^1} = -\frac{-\lambda\left[B_x(t^{LA}_{ind},x^1)+C_x(t^{LA}_{ind},x^1)\right]}{M} < 0$, and $T^{LA}_{x^2} = -\frac{V_x(t^{LA}_{ind},x^{2*})+\lambda B_x(t^{LA}_{ind},x^{2*})}{M} > 0$ (for details, see the proof of Proposition 4 in the main text). Note that $M < 0$. Thus, after some algebraic manipulations, the sufficient condition $T^{LA}_{x^1}\frac{\partial x^{1*}}{\partial \lambda} + T^{LA}_{x^2}\frac{\partial x^{2*}}{\partial \lambda} > 0$ is satisfied if $\lambda B_x(t^{LA}_{ind}, x^{2*}) - \lambda B_x(t^{LA}_{ind}, x^1) + B_x(t^{LA}_{ind}, x^{2*}) - C_x(t^{LA}_{ind}, x^{2*}) - \lambda C_x(t^{LA}_{ind}, x^1) > 0$. The concavity of $B(.)$ in the policy, and the fact that $x^{1*} > x^{2*}$ ensure that $B_x(t^{LA}_{ind}, x^{2*}) - B_x(t^{LA}_{ind}, x^1) > 0$. As for the last three terms, their sum is positive if $C(.)$ is sufficiently flat in the policy or if x^{2*} and x^1 are close enough. In this case, the ideal policy of the indifferent voter is higher than x^{2*} and x^1. This implies that her utility function is increasing above x^1. But x^{2*} is close to x^1 is the case when society is quite old. Perceived loss aversion is strong, which in turn implies that the platform of period 2, x^{2*}, converges to the period 2's status quo, x^1.

6. Demographic Data - Section 4

LIFE EXPECTANCY

Life expectancy has increased substantially in developed countries since 1965, the period data start from. For instance, an American newborn in 2015 lives approximately 9 years more than in 1965, while a 65 years old person lives 5 years longer (see Figures 1. and 2.). In particular, life expectancy of the elderly increased linearly of approximately 1.2 months each year. According to our model, reforms become more likely if on average population live longer. Changing the status quo should be more likely today than it was 50 years ago.

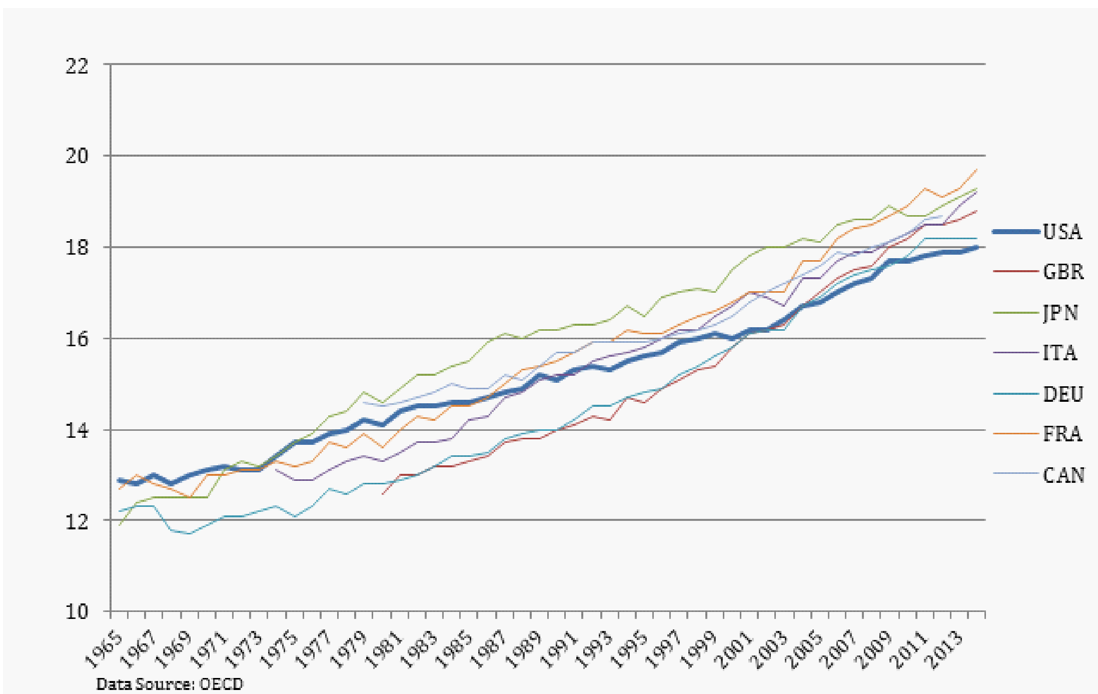


Figure 1. Life expectancy at 65

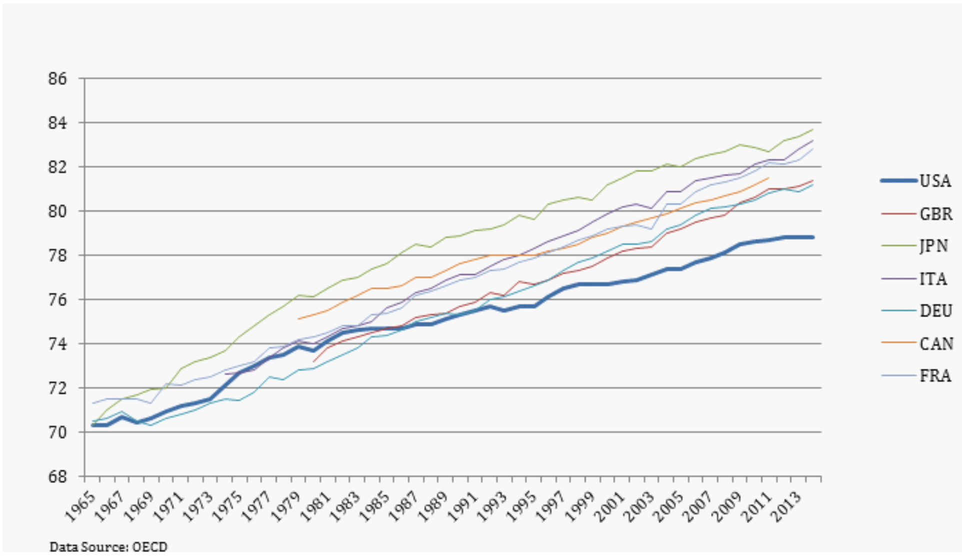


Figure 2. Life Expectancy at birth

FERTILITY RATE

A sharp drop in fertility rate occurred in developed countries in the last fifty years (Figure 3 reports data for USA, UK and Canada). This implies a decrease in the share of the "young generation". This effect goes towards the opposite direction of the increase in life expectancy. Since the share of elderly is larger, reforms become less likely.

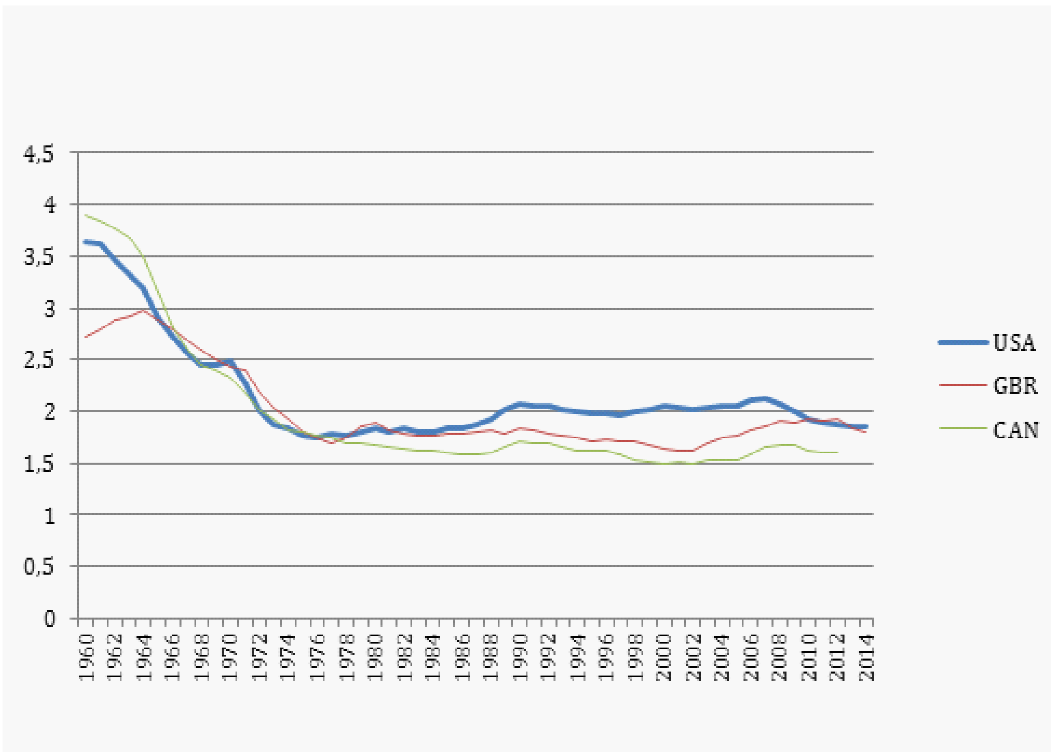


Figure 3. Fertility rate

MIGRATION

Data show that, at least for the US, the effect of naturalized migrants is a decrease in the median age of the population with voting rights. Figure 4 shows that immigration is an increasingly relevant phenomenon in some advanced countries. Figure 5 shows that in U.S. the age distribution of naturalized citizens (the ones with voting rights) is more left-skewed than the age distribution of the entire population older than 18. According to our model, one should observe positive correlation between immigration and the chance of reforms.

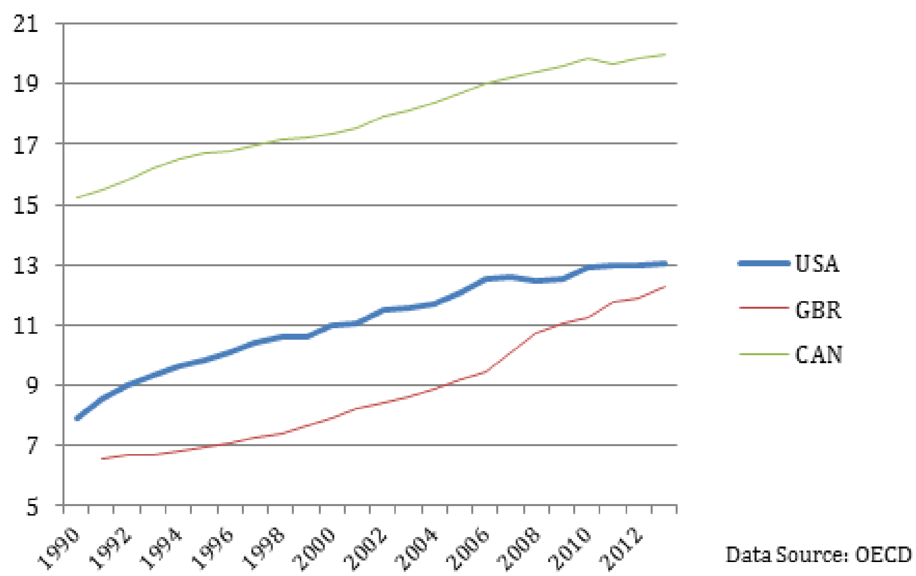
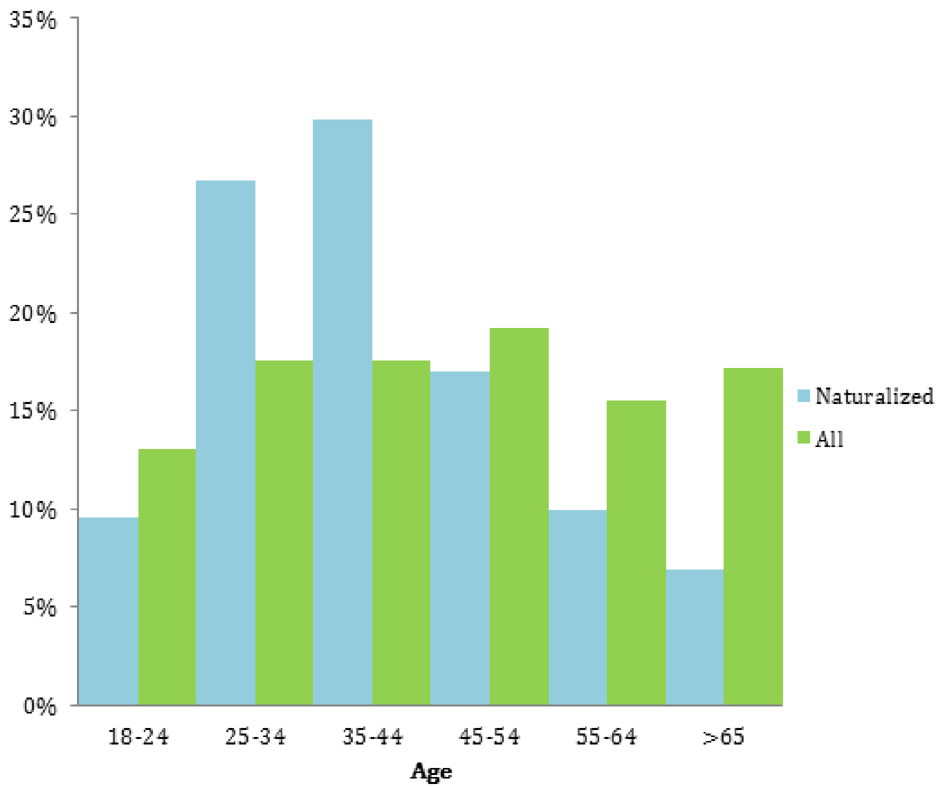


Figure 4. Foreign born population (share of total population)



Data on Naturalizations: 2010. U.S. Department of Homeland Security
Data on All population: 2010. U.S. Census Bureau

Figure 5. Voting age distribution in US

DEVELOPED VS DEVELOPING COUNTRIES

Cohorts in developing countries have shorter life expectancy than in developed countries. But their median age is lower. Table 1 lists median age and life expectancy at that age for a group of countries. Life expectancy at median age in shorter in developed countries (around 38 vs around 45 in developing countries). Our model suggests that reforms should be more likely in developing countries, which may be signalled in those countries by higher political instability.

Table 1: Median age at the closest multiple of five and residual life at that precise age

COUNTRY	Median Population Age	Expected Years left
Japan	45	39
Italy	45	38
Germany	45	37
USA	40	40
Algeria	30	48
Mexico	30	48
Turkey	30	48
Brazil	30	47
Tunisia	30	47
Colombia	30	48
India	30	43
Indonesia	30	43
South Africa	25	41

DATA SOURCES

1. Median age in countries: Central Intelligence Agency
https://www.cia.gov/library/publications/the-world-factbook/fields/2177.html
2. Life Expectancy in countries: wordlifeexpectancy.com
http://www.worldlifeexpectancy.com/
3. Fertility Rate: OECD
https://data.oecd.org/pop/fertility-rates.htm
4. Share of foreign born population: OECD
https://data.oecd.org/migration/foreign-born-population.htm
5. Life expectancy at birth: OECD
https://data.oecd.org/healthstat/life-expectancy-at-birth.htm#indicator-chart
6. Life expectancy at 65: OECD
https://data.oecd.org/healthstat/life-expectancy-at-65.htm
7. Age distribution in USA: 2010 census
http://www.census.gov/prod/cen2010/briefs/c2010br-03.pdf
8. Age distribution of Naturalized Citizens in 2010 in USA: Department of Homeland Security
https://www.dhs.gov/sites/default/files/publications/Naturalization%20Flow%20Report_2014_508.pdf
9. Age and race distribution of voters in USA: National Election Pool Via New York Times
http://www.nytimes.com/interactive/2016/11/08/us/politics/election-exit-polls.html
10. Race distribution of youth voters in USA: National Election Pool Via CIRCLE
http://civicyouth.org/an-estimated-24-million-young-people-vote-in-2016-election/