Supplementary Note to: Legislative committees as information intermediaries: a unified theory of committee selection and amendment rules

Attila Ambrus, Eduardo Azevedo, Yuichiro Kamada, and Yuki Takagi

13th July 2011

This supplementary note extends the model in the main text by departing from the assumptions maintained in the main text that θ is distributed uniformly, and that players have quadratic loss functions.

In particular, for the distribution of θ we only assume that its support is $[0,1]$, and it has a density function f that is C^1 and strictly positive over the support. For loss function l, we only assume that it is twice continuously differentiable and strictly convex, that it attains its minimum value of 0 at point 0, and that it is symmetric around 0. Moreover, $l'(0) > 0$. In short, from the class of preferences considered in Crawford and Sobel (1982; from now on CS), here we consider the ones in which all players have the same symmetric state-independent loss function.[1] We maintain the (innocuous) assumption that $b_L \geq 0$.

Below we generalize some of our results in the main text on the optimal committee bias under closed and open rules.

For the optimal policy under closed rule, first we show that for small enough lobbyist bias it is still optimal to fully delegate decision power to the lobbyist. This result generalizes the first part of Proposition 5 of Dessein (2002) outside the uniform-quadratic specification.

Proposition 1. *If b_L is small enough, full delegation ($b_C^{cl} = b_L$) is optimal under the closed rule.*

Proof. First, note that since the loss function is symmetric, equilibria in direct communication have exactly the same structure as in CS. Denote by $a_z(\theta)$ the action taken at state θ in a direct communication with a sender with bias z and an unbiased receiver. Now, consider a lobbyist with bias $b_L > 0$, and assume that the floor employs a committee with bias of $b_L - z$, where $0 < z < b_L$. The loss to the floor is

$$L = \int_\theta l\left[b_L - z + (a_z(\theta) - \theta)\right] \cdot f(\theta) d\theta.$$

We may approximate this as

$$L = l(b_L - z) + \int_\theta \left[l'(b_L - z)(a_z(\theta) - \theta) + l''(b_L - z)(a_z(\theta) - \theta)^2/2 + O((a_z(\theta) - \theta)^3)\right] \cdot f(\theta) d\theta.$$

We will now bound the terms inside the integral. The first term is always zero, as by definition we have

$$\int_\theta (a_z(\theta) - \theta) dF(\theta) = 0.$$

As for the second term, note that in the CS partition, the integral

$$\int_\theta (a_z(\theta) - \theta)^2 f(\theta) d\theta \geq l_{CS}(z) \cdot \min_{x \in [0,1]} f(x) \geq C' z,$$

for some $C' > 0$, as the loss in the uniform-quadratic specification of the CS model, l_{CS}, is first order with respect to the bias, and f is strictly positive and continuous on $[0,1]$ (hence it takes a strictly positive minimum). Moreover, the constant C' is uniform for all b_L small enough. For the third term, since the largest interval in the CS partition has length of order $z^{1/2}$, we have that

$$\int (a_z(\theta) - \theta)^3 f(\theta) d\theta \leq \max_{x \in [0,1]} f(x) \cdot \max_{x \in [0,1]} (a_z(x) - x) \cdot \int (a_z(\theta) - \theta)^2 d\theta \leq C'' z^{1/2} \cdot z = C'' z^{3/2},$$

[1] We maintain the assumption that all players have the same loss function because it implies that selecting the optimal committee reduces to selecting the optimal $b_C \in \mathbb{R}$. Otherwise the floor would have to choose both the optimal bias and the optimal loss function of the committee (from some feasible set of loss functions), a substantially more difficult problem.

for some bounded constant $C'' \geq 0$, as again f is continuous on $[0,1]$ (hence it takes a bounded maximum). Again, C'' can be taken uniform for small b_L and all $0 < z < b_L$. Consequently, for small $b_L > 0$, we bound the loss function as

$$L \geq l(b_L - z) + l''(b - z) \cdot Cz,$$

where $C > 0$.

Now, we compare this loss with the loss to the floor when appointing a committee with bias exactly equal to b_L, which is $l(b_L)$. The difference between these two losses is

$$l(b_L - z) + l''(b_L - z) \cdot Cz - l(b_L) = l''(b_L - z) \cdot Cz - [l'(b_L)z + O(z^2)]$$

However, the first term of the right hand side is first order with respect to z since $l''(0) > 0$ and l is twice continuously differentiable, while the second term is a higher order than that, because $l'(0) = 0$ and l is continuously differentiable. Thus, overall, this difference is strictly positive for all small enough b_L and $0 < z < b_L$.

This completes the proof. □

Next, we establish that under the same regularity condition that CS use to guarantee various monotonicity properties of the set of equilibria in their sender-receiver game, choosing $b_C < 0$ is always suboptimal under closed rule.

Let $y_{\theta'}^{\theta''}$ be the committee's equilibrium action under closed rule that corresponds to the cell $[\theta', \theta'']$ in the equilibrium partition. For a fixed $b \in \mathbb{R}$ we call a sequence $a = (a_0, ..., a_N)$ a forward solution if $l(y_{a_{i-1}}^{a_i} - a_i - b) = l(y_{a_i}^{a_{i+1}} - a_i - b)$ for every $i \in \{2, ..., N-1\}$, and $a_1 > a_0$.

Assumption 1. *(Assumption (M) CS) For a given value of b, if $\widehat{a}$ and $\widetilde{a}$ are two forward solutions with $\widehat{a}_0 = \widetilde{a}_0$ and $\widehat{a}_1 > \widetilde{a}_1$ then $\widehat{a}_i > \widetilde{a}_i$ for all $i \geq 2$.*

Proposition 2. *If Assumption 1 holds then, under closed rule, $b_C < 0$ cannot be an optimal choice for the floor.*

Proof. Suppose that $b_C < 0$. We show that the floor would be better off by choosing the committee bias of 0. By CS, Assumption 1 implies that $l_{CS}(b)$ is strictly increasing in b. Notice that the floor's loss from $b_C = 0$ is $l_{CS}(b_L)$, while the committee's loss from $b_C < 0$ is $l_{CS}(b_L - b_C)$, which is strictly larger than $l_{CS}(b_L)$ by the strict increasingness. Hence it suffices to show that the floor's loss from choosing $b_C < 0$ is no less than the committee's loss at that b_C. To see this, fix $b_C < 0$ and a cell $[\theta', \theta'']$ in the equilibrium partition. Since the committee takes a best response, we have:

$$y_{\theta'}^{\theta''} = \arg\min_y \int_{\theta'}^{\theta''} l(y - \theta - b_C) f(\theta) d\theta.$$

The first order condition is:

$$\int_{\theta'}^{\theta''} \frac{\partial l(y_{\theta'}^{\theta''} - \theta - b_C)}{\partial y} f(\theta) d\theta = 0.$$

Hence, we must have:

$$\int_{\theta'}^{\theta''} \frac{\partial l(y_{\theta'}^{\theta''} - \theta)}{\partial y} f(\theta) d\theta = \int_{\theta'}^{\theta''} \frac{\partial l([y_{\theta'}^{\theta''} - \theta - b_C] + b_C)}{\partial y} f(\theta) d\theta > 0,$$

where the strict inequality comes from the strict convexity of l. Thus, conditinal on the state lying in $[\theta', \theta'']$, the floor's payoff is not maximized at $y_{\theta'}^{\theta''}$. But notice that the maximized payoff would be exactly equal to the payoff that the committee receives conditinal on the state lying in $[\theta', \theta'']$. Since this is true for all cells contained in $[0, 1]$, the floor's payoff from choosing $b_C < 0$ is strictly smaller than the committee's payoff at that b_C. This completes the proof. □

The optimal committee bias given open rule in this more general environment is an open question, but two qualitative results of the main text apply. The first one establishes that appointing a committee which is biased in the direction of the lobbyist but less so can never be better than appointing an unbiased committee (or, equivalently, the floor talking directly to the lobbyist).

Proposition 3. *(Lemma 4 Ivanov (2010) and Proposition 5 Ambrus et al. (2011)) Under open rule, setting* $b_C = 0$ *is at least weakly better than setting* $b_C \in (0, b_L]$.

The second proposition gives a sufficient condition for the optimal committee under open rule to have a nonzero bias. The sufficient condition covers cases in which in direct communication between the lobbyist and the floor babbling would be the only equilibrium, but there exists a committee with negative bias that facilitates nonzero information transmission.

Let $x_a^b = \arg\max \int_a^b -l(\theta - x) f(\theta) d\theta$. That is, x_a^b is the floor's equilibrium action under closed rule that corresponds to the cell $[a, b]$ in the equilibrium partition.

Proposition 4. *(Proposition 6 Ambrus et al. (2011)) If* $l(-b_L) > l(x_a^b - a - b_L)$ *for every* $a \in [0, 1)$ *and* $b_L < x_0^1$ *then* $b_C = 0$ *cannot be an optimal choice for the floor under open rule.*

1 References

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