

SUPPLEMENT TO “ON THE GEOGRAPHY OF GLOBAL VALUE CHAINS”  
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POL ANTRÀS  
Department of Economics, Harvard University and NBER

ALONSO DE GORTARI

APPENDIX A

A.1. *Mathematical Proofs*

A.1.1. *Increasing Trade-Cost Elasticity*

WE DEMONSTRATE THAT PROPOSITION 1 HOLDS true for arbitrary constant-returns-to-scale production technologies. With that in mind, let the sequential cost function associated with a path of production  $\ell = \{\ell(1), \ell(2), \dots, \ell(N)\}$  be defined by

$$p_{\ell(n)}^n(\ell) = g_{\ell(n)}^n(c_{\ell(n)}, p_{\ell(n-1)}^{n-1}(\ell)\tau_{\ell(n-1)\ell(n)}) \quad \text{for all } n \in \mathcal{N}, \quad (\text{A.1})$$

where the stage- and country-specific cost functions  $g_{\ell(n)}^n$  in equation (A.1) are assumed to feature constant returns to scale and diminishing marginal products. The cost of the first stage depends only on the local composite factor, so constant returns to scale implies  $p_{\ell(1)}^1(\ell) = g_{\ell(1)}^1(c_{\ell(1)})$  for all paths  $\ell$ , with the function  $g_{\ell(1)}^1$  necessarily being linear in  $c_{\ell(1)}$ .

Define  $\tilde{p}_{\ell(n)}^{n-1}(\ell) = p_{\ell(n-1)}^{n-1}(\ell)\tau_{\ell(n-1)\ell(n)}$  to be the price paid in  $\ell(n)$  for the good finished up to stage  $n - 1$  in country  $\ell(n - 1)$ , so that we can express the sequential unit cost function as

$$p_{\ell(n)}^n(\ell) = g_{\ell(n)}^n(c_{\ell(n)}, \tilde{p}_{\ell(n)}^{n-1}(\ell)).$$

Define the elasticity of  $p_j^F(\ell)$  with respect to the trade costs that stage- $n$ 's production faces as

$$\beta_n^j = \frac{\partial \ln p_j^F(\ell)}{\partial \ln \tau_{\ell(n)\ell(n+1)}},$$

with the convention that  $\ell(N + 1) = j$  so that  $\beta_N^j$  is the elasticity of  $p_j^F(\ell)$  with respect to the trade costs faced when shipping assembled goods to final consumers in  $j$ . Because  $\tau_{\ell(n)\ell(n+1)}$  increases  $\tilde{p}_{\ell(n+1)}^n(\ell)$  with a unit elasticity, the following recursion holds for all  $n' > n$ :

$$\frac{\partial \ln p_{\ell(n'+1)}^{n'+1}(\ell)}{\partial \ln \tau_{\ell(n)\ell(n+1)}} = \frac{\partial \ln p_{\ell(n'+1)}^{n'+1}(\ell)}{\partial \ln \tilde{p}_{\ell(n'+1)}^{n'}(\ell)} \frac{\partial \ln p_{\ell(n')}^{n'}(\ell)}{\partial \ln \tau_{\ell(n)\ell(n+1)}}.$$

At the same time, the unit cost elasticity at stage  $n + 1$  satisfies

$$\frac{\partial \ln p_{\ell(n+1)}^{n+1}(\ell)}{\partial \ln \tau_{\ell(n)\ell(n+1)}} = \frac{\partial \ln p_{\ell(n+1)}^{n+1}(\ell)}{\partial \ln \tilde{p}_{\ell(n+1)}^n(\ell)}.$$

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Pol Antràs: pantras@fas.harvard.edu  
Alonso de Gortari: alonso.degortari@gmail.com

Hence, the elasticity of finished-good prices can be decomposed as

$$\beta_n^j = \prod_{n'=n+1}^N \frac{\partial \ln p_{\ell(n')}^{n'}(\boldsymbol{\ell})}{\partial \ln \tilde{p}_{\ell(n')}^{n'-1}(\boldsymbol{\ell})}, \quad (\text{A.2})$$

invoking the convention  $\prod_{n'=N+1}^N f(n') = 1$  for any function  $f(\cdot)$ . Constant returns to scale in production implies that the function  $g_{\ell(n)}^n$  is homogeneous of degree 1. As a result, the elasticity of unit costs with respect to input prices is always less than or equal to 1, so for all  $n > 1$  we have

$$\frac{\partial \ln p_{\ell(n)}^n(\boldsymbol{\ell})}{\partial \ln \tilde{p}_{\ell(n)}^{n-1}(\boldsymbol{\ell})} \leq 1,$$

with strict inequality whenever a stage adds value to the product. From equation (A.2), it is then clear that

$$\beta_n^1 \leq \beta_n^2 \leq \dots \leq \beta_n^N = 1,$$

with strict inequality when value added is positive at all stages.

### A.1.2. *Fighting the Curse of Dimensionality: Dynamic and Linear Programming*

When discussing the lead-firm problem in Section 2.2, we mentioned that there are  $J^N$  sequences that deliver distinct finished-good prices  $p_j^F(\boldsymbol{\ell})$  in country  $j$ . Hence, solving for the optimal sequences  $\boldsymbol{\ell}^j$  for all  $j$  by brute force requires  $J^{N+1}$  computations and is infeasible to do when  $J$  and  $N$  are sufficiently large. However, we show below that use of dynamic programming surmounts this problem by reducing the computation of all sequences to only  $J \times N \times J$  computations. Furthermore, in the special case in which production is Cobb–Douglas, the minimization problem can be modeled with zero-one linear programming, for which very efficient algorithms exist.

### A.1.3. *Dynamic Programming*

Define  $\boldsymbol{\ell}_n^j \in \mathcal{J}^n$  as the optimal sequence for delivering the good completed up to stage  $n$  to producers in country  $j$ . This term can be found recursively for all  $n = 1, \dots, N$  by simply solving

$$\boldsymbol{\ell}_n^j = \arg \min_{k \in \mathcal{J}} p_k^n(\boldsymbol{\ell}_{n-1}^k) \tau_{kj}, \quad (\text{A.3})$$

since the optimal source of the good completed up to stage  $n$  is independent of the local factor cost  $c_j$  at stage  $n$ , of the specifics of the cost function  $g_j^n$ , or of the future path of the good. For this same reason, we have written the pricing function  $p_k^n$  in terms of the  $n - 1$ -stage sequence  $\boldsymbol{\ell}_{n-1}^k$  since it does not depend on future stages of production (though it should be clear that  $p_k^n$  will also be a function of the production costs and technology available for producers at that chosen location  $k$ ). The convention at  $n = 1$  is that there is no input sequence so that  $\boldsymbol{\ell}_0^k = \emptyset$  for all  $k \in \mathcal{J}$  and the price depends only on the composite factor cost:  $p_k^1(\emptyset) = g_k^1(c_k)$ .

The formulation in (A.3) makes it clear that the optimal path to deliver the assembled good to consumers in each country  $j$ , that is,  $\boldsymbol{\ell}^j = \boldsymbol{\ell}_N^j$ , can be solved recursively by comparing  $J$  numbers for each location  $j \in \mathcal{J}$  at each stage  $n \in \mathcal{N}$ , for a total of only  $J \times N \times J$  computations.

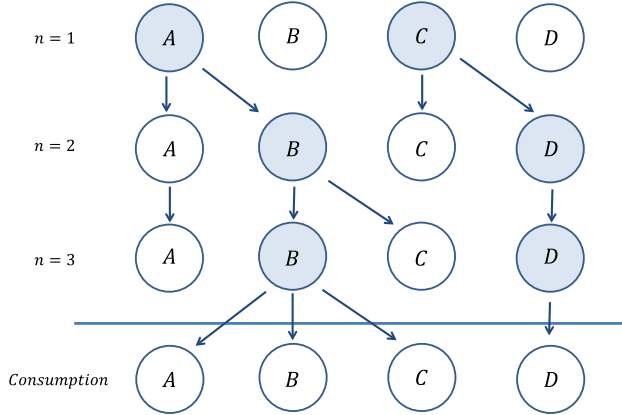


FIGURE A.1.—Dynamic programming—an example with four countries and three stages.

To further understand this dynamic programming approach, Figure A.1 illustrates a case with three stages and four countries. Instead of computing  $J^N = 64$  paths for each of the four locations of consumption, it suffices to determine the optimal source of (immediately) upstream inputs (which entails  $J \times J = 16$  computations at stages  $n = 2$  and  $n = 3$ , and for consumption). In the example, the optimal production path to serve consumers in  $A$ ,  $B$ , and  $C$  is  $A \rightarrow B \rightarrow B$ , while the optimal path to serve consumers in  $D$  is  $C \rightarrow D \rightarrow D$ .

#### A.1.4. Linear Programming

In the special case in which production is Cobb–Douglas, the optimal sourcing sequence can be written as a log-linear minimization problem:

$$\ell^j = \arg \min_{\ell \in \mathcal{J}^N} \sum_{n=1}^{N-1} \beta_n \ln \tau_{\ell(n)\ell(n+1)} + \ln \tau_{\ell(N)j} + \sum_{n=1}^N \alpha_n \beta_n \ln(a_{\ell(n)}^n c_{\ell(n)}).$$

This can in turn be reformulated as the following zero-one integer linear programming problem:

$$\begin{aligned} \ell^j &= \arg \min \sum_{n=1}^{N-1} \beta_n \sum_{k \in \mathcal{J}} \sum_{k' \in \mathcal{J}} \zeta_{kk'}^n (\ln \tau_{kk'} + \alpha_n a_k^n c_k) + \sum_{k \in \mathcal{J}} \zeta_k^N (\ln \tau_{kj} + \alpha_N a_k^N c_k) \\ \text{s.t. } &\sum_{k' \in \mathcal{J}} \zeta_{k'k}^n = \sum_{k' \in \mathcal{J}} \zeta_{kk'}^{n+1}, \forall k \in \mathcal{J}, n = 1, \dots, N-2 \\ &\sum_{k' \in \mathcal{J}} \zeta_{k'k}^{N-1} = \zeta_k^N, \forall k \in \mathcal{J} \\ &\sum_{k \in \mathcal{J}} \zeta_k^N = 1; \zeta_{kk'}^n, \zeta_k^N \in \{0, 1\}. \end{aligned}$$

#### A.1.5. Proof of Proposition 3

If there is free trade or  $\tau$  is constant across all country-pairs (including domestically), then all countries source each variety from the same sequence of countries with  $\pi_{\ell j} = \pi_{\ell}$

for all  $j \in \mathcal{J}$ . Analogously, price indices are the same in all markets so that  $P_j = P$  for all  $j \in \mathcal{J}$ . The probability of sourcing a variety through a given sequence is thus

$$\pi_\ell = \frac{\prod_{n \in \mathcal{N}} (T_{\ell(n)}^n w_{\ell(n)}^{-\gamma\theta})^{1/N}}{\sum_{\ell' \in \mathcal{J}^{\mathcal{N}}} \prod_{n \in \mathcal{N}} (T_{\ell'(n)}^n w_{\ell'(n)}^{-\gamma\theta})^{1/N}}.$$

We will now prove that wages are equalized across countries. Note that the total probability of any country being in a given stage  $n$  is the same regardless of the destination country and equals

$$\sum_{i \in \mathcal{J}} \Pr(\Lambda_i^n) = \sum_{i \in \mathcal{J}} \sum_{\ell \in \Lambda_i^n} \frac{\prod_{n' \in \mathcal{N}} (T_{\ell(n')}^{n'} w_{\ell(n')}^{-\gamma\theta})^{1/N}}{\Theta} = \sum_{i \in \mathcal{J}} (T_i^n w_i^{-\gamma\theta})^{1/N} \times \frac{\prod_{n' \in \mathcal{N} \setminus n} (T_{\ell(n')}^{n'} w_{\ell(n')}^{-\gamma\theta})^{1/N}}{\Theta}.$$

Now, suppose that wages are common across countries with  $w_j = w$  for all  $j \in \mathcal{J}$ . Since the probability of any country being at a given stage  $n$  needs to equal 1, this implies that

$$\sum_{i \in \mathcal{J}} (T_i^n)^{1/N} \times \frac{\prod_{n' \in \mathcal{N} \setminus n} (T_{\ell(n')}^{n'})^{1/N}}{w^{\gamma\theta} \Theta} = 1 \quad \Rightarrow \quad \frac{\prod_{n' \in \mathcal{N} \setminus n} (T_{\ell(n')}^{n'})^{1/N}}{w^{\gamma\theta} \Theta} = \frac{1}{J\bar{T}},$$

where the second line uses our assumption that the geometric mean of  $T_i^n$  across countries is constant across stages of production. Let us now plug this into the right-hand side of the general equilibrium equation together with our guess that wages are equalized across countries:

$$\begin{aligned} w_i &= \sum_{j \in \mathcal{J}} \sum_{n \in \mathcal{N}} \frac{1}{N} \times \Pr(\Lambda_i^n, j) \times w = \sum_{n \in \mathcal{N}} \frac{1}{N} \times \frac{(T_i^n)^{1/N} \times \prod_{n' \in \mathcal{N} \setminus n} (T_{\ell(n')}^{n'})^{1/N}}{w^{\gamma\theta} \Theta} \times Jw \\ &= \sum_{n \in \mathcal{N}} \frac{1}{N} \times \frac{(T_i^n)^{1/N}}{J\bar{T}} \times Jw = \frac{1}{N} \times \frac{N\bar{T}}{J\bar{T}} \times Jw = w, \end{aligned}$$

where the third line uses the previous result and where the fourth line uses our assumption that the geometric mean of  $T_i^n$  across stages of production is constant across countries. Hence, guessing that wages are equalized across countries delivers a fixed point in those wages. Since the equilibrium is unique, this is the only set of wages satisfying the general-equilibrium equation.

To derive the share of goods produced in a domestic supply chain under free trade, rewrite  $\Theta$  as

$$\Theta = \prod_{n \in \mathcal{N}} \sum_{i \in \mathcal{J}} (T_i^n)^{1/N} = (J \times \bar{T})^N = \left( J \times \frac{1}{N} \sum_{n \in \mathcal{N}} (T_j^n)^{1/N} \right)^N,$$

for any  $j \in \mathcal{J}$ . Inserting this into the domestic expenditure share finalizes the proof

$$\pi_j = \left( \frac{\text{GeometricMean}_n[(T_j^n)^{1/N}]}{J \times \text{ArithmeticMean}_n[(T_j^n)^{1/N}]} \right)^N.$$

#### A.1.6. Proof of Proposition 4

If all countries are symmetric, wages are equalized and the domestic expenditure share is

$$\pi_j = \frac{1}{\sum_{\ell' \in \mathcal{J}^N} \prod_{n \in \mathcal{N}} (\tau_{\ell(n)\ell(n+1)})^{-\theta\beta_n}}.$$

The denominator can be rewritten as

$$\begin{aligned} & \sum_{\ell(1) \in \mathcal{J}} \cdots \sum_{\ell(N) \in \mathcal{J}} \prod_{n \in \mathcal{N}} (\tau_{\ell(n)\ell(n+1)})^{-\theta\beta_n} \\ &= \sum_{\ell(1) \in \mathcal{J}} \sum_{\ell(2) \in \mathcal{J}} (\tau_{\ell(1)\ell(2)})^{-\beta_1\theta} \times \cdots \times \sum_{\ell(N) \in \mathcal{J}} (\tau_{\ell(N-1)\ell(N)})^{-\theta\beta_{N-1}} \times (\tau_{\ell(N)j})^{-\theta\beta_N} \\ &= \sum_{\ell(N) \in \mathcal{J}} (1 + (J-1)\tau^{-\beta_1\theta}) \times (1 + (J-1)\tau^{-\beta_2\theta}) \times \cdots \\ & \quad \times (1 + (J-1)\tau^{-\beta_{N-1}\theta}) \times (\tau_{\ell(N)j})^{-\theta\beta_N} \\ &= \prod_{n=1}^N (1 + (J-1)\tau^{-\beta_n\theta}). \end{aligned}$$

Substituting this in the domestic share finishes the proof.

#### A.1.7. Proof of Proposition 5

Let  $(\tau_{ij})^{-\theta} = \rho_i \rho_j$ . In such a case, the probability of country  $j$  sourcing through  $\ell$  reduces to

$$\pi_{\ell j} = \frac{\prod_{m=1}^N (T_{\ell(m)}(C_{\ell(m)})^{-\theta})^{\alpha_m \beta_m} (\rho_{\ell(m)})^{\beta_{m-1} + \beta_m}}{\sum_{\ell \in \mathcal{J}} \prod_{m=1}^N (T_{\ell(m)}(C_{\ell(m)})^{-\theta})^{\alpha_m \beta_m} (\rho_{\ell(m)})^{\beta_{m-1} + \beta_m}}$$

and is thus independent of the destination country  $j$ . The aggregate probability of observing country  $i$  in location  $n$  can thus be expressed as

$$\Pr(\Lambda_i^n) = \sum_{\ell \in \Lambda_i^n} \pi_{\ell j} = \frac{\sum_{\ell \in \Lambda_i^n} \prod_{m=1}^N (T_{\ell(m)}(C_{\ell(m)})^{-\theta})^{\alpha_m \beta_m} (\rho_{\ell(m)})^{\beta_{m-1} + \beta_m}}{\sum_{k \in \mathcal{J}} \sum_{\ell \in \Lambda_k^n} \prod_{m=1}^N (T_{\ell(m)}(C_{\ell(m)})^{-\theta})^{\alpha_m \beta_m} (\rho_{\ell(m)})^{\beta_{m-1} + \beta_m}}. \quad (\text{A.4})$$

But note that we can decompose this as

$$\Pr(\Lambda_i^n) = \frac{(T_i(c_i)^{-\theta})^{\alpha_n \beta_n} (\rho_i)^{\beta_{n-1} + \beta_n} \times \sum_{\ell \in \Lambda_i^n} \prod_{m \neq n} (T_{\ell(m)}(c_{\ell(m)})^{-\theta})^{\alpha_m \beta_m} (\rho_{\ell(m)})^{\beta_{m-1} + \beta_m}}{\sum_{k \in \mathcal{J}} (T_k(c_k)^{-\theta})^{\alpha_n \beta_n} (\rho_k)^{\beta_{n-1} + \beta_n} \times \sum_{\ell \in \Lambda_k^n} \prod_{m \neq n} (T_{\ell(m)}(c_{\ell(m)})^{-\theta})^{\alpha_m \beta_m} (\rho_{\ell(m)})^{\beta_{m-1} + \beta_m}} \quad (\text{A.5})$$

$$= \frac{(T_i(c_i)^{-\theta})^{\alpha_n \beta_n} (\rho_i)^{\beta_{n-1} + \beta_n}}{\sum_{k \in \mathcal{J}} (T_k(c_k)^{-\theta})^{\alpha_n \beta_n} (\rho_k)^{\beta_{n-1} + \beta_n}}, \quad (\text{A.6})$$

where the second line follows from the fact that, for GVCs in the sets  $\Lambda_i^n$  and  $\Lambda_k^n$ , the set of all possible paths excluding the location of stage  $n$  are necessarily identical (and independent of the country where  $n$  takes place), and thus the second terms in the numerator and denominator of the first line cancel out.

For the special symmetric case with  $\alpha_n \beta_n = 1/N$  and  $\alpha_n = 1/n$ , we obtain that

$$\Pr(\Lambda_i^n) = \frac{(T_i(c_i)^{-\theta})^{\frac{1}{N}} (\rho_i)^{\frac{2n-1}{N}}}{\sum_{k \in \mathcal{J}} (T_k(c_k)^{-\theta})^{\frac{1}{N}} (\rho_k)^{\frac{2n-1}{N}}}.$$

Now consider our definition of upstreamness

$$U(i) = \sum_{n=1}^N (N - n + 1) \times \frac{\Pr(\Lambda_i^n)}{\sum_{n'=1}^N \Pr(\Lambda_i^{n'})}. \quad (\text{A.7})$$

This is equivalent to the expected distance from final-good demand at which a country will contribute to global value chains. The expectation is defined over a country-specific probability distribution over stages,  $f_i(n) = \Pr(\Lambda_i^n) / \sum_{n'=1}^N \Pr(\Lambda_i^{n'})$ .

Finally, note that for two countries with  $\rho_{i'} > \rho_i$  and two inputs with  $n' > n$ , we necessarily have

$$\frac{f_{i'}(n')/f_{i'}(n)}{f_i(n')/f_i(n)} = \left( \frac{\rho_{i'}}{\rho_i} \right)^{2(n'-n)/N} > 1.$$

As a result, the probability functions  $f_{i'}(n)$  and  $f_i(n)$  satisfy the monotone likelihood ratio property in  $n$ . As is well known, this is a sufficient condition for  $f_{i'}(n)$  to first-order stochastically dominate  $f_i(n)$  when  $\rho_{i'} > \rho_i$ . But then it is immediate that  $\mathbb{E}_{f_{i'}}[n] > \mathbb{E}_{f_i}[n]$ , and thus the expected value in (A.7), which is simply  $N + 1 - \mathbb{E}_{f_i}[n]$ , will be lower for country  $i'$  than for country  $i$  when  $\rho_{i'} > \rho_i$ . This completes the proof of Proposition 5.

We can finally consider the case with a general path of  $\alpha_n$ , but common technology  $T_i = T$  across countries. From equation (A.6), we have

$$\Pr(\Lambda_i^n, j) = \frac{(c_i)^{-\theta \alpha_n \beta_n} (\rho_i)^{\beta_{n-1} + \beta_n}}{\sum_{k \in \mathcal{J}} (c_k)^{-\theta \alpha_n \beta_n} (\rho_k)^{\beta_{n-1} + \beta_n}}.$$

We then have

$$\frac{\Pr(\Lambda_i^{n'})/\Pr(\Lambda_j^{n'})}{\Pr(\Lambda_i^n)/\Pr(\Lambda_j^n)} = \left(\frac{c_i}{c_j}\right)^{-(\alpha_{n'}\beta_{n'}-\alpha_n\beta_n)} \left(\frac{\rho_i}{\rho_j}\right)^{\beta_{n'-1}+\beta_{n'}-\beta_{n-1}-\beta_n}.$$

Take  $n' = n + 1$ . Then

$$\frac{\Pr(\Lambda_i^{n'})/\Pr(\Lambda_j^{n'})}{\Pr(\Lambda_i^n)/\Pr(\Lambda_j^n)} = \left(\frac{c_i}{c_j}\right)^{-\theta(\alpha_{n+1}\beta_{n+1}-\alpha_n\beta_n)} \left(\frac{\rho_i}{\rho_j}\right)^{\beta_{n+1}-\beta_{n-1}}.$$

Let us inspect the exponents more closely. Note  $\beta_{n-1} = (1 - \alpha_n)\beta_n$ , so  $\alpha_n\beta_n = \beta_n - \beta_{n-1}$  and

$$\frac{\Pr(\Lambda_i^{n'})/\Pr(\Lambda_j^{n'})}{\Pr(\Lambda_i^n)/\Pr(\Lambda_j^n)} = \left(\left(\frac{c_i}{c_j}\right)^{-\theta}\right)^{\beta_{n+1}-2\beta_n+\beta_{n-1}} \left(\frac{\rho_i}{\rho_j}\right)^{\beta_{n+1}-\beta_{n-1}}.$$

But

$$\beta_{n+1} - 2\beta_n + \beta_{n-1} < \beta_{n+1} - \beta_{n-1}$$

because  $\beta_{n-1} < \beta_n$ . This can be iterated starting for  $n'' = n' + 1$ . This result implies that a sufficient condition for

$$\frac{\Pr(\Lambda_i^{n'})/\Pr(\Lambda_j^{n'})}{\Pr(\Lambda_i^n)/\Pr(\Lambda_j^n)} > 1$$

for  $n' > n$  and  $\rho_i > \rho_j$  is that  $(c_i)^{-\theta}\rho_i$  is larger for more central countries. Unfortunately, the general-equilibrium conditions of the model are too complex for us to be able to formally establish that this is indeed the case for all possible parameter values. But, as stated in the main text, we have run millions of simulations and have not found a single case contradicting the claim.

## A.2. General Equilibrium Under Decentralized Approaches

This appendix demonstrates the isomorphism between the general-equilibrium conditions derived under the lead-firm (chain-productivity) formulation in the main text, and the two alternative decentralized approaches outlined in Section 3.2.

### A.2.1. Incomplete Information Approach

We begin with the first approach with stage-specific Fréchet distributions and incomplete information. On the technology side, we now assume that  $1/a_i^n(z)$  is drawn independently (across goods and stages) from a Fréchet distribution satisfying

$$\Pr(a_i^n(z)^{\alpha_n\beta_n} \geq a) = \exp\{-a^\theta (T_i)^{\alpha_n\beta_n}\}. \quad (\text{A.8})$$

To build intuition, we begin by sketching why and how the approach works for the simple case with only two stages, input production (stage 1) and assembly (stage 2). Later, we will show how the approach naturally generalizes to the case  $N > 2$ .

With  $N = 2$ , input producers of a given good  $z$  in a given country  $\ell(1) \in \mathcal{J}$  observe their productivity  $1/a_{\ell(1)}^1(z)$ , and simply hire labor and buy materials to minimize unit production costs, which results in  $p_{\ell(1)}^1(z) = a_{\ell(1)}^1(z)c_{\ell(1)}$ . Assemblers of good  $z$  in any

country  $\ell(2) \in \mathcal{J}$  observe their own productivity  $1/a_{\ell(2)}^2(z)$ , as well as that of all potential input producers worldwide, and solve

$$p_{\ell(2)}^2(z) = \min_{\ell(1) \in \mathcal{J}} \left\{ (a_{\ell(2)}^2(z) c_{\ell(2)})^{\alpha_2} (a_{\ell(1)}^1(z) c_{\ell(1)} \tau_{\ell(1)\ell(2)})^{1-\alpha_2} \right\}.$$

Independently of the values of  $a_{\ell(2)}^2(z)$ ,  $c_{\ell(2)}$ , and  $\alpha_2$ , the solution of this problem simply entails procuring the input from the location  $\ell^*(1)$  satisfying  $\ell^*(1) = \arg \min \{ (a_{\ell(1)}^1(z) \times c_{\ell(1)} \tau_{\ell(1)\ell(2)})^{1-\alpha_2} \}$ . As is well known, the Fréchet assumption in (A.8) will make characterizing this problem fairly straightforward. Consider finally the problem of retailers in each country  $j$  seeking to procure a final good  $z$  to local consumers at a minimum cost. These retailers observe the productivity  $1/a_{\ell(2)}^2(z)$  of all assemblers worldwide, but *not* the productivity of input producers, and thus seek to solve

$$p_j^F(z) = \min_{\ell(2) \in \mathcal{J}} \left\{ (a_{\ell(2)}^2(z) c_{\ell(2)})^{\alpha_2} \mathbb{E} \left[ a_{\ell^*(1)}^1(z) c_{\ell^*(1)} \tau_{\ell^*(1)\ell(2)} \right]^{1-\alpha_2} \tau_{\ell(2)j} \right\}. \quad (\text{A.9})$$

If retailers could observe the particular realizations of input producers, the expectation in (A.9) would be replaced by the realization of  $a_{\ell(1)}^1(z) c_{\ell(1)} \tau_{\ell(1)\ell(2)}$  in all  $\ell(1) \in \mathcal{J}$ , and characterizing the optimal choice would be complicated because it would depend on the product of the distributions  $a_{\ell(2)}^2(z)$  and  $a_{\ell(1)}^1(z)$ , which is not Fréchet under (A.8). Given our incomplete information assumption, however, the expectation in (A.9) does not depend on the particular realizations of upstream productivity draws, and this allows us to apply the well-know properties of the *univariate* Fréchet distribution in (A.8) to characterize the problem of retailers.

To see this, take two countries  $\ell(1)$  and  $\ell(2)$  and consider the probability  $\pi_{\ell j}$  of a GVC flowing through  $\ell(1)$  and  $\ell(2)$  before reaching consumers in  $j$ . This probability is simply the product of (i) the probability of  $\ell(1)$  being the cost-minimizing location of input production conditional on assembly happening in  $\ell(2)$ , and (ii) the probability of  $\ell(2)$  being the cost-minimizing location of assembly for GVC serving consumers in  $j$ . Denoting  $\mathcal{E}_{\ell(2)} = \mathbb{E}[\tau_{\ell^*(1)\ell(2)} a_{\ell^*(1)}^1(z) c_{\ell^*(1)}]^{1-\alpha_2}$ , and using the properties of the Fréchet distribution, it is easy to verify that we can write  $\pi_{\ell j}$  as

$$\pi_{\ell j} = \underbrace{\frac{(T_{\ell(1)})^{1-\alpha_2} (c_{\ell(1)} \tau_{\ell(1)\ell(2)})^{-\theta(1-\alpha_2)}}{\sum_{k \in \mathcal{J}} (T_k)^{1-\alpha_2} (c_k \tau_{k\ell(2)})^{-\theta(1-\alpha_2)}}}_{\Pr(\ell(1)|\ell(2))} \times \underbrace{\frac{(T_{\ell(2)})^{\alpha_2} ((c_{\ell(2)})^{\alpha_2} \tau_{\ell(2)j})^{-\theta} (\mathcal{E}_{\ell(2)})^{-\theta}}{\sum_{i \in \mathcal{J}} (T_i)^{\alpha_2} ((c_i)^{\alpha_2} (\tau_{ij}))^{-\theta} (\mathcal{E}_i)^{-\theta}}}_{\Pr(\ell(2))}. \quad (\text{A.10})$$

A bit less trivially, but also exploiting well-known properties of the Fréchet distribution, it can be shown that

$$\mathcal{E}_{\ell(2)} = \mathbb{E}[\tau_{\ell^*(1)\ell(2)} a_{\ell^*(1)}^1(z) c_{\ell^*(1)}]^{1-\alpha_2} = s \left( \sum_{k \in \mathcal{J}} (T_k)^{1-\alpha_2} (c_k \tau_{k\ell(2)})^{-\theta(1-\alpha_2)} \right)^{-1/\theta},$$

for some scalar  $s > 0$ . This allows us to reduce (A.10) to

$$\pi_{\ell j} = \frac{(T_{\ell(1)})^{1-\alpha_2} (c_{\ell(1)} \tau_{\ell(1)\ell(2)})^{-\theta(1-\alpha_2)} (T_{\ell(2)})^{\alpha_2} ((c_{\ell(2)})^{\alpha_2} \tau_{\ell(2)j})^{-\theta}}{\sum_{k \in \mathcal{J}} \sum_{i \in \mathcal{J}} (T_k)^{1-\alpha_2} (c_k \tau_{ki})^{-\theta(1-\alpha_2)} (T_i)^{\alpha_2} ((c_i)^{\alpha_2} (\tau_{ij}))^{-\theta}}. \quad (\text{A.11})$$



It should be clear that this expression is identical to (8)—plugging in (9)—for the special case  $N = 2$ . It is also straightforward to verify that the distribution of final-good prices  $p_j^F(\ell, z)$  paid by consumers in  $j$  is independent of the actual path of production  $\ell$  and is again characterized, as in equation (7), by  $\Pr(p_j^F(\ell, z) \leq p) = 1 - \exp\{-\tilde{\Theta}_j p^\theta\}$ , where  $\tilde{\Theta}_j$  is the denominator in (A.11), and is the analog of  $\Theta_j$  in (9) when  $N = 2$ .

In sum, this alternative specification of the stochastic nature of technology delivers the exact same distribution of GVCs and of consumer prices as the one in which the overall GVC unit cost is distributed Fréchet.

We next generalize this result to an environment with more than two stages. It should be clear that the input sourcing decisions for the two most upstream stages work in the same way as outlined above. Let  $\ell_z^j(n)$  be the tier-one sourcing decision of a firm producing good  $z$  at stage  $n + 1$  in  $j$ . Generalizing the notation above, define for any  $s > 0$  the expectation

$$\mathcal{E}_j^n[s] = \mathbb{E}_n[(p_{\ell_z^j(n)}^n(z) \tau_{\ell_z^j(n)j})^s],$$

where we have written the expectation with an  $n$  subscript indicating that the expectation takes that unit costs (and prices) from stages  $1, \dots, n$  as unobserved. To be fully clear, a firm at  $n + 2$  observes the productivity draws from stage  $n + 1$  but does not know previous sourcing decisions. Hence, it must form an expectation over the location from which its stage- $n$  suppliers source,  $\ell_z^j(n)$ , and use this to calculate the expected input prices  $\mathcal{E}_j^n[s]$ . As will become clear in the next paragraph, denoting the expectations for a general  $s > 0$  is useful since downstream firms between  $n + 2, \dots, N$  and final consumers will all use the information on expected input prices at  $n$  but in different ways depending on the objective function they seek to minimize.

Substituting in the Cobb–Douglas production process in (1), we can write

$$\mathcal{E}_j^n[s] = \mathbb{E}_n[(a_{\ell_z^j(n)}^n(z) c_{\ell_z^j(n)})^{\alpha_n s} \times \mathcal{E}_{\ell_z^j(n)}^{n-1}[(1 - \alpha_n)s] \times (\tau_{\ell_z^j(n)j})^s].$$

The crucial observation is that, to determine expected input prices from stage  $n$ , a firm must also incorporate expected input prices from stage  $n - 1$ , and so on until input prices from all upstream stages have been incorporated. Note that productivity draws across stages of production are independent, but even more importantly, sourcing decisions across stages of production are also independent. Hence, one can use the law of iterated expectations to compute expected input prices from  $n - 1$ ,  $\mathcal{E}_{\ell_z^j(n)}^{n-1}[\cdot]$ , in the computation of expected prices at  $n$  in  $\mathcal{E}_j^n[\cdot]$ . The latter expectation is over  $\ell_z^j(n)$ , but once we condition on a specific value for  $\ell_z^j(n)$ , the expectation  $\mathcal{E}_{\ell_z^j(n)}^{n-1}[\cdot]$  is a constant. Finally, note also that this recursion starts at  $n = 1$  with  $\mathcal{E}_j^0[s] = 1$  since only labor and materials are used in that initial stage.

Let us next illustrate why these definitions are useful. Consider the optimal sourcing strategies related to procuring the good finished up to stage  $n < N$ . Given the sequential cost function in (1), the problem faced by a stage- $n + 1$  producer in  $j$  can be written as

$$\ell_z^j(n) = \arg \min_{\ell(n) \in \mathcal{J}} \{(a_{\ell(n)}^n(z) c_{\ell(n)})^{\alpha_n(1-\alpha_{n+1})} \times \mathcal{E}_{\ell(n)}^{n-1}[(1 - \alpha_n)(1 - \alpha_{n+1})] \times (\tau_{\ell(n)j})^{1-\alpha_{n+1}}\}.$$

where the  $1 - \alpha_{n+1}$  superscript comes from the stage- $n + 1$  producer wanting to minimize its own expected input price and in which the stage- $n$  input price enters its own unit cost

to this power. Meanwhile, final consumers (or local retailers on their behalf) source their goods by solving

$$\ell_z^j(N) = \arg \min_{\ell(N) \in \mathcal{J}} \{ (a_{\ell(N)}^N(z) c_{\ell(N)})^{\alpha_N} \times \mathcal{E}_{\ell(N)}^{N-1} [1 - \alpha_N] \times \tau_{\ell(N)j} \}.$$

The probability of sourcing inputs from a specific location  $i$  at any stage  $n$  can be determined by invoking the properties of the Fréchet distribution, given that  $1/a_i^n(z)$  is drawn independently (across goods and stages) from a Fréchet distribution satisfying

$$\Pr(a_j^n(z)^{\alpha_n \beta_n} \geq a) = \exp\{-a^\theta (T_j)^{\alpha_n \beta_n}\}.$$

In particular, we obtain

$$\Pr(\ell_z^j(n) = i) = \frac{((T_i)^{\alpha_n} ((c_i)^{\alpha_n} \tau_{ij})^{-\theta})^{\beta_n} \mathcal{E}_i^{n-1} [(1 - \alpha_n)(1 - \alpha_{n+1})]^{-\beta_{n+1} \theta}}{\sum_{l \in \mathcal{J}} ((T_l)^{\alpha_n} ((c_l)^{\alpha_n} \tau_{lj})^{-\theta})^{\beta_n} \mathcal{E}_l^{n-1} [(1 - \alpha_n)(1 - \alpha_{n+1})]^{-\beta_{n+1} \theta}}.$$

These probabilities can now be leveraged in order to compute expected input prices. Define  $\tilde{a}_{ij} = (c_i)^{\alpha_n s} \mathcal{E}_i^{n-1} [(1 - \alpha_n) s] (\tau_{ij})^s$  so that  $1/(a_i^{\alpha_n s} \tilde{a}_{ij}) \sim \text{Fréchet}(T_i^{\alpha_n \beta_n} \tilde{a}_{ij}^{-\frac{\beta_n \theta}{s}}, \frac{\beta_n}{s} \theta)$  (note that the above distribution is the special case in which  $s = 1 - \alpha_{n+1}$ ). Then, using the moment generating formula for the Fréchet distribution, it can be verified that

$$\mathcal{E}_j^n[s] = q \left[ \sum_{l \in \mathcal{J}} T_l^{\alpha_n \beta_n} \tilde{a}_{lj}^{-\frac{\beta_n \theta}{s}} \right]^{-\frac{s}{\beta_n \theta}} \Gamma\left(1 + \frac{\beta_n}{s} \theta\right),$$

where  $\Gamma$  is the gamma function. From this equation, it should also be clear why we are denoting  $E_j^n[s]$  as a function of  $s$ , since, as we move down the value chain, we need to compute the upstream expectations at different “moments.”

We are now ready to determine the equilibrium variables: (1) material prices  $P_j$ , and (2) the distribution of GVCs. Material prices can be derived recursively using our expectations:

$$\begin{aligned} P_j &= (\mathcal{E}_j^N [1 - \sigma])^{\frac{1}{1-\sigma}} \\ &= \left[ \sum_{l \in \mathcal{J}} (T_l)^{\alpha_N} ((c_l)^{\alpha_N} \tau_{lj})^{-\theta} \mathcal{E}_l^{N-1} [(1 - \alpha_N)(1 - \sigma)]^{-\frac{\theta}{1-\sigma}} \right]^{-\frac{1}{\theta}} \Gamma\left(1 + \frac{1 - \sigma}{\theta}\right) \\ &= \varsigma \left[ \sum_{\ell \in \mathcal{J}} \prod_{n=1}^N ((T_{\ell(n)})^{\alpha_n} ((c_{\ell(n)})^{\alpha_n} \tau_{\ell(n)\ell(n+1)})^{-\theta})^{\beta_n} \right]^{-\frac{1}{\theta}}, \end{aligned}$$

where  $\varsigma = \prod_{n=1}^N \Gamma(1 + \frac{1-\sigma}{\beta_n \theta})^{\frac{1}{1-\sigma}}$ . This expression is identical to (10) up to a scalar (which is irrelevant for all equilibrium conditions and that could be “neutralized” by an appropriate rescaling of the stage-specific Fréchet distributions).

Finally, since input decisions from  $n$  are independent from the decisions that firms at  $n - 1$  made, then

$$\begin{aligned}
\pi_{\ell j} &= \Pr(\ell_z^j(N) = \ell(N) | \ell_z^{\ell(N)}(N-1) = \ell(N-1)) \\
&\times \prod_{n=2}^{N-1} \Pr(\ell_z^{\ell(n+1)}(n) = \ell(n) | \ell_z^{\ell(n)}(n-1) = \ell(n-1)) \times \Pr(\ell_z^{\ell(2)}(1) = \ell(1)) \\
&= \Pr(\ell_z^j(N) = \ell(N)) \times \prod_{n=1}^N \Pr(\ell_z^{\ell(n+1)}(n) = \ell(n)) \\
&= \frac{\prod_{n=1}^{N-1} ((T_{\ell(n)})^{\alpha_n} ((c_{\ell(n)})^{\alpha_n} \tau_{\ell(n)\ell(n+1)})^{-\theta})^{\beta_n} \times (T_{\ell(N)})^{\alpha_N} ((c_{\ell(N)})^{\alpha_N} \tau_{\ell(N)j})^{-\theta}}{\sum_{\ell' \in \mathcal{J}} \prod_{n=1}^{N-1} ((T_{\ell'(n)})^{\alpha_n} ((c_{\ell'(n)})^{\alpha_n} \tau_{\ell'(n)\ell'(n+1)})^{-\theta})^{\beta_n} \times (T_{\ell'(N)})^{\alpha_N} ((c_{\ell'(N)})^{\alpha_N} \tau_{\ell'(N)j})^{-\theta}}, \quad (\text{A.12})
\end{aligned}$$

which is identical to equation (8) in the main text obtained in the “randomness-in-the-chain” formulation of technology.

### A.2.2. Oberfield Approach

We next turn to the second decentralized approach inspired by work of Oberfield (2018). To ease the notation, let us define

$$Z_{\ell(n)}^n = (a_{\ell(n)}^n)^{-\alpha_n},$$

so that we can write equation (1) as

$$p_{\ell(n)}^n = \frac{1}{Z_{\ell(n)}^n} (c_{\ell(n)})^{\alpha_n} (p_{\ell(n-1)}^{n-1} \tau_{\ell(n-1)\ell(n)})^{1-\alpha_n}.$$

A key conceptual difference with this approach is that the efficiency level  $Z_{\ell(n)}^n$  is now assumed to be buyer-seller-specific (or *match*-specific). In particular, a firm producing stage  $n$  in location  $\ell(n)$  meets a certain number of potential sellers of stage  $n - 1$  in each location  $\ell(n - 1)$ , with each of these potential sellers being associated with a distinct “match” productivity of combining the good completed up to stage  $n - 1$  with the labor and materials at stage  $n$ . This buyer-seller-specific productivity is drawn from a Pareto distribution with shape parameter  $\theta$  and lower bound  $Z_{\ell(n)}^n$ . Below, we will focus on the limiting case in which  $Z_{\ell(n)}^n \rightarrow 0$ . Given all the available match-specific productivities and production costs, each stage- $n$  producer (or buyer) chooses the supplier offering the lowest cost for the good produced at stage  $n - 1$ . The number of available potential suppliers in each sourcing country  $\ell(n - 1)$  varies across producers, and the precise number  $m_{\ell(n-1)\ell(n)}^n$  of potential suppliers based in country  $\ell(n - 1)$  available to a given firm producing stage  $n$  in country  $\ell(n)$  is assumed to follow a Poisson distribution with arrival rate  $(T_{\ell(n)})^{\alpha_n} (Z_{\ell(n)}^n)^{-\theta}$ . For  $n = 1$ , and for the time being, we assume that productivity in location  $\ell(1)$  is fixed at  $Z_{\ell(1)}^1 = (T_{\ell(1)})^{1/\theta}$ , though we will relax this assumption below.

We now derive the distribution of final-good prices in country  $j$  when sourcing goods through an arbitrary supply chain  $\ell$ . To build intuition, let us first study the case with two stages ( $N = 2$ ). Consider the distribution of prices that a stage-2 producer in country  $\ell(2)$  can offer to consumers in country  $j$  if stage-1 output is bought from country  $\ell(1)$  and the highest matched-pair productivity with suppliers in that country is  $\hat{Z}_{\ell(2)}^2$ . This distribution is given by

$$\begin{aligned} G_j^2(p|\ell(1), \ell(2)) &= \Pr\left(p \leq \frac{1}{\hat{Z}_{\ell(2)}^2} (c_{\ell(2)})^{\alpha_2} (p_{\ell(1)}^1 \tau_{\ell(1)\ell(2)})^{1-\alpha_2} \tau_{\ell(2)j}\right), \\ &= \Pr(\hat{Z}_{\ell(2)}^2 \leq \tilde{Z}(p)), \end{aligned}$$

where  $\tilde{Z}(p) = (c_{\ell(2)})^{\alpha_2} (c_{\ell(1)} (T_{\ell(1)})^{-1/\theta} \tau_{\ell(1)\ell(2)})^{1-\alpha_2} \tau_{\ell(2)j} / p$ .

Now remember that the stage-2 producer has various potential suppliers in each country  $\ell(1)$ , so for the price to be higher than  $p$ , or for  $\max_{\mu=1, \dots, m_{\ell(1)\ell(2)}} \{Z_{\ell(2)}^{2,\mu}\} = \hat{Z}_{\ell(2)}^2 < \tilde{Z}(p)$ , we need  $Z_{\ell(2)}^{2,m} < \tilde{Z}(p)$  for all the draws  $\mu$  associated with all the potential suppliers  $m_{\ell(1)\ell(2)}$  that a specific firm has. Since both the number of suppliers and productivity of each set of suppliers are stochastic, we can obtain the overall distribution of prices invoking the formula for the Poisson probability density function and also plugging in the cumulative density function for the Pareto distribution:

$$\begin{aligned} G_j(p|\ell(1), \ell(2)) &= \sum_{m=0}^{\infty} \prod_{\mu=1}^m \Pr(Z_{\ell(2)}^{2,\mu} \leq \tilde{Z}(p)) \times \Pr(m_{\ell(1)\ell(2)} = m) \\ &= \sum_{m=0}^{\infty} \prod_{\mu=1}^m \left(1 - \left(\frac{Z_{\ell(2)}^2}{\tilde{Z}(p)}\right)^{\theta}\right) \times \frac{((T_{\ell(2)})^{\alpha_2} (Z_{\ell(2)}^2)^{-\theta})^m \exp\{- (T_{\ell(2)})^{\alpha_2} (Z_{\ell(2)}^2)^{-\theta}\}}{m!} \\ &= \exp\{-p^{\theta} (T_{\ell(1)} c_{\ell(1)}^{-\theta} \tau_{\ell(1)\ell(2)}^{-\theta})^{1-\alpha_2} (T_{\ell(2)} c_{\ell(2)}^{-\theta})^{\alpha_2} \tau_{\ell(2)j}^{-\theta}\}. \end{aligned} \tag{A.13}$$

This is the same expression we obtain in the ‘‘Fréchet-in-the-chain’’ formulation in the main text.

Now let us extend these results to the case with  $N = 3$ , and consider the problem of producers of the final assembly stage  $n = 3$  in country  $\ell(3)$ . For such a producer, the distribution of prices it can offer to consumers in  $j$  when stage-2 inputs are bought from country  $\ell(2)$  is more involved than before because it now depends on the product of the distribution of buyer-seller productivity draws  $Z_{\ell(3)}^3$  and the upstream input prices  $p_{\ell(2)}^2$  that each input seller itself sells at (i.e., influenced by the buyer-seller productivity that the stage-2 seller has with its own input suppliers). However, note that the buyer-seller productivity draws at stage 3 are independent of the upstream productivity draws. Instead, what is crucial to take into account is the fact that stage-3 producers that get more stage-2 matches will get, on average, both a better buyer-seller productivity draw but also a better stage-2 input price. Thus, we can split the problem into two parts. We first obtain the expected price distribution conditional on a buyer-seller relationship and then we obtain the price distribution by characterizing the distribution of optimal matches.

Define the distribution of stage-3 prices in  $j$  from a given supply chain  $\ell = \{\ell(1), \ell(2), \ell(3)\}$  conditional on a specific buyer-seller relationship characterized by  $Z_{\ell(3)}^3$  as

$$\begin{aligned} F_j(p|\ell, Z_{\ell(3)}^3) &= \Pr\left(p \leq \frac{1}{Z_{\ell(3)}^3} (c_{\ell(3)})^{\alpha_3} (p_{\ell(2)}^2(\ell) \tau_{\ell(2)\ell(3)})^{1-\alpha_3} \tau_{\ell(3)j}\right) \\ &= \exp\left\{-\Theta_{\ell(1)\ell(2)} \left(\frac{p Z_{\ell(3)}^3}{\tau_{\ell(2)\ell(3)}^{1-\alpha_3} c_{\ell(3)}^{\alpha_3} \tau_{\ell(3)j}}\right)^{\theta/(1-\alpha_3)}\right\}, \end{aligned}$$

where  $\Theta_{\ell(1)\ell(2)} = (T_{\ell(1)} c_{\ell(1)}^{-\theta} \tau_{\ell(1)\ell(2)}^{-\theta})^{1-\alpha_2} (T_{\ell(2)} c_{\ell(2)}^{-\theta})^{\alpha_2}$  and where we have invoked our above distribution (A.13). As in the  $N = 2$ -stage case, with  $N = 3$  the distribution of prices along chain  $\ell$  will be determined by the fact that each producer at the assembly stage chooses the upstream supplier that offers the best combination of buyer-seller productivity and input prices. That is,

$$\begin{aligned} G_j(p|\ell(1), \ell(2), \ell(3)) &= \sum_{m=0}^{\infty} \prod_{\mu=1}^m \int_{\underline{Z}_{\ell(3)}^3}^{\infty} F_j(p|\ell, Z_{\ell(3)}^{3,\mu}) \Pr(Z_{\ell(3)}^{3,\mu} = Z) dZ \times \Pr(m_{\ell(2)\ell(3)} = m) \\ &= \exp\left\{-(T_{\ell(3)})^{\alpha_3} \left(\underline{Z}_{\ell(3)}^3 - \int_{\underline{Z}_{\ell(3)}^3}^{\infty} F_j(p|\ell, Z) \frac{\theta}{Z^{\theta+1}} dZ\right)\right\}, \end{aligned}$$

where we used the fact that  $Z_{\ell(3)}^{3,\mu}$  is a Pareto random variable with lower bound  $\underline{Z}_{\ell(3)}^3$  and shape parameter  $\theta$ . Now, define  $\chi(p)$  such that  $F_j(p|\ell, Z_{\ell(3)}^3) = \exp\{-\chi(p)(Z_{\ell(3)}^3)^{\theta/(1-\alpha_3)}\}$ , and solve the above integral by taking the limit when  $\underline{Z}_{\ell(3)}^3 \rightarrow 0$  and using a change of variable  $\zeta(p) = \chi(p) Z^{\theta/(1-\alpha_3)}$  to obtain

$$\begin{aligned} \int_0^{\infty} F_j(p|\ell, Z) \frac{\theta}{Z^{\theta+1}} dZ &= \frac{\chi(p)^{1-\alpha_3}}{1-\alpha_3} \int_0^{\infty} \exp\{-\zeta(p)\} \zeta(p)^{\alpha_3-1} d\zeta(p) \\ &= \frac{\chi(p)^{1-\alpha_3}}{1-\alpha_3} \Gamma(\alpha_3), \end{aligned}$$

where  $\Gamma(\cdot)$  is the gamma function. Plugging this back in (and remember that we took the limit  $\underline{Z}_{\ell(3)}^3 \rightarrow 0$ ), we obtain that

$$\begin{aligned} G_j(p|\ell(1), \ell(2), \ell(3)) &= \exp\left\{-(T_{\ell(3)})^{\alpha_3} \frac{\chi(p)^{1-\alpha_3}}{1-\alpha_3} \Gamma(\alpha_3)\right\} \\ &= \exp\left\{-p^{\theta} \times \prod_{n=1}^3 (c_{\ell(n)}^{-\theta} T_{\ell(n)})^{\alpha_n \beta_n} \times \prod_{n=1}^2 (\tau_{\ell(n)\ell(n+1)})^{-\theta \beta_n} \times (\tau_{\ell(3)j})^{-\theta} \times \frac{\Gamma(\alpha_3)}{1-\alpha_3}\right\}, \end{aligned}$$

where notation is such that  $\beta_n \equiv \prod_{m=n+1}^N (1-\alpha_m)$  and  $\alpha_1 = 1$ . This last expression is the exact same expression we obtain in the main text for  $\Pr(p_j^F(\ell, z) \geq p)$  in the  $N = 3$  case except for the last scalar term involving the gamma function term. Nevertheless, this scalar term is irrelevant for the main equilibrium conditions in the model.

We have derived this result for stages  $n = 3$  and  $n = 2$ , but it should be clear that the above derivations would work for any two stages  $n$  and  $n - 1$ , as long as the distribution of production costs in upstream stage  $n - 1$  is Fréchet distributed. This has two implications. First, our assumption above that, for  $n = 1$ , productivity in location  $\ell(1)$  is fixed at  $Z_{\ell(1)}^1 = (T_{\ell(1)})^{1/\theta}$  can be relaxed and we can instead assume that  $Z_{\ell(1)}^1$  is Fréchet distributed with shape parameter  $\theta$  and scale parameter  $T_{\ell(1)}$ . Second, one can use induction to conclude from our results above that, for a general  $N$ , we obtain

$$\Pr(p_j^F(\boldsymbol{\ell}, z) \geq p) = \exp \left\{ -p^\theta \times \prod_{n=1}^N (c_{\ell(n)}^{-\theta} T_{\ell(n)})^{\alpha_n \beta_n} \times \prod_{n=1}^{N-1} (\tau_{\ell(n)\ell(n+1)})^{-\theta \beta_n} \times (\tau_{\ell(N)j})^{-\theta} \times \tilde{\xi} \right\},$$

where  $\tilde{\xi}$  is a positive scalar that is irrelevant for all equilibrium conditions and that can be “neutralized” by an appropriate rescaling of the stage-specific Poisson distributions. It should be apparent that this expression coincides with equation (7) in the main text, up to this immaterial scalar  $\tilde{\xi}$ .

Finally, it remains to be shown that this decentralized solution not only delivers the same distribution of final-good prices, but also the same GVC trade shares as in expression (8) in the main text. But this is implied by our previous derivations related to the decentralized approach with incomplete information. In particular, fixing a downstream stage  $n$ , the distribution of upstream costs at  $n - 1$  is again Fréchet distributed, so applying the law of total probability in the same manner as in (A.12) above, it is straightforward to re-derive equation (8) in the main text. And, to reiterate, the scalar  $\tilde{\xi}$  is irrelevant for these equilibrium conditions.

### A.3. Introducing Trade Deficits

Let  $D_j$  be country  $j$ 's aggregate deficit in dollars, where  $\sum_j D_j = 0$  holds since global trade is balanced. The only difference in the model's equations is that the general-equilibrium equation is given by

$$\frac{1}{\gamma_i} w_i L_i = \sum_{j \in \mathcal{J}} \sum_{n \in \mathcal{N}} \alpha_n \beta_n \times \Pr(\Lambda_i^n, j) \times \left( \frac{1 - \gamma_j}{\gamma_j} w_j L_j + w_j L_j - D_j \right),$$

where  $w_j L_j - D_j$  is aggregate final-good consumption in country  $j$ .

A.4. Graphical Description of Multi-Stage Production

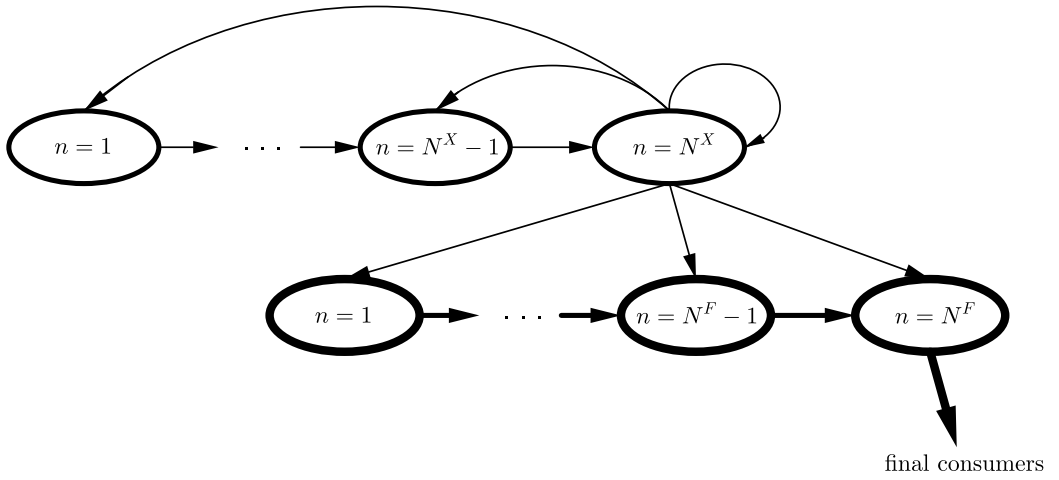


FIGURE A.2.—Multi-stage production with separate intermediate-input and final-good supply chains.

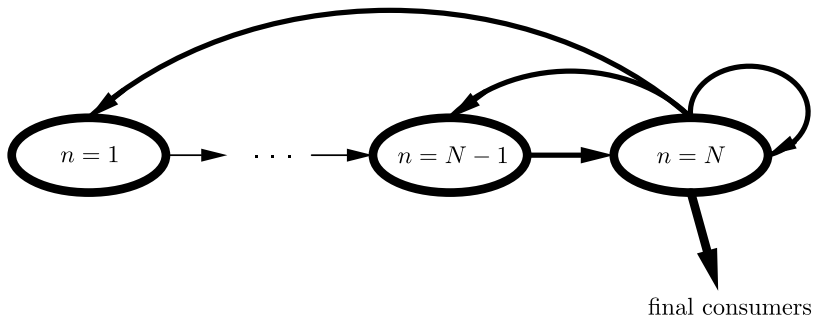


FIGURE A.3.—Multi-stage production with common intermediate-input and final-good supply chains.

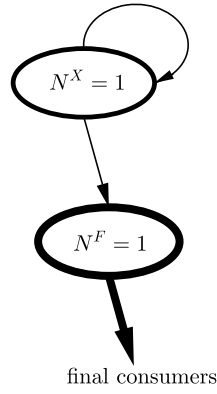


FIGURE A.4.—Single-stage production with separate intermediate-input and final-good technology (Alexander (2017)).

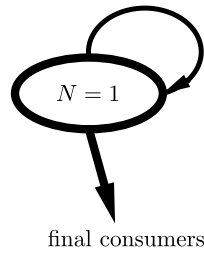


FIGURE A.5.—Single-stage production with common intermediate-input and final-good technology (Eaton and Kortum (2002)).



## A.5. Estimation Results

TABLE A.I  
ESTIMATION RESULTS—ASYMMETRIC PARAMETERIZATIONS

N	$\gamma_j$					$T_j^X$					$T_j^F$				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
AUS	0.54	0.65	0.74	0.79	0.83	7.96	7.97	8.03	8.13	8.29	3.52	4.88	5.01	4.96	4.86
AUT	0.53	0.63	0.73	0.78	0.83	1.42	1.04	0.72	0.55	0.43	2.64	2.07	1.74	1.53	1.40
BEL	0.53	0.62	0.72	0.79	0.84	1.67	0.95	0.65	0.48	0.36	2.01	1.64	1.45	1.33	1.26
BGR	0.36	0.48	0.58	0.65	0.70	0.02	0.01	0.00	0.00	0.00	0.03	0.02	0.01	0.00	0.00
BRA	0.56	0.68	0.76	0.81	0.84	0.10	0.08	0.04	0.03	0.02	0.37	0.12	0.06	0.04	0.03
CAN	0.58	0.68	0.77	0.83	0.87	6.28	5.35	4.59	4.23	4.03	3.44	3.40	3.25	3.08	2.94
CHE	0.53	0.62	0.72	0.78	0.82	9.21	8.00	7.85	7.46	6.92	10.9	12.4	14.5	16.5	17.5
CHN	0.33	0.45	0.55	0.62	0.67	0.16	0.13	0.07	0.05	0.04	0.35	0.12	0.08	0.05	0.04
CZE	0.44	0.53	0.64	0.71	0.76	0.15	0.06	0.03	0.01	0.01	0.22	0.13	0.08	0.05	0.04
DEU	0.54	0.65	0.74	0.80	0.83	3.10	3.16	2.40	1.98	1.66	5.57	4.95	4.69	4.47	4.29
DNK	0.57	0.64	0.73	0.79	0.83	3.01	1.55	1.14	0.86	0.66	5.39	5.08	4.42	4.03	3.76
ESP	0.52	0.63	0.72	0.77	0.80	0.57	0.44	0.27	0.20	0.16	1.25	0.78	0.56	0.44	0.38
FIN	0.51	0.60	0.69	0.74	0.78	1.29	0.70	0.51	0.39	0.32	1.99	2.08	1.61	1.35	1.19
FRA	0.55	0.66	0.74	0.79	0.82	1.87	1.93	1.39	1.12	0.94	4.07	3.07	2.59	2.28	2.07
GBR	0.56	0.67	0.75	0.80	0.84	3.49	3.29	2.59	2.23	2.01	3.32	3.13	2.81	2.57	2.39
GRC	0.58	0.66	0.74	0.78	0.81	0.08	0.03	0.01	0.01	0.01	0.24	0.14	0.07	0.05	0.03
HRV	0.46	0.57	0.68	0.74	0.78	0.02	0.01	0.00	0.00	0.00	0.03	0.03	0.01	0.01	0.00
HUN	0.52	0.59	0.70	0.78	0.83	0.05	0.01	0.00	0.00	0.00	0.09	0.05	0.03	0.01	0.01
IDN	0.53	0.65	0.73	0.79	0.82	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00
IND	0.53	0.65	0.73	0.77	0.80	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00
IRL	0.62	0.67	0.79	0.88	0.95	3.03	0.93	0.56	0.33	0.19	2.97	2.29	2.20	2.22	2.31
ITA	0.51	0.62	0.71	0.76	0.80	0.91	0.82	0.56	0.45	0.38	1.82	1.28	1.01	0.85	0.75
JPN	0.52	0.64	0.72	0.77	0.80	1.32	1.88	1.39	1.16	1.02	6.56	3.66	3.11	2.73	2.47
KOR	0.42	0.53	0.63	0.69	0.74	0.55	0.50	0.34	0.26	0.21	1.72	0.93	0.76	0.64	0.56
LTU	0.46	0.57	0.69	0.76	0.81	0.03	0.01	0.00	0.00	0.00	0.02	0.04	0.02	0.01	0.01
LUX	0.32	0.51	0.67	0.79	0.88	0.96	2.65	3.89	4.76	5.19	0.18	1.61	2.80	4.28	5.90
MEX	0.59	0.70	0.77	0.81	0.84	0.05	0.03	0.01	0.01	0.01	0.38	0.11	0.06	0.04	0.03
NLD	0.60	0.69	0.80	0.88	0.92	5.95	3.74	3.18	2.85	2.62	2.89	2.83	2.83	2.81	2.80
NOR	0.62	0.73	0.83	0.88	0.91	31.8	32.5	40.1	43.7	46.1	14.3	21.2	22.5	23.2	23.2
POL	0.48	0.59	0.68	0.74	0.78	0.20	0.10	0.05	0.03	0.02	0.32	0.18	0.11	0.08	0.06
PRT	0.54	0.63	0.71	0.77	0.80	0.13	0.06	0.03	0.02	0.01	0.27	0.15	0.08	0.05	0.04
ROU	0.49	0.59	0.68	0.74	0.78	0.04	0.02	0.01	0.00	0.00	0.07	0.03	0.02	0.01	0.01
ROW	0.44	0.57	0.67	0.73	0.77	0.06	0.03	0.01	0.01	0.00	0.06	0.02	0.01	0.01	0.00
RUS	0.55	0.69	0.79	0.85	0.89	0.63	0.45	0.29	0.23	0.20	0.06	0.04	0.03	0.02	0.02
SVK	0.44	0.52	0.64	0.72	0.77	0.12	0.04	0.02	0.01	0.00	0.20	0.15	0.08	0.06	0.04
SVN	0.38	0.53	0.65	0.73	0.79	0.05	0.03	0.01	0.00	0.00	0.06	0.08	0.04	0.02	0.02
SWE	0.55	0.65	0.74	0.80	0.83	3.64	2.43	1.98	1.66	1.41	4.30	4.39	3.98	3.71	3.52
TUR	0.53	0.63	0.71	0.76	0.80	0.09	0.04	0.02	0.01	0.01	0.15	0.08	0.04	0.03	0.02
TWN	0.55	0.64	0.75	0.83	0.88	0.79	0.35	0.22	0.17	0.14	0.31	0.24	0.17	0.13	0.11
USA	0.57	0.69	0.77	0.82	0.85	9.15	18.7	17.0	16.6	16.6	17.8	16.5	16.7	16.4	15.9

TABLE A.II  
ESTIMATION RESULTS—SYMMETRIC PARAMETERIZATIONS

$N$	$\gamma_j$		$T_j$	
	1	2	1	2
AUS	0.52	0.88	4.79	3.48
AUT	0.55	0.87	2.14	0.57
BEL	0.54	0.83	1.92	0.45
BGR	0.61	0.95	0.10	0.00
BRA	0.57	0.99	0.15	0.01
CAN	0.55	0.92	3.43	1.65
CHE	0.52	0.81	9.44	6.35
CHN	0.33	0.57	0.18	0.03
CZE	0.48	0.73	0.23	0.02
DEU	0.55	0.87	3.90	1.65
DNK	0.59	0.92	5.04	1.83
ESP	0.54	0.88	0.77	0.14
FIN	0.54	0.87	2.01	0.59
FRA	0.56	0.93	2.58	0.93
GBR	0.55	0.91	2.95	1.24
GRC	0.63	1.00	0.16	0.01
HRV	0.70	1.00	0.22	0.01
HUN	0.61	0.91	0.14	0.01
IDN	0.55	0.93	0.01	0.00
IND	0.56	0.97	0.00	0.00
IRL	0.63	0.92	3.89	0.94
ITA	0.51	0.85	1.15	0.29
JPN	0.54	0.93	2.43	1.08
KOR	0.44	0.71	0.79	0.21
LTU	0.71	1.00	0.58	0.10
LUX	0.45	0.81	8.02	42.5
MEX	0.64	1.00	0.11	0.00
NLD	0.56	0.86	2.83	0.81
NOR	0.61	0.98	22.3	17.8
POL	0.50	0.79	0.24	0.02
PRT	0.57	0.93	0.22	0.01
ROU	0.53	0.84	0.06	0.00
ROW	0.43	0.72	0.05	0.00
RUS	0.50	0.84	0.22	0.02
SVK	0.56	0.85	0.47	0.05
SVN	0.62	1.00	0.68	0.25
SWE	0.56	0.88	4.04	1.50
TUR	0.54	0.88	0.11	0.01
TWN	0.51	0.80	0.43	0.06
USA	0.58	1.00	11.2	15.3

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