Offshoring and the Role of Trade Agreements: Online Appendix

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Abstract

This online Appendix includes various proofs and details that were left out of our manuscript "Offshoring and the Role of Trade Agreements" due to space constraints.

A Secondary Market

In the Benchmark Model we have assumed that the lack of an ex-post contractual agreement leaves both parties with no time to attempt to transact with alternative producers, and thus the outside options in the bargaining are equal to 0. We now explore the robustness of our results to the case in which there exists a secondary market for inputs. For now, we continue to assume that all final good producers in that secondary market are from the home country. Later, we also consider the possibility that the secondary market is located in the foreign country and involves foreign final good producers.

In order to explicitly derive the payoffs associated with the secondary market we now assume that good 1 comes in two types, a customized type T and a generic type G, and that consumer preferences are given by

$$U^j = c_0^j + u\left(c_{1_T}^j + \delta c_{1_G}^j\right), \quad 0 < \delta < 1. \tag{1}$$

Note that the preferences in (1) are such that consumers are willing to buy both types of good 1 only if the price of the generic relative to that of the customized type is equal to δ. This is analogous to consumers perceiving the two goods as perfect substitutes up to a quality shifter. By an appropriate choice of units, we can set the (fixed) price on world markets of the customized type-T final good 1 equal to 1, and that of the generic type-G final good 1 to δ.

The technology for producing final goods and intermediate inputs is as in our Benchmark Model. The only difference between the two types of good 1 is that the production of a generic good G uses an intermediate input x that is *not* customized to the producer's needs.

The game we consider is a straightforward extension of that in our Benchmark Model that incorporates a secondary market for inputs. The sequence of events is as in our Benchmark Model, except that our previous *stage* 4 is now divided into two stages as follows:

stage 4. A small number (formally, a measure-zero countable infinity) n of the bilateral pairs are exogenously dissolved and randomly rematched in a secondary market. They bargain again according to the same generalized Nash bargaining solution as in *stage* 3. No further inputs can be produced; the amount produced in *stage* 2 is perceived as generic in the secondary market because it was tailored to another producer's specifications with probability one.

stage 5. Each producer in H imports x from its partner-supplier and produces the final good with the acquired x, and payments agreed in *stages* 3 and 4 are settled.

We focus directly on deriving Nash policy choices, assuming as before that the home and foreign governments select their respective tariffs simultaneously in a prior *stage* 0. Note that given the specification of the secondary market in *stage* 4, it is easy to see that the breakup of a single bargaining pair in *stage* 3 would result in each member of the pair being rematched with probability 1 with a random partner in *stage* 4. Therefore, *stage* 4 generates an outside option equal to $\alpha\left(\delta\left(1+\tau_1^H\right)y\left(x\right)-\tau_x x\right)$ for the final good producer and $\left(1-\alpha\right)\left(\delta\left(1+\tau_1^H\right)y\left(x\right)-\tau_x x\right)$ for the supplier in their negotiations at *stage* 3. These expressions are valid provided they are non-negative, and throughout this section we characterize results for the case where these non-negativity constraints are non-binding (though we show below that our qualitative results carry through when these constraints bind). Beyond determining outside options, *stage* 4 plays no role in the model, and in particular only the customized type of good 1 will be produced with positive measure in equilibrium.

Following analogous steps as in the main text, it is easy to see that generalized Nash bargaining in *stage* 3 will leave the final good producer in H with a payoff equal to $\alpha\left(\left(1+\tau_1^H\right)y\left(x\right)-\tau_x x\right)$, with the supplier in F now receiving a *stage*-3 payoff of $\left(1-\alpha\right)\left(\left(1+\tau_1^H\right)y\left(x\right)-\tau_x x\right)$. This follows from the fact that the marginal cost of production of generic and customized inputs is the same, so there is no benefit in not customizing the input for the matched producer in *stage* 2. As is apparent, these expressions are identical to those applying in our Benchmark Model and, consequently, they lead to the same choice of $\hat{x}$ and the same trade policy choices by governments.

If we were to assume that the relative bargaining power of suppliers were different in the "primary" and secondary markets of stages 3 and 4 respectively, then the tariff choices would indeed be different, but the main conclusions from our analysis would remain unaltered. To see this, consider the case in which there is generalized Nash bargaining in both stages 3 and 4, but with potentially different bargaining weights $\alpha_P \in (0,1)$ and $\alpha_S \in (0,1)$, respectively. In such a case, the Nash tariff choices are characterized by the following conditions:

$$\begin{aligned}
\hat{\tau}_1^{HN} &= -\frac{\left(1-\bar{\alpha}\right)\hat{x}[\frac{y(\hat{x})}{\hat{x}}-y'\left(\hat{x}\right)]}{\left|\partial D_1/\partial p_1^H\right|}, \\
\hat{\tau}_x^{HN} &= -\frac{\left[\bar{\alpha}-\left(1-\bar{\alpha}\right)\hat{\tau}_1^{HN}\right]y'\left(\hat{x}\right)}{\left(1-\alpha_S\right)}+\frac{\alpha_S\hat{\tau}_x^{FN}}{\left(1-\alpha_S\right)}-\frac{\hat{x}}{\partial\hat{x}/\partial\tau_x^H}, \text{ and} \\
\hat{\tau}_x^{FN} &= -\alpha_S\frac{\hat{x}}{\partial\hat{x}/\partial\tau_x^F},
\end{aligned}$$

where $\bar{\alpha}\equiv\alpha_S\delta+\alpha_P\left(1-\delta\right)$. It is apparent that $\hat{\tau}_1^{HN}<0$ and it can also be verified that $y'\left(\hat{x}\right)>1$.

In describing the Benchmark Model, we have emphasized the role of customization in creating the lock-in effect at the heart of the bilateral determination of prices and the holdup problem. As argued in section 2 of the paper, however, the same lock-in effect could be generated by (ex-post) search frictions even in the absence of any customization. To see this, suppose that $\delta = 1$, so that generic and customized inputs are perfect substitutes, but let search frictions lead to the formation of only κn pairs in *stage* 4, with $\kappa < 1$. It is then clear that the outside option for the final good producer is now $\kappa\alpha\left(\left(1+\tau_1^H\right)y\left(x\right)-\tau_x x\right)$, while that for a supplier is $\kappa\left(1-\alpha\right)\left(\left(1+\tau_1^H\right)y\left(x\right)-\tau_x x\right)$. The resulting *stage* 3 payoffs for these two agents are $\alpha\left(\left(1+\tau_1^H\right)y\left(x\right)-\tau_x x\right)$ and $\left(1-\alpha\right)\left(\left(1+\tau_1^H\right)y\left(x\right)-\tau_x x\right)$, respectively, just as in the case with customized inputs.

Non-Negativity Constraints So far we have ignored situations in which equilibrium trade policies might violate the non-negativity constraints on the outside options and the surplus available to agents in the negotiation. We next explore these situations and show that they do not invalidate the main results of the paper. To this end, recall that the surplus over which the producer and the supplier bargain is given by

$$\left(1+\tau_1^H\right)y\left(\hat{x}\right)-\left(\tau_x^H+\tau_x^F\right)\hat{x}, \tag{2}$$

where the equilibrium $\hat{x}$ satisfies

$$\left(1-\alpha\right)\left(1+\tau_1^H\right)y'\left(\hat{x}\right)=1+\left(1-\alpha\right)\tau_x^H+\left(1-\alpha\right)\tau_x^F. \tag{3}$$

Our first result is that *regardless* of the equilibrium values of τ_1^H, τ_x^H, and τ_x^F, the surplus in equation (2) will always be non-negative. To see this, note that using (3) we can write

$$\left(1+\tau_1^H\right)y\left(\hat{x}\right)\geq\left(1+\tau_1^H\right)\hat{x}y'\left(\hat{x}\right)=\frac{1}{1-\alpha}\hat{x}+\hat{x}\tau_x^H+\hat{x}\tau_x^F\geq\hat{x}\tau_x^H+\hat{x}\tau_x^F,$$

where we have used that the concavity of $y\left(\cdot\right)$ implies $y'\left(\hat{x}\right)\hat{x}<y\left(\hat{x}\right)$. Hence, the non-negativity constraint on the surplus can be ignored hereafter. Intuitively, no matter how distortionary trade taxes are, the level of investment x will adjust to ensure a positive joint surplus of the relationship.

Matters are not as simple with regards to the outside option of each producer. In particular, we are now careful to define this outside option as follows:

$$\begin{aligned}&\max\left\{\alpha\delta(1+\tau_1^H)y\left(\hat{x}\right)-\alpha\left(\tau_x^H+\tau_x^F\right)\hat{x},0\right\};\\&\max\left\{\left(1-\alpha\right)\delta(1+\tau_1^H)y\left(\hat{x}\right)-\left(1-\alpha\right)\left(\tau_x^H+\tau_x^F\right)\hat{x},0\right\}.\end{aligned}$$

It is straightforward to see that whenever $\delta\to 0$, one of the two types of producers (and possibly both types) will find it optimal to ignore the secondary market and simply throw away the amount $\hat{x}$ of input produced. It thus follows that the secondary market will remain inactive (i.e., no matches will succeed) in that case, and the outside options for both producers will be zero. Hence, the (ex-post) payoffs to the final-good producer and the supplier are given, respectively, by

$$\begin{aligned}\pi^F&=\alpha\left((1+\tau_1^H)y\left(\hat{x}\right)-\left(\tau_x^H+\tau_x^F\right)\hat{x}\right),\text{ and}\\\pi^S&=\left(1-\alpha\right)\left((1+\tau_1^H)y\left(\hat{x}\right)-\left(\tau_x^H+\tau_x^F\right)\hat{x}\right)-\hat{x},\end{aligned}$$

which are the same expressions as in our Benchmark model. This implies that the analysis remains unchanged even when the non-negativity constraint is taken into account.

Location of the Secondary Market We consider here the possibility that the secondary market takes place in Foreign and involves foreign final-good producers. This implies that, in the event of disagreement with the "primary" final-good producer in H, the input supplier in F sells the inputs locally in the foreign country rather than exporting to an alternative buyer in H. There are a number of reasons to think that this possibility could be reflected in a richer model (e.g., as a result of search frictions associated with finding international partners on short notice that can be avoided with local matches), but rather than attempting to model these reasons explicitly we simply assume outright that there exists a secondary market in the foreign country (only) where a match with a local producer results in the production of an amount $y(x)$ of the generic good. Without loss of generality, we develop this extension for the case of symmetric bargaining power ($\alpha = 1/2$).

The key difference relative to the Benchmark Model is in the outside options. The home producers now obtain no income in the secondary market, while foreign producers now obtain $\frac{1}{2}\delta(1+\tau_1^F)y(x)$ in that market, where τ_1^F is a foreign trade tax on the final good. Following analogous steps as in the main text, it is easy to see that the final-good producer in H now has a *stage*-3 payoff of

$$\frac{1}{2}\left[(1+\tau_1^H) - \frac{1}{2}\delta(1+\tau_1^F)\right] y(x) - \frac{1}{2}(\tau_x^H + \tau_x^F)x,$$

with the supplier in F now receiving a *stage*-3 payoff of

$$\frac{1}{2}\left[(1+\tau_1^H) + \frac{1}{2}\delta(1+\tau_1^F)\right] y(x) - \frac{1}{2}(\tau_x^H + \tau_x^F)x,$$

so that the *stage*-2 choice of $\hat{x}$ is now defined by

$$\frac{1}{2}\left[(1+\tau_1^H) + \frac{1}{2}\delta(1+\tau_1^F)\right] y'(\hat{x}) = 1 + \frac{1}{2}(\tau_x^H + \tau_x^F), \tag{4}$$

and hence the *stage*-1 payoffs of the home and foreign firm are given by

$$\begin{aligned}
\pi^H &= \frac{1}{2}\left[(1+\tau_1^H) - \frac{1}{2}\delta(1+\tau_1^F)\right] y(\hat{x}) - \frac{1}{2}(\tau_x^H + \tau_x^F)\hat{x}, \text{ and} \\
\pi^F &= \frac{1}{2}\left[(1+\tau_1^H) + \frac{1}{2}\delta(1+\tau_1^F)\right] y(\hat{x}) - \frac{1}{2}(\tau_x^H + \tau_x^F)\hat{x} - \hat{x}.
\end{aligned}$$

Anticipating that F may now have reason to alter p_1^F with its choice of τ_1^F (for reasons analogous to H's incentive to alter p_1^H with its choice of τ_1^H) and hence affect foreign consumer surplus $CS(p_1^F)$, and noting that none (or to be precise, a measure 0) of good 1 is actually produced in F in equilibrium, home and foreign welfare are then given by

$$\begin{aligned}
W^H &= CS(p_1^H) + \pi^H + \tau_1^H[D_1(p_1^H) - y(\hat{x})] + \tau_x^H \hat{x}, \text{ and} \\
W^F &= CS(p_1^F) + \pi^F + \tau_1^F D(p_1^F) + \tau_x^F \hat{x}.
\end{aligned}$$

The first-order conditions that define the Nash policies $\hat{\tau}_1^{HN}$, $\hat{\tau}_x^{HN}$, $\hat{\tau}_1^{FN}$ and $\hat{\tau}_x^{FN}$ can be manipulated to yield

$$\begin{aligned}
\hat{\tau}_1^{HN} &= -\frac{\frac{1}{2}\hat{x}[\frac{y(\hat{x})}{\hat{x}} - y'(\hat{x})]}{|\partial D_1/\partial p_1^H|} < 0, \\
\hat{\tau}_x^{HN} &= -\left[(1-\tau_1^H) - \frac{1}{2}\delta(1+\tau_1^F)\right] y'(\hat{x}) - \frac{\hat{x}}{\partial\hat{x}/\partial\tau_x^H} + \hat{\tau}_x^{FN}, \\
\hat{\tau}_1^{FN} &= \frac{\frac{1}{4}\delta\hat{x}[y'(\hat{x}) + \frac{y(\hat{x})}{\hat{x}}]}{|\partial D_1/\partial p_1^F|} > 0, \text{ and} \\
\hat{\tau}_x^{FN} &= -\frac{1}{2}\frac{\hat{x}}{\partial\hat{x}/\partial\tau_x^F}.
\end{aligned}$$

Again, the expression for $\hat{\tau}_1^{HN}$ is negative, and is similar to the expression derived in the Benchmark Model. The intuition is also analogous to that in the Benchmark Model: the home government finds it optimal to set a negative $\hat{\tau}_1^{HN}$ as a means of shifting surplus from foreign suppliers to the home country. The dual role that $\hat{\tau}_x^{HN}$ plays in alleviating the hold-up problem and at same time transferring surplus implies again that its sign is in general ambiguous.

This extension of the model delivers more interesting implications for the Nash policies adopted by the foreign government. First, as in the Benchmark model, the foreign government has an incentive to set a positive export tax on the intermediate input ($\hat{\tau}_x^{FN} > 0$), because the foreign input supplier can pass part of this cost on to home producers by threatening not to deliver the intermediate input. The key for this is that the outside option for the supplier is not reduced one to one with τ_x^F. In the present variant of the model, this is not only due to less-than-full bargaining power for suppliers but also to the fact that the secondary market does not involve trade flows.

Second, and contrary to the Benchmark Model, foreign taxes on the final good 1 can now affect the distribution of surplus between home and foreign producers. As a result, the foreign government now chooses to optimally balance the relative roles of $\hat{\tau}_x^{FN}$ and $\hat{\tau}_1^{FN}$ in extracting surplus from home firms in the same way that the home government balances $\hat{\tau}_x^{HN}$ and $\hat{\tau}_1^{HN}$ in extracting surplus from foreign firms. For the foreign government this implies the use of a foreign import tariff or export subsidy ($\hat{\tau}_1^{FN} > 0$) on the final good in order to raise p_1^F and thus improve the outside option (and bargaining position) of foreign suppliers.

Although we have shown that the location of the secondary market has implications for the Nash equilibrium values of home and foreign trade policies, it is important again to emphasize the two general features of our model that continue to hold in this extension as well. First, manipulating the above first-order conditions and applying the implicit function theorem to (4), we find

$$y'(\hat{x}) = 1 - \frac{\hat{x}}{\partial\hat{x}/\partial\tau_x^H} > 1,$$

which indicates that again, under Nash policy choices, the international hold-up problem persists and the volume of international input trade is inefficiently low as a consequence. Second, as we have indicated our model predicts the equilibrium use of taxes in the final good market and these distortions arise as a result of each country's attempts to extract bargaining surplus from firms abroad. Once again therefore, the purpose of a trade agreement remains to help governments better solve these two problems.

B Ex-Ante Lump-Sum Transfers

Our Benchmark Model rules out ex-ante lump-sum transfers between home producers and foreign suppliers. Although this seems a plausible assumption in our international framework where the promises associated with these transfers may be hard to enforce, it is useful to study the robustness of our results to this assumption. For that purpose, we consider the following modification of *stage* 1 of our Benchmark Model:

stage 1. The unit measure of producers in H and suppliers in F are randomly matched, producing a unit measure of matches. Each producer in H and its matched supplier in F bargain over whether to continue their relationship or not and lump-sum transfers are allowed in the bargaining. This *stage*-1 bargaining is captured by the generalized Nash bargaining solution with weights β and $(1-\beta)$ for the home producer and foreign supplier, respectively, where $\beta \in (0,1)$. If the relationship is terminated, both firms exit; if an agreement is reached, the producer retains the supplier and provides it with a list of customized input specifications.

For simplicity, we assume that the remaining stages of the game are as in the Benchmark Model (and, in particular, there is no ex-post secondary market). This implies that at *stage* 1, the home producer and the foreign supplier anticipate that if they reach an agreement, they stand to obtain a joint payoff of

$$\pi^H + \pi^F = \left(1 + \tau_1^H\right) y\left(\hat{x}\right) - \tau_x \hat{x} - \hat{x},$$

where $\hat{x}$ is still given by equation (5) in the paper. If instead, an agreement is not reached, both firms exit and are left with a payoff equal to 0. It is straightforward to show that $\pi^H + \pi^F > 0$, which implies that all pairs reach an agreement at *stage* 1. Note, however, that because of the lump-sum transfers, the division of profits between home producers and foreign suppliers is now detached from the ex-post bargaining solution.[1] In particular, we have:

$$\begin{aligned} \pi^H &= \beta\left[\left(1 + \tau_1^H\right) y\left(\hat{x}\right) - \tau_x \hat{x} - \hat{x}\right], \text{ and} \\ \pi^F &= \left(1-\beta\right)\left[\left(1 + \tau_1^H\right) y\left(\hat{x}\right) - \tau_x \hat{x} - \hat{x}\right]. \end{aligned}$$

The values of home and foreign welfare are still given by the same equations as in the Benchmark Model but with these new profit levels π^H and π^F applying.

We can next turn to study the Nash equilibrium policy choices of this variant of the model with lump-sum transfers. Manipulating the first-order conditions related to the choices of $\hat{\tau}_1^{HN}$, $\hat{\tau}_x^{HN}$, and $\hat{\tau}_x^{FN}$ delivers:

$$\begin{aligned} \hat{\tau}_1^{HN} &= -\frac{\left(1-\beta\right)\hat{x}\left[\frac{y(\hat{x})}{x} - y'\left(\hat{x}\right)\right]}{\left|\partial D_1/\partial p_1^H\right|}, \\ \hat{\tau}_x^{HN} &= -\left(1-\beta\right)\frac{\hat{x}}{\partial \hat{x}/\partial \tau_x^H} - \beta + \hat{\tau}_1^{HN} y'\left(\hat{x}\right), \text{ and} \\ \hat{\tau}_x^{FN} &= -\beta\frac{\hat{x}}{\partial \hat{x}/\partial \tau_x^F} - \left(1-\beta\right). \end{aligned}$$

[1] Still, the equilibrium level of $\hat{x}$ will be identical to that in the Benchmark Model, since foreign suppliers choose $\hat{x}$ to maximize ex-post payoffs (thus ignoring ex-ante payments).

Again the role of a trade agreement continues to be to correct both the inefficiently low input trade volume and the inefficiently low home-market final good price that arise under Nash policies, for any $\beta \in (0,1)$. To confirm this, one can manipulate the Nash first-order conditions to derive

$$y'(\hat{x}) = 1 - \frac{\hat{x}}{\partial \hat{x}/\partial \tau_x^F} > 1,$$

which implies that $\hat{x}^N < x^E$. And it is evident that $\hat{\tau}_1^{HN} < 0$ for $\beta < 1$, and so our model continues to predict as well that there are distortions in the final good market (p_1^H is too low) that arise as a result of the home-country's attempts to extract bargaining surplus from foreign suppliers.

C Details on Extension with Multiple Foreign Countries and Search Costs

We next consider multiple foreign countries and search costs, and formalize the associated claims made in the paper. Suppose that there are now two countries populated by potential suppliers: Foreign and South, denote by F and S, respectively. Assume that F contains a measure ρ of potential suppliers, while S contains a measure $1-\rho$. To bring search frictions into our analysis in a simple way, assume that if k home producers search for matches in F, the total measure of successful matches is given by the matching function $m(k,\rho) \leq \min\{k,\rho\}$, where $m(\cdot)$ is increasing in both arguments and features constant returns to scale. For simplicity, we assume that S adopts a laissez-faire policy. Will this force F to give up the use of an export tax, as was the case without search frictions described in the main text? As we now demonstrate, the answer is "No."

To show this, we begin by noting that, for home producers to be indifferent between searching in F and in S, we need:

$$\frac{m(k,\rho)}{k}\left(y\left(\hat{x}^F\right) - \tau_x^F \hat{x}^F\right) = \frac{m(1-k,1-\rho)}{1-k} y\left(\hat{x}^S\right), \tag{5}$$

where $\hat{x}^F$ is such that $y'\left(\hat{x}^F\right) = 1 + \tau_x^F$, while $\hat{x}^S$ is such that $y'\left(\hat{x}^S\right) = 1$. Equation (5) defines a negative relationship between k and τ_x^F: intuitively, an increase in the foreign export tax should be matched by an increase in the probability of finding a match in that country, which in turn requires a decrease in the measure of home producers searching for partners in that country. To see this formally, note that using the assumption of constant-returns-to scale in the matching function, we can write:

$$\frac{dk}{d\tau_x^F} = \frac{\left(y'\left(\hat{x}^F\right) - \tau_x^F\right)\left(-\partial \hat{x}^F/\partial \tau_x^F\right) + \hat{x}^F}{-\frac{\rho\mu'(\rho/k)}{k\mu(\rho/k)}\frac{1}{k}\left(y\left(\hat{x}^F\right) - \tau_x^F \hat{x}^F\right) - \frac{1}{(1-k)^2}\frac{(1-\rho)\mu'((1-\rho)/(1-k))}{\mu(\rho/k)} y\left(\hat{x}^S\right)} < 0, \tag{6}$$

where $\mu(\rho/k) \equiv m(1,\rho/k)$ and thus $\mu'(\rho/k) > 0$.

In order to explore the implications of this framework for the optimal choice of an export tax in F, we first define welfare in F as the sum of consumer surplus and tariff revenue collected from all the matched bilateral pairs:

$$W^F = CS(1) + m(k,\rho)\,\tau_x^F \hat{x}^F.$$

It thus follows that the optimal choice of τ_x^F (denoted $\hat{\tau}_x^F$) will now satisfy:

$$\frac{\partial W^F}{\partial \tau_x^F} = \frac{\partial m(k,\rho)}{\partial k} \frac{dk}{d\tau_x^F} \tau_x^F \hat{x}^F + m(k,\rho)\,\hat{x}^F + m(k,\rho)\,\tau_x^F \frac{\partial \hat{x}^F}{\partial \tau_x^F} = 0,$$

which in turn implies:

$$\hat{\tau}_x^F = \frac{\hat{x}^F}{-\frac{\partial \hat{x}^F}{\partial \tau_x^F} - \frac{\partial m(k,\rho)}{\partial k} \frac{1}{m(k,\rho)} \frac{dk}{d\tau_x^F} \hat{x}^F} > 0.$$

In sum, provided that $\frac{dk}{d\tau_x^F}$ remains bounded, the optimal export tax will be positive. It is straightforward to show that for well-behaved matching functions, the export tax will remain positive even for infinitesimally small countries. In particular, notice from equation (5) that whenever the elasticity of $m(\cdot)$ with respect to both of its arguments is positive, we will have that when $\rho \to 0$, and hence as F becomes infinitesimally small, $\frac{dk}{d\tau_x^F}$ goes to 0 as well, and thus

$$\hat{\tau}_x^F \to \frac{\hat{x}^F}{-\partial \hat{x}^F / \partial \tau_x^F},$$

which corresponds to the expression derived in the previous section (and evaluated at $\beta = 1$) when only F is the source of inputs.[2]

Arguing in this general fashion, it can be seen that the central findings of our Benchmark Model are robust to the introduction of multiple foreign countries where inputs may be sourced and to the associated matching frictions that would naturally arise in this setting.

D Details on Extension with Ad Valorem Tariffs

We next provide details on how our model is modified by the introduction of ad-valorem trade taxes. The main substantive result is that ad valorem tariffs introduce a novel channel through which bargaining between the home producer and foreign supplier can be affected. Despite this novel channel, however, we confirm that the role played by an international trade agreement remains the same.

To this end, with the "international" (foreign exporter) price p_x^* still denoting the price negotiated in *stage* 3 for the exchange of intermediate inputs between the foreign supplier and the home producer, we now let t_x^H and t_x^F denote, respectively, the home-country and foreign-country taxes on trade in the intermediate good x expressed in ad valorem terms. With this notation we highlight explicitly that the *stage*-3 negotiation between producer and supplier divides surplus between them by agreeing on the *price* at which the foreign supplier sells the x units of intermediate input to the home producer.

As will become clear, the novel aspects that arise when tariffs take an ad valorem rather than specific form apply only to input tariffs; nothing substantive changes if the final good tariffs are expressed in ad valorem terms. Therefore, to focus on the novel aspects of ad valorem tariffs, we now ignore tariffs on the final good and set $\tau_1^H = \tau_1^F \equiv 0$. With this assumption, according to the Benchmark Model there would be only one problem for a trade agreement to solve in the presence

[2] In particular, under the maintained assumptions, $\frac{\rho \mu'(\rho/k)}{k \mu(\rho/k)}$ is positive and bounded below 1, while $\frac{(1-\rho)\mu'((1-\rho)/(1-k))}{\mu(\rho/k)}$ goes to infinity when $\rho \to 0$.

of specific tariffs on trade in the intermediate input x, namely, the elimination of the international hold-up problem, and we now confirm that this remains the case when the tariffs take an ad valorem form.

Specifically, if the producer and supplier reach an agreement in their *stage*-3 bargaining that specifies a price level $\tilde{p}_x^*$, then the home-country producer receives a *stage*-3 payoff of $\omega^H = y(x) - (1+t_x^H)\tilde{p}_x^* x$ while the foreign supplier receives a *stage*-3 payoff of $\omega^F = \frac{\tilde{p}_x^*}{(1+t_x^F)}x$. Notice that this implies a bargaining frontier defined by $\omega^H = y(x) - (1+t_x^H)(1+t_x^F)\omega^F$: because the level of the exporter price p_x^* is used by the home producer and foreign supplier to shift surplus between them, a positive ad valorem import tariff or export tax makes the *slope* of the bargaining frontier between the home producer and the foreign supplier steeper, while a negative ad valorem tariff (an import or export subsidy) makes the slope of the bargaining frontier flatter. In effect, then, ad valorem trade taxes penalize the producer and supplier for shifting surplus toward the foreign supplier (with a high p_x^*), while ad valorem trade subsidies encourage surplus-shifting in this direction, suggesting a novel channel through which ad valorem trade taxes can affect the severity of the international hold-up problem. This channel is not present when a specific tariff is instead utilized, because the slope of the bargaining frontier between producer and supplier is -1 independent of the level of the specific tariffs τ_x^H and τ_x^F.

On the other hand, if the producer and supplier fail to reach an agreement in their *stage*-3 bargaining, they will be left with a payoff equal to 0.[3] The *stage*-3 Nash bargaining problem between the home producer and foreign supplier can then be characterized as follows:

$$\begin{aligned}&Max_{\omega^H,\omega^F} \quad \omega^H\omega^F \\ &s.t. \quad \omega^H = y(x) - (1+t_x^H)(1+t_x^F)\omega^F,\end{aligned}$$

where, for simplicity and without loss of generality, we again assume symmetric Nash bargaining. The solution to this bargaining problem yields $\omega^H = \frac{1}{2}y(x)$ and $\omega^F = \frac{1}{2}\frac{y(x)}{(1+t_x^H)(1+t_x^F)}$, and an implied foreign exporter price of $\hat{p}_x^* = \frac{\frac{1}{2}y(x)}{(1+t_x^H)x}$. The choice of x at *stage* 2 is then governed by

$$\frac{1}{2}y'(\hat{x}) = (1+t_x^H)(1+t_x^F), \tag{7}$$

and hence $\hat{x}$ continues to be decreasing in the (ad valorem) tariffs t_x^H and t_x^F, despite the novel channel through which the ad valorem tariffs affect the bargaining between home producer and foreign supplier. With this, we can now write the *stage*-1 payoffs of the home and foreign firm as

$$\begin{aligned}\pi^H &= \frac{1}{2}y(\hat{x}), \text{ and} \\ \pi^F &= \frac{1}{2}\frac{y(\hat{x})}{(1+t_x^H)(1+t_x^F)} - \hat{x}.\end{aligned}$$

We consider next the Nash tariff choices. With $t_1^H = t_1^F \equiv 0$, home and foreign welfare are now

[3] It would be straightforward to extend the analysis to include a secondary market that generates positive outside options for both types of producers.

given by

$$W^H = CS(1) + \pi^H + t_x^H \hat{p}_x^* \hat{x} = CS(1) + \frac{1}{2} y(\hat{x}) + \frac{1}{2} \frac{t_x^H y(\hat{x})}{(1+t_x^H)},$$
$$W^F = CS(1) + \pi^F + \frac{t_x^F}{1+t_x^F} \hat{p}_x^* \hat{x} = CS(1) + \frac{1}{2} \frac{y(\hat{x})}{(1+t_x^H)} - \hat{x}.$$

It is direct to show that the first-order condition for t_x^F implies $y'(\hat{x}) = 2(1+t_x^H)$. Hence, to check whether $\hat{t}_x^{HN}$ might achieve international efficiency in light of $\hat{t}_x^{FN}$, we may observe that, in combination with $\hat{t}_x^{FN}$, international efficiency would require $t_x^H = -\frac{1}{2}$. But differentiating W^H with respect to t_x^H yields

$$\frac{\partial W^H}{\partial t_x^H} = \frac{1}{2} y'(\hat{x}) \frac{(1+2t_x^H)}{(1+t_x^H)} \frac{\partial \hat{x}}{\partial t_x^H} + \frac{1}{2} \frac{y(\hat{x})}{(1+t_x^H)^2},$$

which is strictly positive when evaluated at the internationally efficient level of $t_x^H = -\frac{1}{2}$: by implication, then, $\hat{t}_x^{HN}$ is higher than the internationally efficient level.

Hence, while the mechanisms through which specific and ad valorem tariffs on traded inputs influence the international hold-up problem are distinct, the broad conclusions are the same. Combining this with our earlier observation that the form of the final-good tariff is immaterial, we may conclude that the central findings of our Benchmark Model are robust to the form (ad valorem or specific) that tariffs take, despite the different mechanisms that operate in the two environments.

E Details on Extension with Domestic Suppliers

We now outline some of the details related to the introduction into the analysis of domestic suppliers at Home.

E.1 Separable Case

We begin with a particularly simple example that illustrates the general claims made in the text with regard to domestic suppliers. More specifically, we assume that a fraction $1-\phi$ of the overall unit measure of suppliers is located at Home, while the remaining fraction ϕ continues to be located in Foreign. The matching is random and one-to-one as in our Benchmark Model, so a final-good producer matches with a supplier at Home or a supplier in Foreign, but not both. Furthermore, a contractual breach leaves both parties with a zero outside option (there are no secondary markets). Assume also that the contracts governing transactions between Home final-good producers and Home suppliers are as incomplete as those involving international matches. If contracts between Home agents were perfectly enforceable (or if agents found alternative mechanisms to satisfactorily enforce domestic transactions), then it is almost immediate that the structure of our Benchmark Model would remain intact. The presence of domestic suppliers now opens the role for the use of domestic input subsidies by the Home government (which were useless in our Benchmark Model).

It is clear that given the separability built into our example, the presence of domestic suppliers at Home has no impact on the negotiations between final-good producers at Home and their suppliers in Foreign. In particular, the level of investment $\hat{x}_F$ by Foreign suppliers is implicitly given by

$$(1-\alpha)\left(1+\tau_1^H\right)y'\left(\hat{x}_F\right)=1+(1-\alpha)\left(\tau_x^H+\tau_x^F\right),$$

and the payoffs to the Home final-good producer and the Foreign supplier are

$$\pi^H=\alpha\left(\left(1+\tau_1^H\right)y\left(\hat{x}_F\right)-\left(\tau_x^H+\tau_x^F\right)\hat{x}_F\right),$$

and

$$\pi^F=(1-\alpha)\left(\left(1+\tau_1^H\right)y\left(\hat{x}_F\right)-\left(\tau_x^H+\tau_x^F\right)\hat{x}_F\right)-\hat{x}_F,$$

respectively.

As for pairs involving a final-good producer and a Home (domestic) supplier, we have that the level of investment $\hat{x}_D$ by Home suppliers is implicitly given by

$$(1-\alpha)\left(1+\tau_1^H\right)y'\left(\hat{x}_D\right)=1-s_x^H,$$

where s_x^H denotes a domestic subsidy to the production of the input (alternatively, we could have modeled this policy as a subsidy to the *purchase* of inputs, with no effect on the qualitative results). Note that we are assuming that the bargaining parameters and the marginal cost of production (i.e., 1) are the same regardless of the location of input production. The joint payoff to the Home final-good producer and supplier is in turn given by

$$\hat{\pi}^H=\left(1+\tau_1^H\right)y\left(\hat{x}_D\right)-\left(1-s_x^H\right)\hat{x}_D.$$

Given these expressions, welfare at Home, inclusive of tax revenue, is given by

$$W^H=CS^H(1+\tau_1^H)+\phi\pi^H+(1-\phi)\hat{\pi}^H+\tau_1^H\left[D_1(1+\tau_1^H)-\phi y\left(\hat{x}_F\right)-(1-\phi)y\left(\hat{x}_D\right)\right]+\phi\tau_x^H\hat{x}_F-(1-\phi)s_x^H\hat{x}_D,$$

while welfare in F is equal to

$$W^F=CS^F(1)+\phi\pi^F+\phi\tau_x^F\hat{x}_F.$$

World welfare is hence

$$\begin{aligned}W^W &= W^H+W^F=CS^H(1+\tau_1^H)+CS^F(1)+\left(1+\tau_1^H\right)\left[\phi y\left(\hat{x}_F\right)+(1-\phi)y\left(\hat{x}_D\right)\right]-\phi\hat{x}_F-(1-\phi)\hat{x}_D\\ &\quad+\tau_1^H\left[D_1(1+\tau_1^H)-\phi y\left(\hat{x}_F\right)-(1-\phi)y\left(\hat{x}_D\right)\right],\end{aligned}$$

and depends on τ_1^H and $\tau_x=\tau_x^H+\tau_x^F$ (as in the Benchmark Model), but now also on s_x^H.

The first-order conditions associated with the constrained-efficient policies are given by:

$$\begin{aligned}\frac{\partial W^W}{\partial\tau_1^H} &= \tau_1^H\frac{\partial D_1}{\partial p_1^H}+\phi[y'\left(\hat{x}_F\right)-1]\frac{\partial\hat{x}_F}{\partial\tau_1^H}+(1-\phi)\left[y'\left(\hat{x}_D\right)-1\right]\frac{\partial\hat{x}_D}{\partial\tau_1^H}=0,\text{ and}\\ \frac{\partial W^W}{\partial\tau_x} &= \phi[y'\left(\hat{x}_F\right)-1]\frac{\partial\hat{x}_F}{\partial\tau_x}=0.\\ \frac{\partial W^W}{\partial s_x^H} &= (1-\phi)\left[y'\left(\hat{x}_D\right)-1\right]\frac{\partial\hat{x}_D}{\partial s_x^H}=0.\end{aligned}$$

As is clear from the expressions above, these policies fully resolve the domestic and foreign hold-up problems without distorting final-good trade. The implied policies are:

$$\begin{aligned} \tau_1^{HE} &= 0 \\ \tau_x^{E} &= -\alpha/(1-\alpha) \\ s_x^{HE} &= \alpha. \end{aligned}$$

Consider now the Nash policy choices. The first-order conditions associated with these Nash policy choices are:

$$\begin{aligned} \frac{\partial W^H}{\partial \tau_1^H} &= 0 = \tau_1^H \frac{\partial D_1}{\partial p_1^H} - \phi(1-\alpha) y(\hat{x}_F) + \frac{\partial W^H}{\partial \hat{x}_F}\frac{\partial \hat{x}_F}{\partial \tau_1^H} + (1-\phi)\left(y'(\hat{x}_D) - 1\right)\frac{\partial \hat{x}_D}{\partial \tau_1^H} = 0 \quad (8) \\ \frac{\partial W^H}{\partial \tau_x^H} &= \phi(1-\alpha)\hat{x}_F + \frac{\partial W^H}{\partial \hat{x}_F}\frac{\partial \hat{x}_F}{\partial \tau_x^H} = 0 \\ \frac{\partial W^H}{\partial s_x^H} &= \left[y'(\hat{x}_D) - 1\right]\frac{\partial \hat{x}_D}{\partial s_x^H} = 0 \\ \frac{\partial W^F}{\partial \tau_x^F} &= \phi\alpha\hat{x}_F + \phi\tau_x^F \frac{\partial \hat{x}_F}{\partial \tau_x^F} = 0 \end{aligned}$$

We can draw three broad conclusions from these first-order conditions. First, the third equation in (8) indicates that domestic subsidies are set at their first-best level, i.e., $y'(\hat{x}_D) = 1$. The reason for this is that Home government internalizes the full benefits from providing those subsidies. Second, the Foreign input export tax is set at exactly the same level (for $\hat{x} = \hat{x}_F$) as in the Benchmark Model, that is,

$$\tau_x^{FN} = -\frac{\alpha \hat{x}_F}{\partial \hat{x}_F / \partial \tau_x^F}.$$

Third, the Home final-good trade tax τ_1^{HN} continues to be negative and is given by

$$\tau_1^{HN} = -\frac{\phi(1-\alpha)\hat{x}_F\left(\frac{y(\hat{x}_F)}{\hat{x}_F} - y'(\hat{x}_F)\right)}{\left|\partial D_1/\partial p_1^H\right|} < 0,$$

which is analogous to the expression in the Benchmark Model except for the the shift parameter ϕ. Only when $\phi \to 0$, so that *all* input purchases are domestic, do the distortions in the policy mix identified by our model disappear.

We conclude from these results that introducing domestic inputs in a separable way has no effect on our results.

E.2 Non-Separable Case

The separable case developed above is particularly simple because it shuts down any interaction between domestic input policies and trade taxes (of any form). In order to capture these interactions, assume instead that production of the final good requires both domestic and foreign inputs and that technology is given by

$$y = y(x_D, x_F),$$

where $y(\cdot)$ is increasing in its arguments, is globally concave, and both inputs are essential for production. Assume that the marginal cost faced by suppliers worldwide is equal to 1. Final-good producers now transact with one supplier at Home and another supplier in Foreign. We assume that there is a measure one of suppliers in each country and random matching results in all producers being matched. Assume further that secondary markets do not exist, so outside options are zero in all negotiations (both inputs are essential). Suppose that bargaining between final-good producers and suppliers is bilateral in nature and Nash bargaining leaves each supplier with a share of the marginal return to their investments. Let that share be $1-\alpha$ for Foreign suppliers (as in the Benchmark Model) and $1-\chi$ for Home suppliers (α and χ could well be equal, and we naturally restrict $\alpha+\chi \leq 1$). The resulting investment levels x_D and x_F will now be implicitly given by the following two equations (we omit hats but it should be clear that x_D and x_F are now equilibrium values):

$$\begin{aligned}
(1-\chi)\left(1+\tau_1^H\right)\frac{\partial y(x_D,x_F)}{\partial x_D} &= 1-s_x^H \\
(1-\alpha)\left(1+\tau_1^H\right)\frac{\partial y(x_D,x_F)}{\partial x_F} &= 1+(1-\alpha)\left(\tau_x^H+\tau_x^F\right).
\end{aligned} \tag{9}$$

Note that provided that $y(x_D,x_F)$ features a non-zero cross-partial derivative, s_x^H will affect the Foreign suppliers' choice of x_F, while τ_x^H and τ_x^F will now affect the Home suppliers' choice of x_D. It is straightforward to confirm that the sign of these dependences is governed by the sign of the cross-partial derivative $\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}$ (see below).

The payoffs of Home final-good producers, Home suppliers and Foreign suppliers are, respectively,

$$\begin{aligned}
\pi_f^H &= (\chi+\alpha-1)\left(1+\tau_1^H\right)y(x_D,x_F)-\alpha\left(\tau_x^H+\tau_x^F\right)x_F, \\
\pi_s^H &= (1-\chi)\left(1+\tau_1^H\right)y(x_D,x_F)-\left(1-s_x^H\right)x_D \\
\pi_s^F &= (1-\alpha)\left(\left(1+\tau_1^H\right)y(x_D,x_F)-\left(\tau_x^H+\tau_x^F\right)x_F\right)-x_F.
\end{aligned}$$

Welfare in H, inclusive of tax revenue, is given by

$$W^H = CS^H(1+\tau_1^H)+\pi_f^H+\pi_s^H+\tau_1^H\left[D_1(1+\tau_1^H)-y(x_D,x_F)\right]+\tau_x^H x_F - s_x^H x_D,$$

while welfare in F is

$$W^F = CS^F(1)+\pi_s^F+\tau_x^F x_F.$$

World welfare is then

$$W^W = W^H+W^F = CS^H(1+\tau_1^H)+CS^F(1)+y(x_D,x_F)-x_D-x_F+\tau_1^H D_1(1+\tau_1^H).$$

In light of the above equations, world welfare depends on τ_1^H, $\tau_x=\tau_x^H+\tau_x^F$ and s_x^H.

The first-order conditions associated with the constrained-efficient policies are a bit more cumbersome than in the separable case and are given by:

$$\begin{aligned} \frac{\partial W^W}{\partial \tau_1^H} &= \tau_1^H \frac{\partial D_1}{\partial p_1^H} + \left[\frac{\partial y(x_D, x_F)}{\partial x_D} - 1\right] \frac{\partial x_D}{\partial \tau_1^H} + \left[\frac{\partial y(x_D, x_F)}{\partial x_F} - 1\right] \frac{\partial x_F}{\partial \tau_1^H} = 0, \quad (10) \\ \frac{\partial W^W}{\partial \tau_x} &= \left[\frac{\partial y(x_D, x_F)}{\partial x_F} - 1\right] \frac{\partial x_F}{\partial \tau_x} + \left[\frac{\partial y(x_D, x_F)}{\partial x_D} - 1\right] \frac{\partial x_D}{\partial \tau_x} = 0, \text{ and} \\ \frac{\partial W^W}{\partial s_x^H} &= \left[\frac{\partial y(x_D, x_F)}{\partial x_D} - 1\right] \frac{\partial x_D}{\partial s_x^H} + \left[\frac{\partial y(x_D, x_F)}{\partial x_F} - 1\right] \frac{\partial x_F}{\partial s_x^H} = 0. \end{aligned}$$

Noting that the two expressions in (9) imply

$$\begin{aligned} \frac{\partial x_D}{\partial \tau_x} &= \frac{-\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}}{\left(1+\tau_1^H\right)\left(\frac{\partial^2 y(x_D,x_F)}{\partial (x_D)^2}\frac{\partial^2 y(x_D,x_F)}{\partial (x_F)^2} - \frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\right)} \\ \frac{\partial x_D}{\partial s_x^H} &= \frac{-\frac{\partial^2 y(x_D,x_F)}{\partial (x_F)^2}}{\left(1+\tau_1^H\right)\left(\frac{\partial^2 y(x_D,x_F)}{\partial (x_D)^2}\frac{\partial^2 y(x_D,x_F)}{\partial (x_F)^2} - \frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\right)} \\ \frac{\partial x_F}{\partial \tau_x} &= \frac{\frac{\partial^2 y(x_D,x_F)}{\partial (x_D)^2}}{\left(1+\tau_1^H\right)\left(\frac{\partial^2 y(x_D,x_F)}{\partial (x_D)^2}\frac{\partial^2 y(x_D,x_F)}{\partial (x_F)^2} - \frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\right)} \\ \frac{\partial x_F}{\partial s_x^H} &= \frac{\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}}{\left(1+\tau_1^H\right)\left(\frac{\partial^2 y(x_D,x_F)}{\partial (x_D)^2}\frac{\partial^2 y(x_D,x_F)}{\partial (x_F)^2} - \frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\right)}, \end{aligned}$$

we can conclude that

$$\frac{\partial x_F}{\partial \tau_x}\frac{\partial x_D}{\partial s_x^H} - \frac{\partial x_D}{\partial \tau_x}\frac{\partial x_F}{\partial s_x^H} \neq 0,$$

which in turn implies that the second and third conditions in (10) necessarily imply that the constrained-efficient policies are such that

$$\frac{\partial y(x_D, x_F)}{\partial x_D} = \frac{\partial y(x_D, x_F)}{\partial x_F} = 1.$$

In sum, these policies solve the (two-sided) hold up problem and, in light of the first equation in (10), there is no intervention in final good markets. In particular, the constrained-efficient policies are analogous to those in the separable case and are given by:

$$\begin{aligned} \tau_1^{HE} &= 0 \\ \tau_x^E &= -\alpha/(1-\alpha) \\ s_x^{HE} &= \chi. \end{aligned}$$

The characterization of the Nash policy choices is a bit more complicated than in the separable case. The first-order conditions associated with the four policy choices (τ_1^H, τ_x^H, s_x^H, τ_x^F) are given by:

$$\begin{aligned}
\frac{\partial W^H}{\partial \tau_1^H} &= 0 = \tau_1^H \frac{\partial D_1}{\partial p_1^H} - (1-\alpha)\, y(x_D, x_F) + \frac{\partial W^H}{\partial x_D}\frac{\partial x_D}{\partial \tau_1^H} + \frac{\partial W^H}{\partial x_F}\frac{\partial x_F}{\partial \tau_1^H} = 0 \qquad (11)\\
\frac{\partial W^H}{\partial \tau_x^H} &= (1-\alpha)\, x_F + \frac{\partial W^H}{\partial x_D}\frac{\partial x_D}{\partial \tau_x^H} + \frac{\partial W^H}{\partial x_F}\frac{\partial x_F}{\partial \tau_x^H} = 0\\
\frac{\partial W^H}{\partial s_x^H} &= \frac{\partial W^H}{\partial x_D}\frac{\partial x_D}{\partial s_x^H} + \frac{\partial W^H}{\partial x_F}\frac{\partial x_F}{\partial s_x^H} = 0\\
\frac{\partial W^F}{\partial \tau_x^F} &= \alpha x_F + \tau_x^F \frac{\partial x_F}{\partial \tau_x^F} + \frac{\partial W^F}{\partial x_D}\frac{\partial x_D}{\partial \tau_x^F} = 0
\end{aligned}$$

After cumbersome algebra, we can express the term $\frac{\partial W^H}{\partial x_D}\frac{\partial x_D}{\partial \tau_1^H} + \frac{\partial W^H}{\partial x_F}\frac{\partial x_F}{\partial \tau_1^H}$ in the first equation in (11) as:

$$\frac{\partial W^H}{\partial x_D}\frac{\partial x_D}{\partial \tau_1^H} + \frac{\partial W^H}{\partial x_F}\frac{\partial x_F}{\partial \tau_1^H} = -(1-\alpha)\, x_F \left[-\frac{\partial y(x_D, x_F)}{\partial x_F} + \frac{\left(\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F} - \frac{\partial y(x_D,x_F)}{\partial x_D}\right)\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\frac{\partial^2 y(x_D,x_F)}{\partial (x_F)^2}}{\frac{\partial^2 y(x_D,x_F)}{\partial (x_D)^2}\frac{\partial^2 y(x_D,x_F)}{\partial (x_F)^2} - \left(\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\right)^2} \right],$$

which implies that

$$\tau_1^{HN} = -\frac{(1-\alpha)\left(y(x_D, x_F) - x_F \frac{\partial y(x_D,x_F)}{\partial x_F} - \frac{\left(\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F} - \frac{\partial y(x_D,x_F)}{\partial x_D}\right)\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\frac{\partial^2 y(x_D,x_F)}{\partial (x_F)^2}}{\frac{\partial^2 y(x_D,x_F)}{\partial (x_D)^2}\frac{\partial^2 y(x_D,x_F)}{\partial (x_F)^2} - \left(\frac{\partial^2 y(x_D,x_F)}{\partial x_D \partial x_F}\right)^2} \right)}{\left|\partial D_1/\partial p_1^H\right|}.$$

Notice that when we set the cross-partial derivative of $y(x_D, x_F)$ equal to 0, we have

$$\tau_1^{HN} = -\frac{(1-\alpha)\, x_F \left(\frac{y(x_D,x_F)}{x_F} - \frac{\partial y(x_D,x_F)}{\partial x_F}\right)}{\left|\partial D_1/\partial p_1^H\right|} < 0,$$

which is an expression analogous to that in the Benchmark Model. When the cross-partial is not 0, however, the sign of the final-good tariff is ambiguous. More importantly, τ_1^{HN} will not equal 0 except in knife-edge cases, and thus we can conclude that the Nash policy choices continue to be inefficient. In a similar manner, one can show that the remaining policy instruments, including the domestic input subsidy s_x^H, will not be set at their constrained-efficient level. The intuition for this is that, because of Nash bargaining and non-separabilities, the Home government will not internalize the full (marginal) effect of these subsidies. For instance, if x_D and x_F are highly complementary, the Home government will fail to take into account that the provision of a subsidy to domestic suppliers helps ameliorate the hold-up problem faced by Foreign suppliers.

It is interesting to note that it is not only the case that trade policy choices will continue to be inefficient in this case, but the design of trade agreements is actually *even more complicated* in this extension. The reason for this is that the Home country now has the ability to affect the division of surplus (i.e., the international price p_x^*) through three different policy instruments, so it becomes important that domestic input subsidies also be part of the trade agreement (this is a further illustration of the point described in note 29 of the paper).

F Details on Extension with Two-Sided Investments

Here we provide some details on the variant of the model with two-sided investments discussed in the paper. In particular, we consider the case in which transforming the supplier's intermediate input into a final good requires an additional relationship-specific investment (or input) on the part of the final-good producer, as in the property-rights model of Antràs (2003, 2005) and Antràs and Helpman (2004).

Fortunately, the analysis is essentially identical to a variant of the model with domestic suppliers described before, where final-good producers play the role of these domestic suppliers. Formally, we again have that

$$y = y\left(x_D, x_F\right),$$

but x_D is now a relationship-specific investment undertaken by the Home final-good producer. Suppose that Foreign suppliers have a bargaining weight equal to $1-\alpha$, there are no secondary markets, and Home subsidies to the provision of x_D are allowed. We then have that the Home final-good producer and the Foreign supplier obtain payoffs equal to

$$\begin{aligned} \pi_f^H &= \alpha\left(1+\tau_1^H\right) y\left(x_D, x_F\right) - \alpha\left(\tau_x^H + \tau_x^F\right) x_F - \left(1 - s_x^H\right) x_D, \text{ and} \\ \pi_s^F &= (1-\alpha)\left(\left(1+\tau_1^H\right) y\left(x_D, x_F\right) - \left(\tau_x^H + \tau_x^F\right) x_F\right) - x_F, \end{aligned}$$

and choose investment levels that are implicitly given by

$$\begin{aligned} \alpha\left(1+\tau_1^H\right) \frac{\partial y\left(x_D, x_F\right)}{\partial x_D} &= 1 - s_x^H, \text{ and} \\ (1-\alpha)\left(1+\tau_1^H\right) \frac{\partial y\left(x_D, x_F\right)}{\partial x_F} &= 1 + (1-\alpha)\left(\tau_x^H + \tau_x^F\right). \end{aligned}$$

It is immediate that these expressions are identical to those in the previous section whenever $\chi = 1-\alpha$. Because none of the conclusions in the previous section depended on the particular value of χ it is clear that (i) the constrained-efficient policies will again be identical to those in our Benchmark Model (except for the introduction of a subsidy to the provision of the final-good producer's input), and (ii) the Nash policy choices will depart from those in our Benchmark Model, but they will do so in an analogous manner to the case with domestic suppliers.

G Details on Three-Country Model Extension

We consider here the possibility that the foreign country is large in the world market for the final good 1, so that it is able to use its final good tariff to alter final good prices in the home-country market through its impact on the world price for final good 1. We establish in this extended setting that the political optimum continues to be inefficient in the presence of offshoring when political economy motives are present, as we emphasize in the two-small-country-setting of the paper. We accomplish this by considering a three-country model version of our Benchmark Model.

For simplicity (and wlog), let H be the sole producer of good 1 in the world. F and ROW each import good 1. And F is the sole producer of x in the world. Let τ_1^H be $H's$ export tax on good 1 (export subsidy if negative), τ_1^F be Fs import tariff on good 1 (import subsidy if negative), and τ_1^{ROW} be $ROW's$ import tariff on good 1 (import subsidy if negative). And as in the paper, τ_x^H and τ_x^F are the trade taxes on exports of x from F to H. With p_1^W denoting the world price of good 1, the following pricing relationships hold by arbitrage for non-prohibitive trade taxes:

$$p_1^H = p_1^W - \tau_1^H;\ \ p_1^F = p_1^W + \tau_1^F;\ \ p_1^{ROW} = p_1^W + \tau_1^{ROW}.$$

Market clearing in good 1 is given by

$$y(\hat{x}) - D_1^H(p_1^H) = D_1^F(p_1^F) + D_1^{ROW}(p_1^{ROW}).$$

Substitution yields

$$y(\hat{x}) - D_1^H(p_1^W - \tau_1^H) = D_1^F(p_1^W + \tau_1^F) + D_1^{ROW}(p_1^W + \tau_1^{ROW}),$$

implying the market-clearing world price $p_1^W(\hat{x}, \tau_1^H, \tau_1^F, \tau_1^{ROW})$.

Consider for the moment a world of free trade policies $\tau_1^H = \tau_1^F = \tau_1^{ROW} = \tau_x^H = \tau_x^F \equiv 0$. The efficient level of production of x is now defined implicitly (analogue of (3) in the paper) by

$$p_1^W(x^E, 0, 0, 0) \cdot y'(x^E) = 1,$$

while under free trade policies the equilibrium level of x is given by (analogue of (4) in the paper)

$$p_1^W(\hat{x}, 0, 0, 0) \cdot y'(\hat{x}) = \frac{1}{1-\alpha},$$

implying $\hat{x} < x^E$.

We next derive expressions for the local prices of good 1 in terms of the tariffs. To this end, notice that, if we define $\tau_1^{HF} \equiv [\tau_1^H + \tau_1^F]$ and $\tau_1^{HROW} \equiv [\tau_1^H + \tau_1^{ROW}]$, then using $p_1^H = p_1^F - \tau_1^{HF} = p_1^{ROW} - \tau_1^{HROW}$ we could also write the market clearing condition for good 1 as

$$y(\hat{x}) - D_1^H(p_1^H) = D_1^F(p_1^H + \tau_1^{HF}) + D_1^{ROW}(p_1^H + \tau_1^{HROW}),$$

which then implies $p_1^H(\hat{x}, \tau_1^{HF}, \tau_1^{HROW})$. Letting $\tau_x \equiv [\tau_x^H + \tau_x^F]$, we may then write the determination of $\hat{x}$ as (notice that each of the unit measure of suppliers ignores the impact of his supply on the price of the final good in this expression)

$$(1-\alpha)\, p_1^H(\hat{x}, \tau_1^{HF}, \tau_1^{HROW}) \cdot y'(\hat{x}) = 1 + (1-\alpha)\, \tau_x,$$

implying $\hat{x}(\tau_1^{HF}, \tau_1^{HROW}, \tau_x)$ and therefore

$$p_1^H(\hat{x}(\tau_1^{HF}, \tau_1^{HROW}, \tau_x), \tau_1^{HF}, \tau_1^{HROW}) \equiv p_1^H(\tau_1^{HF}, \tau_1^{HROW}, \tau_x).$$

Proceeding similarly for p_1^F and p_1^{ROW}, we have

$$p_1^F(\hat{x}(\tau_1^{HF}, \tau_1^{HROW}, \tau_x), \tau_1^{HF}, \tau_1^{HROW}) \equiv p_1^F(\tau_1^{HF}, \tau_1^{HROW}, \tau_x),$$

and

$$p_1^{ROW}(\hat{x}(\tau_1^{HF},\tau_1^{HROW},\tau_x),\tau_1^{HF},\tau_1^{HROW}) \equiv p_1^{ROW}(\tau_1^{HF},\tau_1^{HROW},\tau_x).$$

Finally, substituting the expression for $\hat{x}$ into the expression for $p_1^W(\hat{x},\tau_1^H,\tau_1^F,\tau_1^{ROW})$ yields

$$p_1^W(\hat{x}(\tau_1^{HF},\tau_1^{HROW},\tau_x),\tau_1^H,\tau_1^F,\tau_1^{ROW}) \equiv p_1^W(\tau_1^H,\tau_1^F,\tau_1^{ROW},\tau_x).$$

We now have an expression for the world price of good 1 and each of the local prices of good 1 in terms of the tariffs.

Note that with $p_1^F - p_1^H = \tau_1^{HF}$, $p_1^{ROW} - p_1^H = \tau_1^{HROW}$ and $p_x^H - p_x^F = \tau_x$, we may alternatively write $\hat{x}(\tau_1^{HF},\tau_1^{HROW},\tau_x) = \bar{x}(p_1^F - p_1^H, p_1^{ROW} - p_1^H, p_x^H - p_x^F)$. We will sometimes use this representation in what follows.

Next recall that

$$\pi^F = (1-\alpha)\,p_1^H(\tau_1^{HF},\tau_1^{HROW},\tau_x)\cdot y(\hat{x}) - [1 + (1-\alpha)\,\tau_x\hat{x}],$$

and that $p_x^* \equiv \frac{\pi^F}{\hat{x}} + [1+\tau_x^F]$, and so

$$\begin{aligned} p_x^* &\equiv (1-\alpha)\,p_1^H(\tau_1^{HF},\tau_1^{HROW},\tau_x)\cdot\frac{y(\hat{x}(\tau_1^{HF},\tau_1^{HROW},\tau_x))}{\hat{x}(\tau_1^{HF},\tau_1^{HROW},\tau_x)} - (1-\alpha)\,\tau_x^H + \alpha\tau_x^F \\ &\equiv p_x^*(\tau_1^{HF},\tau_1^{HROW},\tau_x^H,\tau_x^F). \end{aligned}$$

And substituting yields

$$p_x^H = p_x^*(\tau_1^{HF},\tau_1^{HROW},\tau_x^H,\tau_x^F) + \tau_x^H \equiv p_x^H(\tau_1^{HF},\tau_1^{HROW},\tau_x),$$

and

$$p_x^F = p_x^*(\tau_1^{HF},\tau_1^{HROW},\tau_x^H,\tau_x^F) - \tau_x^F \equiv p_x^F(\tau_1^{HF},\tau_1^{HROW},\tau_x).$$

We now have expressions for world and local prices of the input x in terms of the tariffs.

Summarizing, we have:

$$\begin{aligned} \hat{x} &= \hat{x}(\tau_1^{HF},\tau_1^{HROW},\tau_x) = \bar{x}(p_1^F - p_1^H, p_1^{ROW} - p_1^H, p_x^H - p_x^F); \\ p_1^W &\equiv p_1^W(\tau_1^H,\tau_1^F,\tau_1^{ROW},\tau_x) \text{ and } p_x^* = p_x^*(\tau_1^{HF},\tau_1^{HROW},\tau_x^H,\tau_x^F); \\ p_1^H &= p_1^H(\tau_1^{HF},\tau_1^{HROW},\tau_x),\;\; p_1^F = p_1^F(\tau_1^{HF},\tau_1^{HROW},\tau_x) \text{ and } p_1^{ROW} = p_1^{ROW}(\tau_1^{HF},\tau_1^{HROW},\tau_x); \\ p_x^H &= p_x^H(\tau_1^{HF},\tau_1^{HROW},\tau_x), \text{ and } p_x^F = p_x^F(\tau_1^{HF},\tau_1^{HROW},\tau_x). \end{aligned}$$

G.1 Welfare expressions in the 3-country model

We next write welfare for the three countries, individually and in total. We allow political economy motives in H and F, but abstract from them in ROW as this country only has demand for the non-numeraire good 1.

Using the pricing definitions above, and letting $\bar{x}(\cdot) = \bar{x}(p_1^F - p_1^H, p_1^{ROW} - p_1^H, p_x^H - p_x^F)$, Home welfare is given by

$$\begin{aligned} W^H &= CS(p_1^H) + \gamma^H [p_1^H \cdot y(\bar{x}(\cdot)) - p_x^H \cdot \bar{x}(\cdot)] \\ &\quad + [p_1^W - p_1^H][y(\bar{x}(\cdot)) - D_1^H(p_1^H)] + [p_x^H - p_x^*]\bar{x}(\cdot) \\ &\equiv W^H(p_1^F, p_1^H, p_1^{ROW}, p_x^H, p_x^F, p_x^*, p_1^W). \end{aligned}$$

Likewise, Foreign welfare is given by

$$\begin{aligned} W^F &= CS(p_1^F) + \gamma^F [p_x^F - 1]\bar{x}(\cdot) \\ &\quad + [p_1^F - p_1^W] D_1^F(p_1^F) + [p_x^* - p_x^F]\bar{x}(\cdot) \\ &\equiv W^F(p_1^F, p_1^H, p_1^{ROW}, p_x^H, p_x^F, p_x^*, p_1^W). \end{aligned}$$

Welfare in ROW is given by

$$\begin{aligned} W^{ROW} &= CS(p_1^{ROW}) + [p_1^{ROW} - p_1^W] D_1^{ROW}(p_1^{ROW}) \\ &\equiv W^{ROW}(p_1^{ROW}, p_1^W). \end{aligned}$$

Finally, joint (global) welfare is given by

$$W^G \equiv W^H + W^F + W^{ROW} = W^G(p_1^F, p_1^H, p_1^{ROW}, p_x^H, p_x^F).$$

Notice that W^G is a function only of local prices, and local prices are functions only of τ_1^{HF}, τ_1^{HROW} and τ_x, the sum total of the trade taxes on any channel of trade.

G.2 Efficiency of the Political Optimum in the 3-country model

We first characterize efficient policies, looking for trade taxes that maximize world welfare W^G. Recall that W^G is a function only of local prices, and local prices are functions only of τ_1^{HF}, τ_1^{HROW} and τ_x, the sum total of the trade taxes on any channel of trade, so efficiency ties down τ_1^{HFE}, τ_1^{HROWE} and τ_x^E. But using $\tau_1^{HF} \equiv [\tau_1^H + \tau_1^F]$ and $\tau_1^{HROW} \equiv [\tau_1^H + \tau_1^{ROW}]$ and the fact that $\frac{\partial \tau_1^{HF}}{\partial \tau_1^H} = 1 = \frac{\partial \tau_1^{HROW}}{\partial \tau_1^H}$, we can define:

$$\begin{aligned} \frac{dp_1^F}{d\tau_1^H} &\equiv \frac{\partial p_1^F(\tau_1^{HF}, \tau_1^{HROW}, \tau_x)}{\partial \tau_1^{HF}} + \frac{\partial p_1^F(\tau_1^{HF}, \tau_1^{HROW}, \tau_x)}{\partial \tau_1^{HROW}}, \\ \frac{dp_1^H}{d\tau_1^H} &\equiv \frac{\partial p_1^H(\tau_1^{HF}, \tau_1^{HROW}, \tau_x)}{\partial \tau_1^{HF}} + \frac{\partial p_1^H(\tau_1^{HF}, \tau_1^{HROW}, \tau_x)}{\partial \tau_1^{HROW}}, \\ \frac{dp_1^{ROW}}{d\tau_1^H} &\equiv \frac{\partial p_1^{ROW}(\tau_1^{HF}, \tau_1^{HROW}, \tau_x)}{\partial \tau_1^{HF}} + \frac{\partial p_1^{ROW}(\tau_1^{HF}, \tau_1^{HROW}, \tau_x)}{\partial \tau_1^{HROW}}, \\ \frac{dp_x^H}{d\tau_1^H} &\equiv \frac{\partial p_x^H(\tau_1^{HF}, \tau_1^{HROW}, \tau_x)}{\partial \tau_1^{HF}} + \frac{\partial p_x^H(\tau_1^{HF}, \tau_1^{HROW}, \tau_x)}{\partial \tau_1^{HROW}}, \text{ and} \\ \frac{dp_x^F}{d\tau_1^H} &\equiv \frac{\partial p_x^F(\tau_1^{HF}, \tau_1^{HROW}, \tau_x)}{\partial \tau_1^{HF}} + \frac{\partial p_x^F(\tau_1^{HF}, \tau_1^{HROW}, \tau_x)}{\partial \tau_1^{HROW}}. \end{aligned}$$

Then we may think of the social planner setting $\tau_1^{ROW} \equiv 0$ and choosing the three instruments τ_1^H, τ_1^F and τ_x to achieve efficiency (with $\tau_1^{ROW} \equiv 0$, the choice of $\tau_1^H$and τ_1^F will determine τ_1^{HF} and τ_1^{HROW}) according to the following FOC's:

$$\begin{aligned} W^G_{p_1^F}\frac{dp_1^F}{d\tau_1^H} + W^G_{p_1^H}\frac{dp_1^H}{d\tau_1^H} + W^G_{p_1^{ROW}}\frac{dp_1^{ROW}}{d\tau_1^H} + W^G_{p_x^H}\frac{dp_x^H}{d\tau_1^H} + W^G_{p_x^F}\frac{dp_x^F}{d\tau_1^H} &= 0, \\ W^G_{p_1^F}\frac{\partial p_1^F}{\partial\tau_1^{HF}} + W^G_{p_1^H}\frac{\partial p_1^H}{\partial\tau_1^{HF}} + W^G_{p_1^{ROW}}\frac{\partial p_1^{ROW}}{\partial\tau_1^{HF}} + W^G_{p_x^H}\frac{\partial p_x^H}{\partial\tau_1^{HF}} + W^G_{p_x^F}\frac{\partial p_x^F}{\partial\tau_1^{HF}} &= 0, \text{ and} \\ W^G_{p_1^F}\frac{\partial p_1^F}{\partial\tau_x} + W^G_{p_1^H}\frac{\partial p_1^H}{\partial\tau_x} + W^G_{p_1^{ROW}}\frac{\partial p_1^{ROW}}{\partial\tau_x} + W^G_{p_x^H}\frac{\partial p_x^H}{\partial\tau_x} + W^G_{p_x^F}\frac{\partial p_x^F}{\partial\tau_x} &= 0. \end{aligned} \tag{12}$$

To define the politically optimal policies, we suppose that each country chooses its policies unilaterally but the Home country acts "as if" $W^H_{p_x^*} \equiv 0 \equiv W^H_{p_1^W}$, the Foreign country acts as if $W^F_{p_x^*} \equiv 0 \equiv W^F_{p_1^W}$, and the ROW acts as if $W^{ROW}_{p_1^W} \equiv 0$. Then we also have the conditions for politically optimal policies:

$$\begin{aligned} W^H_{p_1^F}\frac{dp_1^F}{d\tau_1^H} + W^H_{p_1^H}\frac{dp_1^H}{d\tau_1^H} + W^H_{p_1^{ROW}}\frac{dp_1^{ROW}}{d\tau_1^H} + W^H_{p_x^H}\frac{dp_x^H}{d\tau_1^H} + W^H_{p_x^F}\frac{dp_x^F}{d\tau_1^H} &= 0, \\ W^H_{p_1^F}\frac{\partial p_1^F}{\partial\tau_x} + W^H_{p_1^H}\frac{\partial p_1^H}{\partial\tau_x} + W^H_{p_1^{ROW}}\frac{\partial p_1^{ROW}}{\partial\tau_x} + W^H_{p_x^H}\frac{\partial p_x^H}{\partial\tau_x} + W^H_{p_x^F}\frac{\partial p_x^F}{\partial\tau_x} &= 0, \\ W^F_{p_1^F}\frac{\partial p_1^F}{\partial\tau_1^{HF}} + W^F_{p_1^H}\frac{\partial p_1^H}{\partial\tau_1^{HF}} + W^F_{p_1^{ROW}}\frac{\partial p_1^{ROW}}{\partial\tau_1^{HF}} + W^F_{p_x^H}\frac{\partial p_x^H}{\partial\tau_1^{HF}} + W^F_{p_x^F}\frac{\partial p_x^F}{\partial\tau_1^{HF}} &= 0, \\ W^F_{p_1^F}\frac{\partial p_1^F}{\partial\tau_x} + W^F_{p_1^H}\frac{\partial p_1^H}{\partial\tau_x} + W^F_{p_1^{ROW}}\frac{\partial p_1^{ROW}}{\partial\tau_x} + W^F_{p_x^H}\frac{\partial p_x^H}{\partial\tau_x} + W^F_{p_x^F}\frac{\partial p_x^F}{\partial\tau_x} &= 0, \text{ and} \\ W^{ROW}_{p_1^{ROW}}\frac{\partial p_1^{ROW}}{\partial\tau_1^{HROW}} &= 0. \end{aligned} \tag{13}$$

Comparing the efficiency conditions in (12) with the conditions for the political optimum in (13), and using the definitions of W^G, W^H, W^F and W^{ROW}, it is then clear that the conditions for the political optimum satisfy the efficiency conditions if and only if, evaluated at politically optimal policies, we have:

$$\begin{aligned} W^F_{p_1^F}\frac{dp_1^F}{d\tau_1^H} + W^F_{p_1^H}\frac{dp_1^H}{d\tau_1^H} + W^F_{p_1^{ROW}}\frac{dp_1^{ROW}}{d\tau_1^H} + W^F_{p_x^H}\frac{dp_x^H}{d\tau_1^H} + W^F_{p_x^F}\frac{dp_x^F}{d\tau_1^H} &= 0, \text{ and} \\ W^H_{p_1^F}\frac{\partial p_1^F}{\partial\tau_1^{HF}} + W^H_{p_1^H}\frac{\partial p_1^H}{\partial\tau_1^{HF}} + W^H_{p_1^{ROW}}\frac{\partial p_1^{ROW}}{\partial\tau_1^{HF}} + W^H_{p_x^H}\frac{\partial p_x^H}{\partial\tau_1^{HF}} + W^H_{p_x^F}\frac{\partial p_x^F}{\partial\tau_1^{HF}} &= 0. \end{aligned} \tag{14}$$

It is direct to show that, if both H and F maximize national income, so that $\gamma^H = 1 = \gamma^F$, politically optimal policies satisfy

$$W^F_{p_1^F} = W^F_{p_1^H} = W^F_{p_1^{ROW}} = W^F_{p_x^H} = 0 = W^H_{p_1^F} = W^H_{p_1^H} = W^H_{p_1^{ROW}} = W^H_{p_x^F}, \tag{15}$$

which satisfy (14) and are hence efficient. This mirrors our finding in the paper that politically optimal policies are indeed efficient in the absence of (foreign) political motives.[4]

If political motives are present in H and/or F, then (15) does not hold at politically optimal policies. In this case, depending on which of the welfare price derivatives in (15) are non-zero at the political optimum, it is direct to show that some combination of the following equalities must hold (and all equalities must hold if all the welfare price derivatives are non-zero at the political optimum) if the politically optimal policies are nevertheless efficient (and therefore satisfy (14)):

$$\frac{\frac{dp_1^F}{d\tau_1^H}}{\frac{\partial p_1^F}{\partial \tau_1^{HF}}} = \frac{\frac{dp_1^H}{d\tau_1^H}}{\frac{\partial p_1^H}{\partial \tau_1^{HF}}} = \frac{\frac{dp_1^{ROW}}{d\tau_1^H}}{\frac{\partial p_1^{ROW}}{\partial \tau_1^{HF}}} = \frac{\frac{dp_x^H}{d\tau_1^H}}{\frac{\partial p_x^H}{\partial \tau_1^{HF}}} = \frac{\frac{dp_x^F}{d\tau_1^H}}{\frac{\partial p_x^F}{\partial \tau_1^{HF}}}. \tag{16}$$

If (16) were to hold, then the third condition for political optimality in (13) would imply that the first condition in (14) holds, while the first condition for political optimality in (13) would imply that the second condition in (14) holds; and therefore under (16) the efficiency of the political optimum would be assured also in this case. But (16) implies that τ_1^H and τ_1^F are perfect substitutes and thus have identical effects on all local prices, which (as in the two-small-country-setting of our paper) is not the case in our 3-country world.[5] Hence, in the extended 3-country setting (16) is not satisfied, and so we have established that the politically optimal policies are inefficient (efficient) in the presence of offshoring when political economy motives are present (absent), as we emphasize in the two-small-country-setting of the paper.

H A Competitive Benchmark Model

For comparison, we now develop the competitive analogue of our (political-economy augmented) model. We suppose that foreign inputs are competitively supplied according to the supply curve

$$x_S^F \equiv x_S^F(p_x^F).$$

In country H, the final good 1 is produced according to the concave production function $y(x)$, and the marginal cost of production of final good 1 is given by

$$mc_1^H = \frac{p_x^H}{y'(x)}.$$

[4]It is interesting that politically optimal tariffs are efficient according to this argument only if both $\gamma^F = 1$ *and* $\gamma^H = 1$, whereas in the two-small-country-setting of our paper efficiency of the political optimum only requires that foreign political motives be absent. The reason that home political economy motives must also be absent in our 3-country setting is that, in the 3-country world, the tariffs τ_1^H, τ_1^F and τ_1^{ROW} are not perfect substitutes for each other and they can all impact home prices, which in our two-small-country model is not the case: in our two-small-country model, there is no τ_1^{ROW} by assumption and τ_1^F cannot impact home prices because F is small in world markets, and so H has no possibility of gaining from coordinating F's trade policies to help it redistribute even if it has political economy motives. But in the 3-country model, H can now gain in this fashion if it has political economy motives, just as F can gain.

[5]In fact, there is one special case in which this statement would not hold, and that is if $D_1^{ROW} \equiv 0$ so that our 3-country model then collapses to a 2–large-country world. In such a 2-country environment, the policies τ_1^H and τ_1^F are perfect substitutes and thus have identical effects on all local prices, and in this special case the politically optimal policies would then be efficient in the presence of offshoring even when governments have political motivations.

Competitive supply of final good 1 in country H is then determined according to $p_1^H = mc_1^H$ or

$$p_1^H = \frac{p_x^H}{y'(x_D^H)},$$

which implicitly defines x_D^H, the derived demand for the input x, as

$$x_D^H = y'^{-1}\left(p_x^H/p_1^H\right) \equiv x_D^H(p_1^H, p_x^H).$$

The pricing relationships are (with p_x^* the international or world/untaxed price):

$$p_1^H = 1 + \tau_1^H \equiv p_1^H(\tau_1^H); \quad p_x^H = p_x^* + \tau_x^H \equiv p_x^H(\tau_x^H, p_x^*); \quad p_x^F = p_x^* - \tau_x^F \equiv \; p_x^F(\tau_x^F, p_x^*).$$

The market-clearing condition in the world (home and foreign) x market is then given by $x_D^H = x_S^F$, or

$$x_D^H(p_1^H(\tau_1^H), p_x^H(\tau_x^H, p_x^*)) = x_S^F(p_x^F(\tau_x^F, p_x^*)), \tag{17}$$

which determines $p_x^*(\tau_1^H, \tau_x^H, \tau_x^F)$. Market-clearing input trade volume may then be written as $\hat{x}(p_1^H, p_x^H) \equiv x_D^H(p_1^H(\tau_1^H), p_x^H(\tau_x^H, p_x^*(\tau_1^H, \tau_x^H, \tau_x^F)))$ or equivalently $\hat{x}(p_x^F) \equiv x_S^F(p_x^F(\tau_x^F, p_x^*(\tau_1^H, \tau_x^H, \tau_x^F)))$. We also have $y(p_1^H, p_x^H) \equiv y(\hat{x}(p_1^H, p_x^H))$. Notice that (17) can be differentiated to yield

$$\frac{\partial p_x^*}{\partial \tau_x^H} = \frac{-\frac{\partial x_D^H(p_1^H, p_x^H)}{\partial p_x^H}}{\frac{\partial x_D^H(p_1^H, p_x^H)}{\partial p_x^H} - \frac{\partial x_S^F(p_x^F)}{\partial p_x^F}} < 0; \quad \frac{\partial p_x^*}{\partial \tau_x^F} = \frac{-\frac{\partial x_S^F(p_x^F)}{\partial p_x^F}}{\frac{\partial x_D^H(p_1^H, p_x^H)}{\partial p_x^H} - \frac{\partial x_S^F(p_x^F)}{\partial p_x^F}} > 0,$$

and so we have that

$$1 = \frac{\partial p_x^*}{\partial \tau_x^F} - \frac{\partial p_x^*}{\partial \tau_x^H}. \tag{18}$$

The home welfare function may now be written as:

$$W^H = CS(p_1^H) + \gamma^H \int_0^{p_1^H} y(p, p_x^H)dp + (p_1^H - 1)[D_1^H(p_1^H) - y(p_1^H, p_x^H)] + (p_x^H - p_x^*)\hat{x}(p_1^H, p_x^H),$$

or

$$W^H \equiv W^H(p_1^H, p_x^H, p_x^*).$$

Similarly, the foreign welfare function may now be written as:

$$W^F = CS(1) + \gamma^F \int_0^{p_x^F} x_S^F(p)dp + (p_x^* - p_x^F)\hat{x}(p_x^F),$$

or

$$W^F \equiv W^F(p_x^F, p_x^*).$$

Using the fact that $W^F_{p^*_x} = -W^H_{p^*_x} = \hat{x}$, the efficiency frontier is defined by the three conditions:

$$\begin{aligned} W^H_{p^H_x} + [W^H_{p^H_x} + W^F_{p^F_x}]\frac{\partial p^*_x}{\partial \tau^H_x} &= 0, \\ -W^F_{p^F_x} + [W^F_{p^F_x} + W^H_{p^H_x}]\frac{\partial p^*_x}{\partial \tau^F_x} &= 0, \text{ and} \\ W^H_{p^H_1} + [W^H_{p^H_x} + W^F_{p^F_x}]\frac{\partial p^*_x}{\partial \tau^H_1} &= 0. \end{aligned}$$

Using (18), it is easy to show that the first two first-order conditions are identical, and therefore determine the sum of τ^H_x and τ^F_x that is consistent with international efficiency.

To further interpret the conditions for efficiency, we multiply the first efficiency condition by $-[\frac{\partial p^*_x/\partial \tau^H_1}{\partial p^*_x/\partial \tau^H_x}]$ and add it to the third efficiency condition, so that we may then restate the two conditions for international efficiency as

$$\begin{aligned} W^H_{p^H_x} + [W^H_{p^H_x} + W^F_{p^F_x}]\frac{\partial p^*_x}{\partial \tau^H_x} &= 0, \text{ and} \\ W^H_{p^H_1} - W^H_{p^H_x} \cdot \frac{\partial p^*_x/\partial \tau^H_1}{\partial p^*_x/\partial \tau^H_x} &= 0. \end{aligned} \tag{19}$$

The interpretation of (19) is as follows. Let us begin with the second efficiency condition. On the left-hand side is the impact on home welfare of (infinitessimal) changes in the *mix* of τ^H_1 and τ^H_x which hold fixed p^*_x – and hence, by (17) and with τ^F_x and thus $p^F_x(\tau^F_x, p^*_x)$ unchanged, hold fixed as well the level of x^H_D and therefore the equilibrium level of input trade volume $\hat{x}$. Notice, though, that foreign welfare $W^F(p^F_x(\tau^F_x, p^*_x), p^*_x)$ is unaffected by such changes, because p^*_x is held fixed and τ^F_x is not changed and so, as already mentioned, $p^F_x(\tau^F_x, p^*_x)$ is held fixed as well. Hence, the second efficiency condition in (19) says simply that, at internationally efficient choices of τ^H_1 and τ^H_x, such changes can have no first-order effect on home welfare either. The first efficiency condition in (19) then ensures that the sum of τ^H_x and τ^F_x achieves the efficient level of p^F_x, and hence the efficient level of input trade volume in light of the mix of τ^H_1 and τ^H_x that the home country employs to deliver the chosen level of p^*_x and (with τ^F_x fixed) p^F_x.

Next consider the Nash policies. The associated first-order conditions are

$$\begin{aligned} W^H_{p^H_x} + [W^H_{p^H_x} + W^H_{p^*_x}]\frac{\partial p^*_x}{\partial \tau^H_x} &= 0, \\ -W^F_{p^F_x} + [W^F_{p^F_x} + W^F_{p^*_x}]\frac{\partial p^*_x}{\partial \tau^F_x} &= 0, \text{ and} \\ W^H_{p^H_1} + [W^H_{p^H_x} + W^H_{p^*_x}]\frac{\partial p^*_x}{\partial \tau^H_1} &= 0. \end{aligned} \tag{20}$$

Using (18) and $W^F_{p^*_x} = -W^H_{p^*_x}$, the first two Nash first-order conditions can be added together to yield:

$$W^H_{p^H_x} + [W^H_{p^H_x} + W^F_{p^F_x}]\frac{\partial p^*_x}{\partial \tau^H_x} + W^F_{p^*_x} = 0. \tag{21}$$

Comparing (21) to the first efficiency condition in (19), the difference is the additional term $W^F_{p^*_x} > 0$ on the left-hand side of (21), which implies that the sum $\tau^H_x + \tau^F_x$ is *inefficiently high* (the first-order condition for efficiency is negative at the Nash taxes), and therefore that the Nash level of input trade volume is inefficiently low in light of the mix of τ_1^H and τ_x^H that the home country employs in the Nash equilibrium to deliver the chosen level of p_x^* and (with τ_x^F fixed) p_x^F.

Next we multiply the initial first-order condition in (20) by $-[\frac{\partial p_x^*/\partial \tau_1^H}{\partial p_x^*/\partial \tau_x^H}]$ and add it to the last first-order condition to get

$$W_{p_1^H}^H - W_{p_x^H}^H \cdot \frac{\partial p_x^*/\partial \tau_1^H}{\partial p_x^*/\partial \tau_x^H} = 0. \tag{22}$$

Comparing (22) to the second efficiency condition in (19), we may conclude that the *mix* of τ_1^H and τ_x^H that the home country employs in the Nash equilibrium to deliver its chosen level of p_x^* and hence p_x^F – and therefore by (17), x_D^H and hence $\hat{x}$ – is internationally *efficient*.

Therefore, we may conclude that the single inefficiency in the Nash equilibrium in our competitive benchmark model is that the sum $\tau_x^H + \tau_x^F$ is *inefficiently high*, and hence that there is too little equilibrium input trade volume/input "market access": in the competitive benchmark model, the task of a trade agreement is thus to expand and secure market access to internationally efficient levels (see Bagwell and Staiger, 2001, 2002, for an interpretation of analogous findings from a market access perspective).

Next consider the political optimum conditions. Specifically, following Bagwell and Staiger (1999) we consider the hypothetical situation that governments are *not motivated* by the impact of their tariff choices on p_x^*, in the specific sense that $W_{p_x^*}^H \equiv 0$ and $W_{p_x^*}^F \equiv 0$. We then identify the tariffs that would be chosen unilaterally (i.e., non-cooperatively) by governments with these hypothetical preferences and ask whether these tariffs are efficient with respect to the actual government preferences. It is direct to show using (20) that in our competitive benchmark model the following conditions define the political optimum:

$$W_{p_x^H}^H = 0, \; W_{p_x^F}^F = 0, \;\text{ and }\; W_{p_1^H}^H = 0. \tag{23}$$

Clearly, as an examination of (19) indicates, the political optimum defined in (23) is efficient in this setting, whether or not governments are motivated by political economy concerns, so we now have shown that the standard terms-of-trade theory applies in a competitive-supplier version of our set-up.

References

Antràs, Pol. (2003), "Firms, Contracts, and Trade Structure," *Quarterly Journal of Economics*, 118:4, pp. 1375-1418.

Antràs, Pol (2005), "Incomplete Contracts and the Product Cycle," *American Economic Review*, 95:4, pp. 1054-1073.

Antràs, Pol, and Elhanan Helpman (2004), "Global Sourcing," *Journal of Political Economy* 112:3, pp. 552-580.

Bagwell, Kyle and Robert W. Staiger (1999), "An Economic Theory of GATT," *American Economic Review*, Vol. 89, No. 1, pp. 215-248.

Bagwell, Kyle and Robert W. Staiger (2001), "Domestic Policies, National Sovereignty and International Economic Institutions," *Quarterly Journal of Economics*, Vol. 116, No. 2, pp. 519-562.

Bagwell, Kyle and Robert W. Staiger (2002), *The Economics of the World Trading System*, MIT Press.