Globalization and Pandemics

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Online Appendix (Not for Publication)

C Extensions of Economic Model

In this Appendix, we flesh out some of the details of the four extensions of our framework mentioned in Section 2.3 of the main text.

C.1 Traveling Costs in Terms of Labor

If traveling costs are specified in terms of labor (rather than utility), welfare at the household level depends only on consumption

$$W_i = \left(\sum_{j \in \mathcal{J}} \int_0^{n_{ij}} q_{ij}(k)^{\frac{\sigma-1}{\sigma}} dk \right)^{\frac{\sigma}{\sigma-1}},$$

and the implied demand (for a given n_{ii} and n_{ij}) is given by

$$q_{ij}(k) = \left(\frac{p_{ij}}{P_i} \right)^{-\sigma} \frac{\mathcal{I}_i}{P_i},$$

where $\mathcal{I}_i$ is household income, which is given by

$$\mathcal{I}_i = w_i \left(1 - \frac{c}{\phi} \sum_{j \in \mathcal{J}} \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi} \right),$$

since the household now needs to hire labor to be able to secure final-good differentiated varieties, and where

$$P_i = \left(\sum_{j \in \mathcal{J}} n_{ij} p_{ij}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

Welfare can therefore be rewritten as

$$W_i = \frac{\mathcal{I}_i}{P_i} = w_i \left(1 - \frac{c}{\phi} \sum_{j \in \mathcal{J}} \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi} \right) \left(\sum_{j \in \mathcal{J}} n_{ij} p_{ij}^{1-\sigma} \right)^{\frac{1}{\sigma-1}}$$

The first-order condition for the choice of n_{ij} delivers:

$$n_{ij} = (c(\sigma-1))^{-\frac{1}{\phi-1}} \left(\frac{\mathcal{I}_i}{w_i} \right)^{\frac{1}{\phi-1}} \left(\frac{t_{ij} w_j}{Z_j P_i} \right)^{-\frac{\sigma-1}{\phi-1}} \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\delta(\sigma-1)}{\phi-1}}$$

Bilateral import flows by country i from country j are given by

$$X_{ij} = n_{ij} p_{ij} q_{ij} L_i = (c(\sigma-1))^{-\frac{1}{\phi-1}} \left(\frac{\mathcal{I}_i}{w_i} \right)^{\frac{1}{\phi-1}} \left(\frac{t_{ij} w_j}{Z_j P_i} \right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} \mathcal{I}_i L_i,$$

and the trade share can be written as

$$\pi_{ij} = \frac{X_{ij}}{\sum_{l\in\mathcal{J}} X_{il}} = \frac{\left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} t_{ij}^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{l\in\mathcal{J}} \left(\frac{w_l}{Z_l}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times \mu_{il}^{-\frac{1}{\phi-1}} d_{il}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} t_{il}^{-\frac{\phi(\sigma-1)}{\phi-1}}} = \frac{S_j}{\Phi_i} \times \Gamma_{ij}^{-\varepsilon},$$

where

$$\Gamma_{ij}^{-\varepsilon} = \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} t_i^{-\frac{\phi(\sigma-1)}{\phi-1}},$$

which is identical to equation (9) applying to our baseline model with traveling costs in terms of labor.

The price index is in turn given by

$$P_i = (c(\sigma-1))^{\frac{1}{\phi(\sigma-1)}} \left(\frac{\mathcal{I}_i}{w_i}\right)^{-\frac{1}{\phi(\sigma-1)}} \left(\sum_{j\in\mathcal{J}} \left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

and using this expression together for the one for π_{ij}, one can verify that we can write

$$n_{ij} = \left(\frac{t_{ij} d_{ij}^{\delta} w_j}{Z_j P_i}\right)^{\sigma-1} \pi_{ij},$$

just as in equation (15) of the main text.

Plugging this expression back into the budget constraint yields

$$\mathcal{I}_i = \frac{\phi(\sigma-1)}{\phi(\sigma-1)+1} w_i,$$

and a resulting price index equal to

$$P_i = \left(\frac{c\phi}{\phi(\sigma-1)+1}\right)^{\frac{1}{\phi(\sigma-1)}} \left(\sum_{j\in\mathcal{J}} \left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

which is only slightly different than expression (11) in the main text,

The labor-market conditions are given by

$$\pi_{ii}\mathcal{I}_i L_i + \pi_{ji}\mathcal{I}_j L_j = \mathcal{I}_i L_i$$

or, equivalently,

$$\pi_{ii} w_i L_i + \pi_{ji} w_j L_j = w_i L_i,$$

just as in the main text, and remember that the expressions for π_{ii} and π_{ji} are also left unchanged.

We next turn to verifying that Propositions 1 through 4 in the main text continue to hold whenever travel costs in equation (1) are specified in terms of labor rather than being modelled as a utility cost.

Proposition 1': *As long as trade frictions (Γ_{ij}) are bounded, there exists a unique vector of equilibrium wages $w^* = (w_i, w_j) \in R^2_{++}$ that solves the system of equations above.*

Proof. By results in standard gravity models in Alvarez and Lucas (2007), Allen and Arkolakis (2014), and Allen et al. (2020). ■

Proposition 2': *A decline in any international trade or mobility friction ($d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji}$) leads to: (a) a decline in the rates (n_{ii} and n_{jj}) at which individuals will meet individuals in their own country; and (b) an increase in the rates at which individuals will meet individuals from the other country (n_{ij} and n_{ji}).*

Proof. **(a)** Given that $\mathcal{I}_i = \frac{\phi(\sigma-1)}{\phi(\sigma-1)+1} w_i$,

$$n_{ii} = (c(\sigma-1))^{-\frac{1}{\phi-1}} \left(\frac{\mathcal{I}_i}{w_i}\right)^{\frac{1}{\phi-1}} \left(\frac{t_{ii} w_i}{Z_i P_i}\right)^{-\frac{\sigma-1}{\phi-1}} \mu_{ii}^{-\frac{1}{\phi-1}} d_{ii}^{-\frac{\rho+\delta(\sigma-1)}{\phi-1}} = const \times \left(\frac{P_i}{w_i}\right)^{\frac{\sigma-1}{\phi-1}}$$

Then

$$\frac{P_i}{w_i} = \left(\frac{c\phi}{\phi(\sigma-1)+1}\right)^{\frac{1}{\phi(\sigma-1)}} \left(Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

where $\omega = w_j / w_i$ is the relative wage in country j .

Note that the labor constraint can be rewritten as

$$\frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}} L_i + \frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{jj}^{-\varepsilon}} \omega L_j = L_i$$

Consider a case when Γ_{ij} decreases, while other Γ_{kl} remain constant. That means that the first term in the sum goes down, while the second term is constant. For the equality to hold, ω should increase. After re-equilibration, the second term in the sum increased, which means that the first term decreased. This means that P_i/w_i decreased, and n_{ii} as well.

Consider now a case when Γ_{ji} decreases, while other Γ_{kl} remain constant. The second term increases, so ω needs to go down to equilibrate the model. That means that the first term decreases, and P_i/w_i and n_{ii} decrease by extension.

Therefore, whenever one decreases any international friction ($d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji}$), Γ_{ij} or Γ_{ji} goes down, and, hence, n_{ii} and n_{jj} go down.

(b) Note that

$$\frac{\mathcal{I}_i}{w_i} = 1 - \frac{c}{\phi} \sum_{j \in \mathcal{J}} \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi}$$

Since $\mathcal{I}_i = \frac{\phi(\sigma-1)}{\phi(\sigma-1)+1)} w_i$, the left-hand side is constant. Since n_{ii} and n_{jj} decrease, n_{ij} and n_{ji} must increase. ■

Proposition 3': *Suppose that countries are symmetric, in the sense that $L_i = L$, $Z_i = Z$, and $\Gamma_{ij} = \Gamma$ for all i. Then a decline in any (symmetric) international trade frictions leads to an overall increase in human interactions ($n_{dom} + n_{for}$) experienced by both household buyers and household sellers.*

Proof. We begin by considering the case with general country asymmetries. Consider the sum

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi} + \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi} = \frac{1}{\phi(\sigma - 1) + 1}$$

Differentiating yields

$$\phi \mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi - 1} dn_{ii} + \phi \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi - 1} dn_{ij} + \phi n_{ij}^{\phi} \underbrace{d\left(\mu_{ij} d_{ij}^{\rho}\right)}_{\leq 0} = 0$$

Hence,

$$\phi \mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi - 1} dn_{ii} + \phi \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi - 1} dn_{ij} \geq 0,$$

and if $\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} > \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi-1}$, then $dn_{ij} > -dn_{ii}$.

From the FOC for the choice of n_{ii} and n_{ij},

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi - 1} = \frac{1}{c(\sigma - 1)} \frac{\mathcal{I}_i}{w_i} \left(\frac{p_{ii}}{P_i}\right)^{1-\sigma}$$

$$\mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi - 1} = \frac{1}{c(\sigma - 1)} \frac{\mathcal{I}_i}{w_i} \left(\frac{p_{ij}}{P_i}\right)^{1-\sigma}$$

Therefore, $\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} > \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi-1}$ is satisfied if and only if $p_{ii} < p_{ij}$.

When countries are symmetric, this holds trivially because of international trade costs $t_{ij} > t_{ii}$ and $d_{ij} > d_{ii}$. Hence, $dn_{ij} > -dn_{ii}$, and $n_{dom} + n_{for}$ increases. ■

Proposition 4': *An increase in the relative size of country i's population leads to an increase in the rate n_{ii} at which individuals from i will meet individuals in their own country, and to a decrease in the rate n_{ij} at which individuals will meet individuals abroad.*

Proof. Consider again

$$\frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}} L_i + \frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{jj}^{-\varepsilon}} \omega L_j = L_i$$

An increase in L_i makes the left-hand side smaller then the right-hand side. Therefore, ω grows to re-equilibrate. Then

$$\frac{P_i}{w_i} = \left(\frac{c\phi}{\phi(\sigma-1)+1}\right)^{\frac{1}{\phi(\sigma-1)}} \left(Z_i^{\frac{\phi(\sigma-1)}{\phi-1}}\Gamma_{ii}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}}\Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

increases, and $n_{ii} = const \times \left(\frac{P_i}{w_i}\right)^{\frac{\sigma-1}{\phi-1}}$ increases with it.

Since

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi} + \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi} = \frac{1}{\phi(\sigma-1)+1},$$

n_{ij} decreases.

Therefore, following a growth in population L_i, n_{ii} increases while n_{ij} decreases. ■

C.2 International Sourcing of Inputs

The assumption that households travel internationally to procure final goods may seem unrealistic. Perhaps international travel is better thought as being a valuable input when firms need specialized inputs and seek potential providers of those inputs in various countries. It is straightforward to re-interpret our model along those lines. In particular, suppose now that all households in country i produce a homogeneous final good but also produce differentiated intermediate input varieties. The household's final good is produced combining a bundle of the intermediate inputs produced by other households. Technology for producing the final good is given by

$$Q_i = \left(\sum\nolimits_{j\in\mathcal{J}} \int_0^{n_{ij}^I} q_{ij}^I(k)^{\frac{\sigma-1}{\sigma}}\, dk\right)^{\frac{\sigma}{\sigma-1}}$$

and this final good is not traded (this is without loss of generality if households are homogeneous in each country and trade costs for final goods are large enough). Household welfare is linear in consumption of the final good and is reduced by the disutility cost of a household's member having to travel to secure intermediate inputs. In particular, we have

$$W_i = \left(\sum\nolimits_{j\in\mathcal{J}} \int_0^{n_{ij}^I} q_{ij}^I(k)^{\frac{\sigma-1}{\sigma}}\, dk\right)^{\frac{\sigma}{\sigma-1}} - \frac{c}{\phi}\sum\nolimits_{j\in\mathcal{J}} \mu_{ij}\left(d_{ij}\right)^{\rho} \times \left(n_{ij}^I\right)^{\phi}.$$

Under this model is isomorphic to the one above, except that trade will be in intermediate inputs rather than in final goods.

C.3 Multi-Country Model

We next consider a version of our model with a world economy featuring multiple countries. It should be clear that all our equilibrium conditions, except for the labor-market clearing condition (14) apply to that multi-country environment once the set of countries $\mathcal{J}$ is redefined to include multiple countries. The labor-market condition is in turn simply given by

$$\sum\nolimits_{j\in\mathcal{J}} \pi_{ij}\left(\mathbf{w}\right) w_j L_j = w_i L_i,$$

where $\pi_{ij}\left(\mathbf{w}\right)$ is defined in (9) for an arbitrary set of countries $\mathcal{J}$. Similarly, the model is also easily adaptable to the case in which there is a continuum of locations $i \in \Omega$, where Ω is a closed and bounded set of a finite dimensional Euclidean space. The equilibrium conditions are again unaltered, with integrals replacing summation operators throughout.

From the results in Alvarez and Lucas (2007), Allen and Arkolakis (2014), and Allen et al. (2020), it is clear that Proposition 1 in the main text on existence and uniqueness will continue to hold. In the presence of arbitrary asymmetries across countries, it is hard however to derive crisp comparative static results of the type in Propositions 2 and 4. Nevertheless, our result in Proposition 3 regarding the positive effect of declines of trade and mobility barriers on the overall level of human interactions between symmetric countries is easily generalizable to the case of many countries (details available upon request - future versions of the paper will include an Online Appendix with the details).

C.4 Traveling Salesman Model

Finally, we explore a variant of our model in which it is the household's seller rather than the buyer who travels to other locations. We model this via a framework featuring scale economies, monopolistic competition and fixed cost of exporting, as in the literature on selection into exporting emanating from the seminal work of Melitz (2003), except that the fixed costs of selling are defined at the buyer level rather than at the country level, as in the work of Arkolakis (2010).

On the consumption side, households maximize their utility, given by

$$W_i = \left(\sum_{j\in\mathcal{J}} \int_0^{\eta_{ij}} q_{ij}(k)^{\frac{\sigma-1}{\sigma}} dk\right)^{\frac{\sigma}{\sigma-1}},$$

where η_{ij} is the measure of varieties available to them, subject to the household budget constraint. This yields

$$q_{ij}(k) = \left(\frac{p_{ij}}{P_i}\right)^{-\sigma} \frac{\mathcal{I}_i}{P_i},$$

where $\mathcal{I}_i$ is household income and the price index is

$$P_i = \left(\sum_{j\in\mathcal{J}} \eta_{ij} p_{ij}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

Household sellers in country j produce N_j varieties and make them available to n_{ij} consumers. Both N_j and n_{ij} are endogenous and pinned down as part of the equilibrium. Note that because there are L_i and L_j households in i and j, respectively, the measure of varieties available from j to consumers in i is given by $\eta_{ij} = n_{ij} N_j L_j / L_i$ (where implicitly we assume that which $n_{ij} < L_j$

consumers in j get access to a seller's varieties is chosen at random).

The level of output and price of each variety, as well as the measure of consumers n_{ij} sellers reach out to follows from profit maximization:

$$\max_{n_{ij},p_{ij}} n_{ij}\left(p_{ij}-\frac{\tau_{ij}w_j}{Z_j}\right)q_{ij}-w_j\frac{c}{\phi}\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi},$$

where again n_{ij} is the number of customers served, and where the remaining parameters are analogous to those in our baseline model.

Sellers naturally charge a constant markup over marginal cost,

$$p_{ij}=\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_j}{Z_j},$$

so the choice of n_{ij} boilds down to

$$\max_{n_{ij}} \frac{n_{ij}}{\sigma}p_{ij}q_{ij}-w_j\frac{c}{\phi}\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi}.$$

The first-order condition of this problem yields

$$\frac{p_{ij}q_{ij}}{\sigma}=w_jc\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi-1}\Rightarrow n_{ij}=\left(\frac{p_{ij}q_{ij}}{c\sigma\mu_{ij}d_{ij}^{\rho}w_j}\right)^{\frac{1}{\phi-1}}$$

Developing a new variety costs w_if. Hence, by free entry, $\sum_k \Pi_{ki}=w_if$, and the zero-profit condition also entails $\mathcal{I}_i=w_i$. As a result, we can express n_{ij} as

$$n_{ij}=(c\sigma)^{-\frac{1}{\phi-1}}\mu_{ij}^{-\frac{1}{\phi-1}}d_{ij}^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}}\left(\frac{\sigma}{\sigma-1}\frac{t_{ij}w_j}{P_iZ_j}\right)^{-\frac{\sigma-1}{\phi-1}}\left(\frac{w_i}{w_j}\right)^{\frac{1}{\phi-1}}.$$

With this expression at hand, we can compute the import volume of country i from country j:

$$\begin{aligned}
X_{ij} &= \eta_{ij}p_{ij}q_{ij}L_i=n_{ij}p_{ij}q_{ij}N_jL_j\\
&= w_jc\sigma\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi}N_jL_j\\
&= (c\sigma)^{-\frac{1}{\phi-1}}\mu_{ij}^{-\frac{1}{\phi-1}}d_{ij}^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}}\left(\frac{\sigma}{\sigma-1}\frac{t_{ij}w_j}{P_iZ_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\frac{w_i}{w_j}\right)^{\frac{1}{\phi-1}}w_iN_jL_j\\
&= (c\sigma)^{-\frac{1}{\phi-1}}\Gamma_{ij}^{-\varepsilon}\left(\frac{\sigma}{\sigma-1}\frac{w_j}{P_iZ_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}\left(\frac{w_i}{w_j}\right)^{\frac{1}{\phi-1}}w_iN_jL_j
\end{aligned}$$

Hence, the share of country j in country i's import is

$$\pi_{ij}=\frac{\Gamma_{ij}^{-\varepsilon}\left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}w_j^{-\frac{1}{\phi-1}}N_jL_j}{\sum_k\Gamma_{ik}^{-\varepsilon}\left(\frac{w_k}{Z_k}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}w_k^{-\frac{1}{\phi-1}}N_kL_k}.$$

Solving for price index yields

$$w_i L_i = \sum_j X_{ij}$$

$$w_i L_i = \sum_j (c\sigma)^{-\frac{1}{\phi-1}} \Gamma_{ij}^{-\varepsilon} \left(\frac{\sigma}{\sigma-1} \frac{w_j}{P_i Z_j} \right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \left(\frac{w_i}{w_j} \right)^{\frac{1}{\phi-1}} w_i N_j L_j$$

$$P_i = \frac{\sigma}{\sigma-1} (c\sigma)^{\frac{1}{\phi(\sigma-1)}} L_i^{\frac{\phi-1}{\phi(\sigma-1)}} \left(\sum_j \Gamma_{ij}^{-\varepsilon} \left(\frac{w_j}{Z_j} \right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \left(\frac{w_i}{w_j} \right)^{\frac{1}{\phi-1}} N_j L_j \right)^{-\frac{\phi-1}{\phi(\sigma-1)}}.$$

We can next study the choice of the number of varities N_j produced by sellers. Profits of sellers are given by

$$\Pi_{ij} = \frac{\phi-1}{\phi} \frac{n_{ij} p_{ij} q_{ij}}{\sigma} = \frac{\phi-1}{\phi} \frac{X_{ij}}{\sigma N_j L_j},$$

so the zero-profit condition implies

$$\sum_k \Pi_{ki} = w_i f \Rightarrow \frac{\phi-1}{\phi} \frac{1}{\sigma N_i L_i} \sum_k X_{ki} = w_i f.$$

Since $\sum_k X_{ki} = w_i L_i$,

$$\frac{\phi-1}{\phi} \frac{w_i L_i}{\sigma N_i L_i} = w_i f \Rightarrow N_i = \frac{\phi-1}{\phi \sigma f}$$

Hence, the number of varieties is constant and independent of many of the parameters of the model.

We finally turn to the general equilibrium of the model, which is associated with the condition:

$$\pi_{ii} w_i L_i + \pi_{ji} w_j L_j = w_i L_i$$

Plugging in the expressions for trade shares yields

$$\frac{\Gamma_{ii}^{-\varepsilon} \left(\frac{w_i}{Z_i} \right)^{-\frac{\phi(\sigma-1)}{\phi-1}} w_i^{-\frac{1}{\phi-1}} L_i}{\sum_k \left(\Gamma_{ik}^{-\varepsilon} \frac{w_k}{Z_k} \right)^{-\frac{\phi(\sigma-1)}{\phi-1}} w_k^{-\frac{1}{\phi-1}} L_k} w_i L_i + \frac{L_i \left(\frac{w_i}{Z_i} \right)^{-\frac{\phi(\sigma-1)}{\phi-1}} w_i^{-\frac{1}{\phi-1}} \Gamma_{ji}^{-\varepsilon}}{\sum_k L_k \left(\frac{w_k}{Z_k} \right)^{-\frac{\phi(\sigma-1)}{\phi-1}} w_k^{-\frac{1}{\phi-1}} \Gamma_{jk}^{-\varepsilon}} w_j L_j = w_i L_i.$$

We are now ready to state and proof results analogous to those in Propositions 1 and 4 in the main text.

Proposition 1": As long as trade frictions (Γ_{ij}) are bounded, there exists a unique vector of equilibrium wages $\mathbf{w}^* = (w_i, w_j) \in \mathbb{R}_{++}^2$ that solves the system of equations above.

Proof. This follows again from results in standard gravity models in Alvarez and Lucas (2007), Allen and Arkolakis (2014), and Allen et al. (2020), and the fact that if there exists a unique wage vector, the remaining equilibrium variables in this single-sector economy are uniquely determined. ■

Proposition 2": A decline in any international trade or mobility friction ($d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji}$) leads to: (a) a decline in the rates (n_{ii} and n_{jj}) at which individuals will meet individuals in their own country; and (b) an increase in the rates at which individuals will meet individuals from the other country (n_{ij} and n_{ji}).

Proof. **(a)** First, note that

$$n_{ii} = \xi \mu_{ii}^{-\frac{1}{\phi-1}} d_{ii}^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ii} w_i}{P_i Z_i}\right)^{-\frac{\sigma-1}{\phi-1}} = const \times \left(\frac{P_i}{w_i}\right)^{\frac{\sigma-1}{\phi-1}}$$

Then

$$\frac{P_i}{w_i} = const \times L_i^{\frac{\phi-1}{\phi(\sigma-1)}} \left(L_i \Gamma_{ii}^{-\varepsilon} Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} + L_j \Gamma_{ij}^{-\varepsilon} \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}} \right)^{-\frac{\phi-1}{\phi(\sigma-1)}}$$

where $\omega = w_j / w_i$ is the relative wage in country j .

Note that the equilibrium equations can be rewritten as

$$\frac{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon}}{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + L_j \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}} \Gamma_{ij}^{-\varepsilon}} L_i \tag{C.1}$$

$$+ \frac{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon}}{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon} + L_j \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}} \Gamma_{jj}^{-\varepsilon}} \omega L_j = L_i \tag{C.2}$$

Consider a case when Γ_{ij} decreases, while other Γ_{kl} remain constant. That means that the first term in the sum goes down, while the second term is constant. For the equality to hold, ω should increase. After re-equilibration, the second term in the sum increased, which means that the first term decreased. This means that P_i/w_i decreased, and n_{ii} as well.

Consider now a case when Γ_{ji} decreases, while other Γ_{kl} remain constant. The second term increases, so ω needs to go down to equilibrate the model. That means that the first term decreases, and P_i/w_i and n_{ii} decrease by extension.

Therefore, whenever one decreases any international friction ($d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji}$), Γ_{ij} or Γ_{ji} goes down, and, hence, n_{ii} and n_{jj} go down.

(b) Note that $\Pi_{ii} + \Pi_{ji} = w_i f$. That can be rewritten as

$$\frac{\phi-1}{\phi} \frac{n_{ii} p_{ii} q_{ii}}{w_i \sigma} + \frac{\phi-1}{\phi} \frac{n_{ji} p_{ji} q_{ji}}{w_i \sigma} = f$$

Using the FOC for n_{ij}, that yields

$$\frac{\phi-1}{\phi} c \mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi} + \frac{\phi-1}{\phi} c \mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi} = f$$

Since n_{ii} and n_{jj} decrease and frictions do not increase, n_{ij} and n_{ji} have to increase. ■

Proposition 3": Suppose that countries are symmetric, in the sense that $L_i = L$, $Z_i = Z$, and $\Gamma_{ij} = \Gamma$ for all i. Then a decline in any (symmetric) international trade frictions leads to an overall increase in human interactions ($n_{dom} + n_{for}$) experienced by both household buyers and household sellers.

Proof. We begin by considering the case with general country asymmetries. Consider the sum

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi} + \mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi} = const$$

Differentiating yields

$$\phi \mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} dn_{ii} + \phi \mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi-1} dn_{ji} + n_{ji}^{\phi} \underbrace{d\left(\mu_{ji} d_{ji}^{\rho}\right)}_{\leq 0} = 0$$

Hence,

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} dn_{ii} + \mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi-1} dn_{ji} \geq 0,$$

and if $\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} > \mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi-1}$, then $dn_{ji} > -dn_{ii}$.

From the FOC for the choice of n_{ii} and n_{ji},

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi-1} = const \times \frac{p_{ii} q_{ii}}{w_i} = const \times \left(\frac{p_{ii}}{P_i}\right)^{1-\sigma}$$

$$\mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi-1} = const \times \frac{p_{ji} q_{ji}}{w_i} = const \times \left(\frac{p_{ji}}{P_j}\right)^{1-\sigma} \left(\frac{w_j}{w_i}\right)$$

Since the countries are symmetrics, $P_i = P_j$ and $w_i = w_j$, so the inequality is satisied if and only if $p_{ii} < p_{ji}$.

When countries are symmetric, this holds trivially because of international trade costs $t_{ji} > t_{ii}$ and $d_{ji} > d_{ii}$. Hence, $dn_{ji} > -dn_{ii}$, and $n_{dom} + n_{for}$ increases. ■

Proposition 4": An increase in the relative size of country i's population leads to an increase in the rate n_{ii} at which individuals from i will meet individuals in their own country, and to a decrease in the rate n_{ji} at which individuals will meet individuals abroad.

Proof. Consider again

$$\frac{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon}}{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + L_j \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}} \Gamma_{ij}^{-\varepsilon}} L_i \tag{C.3}$$

$$+ \frac{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon}}{L_i Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon} + L_j \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}} \Gamma_{jj}^{-\varepsilon}} \omega L_j = L_i \tag{C.4}$$

An increase in L_i makes the left-hand side smaller then the right-hand side. Therefore, ω grows to re-equilibrate. Then

$$\frac{P_i}{w_i} = const \times L_i^{\frac{\phi-1}{\phi(\sigma-1)}} \left(L_i \Gamma_{ii}^{-\varepsilon} Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} + L_j \Gamma_{ij}^{-\varepsilon} \left(\frac{Z_j}{\omega} \right)^{\frac{\phi(\sigma-1)}{\phi-1}} \omega^{-\frac{1}{\phi-1}} \right)^{-\frac{\phi-1}{\phi(\sigma-1)}}$$

increases, and $n_{ii} = const \times \left(\frac{P_i}{w_i} \right)^{\frac{\sigma-1}{\phi-1}}$ increases with it.

Since

$$\frac{\phi-1}{\phi} c\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi} + \frac{\phi-1}{\phi} c\mu_{ji} d_{ji}^{\rho} n_{ji}^{\phi} = f,$$

n_{ji} decreases.

Therefore, following a growth in population L_i, n_{ii} increases while n_{ji} decreases. ■

D Proof of Proposition 5

Proposition 5: *Assume that there is trade between the two countries (i.e., $\alpha_j n_{ij} + \alpha_i n_{ji} > 0$), which implies that the next generation matrix FV^{-1} is irreducible. If $\mathcal{R}_0 \leq 1$, the no-pandemic equilibrium is the unique stable equilibrium. If $\mathcal{R}_0 > 1$, the no-pandemic equilibrium is unstable, and there exists a unique stable endemic equilibrium.*

Proof. The proof of existence and uniqueness, depending on whether $\mathcal{R}_0 \leq 1$ or $\mathcal{R}_0 > 1$, follows standard arguments for a two-group SIR model, as in Magal et al. (2016). We proceed in the following steps. **(A)** The system of dynamic equations for susceptibles, infected and recovered is given by:

$$\dot{S}_i(t) = -2\alpha_i n_{ii} S_i(t) I_i(t) - \alpha_j n_{ij} S_i(t) I_j(t) - \alpha_i n_{ji} S_i(t) I_j(t), \tag{D.1}$$

$$\dot{S}_j(t) = -2\alpha_j n_{jj} S_j(t) I_j(t) - \alpha_i n_{ji} S_j(t) I_i(t) - \alpha_j n_{ij} S_j(t) I_i(t), \tag{D.2}$$

$$\dot{I}_i(t) = 2\alpha_i n_{ii} S_i(t) I_i(t) + \alpha_j n_{ij} S_i(t) I_j(t) + \alpha_i n_{ji} S_i(t) I_j(t) - \gamma_i I_i(t), \tag{D.3}$$

$$\dot{I}_j(t) = 2\alpha_j n_{jj} S_j(t) I_j(t) + \alpha_i n_{ji} S_j(t) I_i(t) + \alpha_j n_{ij} S_j(t) I_i(t) - \gamma_j I_j(t), \tag{D.4}$$

$$\dot{R}_i(t) = \gamma_i I_i(t), \tag{D.5}$$

$$\dot{R}_j(t) = \gamma_j I_j(t). \tag{D.6}$$

Note that we can re-write the dynamic equations for infections (D.3) and (D.4) as:

$$\begin{bmatrix} \dot{I}_i(t) \\ \dot{I}_j(t) \end{bmatrix} = \left\{ \begin{bmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i(t) & \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} S_i(t) \\ \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i} S_j(t) & \frac{2\alpha_j n_{jj}}{\gamma_j} S_j(t) \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} \gamma_i I_i(t) \\ \gamma_j I_j(t) \end{bmatrix}. \tag{D.7}$$

The properties of this dynamic system depend crucially on the properties of the matrix B:

$$B \equiv \begin{bmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i(t) & \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} S_i(t) \\ \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i} S_j(t) & \frac{2\alpha_j n_{jj}}{\gamma_j} S_j(t) \end{bmatrix}.$$

We assume that there is trade between the two countries:

$$\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i} > 0, \qquad \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} > 0,$$

which implies that the matrix B is irreducible for all strictly positive susceptibles $S_i(t), S_j(t) > 0$.
(B) Re-writing equations (D.1) and (D.2) in proportional changes, and using equations (D.5) and (D.6), we have:

$$\frac{\dot{S}_i(t)}{S_i(t)} = -\frac{2\alpha_i n_{ii}}{\gamma_i}\dot{R}_i(t) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\dot{R}_j(t),$$

$$\frac{\dot{S}_j(t)}{S_j(t)} = -\frac{2\alpha_j n_{jj}}{\gamma_j}\dot{R}_j(t) - \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i}\dot{R}_i(t).$$

Integrating from 0 to t, we have:

$$\log S_i(t) - \log S_i(0) = -\frac{2\alpha_i n_{ii}}{\gamma_i}(R_i(t) - R_i(0)) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}(R_j(t) - R_j(0)),$$

$$\log S_j(t) - \ln S_j(0) = -\frac{2\alpha_j n_{jj}}{\gamma_j}(R_j(t) - R_j(0)) - \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i}(R_i(t) - R_i(0)).$$

Using the accounting identities, $S_i(t) + I_i(t) + R_i(t) = 1$ and $S_j(t) + I_j(t) + R_j(t) = 1$, we obtain:

$$\log S_i(t) - \log S_i(0) = \frac{2\alpha_i n_{ii}}{\gamma_i}\left[(S_i(t) + I_i(t)) - (S_i(0) + I_i(0))\right] + \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\left[(S_j(t) + I_j(t)) - (S_j(0) + I_j(0))\right],$$

$$\log S_j(t) - \ln S_j(0) = \frac{2\alpha_j n_{jj}}{\gamma_j}\left[(S_j(t) + I_j(t)) - (S_j(0) + I_j(0))\right] + \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i}\left[(S_i(t) + I_i(t)) - (S_i(0) + I_i(0))\right].$$

In steady-state as $t \to \infty$, we have $I_i(\infty) = I_j(\infty) = 0$, and hence:

$$S_i(\infty) = S_i(0)\exp\left[\frac{2\alpha_i n_{ii}}{\gamma_i}\left[S_i(\infty) - V_i\right] + \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\left[S_j(\infty) - V_j\right]\right], \tag{D.8}$$

$$S_j(\infty) = S_j(0)\exp\left[\frac{2\alpha_j n_{jj}}{\gamma_j}\left[S_j(\infty) - V_j\right] + \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i}\left[S_i(\infty) - V_i\right]\right], \tag{D.9}$$

where $V_i \equiv S_i(0) + I_i(0)$ and $V_j(0) \equiv S_j(0) + I_j(0)$. We now define the following notation:

$$X \leq Y \quad \Leftrightarrow \quad X_k \leq Y_k \text{ for all } k \in \{i, j\},$$

$$X < Y \quad \Leftrightarrow \quad X \leq Y \text{ and } X_k < Y_k \text{ for some } k \in \{i, j\},$$

$$X \ll Y \quad \Leftrightarrow \quad X_k < Y_k \text{ for all } k \in \{i, j\},$$

and represent the system (D.8)-(D.9) as the following map:

$$X = T(X),$$

$$\begin{pmatrix} x_i \\ x_j \end{pmatrix} = T \begin{pmatrix} x_i \\ x_j \end{pmatrix} = \begin{pmatrix} T_i(x_i, x_j) \\ T_j(x_i, x_j) \end{pmatrix},$$

with

$$T_i(x_i, x_j) = S_i(0) \exp\left[\frac{2\alpha_i n_{ii}}{\gamma_i}[x_i - V_i] + \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}[x_j - V_j]\right],$$

$$T_j(x_i, x_j) = S_j(0) \exp\left[\frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i}[x_i - V_i] + \frac{2\alpha_j n_{jj}}{\gamma_j}[x_j - V_j]\right].$$

(C) Using this notation, we begin by establishing that all the fixed points of T in $[0, S(0)]$ are contained in the smaller interval $[S^-, S^+]$. To establish this result, note that T is monotone in increasing, which implies that:

$$X \leq Y \quad \Rightarrow \quad T(X) \leq T(Y).$$

Using our assumption of positive trade, $\frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i} > 0$ and $\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} > 0$, this implies:

$$X \ll Y \quad T(X) \ll T(Y).$$

For $S(0) \gg 0$, and using the definitions of V_i and V_j above, this implies:

$$0 \ll T(0) < T(S(0)) < S(0).$$

Therefore, by induction arguments, we have the following result for each $n \geq 1$:

$$0 \ll T(0) \cdots \ll T^n(0) \ll T^{n+1}(0) \leq T^{n+1}(S(0)) < \cdots T^n(S(0)) < S(0).$$

By taking the limit as n does to $+\infty$, we obtain:

$$0 \ll \lim_{n \to +\infty} T^n(0) =: S^- \leq S^+ := \lim_{n \to +\infty} T^n(S(0)) < S(0).$$

Then, by continuity of T, we have:

$$T\left(S^-\right) = S^- \quad \text{and} \quad T\left(S^+\right) = S^+.$$

(D) We next establish that if $S^- < S^+$ then $S^- \ll S^+$. This property follows from our assumption that the matrix B above is irreducible. Assume, for example, that $S_i^- < S_i^+$. Then, since $\frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i} > 0$, we have:

$$S_j^- = T_j\left(S_i^-, S_j^-\right) \leq T_j\left(S_i^-, S_j^+\right) < T_2\left(S_i^+, S_j^+\right) = S_j^+.$$

Hence,

$$S_i^- < S_i^+ \Rightarrow S_j^- < S_j^+.$$

By the same argument, $\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j} > 0$ implies,

$$S_j^- < S_j^+ \Rightarrow S_i^- < S_i^+.$$

(E) We now establish the following result for $\lambda > 1$ and $X \gg 0$:

$$T\left(\lambda X + S^-\right) - T\left(S^-\right) \gg \lambda\left[T\left(X + S^-\right) - T\left(S^-\right)\right].$$

Note that we can write the left-hand side of this inequality as follows:

$$T\left(\lambda X + S^-\right) - T\left(S^-\right) = \int_0^1 DT\left(l\lambda X + S^-\right)(\lambda X)\, dl = \lambda \int_0^1 DT\left(l\lambda X + S^-\right) X dl,$$

where the differential of T is given by:

$$DT(X) = \begin{pmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} T_i(x_i, x_j) & \frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j} T_i(x_i, x_j) \\ \frac{\alpha_i n_{ji}+\alpha_j n_{ij}}{\gamma_i} T_j(x_i, x_j) & \frac{2\alpha_j n_{jj}}{\gamma_j} T_j(x_i, x_j) \end{pmatrix}. \tag{D.10}$$

Since $\lambda > 1$ and $X \gg 0$, we have:

$$DT\left(l\lambda X + S^-\right) X \gg DT\left(lX + S^-\right) X \qquad \forall \quad l \in [0, 1].$$

It follows that:

$$\begin{aligned} T\left(\lambda X + S^-\right) - T\left(S^-\right) &\gg \lambda \int_0^1 DT\left(lX + S^-\right) X dl, \\ &= \lambda\left[T\left(X + S^-\right) - T\left(S^-\right)\right], \end{aligned}$$

which establishes the result.

(F) We now show that the map T has at most two equilibria such that either:

(i) $S^- = S^+$ and T has only one equilibrium in $[0, S(0)]$;

(ii) $S^- \ll S^+$ and the only equilibria of T in $[0, S(0)]$ are S^- and S^+.

We prove this result by contradiction. Assume that $S^- \neq S^+$. Then $S^- < S^+$, which implies $S^- \ll S^+$. Now suppose that there exists $\bar{X} \in [S^-, S^+]$ a fixed point T such that:

$$S^- \neq \bar{X} \qquad \text{and} \qquad \bar{X} \neq S^+.$$

Then, by using the same arguments as in **(D)** above, we have:

$$S^- \ll \bar{X} \ll S^+.$$

Now define:

$$\gamma := \sup\left\{\lambda \geq 1 : \lambda\left(\bar{X} - S^-\right) + S^- \leq S^+\right\}. \tag{D.11}$$

Since $\bar{X} \ll S^{+}$ this implies that

$$\gamma > 1.$$

We have

$$\gamma \left(\bar{X} - S^{-}\right) + S^{-} \leq S^{+},$$

and, by applying T to both sides of this inequality, we obtain:

$$T\left(\gamma\left(\bar{X} - S^{-}\right) + S^{-}\right) \leq S^{+}.$$

Now, using **(E)**, we have:

$$\begin{aligned} T\left(\gamma\left(\bar{X} - S^{-}\right) + S^{-}\right) - T\left(S^{-}\right) &\gg \gamma\left[T\left(\left(\bar{X} - S^{-}\right) + S^{-}\right) - T\left(S^{-}\right)\right], \\ &= \gamma\left[T\left(\bar{X}\right) - T\left(S^{-}\right)\right], \\ &= \gamma\left[\bar{X} - S^{-}\right]. \end{aligned}$$

Therefore we have shown that:

$$S^{+} \geq T\left(\gamma\left(\bar{X} - S^{-}\right) + S^{-}\right) \gg \gamma\left[\bar{X} - S^{-}\right],$$

which contradicts the definition of gamma as the supremum of the set in equation (D.11), since $S^{-} \geq 0$. Therefore we cannot have another fixed point $\bar{X} \in \left[S^{-}, S^{+}\right]$.

(G) Now consider the case where:

$$S^{-} \ll S^{+}.$$

In this case of two equilibria, the differential of T can be written as:

$$DT\left(S^{\pm}\right) = B\left(S_i^{\pm}\right) = \begin{pmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i^{\pm} & \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} S_i^{\pm} \\ \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i} S_j^{\pm} & \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^{\pm} \end{pmatrix}.$$

(H) We now establish the following property of the spectral radius of the matrices $DT\left(S^{-}\right)$ and $DT\left(S^{+}\right)$:

$$\rho\left(DT\left(S^{-}\right)\right) < 1 < \rho\left(DT\left(S^{+}\right)\right).$$

To prove this result, note that:

$$\begin{aligned} S^{+} - S^{-} &= T\left(S^{+}\right) - T\left(S^{-}\right), \\ &= T\left(\left(S^{+} - S^{-}\right) + S^{-}\right) - T\left(S^{-}\right), \\ &= \int_0^1 DT\left(l\left(S^{+} - S^{-}\right) + S^{-}\right)\left(S^{+} - S^{-}\right) dl. \end{aligned}$$

Since $S^+ - S^- \gg 0$, we also have:

$$\begin{aligned} DT\left(S^+\right)\left(S^+ - S^-\right) &\gg \int_0^1 DT\left(l\left(S^+ - S^-\right) + S^-\right)\left(S^+ - S^-\right) dl, \\ &\gg DT\left(S^-\right)\left(S^+ - S^-\right). \end{aligned}$$

Combining these two results, we obtain:

$$DT\left(S^+\right)\left(S^+ - S^-\right) \gg \left(S^+ - S^-\right) \gg DT\left(S^-\right)\left(S^+ - S^-\right). \tag{D.12}$$

which can be equivalently written as:

$$\left[DT\left(S^+\right) - I\right]\left(S^+ - S^-\right) > 0,$$

$$\left[DT\left(S^+\right) - \xi^+ I\right]\left(S^+ - S^-\right) = 0, \qquad \xi^+ > 1,$$

and

$$\left[DT\left(S^-\right) - I\right]\left(S^+ - S^-\right) < 0,$$

$$\left[DT\left(S^-\right) - \xi^- I\right]\left(S^+ - S^-\right) = 0, \qquad \xi^- < 1,$$

where I is the identity matrix. Noting that the matrices $DT\left(S^+\right)$ and $DT\left(S^-\right)$ are non-negative and irreducible, the Perron-Frobenius theorem implies:

$$\xi^- = \rho\left(DT\left(S^-\right)\right) < 1 < \rho\left(DT\left(S^+\right)\right) = \xi^+.$$

(I) We now solve explicitly for the spectral radius of the matrices $DT\left(S^\pm\right)$. We find the eigenvalues of the matrix $DT\left(S^\pm\right)$ by solving the characteristic equation:

$$\left|DT\left(S^\pm\right) - \xi^\pm I\right| = \left|\begin{bmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i^\pm & \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} S_j^\pm \\ \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i} S_i^\pm & \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^\pm \end{bmatrix} - \begin{bmatrix} \xi^\pm & 0 \\ 0 & \xi^\pm \end{bmatrix}\right| = 0.$$

The characteristic polynomial is:

$$\left(\xi^\pm\right)^2 - \left(\frac{2\alpha_i n_{ii}}{\gamma_i} S_i^\pm + \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^\pm\right)\xi^\pm + \left(\frac{2\alpha_i n_{ii}}{\gamma_i}\frac{2\alpha_j n_{jj}}{\gamma_j} S_i^\pm S_j^\pm - \frac{\left(\alpha_j n_{ij} + \alpha_i n_{ji}\right)^2}{\gamma_i \gamma_j} S_i^\pm S_j^\pm\right) = 0.$$

The spectral radius is the largest eigenvalue:

$$\rho\left(DT\left(S^\pm\right)\right) = \frac{1}{2}\left(\frac{2\alpha_i n_{ii}}{\gamma_i} S_i^\pm + \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^\pm\right) + \frac{1}{2}\sqrt{\left(\frac{2\alpha_i n_{ii}}{\gamma_i} S_i^\pm - \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^\pm\right)^2 + 4\frac{\left(\alpha_j n_{ij} + \alpha_i n_{ji}\right)^2}{\gamma_i \gamma_j} S_i^\pm S_j^\pm}.$$

(J) We now use the results in **(H)** and **(I)** to examine the local stability of the two steady-state equilibria. From the dynamics of infections in equation (D.7), we have:

$$\begin{bmatrix} \dot{I}_i^{\pm} \\ \dot{I}_j^{\pm} \end{bmatrix} = \left\{ \begin{bmatrix} \frac{2\alpha_i n_{ii}}{\gamma_i} S_i^{\pm} & \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j} S_j^{\pm} \\ \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i} S_i^{\pm} & \frac{2\alpha_j n_{jj}}{\gamma_j} S_j^{\pm} \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} I_i^{\pm} \\ I_j^{\pm} \end{bmatrix}. \tag{D.13}$$

Therefore the spectral radius of the matrix $DT\left(S^{\pm}\right)$ corresponds to the global $\mathcal{R}_0$ that determines the local stability of the two steady-state equilibria. As we have shown that $\rho\left(DT\left(S^{+}\right)\right) > 1$, the steady-state S^{+} is locally unstable. As we have shown that $\rho\left(DT\left(S^{-}\right)\right) < 1$, the steady-state S^{-} is locally stable.

■

E Computational Appendix

In this Appendix we describe the algorithms we use to do the numerical simulations in each section of the paper.

E.1 A Two-Country SIR Model with Time-Invariant Interactions

Solution Algorithm

1. Compute the value of n_{ii}, n_{ij}, n_{ij}, and n_{jj} as the outcome of the equilibrium that solves

$$n_{ij} = \left(c\left(\sigma-1\right)\mu_{ij}\right)^{-1/(\phi-1)} \left(d_{ij}\right)^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ij} w_j}{Z_j P_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}$$

$$\pi_{ii} w_i L_i + \pi_{ji} w_j L_j = w_i L_i,$$

   where π_{ij} is given by

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell\in\mathcal{J}} X_{i\ell}} = \frac{\left(w_j/Z_j\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times \left(\mu_{ij}\right)^{-\frac{1}{\phi-1}} \left(d_{ij}\right)^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} \left(t_{ij}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell\in\mathcal{J}} \left(\mu_{i\ell}\right)^{-\frac{1}{\phi-1}} \left(d_{i\ell}\right)^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} \left(t_{i\ell} w_\ell/Z_\ell\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}},$$

   corresponding to equation (9) in the paper. Call them $\bar{n}_{ii}$, $\bar{n}_{ij}$, $\bar{n}_{ij}$, and $\bar{n}_{jj}$. Provided population, technology, and relative wages are time invariant, these quantities will be fixed.

2. Set $I_i(0) = 0.1 \times 10^{-4}$, $S_i(0) = 1 - I_i(0)$, and $R_i(0) = 0$ for all i. For each $t \in [1, T]$ solve the following system of equations:

$$\begin{bmatrix} S_i(t+1) \\ I_i(t+1) \\ R_i(t+1) \\ S_j(t+1) \\ I_j(t+1) \\ R_j(t+1) \end{bmatrix} = \begin{bmatrix} -\Omega_i & \Omega_i & 0 & 0 & 0 & 0 \\ 0 & -\gamma_i & \gamma_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\Omega_j & \Omega_j & 0 \\ 0 & 0 & 0 & 0 & -\gamma_j & \gamma_j \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times (1/step) \times \begin{bmatrix} S_i(t) \\ I_i(t) \\ R_i(t) \\ S_j(t) \\ I_j(t) \\ R_j(t) \end{bmatrix} + \begin{bmatrix} S_i(t) \\ I_i(t) \\ R_i(t) \\ S_j(t) \\ I_j(t) \\ R_j(t) \end{bmatrix}$$

where

$$\Omega_i = \alpha_i \times 2\bar{n}_{ii} \times I_i(t) + \alpha_j \times \bar{n}_{ij} \times I_j(t) + \alpha_i \times \bar{n}_{ji} \times I_j(t).$$

This system corresponds to equations $(16) - (18)$ in the paper. The variable *step* marks the number of steps taken within each time period, in this section we use $step = 2$.

Associated Figures

This section in the paper uses three sets of parameters. Figures 1, 2, and 3 present a general specification in which international trade favors the onset of a pandemic, with standard parameters as listed in Table E.1 for Figure 1 and Table E.2 for Figures 2 and 3. Figures 4 and 5 look at an example in which free trade prevents the onset of a pandemic, using parameters listed in Table E.3. Figure 6 presents the possibility of second waves of infection, using parameters listed in Table E.4.

If no other mention is made, trade frictions are set at baseline values $\mu_{ij} = \mu_{ji} = 1$, $t_{ij} = t_{ji} = 1$, $d_{ij} = d_{ij} = 1.1$. Some of these figures study changes in trade frictions moving one of these parameters. All other parameters are kept at baseline value.

Table E.1: Baseline parameters - Figure 1 in draft.

Parameter	Value
σ	5
ϕ	2
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.15
α_1	0.04
α_2	$\{0.04, 0.10\}$
γ_1, γ_2	$0.20, 0.20$
η_1, η_2	$0.0, 0.0$

In order to obtain the result described for the second set of parameters, $\phi = 1.5$ is crucial. The only other difference with respect to the general scenario is a decrease of c to 0.1. This is not necessary: the qualitative result also holds for $c = 0.15$ but it was originally changed so that n_{ii} would be approximately the same in both cases.

There are more parameters that will generate a second wave of infections. The ones presented here were picked to obtain *reasonable* values for R^{0i} and R^0. What is essential for this feature to occur is that both countries have different timings for their own pandemics in autarky. One (small) country has very fast contagion rates (α) and very short recovery periods (high γ), while in the other (big) country the disease must progress much slower so that when the cycle starts it will drag the first country with it once again. The difference in size is there so that when the small country goes through its first cycle, the big country will remain mostly unaffected.

Table E.2: Baseline parameters - Figures 2, 3 in draft.

Parameter	Value
σ	5
ϕ	2
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21},\ t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.15
α_1, α_2	$0.04, 0.07$
γ_1, γ_2	$0.20, 0.20$
η_1, η_2	$0.0, 0.0$

Table E.3: "*Better trade*" parameters - Figures 4, 5 in draft.

Parameter	Value
σ	5
ϕ	1.5
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21},\ t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.10
$\alpha_1,\ \alpha_2$	$0.04, 0.07$
γ_1, γ_2	$0.20, 0.20$
η_1, η_2	$0.0, 0.0$

E.2 General-Equilibrium Induced Responses

Solution Algorithm

1. Compute the value of $n_{ii}(0)$, $n_{ij}(0)$, $n_{ij}(0)$, and $n_{jj}(0)$ as the outcome of the equilibrium that solves

$$n_{ij} = \left(c\left(\sigma - 1\right)\mu_{ij}\right)^{-1/(\phi-1)} \left(d_{ij}\right)^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ij}w_j}{Z_jP_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}$$

$$\pi_{ii}w_iL_i(1 - D_i(t)) + \pi_{ji}w_jL_j(1 - D_j(t)) = w_iL_i(1 - D_i(t)),$$

where π_{ij} is once again given by

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell\in\mathcal{J}} X_{i\ell}} = \frac{(w_j/Z_j)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times (\mu_{ij})^{-\frac{1}{\phi-1}} \left(d_{ij}\right)^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} \left(t_{ij}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell\in\mathcal{J}} (\mu_{i\ell})^{-\frac{1}{\phi-1}} \left(d_{i\ell}\right)^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} \left(t_{i\ell}w_\ell/Z_\ell\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}},$$

Table E.4: Second-wave parameters - Figure 6 in draft

Parameter	Value
σ	4.5
ϕ	2
Z_1, Z_2	1
L_1, L_2	$2, 20$
$d_{12} = d_{21}$	1
δ	1
ρ	1
c	0.12
α_1, α_2	$0.69, 0.09$
β_1, β_2	$2.29, 0.30$
γ_1, γ_2	$2.1, 0.18$

corresponding to equation (9) in the paper. These values are no longer fixed and will evolve as the pandemic progresses.

2. Set $I_i(0) = 0.1 \times 10^{-4}$, $S_i(0) = 1 - I_i(0)$, and $R_i(0) = 0$ for all i. For each $t \in [1, T]$:

   (a) Solve the following system of equations:

$$\begin{bmatrix} S_i(t+1) \\ I_i(t+1) \\ R_i(t+1) \\ D_i(t+1) \\ S_j(t+1) \\ I_j(t+1) \\ R_j(t+1) \\ D_j(t+1) \end{bmatrix} = \begin{bmatrix} -\Omega_i & \Omega_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\kappa_i & \gamma_i & \eta_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega_j & \Omega_j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\kappa_j & \gamma_j & \eta_j \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times (1/step) \times \begin{bmatrix} S_i(t) \\ I_i(t) \\ R_i(t) \\ D_i(t) \\ S_j(t) \\ I_j(t) \\ R_j(t) \\ D_j(t) \end{bmatrix} + \begin{bmatrix} S_i(t) \\ I_i(t) \\ R_i(t) \\ D_i(t) \\ S_j(t) \\ I_j(t) \\ R_j(t) \\ D_j(t) \end{bmatrix}$$

   where $\kappa_i = \gamma_i + \eta_i$ and

$$\Omega_i = \alpha_i \times 2n_{ii}(t) \times I_i(t) + \alpha_j \times n_{ij}(t) \times I_j(t) + \alpha_i \times n_{ji}(t) \times I_j(t).$$

   This system corresponds to equations (28) – (31) in the paper. The variable *step* marks the number of steps taken within each time period, in this section we use $step = 2$.

   (b) Update $n_{ij}(t+1)$ and $w_i(t+1)$ as the values that solve:

$$n_{ij}(t+1) = (c(\sigma-1)\mu_{ij})^{-1/(\phi-1)} (d_{ij})^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ij} w_j(t+1)}{Z_j P_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i(t+1)}{P_i}\right)^{1/(\phi-1)}$$

$$\pi_{ii} w_i(t+1) L_i(1 - D_i(t+1)) + \pi_{ji} w_j(t+1) L_j(1 - D_j(t+1)) = w_i(t+1) L_i(1 - D_i(t+1)).$$

Associated Figures

This section in the paper is associated with Figure 7, which uses the parameters described in Table E.5. These correspond to the first set of parameters in the previous section (associated to Figures 1, 2, and 3). The duration of the disease remains the same, as the exit rate from the infected stage $(\gamma_i + \eta_i)$ is unchanged, but now both countries experience deaths, with one of them having a much higher death rate than the other (η_i marks the entry into the dead stage, so $\eta_i/(\gamma_i + \eta_i)$ marks how many of those that were infected will end up dying).

Table E.5: Section 4 parameters - Figure 7.

Parameter	Value
σ	5
ϕ	2
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.15
α_1, α_2	$0.04, 0.07$
$(\gamma_i + \eta_i)$	0.20
$\eta_i/(\gamma_i + \eta_i)$	$0.01, 0.50$

E.3 Behavioral Responses - Symmetric Case

Solution Algorithm

1. Choose $T(\infty) = 500,000$ (some large number), and $T = 10,000$. Guess $D(\infty) = \mathcal{D}_i$.

2. Compute the value of $n_{ii}(\infty)$, $n_{ij}(\infty)$, $n_{ij}(\infty)$, and $n_{jj}(\infty)$ as the outcome of the equilibrium that solves

$$n_{ij} = \left(c\left(\sigma - 1\right)\mu_{ij}\right)^{-1/(\phi-1)} \left(d_{ij}\right)^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ij}w_j}{Z_j P_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}$$

$$\pi_{ii} w_i L_i (1 - \mathcal{D}_i) + \pi_{ji} w_j L_j (1 - \mathcal{D}_j) = w_i L_i (1 - \mathcal{D}_i),$$

where π_{ij} is given by

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell \in \mathcal{J}} X_{i\ell}} = \frac{\left(w_j/Z_j\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times \left(\mu_{ij}\right)^{-\frac{1}{\phi-1}} \left(d_{ij}\right)^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} \left(t_{ij}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell \in \mathcal{J}} \left(\mu_{i\ell}\right)^{-\frac{1}{\phi-1}} \left(d_{i\ell}\right)^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} \left(t_{i\ell} w_\ell / Z_\ell\right)^{-\frac{\phi(\sigma-1)}{\phi-1}}}$$

corresponding to equation (9) in the paper.

3. Transversality conditions are satisfied if

$$\lim_{t\to\infty} \theta_i^k(t) = 0$$
$$\lim_{t\to\infty} \theta_i^i(t) = 0$$
$$\lim_{t\to\infty} \theta_i^s(t) = 0$$

Set $\theta_i^k(\infty) = \theta_i^i(\infty) = \theta_i^s(\infty) = 0$ and let the economy run without infections between T and $T(\infty)$, that is, for each time period $t \in [T, T(\infty)]$ update the Lagrange multipliers as

$$\theta_i^k(t) = \theta_i^k(t+1) - [Q_i(n_{ii}(\infty), n_{ij}(\infty)) - C_i(n_{ii}(\infty), n_{ij}(\infty))]\, e^{-\xi t}\Delta t$$
$$\theta_i^i(t) = \frac{1}{1+(\gamma_i+\eta_i)\Delta t}[\eta_i\theta_i^k(t)\Delta t + \theta_i^i(t+1)]$$

where Δt is the step size (one over how many times you update within each day). Keep $\theta^k(T)$ and $\theta^i(T)$ as the terminal values of the Lagrange multipliers.

4. Set $I_i(T) = 10^{-6}$, $\theta_i^s(T) = 0$ and $S_i(T) = 1 - I_i(T) - \mathcal{D}_i/(\eta_i/(\gamma_i+\eta_i))$. Recompute $n_{i\cdot}(T)$ as the values that solve

$$\left[\frac{\partial Q_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}}\right](1-\mathcal{D}_i)e^{-\xi T} = [\theta_i^s(T) - \theta_i^i(T)]S_i(T)\alpha_j I_j(T),$$

corresponding to equation (32) in the paper. Given perfect symmetry between countries, we will have $w_i = 1$ for all i.

5. For each $t \in [T, 0]$ solve the following system of equations, where all values evaluated at $t+1$ are known, to obtain values at t:

$$\theta_i^s(t+1) - \theta_i^s(t) = [\theta_i^s(t) - \theta_i^i(t)][2\alpha_i n_{ii}(t)I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t))I_j(t)]\Delta t$$
$$\theta_i^s(t+1) - \theta_i^i(t) = (\gamma_i+\eta_i)\theta_i^i(t)\Delta t - \eta_i\theta_i^k(t)\Delta t$$
$$\theta_k^s(t+1) - \theta_i^k(t) = [Q_i(n_{ii}(t), n_{ij}(t)) - C_i(n_{ii}(t), n_{ij}(t))]\, e^{-\xi t}\Delta t$$
$$I_i(t+1) - I_i(t) = S_i(t)[2\alpha_i n_{ii}(t)I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t))I_j(t)]\Delta t - (\gamma_i+\eta_i)I_i(t)\Delta t$$
$$S_i(t+1) - S_i(t) = -S_i(t)[2\alpha_i n_{ii}(t)I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t))I_j(t)]\Delta t$$
$$D_i(t+1) - D_i(t) = \eta_i I_i(t)\Delta t$$

and where $n_{i\cdot}(t)$ is again obtained as the value that solves:

$$\left[\frac{\partial Q_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}}\right](1-D_i(t))e^{-\xi t} = [\theta_i^s(t) - \theta_i^i(t)]S_i(t)\alpha_j I_j(t).$$

These correspond to equations (32)-(35) in the paper plus the equations determining the evolution of the epidemiological variables, once we have imposed equilibrium conditions.

6. Repeat for all periods until $I(t)$ reaches the desired initial condition, that is, $I(t) = 10^{-5}$. If at this t we have $|D(t)| < 10^{-5}$ stop. Otherwise, adjust guess $\mathcal{D}_i$.

Associated Figures

This section in the paper is associated with Figures 8 and 9, which uses the parameters described in Table E.6.

Table E.6: Behavioral response parameters - Figures 8, 9 in draft.

Parameter	Value
σ	5
ϕ	1.5
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.10
α_1, α_2	$0.1, 0.1$
$\gamma_i + \eta_i$	$0.20, 0.20$
$\eta_i/(\gamma_i + \eta_i)$	$0.0062, 0.0062$
Δt	$1/5$
ξ	$0.05/(365 \times \Delta t)$

The initial guess used in the code Figure 8 is $\mathcal{D}_i = 0.0022$, and the initial guess for Figure 9 is $\mathcal{D}_i = 0.004$.

E.4 Behavioral Responses - Asymmetric Case

Solution Algorithm

1. Choose $T(\infty) = 500,000$ (some large number), and $T =$ 10,000. Guess $D_1(\infty) = \mathcal{D}_1$. Fix $I_1(T) = 10^{-7}$.

2. Generate a grid for $D_2(\infty) = \mathcal{D}_2$ wide enough to contain the solution (use solution without behavioral responses as an upper bound for this guess). For each of the points in this grid

   (a) Compute the value of $n_{ii}(\infty)$, $n_{ij}(\infty)$, $n_{ij}(\infty)$, and $n_{jj}(\infty)$ as the outcome of the equilibrium that solves

$$n_{ij} = (c\,(\sigma-1)\,\mu_{ij})^{-1/(\phi-1)}\,(d_{ij})^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ij}w_j}{Z_jP_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}$$

$$\pi_{ii}w_iL_i(1-\mathcal{D}_i) + \pi_{ji}w_jL_j(1-\mathcal{D}_j) = w_iL_i(1-\mathcal{D}_i),$$

where π_{ij} is once again given by

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell \in \mathcal{J}} X_{i\ell}} = \frac{(w_j/Z_j)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times (\mu_{ij})^{-\frac{1}{\phi-1}} (d_{ij})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{ij})^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell \in \mathcal{J}} (\mu_{i\ell})^{-\frac{1}{\phi-1}} (d_{i\ell})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{i\ell} w_\ell/Z_\ell)^{-\frac{\phi(\sigma-1)}{\phi-1}}}$$

corresponding to equation (9) in the paper.

(b) Transversality conditions are satisfied if

$$\lim_{t \to \infty} \theta_i^k(t) = 0$$
$$\lim_{t \to \infty} \theta_i^i(t) = 0$$
$$\lim_{t \to \infty} \theta_i^s(t) = 0$$

Set $\theta_i^k(\infty) = \theta_i^i(\infty) = \theta_i^s(\infty) = 0$ and let the economy run without infections between T and $T(\infty)$, that is, for each time period $t \in [T, T(\infty)]$ update the multipliers as

$$\theta_i^k(t) = \theta_i^k(t+1) - [Q_i(n_{ii}(\infty), n_{ij}(\infty)) - C_i(n_{ii}(\infty), n_{ij}(\infty))] e^{-\xi t} \Delta t$$
$$\theta_i^i(t) = \frac{1}{1 + (\gamma_i + \eta_i)\Delta t} [\eta_i \theta_i^k(t) \Delta t + \theta_i^i(t+1)]$$

where Δt is the step size (one over how many times you update within each day). Keep $\theta^k(T)$ and $\theta^i(T)$ as the terminal values of the Lagrange multipliers.

(c) Guess a value for $I_2(T)$. Set $\theta_i^s(T) = 0$ and $S_i(T) = 1 - I_i(T) - \mathcal{D}_i/(\eta_i/(\gamma_i + \eta_i))$. Recompute $n_{i\cdot}(T)$ as the values that solve

$$\left[\frac{\partial Q_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}} \right] (1 - \mathcal{D}_i) e^{-\xi T} = [\theta_i^s(T) - \theta_i^i(T)] S_i(T) \alpha_j I_j(T),$$

corresponding to equation (32) in the paper. Given perfect symmetry between countries, we will have $w_i = 1$ for all i.

i. Given a value for $I_2(T)$, for each $t \in [T, 0]$ solve the following system of equations, where all values evaluated at $t+1$ are known, to obtain values at t:

$$\theta_i^s(t+1) - \theta_i^s(t) = [\theta_i^s(t) - \theta_i^i(t)][2\alpha_i n_{ii}(t) I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t)) I_j(t)] \Delta t$$
$$\theta_i^s(t+1) - \theta_i^i(t) = (\gamma_i + \eta_i) \theta_i^i(t) \Delta t - \eta_i \theta_i^k(t) \Delta t$$
$$\theta_k^s(t+1) - \theta_i^k(t) = [Q_i(n_{ii}(t), n_{ij}(t)) - C_i(n_{ii}(t), n_{ij}(t))] e^{-\xi t} \Delta t$$
$$I_i(t+1) - I_i(t) = S_i(t)[2\alpha_i n_{ii}(t) I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t)) I_j(t)] \Delta t - (\gamma_i + \eta_i) I_i(t) \Delta t$$
$$S_i(t+1) - S_i(t) = -S_i(t)[2\alpha_i n_{ii}(t) I_i(t) + (\alpha_j n_{ij}(t) + \alpha_i n_{ji}(t)) I_j(t)] \Delta t$$
$$D_i(t+1) - D_i(t) = \eta_i I_i(t) \Delta t$$

and where $n_{i\cdot}(t)$ is again obtained as the value that solves:

$$\left[\frac{\partial Q_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}}\right](1-D_i(t))e^{-\xi t} = [\theta_i^s(t)-\theta_i^i(t)]S_i(t)\alpha_j I_j(t).$$

These correspond to equations (32)-(35) in the paper plus the equations determining the evolution of the epidemiological variables, once we have imposed equilibrium conditions.

ii. Given a particular grid, two adjacent guesses of $\mathcal{D}_2$ may lead to diverging paths for I_i. If this is the case, pick the two guesses that split the paths between those diverging upwards and downwards and re-draw a finer grid for $\mathcal{D}_2$ within these bounds.

iii. Repeat for all periods until $I_i(t)$ reaches the desired initial condition, that is, $I(t) = 10^{-5}$ and $I_i(t) < I_i(t+1)$ in a flat line (meaning it does not diverge to plus or minus infinity). If at this t we have $D_1(t) = D_2(t)$ go back to outside layer of the loop. Otherwise, adjust guess $I_2(T)$.

3. If at this t we have $|D_i(t)| < 10^{-5}$ stop. Otherwise, adjust guess $\mathcal{D}_1$.

Associated Figures

This section in the paper is associated with Figure 10, which uses the parameters described in Table E.7.

Table E.7: Behavioral response parameters - Figure 10 in draft.

Parameter	Value
σ	5
ϕ	1.5
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.10
α_1, α_2	$0.1, 0.1$
$\gamma_i + \eta_i$	$0.20, 0.20$
$\eta_i/(\gamma_i + \eta_i)$	$0.003, 0.0062$
Δt	$1/3$
ξ	$0.05/(365 \times \Delta t)$

Notes about the Algorithm

This algorithm is not closed, as it still requires a mechanism that will automatically define which are the bounds for $\mathcal{D}_2$ in step 2(c)ii.

E.5 Adjustment Costs and the Risk of a Pandemic

Solution Algorithm

1. Choose $T(\infty) = 500,000$ (some large number), and T = 10,000. Guess $D(\infty) = \mathcal{D}_i$.

2. Compute the value of $n_{ii}(\infty)$, $n_{ij}(\infty)$, $n_{ij}(\infty)$, and $n_{jj}(\infty)$ as the outcome of the equilibrium that solves

$$n_{ij} = \left(c\left(\sigma - 1\right)\mu_{ij}\right)^{-1/(\phi-1)} \left(d_{ij}\right)^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ij}w_j}{Z_j P_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)}$$
$$\pi_{ii} w_i L_i(1-\mathcal{D}_i) + \pi_{ji} w_j L_j (1-\mathcal{D}_j) = w_i L_i (1-\mathcal{D}_i),$$

where π_{ij} is once again given by

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell\in\mathcal{J}} X_{i\ell}} = \frac{(w_j/Z_j)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times (\mu_{ij})^{-\frac{1}{\phi-1}} (d_{ij})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{ij})^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell\in\mathcal{J}} (\mu_{i\ell})^{-\frac{1}{\phi-1}} (d_{i\ell})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{i\ell} w_\ell / Z_\ell)^{-\frac{\phi(\sigma-1)}{\phi-1}}}$$

corresponding to equation (9) in the paper.

3. Transversality conditions are satisfied if

$$\lim_{t\to\infty} \theta_i^k(t) = 0$$
$$\lim_{t\to\infty} \theta_i^i(t) = 0$$
$$\lim_{t\to\infty} \theta_i^s(t) = 0$$

Set $\theta_i^k(\infty) = \theta_i^i(\infty) = 0$ and let the economy run without infections between T and $T(\infty)$, that is, for each time period $t \in [T, T(\infty)]$ update the multipliers as

$$\theta_i^k(t) = \theta_i^k(t+1) - \left[Q_i(n_{ii}(\infty), n_{ij}(\infty)) - C_i(n_{ii}(\infty), n_{ij}(\infty))\right] e^{-\xi t} \Delta t$$
$$\theta_i^i(t) = \frac{1}{1 + (\gamma_i + \eta_i)\Delta t} [\eta_i \theta_i^k(t) \Delta t + \theta_i^i(t+1)]$$

where Δt is the step size (one over how many times you update within each day). Keep $\theta^k(T)$ and $\theta^i(T)$ as the terminal values of the Lagrange multipliers.

4. Set $I_i(T) = 10^{-7}$, $\theta_i^s(T) = 0$ and $S_i(T) = 1 - I_i(T) - \mathcal{D}_i/(\eta_i/(\gamma_i + \eta_i))$. Recompute $n_{i\cdot}(T)$ as the values that solve

$$\left[\frac{\partial Q_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(T), n_{ij}(T))}{\partial n_{ij}}\right] (1-\mathcal{D}_i) e^{-\xi T} = [\theta_i^s(T) - \theta_i^i(T)] S_i(T) \alpha_j I_j(T),$$

corresponding to equation (32) in the paper. Given perfect symmetry between countries, we will have $w_i = 1$ for all i.

5. For each $\tau - 1 \in [T, 0]$ solve the following system of equations, where all values evaluated at τ are known and we have imposed perfect symmetry between countries, to obtain values at t:

$$\begin{aligned}
\theta^s(\tau) - \theta^s(\tau-1) &= [\theta^s(\tau) - \theta^i(\tau)][2\alpha n_i(\tau)I(\tau) + 2\alpha n_j(\tau)I(\tau)]\Delta\tau \\
\theta^s(\tau) - \theta^s(\tau-1) &= (\gamma+\eta)\theta^i(\tau)\Delta\tau - \eta\theta^k(\tau)\Delta\tau \\
\theta^k(\tau) - \theta^k(\tau-1) &= \left[Q(n_j(\tau), n_j(\tau)) - C(n_i(\tau), n_j(\tau)) - \psi_1(|g_{ii}(t)|^{\psi_2} + |g_{ij}(t)|^{\psi_2})\right] e^{-\xi\tau}\Delta\tau \\
I(\tau) - I(\tau-1) &= S(\tau)[2\alpha n_i(\tau)I(\tau) + 2\alpha n_j(\tau)I(\tau)]\Delta\tau - (\gamma+\eta)I(\tau)\Delta\tau \\
S(\tau) - S(\tau-1) &= -S(\tau)[2\alpha n_i(\tau)I(\tau) + 2\alpha n_j(\tau))I(\tau)]\Delta\tau \\
D(\tau) - D(\tau-1) &= \eta I(\tau)\Delta\tau
\end{aligned}$$

and where $n_{i\cdot}(\tau)$ is obtained as $n_{i\cdot}(\tau+1) - g_{i\cdot}(\tau) \times \Delta t$ for the value of $g_{i\cdot}(\tau)$ that solves:

$$\begin{aligned}
& e^{-\xi\tau}\left[\frac{\partial Q_i}{\partial n_{ij}}(n_{ij}(\tau)) - \frac{\partial C_i}{\partial n_{ij}}(n_{ij}(\tau))\right] \times (1 - D(\tau)) \\
& + \sum_{t=\tau+1}^{\infty} e^{-\xi t}\left[\frac{\partial Q_i}{\partial n_{ij}}(n_{ij}(t)) - \frac{\partial C_i}{\partial n_{ij}}(n_{ij}(t))\right] \times (1 - D(t)) \\
& - (\theta^s(\tau) - \theta^i(\tau)) \times S(\tau) \times \alpha \times I(\tau) - \sum_{t=\tau+1}^{\infty} (\theta^s(t) - \theta^i(t)) \times S(t) \times \alpha \times I(t) \\
& - \psi_1\psi_2 \left|\frac{n_{ij}(\tau+1) - n_{ij}(\tau)}{\Delta\tau}\right|^{\psi_2 - 1} \times (1 - D(\tau))e^{-\xi\tau} \\
& = 0.
\end{aligned}$$

Note that, in contrast to the other cases above, we compute changes as happening between τ and $\tau - 1$, rather $\tau + 1$ and τ. This makes the system easier to solve backwards, although the difference in solutions is negligible for small enough step size.

6. Repeat for all periods until $I(\tau)$ reaches the desired initial condition, that is, $I(\tau) = 10^{-5}$. If at this τ we have $D(\tau) = 0$ stop. Otherwise, adjust guess $\mathcal{D}_i$.

Associated Figures

This section in the paper is associated with Figure 11, which uses the parameters described in Table E.8.

Table E.8: Behavioral response parameters - Figure 10 in draft.

Parameter	Value
σ	5
ϕ	1.5
Z_1, Z_2	1
L_1, L_2	$3, 3$
$d_{12} = d_{21}$	1.1
$\mu_{12} = \mu_{21}$, $t_{12} = t_{21}$	1
δ	1
ρ	1
c	0.10
α_1, α_2	$0.1, 0.1$
$\gamma_i + \eta_i$	$0.20, 0.20$
$\eta_i/(\gamma_i + \eta_i)$	$0.0062, 0.0062$
ξ	$0.05/(365 \times \Delta t)$
ψ_1	1
ψ_2	4
Δt	$1/10$