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Case Study II: The Physics of Ocean Circulation and its Relation to Adjoint Modeling

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I Introduction

Climate change, perhaps the most urgent and universal issue of our time, involves a complicated web of interrelated processes, many of which are quite complicated, themselves. As a result, although there is consensus that climate change is occurring, it is difficult to determine what its effects will ultimately be or when they will be realized. As this has potentially critical consequences for civilization, as we know it, more accurate predictions obtained through a deeper understanding of the processes at play must be pursued in the coming years.

One such process, complicated yet critical to climate change, is ocean circulation. Covering about 71% of the Earth and interacting strongly with ice and the atmosphere, oceans are an active component of the climate system (Rahmstorf, Ocean Currents and Climate Change, 1997). Sediment at the ocean floor gives us information about past environmental conditions, and it appears that most of the spikes in oceanic conditions corresponded with climate shifts on land (Bond et al. 1993 as cited in Rahmstorf, Ocean Currents and Climate Change, 1997). This makes sense, as oceanic currents are responsible for much of the heat transfer that occurs between different parts of the globe, yet also respond to changes in environmental conditions (Ocean and Climate - The Odd Couple). In this way, they are intimately related to the global climate system.

Like many other components of the global climate system, ocean circulation is driven by various competing forces, and is thus quite difficult to characterize in a simple, reliable manner. Small-scale mixing processes, such as wind, buoyancy, shear and internal waves, and frictional processes at the ocean floor, as well as large-scale processes, such as heat uptake, carbon dioxide uptake, precipitation, and melting affect ocean circulation and each other (Ocean Mixing). Any model that attempts to describe ocean circulation must account for each of these interconnected processes, in addition to each parameter's spatial variance. The result is an incredibly high-order problem with a huge number of unknowns and linear equations, limited by computation power and storage (Majda, 2012; Tziperman & Thacker, 1989).

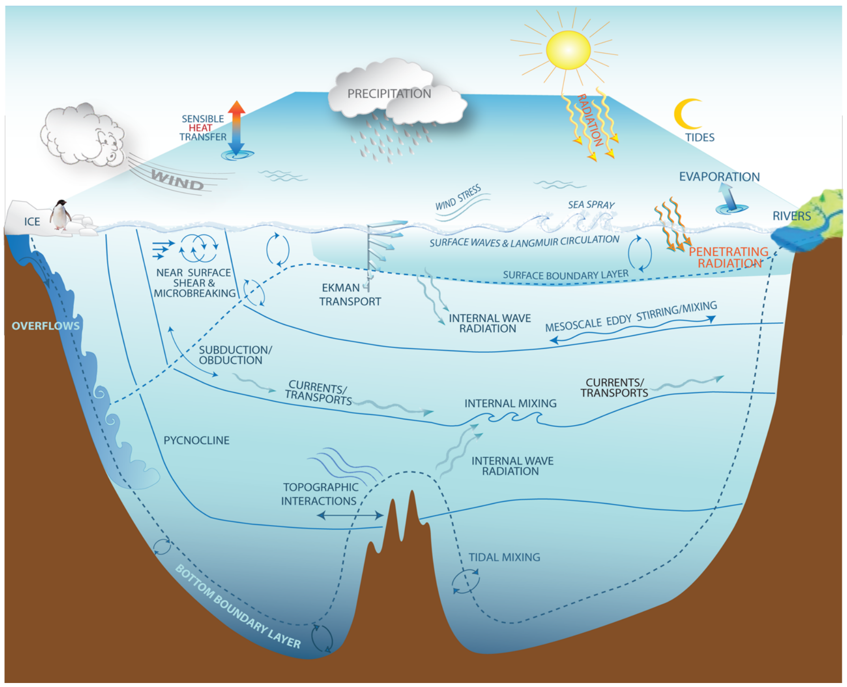

Figure 1: A schematic of the small-scale mixing processes in the ocean. Source: http://www.gfdl.noaa.gov/ocean-mixing

One method used to cope with these limitations is the "adjoint" method. Adjoint models give the sensitivity of a diagnostic to all forcing fields in a single integration. It has been shown to be extremely valuable in highlighting the processes important to ocean circulation, as well as the time scales on which they exert their effects. (Bugnion & Hill, 2006). In the context of ocean circulation, adjoint modeling has many applications. One such application is the evaluation of carbon sequestration efficiency (Hill, Bugnion, Follows, & al., 2004).

In this case study, I will describe some of the forces behind ocean circulation, and its importance in the context of climate studies. I will then discuss the importance of reliable ocean circulation models, and the problems with the traditional methods of modeling ocean circulation. I will finally discuss the adjoint method as a feasible solution, the mathematics behind it, and a potential application in the quest to preserve our environment.

II Ocean Circulation in the Context of Climate Change

The Physics

Part of the ocean system's effect on climate stems from its sheer size. Covering about 71% of the Earth, the oceans absorb twice as much of the sun's radiation as the atmosphere or land surface (Rahmstorf, Ocean Currents and Climate Change, 1997). Not only do the oceans absorb energy from the sun, but they also store it in the form of heat with little change in temperature (Ocean and Climate - The Odd Couple). This is due to the large heat capacity of the oceans, which allows them to damp global temperature fluctuations.

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To find the amount of heat required to raise the upper 1000 M of ocean by 1°C:

Area of Earth: $5.1 \times 10^{14}\ m^2$

71% of Earth covered by ocean

Water's heat capacity: C = $4.2\,\frac{KJ}{kg*K}$

Volume of upper 1000 m: $(0.71)(5.1 \times 10^{14}\ m^2)(10^3) = \ 3.6\times 10^{17}\ m^3$

Heat = q = $(C)(\Delta m)(\Delta T) = \ 4.1 \times 10^{17}\ kWh\ \approx \mathbf{1.5 \times 10^{24}}\ \boldsymbol{W}$

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Furthermore, as the oceans circulate and mix, they transport vast amounts of heat around the globe. In fact, they transport about the same amount of heat as the atmosphere, though they are more confined than the atmosphere due to the presence of landmasses. Therefore, the heat transport is localized and channeled into specific regions, resulting in currents. (Rahmstorf, Ocean Currents and Climate Change, 1997)

The formation of these ocean currents is mainly driven by density, which depends on temperature and salinity. Thus, it is generally called thermohaline circulation. Salty seawater is denser than pure freshwater, and cold seawater is denser than warm seawater due to tighter packing of molecules. Heat moves towards cooler regions, and highly concentrated solutions diffuse into regions of low concentration; this generally governs the direction and location of currents. Solutions of low density rise above solutions of high density; this generally governs the depth of the currents. This process can be seen in the polar regions, where sea ice forms due to low temperatures, increasing the salinity of the surrounding water, causing it to sink. (Ocean and Climate - The Odd Couple)

The North Atlantic is at the receiving end of one such circulation system linking the two poles. Colloquially, it is known as the great ocean conveyor belt, which is a rather fitting name. An upper branch loaded with heat moves north, delivers the heat to the atmosphere, cools and sinks, and then returns south at about 2-3 kilometers below the sea surface as North Atlantic Deep Water (NADW).

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The quantity of heat transported to the northern North Atlantic by
the great ocean conveyor belt is about the equivalent of
1 petawatt (PW) (Rahmstorf, Ocean Currents and Climate Change, 1997).
Assuming a fossil fuel burning power plant ~ 500MW = $5\times10^8\ W$:

1 PW = $\mathbf{1\times10^{15}}\ W \approx$ 2 million power plants

Global energy demand in 2014:

$$Power = Population * \frac{Income}{capita} * \frac{watts * year}{\$GDP}$$

$$Power = (7\times10^9 p)(\frac{\$5000}{p})(0.5\ \frac{W * yr}{\$} = 17.5\times10^9\ \frac{J}{s} = 17.5\ TW$$

1 PW = about 60 X total global energy consumption

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The effect of the great ocean conveyor belt can be demonstrated by comparing the temperature between locations of the same latitudes in North America and Europe. Regional climates are drastically affected by the trajectory of the warm water transported in this circulation system

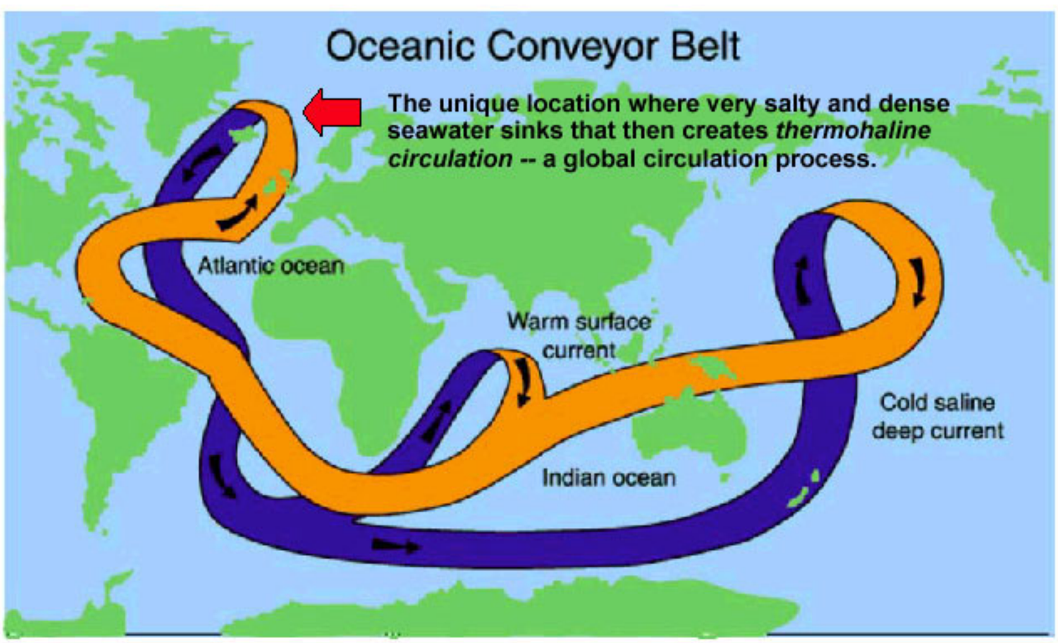

Figure 2: Oceanic Conveyor Belt. Image from http://www.groundtruthinvestigations.com/documents/climatechange.html

Implications for Climate

The great ocean conveyor belt's role in determining regional climate hints at its profound implications for global climate change. Moderate temperatures in Western Europe are not the only effect of this ocean circulation pattern; the increase in water density caused by strong trade winds and evaporation in the North Atlantic cause the warm northbound water to sink, preventing it from warming the North Pole too drastically. In fact, when this sinking pattern arose, it greatly contributed to the onset of an ice age (Crouse). That said, models have produced results indicating two stable climate states (one with the conveyor belt, and one without it), implying indicate that the northern hemisphere would experience significant cooling if the heat transfer from the conveyor belt were to stop (Rahmstorf, Ocean Currents and Climate Change, 1997).

Climate scientists are working hard to determine what aspects of the Earth's climate system could be tipping points with the potential to cause abrupt shifts between relatively stable climate states. There is a strong sentiment that events related to ocean circulation, such as a shift in the Gulf Stream, or the shutoff of the North Atlantic deep-water formation leading to re-organization of Atlantic thermohaline circulation, might be a few of these tipping points. There is ample concern that the rise in greenhouse gas concentrations and global temperatures could influence the ocean's deep-water circulation patterns, initiating a positive feedback loop of further climate change. (Crouse) (Rahmstorf, Ocean Currents and Climate Change, 1997) (Research Focus Groups, 2013) Information about past environmental conditions, drawn from ocean floor sediment and paleo-data, indicates that shifts in oceanic conditions do indeed correspond to shifts in overall climatic conditions. Scientists believe that certain cold climate episodes, occurring over a few decades or even less, were caused by abrupt disturbances in the ocean currents of the North Atlantic. (Rahmstorf, Ocean Currents and Climate Change, 1997)

III Modeling of Ocean Circulation

In light of this evidence of ocean circulation's relationship to climate change and environmental conditions, the need for robust and reliable models of ocean circulation is clear. Knowledge is power, and the ability to predict climate phenomena like El Niño or more drastic shifts in circulation patterns could allow governments and individuals to better prepare for them. Ideally, predictions could also lead to environmental policy measures that are quick and powerful enough to improve the situation before it is too late.

Unfortunately, modeling of ocean circulation is not as simple as one would hope. While ocean circulation models do exist and have absolutely improved scientists understanding of the process, they are not powerful or accurate enough to be as effective as they could be. Complications include, but are not limited to: parameterization of ocean processes, nonlinearity of thermohaline circulation, poorly known parameters, and vast amounts of relevant data.

Computational power and storage are also significant limiting factors. For example, in modeling the ocean circulation of the North Atlantic, a model with a grid of 30 x 30, eight vertical levels, and five parameter fields, the number of unknowns and model equations will be 30 x 30 x 8 x 5 = 36000. Traditional models use a matrix with dimensions of (number of unknowns) x (number of equations), meaning the matrix would have a dimension on the order of 10^9. (Tziperman & Thacker, 1989) This is an extremely high computational cost for a model of low resolution with few parameters.

IV Adjoint Modeling – An Effective Method

Ocean general circulation models (OGCMs) generally consist of a set of discretized partial differential equations, prognostic variables, and parameters. Input parameters can be determined through what is called an "inverse" problem, which is optimized to minimize the "cost function," which describes how well a model matches observations (Marotzke, Giering, Zhang, & al., 1999) (Tziperman & Thacker, 1989).

The OGCM is sensitive to changes in each of its parameters, boundary conditions, and initial conditions. The goal is to assess these sensitivities and improve the model, accordingly. The problem is that minimizing the cost function is extremely slow and inefficient without information about its gradient, or the directional behavior of the function (Tziperman & Thacker, 1989). Thankfully, the adjoint method is an efficient way of calculating this gradient.

The adjoint method is as follows: A numerical algorithm is used to obtain the value of a cost function, given initial guesses for the parameters. The adjoint model, in a single adjoint integration step, outputs the gradient, or the sensitivity of the cost function to its initial conditions, control variables, and physical parameters (Marotzke, Giering, Zhang, & al., 1999) (Bugnion & Hill, 2006). This is generally calculated using technology called Automatic Differentiation, which provides computer code for efficiently generating accurate derivatives of a given function (Tangent Linear and Joint Code Generation Via Automatic Differentiation). This information is then used to improve the parameter values and further reduce the cost function. This process is repeated several times until a minimum value for the cost function is obtained.

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For a general model in which $\boldsymbol{X}_n$ denotes the state of a model at time step n, where $0 \leq n \leq N$, and initial conditions are $\boldsymbol{X}_0$, but the cost function J depends only on the final state $\boldsymbol{X}_N$, we can write:

$$J = f \circ \boldsymbol{X}_N$$

where f is a scalar function mapping $\boldsymbol{X}_N$ onto the real axis. J is linked to the control variables by iteration, or repeated application of the numerical model:

$$J = f \circ \psi_N \circ \psi_{N-1} \circ \psi_{N-2} \circ \ldots \circ \psi_2 \circ \psi_1 \circ \boldsymbol{X}_0 = J\left(f\left(\psi_N\left(\psi_{N-1}(\ldots(\psi_1(\boldsymbol{X}_0)\ldots))\right)\right)\right)$$

The sensitivity of J to the control vector is given by the chain rule:

$$\frac{\partial J}{\partial \boldsymbol{X}_0} = 1 \circ f' \circ \psi'_N \circ \psi'_{N-1} \circ \psi'_{N-2} \circ \dots \circ \psi'_2 \circ \psi'_1 \circ \boldsymbol{I}$$

where $\boldsymbol{I}$ is the unit matrix, and every ψ'_n is the Jacobian matrix of the model at *n*, or the sensitivity of the state *after* time step *n* to the state *before* time step n.

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In the case where the cost function represents the discrepancy between model and observations, the value of the cost function will equal zero if the observations exactly match the steady-state model solution. (Tziperman & Thacker, 1989) Using the adjoint method when solving the inverse problem allows one to avoid working with large matrices, and significantly reduces computation time. For example, in their ECCO2 project, Brix, Menemenlis, Hill et al. were able to adjust about two billion model parameters using this method – something that would not have been feasible without the adjoint technique (Brix, Menemenlis, Hill, & al., 2015).

As more types of data from satellites and new technologies emerge, and as numerical models are further improved, the adjoint technique will be useful in modeling the processes of ocean circulation. Furthermore, these new types of observations will be largely driven by the results of adjoint models, which can indicate the relative effectiveness of particular data types in determining unknown parameters. (Tziperman & Thacker, 1989)

Potential Applications

This adjoint technique has been applied to many areas of oceanography, meteorology, and other fields. Specifically in oceanography, it has been used to improve models of time-mean and seasonally varying general circulations of the North Atlantic, heat transport across a transoceanic section, circulation's dependence on bottom topography, and many other topics.

One application that I find to be particularly intriguing is the efficiency of carbon sequestration, or the process of removing atmospheric CO_2 and storing it, in this case in the oceans (Carbon Dioxide Capture and Sequestration). Hill, Bugnion, Follows and Marshall used the adjoint method to develop a three-dimensional map of carbon sequestration efficiency and mean residence time in an ocean general circulation model. They found the adjoint technique to be effective in describing temporal and spatial variations in their parameters. It allowed them to compute about 50,000 perturbation experiments in a single numerical integration, while only increasing the computation cost by a factor of five over the cost of a single perturbation experiment. (Hill, Bugnion, Follows, & al., 2004)

In their project, they use a carbon-like tracer *C* in their model to mimic the behavior of carbon, with the cost function measuring the amount of *C* lost.

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The behavior of *C* is governed by the following equation, where $\vec{U}$ represents velocity and model currents, *K* represents isopycnal stirring, $\Gamma(C)$ represents transport during convective adjustment, and *S* represents injection sources:

$$\frac{\partial C}{\partial t} = -\vec{U} \cdot \nabla C + \nabla \cdot (K \nabla C) + \Gamma(C) - \mu C + S$$

The model's cost function represents the total amount of *C* lost by outgassing, or vapor release across the sea surface, where *A* is the area, and Δz is surface layer thickness

$$J(t = T) = \int_{t=0}^{t=T} \int_{A} \mu C \Delta z \, dA dt$$

This represents the number of moles of CO_2 that would be released into the atmosphere in time interval *T*. The rate at which *J(t)* rises depends not only on injection rate, but also on the rate of sea surface exposure, governed by mixing and circulation processes.

Two sensitivity parameters describe the air-sea flux of injected carbon:

$$\tilde{C} = \frac{1}{V} \frac{\partial J}{\partial C}(\lambda, \phi, z, t) \qquad \tilde{S} = \frac{1}{tV} \frac{\partial J}{\partial S}(\lambda, \phi, z, t)$$

where $\tilde{C}$ represents impulsive local injections of carbon, while $\tilde{S}$ is continuous. These express the ratio of total (time integrated) outgassed moles of carbon to the total (impulsively or continuously) injected input moles of carbon

The adjoint model yields the sensitivities:

$$C^*(\lambda, \phi, z, t) = \frac{\partial J}{\partial C}(\lambda, \phi, z, t) \qquad S^*(\lambda, \phi, z, t) = \frac{\partial J}{\partial S}(\lambda, \phi, z, t)$$

They define efficiency as the amount of injected carbon remaining in the ocean at time *t* = *T*. For a continuous source, efficiency can be derived from $\tilde{S}$ as:

$$E(t = T) = 100 \cdot \left(1 - \tilde{S}(t = T)\right)$$

They also consider mean residence time, $\bar{R}$, or the average time from impulsive injection that carbon stays in the ocean before being lost into the atmosphere:

$$R(t = T) = \left(1 - \tilde{C}(t = T)\right)$$

$$\bar{R} = \int_{t=0}^{t=T_f} R(t) dt = \int_{t=0}^{t=T_f} \left(1 - \tilde{C}(t)\right) dt$$

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The results of this study showed that the North Atlantic basin is more efficient at sequestering the tracer over timescales of several hundred years, while the Pacific is more efficient over shorter time scales. This information could be extremely valuable in analyzing possible solutions to the large and increasing concentration of CO_2 in the atmosphere.

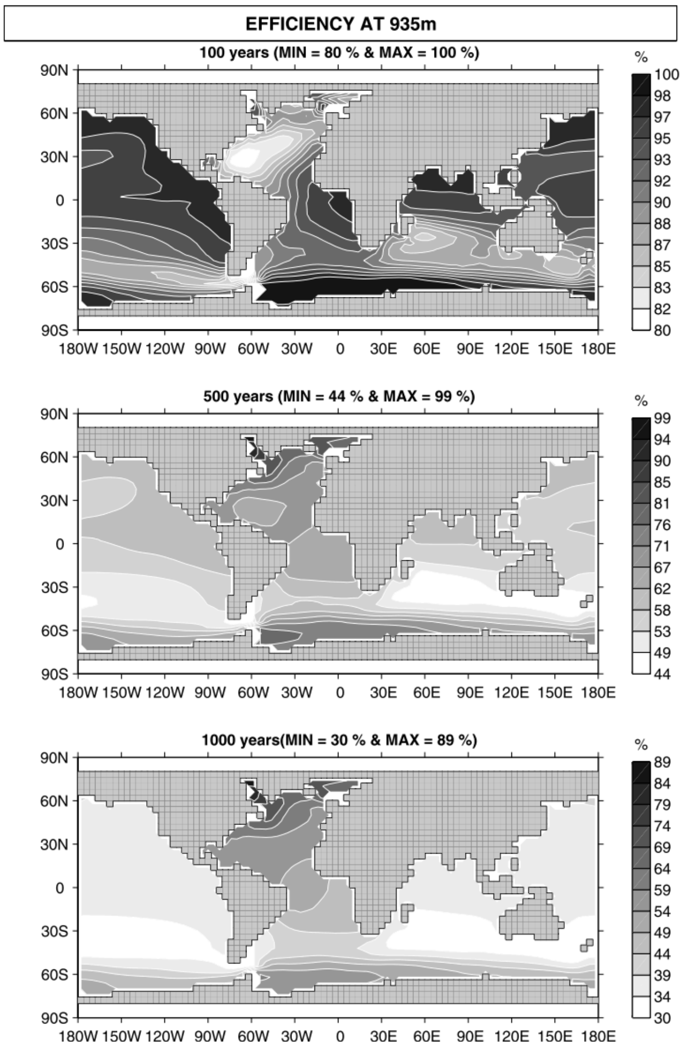

Figure 3: Sequestration efficiency, E, mapped as a function of latitude and longitude for continuous injection sources at 935 m in the ocean model at three time intervals (100, 500, and 1000 years) following the commencement of injection. Image from Hill et al.

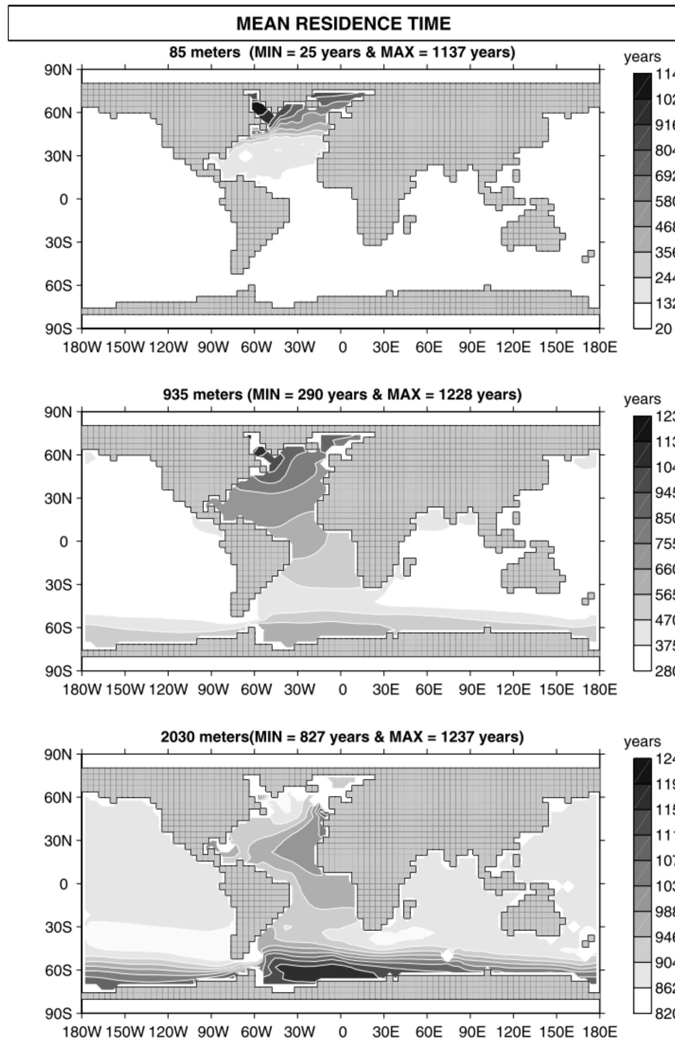

Figure 4: Mean residence time R, mapped for continuous injection sources at three depth levels in the ocean model.. Image from Hill et al.

The authors of this study propose a few future directions that might improve the results of their work. For example, they state that future studies should include a more comprehensive description of the ocean-atmosphere carbon system, to account for atmospheric push back (Hill, Bugnion, Follows, & al., 2004). They also propose extending this study to more closely examine the dependence of sequestration efficiency on certain parameters of the ocean model. In fact, this is one of the most important benefits of adjoint modeling: the assisting in the "targeting" of observations (Errico, 1997). In other words, the sensitivity fields given by the adjoint model indicate where it is most important to produce an accurate initial condition for a forecast. For parameters or geographic regions with large sensitivity, a large amount of error will greatly reduce the accuracy of the model. In this way, adjoint models not only allow for more efficient and accurate modeling, but also have a sort of built-in mechanism for improvement.

V Conclusion

Modeling of ocean circulation will continue to draw attention from scientists in coming years due to its inextricable ties with global climate. Ocean circulation both affects and is affected by shifts in environmental conditions. Although the oceans often tend to damp climatic changes, they can also augment them if conditions are disturbed enough to push ocean circulation over a "tipping point". In fact, scientists believe that ocean behavior has been extremely variable for the last hundreds of thousands of years – it is only in the past 8,000 years or so that oceans have been relatively stable. (Rahmstorf, Ocean Currents and Climate Change, 1997) Despite what we know about the massive amounts of heat required to actually warm the ocean, circulation is governed by a host of other delicate, complicated, interrelated processes. It is critical to understand the manner in which human-induced climate change is affecting ocean circulation in the coming years, as its effects are becoming more pronounced.

In order to understand such large and complicated processes, scientists must simulate their behavior using mathematical models. In the case of ocean circulation, the large amount of parameters and large spatial grid make the development and use of these models computationally difficult. Adjoint modeling is an effective solution to this problem, offering the ability to investigate multiple perturbations in one step. Further, the sensitivity information it provides gives insight into the relative importance of certain observations and their required precision.

There have already been several models created which use adjoint modeling techniques. However, there is still much to be done in the way of accounting for all of the important parameters, and acquiring enough relevant, high-quality data. Even a simple model, such as the one described in the previous section, in which there are only a few main variables, depends heavily on complex and largely unknown parameters. The adjoint method will be useful in guiding future observations and developing more reliable and accurate models of ocean circulation for climate studies.

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