The brighter-fatter effect

Observation, Interpretation and Correction

Based on Guyonnet et Al. 2015
Context

Multi-probes Cosmology from Wide Field Imagers

Wide field imagers:
- DeCam @ CTIO
- H(SC) @ Subaru
- MegaCam @ CFHT
- LSST soon @ CTIO

2 cosmological probes:
- Type Ia SuperNovae (SNIa)
- Weak lensing
  - Clusters of galaxies
  - Cosmic Shear
Observation from an optical imager

celestial objects: Which ones are stars and which ones are galaxies?

SuprimeCam @ Subaru Telescope in Hawaii
celestial objects: Which ones are stars and which ones are galaxies?

Galaxies are extended sources

Stars are point like sources
The Point Spread Function (PSF)

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<td>Focal Plane</td>
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![Diagram of the point spread function (PSF)](image)
Using the Point Spread Function (PSF), it is possible to separate stars from galaxies.
Prologue

Separating stars from galaxies using objects’ size

Optical Imager

Astier et al. 2013

Size in X direction (pixel)

Size in Y direction (pixel)

Stars

Galaxies
The PSF is useful to measure the flux of faint objects, such as Type Ia Supernovae.
PSF is used to measure flux of SNIa

Crowded field, low S/N, varying background …
So, there is this paradigm in image analysis:
The apparent size of the stars are independent of their brightness. Then, they can be used to determine one PSF per exposure.

How true is this?
... Well, it is not: It is called the brighter-fatter effect (i).

**DECam**

\[
\tilde{M}_g = 2 \sum_{\text{pixels}} (\vec{x}_i - \vec{x}_c)(\vec{x}_i - \vec{x}_c)^T e^{-\frac{1}{2}(\vec{x}_i - \vec{x}_c)^T \tilde{M}_g^{-1}(\vec{x}_i - \vec{x}_c)} I_i \\
\sum_{\text{pixels}} e^{-\frac{1}{2}(\vec{x}_i - \vec{x}_c)^T \tilde{M}_g^{-1}(\vec{x}_i - \vec{x}_c)} I_i
\]
… Well, no : It is called the brighter-fatter effect (iii)

LSST

Camera sensor candidates

![Diagram showing relative variation of spot width as a function of spot peak flux on CCD for E2V-250 at two wavelengths, 550 nm (left panel) and 900 nm (right panel).]
The brighter-fatter effect is the observation that the width of stellar images or laboratory luminous spots increases with the flux of the object.

- It has been seen by all the telescopes that looked for it,
- This is linear with the flux,
- This is slightly asymmetric,
- There is no chromaticity detected.

What is the impact on science?
Where does it come from?
What is the impact on observational cosmology using optical telescopes?
Impact on Science

Dark energy with SNIa, in 1 slide

Hubble Diagram

Astier & al. 2007

$\mu_B \propto -2.5 \log_{10}(\Phi_{PSF})$

PSF photometry
Overestimation of integrated flux of faint objects

\[ \hat{\Phi} = P \hat{PSF} = \sum_p (w_p I_p \ PSF) \]

Assuming that the faint object has an actual PSF smaller than the one of the model:

\[ I_p = P \hat{PSF} \]

An error in the PSF translates in an error on the flux the following way:

\[ \frac{\hat{\Phi}}{\Phi} = \frac{\sum_p (PSF \cdot P \hat{PSF})}{\sum_p PSF^2} \]
Impact on Science

Weak Lensing

Weak lensing principle

Effects on the image:
→ Rotation
→ Displacement
→ Magnification
→ Shear

Deflection potential

\[ \nabla^2_\theta \psi = \frac{8 \pi G}{c^2} \int \frac{D_L D_{LS}}{D_S} \rho d\ell \]

Lensing can be used to infer the distant gravitational potential
Shear measurements: round objects appear elongated

- Statistically, an average net elongation indicates shear,
  ➡ after accounting for the PSF
  ➡ that is measured using star images

- So, shear is encoded in the difference of the average shape of galaxies and stars.
Impact on Science

An example from the Subaru Telescope

Difference of the average size of stars and galaxies

SuprimeCam @ Subaru

Wheighing the gas clusters
Impact on Science
Impact on morphometry

Impact on the observed shape of a galaxy

Observation

« true » PSF
brighter-fatter

« true » galaxy
Reconstructed ellipticity

Quantifying the problem:

Usual parametrization of shear bias

\[ \hat{\gamma} = (1 + m)\gamma + c \]

Impact of a 1% « brighter-fatter » on the LSST

\[ m \approx 0.027 \]

Meanwhile, the requirement is

\[ m_{req} \approx 0.003 \]

Ellipticity:

\[ \epsilon_{gal} = \left( \frac{g_{mx} - g_{my}}{g_{mx} + g_{my}} \right) + i \left( \frac{2 g_{mx}y}{g_{mx} + g_{my}} \right) \]

Relation between ellipticity and shear:

\[ \epsilon_{gal} = \epsilon_{int} + \gamma \]
Weighing the Giants I (1208.0597)
Contours of mass distribution

The shear profile turns into a measurement of the mass of the cluster

\[ \gamma_1 = \frac{g_{xx} - g_{yy}}{g_{xx} + g_{yy}} \quad \gamma_2 = \frac{g_{xy}}{g_{xx} + g_{yy}} \]

\[ \gamma_t = - (\gamma_1 \cos(2\theta_c) + \gamma_2 \sin(2\theta_c)) \]

Shear VS radius

Contour of mass distribution

Impact on Science

Cosmology with weak lensing: Cluster Count

Weak Lensing
Impact on Science

Cosmology with weak lensing: Cluster Count and Cosmic Shear

Cluster count

Mantz et al. 2014

Cosmic shear

Kitching et al. (2014)

Mass of Clusters of galaxies as a function of redshift (strong signal)

Two points correlation function of galaxies’ shape (weak signal)
Where does the effect come from?
Brighter-fatter effect

Uniform illumination : Flatfield image

CTIO Screen

The contrast on the image comes from Photon noise

http://decamctio.blogspot.fr/
Brighter-fatter effect

Uniform illumination : Flatfield image

C. Laige 2016

The noise should follow a Poissonian statistic …
Photon transfer curve: « Sub » Poissonian noise?

Variance \( \equiv \frac{1}{\text{gain}} \times \text{Nadu} \)

Flux \( (\text{ke}^-) \)

Variances \( (\text{ke}^-) \)

LSST

Photon transfer curve

Poissonian Distribution

Brighter-fatter effect

Interpretation

PTCs residuals to Poissonian noise \( (\text{ke}^-) \)
**Brighter-fatter effect**

**Pixel spatial correlations**

![Graph showing flux vs. variance for different pixel spatial correlations.](image)

**Interpretation**

\[
R(k, l) = \left( \frac{\sum_{i=K}^{I} \sum_{j=L}^{J} \frac{\text{pix}_{i,j} - \mu}(\text{pix}_{i+k,j+l} - \mu)}{\sigma^2} \times \frac{\text{pix}_{i+k,j+l} - \mu}{(I-K) \times (J-L)} \right)
\]

**LSST E2V candidate [zoom]**

- Blue dots: \(R(0,1), \text{Amp1-8}\)
- Red triangles: \(R(1,0), \text{Amp1-8}\)
- Green squares: \(R(1,1), \text{Amp1-8}\)

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Brighter-fatter effect

Pixel spatial correlations

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<th>DECam</th>
<th>MegaCam</th>
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<tr>
<td><img src="image1" alt="LSST graph" /></td>
<td><img src="image2" alt="DECam graph" /></td>
<td><img src="image3" alt="MegaCam graph" /></td>
</tr>
</tbody>
</table>

**Interpretation**

1. **Brighter-fatter effect**
2. **Interpretation**

- **Pixel spatial correlations**
- **2-D correlation map in flatfields at a 50 ke level**

**DES Collab.**
Gruen & al. 2015

1200 flatfields, 59 chips
<table>
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<th>Interpretation</th>
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<tr>
<td>Photon Transfer curve and Pixel spatial correlations</td>
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- Large distance correlations (up to 20 pixels separation !)
- The perturbation exactly conserves charges.
- The integral of the correlation function in flatfields is conserved.
How can we combine these two observations?

- Bright spots are broader than faint ones.
- Contrast from photon noise does not linearly increase with the flux, while pixels’ correlation are non-zero, and linearly increase.
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<td>CCD pixels</td>
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</table>

Inhomogenous distribution of the charges resulting from:
- Contrast from the photon noise in flatfield images.
- PSF of a star.

![Diagram of CCD pixels with photons and electrons]
Coulomb forces in a CCD

Empty

Filled with 50 keV
Coulomb forces in a CCD

Field lines displacement

Dynamical pixel boundaries
CCD’s pixels: top view, illumination from a star
Brighter-fatter effect

CCD’s pixels: top view, illumination from a star
Effective boundaries displacement: Pixel size variation
Brighter-fatter effect

Coulomb forces explain both observations

Correlations and brighter-fatter share the same physical origin: charges accumulated in the CCD alter the field lines.
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<th>Correction</th>
<th>Inverse problem</th>
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<td><strong>Implementation of a correction at the pixel level</strong></td>
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- The effect can be described as dynamical pixel boundary displacements caused by the evolution of the electrostatic potential due to collection of charges.

We have to figure out:

1. The relations between the stored charge and the induced boundaries’ displacement.
2. A way to recover the undisturbed images.

By solving the inverse problem
For a pixel in the surrounding, it results in a boundary displacement:

\[
\frac{\delta X}{p} = \frac{1}{2} \sum_{i,j} Q_{i,j} a_{i,j}^X
\]
Collected charges affects incoming charges.

In the perturbed image:

\[ Q'_{0,0} = Q_{0,0} + \sum_X \sum_{i,j} \frac{1}{2} a_{i,j}^X Q_{i,j} \left( \frac{Q_{0,0} + Q_X}{2} \right) \]

Charge «transfer»
The coefficients of the model are determined from the correlation measurements:

The slope of a correlation $R(i,j)$:

$$\frac{R_i,j}{\mu} = \sum_X a_i,j^X$$
But there are more parameters than observables, For instance:

For $i,j = 0, ..., 4$ there are 50 (a) terms to evaluate,

There are 24 $R_{i,j}$ measurements.

<table>
<thead>
<tr>
<th>$Q_{i,j}$</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
We suppose that the electrostatic force is isotropic and that it is a smooth function of the distance. The (a) coefficients correspond to its projection normal to the boundary:

\[ a_{i,j}^X = f(r_{ij}) \cos \theta_{i,j} \]
Measurement of pixel correlation coefficients on flatfields

pixel effective size model

An assumption on boundaries displacement

Projection of the electrostatic effect

solving the model

Charge density approximation

Raw Images → redistribution of the charges → Processed Images
Redistribution of charges: Comparing the model to the data

Reverse model as a post-processing method to move flux back to where it belongs

$$Q_{0,0} = Q'_{0,0} - \frac{1}{4} \sum_{X} \sum_{i,j} a_{i,j}^{X} Q_{i,j} (Q_{0,0} + Q_X)$$
Assessment of the quality of the correction

After correction  >90% of the effect is gone.

Evaluating the impact of the charge density approximation on the correction:

The approximation underestimate the effect by about 4% @ IQ =1.6 pix  2% @ IQ =2 pix.
Result: SuprimeCam @ Subaru Telescope

Subaru Camera - Stars second moments

Correction
Inverse problem

Wheighing the gas clusters
**Summary of the correction at the pixel level**

**Correction**

- Measurement of pixel correlation coefficients on flatfields
  - Correlations measurements at 0.0001 level
  - *Pixel effective size model*

- Solving the model
  - Turns into a 5% uncertainty on bf
  - *Charge density approximation*
    - Local charge density approximation (few % bias)

- Redistribution of the charges
  - *Inverse problem*
    - An assumption on boundaries displacement
      - Negligible
    - Projection of the electrostatic effect

**Inverse problem**

- Raw Images ➔ Redistribution of the charges ➔ Processed Images
  - *m = 0.027 for 1% bf* ➔ *bf = 0.0005 +/- 0.0005* ➔ *shear requires m < 0.003*
Result: The correction is good for flux but not so good for shape.
Simulation: Next to leading order effects?

Simulation

Charge density approximation

Raw Images → redistribution of the charges → Processed Images
Simulation: Non-linear Brighter-Fatter?

Simulation: Next to leading order brighter-fatter?

C. Laige 2016

Well Filling with One Collecting Phase

Well Filling with Two Collecting Phases
Simulation challenge:

- We do not know all the details of CCD fabrication.

**Actual Situation.**

C. Lage, 2016 DESC meeting
How do the pixel boundaries (and their areas) respond to accumulating electrons during integration?

Direct drift calculations departure from the simulation to the linear model.

Red : direct calculation.
Black : scaled template
Relative area loss in $ij=00$: direct/scaled $\sim 1.065$
A potentially larger correction to the instantaneous area functions may be due to a varying channel depth as charges accumulate.

Rasmussen 2016
Simulation challenge:

- In many scenarios, it predicts non-linear effects.

Measurements:

- No Next to Leading Order effect has been observed.

Intriguing!
Conclusion

Photometry and morphometry of astrophysical objects are extracted from the image of their shape on a camera.

A shape is determined from the projection of an image onto a grid:

- Mapping the pixel grid is critical for ongoing and upcoming surveys.
- A combination of simulations and measurements is required to calibrate the pixel lattice.