Hybrid averaging shows that within-stance symmetry helps mitigate coupling interactions between degrees of freedom in a sagittal 2DOF monoped and 3DOF biped

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Introduction

Raibert described his method for planar hopping in three "parts" (controlled vertical hopping, controlled forward speed, and controlled body attitude), which he synthesized by developing controllers for each of these parts in isolation (e.g. vertical hopping controller ignores fore-aft motion, etc.). We have used similar strategies successfully on a tailed biped (De and Koditschek, 2015a) and a quadruped (De and Koditschek, 2016). We are interested in knowing why and when we can ignore these coupling interactions, and also how this fact can be leveraged to synthesize useful running behavior.

Hybrid averaging

We have recently introduced a tool for analyzing stability of hybrid limit cycles (De et al., 2016) using a perturbation technique borrowed from smooth dynamical systems theory: dynamical averaging (Guckenheimer and Holmes, 1990). So far, we have provided sets of sufficient conditions for local stability for tailed SLIP (De and Koditschek, 2015b) and in-place virtual bipedal gaits (De and Koditschek, 2016) using hybrid averaging as an analytical tool.

Intuitively, we reinterpret the hopping/running locomotor as a system of coupled 1DoF oscillators that must (a) be regulated (maintain their energy), and (b) be coordinated (maintain the appropriate relative phase). These coupled oscillators are then analyzed as an ϵ-perturbation of a set of decoupled oscillators, affording the following benefits:

1) Equivalence – the original system has a periodic orbit spatially "close" to the averaged system, and these orbits have the same (regulated and coordinated) stability properties.
2) Simplified Dynamics – the averaged system has co-dimension 1, and the removed dimension corresponds to one of the "fast" modes of the system. Removing these fast dynamics results in analytical as well as numerical computational benefits.
3) Implicit Form – deriving the expression for and then analyzing the simplified linearization requires neither computing the periodic orbit nor the return map.

However, in both applications so far, (a) a coordinate change to averageable coordinates had to be hand-crafted, and (b) the eigenvalues of the linearization of the return map had to be explicitly computed to verify stability.

Within-stance symmetry in running

In Appendix 5B of (Raibert, 1986), the author makes the intuitive observation that forces acting on the center of mass through stance must "integrate out" in the horizontal plane for steady horizontal progress. (Altendorfer et al., 2004) formalized (and (Razavi et al., 2016) later extended) these observations in terms of time-reversal symmetry that seems to commonly seen on the stance portion of the periodic orbit of SLIP-like running/hopping models (Razavi et al., 2016), and is intricately related to Hamiltonian or equivariant systems (Lamb and Roberts, 1998).

Our newest results show that time-reversal symmetry manifests as cancellation of cross-terms in the return map linearization of the averaged system. These ideas allow us to eliminate both difficulties (a) and (b) in the application of hybrid averaging while retaining the benefits 1), 2), 3) listed above. Additionally, we hypothesize that the "averaging out" of coupling forces between different degrees-of-freedom is key in the success of the "decoupled controllers" (as described in the Introduction).

Results

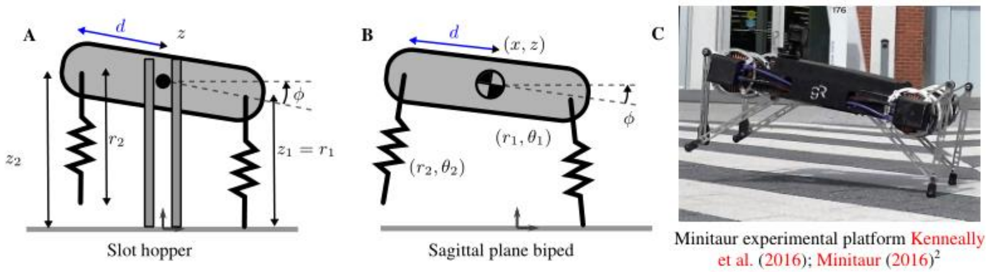

Figure 1: Virtual biped models of increasing complexity (figure from (De and Koditschek, 2016)).

So far, our mode of analysis not only allows for stability proofs for in-place virtual bipedal hopping (using the vertically-constrained bipedal "slot hopper" model in Figure 1**A**), but also helps us identify and prove stability of both in-phase and out-of-phase coordination between the two legs, without any informational coupling between them whatsoever. Our proof of coordination (De and Koditschek, 2016) shows how these regimes emerge only as a function of the non-dimensional rotational inertia of the body. We demonstrated these results on the physical quadruped, Minitaur (Figure 1**C**), snapshots of which are shown in Figure 2.

Figure 2: Preflexive bounding (left) and pronking (right) on Minitaur; stability of both are proved in (De and Koditschek, 2016).

Our latest improvements to hybrid averaging—leveraging time-reversal symmetry—promise to enable (work in progress to be completed by the conference) for the first time a proof of stability of a 5-DOF sagittal runner (Figure 1**B**) using decoupled controllers with few parameters and a great deal of empirical robustness.

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