

## **On the Welfare Costs of Consumption Uncertainty\***

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### **Abstract**

Satisfactory calculations of the welfare cost of aggregate consumption uncertainty require a framework that replicates major features of asset prices and returns, such as the high equity premium and low risk-free rate. A Lucas-tree model with rare but large disasters is such a framework. In the baseline simulation, the welfare cost of disaster risk is large—society would be willing to lower real GDP by about 20% each year to eliminate all disaster risk. In contrast, the welfare cost from usual economic fluctuations is much smaller, though still important—corresponding to lowering GDP by around 1.5% each year.

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Lucas (1987, Ch. 3; 2003, section II) argued that the welfare gain from eliminating uncertainty in aggregate consumption is trivial. He got this answer by using parameters for the time series of real per capita consumer expenditure from U.S. post-World War II macroeconomic data, along with plausible values for the coefficient of relative risk aversion.

One problem with this calculation, apparent from Mehra and Prescott (1985), is that simulations with the same model and parameters do not get into the right ballpark for explaining well-known asset-pricing puzzles, such as the high equity premium and low risk-free rate. These failures with respect to asset returns suggest, as observed by Atkeson and Phelan (1994), that the model misses important aspects of consumption uncertainty. Hence, the model's estimates of welfare effects from consumption uncertainty are likely to be inaccurate.

A possibly satisfactory framework, used here, is one with rare economic disasters, as in Rietz (1988). Barro (2006) shows that this model can replicate prominent features of asset returns. Salyer (2007) demonstrates that the allowance for low-probability crash states amplifies the welfare costs computed by Lucas. The present analysis builds on this work to show that, in a rare-disasters setting, changes in consumption uncertainty that reflect shifts in the probability of disaster have major implications for welfare.

Section I works out the baseline, Lucas-tree model with rare disasters. Key features are i.i.d. shocks to output growth and iso-elastic preferences. Section II computes welfare costs within this model; first for marginal changes in uncertainty and then for large changes. Section III discusses the dependence of welfare costs on the key preference parameter—the coefficient of relative risk aversion, which equals the

reciprocal of the elasticity of intertemporal substitution. Section IV allows for endogenous saving and investment and shows how adjustments of saving affect welfare costs. In Section V, the use of Epstein and Zin (1989)/Weil (1990) preferences allows for distinct effects on welfare costs from the coefficient of relative risk aversion and the elasticity of intertemporal substitution. Section VI discusses the asset-price-based calculations of the welfare costs of uncertainty in Alvarez and Jermann (2004). Section VII concludes by emphasizing the effects of policies and institutions on disaster probabilities and sizes.

### **I. A Lucas Fruit-Tree Model**

The initial model is a version of Lucas's (1978) representative-agent, fruit-tree economy with exogenous, stochastic production. Output of fruit in period  $t$  equals real GDP,  $Y_t$ . Population is constant. The number of trees is fixed; that is, there is neither investment nor depreciation. (A later model allows for investment.) Government purchases are nil. Since the economy is closed and all output is consumed, consumption,  $C_t$ , equals  $Y_t$ .

The log of output evolves as a random walk with drift:

$$(1) \quad \log(Y_{t+1}) = \log(Y_t) + g - (1/2)\sigma^2 + u_{t+1} + v_{t+1}.$$

The random term  $u_{t+1}$  is i.i.d. normal with mean 0 and variance  $\sigma^2$ . This term reflects "normal" economic fluctuations. The parameter  $g \geq 0$  is a constant that reflects exogenous productivity growth. The subtraction of  $(1/2)\sigma^2$  (quantitatively unimportant in the calibrations) eliminates the effect of  $\sigma^2$  on the expected growth rate of real GDP.

The random term  $v_{t+1}$  in Eq. (1) picks up low-probability disasters, as in Rietz (1988) and Barro (2006). In these rare events, output and consumption jump down sharply. The probability of a disaster is the constant  $p \geq 0$  per unit of time. The probability of more than one disaster in a period is assumed to be small enough to neglect; later, the arbitrary period length shrinks to zero. In a disaster, output contracts by the fraction  $b$ , where  $0 < b < 1$ . The distribution of  $v_{t+1}$  is given by

$$\begin{aligned} \text{probability } 1-p: v_{t+1} &= 0, \\ \text{probability } p: v_{t+1} &= \log(1-b). \end{aligned}$$

The disaster size,  $b$ , follows some probability distribution (gauged by the empirical distribution of disaster sizes).

Unlike Lucas (1987, Ch. 3), but in line with Obstfeld (1994), the shocks  $u_{t+1}$  and  $v_{t+1}$  in Eq. (1) represent permanent effects on the level of output, rather than transitory disturbances to the level. That is, the economy has no tendency to revert to a deterministic trend line.

Cochrane (1988, Table 1) used variance-ratio statistics for  $k$ -year differences to assess the extent of reversion to a deterministic trend in the log of U.S. real per capita GNP for 1869-1986. He found evidence for reversion in that the ratio of the  $k$ -year variance (divided by  $k$ ) to the 1-year variance was between 0.30 and 0.36 for  $k$  between 20 and 30 years. Therefore, at large  $k$ , the empirical variance ratio was much less than the value 1.0 predicted by Eq. (1). However, Cogley (1990, Table 2) showed that the Cochrane finding was particular to the United States. For 9 OECD countries, including the United States, from 1871 to 1985, the mean of the variance ratio at 20 years was 1.1; hence, close to the value 1.0 predicted by Eq. (1).

Cogley's results hold up for a broader sample comprising 19 OECD countries. The data on per capita GDP are for 1870-2004 from Maddison (2003), updated from World Bank, *World Development Indicators* (and using U.S. data from Balke and Gordon [1989] before 1929). For  $k=20$ , the mean of the variance ratios for the 19 countries is 1.22 and the median is 1.00, while for  $k=30$ , the corresponding values are 1.30 and 0.96. These values accord with Eq. (1). The United States—with variance ratios of 0.42 when  $k=20$  and 0.38 when  $k=30$ —has the lowest ratios at these values of  $k$  among the 19 countries.<sup>1</sup> The critical factor for the United States is that the turbulence of the Great Depression and World War II happened to be followed by the log of per capita GDP reverting roughly to the pre-1930 and pre-1914 trend lines. Most other countries do not look like this.

My inference from the long-term GDP data for the OECD countries is that the evidence conflicts with reversion to a fixed, deterministic trend. The key, counter-factual prediction from this model is the comparatively low uncertainty about the distant future. In contrast, the variance-ratio results are consistent with the stochastic-trend specification in Eq. (1). Therefore, I use this model for the present analysis. Richer models worth considering would allow for trend breaks (analyzed starting from Banerjee, Lumsdaine, and Stock [1992]) and for gradual reversion to past levels after major crises, such as wars.

Previous research (Barro [2006, Table 1 and Figure 1]) gauged the probability and size of disaster events from time series on per capita GDP for 35 countries for the full

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<sup>1</sup> The next smallest values for  $k=20$  are 0.55 for New Zealand, 0.68 for Germany, and 0.77 for Switzerland. At  $k=30$ , the next smallest values are 0.40 for New Zealand, 0.53 for Germany, and 0.54 for Canada. For smaller values of  $k$ , the mean and median of the variance ratios are, respectively, 1.16 and 1.18 at  $k=2$ , 1.23 and 1.31 at  $k=5$ , and 1.13 and 1.06 at  $k=10$ . The U.S. ratios at these values of  $k$  are, respectively, 1.30, 1.34, and 0.94.

20<sup>th</sup> century from Maddison (2003).<sup>2</sup> For contractions of 15% or more over consecutive years (such as 1939-44 for France and 1929-33 for the United States), 60 events were found.<sup>3</sup> For the 35 countries, the main global disasters were World War II (18 countries with large GDP contractions), the Great Depression (16 countries), World War I (13 countries), and post-World War II depressions in Latin America and Asia (11 country-events). The empirical frequency—60 events for 35 countries over 100 years—corresponds to a disaster probability,  $p$ , of 1.7% per year.

The contraction proportion  $b$  for the observed 20<sup>th</sup> century disasters ranged from 15% to 64%, with a mean of 29%.<sup>4</sup> However, with substantial risk aversion (for example, with a coefficient of relative risk aversion of 3 or 4), the effective average value of  $b$  is substantially higher than the mean. A constant  $b$  of around 40% generates about the same equity premium and welfare effects as the empirically observed frequency distribution of  $b$ .<sup>5</sup>

The formulation neglects rare bonanzas. With substantial risk aversion, bonanzas do not count nearly as much as disasters for the pricing of assets and for welfare effects. Moreover, long-term data on annual growth rates of per capita GDP tend to exhibit negative skewness. For 19 OECD countries from 1880 to 2004, 14 exhibit negative

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<sup>2</sup> In the fruit-tree model, GDP and consumption coincide. More generally, consumption would be more appropriate than GDP for analyses of asset pricing and welfare costs. However, long-term data on real consumer expenditure are not reported by Maddison (2003) and are not readily available for many countries. In ongoing research, Jose Ursua and I are assembling a data set on long-term real personal consumer expenditure for as many countries as possible.

<sup>3</sup> This analysis excludes five post-war GDP contractions that did not involve large declines in real personal consumer expenditure. The lower limit of 15% is arbitrary. Extending to 10% brings in another 21 contractions for the 35 countries. However, the inclusion of these smaller contractions has a minor effect on the results.

<sup>4</sup> The 29% figure refers to raw levels of per capita GDP. With an adjustment for trend growth, the mean contraction size was 35%.

<sup>5</sup> I neglect two other features of my earlier analysis that matter for the equity premium but not for the welfare calculations—leverage in the ownership structure for trees and default possibilities on bonds.

skewness, and the only substantially positive values are for France, the Netherlands, and Switzerland.

The expected growth rate of real GDP depends not only on the growth-rate parameter,  $g$ , but also on  $p$  and  $b$ . As the length of the period approaches zero, the specification in Eq. (1) implies that the expected growth rate of GDP and consumption, denoted by  $g^*$ , is given by

$$(2) \quad g^* = g - p \cdot E_b,$$

where  $E_b$  is the expected value of  $b$  (0.29 in the sample of 60 observed crises).

The representative consumer maximizes a familiar time-additive utility function with iso-elastic preferences:

$$(3) \quad U_t = E_t \sum_{i=0}^{\infty} \frac{1}{(1+\rho)^i} \cdot [(C_{t+i})^{1-\theta} - 1]/(1-\theta),$$

where  $\rho \geq 0$  is the rate of time preference, and  $\theta > 0$  is the coefficient of relative risk aversion and the reciprocal of the elasticity of intertemporal substitution for consumption. (A later section modifies the utility function to allow for departures of the coefficient of relative risk aversion from the reciprocal of the elasticity of substitution.) The simplicity of the underlying structure (i.i.d. shocks, representative consumer with time-additive and iso-elastic preferences, closed economy with no investment) allows for a closed-form solution for expected utility as a function of the underlying parameters of preferences and of the stochastic process for output. Obstfeld (1994) derived analogous closed forms in a model without disaster risk.

A key variable is the market value,  $V$ , of a tree that initially produces one unit of fruit. This value can be calculated by summing the prices of equity claims on future “dividends,”  $C_{t+i} = Y_{t+i}$ . (In order to correspond to the summation in Eq. [3], it is

convenient to treat  $C_t$ , rather than  $C_{t+1}$ , as the first payout on tree equity.) These prices follow from the usual first-order conditions for maximizing  $U_t$ . As the arbitrary period length approaches zero, the reciprocal of  $V$  turns out to be<sup>6</sup>

$$(4) \quad 1/V = \rho + (\theta-1) \cdot g - (1/2) \cdot \theta \cdot (\theta-1) \cdot \sigma^2 - p \cdot [E(1-b)^{1-\theta} - 1],$$

where  $E(1-b)^{1-\theta}$  is the expected value of  $(1-b)^{1-\theta}$ . We can think of  $V$  as the price-dividend ratio for an unlevered equity claim on a tree.

The right-hand side of Eq. (4) equals the difference between the expected rate of return on unlevered equity, given by<sup>7</sup>

$$(5) \quad r^e = \rho + \theta g - (1/2) \cdot \theta \cdot (\theta-1) \cdot \sigma^2 - p \cdot [E(1-b)^{1-\theta} - 1 + Eb],$$

and the expected growth rate,  $g^*$ , given in Eq. (2). The transversality condition, which guarantees that the market value of a tree is positive and finite, is that the right-hand side of Eq. (4) be positive—that is,  $r^e > g^*$ .

Expected utility as of period  $t$ , given in Eq. (3), can be determined to be<sup>8</sup>

$$(6) \quad U_t = \left(\frac{1}{1-\theta}\right) \cdot V \cdot Y_t^{1-\theta},$$

up to an inconsequential additive constant. Equation (4) determines  $V$ . However, for subsequent purposes, we also want a formula for  $V$  in terms of the expected growth rate,  $g^*$ , rather than the growth-rate parameter,  $g$ . The result follows from Eq. (2):

$$(7) \quad 1/V = \rho + (\theta-1) \cdot g^* - (1/2) \cdot \theta \cdot (\theta-1) \cdot \sigma^2 - p \cdot [E(1-b)^{1-\theta} - 1 - (\theta-1) \cdot Eb].$$

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<sup>6</sup> See Barro (2006, Eq. [17]).

<sup>7</sup> See Barro (2006, Eq. [9]).

<sup>8</sup> For  $\theta=1$ , expected utility is  $U_t = (1/\rho) \cdot \log(Y_t)$ . The derivation of Eq. (6) depends on the condition  $C_t=Y_t$  and on time-additive, iso-elastic preferences but not on the particular stochastic process for output in Eq. (1). However, the constancy of the price-dividend ratio,  $V_t=V$ , depends on the i.i.d. form of the shocks,  $u_t$  and  $v_t$ . A constant  $V$  conflicts with the observed volatility of price-dividend ratios for stock-market claims. The model can match this volatility if the disaster probability,  $p_t$ , moves around. Gabaix (2006) shows that the main implications of the model for asset pricing go through if  $p_t$  evolves exogenously in random-walk-like fashion.

Note that, if  $\theta > 1$ ,  $V$  rises with increased uncertainty, in the sense of an increase in  $\sigma$  or  $p$  or a shift in the distribution of  $b$  toward larger values. This property depends on time-additive, iso-elastic preferences, as discussed in section V.

## II. Baseline Calculation of Welfare Effects

Equations (6) and (7) determine the effects on expected utility from changes in the expected growth rate,  $g^*$ , and the parameters that govern consumption risk:  $\sigma$ ,  $p$ , and the probability distribution of  $b$ . These effects can be compared with those from proportionate shifts in the initial level of GDP and consumption,  $Y_t$ .

The marginal effect on utility from a proportionate change in  $Y_t$  is given from Eq. (6) by

$$(8) \quad \frac{\partial U_t}{\partial Y_t} \cdot Y_t = (Y_t)^{1-\theta} \cdot V.$$

The marginal effect from a change in  $g^*$  follows from Eqs. (6) and (7) as

$$(9) \quad \frac{\partial U_t}{\partial g^*} = (Y_t)^{1-\theta} \cdot V^2.$$

Therefore, the utility rate of transformation between proportionate changes in  $Y_t$  and changes in  $g^*$  is given by

$$(10) \quad \frac{-\partial U_t / \partial g^*}{(\partial U_t / \partial Y_t) \cdot Y_t} = -V.$$

This result gives the proportionate decrease in  $Y_t$  that compensates, at the margin, for an increase in  $g^*$ —in the sense of preserving expected utility. Equation (10) shows that this compensating output change depends only on the combination of parameters that enter into the price-dividend ratio,  $V$ , as determined in Eq. (4) or Eq. (7).

To pin down a reasonable magnitude for  $V$ , start with the already mentioned specification  $p=0.017$  per year. The probability distribution for  $b$  is the historical one mentioned before. The other parameters equal the values used in the main calibration exercise in Barro (2006, Table 5, col. 2). The rate of time preference is  $\rho=0.03$  per year, the coefficient of relative risk aversion (and reciprocal of the elasticity of intertemporal substitution) is  $\theta=4$ , the growth parameter is  $g=0.025$  per year, and the standard deviation of the  $u_t$  shocks is  $\sigma=0.02$  per year.<sup>9</sup> These parameters imply that the expected growth rate in Eq. (2) is  $g^*=0.020$  per year. The price-dividend ratio in Eq. (7) is  $V=19.7$ .

With  $V=19.7$ , Eq. (10) implies that a small rise in the expected growth rate,  $g^*$ —for example, by 0.1% per year—has to be compensated by a fall in the initial level of GDP,  $Y_t$ , by 2.0%. Despite differences in specification, this result accords with the one found by Lucas (1987, Ch. 3, p. 24). An economy should be willing to give up a lot in its initial level of GDP to obtain a small increase in its long-term growth rate.

The Lucas calculations about consumption uncertainty relate in the present model to the parameter  $\sigma$ . The marginal effect on expected utility,  $U_t$ , from a change in  $\sigma$  is given from Eqs. (6) and (7) by

$$(11) \quad \frac{\partial U_t}{\partial \sigma} = -(Y_t)^{1-\theta} \cdot \theta \sigma V^2.$$

The utility rate of transformation between proportionate changes in  $Y_t$  and changes in  $\sigma$  is given by

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<sup>9</sup> The values for  $g$  and  $\sigma$  come from data on real personal consumer expenditure for 21 OECD countries for 1954-2004, a tranquil period with no disaster events for these countries. The largest contraction was 14% for per capita real consumer expenditure (12% for per capita GDP) for Finland in 1989-93. For 1954-2004, the median of the growth rates of real per capita personal consumer expenditure for the 21 countries was 0.026 per year, and the median standard deviation of the growth rates was 0.024. The U.S. values were 0.024 and 0.018, respectively. With  $\theta=4$ , the expectations associated with the historical distribution of disaster sizes,  $b$ , are  $Eb=0.29$ ,  $E(1-b)^{-\theta} = 7.69$ , and  $E(1-b)^{1-\theta} = 4.05$ .

$$(12) \quad \frac{-(\partial U_t / \partial \sigma)}{(\partial U_t / \partial Y_t) \cdot Y_t} = \theta \sigma V.$$

This expression gives the proportionate increase in initial GDP required to compensate, at the margin, for a rise in  $\sigma$ . The parameters specified before imply  $\theta \sigma V = 1.58$ .

Therefore, to maintain expected utility, an increase in  $\sigma$  by, say, 10% (from 0.020 to 0.022) requires a rise in the initial level of GDP by approximately 0.32%. Since the expected growth rate,  $g^*$ , is held fixed, this proportionate rise in GDP level should be viewed as applying each year.<sup>10</sup>

These calculations apply for small changes in  $\sigma$ . Large changes, considered later, recognize that the utility rate of transformation rises with  $\sigma$  on the right-hand side of Eq. (12). This consideration means that the welfare gain from setting  $\sigma$  to zero is smaller in magnitude than the amount—3.2%—that would be calculated from Eq. (12) if the utility rate of transformation were constant.

Consider now the welfare consequences from a change in the disaster probability,  $p$ , for a given distribution of disaster sizes,  $b$ . Equations (6) and (7) imply

$$(13) \quad \frac{\partial U_t}{\partial p} = -(Y_t)^{1-\theta} \cdot V^2 \cdot [E(1-b)^{1-\theta} - 1 - Eb \cdot (\theta - 1)] / (\theta - 1).$$

This formula applies while holding fixed the expected growth rate,  $g^*$ ; that is, it does not allow for the negative effect of  $p$  on  $g^*$ , for given  $g$ , in Eq. (2). The utility rate of transformation between proportionate changes in  $Y_t$  and changes in  $p$  is given by

$$(14) \quad \frac{-(\partial U_t / \partial p)}{(\partial U_t / \partial Y_t) \cdot Y_t} = V \cdot [E(1-b)^{1-\theta} - 1 - Eb \cdot (\theta - 1)] / (\theta - 1).$$

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<sup>10</sup> Obstfeld (1994) observes that Lucas (1987, Ch. 3) gets far smaller estimates for the welfare cost of consumption uncertainty because he treats the shock, analogous to  $u_t$  in the present model, as a transitory disturbance to the level of output.

With the parameter values used before, the right-hand side of Eq. (14) equals 14.3. As before, the result applies to small changes. An increase in  $p$  by 10% (from 0.0170 to 0.0187) matches up approximately with a proportionate rise in initial GDP by 2.4%. Again, this change in GDP level applies each year.

We can modify the calculations to allow for the growth effect from a change in  $p$ ; that is, for given  $g$ ,  $g^*$  falls with  $p$  in Eq. (2).<sup>11</sup> Using Eq. (4), the result in Eq. (14) is modified to

$$(15) \quad \frac{-(\partial U_t / \partial p)}{(\partial U_t / \partial Y_t) \cdot Y_t} = V \cdot [E(1-b)^{1-\theta} - 1] / (\theta - 1).$$

With the same parameter values as before, the right-hand side equals 20.0. Therefore, a rise in  $p$  by 10% now matches up with a proportionate increase in GDP by 3.4%—larger than before because of the decline in  $g^*$ .

As already noted, the formulas in Eqs. (10), (12), (14), and (15) apply locally; that is, to small changes in  $Y_t$ ,  $g^*$ ,  $\sigma$ , and  $p$ . We can instead use Eqs. (6) and (7) to assess the effects on expected utility from large changes. Let  $V$  and  $Y_t$  be the values that apply for the baseline specification of parameters. Let  $V^*$  and  $(Y_t)^*$  be values that apply in an alternative situation that delivers the same expected utility,  $U_t$ . Then the formula for  $U_t$  in Eq. (6) implies<sup>12</sup>

$$(16) \quad (Y_t)^* / Y_t = (V^* / V)^{1/(\theta-1)}.$$

As an aside, Alvarez and Jermann (2004) try to go as far as possible to gauge the welfare costs of consumption uncertainty by observing or estimating various asset prices. Equation (16) provides insight for the present model on the extent to which welfare costs

<sup>11</sup> See Barlevy (2004) for a discussion of models in which uncertainty affects the expected growth rate of GDP.

<sup>12</sup> Equation (16) gives the compensating income change in the sense of Hicks (1946, pp. 330-331) for a shift in a parameter, such as  $g^*$ ,  $\sigma$ , or  $p$ .

can be assessed from observations of asset prices related to equity shares. The price  $V$  may be observable—in the Lucas-tree economy,  $V$  is the price-dividend ratio for unlevered equity claims on trees. However, the price  $V^*$  is unlikely to be observable:  $V^*$  is the price-dividend ratio for unlevered tree equity in a hypothetical economy, such as one with zero uncertainty. If  $V^*$  could be observed or estimated, Eq. (16) shows that the welfare cost, measured by the compensating output change  $(Y_t)^*/Y_t$ , depends separately on the parameter  $\theta$ , for given  $V$  and  $V^*$ .

Lucas focused on the consequences of eliminating all consumption uncertainty associated with usual business fluctuations—in the present context, this exercise corresponds to setting  $\sigma=0$ . The formula for  $V$  in Eq. (7) implies for this case

$$1/V^* = 1/V + (1/2) \cdot \theta \cdot (\theta-1) \cdot \sigma^2.$$

Substituting into Eq. (16) yields<sup>13</sup>

$$(17) \quad (Y_t)^*/Y_t = [1 + (1/2) \cdot \theta \cdot (\theta-1) \cdot \sigma^2 V]^{1/(1-\theta)}.$$

With the parameter values assumed before,  $(Y_t)^*$  is 1.5% below  $Y_t$ . That is, society would be willing to give up 1.5% of output each year to eliminate all of the customary economic fluctuations represented by  $\sigma$ . As noted before (n. 10), this effect is much larger than that found by Lucas (1987) mainly because the impact of a shock,  $u_t$ , on the GDP level is permanent in the present model.

Setting the disaster probability,  $p$ , to zero (or, equivalently, the disaster size,  $b$ , to zero) has much greater consequences for welfare. The formula, based on Eqs. (7) and (16), is

$$(18) \quad (Y_t)^*/Y_t = \{1 + pV \cdot [E(1-b)^{1-\theta} - 1 - (\theta-1) \cdot Eb]\}^{1/(1-\theta)}.$$

<sup>13</sup> If the magnitude of  $(1/2) \cdot \theta \cdot (\theta-1) \cdot \sigma^2 V$  is much less than one, the result simplifies to  $\log[(Y_t)^*/Y_t] \approx -(1/2) \cdot \theta \cdot \sigma^2 V$ ; that is, one-half the size of the local effect implied by Eq. (12). This approximation is satisfactory at the baseline specification, where  $(1/2) \cdot \theta \cdot (\theta-1) \cdot \sigma^2 V = 0.047$ .

Note that this formula holds fixed the expected growth rate,  $g^*$ ; that is, it does not allow for the inverse relation between  $p$  and  $g^*$  in Eq. (2), for given  $g$ . With the same parameter values as before,  $(Y_t)^*$  is 16.7% below  $Y_t$ . Hence, when gauged by the compensating proportionate change in output, eliminating disaster risk is worth about 10 times as much as eliminating normal economic fluctuations.

Using Eq. (4), we can allow also for the negative growth effect of  $p$  (in Eq. [2]).

The formula modifies Eq. (18) to

$$(19) \quad (Y_t)^*/Y_t = \{1 + pV \cdot [E(1-b)^{1-\theta} - 1]\}^{1/(1-\theta)}.$$

With the usual parameter values,  $(Y_t)^*$  is 20.9% below  $Y_t$ . This result is larger than before because the reduction in  $p$  raises  $g^*$ .

We can also consider the elimination of all consumption uncertainty by setting  $\sigma=0$  and  $p=0$  (or  $b=0$ ) simultaneously. If  $g^*$  is held fixed,  $(Y_t)^*$  is 17.4% below  $Y_t$ . Allowing for the inverse relation between  $p$  and  $g^*$ , the result is 21.4%. Not surprisingly, the main effects in each case come from setting  $p=0$ .

### III. Sensitivity of the Welfare-Cost Estimates

The welfare-cost estimates, including the effects from eliminating all disaster risk, depend particularly on the preference parameter,  $\theta$ . In the baseline specification with  $\theta=4$ , the elimination of all disaster risk (setting  $p=0$ ) balances with a proportionate cut in initial output by 20.9% (for the case that includes the inverse relation between  $p$  and  $g^*$ ).

For lower  $\theta$ , with the rest of the specification unchanged, the compensating proportionate decline in initial GDP is 15.7% when  $\theta=3$ , 14.8% when  $\theta=2$ , and 18.6% when  $\theta=1$ . Thus, the results are not highly sensitive to variations in  $\theta$ . The reason that

the impact of  $\theta$  is non-monotonic is that this parameter picks up two effects—lower  $\theta$  means a smaller coefficient of relative risk aversion and a larger elasticity of intertemporal substitution. The discussion in section V shows that these two forces tend to have offsetting implications for welfare effects. That section goes into more detail on the dependence of welfare costs on parameter values.

#### **IV. A Model with Endogenous Saving**

In the endowment economy, agents do not react to changes in uncertainty—shifts in  $\sigma$ ,  $p$ , and the probability distribution of  $b$ —by altering saving ratios or other choices. Generally, the potential for such adjustments affects the welfare costs—not at the margin (by the envelope theorem) but for large changes in parameters. This section illustrates this process by using a version of the tractable AK model of endogenous saving developed in Barro (2006, section VIII).<sup>14</sup>

The quantity of trees is now variable and corresponds to the capital stock,  $K_t$ . Production of fruit is given by an AK production function:

$$(20) \quad Y_t = AK_t.$$

Unlike the previous model, the productivity level,  $A > 0$ , is constant. Output can be consumed as fruit or invested as seed. The creation of new trees through planting seeds is rapid enough so that, as in the conventional one-sector production framework, the fruit price of trees (capital) is pegged at a price normalized to one. This setting corresponds to “Tobin’s  $q$ ” equaling one—unlike in the previous model, where the market price of trees was variable.

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<sup>14</sup>Epaulard and Pommeret (2003) modified the calculations of welfare costs of uncertainty to allow for adjustments of saving in an AK model. However, because they excluded disaster risks, the quantitative significance of the saving adjustment was small.

The capital stock varies because of gross investment,  $I_t$ , and depreciation,  $\delta_{t+1}K_t$ :

$$(21) \quad K_{t+1} = K_t + I_t - \delta_{t+1}K_t.$$

The depreciation rate is stochastic and equal to

$$(22) \quad \delta_{t+1} = \delta + u_{t+1} + v_{t+1},$$

where  $0 < \delta < 1$ . The  $u_{t+1}$  shock, normally distributed with mean 0 and variance  $\sigma^2$ , represents normal fluctuations, as in the previous setting. The  $v_{t+1}$  shock represents rare disasters, again as in the earlier model. With probability  $1-p$ ,  $v_{t+1} = 0$ , and with probability  $p$ ,  $v_{t+1} = -b$ ; that is, the fraction  $b$  ( $0 < b < 1$ ) of the trees is destroyed. The analysis requires  $0 < \delta_{t+1} < 1$ . However, this restriction holds with probability one as the length of the period approaches zero, assuming that  $\sigma^2$  and  $p$  are proportional to the length of the period.

Since the market price of trees is pegged at one, the expected rate of return on equity shares is given immediately by

$$(23) \quad r^e = A - \delta - p \cdot E b.$$

Because the shocks,  $u_{t+1}$  and  $v_{t+1}$ , are i.i.d. (permanent to the levels of capital stock and GDP), the ratio of gross investment (and gross saving) to the capital stock will be optimally chosen as a constant, denoted by  $v$ . One way to determine  $v$  is to use the usual consumption-based asset-pricing formula for equity shares, combined with the condition that the price of these shares equals unity. As the length of the period approaches zero, the saving ratio is given by

$$(24) \quad v = \delta + (1/\theta) \cdot [A - \delta - \rho + (1/2) \cdot \theta \cdot (\theta - 1) \sigma^2 - p + p \cdot E(1-b)^{1-\theta}].$$

If  $\theta > 1$ ,  $v$  rises with  $\sigma$  and  $p$  (and with a shift in the distribution of  $b$  toward larger values).

The expected growth rate of the economy is

$$(25) \quad E_t(K_{t+1}/K_t - 1) = v - \delta - p \cdot E_b.$$

I calibrate this model using the same parameter values as for the endowment economy. To get a full match, the expected growth rate, given by Eq. (25), has to equal the value  $g^* = 0.020$ , given by Eq. (2). This condition pins down the parameter combination  $A - \delta$ , which turns out to equal 0.076 per year. It is then straightforward to show that the expected rate of return on equity, the risk-free rate,<sup>15</sup> and the equity premium in the endogenous-saving model coincide with those in the endowment economy. Substitution of the assumed parameter values into Eq. (24) yields a gross saving ratio,  $v$ , equal to  $0.025 + \delta$ . For example, if  $\delta = 0.05$ ,  $v = 0.075$ . That is, annual gross saving and investment equal 7.5% of the capital stock.

The interesting new results on welfare costs apply to large changes; for example, setting  $\sigma=0$  or  $p=0$ . We can again express the effects in terms of proportionate declines in levels of GDP (and capital stock) that would be willingly exchanged for each kind of reduction in uncertainty. *These welfare effects in the endogenous-saving model coincide with those for the endowment economy if the gross saving ratio,  $v$ , is constrained to remain fixed at its initial value (0.075).* Specifically, the offsetting proportionate reductions in GDP are 1.53% for setting  $\sigma=0$ , 20.9% for  $p=0$ , and 21.4% for  $\sigma=p=0$ .<sup>16</sup> In effect, with  $v$  held fixed, the endogenous-saving model operates like an endowment economy.

The results are different if the saving ratio,  $v$ , is free to adjust to the changes in  $\sigma$  and  $p$ , in accordance with the optimal response given by Eq. (24). Since the optimal

<sup>15</sup>In the endowment model, the risk-free rate is  $r^f = \rho + \theta g - (1/2) \cdot \theta \cdot (\theta + 1) \cdot \sigma^2 - p \cdot [E(1-b)^{-\theta} - 1]$ —see Barro (2006, Eq. [12]). The formula in the endogenous-saving model is  $r^f = A - \delta - \theta \sigma^2 - p \cdot [E(1-b)^{-\theta} - E(1-b)^{1-\theta}]$ .

<sup>16</sup>These effects are the same as before if the calculation for the endowment economy (as in Eq. [19]) includes the growth effect from  $p$  through the term  $-p \cdot E_b$  in Eq. (2).

saving response cannot make the situation worse, the compensating output variations for eliminating uncertainty must be at least as large as those in the endowment economy.<sup>17</sup>

For the calibration parameters already mentioned, the results are

setting  $\sigma=0$ : saving ratio,  $v$ , falls from 0.0750 to 0.0744, welfare effect = 1.55%,

setting  $p=0$ :  $v$  falls to 0.0620, welfare effect = 26.1%,

setting  $p=\sigma=0$ :  $v$  falls to 0.0614, welfare effect = 27.0%.

For the case where  $\sigma=0$ , the impact of allowing for the small decline in the saving ratio (from 0.0750 to 0.0744) is minor. Hence, the welfare effect, 1.55%, differs negligibly from that, 1.53%, in the endowment economy. However, when considering  $p=0$ , the significant reduction in the saving ratio (from 0.075 to 0.062) raises the welfare effect substantially. The output that would be willingly relinquished to eliminate disaster risk rises from 20.9% to 26.1%. Note that we could instead start from  $\sigma=0$  or  $p=0$  and compute the proportionate increase in GDP required to compensate for an increase in  $\sigma$  or  $p$ . In this case, the optimal adjustment of the saving ratio (upward) reduces the welfare effect in the sense of the compensating, proportionate change in GDP.

## V. Epstein-Zin-Weil Preferences

The results apply thus far under time-separable, iso-elastic preferences, for which the coefficient of relative risk aversion equals the inverse of the elasticity of intertemporal substitution. Obstfeld (1994) observed that welfare effects from uncertainty differ under the preference formulations of Epstein and Zin (1989) and Weil

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<sup>17</sup> If  $\theta=1$ ,  $v$  does not depend on  $\sigma$  or  $p$ , and the results are the same as those as in the case where  $v$  is constrained not to vary.

(1990), in which the coefficient of relative risk aversion is de-linked from the elasticity of intertemporal substitution.

Using a minor modification of the Weil (1990) formulation, the utility formula is

$$(26) \quad U_t = \frac{\left\{ (1-\beta)C_t^{1-\theta} + \beta[(1-\beta)(1-\gamma)E_t U_{t+1}]^{(1-\theta)/(1-\gamma)} \right\}^{(1-\gamma)/(1-\theta)}}{(1-\beta)(1-\gamma)},$$

where the discount factor  $\beta$  equals  $1/(1+\rho)$ ,  $1/\theta > 0$  is the elasticity of intertemporal substitution, and  $\gamma > 0$  is the coefficient of relative risk aversion. Equation (3) is the special case of Eq. (26) with  $\theta = \gamma$ .

Because the underlying shocks are i.i.d., attained utility,  $U_t$ , ends up as a simple function of contemporaneous consumption,  $C_t$ :

$$(27) \quad U_t = \Phi C_t^{1-\gamma},$$

where the constant  $\Phi$  depends on the parameters of the model.<sup>18</sup> Using Eq. (27), a standard perturbation argument generates the first-order conditions for utility maximization:

$$(28) \quad C_t^{-\gamma} = \left( \frac{1}{1+\rho^*} \right) \cdot E_t(R_t \cdot C_{t+1}^{-\gamma}),$$

where  $R_t$  is the gross, one-period return on any asset. A key property of Eq. (28) is that the exponents on  $C_t$  and  $C_{t+1}$  involve  $\gamma$ , the coefficient of relative risk aversion, not  $\theta$ , the reciprocal of the elasticity of intertemporal substitution. The second point is that the effective rate of time preference,  $\rho^*$ , differs from  $\rho$  when  $\gamma$  and  $\theta$  diverge. The formula for  $\rho^*$  is (if  $\gamma \neq 1$ )

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<sup>18</sup> Giovannini and Weil (1989, appendix) show that, with the utility function in Eq. (26), attained utility,  $U_t$ , is proportional to wealth raised to the power  $1-\gamma$ . The form in Eq. (27) follows because  $C_t$  is optimally chosen as a constant ratio to wealth in the i.i.d. case. The formula for  $\Phi$  is, if  $\gamma \neq 1$  and  $\theta \neq 1$ ,

$$\Phi = \left( \frac{\rho^{(\theta-\gamma)/(1-\theta)}}{1-\gamma} \right) \cdot \left\{ \rho + (\theta-1)g - (1/2)\gamma(\theta-1)\sigma^2 - \left( \frac{\theta-1}{\gamma-1} \right) \cdot p \cdot [E(1-b)^{1-\gamma} - 1] \right\}^{(1-\gamma)/(1-\theta)}.$$

$$(29) \quad \rho^* = \rho - (\gamma - \theta) \cdot \left\{ g - (1/2) \cdot \gamma \sigma^2 - \left( \frac{p}{\gamma - 1} \right) \cdot [E(1 - b)^{1-\gamma} - 1] \right\}.$$

Equations (28) and (29) imply that the asset-pricing formulas derived from the model with time-additive, iso-elastic preferences continue to apply if  $\gamma$  replaces  $\theta$  and  $\rho^*$  replaces  $\rho$ . The reciprocal of the elasticity of intertemporal substitution,  $\theta$ , affects levels of rates of return and the price-dividend ratio,  $V$ —through influences on  $\rho^*$ —but not the equity premium. Therefore, with  $\gamma=4$  (and the same parameters as before for the GDP process), the extended version of preferences in Eq. (26) fits the equity premium exactly as the original model.

The formula for the price-dividend ratio,  $V$ , changes from Eq. (4) to

$$(30) \quad 1/V = \rho + (\theta - 1) \cdot g - (1/2) \cdot \gamma \cdot (\theta - 1) \cdot \sigma^2 - p \cdot \left( \frac{\theta - 1}{\gamma - 1} \right) \cdot [E(1 - b)^{1-\gamma} - 1],$$

assuming  $\gamma \neq 1$ . For any  $\gamma$ ,  $V$  now falls with increased uncertainty (a rise in  $\sigma$  or  $p$  or a shift of the  $b$ -distribution toward higher values) if  $\theta < 1$ .<sup>19</sup>  $V$  is invariant with the uncertainty parameters if  $\theta = 1$ .<sup>20</sup>

Utility,  $U_t$ , can again be expressed in terms of the price-dividend ratio,  $V$ . The result, which extends Eq. (6) to the case where  $\theta \neq \gamma$ , is<sup>21</sup>

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<sup>19</sup> These results apply when the price-dividend ratio,  $V$ , pertains to unlevered equity. We can instead consider levered equity, as in Barro (2006, section III). The effect of increased uncertainty on the price of levered equity can be negative even if  $\theta > 1$ . The condition for increased  $\sigma$  to reduce the levered equity price is  $\theta < 1 + 2\lambda$ , where  $\lambda$  is the debt-equity ratio for claims on trees. For increased  $p$ , the condition depends on the distribution of disaster sizes,  $b$ , and the coefficient of relative risk aversion,  $\gamma$ . For the baseline specification with  $\gamma=4$  and the historical distribution of  $b$ , the condition is  $\theta < 1 + 3.6 \cdot \lambda$ .

<sup>20</sup> In the AK model with endogenous saving from the previous section, the saving ratio under Epstein-Zin-Weil preferences generalizes from Eq. (24) to

$$v = \delta + (1/\theta) \cdot \left\{ A - \delta - \rho + (1/2) \cdot \gamma \cdot (\theta - 1) \cdot \sigma^2 + \left( \frac{\theta - 1}{\gamma - 1} \right) \cdot p \cdot [E(1 - b)^{1-\gamma} - 1] \right\}, \text{ assuming } \gamma \neq 1. \text{ The}$$

saving ratio rises with more uncertainty (higher  $\sigma$  or  $p$  or a shift of the  $b$ -distribution toward larger values) if  $\theta > 1$ . That is, the sign of this precautionary saving effect depends on intertemporal substitution, not risk aversion.

<sup>21</sup> As usual, cases where  $\gamma$  or  $\theta$  approach 1 can be handled as limits.

$$(31) \quad U_t = \left( \frac{\rho^{(\theta-\gamma)/(1-\theta)}}{1-\gamma} \right) \cdot V^{(1-\gamma)/(1-\theta)} \cdot Y_t^{1-\gamma}.$$

Equations (30) and (31) can be used, as before, to assess welfare effects from changes in parameters. The result is that the form of Eq. (16) continues to apply (see n. 21):

$$(32) \quad (Y_t)^*/Y_t = (V^*/V)^{1/(\theta-1)}.$$

The difference from before is that  $\theta$  has to be interpreted as the reciprocal of the elasticity of intertemporal substitution, not the coefficient of relative risk aversion,  $\gamma$ . However,  $\gamma$  affects the calculations of welfare effects through its influence on the price-dividend ratio,  $V$ , determined in Eq. (30).

Table 1 shows how the preference parameters,  $\gamma$  and  $\theta$ , affect the computed welfare effects from eliminating uncertainty ( $p=0$  or  $\sigma=0$ ). The first line, where  $\gamma=\theta=4$ , corresponds to the results derived before for the model with time-additive, iso-elastic preferences. The welfare effect from setting  $p=0$  corresponds to a proportionate reduction in initial GDP by 20.9%. (This calculation includes the positive effect on the growth rate,  $g^*$ , in Eq. [2].) The next three lines stay within the time-additive, iso-elastic model by reducing  $\gamma$  and  $\theta$  together from 4 to 1. As noted before, the welfare effect from setting  $p=0$  is 15.7% at  $\gamma=\theta=3$ , 14.8% at  $\gamma=\theta=2$ , and 18.6% at  $\gamma=\theta=1$ .

The next part of the table considers reductions in  $\gamma$  from 4 to 1, while holding fixed  $\theta=4$ . Hence, these exercises isolate changes in the coefficient of relative risk aversion, holding fixed the elasticity of intertemporal substitution. Equations (30) and (32) imply that a reduction in  $\gamma$  lowers the welfare effect from uncertainty—that is, lower risk aversion makes uncertainty less costly. Quantitatively, the table shows that the effect from setting  $p=0$  declines from 20.9% at  $\gamma=4$  to 12.6% at  $\gamma=3$ , 8.5% at  $\gamma=2$ , and 6.3% at  $\gamma=1$ .

A problem with these calculations is that decreases in  $\gamma$  substantially lower the equity premium. The model's predicted premium is 5.9% at  $\gamma=4$ , 2.5% at  $\gamma=3$ , 1.1% at  $\gamma=2$ , and 0.4% at  $\gamma=1$ . Hence, if  $\gamma$  is much below 4, the predictions deviate sharply from observed equity premia of around 6% (see Table 2).<sup>22</sup> The model's implications for welfare costs of uncertainty likely should not be taken seriously in the range of values for  $\gamma$  where the model fails to get into the right ballpark for explaining the equity premium.

It is possible to restore reasonable predictions for the equity premium at low  $\gamma$  if the disaster probability,  $p$ , is raised above 1.7% per year. At  $\gamma=3$ ,  $p$  has to be 4.1% to generate the same equity premium, 5.9%, as originally. With this unrealistically high  $p$ , the elimination of all disaster risk (setting  $p$  or  $b$  to zero) turns out to balance against a proportionate decline in initial output by 41.5%, well above the 20.9% calculated originally.

The final lines of Table 1 consider reductions in  $\theta$  from 4 to 1, holding fixed  $\gamma=4$ . Hence, these exercises isolate changes in the elasticity of intertemporal substitution, for a given coefficient of relative risk aversion. Equations (30) and (32) imply that a decrease in  $\theta$  generally has an ambiguous impact on the magnitude of welfare effects. However, in the relevant range, a reduction in  $\theta$  raises welfare effects, along the lines discussed by Obstfeld (1994, section 3). The main effect is that a lower  $\theta$  reduces the effective rate of time preference,  $\rho^*$ , in Eq. (29) and, thereby, works like a lower  $\rho$  in magnifying the present value of the stream of welfare costs. Quantitatively, the table shows that, if  $\gamma=4$ , the welfare effect from setting  $p=0$  rises from 20.9% when  $\theta=4$  to 25.2% at  $\theta=3$ , 31.9% at  $\theta=2$ , and 43.7% at  $\theta=1$ .

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<sup>22</sup> The prediction is still at odds with the data if one allows for leverage in the observed stock returns.

Unlike the calculations for alternative values of  $\gamma$ , the results for alternative values of  $\theta$  do not interact with the equity premium, which depends on  $\gamma$ . However, changes in  $\theta$  affect levels of rates of return, including the risk-free rate,  $r^f$ , by altering  $\rho^*$  in Eq. (29). Specifically, if we fix the other parameters, including  $\rho=0.03$ , the model implies  $r^f=0.012$  when  $\theta=4$ . At  $\theta=3$ ,  $\rho^*$  falls from 0.030 to 0.023 and  $r^f$  declines to 0.005. At  $\theta=2$ ,  $\rho^*=0.016$  and  $r^f=-0.002$ . At  $\theta=1$ ,  $\rho^*=0.009$  and  $r^f=-0.009$ . Since the setting of  $\rho=0.03$  is largely arbitrary, we could instead vary  $\rho$  each time we change  $\theta$  to keep  $\rho^*$  fixed at 0.03 and, hence, the risk-free rate,  $r^f$ , fixed at 0.012.<sup>23</sup> As Table 1 shows, this adjustment eliminates much of the inverse relation between  $\theta$  and the welfare costs from eliminating uncertainty. The welfare effect from setting  $p=0$  is 20.9% at  $\theta=4$ , 22.9% at  $\theta=3$ , 25.4% at  $\theta=2$ , and 28.9% at  $\theta=1$ .

## VI. Alvarez-Jermann and Risk-Free Interest Rates

As mentioned before, Alvarez and Jermann (2004), henceforth AJ, gauge the welfare cost of aggregate consumption uncertainty from an approach that tries to rely on asset prices. In a general way, my analysis follows their idea that welfare-cost estimates should be disciplined to be consistent with observed patterns in asset prices and returns. An earlier expression of this idea is in Atkeson and Phelan (1994).

AJ observe (p. 1225) that the “marginal cost of consumption fluctuations [equals] the ratio of the values of two securities: a claim to the consumption trend ... and a claim

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<sup>23</sup> Given the process that generates GDP and consumption, observations of the equity premium and the risk-free rate pin down  $\gamma$  and  $\rho^*$ , which depends on the combination of  $\rho$  and  $\theta$  given by Eq. (29). The inability to identify  $\rho$  and  $\theta$  separately relates to the observational-equivalence point of Kocherlakota (1990). The parameters  $\rho$  and  $\theta$  could be separately identified from other data; for example, if we knew the slope coefficients of the dividend-price ratio,  $1/V$ , with respect to the uncertainty parameters  $\sigma$  and  $p$  in Eq. (30). Alternatively, identification would follow from the slope coefficients of the saving ratio,  $v$ , with respect to  $\sigma$  and  $p$  in the formula given in n. 20.

to aggregate consumption ...” In the Lucas-tree economy, the claim to aggregate consumption corresponds to an equity share on a tree and has the price  $V$  determined in Eq. (7) (for the case of time-additive, iso-elastic utility). This price is positive and finite if the expected rate of return on equity,  $r^e$  in Eq. (5), exceeds the expected growth rate of GDP and consumption,  $\gamma^*$  in Eq. (2). This inequality is the transversality condition for the model.

A problem arises with the price of a claim to the consumption trend—the payments on this claim would start at the initial level of per capita GDP and consumption and then grow at a constant rate,  $g^*=2\%$  per year in the models considered before. The rate of return on this security equals the risk-free rate,  $r^f$ . The price of such a claim is finite only if  $r^f > g^*$ . However, there is no reason for this inequality to hold—in fact,  $r^f$  can be negative. In the baseline calibrations of the endowment and endogenous-saving models,  $r^f$  turns out to be positive—1.2%—but less than  $g^*$ , 2.0%.

Table 2 shows growth rates and real rates of return for 1880-2005 for 11 OECD countries with available data.<sup>24</sup> The table reports four concepts of growth rates: for per capita and levels of real personal consumer expenditure and GDP. In the Lucas-tree model with constant population, the four concepts coincide. When consumption and GDP diverge, consumption—proxied by real personal consumer expenditure in the data—would be the relevant variable for asset pricing. However, this distinction is

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<sup>24</sup>I excluded countries that were missing data on asset returns during major crises—Austria, Belgium, and the Netherlands around World Wars I and II; Finland, New Zealand, Portugal, and Switzerland around World War I; and Spain during the Spanish Civil War. See the notes to Table 2 for a listing of years of missing data for the 11 included countries. From a selection standpoint, the most serious concerns are the missing data on bond returns in Germany for 1880-1923 and Sweden for 1880-1921. These omissions cover World War I and the German hyperinflation and, therefore, miss times of low real rates of return on bonds. The mean values shown in Table 2 for bond returns in Germany and Sweden for 1880-2005 use bill returns for the periods of missing bond data.

empirically unimportant in the present context, because the long-term growth rates of real personal consumer expenditure and GDP are similar.

When population is growing, assets would still be priced in relation to variations over time in per capita consumption. However, if new people enter the economy as children tied altruistically to parents, the AJ claim to the consumption trend would relate to the trend of the level of consumption, not consumption per capita. (A family dynasty would want to provide in a risk-free way for new members as well as for rising consumption per person.) Thus, in this context, the relevant growth rates in Table 2 are those for the levels variables. These growth rates include population growth and are, not surprisingly, substantially higher than the per capita growth rates.

Table 2 shows three real rates of return—on stocks, short-term bills (analogous to U.S. Treasury bills<sup>25</sup>), and long-term bonds (typically 10-year government bonds). The first observation is that the average real rate of return on stocks—7.4% per year—clearly exceeded the average growth rates. This pattern would still hold if we took account of leverage to estimate the average real rate of return on unlevered equity. A second observation is that the average real rate of return on bills—1.0%—fell short of the average growth rates. Finally, the average real rate of return on bonds—2.2%—was close to the average per capita growth rates—1.9% for personal consumer expenditure and 2.1% for GDP—but less than the average growth rates of levels—2.8% and 3.0%, respectively.

In the models (with i.i.d. shocks to GDP and consumption), the term structure of risk-free rates is flat. Therefore, in this setting, one could equally well examine short-term or long-term risk-free rates. More generally, a long-term risk-free rate would be

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<sup>25</sup> In some cases, such as the United States before 1922, the data are for commercial paper.

appropriate for pricing a claim on the consumption trend. A problem in the data is the absence of risk-free assets—indexed bonds issued by the U.S., U.K. and some other governments come close, but these securities exist only recently. The observed real returns on conventional short-term bills or long-term bonds are not risk-free returns, given especially the uncertainty about inflation. Unexpected inflation amounts to a form of partial default on nominal debt and tends to be high in bad economic times, especially wartime economic crises. This pattern means that hypothetical risk-free rates tend to be lower than observed averages of real rates of return on bills and bonds. Thus, my inference from Table 2 is that, for the OECD countries in the long run, risk-free rates were likely lower than expected growth rates, particularly growth rates of levels of consumption and GDP.<sup>26</sup>

When the risk-free rate is below the expected growth rate, the AJ measure of the “marginal cost of all consumption uncertainty” is infinity. AJ demonstrate (section V) that their marginal cost is an upper bound for the total cost of all consumption uncertainty in the sense of Lucas (1987) and in my estimates (with  $p$  and  $\sigma$  both set to zero). Obviously, this result is consistent with the finding that the AJ measure of the marginal cost of all consumption uncertainty is likely to be infinite. My conclusion is that it is unclear how to estimate welfare costs purely from observations of asset prices (even ones on hypothetical claims).

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<sup>26</sup> Another consideration that reinforces this conclusion is that after-tax real returns tend to be lower than the measured gross returns.

## VII. Concluding Observations

The baseline parameter value  $\sigma=0.02$  per year represents the extent of business fluctuations during the tranquil post-World War II years in the United States and other OECD countries. This period was calm for the OECD countries when considered in comparison to the first half of the 20<sup>th</sup> century, a turbulent time that featured World Wars I and II and the Great Depression. Hence, a reduction in  $\sigma$  amounts to making milder the business fluctuations that were already strikingly tame. Not surprisingly, the benefit from this change—corresponding to around 1.5-2% of GDP each year—is only moderate, though still important.

In contrast, the probability parameter  $p$  and size parameter  $b$  refer to major economic disasters, such as those that occurred in many countries during World Wars I and II and the Great Depression. Outside of the OECD, we can also think of  $p$  and  $b$  as relating to events such as the Asian financial crisis of the late 1990s, the Latin-American debt crisis of the early 1980s, and the Argentine exchange-rate crisis of 2001-02. A reduction in  $p$  amounts to lowering the chance of repeating these kinds of extreme events, and a fall in  $b$  amounts to decreasing the likely size of these events. To go further, decreases in  $p$  or  $b$  constitute reductions in the probability or size of disasters not yet seen or, at least, not seen in the 20<sup>th</sup> century. Included here would be nuclear conflicts, large-scale natural disasters (tsunamis, hurricanes, earthquakes, asteroid collisions), and epidemics of disease (Black Death, avian flu). My estimates indicate that the welfare consequences from eliminating all uncertainty of this kind are large—10 times or more the effects for normal economic fluctuations. Moreover, these large welfare effects arise

even though my analysis limits the impacts of disasters to utility losses from reduced consumption.<sup>27</sup>

Macroeconomic stabilization policies, including monetary policy, relate to both types of uncertainty— $\sigma$  on the one hand and  $p$  or  $b$  on the other hand. The policies may also affect the long-term expected growth rate,  $g^*$ . Well known is the success of OECD countries in achieving low and stable inflation since the mid-1980s. This success is sometimes argued to have contributed to milder business fluctuations (lower  $\sigma$ ) and perhaps to stronger average economic growth (higher  $g^*$ ). However, commentaries on monetary policy frequently also stress the roles of central banks in exacerbating or moderating major economic crises. For example, Friedman and Schwartz (1963) blame the Federal Reserve for the severity of the Great Depression in the United States, as well as for the sharp recession of 1937-38. Observers of Alan Greenspan's tenure as Fed chair often focus on his role in apparently moderating the consequences of the global stock-market crash of 1987 and the Long-Term Capital Management/Russian crisis of 1998. These policy actions—if actually effective—have more to do with lowering  $p$  and  $b$  than decreasing  $\sigma$ . A key, unresolved issue is whether and how a monetary authority can reduce the probability,  $p$ , and size,  $b$ , of economic collapses.

Other governmental institutions and policies can also affect disaster probabilities and sizes. For example, the formation of the European Union and the adoption of the euro have often been analyzed as influences on the extent of business fluctuations ( $\sigma$ ) and the average rate of economic growth ( $g^*$ ), sometimes focusing on the role of international trade in goods and assets. However, from a political perspective, the main force behind

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<sup>27</sup> A broader analysis would include deaths, injuries, and so on. For discussion in the context of conflicts, see Hess (2003).

the adoption of these institutions was likely the desire to avoid a repetition of World War II; that is, to reduce the disaster probability,  $p$ , applicable to war. This perceived impact on disaster probability related to war is likely to be a key element in explaining why these institutions exist in Western Europe. Of course, this perception may be inaccurate—forcing Germany and France to share monetary, fiscal, and other policies may ultimately create more conflict than it eliminates. Thus, an important research topic is the actual influence of various policies and institutions on the probability and size of disasters, including wars.

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<b>Table 1</b>			
<b>Effects of Preference Parameters on Welfare Costs</b>			
		<b>welfare effect from eliminating uncertainty [1 - (Y<sub>t</sub>)*/Y<sub>t</sub>]</b>	
<b>γ</b>	<b>θ</b>	<b>p=0</b>	<b>σ=0</b>
4	4	20.9%	1.5%
3	3	15.7%	1.1%
2	2	14.8%	0.9%
1	1	18.5%	0.7%
3	4	12.6%	0.9%
2	4	8.5%	0.5%
1	4	6.3%	0.2%
4	3	25.2%	1.8%
4	2	31.9%	2.1%
4	1	43.8%	2.6%
4	3*	22.9%	1.5%
4	2*	25.4%	1.6%
4	1*	28.9%	1.6%

Note: Each term gives the proportionate reduction in initial output,  $1 - (Y_t)^*/Y_t$ , that maintains utility while setting  $p=0$  or  $\sigma=0$ .  $\gamma$  is the coefficient of relative risk aversion and  $\theta$  is the reciprocal of the elasticity of intertemporal substitution in the formula for utility in Eq. (26).  $(Y_t)^*/Y_t$  is calculated from Eq. (32), using the formula for the price-dividend ratio,  $V$ , in Eq. (30).

\*These results incorporate the change in  $\rho$  necessary to hold constant the effective rate of time preference,  $\rho^*$ , given in Eq. (29).

**Table 2**  
**Growth Rates and Rates of Return for OECD Countries, 1880-2005**

Country	Mean growth rates				Mean real rates of return		
	$\Delta c/c$	$\Delta y/y$	$\Delta C/C$	$\Delta Y/Y$	stocks	bills	bonds
<b>Australia</b>	0.015	0.015	0.032	0.033	0.103	0.012	0.035
<b>Canada</b>	0.021	0.022	0.038	0.039	0.077	0.016	0.035
<b>Denmark</b>	0.018	0.020	0.026	0.028	0.074	0.030	0.038
<b>France</b>	0.016	0.021	0.020	0.025	0.056	-0.011	0.002
<b>Germany</b>	0.023	0.022	0.030	0.027	0.073	-0.018	-0.002*
<b>Italy</b>	0.019	0.022	0.025	0.027	0.049	0.002	0.011
<b>Japan</b>	0.024	0.029	0.034	0.040	0.093	0.004	0.020
<b>Norway</b>	0.020	0.023	0.027	0.030	0.069	0.018	0.026
<b>Sweden</b>	0.020	0.020	0.025	0.026	0.091	0.023	0.031*
<b>U.K.</b>	0.015	0.016	0.019	0.020	0.064	0.017	0.029
<b>U.S.</b>	0.020	0.020	0.034	0.034	0.080	0.014	0.023
<b>Means</b>	<b>0.019</b>	<b>0.021</b>	<b>0.028</b>	<b>0.030</b>	<b>0.075</b>	<b>0.010</b>	<b>0.023</b>

Note: c is real per capita personal consumer expenditure, y is real per capita GDP, C is real personal consumer expenditure, and Y is real GDP. Growth rates are for annual data for year t compared to year t-1. Real rates of return are calculated from arithmetic annual returns during each year, based on nominal total return indexes and consumer price indexes. (For some country-years, stock returns are based on stock-price indexes and estimates of dividend yields.) Periods for growth rates are 1881-2005 and for returns are 1880-2005, except for the following missing data. Australia is missing growth rates of c and C for 1881-1901. Canada is missing stock returns for 1880-1915 and bill returns for 1880-1899. Denmark is missing growth rates of c and C for 1915-21 and stock returns for 1880-1914. France is missing growth rates of c and C for 1947-49 and stock returns for 1940-41. Germany is missing growth rates of c and C for 1914-25, 1939-40, and 1945-48 and bond returns for 1880-1923. Italy is missing growth rates of c and C for 1881-85 and stock returns for 1880-1905. Japan is missing growth rates of c and C for 1881-85, stock returns for 1880-1914, and bill returns for 1880-82. Norway is missing stock returns for 1880-1914. Sweden is missing stock returns for 1880-1901 and bond returns for 1880-1921. Data on real GDP and population are from Maddison (2003), updated from World Bank, *World Development Indicators* and Economist Intelligence Unit, *Country Data*. U.S. GDP data before 1929 are from Balke and Gordon (1989). Real personal consumer expenditure is from various sources, based on ongoing research. Data on asset returns and consumer price indexes are from Global Financial Data, discussed in Taylor (2005).

\*German bond returns are available only for 1924-2005. The mean value shown is calculated from bill returns for 1880-1923 and bond returns for 1924-2005. Swedish bond returns are available only for 1922-2005. The mean value shown is calculated from bill returns for 1880-1921 and bond returns for 1922-2005.