Double-Counting of Investment*

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Abstract

The national-income accounts double-count investment, which enters once when it occurs and again in present value as rental income on added capital. The double-counting implies that GDP and national income overstate sustainable consumption. An alternative measure, "permanent income," equals consumption in the steady state but deviates from consumption outside of the steady state because expensing of gross investment applies to the long-run flow, not the current value. The permanent-income perspective substantially affects measured factor-income shares. When computed in relation to permanent income, the U.S. labor-income share has been reasonably stable, in contrast to the declining share based on GDP.

Keywords: national-income accounting, investment, double-counting, permanent income, depreciation, economic growth, labor-income share.

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In setting up a system of national accounts, Kuznets (1941) stressed that the only true final goods are consumption at various dates. Therefore, a reasonable test for a national accounting system for production and income is how well it measures the potential for consumption over time. As Kuznets put it (p. 46): "Is it the value of goods *produced* that leads to the most valid appraisal of the positive contents of economic activity? Since the final aim is to satisfy the wants of ultimate consumers, we might perhaps more properly center attention on ultimate consumption."

If the only final goods are consumption in various periods, a reasonable requirement for a measure of national product or income is that it accurately reflect, subject to data limitations, the resources available for consumption. Specifically, a necessary condition from an intertemporal perspective is that—at least conceptually—a measure of current income or product, when appropriately capitalized, should equal the present value of consumption. However, the usual measures of national or domestic product and income fail this test because they double-count investment. Gross or net product includes gross or net investment when it occurs and includes the corresponding present value a second time when additional rental income results from the enhanced stocks of capital. Although depreciation is a significant part of the analysis, the double-counting issue is a distinct phenomenon. Double-counting of investment would apply even if capital were infinitely lived, so that gross and net product would coincide.

The standard national accounts, discussed in Bureau of Economic Analysis (2019), avoid the double-counting of GDP when it comes to intermediate inputs, which are netted out in the calculation of value-added. For example, value-added in automobile manufacturing is computed net of the cost of the steel, tires, etc. used up in the production of automobiles. But a basic economic question is why these intermediates differ from a durable machine, the services of which contribute to the production of other goods. The main answer is that they are not different. The machine counts once in GDP when it is produced—when it is treated as a final good in the form of gross fixed investment.[1] But then the machine generates a flow of services, which enter into future GDP. Since the present value of this service flow equals the initial cost, the investment in a machine counts twice in GDP in a present-value sense. Moreover, this conclusion holds independently of the durability of the machine—the double-counting applies even when the machine lasts forever.

In a model with a representative agent, such as the one described in the next section, welfare corresponds to the single agent's attained utility. Then, with no labor-leisure choice, welfare depends only on the time path of consumption. More generally, measured welfare would factor in other considerations, including the path of leisure time and the distribution of income, as suggested in a report written for the U.S. Senate by Kuznets (1934, pp. 6-7):

> "Economic welfare cannot be adequately measured unless the personal distribution of income is known. And no income measurement undertakes to estimate the reverse side of income, that is, the intensity and unpleasantness of effort going into the earning of income. The welfare of a nation can, therefore, scarcely be inferred from a measurement of national income … "

The present paper deals with how the reported aggregates of product and income relate to the path of aggregate consumption. Therefore, the analysis relates to welfare in so far as welfare depends on the path of aggregate consumption.

Stiglitz, Sen, and Fitoussi (2009) provide a survey of issues concerning measured product and income as gauges of welfare. Adjustments of real GDP and consumption for specific issues, including income distribution, environmental damage, life expectancy, and work hours have been carried out by Osberg and Sharpe (2002) and Jones and Klenow (2016), among others.

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[1] In practice, the BEA treats goods that last for at least one year as fixed assets.

However, the dimensions of welfare considered in these studies are conceptually distinct from the issue of double-counting of investment, which is the focus of the present analysis.

The issues matter not only for measurement of product and income but also for computations of factor-income shares and for input-output analyses. The main points emerge clearly within a simple, well-known framework, the neoclassical growth model. However, the results generalize beyond this setting. The key element of the model is its respect for intertemporal budget constraints.

1. Literature Review

Kuznets (1941) set out the basics of national-income accounting and argued that consumption is the only true final good. This insight suggests that gross investment is an intermediate good that should, in some sense, be expensed out of GDP in order to obtain an income measure that gauges the intertemporal possibilities for consumption. But this idea, not pursued by Kuznets, does not provide guidance on how to carry out the expensing. In the steady state, the full expensing of gross investment is appropriate, and current consumption satisfactorily gauges intertemporal consumption possibilities. But this approach fails outside of the steady state. As shown in this paper, the correct expensing at date t is the steady-state flow of gross investment associated with date t's capital stock, not date t's actual gross investment.

The closest paper to the present one is Weitzman (1976). He shows in his equation (10) that, under some conditions, current national income (equal to net domestic product for a closed economy) is the current income measure that corresponds to the present value of future consumption. However, his analysis does not account for levels and growth rates of labor income. With this addition, national income double-counts the flow of net investment even in the steady state. At date t, the income measure that corresponds to the present value of consumption expenses out of GDP not only depreciation but also the steady-state flow of net investment associated with date *t*'s capital stock.

Hulten (1979) emphasizes that investment is an intermediate good, which should not appear in an intertemporal budget constraint that determines the present value of consumption. Therefore, the budget constraint over a four-period horizon in his equation (13) excludes investments made at intermediate dates. However, Hulten does not relate this result to standard measures of income and product (such as GDP and national income) or to alternative measures that correspond to an economy's consumption potential.

The analysis of national-accounting aggregates in this paper also relates to a large literature associated with factor-income shares. This literature is discussed and extended in Karabarbounis and Neiman (2014), which focuses on the apparent decline of the labor-income share in recent years in the United States and other countries. The present analysis suggests major modifications to the way that factor-income shares should be calculated and, therefore, calls into question the empirical analysis in Karabarbounis and Neiman (2014) and other studies. This critique is a generalization of the one presented in Koh, Santaeulalia-Llopis, and Zheng (2020), which focuses on effects from recent changes in accounting practices to capitalize parts of intellectual-property products.

2. Intertemporal Framework

2.1. GDP, National Income, Permanent Income

The setup is standard, corresponding to the well-known infinite-horizon neoclassical growth model for a closed economy.[2] The representative agent's assets are held at time *t* as internal loans (private bonds that aggregate to zero) or claims on capital, $K(t)$, which depreciates

[2]This setup is often called the Ramsey model or the Cass-Koopmans model, following Ramsey (1928), Cass (1965), and Koopmans (1965). Cass and Koopmans generalized the models of Solow (1956) and Swan (1956).

at the constant rate $\delta>0$. Perfectly competitive producers of goods produce output, $Y(t)$, using $K(t)$ and labor, $L(t)$, through a constant-returns-to-scale production function, $F(\cdot)$, which satisfies the usual neoclassical properties. Output for a closed economy without a government sector divides between consumption, $C(t)$, and gross investment, $I(t)$.[3]

The representative agent's budget constraint at a point in time is:

$$Y(t) = F[K(t), L(t)] = C(t) + I(t) = w(t)L(t) + R(t)K(t), \quad (1)$$

where $w(t)$ is the real wage rate, equaling the marginal product of labor, and $R(t)$ is the real rental price of capital, equaling the marginal product of capital. The real rate of return on capital and internal loans in this one-sector-production-function model is[4]

$$r(t) = R(t) - \delta. \quad (2)$$

The representative household chooses $C(t)$ over time to maximize overall utility. In the usual setup, as in Barro and Sala-i-Martin (2004, Ch. 2), the steady-state real interest rate is determined by parameters related to time preference and intertemporal substitution and by technical progress, assumed to occur at a constant rate in a labor-augmenting form. Correspondingly, the common steady-state growth rate of $Y(t)$, $C(t)$, $K(t)$, and $w(t)L(t)$ is a constant, $g\geq 0$, which is the sum of the constant growth rate of labor, corresponding to constant population growth, and the constant rate of labor-augmenting technical progress. In a stochastic

[3]The focus here is on a closed economy without government. But C can include net imports and government consumption, I can include net foreign investment and public investment, and K can include net foreign assets and government-owned capital.

[4]The uses of output in this one-sector model can be expanded to include perishable intermediate goods, which enter into production of final goods. The production function $F(\cdot)$ in equation (1) can then be viewed as applying to value added. This setup works exactly if intermediates enter in fixed proportions with gross output; for example, if an automobile always requires one ton of steel.

version of this model, the steady-state expected real rate of return on capital depends also on risk aversion and on parameters related to uncertainty in the production process.[5]

The present analysis simplifies by assuming that the expected real rate of return on capital or equity, *r(t)*, is constant throughout at its steady-state value, denoted by *r*. The rate *r* exceeds the constant safe real interest rate by an amount that corresponds to the equity premium. More generally, *r* does not have to be constant and can be viewed as an average of present and expected future real rates of return on capital.

The intertemporal budget constraint for the representative household, starting from the current date *t*, is

$$K(t) + \int_t^\infty w(T)L(T)e^{-r(T-t)}\,dT = \int_t^\infty C(T)e^{-r(T-t)}\,dT. \quad (3)$$

This constraint reflects the usual transversality or dynamic-efficiency condition, $r>g$. This condition ensures that the "terminal value" of *K(T)* has a present value that asymptotes to zero as *T* tends to infinity.[6] Empirically, $r>g$ is satisfied if we measure *r* by the long-run average real rate of return on capital or equity, not the safe real interest rate, and *g* by the long-run average growth rate of real GDP or consumption.[7] Reasonable values, discussed later, are *r* around 0.08 per year and *g* between 0.02 and 0.03 per year.

The left side of equation (3) gives the household's overall wealth or capitalized income, which is used to purchase the present value of consumption, given on the right side. Similarly, Weitzman (1976) emphasizes that this present value of consumption is the key object to

[5]See, for example, Barro (2009).

[6]See Barro and Sala-i-Martin (2004, p.93). The dynamic-efficiency condition, $r>g$, reflects the representative consumer's optimization problem (which underlies the transversality condition) and is needed to make sense of the infinite-horizon, representative-agent model related to Ramsey (1928).

[7]Barro (2009) shows, in a rare-disasters model, that the condition $r^f<g$, where r^f is the safe real interest rate, can hold in the theoretical model and tends to apply empirically. The key transversality condition in the model is $r^e>g$, where r^e is the expected real rate of return on unlevered equity and capital. This restriction clearly holds empirically. The key element here is that r^e is much larger than r^f; that is, the equity premium is large.

consider; that is, measures of wealth and income should be assessed in relation to the present value of consumption that can be financed.

Assume to simplify that real labor income, $w(T)L(T)$ for $T \geq t$, always grows at the steady-state rate, g. In this case, equation (3) becomes

$$(r-g) \cdot K(t) + w(t)L(t) = (r-g) \cdot \int_t^{\infty} C(T)e^{-r(T-t)}\, dT. \tag{4}$$

If consumption, $C(T)$, also grows at rate g (as it does in the steady state), the right side of equation (4) simplifies to $C(t)$. However, the present analysis does not require $C(T)$ and $K(T)$ to be on a steady-state path. The right side of equation (4) can be viewed, in the spirit of Milton Friedman (1957, Ch.2), as "permanent consumption." Permanent consumption at time t is defined as the flow that, when growing at the steady-state rate, g, has a present value equal to the actual one, $\int_t^{\infty} C(T)e^{-r(T-t)}\, dT$.

The intertemporal budget constraint in equation (4) corresponds to Weitzman (1976, equation [10]) if real labor income, $w(t)L(t)$, or the growth rate, g, of this income equals zero. If $w(t)L(t)=0$ for all t, the equation implies that $K(t)$ equals the present value of consumption. Since date t's national income in this case equals $rK(t)$, the ratio of national income to r equals $K(t)$, which equals the present value of consumption. Alternatively, if $w(t)L(t)>0$ for all t but $g=0$, the left side of the equation equals date t's national income (equal to net domestic product for a closed economy). Then the equation implies that the ratio of this national income to r equals the present value of consumption, evaluated starting from date t.

More generally, neither g nor $w(t)L(t)$ equals zero.[8] In this situation, equation (4) still yields a concept of current "income" that, when appropriately capitalized, equals the present

[8]Hulten's (1979) equation (13) allows for labor income over a four-period horizon. However, the analysis does not explain how to deal with the present value of the capital stock left over at the end of the horizon (on the right side

value of consumption. This income concept, which I call "permanent income," $Y^*(t)$, equals the permanent consumption mentioned before and corresponds to the left side of equation (4):

$$(5) \qquad \text{Permanent income} \equiv Y^*(t) = (r\text{-}g)\cdot K(t) + w(t)L(t).$$

Equation (4) implies that capitalizing $Y^*(t)$ by dividing by r-g delivers the present value of consumption. Moreover, this equality arises not just in the steady state but for any paths of future consumption and capital stock, $C(T)$ and $K(T)$. The only assumptions needed are that r is constant, labor income, $w(T)L(T)$, always grows at the constant rate g, and $r>g$.

In the steady state, where $C(T)$ also grows at rate g, equation (4) implies $Y^*(t)=C(t)$. Thus, in the steady state, we could follow Kuznets (1941) and think of current consumption, $C(t)$, as the appropriate measure of permanent income. However, this treatment of consumption fails outside of the steady state. Suppose, for example, that, starting from a steady-state path, $C(t)$ declines and $I(t)$ rises for some length of time, so that the expanded capital stock allows for higher $C(T)$ in the future. The identification of permanent income with consumption, $C(t)$, would imply that permanent income falls in this example, although the present value of consumption dictated by the intertemporal budget constraint in equations (3) and (4) does not change. In contrast, permanent income, $Y^*(t)$, as defined in equation (5), gives the right answer because this measure is unchanged.

The results for permanent income can be compared with those for standard measures of income and product. The formula for GDP is

$$(6) \qquad Y(t) = RK(t) + w(t)L(t) = (r+\delta)\cdot K(t) + w(t)L(t).$$

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of the equation). There is also no discussion of what measure of current income, appropriately capitalized, equals the present value of consumption.

Permanent income, $Y^*(t)$ in equation (5), corresponds to subtracting from $Y(t)$ in equation (6) the flow $(\delta+g)\cdot K(t)$, which equals gross investment along a steady-state path. That is, permanent income effectively expenses out steady-state gross investment from GDP. In contrast, consumption, $C(t)$, expenses out actual gross investment, $I(t)$, from GDP. The two procedures coincide in the steady state. But, outside of the steady state, the calculation of permanent income corresponds to subtracting from GDP the steady-state flow of gross investment, not the actual flow.

The excess of $Y(t)$ over $Y^*(t)$, which equals $(\delta+g)\cdot K(t)$, gives (when divided by r-g) the extent to which current GDP overstates the present value of consumption that can be financed. In the steady state, this excess corresponds to gross investment (divided by r-g). An interpretation is that GDP effectively counts gross investment twice—once when it occurs and a second time when it generates gross rentals from stocks of capital.

Similarly, the formula for national income is

$$Y^n(t) = (R-\delta)\cdot K(t) + w(t)L(t) = rK(t) + w(t)L(t). \tag{7}$$

Permanent income, $Y^*(t)$ in equation (5), corresponds to subtracting from national income in equation (6) the flow $gK(t)$, which equals net investment along a steady-state path. That is, permanent income expenses out steady-state net investment from national income, whereas $C(t)$ expenses out actual net investment. Again, the two procedures coincide in the steady state. But, outside of the steady state, the calculation of permanent income corresponds to subtracting from national income the steady-state flow of net investment, not the actual flow.

The excess of $Y^n(t)$ over $Y^*(t)$, which equals $gK(t)$, gives (when divided by r-g) the extent to which current national income overstates the present value of consumption that can be financed. In the steady state, this excess corresponds to net investment (divided by r-g). An interpretation is that national income counts net investment twice—once when it occurs and a second time when it generates net rentals from stocks of capital.

The assumed constancy of r and g is unimportant for the basic approach. More generally, r corresponds to the average of current and expected future real rates of return on capital, and g corresponds to the average of expected future growth rates of real labor income. The identification of r with the (constant) steady-state real rate of return and of g with the (constant) steady-state growth rate are just approximations.

The key result is that permanent income, $Y^*(t)$, equals GDP, $Y(t)$, adjusted for "effective depreciation," $(\delta+g)\cdot K(t)$:

$$Y^*(t) = Y(t) - (\delta+g)\cdot K(t). \tag{8}$$

The computation of permanent income is, therefore, an extension of the usual practice of subtracting depreciation, $\delta K(t)$, from GDP to compute net domestic product, NDP, which equals national income for a closed economy. The difference is that the effective depreciation rate is now $\delta+g$, rather than δ, where g corresponds to the prospective long-run growth rate. Note that an adjustment to GDP to calculate permanent income is necessary even when capital lasts forever; that is, if $\delta=0$.

For given values of $Y(t)$, $K(t)$, and δ, a rise in g lowers permanent income, $Y^*(t)$, in equation (8). However, the present value of consumption equals $Y^*(t)$ divided by r-g. Taking into account the change in the denominator, r-g, leads to the expected result that the present value of consumption rises with g, for given values of $Y(t)$, $K(t)$, and δ.

From a measurement perspective, an important result is that the standard GDP, $Y(t)$, enters on the right-hand side of equation (8). GDP corresponds to the current production of goods and services[9] and is likely to be the part of permanent income that is most accurately measured in practice. The other part, $-(\delta+g)\cdot K(t)$, reflects the effective depreciation flow, which corresponds partly to literal depreciation and partly to the investment outlay required to maintain growth of the capital stock at its long-run rate of g. Permanent income will contain measurement error related to the capital stock, $K(t)$, the depreciation rate, δ (which could vary over time), and the long-run growth rate, g (which could also vary over time). In practice, the measured $K(t)$, corresponding say to measures of fixed assets in the national accounts, is slowly moving, and the measured δ changes only because of (slowly-moving) shifts in the composition of capital (for given rates of depreciation on each type of capital). The assumed value of g would be constant or slowly-moving. Therefore, the short-term variations in measured $Y^*(t)$ would be dominated by the observed short-term variations in $Y(t)$. Specifically, the business-cycle fluctuations reflected in measured permanent income would correspond closely to those in observed GDP.

Figure 1 shows the empirical relation between the logs of real GDP, real NDP, and real permanent income, computed from U.S. BEA data from 1948 to 2019. The series for NDP uses BEA depreciation on fixed assets (including government assets but excluding consumer durables other than residential housing). The calculation for permanent income uses the fixed value $g=0.031$ per year, which equals the average growth rate of real GDP from 1948 to 2019.[10] Figure 1 shows that the short-term proportionate movements in calculated real NDP and real permanent income mirror those of real GDP. In this sense, the standard GDP measure remains the key variable for gauging business fluctuations.

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[9]GDP includes net inventory investment—including changes in goods-in-process and materials—and does not correspond to production of finished goods. For example, the production of steel intended for use in automobile production adds to GDP. But then the existing steel used up in auto production subtracts from GDP.
[10]To calculate gK for year t, the value $g=0.031$ is multiplied by the average of fixed assets at the ends of years t and $t-1$.

The most important distinctions among the three macroeconomic aggregates shown in Figure 1 are the substantial and persistent differences in levels. For example, the averages of the log differences from 1948 to 2019 are 0.15 between GDP and NDP and 0.11 between NDP and permanent income, corresponding to an average log difference between GDP and permanent income of 0.26, implying a 30% difference in levels. Thus, for long-term averages, the GDP measure overstates permanent income and, hence, consumption potential by around 30%.

The log gaps among the macro aggregates are not constant: from 1948 to 2019, the GDP-NDP gap rose from 0.12 in 1948 to 0.17, and the NDP-permanent income gap rose from 0.10 to 0.12. The first change reflects mostly the rise from 1948 to 2019 in the BEA value of δ (calculated as the ratio of depreciation to fixed assets) from 0.041 to 0.054. This change reflects shifts in the composition of capital, particularly toward intellectual-property products. Another force, which affects both log gaps, is the rise in the ratio of fixed assets to GDP from 2.98 in 1948 to 3.09 in 2019.

Figure 2 compares the estimated series for permanent income with observed "consumption," gauged by the sum of real personal consumer expenditure and real government consumption.[11] For most of the sample, the consumption series (solid line) corresponds well in levels and changes to permanent income (dotted line), as would be anticipated if the U.S. economy were moving mostly along a steady-state path. Permanent income does fall notably relative to consumption during recessions, such as those in 1982-84 and 1975. The key factor here is that gross investment and net exports tend to be especially low during recessions. A more

[11]Since personal consumer expenditure includes purchases of consumer durables, this approach corresponds to the exclusion of stocks of consumer durables from fixed assets in the calculations of depreciation for the construction of Figure 1. The inclusion of government consumption corresponds to the inclusion of public capital for the fixed assets used in the calculations of depreciation.

persistent positive gap between consumption and permanent income shows up from 2000 on, partly because of the recession in 2009-10.

An important element in the construction of permanent income is the assumed value for the long-run growth rate, taken to be $g=0.031$ per year. There is substantial uncertainty about this growth rate because of uncertainty about future rates of technological progress and population growth. Specifically, there is an ongoing debate about whether future U.S. growth rates of real GDP and other macroeconomic aggregates will match the long-term historical average of around 3% per year or will instead be only about 2%, the rate since 2000. If the value $g=0.02$ is used instead of 0.031 in the calculations of permanent income from 2000 on, the revised series, shown by the dashed graph in Figure 2, parallels the dotted graph but is higher on average. Hence, the gap since 2000 between consumption and permanent income is smaller with this alternative version of permanent income. Possibly, the remaining gap reflects exaggeration of depreciation on intellectual-property products in recent years.

The roles of double-counting of investment in GDP and national income parallel the well-known idea in public finance that a tax on capital income amounts to double taxation. This argument has been used to argue that a tax on consumption (or expenditure) is preferable to a tax on income, broadly defined to include capital income. This idea goes back at least to Mill (1857, pp. 380-81), who said " … the proper mode of assessing an income tax would be to tax only the part of income devoted to expenditure, exempting that which is saved. For when saved and invested … it thenceforth pays income tax on the interest or profit which it brings, notwithstanding that it has already been taxed on the principal. Unless, therefore, savings are exempted from an income tax, the contributors are twice taxed on what they save, and only once on what they spend. … The principal and the interest cannot both together form part of [a person's] resources; they are the same portion twice counted …"

2.2. Factor-income Shares

A natural measure of capital income is the net income on capital, $rK(t)$, which enters along with labor income, $w(t)L(t)$, in national income, $Y^n(t)$, in equation (7). This measure uses r, the net real rate of return on capital, to gauge capital income, and the capital-income share of national income is the ratio of $rK(t)$ to $Y^n(t)$. The labor-income share of national income is then one minus this capital-income share. Equivalently, the labor-income share can be calculated directly as the ratio of $w(t)L(t)$ to $Y^n(t)$.

The problem with this approach is that, as shown in the previous section, national income, $Y^n(t)$, overstates the resources available intertemporally for consumption. From the standpoint of consumption potential, the appropriate measure is permanent income, $Y^*(t)$, given in equation (5). The labor-income share of permanent income is the ratio of $w(t)L(t)$ to $Y^*(t)$, and the capital-income share of permanent income is one minus this labor-income share. Equivalently, the capital-income share of permanent income is the ratio of $(r\text{-}g)\cdot K(t)$ to $Y^*(t)$. Since $Y^*(t) < Y^n(t)$, the labor-income share of national income is smaller than that for permanent income and, correspondingly, the capital-income share of national income is larger than that for permanent income.

The standard approach measures capital income by the gross income on capital, $RK(t)$, which enters along with $w(t)L(t)$ in GDP, $Y(t)$, in equation (6). The capital-income share for GDP is the ratio of $RK(t)$ to $Y(t)$, and the labor-income share is one minus this capital-income share. Since GDP exceeds national income, the labor-income share of GDP is smaller than that for national income (and this share is smaller than that for permanent income). Correspondingly, the capital-income share for GDP is larger than that for national income (and this share is larger than that for permanent income).

The analysis below shows that, in terms of average levels, factor-income shares computed from GDP or national income tend to differ substantially from those calculated from permanent income. Moreover, in non-steady-state situations where the depreciation rate, δ, and the ratio of $K(t)$ to $Y(t)$ are changing over time, the factor-income shares computed from GDP or national income can exhibit trends even when no trends apply to the shares calculated from permanent income.

2.3. Quantitative Estimates in the Steady State

Equations (5)-(7) implied that, from the standpoint of sustainable consumption, the overstatements compared to permanent income equaled $(\delta+g)\cdot K$ for GDP and gK for national income (where the time subscripts are omitted for convenience). The proportionate sizes of these overstatements depend on the ratios of K to GDP and national income, respectively. This section evaluates these ratios and, hence, the proportionate sizes of overstatements in a steady state when the production function $F(\cdot)$ in equation (1) is Cobb-Douglas with exponents α on K and $1-\alpha$ on L:

(9) $$Y = A \cdot K^{\alpha} L^{1-\alpha},$$

where $A>0$ and $0<\alpha<1$.

Perfect competition among goods producers implies the equation of the gross marginal product of capital, given by $\alpha Y/K$, to $R=r+\delta$. This condition implies from equation (9) that K/Y is constant in the steady state (in which r is constant) and given by

(10) $$\frac{K}{Y} = \frac{\alpha}{r+\delta}.$$

Equation (10) implies that the ratio of GDP overstatement, $(\delta+g)\cdot K$, to Y is constant in the steady state and given by

$$(11) \qquad \text{Ratio of GDP overstatement to GDP} = \frac{\alpha\cdot(\delta+g)}{r+\delta}.$$

Because the source of the GDP overstatement is the double-counting of gross investment, the amount given in equation (11) is nil when $\delta=g=0$, in which case there is no gross investment in the steady state. As a quantitative example, suppose that $\alpha=0.40$, $\delta=0.05$ per year, $g=0.03$ per year, and $r=0.08$ per year.[12] In this case, the ratio in equation (10) equals 0.25.[13] That is, in the steady state with reasonable parameters, GDP overstates permanent income by 25%. In contrast, the empirical average constructed from the BEA data described in section 2.1 was around 30%.

An analogous exercise applies to the double-counting ratio for national income. The result in the steady state is

$$(12) \qquad \text{Ratio of national-income overstatement to national income} = \frac{\alpha g}{r+\delta(1-\alpha)}.$$

This overstatement is nil when $g=0$, in which case there is no net investment in the steady state. Using the parameters noted above, national income overstates permanent income in the steady state by 11%. Hence, the netting out of depreciation substantially reduces the overstatement of income (from 25% to 11%) but does not eliminate it.

Consider now the various measures of factor-income shares, as discussed in Section 2.2. For GDP, equation (10) implies, as usual, that the ratio of capital's gross rental income, $(r+\delta)\cdot K$,

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[12]These parameters are motivated mostly by U.S. data. The values of δ and g correspond to the averages for BEA data discussed in the previous section. The value for α approximates estimates of Fernald (2014) for the U.S. corporate sector in recent years. The value for r corresponds to an average of (arithmetic) real rates of return on equity (0.083 for the United States, 0.070 for 14 OECD countries with long-term data) from an updated version of the numbers in Barro and Ursúa (2008, Table 5). The underlying data come mostly from *Global Financial Data*. The calculated real rates of return are net of taxes levied on corporations; a value for r gross of business taxes (corresponding more closely to the marginal product of capital) would be higher. However, a lower value of r would be appropriate to allow for the influence of leverage on the rate of return on equity.

[13]With the assumed parameter values, the steady-state K/Y from equation (10) equals 3.1 (in units of years).

to Y equals α, which was set at 0.40 in the previous examples. As noted before, this standard concept does not correspond to a capital-income share because gross rental income is not a measure of net income. However, if one takes seriously the Cobb-Douglas form of the production function in equation (9), then this standard "capital-income share" is interesting as a measure of the exponent α, which gives the elasticity of output, Y, with respect to the capital stock, K.

For national income, the ratio of rK to national income, Y-δK, in the steady state can be determined to equal

(13) National-income capital share = $\frac{r\alpha}{r+\delta(1-\alpha)}$.

This share equals α when $\delta=0$ (in which case national income equals GDP, and net investment equals gross investment). Using the parameter values assumed before for r, δ, and α, the national-income capital share in equation (13) is 0.29, well below the standard number of $\alpha=0.40$.

For permanent income, the income on capital in equation (5) is $(r\text{-}g)\cdot K$. The ratio to permanent income, Y^*, in the steady state can be determined to equal:

(14) Permanent-income capital share = $\frac{(r-g)\cdot\alpha}{r+\delta(1-\alpha)-g\alpha}$.

This share equals α when $\delta=g=0$ (in which case permanent income equals GDP, and net and gross investment are both zero in the steady state). Using the parameter values assumed before for r, δ, g, and α, the permanent-income capital share in equation (14) is 0.20, half of the standard share number of $\alpha=0.40$.

As mentioned before, Karabarbounis and Neiman (2014) summarize and extend a substantial literature concerned with the empirical evolution of labor-income shares. They find that the labor-income share has been declining for some time in the United States and many other countries. In their research, the labor share is gauged by the ratio of labor income to GDP. They focus on data for the corporate sector to avoid issues about how to allocate entrepreneurial income between labor and capital. Since GDP includes gross rental payments on capital, the implicit assumption is that these gross rentals are a satisfactory measure of the income from capital. The argument above is that the income from capital should be scaled down to incorporate effective depreciation of capital at the rate $\delta+g$. Correspondingly, the level of labor's income share should be scaled up—because the denominator, GDP, in the formula for the labor share should be scaled down to reflect effective depreciation.

These adjustments affect not only average levels of labor-income shares but also changes over time in these shares. Specifically, if δ or K/Y is rising, the labor-income share based on GDP tends to fall compared to that based on permanent income. Figure 3 shows how these adjustments affect the calculated labor-income shares for the U.S. corporate sector[14] from 1948 to 2019. The bottom graph is the standard measure, given by the ratio of labor compensation to GDP (all originating in the corporate sector). The middle and upper graphs show the corresponding ratios of labor compensation to national income and permanent income.[15]

The results do not support the view that the U.S. labor-income share has been systematically declining. The standard labor-share number (bottom graph) was 62.4% in 1948, 64.5% in 1990, 56.1% in 2014 (the low point), and 58.7% in 2019. Hence, there was a fall by 3.7 percentage points from 1948 to 2019 and by 5.8 percentage points from 1990 to 2019. In contrast, the labor share based on permanent income is 71.1% in 1948, 78.5% in 1990, 70.5% in 2014, and 73.9% in 2019. These numbers show a *rise* by 2.8 percentage points from 1948 to 2019 and a fall by 4.6 percentage points from 1990 to 2019. Overall, the calculations based on

[14]In the BEA data, the corporate sector includes S-corporations along with C-corporations.
[15]For the calculation of permanent income, the growth rate, g, was again assumed to be *0.031* per year.

permanent income seem consistent with moderate fluctuations of the labor-income share around a reasonably stable long-run value.[16]

2.4. Practical measurement

In terms of practical measurement, it is worth stressing that, starting from the computed GDP, $Y(t)$, the calculation of permanent income, $Y^*(t)$, in equation (8) is a straightforward extension of the usual practice of deducting depreciation to compute net domestic product or national income, $Y^n(t)$. That is, national income subtracts $\delta K(t)$ from $Y(t)$,[17] and permanent income goes further to subtract "effective depreciation," $(\delta+g)K(t)$. In other words, the effective depreciation rate is $\delta+g$, rather than δ, and g corresponds to an average of prospective future growth rates of real labor income. In the neoclassical growth model, the long-run growth rate of real labor income, g, equals the steady-state growth rate of various aggregates, including real GDP, real national income, consumption, and the capital stock.

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[16]These conclusions accord with those of Koh, Santaeulalia-Llopis, and Zheng (2020), discussed below, who focus on the role of the recently implemented capitalization of investment in some forms of intellectual-property products. Their major point is that the rising trend of this form of capital relative to GDP can explain the apparent negative trend in the measured U.S. labor-income share.

[17]Kuznets stressed that estimating depreciation is crucial for computing net concepts of product and income but that the measurement of depreciation is difficult. (Much later, Hulten and Wykoff [1981] pioneered the use of observed prices of used capital goods of different vintages to infer depreciation rates.) Kuznets (1941, pp. 41-42) said: "What fraction of the durable capital good is consumed during the given period? The signs that would indicate that this or that fraction of a machine's total useful life or capacity has been absorbed are few. There are few reliable data even on total useful life and capacity. Consequently, estimates by business enterprises of current consumption of durable capital are exceedingly crude … The investigator must accept these estimates … To prevent distortion of the national income total and its distribution, estimates of intermediate consumption must be complete." In comparing net product with gross product, Kuznets (1937, p. 3) noted some advantages of the gross measures: "Of … several possible concepts of gross national product one appears of greater importance than the others, that in which the value of commodities and services produced is not adjusted for the value of durable capital goods consumed in the process of production, but is adjusted for raw materials, partly fabricated products and fuel consumed. It is this concept that is referred to … as gross national product … [which] has the advantage of being a variable that can be measured more accurately than net national product. More important is the fact that the replacement of durable capital goods in use by new commodities is not as rigidly controlled by technical considerations as is the replacement of raw materials … over short periods the stock of capital equipment may be treated as indestructible, and its consumption in the process of production neglected. … Consequently, in addition to net national product or national income, we also measure gross national product …"

As already mentioned, there is uncertainty about the economy's long-run growth rate, *g*. The problem, however, is that ignoring the role of *g* in the calculation of permanent income amounts to assuming $g=0$. Using a value such as 2-3% per year seems preferable. If the value $g=0.02$ is used instead of $g=0.031$ in the calculation of permanent income from 1948 to 2019, the mean of the gap between log(real NDP) and log(real permanent income) is *0.07*, rather than *0.11*.

Note that, as long as the key parameters and measurement procedures are held fixed, the overstatement issues for GDP and national income affect computed levels of macroeconomic aggregates but need not affect growth rates, including the steady-state growth rate, *g*. In the steady state, all of the measures related to product and income—including real GDP, real national income, permanent income, capital stock, real labor income, and consumption—grow at the same rate, *g*.

The level effects are important, however, and do not involve merely a normalization for the calculated level of real GDP. The double-counting issue affects ratios of GDP or national income to consumption. For example, in the steady state, the ratio of *C* to GDP is:

$$(15) \qquad \frac{C}{Y} = 1 - \frac{I}{Y} = 1 - \frac{(\delta+g)K}{Y} = 1 - \frac{\alpha\cdot(\delta+g)}{(r+\delta)},$$

where the last result uses the formula for steady-state *K/Y* with a Cobb-Douglas production function from equation (10). The last term on the right side of equation (15) is the ratio of GDP overstatement to GDP, as contained in equation (11). This term reveals the proportionate extent to which *C* falls short of *Y* in the steady state; that is, it indicates by how much production (real GDP) proportionately overstates the resources available for consumption.

The level effects are also important for comparisons across countries. The central idea of the International Comparison Program (ICP) is to use estimated purchasing-power parities

(PPPs) to construct levels of real per capita GDP that can be compared among countries at a point in time.[18] These cross-sectional comparisons would be affected by the double-counting issue if the problem were more serious in some countries than in others. Specifically, countries with higher capital-output ratios, *K/Y*—reflecting higher propensities to save and invest—would tend to have greater proportionate overstatement in levels of real per capita GDP. The "contribution" of saving and investment to a central measure of economic development—the level of PPP adjusted real per capita GDP—is, therefore, mechanically exaggerated.[19]

2.5. Input-output tables

Another issue is that the treatment of investment as a final good affects the BEA's main input-output tables, which apply across industries.[20] Goods treated as perishable (materials and intermediates) show up as flows from one sector to another. Goods viewed as durable investments are treated instead as final goods, which appear, along with consumption, as a final-good use of products. The BEA does generate *Capital Flow Tables*, last produced publicly for 1997, which provide an input-output analysis for newly produced structures, equipment, and software. These tables apply to investment, not to the flows of rental services on capital that would enter as inputs into production. Vom Lehn and Winberry (2020, Section 2.1) estimate the capital-flow tables annually from 1947 to 2017.

As discussed in the next section, the BEA revised its accounting procedures in 1999 and 2013 to capitalize some types of intellectual-property products. Before the revisions, these products—such as software and R&D—entered into the BEA's main input-output tables as flows

[18]This program was begun in 1968 as a joint venture of the United Nations and the University of Pennsylvania and is now directed by the World Bank in connection with the United Nations. For a conceptual discussion, see Summers and Heston (1991) and Feenstra, Inklaar, and Timmer (2015).
[19]This conclusion applies also to level comparisons across countries based on observed exchange rates, rather than purchasing-power parities.
[20]For a discussion of these tables, see Young, et al. (2015).

of non-durable goods from one industry to another. After the revisions, these outlays were treated as final goods (investments) that no longer appeared in the flows across sectors.[21]

An alternative procedure would treat estimated rental services on durable investments, such as machines, as flows of services from one sector to another. This calculation would apply automatically if capital goods were literally rented by producers (say manufacturers of machines) to users (say construction companies or farmers). Importantly, these rental flows would not be contemporaneous with the investments themselves. Conceivably, the information that underlies the BEA's *Capital Flow* Tables, as extended by Vom Lehn and Winberry (2020, Section 2.1), could be used to construct an input-output analysis for rental services.

3. Capital in the National Accounts

The model applies directly to several forms of capital in the national accounts, including business equipment and non-residential structures. Issues arise for other types of "capital." Remarkably, the standard treatment of investment and capital varies substantially across forms of capital and not in ways that are readily understandable from the viewpoints of economic concepts or practicalities of measurement.

The model can address issues with different forms of capital from an extension to allow for two types, K_1 and K_2. Hence, the Cobb-Douglas production function from equation (9) is extended to:

$$Y = A \cdot K_1^{\alpha_1} K_2^{\alpha_2} L^{1-\alpha_1-\alpha_2}, \tag{16}$$

where $\alpha_1>0$, $\alpha_2\geq 0$, and $0<\alpha_1+\alpha_2<1$. GDP now equals consumption, C, plus the two types of gross investment, I_1 and I_2. To simplify the algebra, the two depreciations rates, δ_1 and δ_2, are assumed to be the same and equal to δ. In this case, the previous results on overstatement of

[21] Xiang Ding brought this point to my attention.

GDP and national income and on factor-income shares continue to apply, with the substitution of $\alpha_1+\alpha_2$ for α in the various equations. If $\alpha_1+\alpha_2 = 0.40$, the value assumed before for α, the previous numerical results continue to hold.

3.1. Intellectual property

In the first application, K_2 is identified with intellectual property, a form of intangible capital. In its 1999 and 2013 revisions to the national accounts, the Bureau of Economic Analysis (BEA) included categories of intellectual property—software in 1999, research & development and artistic originals in 2013—as capital goods, analogous to equipment and structures.[22] Intellectual property is now a large item, constituting 27% of U.S. private fixed domestic investment in 2019. This category has grown substantially over time, from 4% of private fixed investment in 1950 and 10% in 1980. Moreover, Bhandari and McGrattan (2021) propose a further extension of intangible capital to include "sweat equity," which they define (p.1) as "the value of business owners' time and expenses to build customer bases, client lists, and other intangible assets."

Barro and Furman (2018, Table 3) estimated for 2017 that the intellectual-property category constituted 29% of the standard measure of the total capital-income share of GDP, which corresponds to $\alpha_1+\alpha_2$ in equation (16). Therefore, the parameters $\alpha_1=0.28$ and $\alpha_2=0.12$ (which add to 0.40) should provide a reasonable approximation to the production-function parameters.

Suppose now that equation (16) represents the true production function, but that the BEA changed its measurement procedure at some point to capitalize the investment expenses that

[22] See Bureau of Economic Analysis (2019). Similarly, the U.N.'s *System of National Accounts*, which evolved from Meade and Stone (1941), capitalizes some outlays on intellectual property products, including software, R&D, and artistic originals. See United Nations (2009).

underlie K_2. Capitalization for investments underlying K_1 applies throughout. The assumption is that the BEA is now generating "accurate" measures of the two gross investment measures, I_1 and I_2, and of the associated capital stocks, K_1 and K_2. However, prior to 1999, the BEA reported $I_2=K_2=0$; that is, outlays on intellectual property were treated as current expenses and were, therefore, netted out in the calculation of business value added.

In the old system, measured GDP equals $C+I_1$; that is, I_2 is excluded. In this sense, measured GDP understates true GDP. Recall, however, that true GDP is an overestimate not only of national income but also of permanent income, which is superior to GDP or national income as an intertemporal gauge of consumption possibilities. Therefore, it is possible (actually likely) that the improvement in measured GDP is accompanied by a worsening of measurement for purposes of gauging permanent income and the potential for consumption.

On the income side, the old system has labor income wL and rental income RK_1 arising from the first type of capital and treated here as observable. With this approach, a residual capital income attaches to the second type of capital, K_2—because this intellectual property is actually durable and productive but was not measured that way. Measured GDP in the old system is:

$$(17)\qquad measured\ GDP = C + I_1 = wL + \mathrm{R}K_1 + residual\ capital\ income_2.$$

True GDP (assumed to be measured in accordance with current BEA practice) includes I_2 and $\mathrm{R}K_2$ and is given by:

$$(18)\qquad true\ GDP = C + I_1 + I_2 = wL + \mathrm{R}K_1 + \mathrm{R}K_2.$$

Note that the factor incomes on the right side of equation (18) are assumed to exhaust the true GDP. Also, the factor prices, w and R, and the quantities, C, I_1, and K_1, are the same in the measured and true cases, given the assumption that BEA measurement choices affect nothing real. A further assumption is that the same rental price, R, applies to K_1 and K_2 (although differences in depreciation rates would create a divergence here).

The residual capital income of type 2 can be computed from equations (17) and (18) as

$$(19) \quad residual\ capital\ income_2 = RK_2 - I_2 = (r+\delta)K_2 - I_2.$$

In the steady state, I_2=(δ+g)K_2, and the expression on the right simplifies to *(r-g)*K_2. Note that this expression for type-2 capital income takes exactly the form of the "net" capital income that enters into the definition of permanent income in equation (5). That is, the old system of national accounts gets the right answer for the K_2 part of capital income in the steady state because it effectively expenses out the investment outlay I_2 in the measurement of GDP in equation (17).

Another way to look at the results is that the revisions of the national accounts to capitalize intellectual property worsened the overstatement problem for GDP. The computations in Section 2.3 in equation (11) found that, in the steady state, the BEA's current procedure leads to an overstatement of GDP when compared to permanent income by 25%. The pre-1999 BEA system avoided this problem with respect to the intellectual-property part of investment and capital stock. Using the parameters already described, including $\alpha_1=0.28$, the overstatement of GDP compared to permanent income in the pre-1999 system turns out to be 17%, rather than 25%.[23] To put it another way, the combined impact of the 1999 and 2013 revisions of the national accounts with respect to intellectual property is to worsen the GDP overstatement by 8 percentage points.

The BEA's revised treatment of intellectual property also affects measured factor-income shares based on GDP. This idea is stressed by Koh, Santaeulalia-Llopis, and Zheng (2020). In

[23]This result follows from equation (11), using $\alpha_1=0.28$, rather than $\alpha=0.40$.

the BEA's current system, the standard capital-income share is $\alpha_1+\alpha_2=0.40$. In the old system, type-2 gross investment, $(\delta+g)K_2$, is subtracted from GDP and from rental income. Therefore, the capital-income share of GDP in the steady state is

$$old\ system\ capital\ share = \frac{R\cdot(K_1+K_2)-(\delta+g)K_2}{Y-(\delta+g)K_2}$$

With the Cobb-Douglas form of the production function in equation (16), this result becomes:

$$(20)\qquad old\ system\ capital\ share = \frac{\alpha_1(r+\delta)+\alpha_2(r-g)}{r+\delta-\alpha_2(r+g)}.$$

Using the parameter values from before, this expression equals 0.36, as contrasted with the standard number of 0.40. That is, the combined 1999 and 2013 revisions in the BEA's measurement of intellectual property can "explain" a rise in the capital-income share of GDP from 0.36 to 0.40.

This result accords with the calculations in Koh, Santaeulalia-Llopis, and Zheng (2020, Figure 1). They also show that the change in capitalization procedure affects measured trends in factor-income shares for a given procedure for capitalization. Notably, the rising trend in K/Y for the covered parts of intellectual property creates the appearance of a rising capital-income share and a falling labor-income share—in the approach that capitalizes these parts of intellectual property in the full time series (as reflected in currently reported BEA data going back to 1929).

3.2. Home production and consumer durables

Not all aspects of conventional national accounting generate overstatements of product and income from the perspective of gauging consumption possibilities. The most important sources of understatement likely involve the neglect of most home production and the informal or black-market economy.[24] The BEA recognizes these sources of understatement of production

[24]Kuznets (1941, p. 20) argued for the exclusion of most non-market activities, including illegal transactions: " … of the net money receipts by individuals from ordinary market transactions or other sources the following are

and income and has tried to rectify this problem with respect to home production through the construction of "satellite accounts" that include estimates of the amount of this form of economic activity. (See Bridgman, et al. [2012] and Bridgman [2016].) This section deals with home production, though an analysis of the informal sector would be similar in form.

Household production involves consumer durables, including households' ownership of automobiles, furniture, appliances, and so on. Households' ownership of houses is treated differently in the national accounts, as discussed in the next section.

Suppose in equation (16) that K_1 represents capital goods used in the market (such as businesses' equipment and structures), and K_2 represents capital goods used in home production. The standard GDP comprises consumption of market goods, C_1; gross investment, I_1, in market capital; and gross investment, I_2, in home capital. The last item corresponds to purchases of consumer durables and is included in the standard national accounts as part of personal consumer expenditure, rather than gross investment.

The standard GDP is generated by market capital and labor and is given as an extension of equation (1) by:

(21) $$Y_1 = F(K_1, L_1) = C_1 + I_1 + I_2 = wL_1 + RK_1.$$

Analogously, home production depends on home capital and labor and is given by:

(22) $$Y_2 = G(K_2, L_2) = C_2 = wL_2 + RK_2.$$

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excluded: … products of illegal activities, such as smuggling, racketeering, bootlegging, and drug peddling." This measurement procedure is still followed in the United States; for example, the BEA (2019, p. 2-2) says: "… illegal activities, such as gambling and prostitution in some states, should in principle be included in measures of production. However, these activities are excluded from the U.S. accounts because they are by their very nature conducted out of sight of public scrutiny and so data are not available to measure them." In contrast to the U.S. procedure, estimates of the underground or informal economy and of illegal activity are included in GDP to some extent in some countries. For example, these activities are included in the U.N.'s *System of National Accounts* for 2008 (see United Nations [2009, Section 6.9 and Ch. 25]).

The assumption here is that all of home production goes to home consumption, C_2. The production functions $F(\cdot)$ and $G(\cdot)$ satisfy the usual neoclassical properties. The change in each capital stock equals gross investment less depreciation, where the two depreciation rates are assumed to be the same and equal to δ. The assumption in equation (22) is that home labor and capital have associated shadow prices, w and R, that are the same as those of market labor and capital, respectively. This assumption would be valid if households can readily shift time and capital between home and market uses (ignoring taxes and assuming that time spent at market and home work is the same from a utility perspective). Note in equation (21) that measured GDP excludes home production and consumption, Y_2 and C_2, and gross income excludes home wage and rental income, wL_2 and RK_2.

As before, the market production function $F(\cdot)$ in equation (21) is assumed to be Cobb-Douglas with capital exponent α, as in equation (9). The home production function $G(\cdot)$ in equation (22) is also assumed to be Cobb-Douglas with capital exponent α. In this setup, which features equal capital intensities for producing market and home goods, the relative price of market and home consumption goods is fixed on the supply side, with the division between C_1 and C_2 determined by preferences. In this case, the intertemporal budget constraint from equation (3) can be expressed directly in terms of total consumption, C_1+C_2:

$$(23) \quad K_1(t) + K_2(t) + \int_t^{\infty} w(T) \cdot [L_1(T)+L_2(T)]e^{-r(T-t)}\, dT = \int_t^{\infty}[C_1(T) + C_2(T)]e^{-r(T-t)}\, dT.$$

If $w(T)L_1(T)$ and $w(T)L_2(T)$ each grow at every date at the rate g, equation (23) becomes (as an extension of equation [4]):

$$(24) \quad (r-g) \cdot [K_1(t) + K_2(t)] + w(t) \cdot [L_1(t) + L_2(t)] = (r-g) \cdot \int_t^{\infty}[C_1(T) + C_2(T)]e^{-r(T-t)}\, dT.$$

The left side of equation (24) equals permanent income, $Y^*(t)$, as discussed previously. The difference between GDP and permanent income includes, as before, the difference between $RK_1(t)$ and $(r\text{-}g)\cdot K_1(t)$; that is, $(\delta+g)\cdot K_1(t)$. A new effect is that GDP excludes the home production terms, $(r\text{-}g)\cdot K_2(t)+w(t)L_2(t)$. The net amount of GDP overstatement can be written as

(25) $$\text{GDP overstatement} = (\delta + g)\cdot[K_1(t) + K_2(t)] - Y_2(t).$$

The usual conditions for a steady state (with Cobb-Douglas production functions) give the capital-output ratios as $K_1/Y_1 = K_2/Y_2 = \alpha/(r+\delta)$, as in equation (10). These results imply that, in the steady state, GDP overstatement in equation (25) can be expressed relative to GDP, Y_1, as:

(26) $$\text{(GDP overstatement)/GDP} = \frac{\alpha\cdot(\delta+g)}{(r+\delta)} - \left(\frac{Y_2}{Y_1}\right)\cdot\left[1 - \frac{\alpha\cdot(\delta+g)}{(r+\delta)}\right].$$

In the steady state, $Y_2/Y_1 = K_2/K_1 = I_2/I_1$, and the last ratio can be calculated from numbers on purchases of consumer durables (I_2) and gross private fixed domestic investment (taken as a measure of I_1). For example, for 2019, the ratio I_2/I_1 equals 0.41. In that case, with the previously used parameters ($\alpha=0.40$, $\delta=0.05$, $g=0.03$, $r=0.08$), the first term on the right side of equation (26) is 0.25, as before, and the second term is -0.31. Therefore, on net, GDP is understated in the steady state by 6%; that is, the effect from the omission of home production more than offsets that from the double-counting of investment.

An alternative procedure uses more detailed estimates of home production; for example, Bridgman, et al. (2012) and Bridgman (2016) estimate from data on household time-use and consumer durables that the ratio of U.S. home production to GDP was 0.37 in 1965 but only 0.23

in 2014.[25] Using the recent value of 0.23 to gauge Y_2/Y_1 in equation (26),[26] the second term on the right side of the equation becomes -0.14. In this case, the net effect is an overstatement of GDP by 11%.

Although the consideration of home production and consumer durables has a significant impact on GDP overstatement, it tends not to have major implications for the calculations of factor-income shares. For example, since home production is assumed to have the same capital intensity as market production, the "capital-income share" computed by expanding GDP to include estimated home production is still $\alpha=0.40$.

3.3. Housing

Suppose now in equation (16) that the analysis treats K_2 as housing and neglects other forms of consumer durables. The variable Y_2, which equals C_2, now refers to household production of housing services. Unlike other consumer durables, the standard national accounts include in GDP and consumption the imputed rental income, RK_2, on owner-occupied housing.[27] The rental income on rental housing is also included in GDP. What is unclear is the treatment of

[25]Bridgman, et al. (2012, p. 23) say: "The decline reflects the steadily decreasing number of hours households spent on home production. In 1965, men and women spent an average of 27 hours in home production, and by 2010, they spent 22 hours. This overall decline reflects a drop in women's home production from 40 hours to 26 hours, which more than offset an increase in men's hours from 14 hours to 17 hours."

[26]However, the value 0.23 may be an under-estimate, because Bridgman (2016, p. 2) uses a low shadow wage rate to value household time used in home production: "The value of general services is the product of wage rate of general-purpose domestic workers and the number of hours of work." For many household members, the value of time would exceed the low wage rate received by domestic workers.

[27]This treatment appears in Kuznets (1941, p. 20). However, in an earlier analysis, Kuznets (1934, p. 12) suggested that it might be better to omit this item: " … there is some doubt as to the propriety of including this item [imputed net rental income accruing to people living in their own homes] since the ownership of a home combined with its possession does not constitute a participation by the proprietor in the economic activity of the nation in the same recognized fashion as does his work for wages, profit, or salary, or his capital investment in industry. For similar reasons, such an item as interest on durable goods owned has also been omitted." The BEA says that one motivation for its methodology on owner-occupied housing is "for GDP to be invariant when housing units shift between tenant occupancy and owner occupancy" (Mayerhauser and Reinsdorf [2007]). (GDP is not invariant to other shifts between renting or leasing and owning, such as for household automobiles and furniture.) The BEA procedure estimates the imputed rental income on owner-occupied housing by observing "rents charged for similar tenant-occupied buildings" (op. cit.). The owner-occupied housing part of housing services in personal consumer expenditure includes also outlays on maintenance & repairs, property insurance, and a few other items.

home labor input, L_2, associated with the production of housing services. Much of the associated payments (or shadow payments), wL_2, would not appear in GDP—in particular, the value of occupants' time expended on housing services is excluded. This omission is likely to be greater for owner-occupied housing than for rental housing—if landlord-provided labor services (which appear in GDP) go beyond those purchased on the market by owner-occupiers (and, therefore, included in GDP). However, the assumption here is that wL_2 is fully absent from GDP in both contexts.

Because imputed or explicit rental income on housing appears in GDP, the expression for GDP overstatement in equation (25) no longer involves subtraction of the full home production, Y_2, on the right side. Instead, the subtraction involves $Y_2\text{-}RK_2$, which equals the unmeasured home labor income, wL_2. Again assuming Cobb-Douglas production functions (with the same capital intensities for market goods and housing services), equation (26) is modified to:

$$\text{(27)}\quad \text{(GDP overstatement)/GDP} = \frac{\alpha\cdot(\delta+g)}{(r+\delta)} - \left(\frac{Y_2}{Y_1}\right)\cdot\left[(1-\alpha) - \frac{\alpha\cdot(\delta+g)}{(r+\delta)}\right].$$

Note that the term $1\text{-}\alpha$ (corresponding to home labor earnings) appears within the brackets on the right side of equation (27), whereas the term *1* appears in the comparable position in equation (26).

In the steady state, $Y_2/Y_1 = K_2/K_1 = I_2/I_1$ again applies. The last ratio can now be calculated from numbers on gross private fixed domestic residential investment (I_2) and gross private fixed domestic non-residential investment (I_1). For example, for 2019, the ratio I_2/I_1 equals 0.28. In that case, with the previously used parameters ($\alpha=0.40$, $\delta=0.05$, $g=0.03$, $r=0.08$), the first term on the right side of equation (27) is 0.25, as before, and the second term is -0.10. Therefore, on net, GDP would be overstated in the steady state by 15%. This net overstatement is larger than that associated with consumer durables because GDP includes the rental income on housing.

3.4. Inventories

Inventories comprise materials, goods-in-process, and finished goods. Net increases in stocks are classified as investment. Since inventories entail holding costs, a corresponding rental income associated with these holdings must appear in GDP. The rentals might reflect the benefit from having a larger stock available to meet customer demand (in the case of finished or nearly finished goods) or might represent cost reductions from employing a production process that allows large average holdings of materials, etc. (as opposed to a process that utilizes just-in-time inventory management).

Since GDP counts investment in inventories and also counts the rental income on stocks of inventories, the usual double-counting issue applies. However, because inventory change is, on average, a small fraction of GDP (0.55% from 1950 to 2019), the overstatement is minor when considered relative to GDP.

3.5. Government fixed capital

Denote by K_2 in equation (16) the amount of government owned capital. The new element here is that the national accounts allow for depreciation of public capital, δK_2, but assume a net real rate of return, r, of zero on this capital.[28] The associated overstatement of GDP in the steady state equals $\delta K_2 - (r-g)K_2 = (\delta+g-r)K_2$. With the parameters used before, this net

[28] According to BEA (2017, p. 9-4): "Alternatively, BEA could augment its measure of capital services by including a net return on assets, a change that would tend to raise the overall level of government output and consumption expenditures, and thus GDP. Several approaches have been suggested: using a private sector rate of return, a municipal bond rate, the Office of Management and Budget hurdle rate for investment, or others."

term turns out to equal zero. However, for national income, the overstatement equals $(g-r)K_2$, which is negative.

3.6. Human capital

This section is a sketch of how human capital would enter into the analysis. Formal schooling and on-the-job training contribute to human capital and, thereby, to higher labor productivity. The returns on this form of capital would show up in GDP, though directly as income from labor, $w(t)L(t)$, rather than capital. Human capital would also enter into home production. As with the implicit rentals on consumer durables, the implicit rentals on human capital used at home would not appear in measured GDP.

On the investment side, part of the outlays for accumulating human capital show up in GDP. Included here are costs of schooling, notably for teachers and school buildings. The bulk of these expenses appear in the national accounts as government consumption and investment and are valued at cost of production (including a net real rate of return of zero on government-owned capital). Another part of the expenditure on education involves private market purchases, and these outlays enter into the national accounts as personal consumer expenditure. An important omission on the expenditure side is the shadow value of student time used as an input into the educational process. From the perspective of GDP measurement, the omission of student time lessens the double-counting effect for human capital compared to that for physical capital.

For factor-income shares, a key issue is whether the returns to human capital should be construed as part of the income from labor or instead as part of the income from a broad concept of capital. The inclusion with labor income matches up with the observation that the income accrues to workers (in an amount consistent with the enhanced productivity derived from more human capital). However, the inclusion with capital income is appropriate from the perspective that the income reflects in part the services from a form of "capital," which is accumulated through a costly process of investment. In this respect, human capital and investments in this capital are analogous to physical capital and investments in that capital.

3.7. Net Foreign Assets

An open economy allows for net foreign assets and, therefore, for net flows of asset income between the United States and the rest of the world. The underlying assets can be thought of as U.S.-owned claims on capital used abroad and foreign-owned claims on capital used in the United States.

In the BEA data, the difference between net national product, NNP, and net domestic product, NDP, is primarily the net asset income from abroad—U.S. receipts on foreign assets less foreign receipts on U.S. assets. These flows correspond mostly to incomes on asset portfolios (including interest, dividends, and retained earnings) and earnings on direct investments. These amounts are effectively net of depreciation on the underlying capital and, therefore, correspond to the net capital income included in NDP.

In the BEA procedure, the gap between GNP and GDP is assumed to equal that between NNP and NDP; that is, the same net asset income from abroad enters into both calculations. A logically more consistent approach for GNP would add an estimate of depreciation on the underlying capital associated with U.S. ownership of foreign assets and subtract an estimate of depreciation on the underlying capital associated with foreign ownership of U.S. assets. An estimate of these flows requires a depreciation rate for each country, such as the value $\delta=0.05$ used before for the United States. The product of δ and each stock of assets (U.S.-owned foreign assets and foreign-owned U.S. assets) gives an estimate of each depreciation flow. The net of these flows would then add to the measured net asset income from abroad, and this total would give a revised version of the gap between GNP and GDP.

Similarly, terms involving $g \cdot K(t)$, which enters into the computation of permanent income in equation (8), can be calculated from the long-run growth rate, g, for each country. One possibility is that each growth rate equals the value $g=0.03$ used before for the United States. The product of g and each stock of assets gives an estimate of the two relevant terms. The net of these terms could then be subtracted from the measured net asset income from abroad to estimate the gap between national and domestic values of permanent income.

Although these adjustments to the net asset income from abroad can be made, it should be noted that there are other sources of measurement error in cross-country asset holdings and returns that are likely far more important in practice. These issues are considered in, among others, Gourinchas and Rey (2007); Curcuru, Dvorak, and Warnock (2008); Zucman (2016); and Coppola, et al. (2020). The large measurement errors in net asset income from abroad mean that the reported gaps between national and domestic product or income are highly imprecise.

4. Observations

The basic structure of the national income and product accounts features double-counting of investment. Gross (or net) fixed investment counts once in gross (or net) product when the investment occurs and a second time in present value when the cumulated capital leads to more gross (or net) rental income. This double-counting leads to overstatements of levels of aggregates such as GDP and national income when compared to permanent income and sustainable consumption. The standard approach also overstates capital-income shares.

The overstatement issue is straightforward for businesses' equipment and structures but takes on different forms for other types of capital, such as intellectual property, household durables, housing, inventories, government capital, human capital, and net foreign assets. As an example, the recent revisions of the national accounts to capitalize intellectual property resulted in substantial increases in reported levels of product and income and in capital-income shares. Although reasonable for some purposes, this expansion of the capitalization of investment flows magnifies the double-counting problem.

A remedy for the double-counting of investment involves the calculation of "permanent income," which adjusts GDP by expensing out the long-run flow of gross investment. The standard calculations of net product and national income feature deductions for depreciation of the existing capital stock. The purging of the remaining double-counting requires a further downward adjustment that reflects the long-term flow of net investment associated with the existing capital stock. In the steady state, permanent income equals consumption, which corresponds to the full subtraction of gross investment from GDP. Outside of the steady state, permanent income differs from consumption because the expensing relates to the long-run flow of gross investment, rather than the current flow. At a practical level, the permanent-income concept seems implementable because it requires only an extension from the standard depreciation rate to an effective rate that adds in the economy's expected long-run rate of economic growth.

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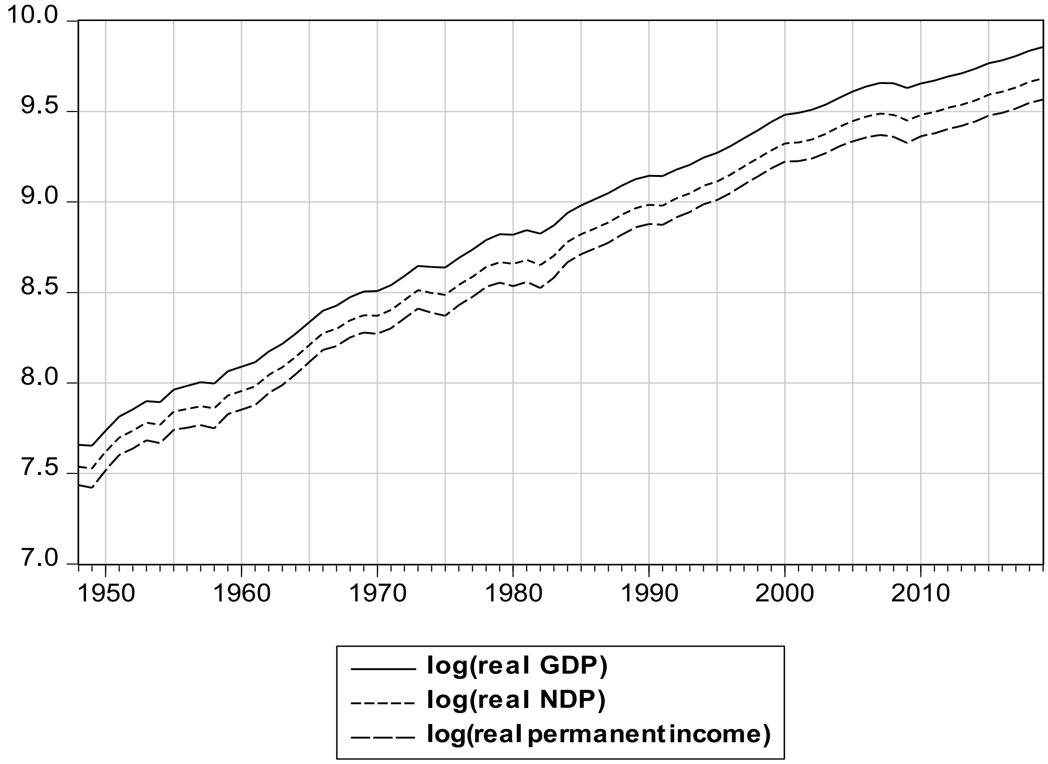
Figure 1

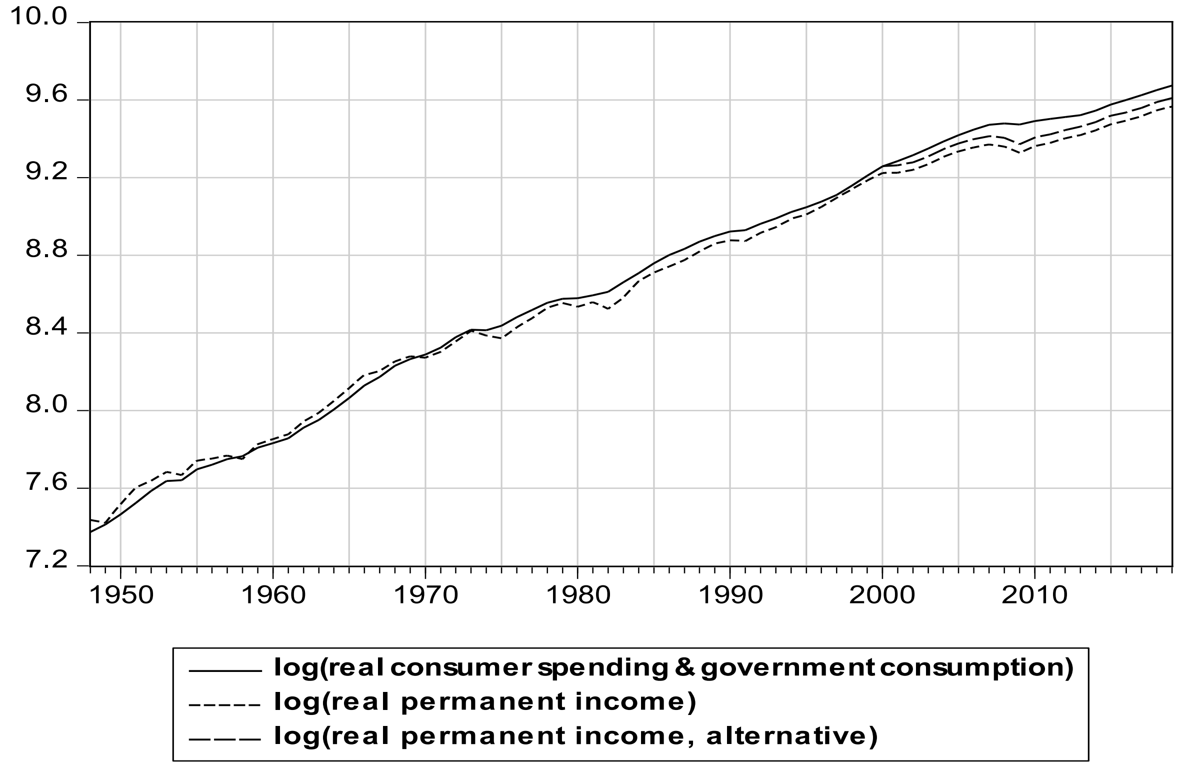
Relations Among U.S. GDP, NDP, and Permanent Income, 1948-2019



Note: All data are for the United States from the Bureau of Economic Analysis. Real GDP is the BEA chain series in millions of 2012 U.S. dollars. Nominal NDP equals nominal GDP less BEA nominal depreciation on fixed assets (including government assets but excluding consumer durables other than residential housing). The average depreciation rate (ratio of nominal depreciation on fixed assets to nominal fixed assets) from 1948 to 2019 is 0.050. The average ratio of nominal fixed assets to nominal GDP from 1948 to 2019 is 2.90. Real NDP is the nominal value multiplied by the ratio of real to nominal GDP. Nominal permanent income subtracts from nominal NDP the term $g \cdot K(t)$, where $K(t)$ is nominal fixed assets (averages of year-end data for years t and $t-1$) and g is assumed to be 0.031 per year, the mean growth rate of real GDP from 1948 to 2019. Real permanent income is the nominal value multiplied by the ratio of real to nominal GDP.

Figure 2

Relation of U.S. Consumption Spending to Permanent Income, 1948-2019

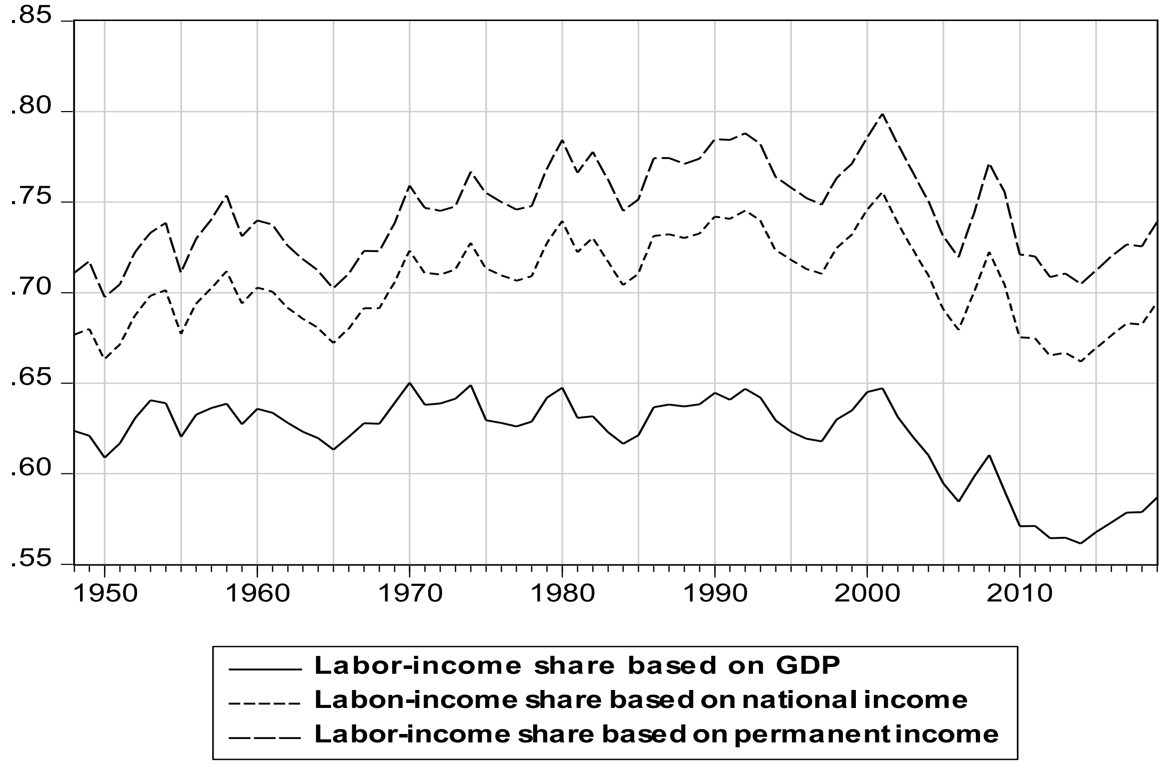


Note: The solid graph shows the log of the sum of real personal consumer expenditure and real government consumption. Real personal consumer expenditure is the BEA chain series in millions of 2012 U.S. dollars. Real government consumption is the nominal value multiplied by the ratio of real to nominal GDP. The log of real permanent income (dotted graph) corresponds to the dashed graph in Figure 1. The alternative series (dashed graph) uses the value $g=0.020$, rather than 0.031, from 2000 to 2019.

Figure 3

Alternative Measures of U.S. Labor-Income Shares

Based on BEA Corporate Data, 1948-2019



Note: The graphs uses BEA corporate-sector data on labor compensation, GDP, and national income. For the calculation of permanent income, the growth rate, g, was taken to be 0.031 per year.