Partial Equilibrium Thinking, Extrapolation, and Bubbles*

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Abstract

We model a financial market where some agents mistakenly attribute any price change they observe to new information alone, when in reality part of the price change is due to other agents' buying/selling pressure, a form of bounded rationality that we refer to as "Partial Equilibrium Thinking" (PET). PET provides a micro-foundation for price extrapolation, where the degree of extrapolation depends on the informational edge of informed agents. In normal times, this edge is constant and bubbles and crashes do not arise. By contrast, following a large one-off innovation in fundamentals that temporarily wipes out informed agents' edge (a "displacement event"), extrapolation by PET traders is initially very aggressive but then gradually dies down, leading to bubbles and endogenous crashes. Micro-founding the degree of extrapolation in this way allows us to shed light on both normal market dynamics and on the Kindleberger (1978) narrative of bubbles within a unified framework.

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Sustained periods of over-optimism that eventually end in a crash are at the heart of many macro-economic events, such as stock market bubbles, house price bubbles, investment booms, or credit cycles (Mackay (1841), Bagehot (1873), Galbraith (1954), Kindleberger (1978), Shiller (2000), Jordà et al. (2015), Greenwood et al. (2021)). Given the real consequences of bubbles and crashes, there has been widespread interest in understanding their anatomy and the beliefs that support them.

Perhaps the best known narrative of bubbles and crashes comes from Kindleberger (1978), who identifies three key stages of bubbles. The first stage is characterized by a *displacement*, which Kindleberger defines as "some outside event that changes horizons, expectations, anticipated profit opportunities, behavior." Examples include technological revolutions, such as the railroads in the 1840s, the radio and automobiles in the 1920s, and the internet in the 1990s, or financial innovations such as securitization prior to the 2008 financial crisis. As investors respond to such shocks, displacements lead to a wave of optimism and rising prices. The second stage is characterized by *euphoria*, in which higher prices encourage further buying in a self-sustaining feedback-loop between prices and beliefs that decouples prices from fundamentals. This stage is also characterized by destabilizing speculation (De Long et al. (1990), Brunnermeier and Nagel (2004)), accelerating and convex price paths (Greenwood et al. (2019)), and heavy trading (Ofek and Richardson (2003), Hong and Stein (2007), Barberis (2018), DeFusco et al. (2020)). Eventually, in the third stage of the bubble, sophisticated agents who rode the bubble exit, leading to a *crash*.

Early theories of bubbles maintain the assumption of rational expectations (Blanchard and Watson (1982), Tirole (1985), Martin and Ventura (2012)). However, as well as being at odds with empirical evidence on prices (Giglio et al. (2016)), these theories are also unable to speak to the pervasive empirical and experimental evidence on extrapolative beliefs (Smith et al. (1988), Haruvy et al. (2007), Case et al. (2012), Greenwood and Shleifer (2014)). Behavioral theories have instead turned to over-confidence and short-sale constraints (Harrison and Kreps (1978), Scheinkman and Xiong (2003)), and more recently to modeling extrapolative expectations themselves (Cutler et al. (1990), De Long et al. (1990), Hong and Stein (1999), Barberis et al. (2015), Hirshleifer et al. (2015), Glaeser and Nathanson (2017), Barberis et al. (2018), Bordalo et al. (2021), Liao et al. (2021), Chodorow-Reich et al. (2021)). Following a sequence of positive news, investors extrapolate recent price rises, and become more optimistic. This then translates into even higher prices, and even more optimistic future beliefs. By directly modeling the self-sustaining feedback between outcomes and beliefs that is characteristic of bubbles, these models generate many features of the Kindleberger (1978) narrative.[1]

At the same time, the reduced form nature of extrapolation considered in these theories leaves several questions open. First, what are the foundations of extrapolative expectations, and what determines how strongly traders extrapolate price changes in updating their future beliefs? Second, why is it that "[b]y no means does every upswing in business excess lead inevitably to mania and panic" (Kindleberger (1978))? In other words, why is it that the same type of extrapolative beliefs sometimes leads prices and beliefs to become extreme and decoupled from fundamentals, while in normal times we don't observe such extreme responses to shocks?

To answer these questions we provide a micro-foundation for the degree of price extrapolation with a theory of "Partial Equilibrium Thinking" (PET) (Bastianello and Fontanier (2021*b*)) in which traders fail to realize the general equilibrium consequences of their actions when learning information from prices. This cognitive failure leads to constant price extrapolation in normal times, and to stronger and time-varying extrapolation in response to displacement events.

Micro-founding the degree of extrapolation in this way provides a unifying theory in which the two-way feedback between prices and beliefs is present at all times, but only manifests itself in explosive ways under very specific circumstances. According to Soros (2015): "[...] in most situations [the two-way feedback] is so feeble that it can safely be ignored. We may distinguish between near-equilibrium conditions where certain corrective mechanisms prevent perceptions and reality from drifting too far apart, and far-from equilibrium conditions where a reflexive double-feedback mechanism is at work and there is no tendency for perceptions and reality to come closer together [...]." We formalize

[1]See Brunnermeier and Oehmke (2013), Xiong (2013) and Barberis (2018) for exhaustive surveys on bubbles and crashes.

this notion of "near-equilibrium" and "far-from equilibrium" conditions by modeling the distinction between normal times shocks which do not generate large changes to the environment, and Kindleberger-type displacements which instead do.

To illustrate our notion of partial equilibrium thinking, consider some investors who see the price of a stock rise, but do not know what caused this. They may think that some informed investors in the market have received positive news about this stock and decided to buy, pushing up its price. Given this thought process, they infer positive news about it, and also buy, leading to a further price increase. At this point, rational agents perfectly understand that this additional price rise is not due to further good news, but simply reflects the lagged response of uninformed agents who are thinking and behaving just like them. As a result, they no longer update their beliefs in response to this second price rise, and the two-way feedback between prices and beliefs fails to materialize, as shown in the top panel of Figure 1.

However, for uninformed agents to learn the right information from prices, they must perfectly understand what generates the price changes they observe at each point in time, which in turn requires them to perfectly understand other agents' actions and beliefs. Theories of rational expectations model this level of understanding by assuming common knowledge of rationality, which has been widely rejected by experimental evidence (Crawford et al. (2013), Kübler and Weizsäcker (2004), Penczynski (2017), Eyster et al. (2018)). We relax this assumption by instead assuming that agents think in partial equilibrium, whereby "otherwise rational expectations fail to take into account the strength of similar responses by others" (Kindleberger (1978)). PET agents neglect that all other uninformed agents are thinking and behaving just like them, and attribute any price change they observe to new information alone. Following the second price rise in the example in Figure 1, PET agents attribute it to further good news, encouraging further buying and inducing further price rises in a self-sustaining feedback between prices and beliefs. In this paper we formalize the intuition behind this example and show how, depending on the information structure, the strength of this feedback effect may be time-varying.

Partial equilibrium thinking is an example of substitution bias (DeMarzo et al. (2003), Kahneman (2011), Greenwood and Hanson (2015), Glaeser and Nathanson (2017)), where

Figure 1: The Feedback-Loop Theory of Bubbles. Changes in prices and beliefs after a one-off shock to fundamentals, under rational expectations (top panel) and under partial equilibrium thinking (bottom panel).

$$\text{REE:} \qquad \Delta\mathbb{E}_{I,1} \implies \Delta P_1 \implies \Delta\mathbb{E}_{U,2} \implies \Delta P_2$$

$$\text{PET:} \qquad \begin{tikzcd} \Delta\mathbb{E}_{I,1} \arrow[r, Rightarrow] & \Delta P_1 \arrow[r, Rightarrow] & \Delta\mathbb{E}_{U,2} \arrow[r, Rightarrow] & \Delta P_2 \arrow[r, Rightarrow] & \Delta\mathbb{E}_{U,t+1} \arrow[r, Rightarrow, bend left] & \Delta P_{t+1} \arrow[l, Rightarrow, bend left] \end{tikzcd}$$

traders replace a complicated general equilibrium inference problem with a simpler one that is driven by partial equilibrium intuitions. In so doing, PET agents effectively think they are the only ones learning information from prices, which is consistent with psychological evidence on the Lake Wobegan effect, where all agents incorrectly think they have an edge relative to others (Svenson (1981), Maxwell and Lopus (1994)). One of the most telling pieces of evidence of such behavior in financial markets comes from the work of Liu et al. (2021), who survey retail traders in China about their trading motives and combine these responses with observational data on their trading behavior. Interestingly, they find that a perceived information advantage is a dominant trading motive. Finally, the bias which underlies partial equilibrium thinking has also been widely studied in social learning contexts with models of naive inference and correlation neglect (Eyster and Rabin (2005), Eyster and Rabin (2010)).[2] We model this type of bias in a general equilibrium environment and study how it interacts differently with normal times shocks and with the type of shocks that Kindleberger (1978) identifies as displacements.[3]

We begin by introducing partial equilibrium thinking into a standard infinite horizon model of a financial market where each period a continuum of investors solve a portfolio choice problem between a risky and a riskless asset. Our agents differ in their ability

[2] See also Bohren (2016), Esponda and Pouzo (2016), Gagnon-Bartsch and Rabin (2016), Fudenberg et al. (2017), Bohren and Hauser (2021), Frick et al. (2020), Fudenberg et al. (2021), Gagnon-Bartsch et al. (2021) among others.

[3] By considering general equilibrium environments, the outcomes agents learn from have not only an informational role, but also a market feedback effect role, and it is the interaction of these two forces which determines the strength of the feedback effect (Bastianello and Fontanier (2021*b*)). Displacements then make both these forces and the strength of the feedback effect time-varying, thus allowing for endogenous reversals even after outcomes and beliefs have become extreme and decoupled from fundamentals.

to observe fundamental news: a fraction of agents are informed and observe fundamental shocks, and the remaining fraction of agents are uninformed and instead infer information from prices. Motivated by empirical and experimental evidence that traders extrapolate trends as opposed to instantaneous price movements, we assume that traders learn information from past as opposed to current prices (Andreassen and Kraus (1990), Case et al. (2012)), as with the positive feedback traders in De Long et al. (1990), Hong and Stein (2007) and Barberis et al. (2018).[4]

Given this information structure, in each period price changes reflect both the contemporaneous response of informed agents to news, and the lagged response of uninformed agents who learn from past prices. However, when uninformed agents think in partial equilibrium, they neglect the second source of variation and attribute any price change to new information alone, leading to a simple type of price extrapolation.

The key prediction of the model which leads to different dynamics in response to different types of shocks is that the degree of extrapolation and the bias that partial equilibrium thinking generates are decreasing in informed traders' informational edge. This edge is simply defined as the aggregate confidence of informed traders relative to uninformed traders, and is higher when there are more informed traders in the market, and when the precision of the additional information informed traders hold is higher. When this informational edge is high, informed traders trade more aggressively, and the influence on prices of uninformed traders' beliefs is lower. This leads partial equilibrium thinkers to neglect a smaller source of price variation, therefore leading to a smaller bias and a smaller strength of the feedback between prices and beliefs. Conversely, when informed traders' edge is low, partial equilibrium thinkers neglect a bigger source of price variation, leading to a larger bias and a stronger feedback effect. By understanding how this edge varies in response to different types of shocks, we can then understand how partial equilibrium thinking generates different dynamics in normal times, and following a displacement.

We show that in normal times informed agents' edge is high and constant over time.

[4]This assumption allows us to model the evolution of the two-way feedback between outcomes and beliefs dynamically. We explore the implications of partial equilibrium thinking when agents learn from current prices in Bastianello and Fontanier (2021*b*).

For example, normal times shocks may come in the form of earnings announcements: sophisticated traders are better able to understand the long run implications of such shocks, and uninformed retail traders can learn about them more slowly by observing how the market reacts to such news. When this is the case, informed traders are always one step ahead of uninformed traders, and their edge is high and constant, meaning that partial equilibrium thinkers neglect a small source of price variation, thus leading to weak departures from rationality.

This is no longer true following a Kindleberger-type displacement, when we show that the informational edge becomes time-varying. Specifically, displacements are "something new under the sun", and the implications of such shocks for long term outcomes can be learnt only gradually over time. These shocks wipe out much of informed agents' edge as not even the most informed of informed agents are able to immediately grasp the full long-term implications of such events. This leads informed agents to trade less aggressively, and to a rise in the influence on prices of uninformed traders' beliefs. Partial equilibrium thinkers then neglect a greater source of price variation, leading to a stronger bias. This fuels the strength of the feedback between prices and beliefs, allowing both to accelerate away from fundamentals. As informed traders learn more about the displacement over time, they regain their edge, leading to a gradual fall in the degree of extrapolation, and in the strength of the feedback effect. When the feedback effect runs out of steam, the bubble bursts, and prices and beliefs converge back towards fundamentals. The exact shape of the bubble then depends on the speed with which informed traders learn more about the displacement.

Finally, we study how our bias interacts with speculative motives, and show that whether speculators amplify bubbles or arbitrage them away depends on their beliefs of whether mispricing is temporary or predictable. If they think that mispricing is temporary, they arbitrage it away immediately, and bubbles and crashes do not arise. If instead they realize that mispricing is predictable and that they will be able to sell the asset to "a greater fool" at a higher price in the future, they increase their position in the asset, thus pushing prices up further, and amplifying the bubble (De Long et al. (1990)). These predictions are consistent with bubbles being associated with the type of destabilizing speculation described in the latter case (Keynes (1936)), and with more sophisticated traders initially riding the bubble (Brunnermeier and Nagel (2004)).

This paper proceeds as follows. In Section 1 we illustrate our notion of partial equilibrium thinking with a reduced form model. Section 2 provides a full micro-foundation of this model and considers the implications of partial equilibrium thinking in normal times. Section 3 models displacements and shows how these shocks interact with partial equilibrium thinking in generating bubbles and crashes. In Section 4 we add speculative motives. Section 5 concludes and discusses some directions of future research. While prices are a very natural equilibrium outcome agents may learn from, partial equilibrium thinking can be applied more broadly to any setup where agents learn information from a general equilibrium variable, thus lending itself to a variety of other macro and finance applications, such as credit cycles and investment booms (Bastianello and Fontanier (2020)).

1 The Feedback Loop Theory of Bubbles

In this section we start with a reduced-form model to introduce our notion of partial equilibrium thinking (PET), and we show how PET gives rise to the natural self-sustaining feedback between outcomes and beliefs that lies at the heart of the Kindleberger narrative of bubbles. We micro-found this model fully in Section 2.

1.1 Reduced-Form Setup

Consider an asset that is in fixed supply and whose fundamental value is determined by its terminal payoff D_T, which evolves as a random walk:[5]

$$D_T = \bar{D} + \sum_{j=0}^{T} u_j \tag{1}$$

where $u_j \overset{iid}{\sim} N(0, \sigma_u^2)$, and $\bar{D}$ is a positive constant. Moreover, suppose the market for this risky asset is populated by two types of risk averse agents $i \in \{I, U\}$, who decide how much of the risky asset to hold based on their beliefs about its fundamental value. The

[5]By assuming the asset is in fixed supply we consider the simple case where prices are fully revealing.

two types of traders differ in their ability to observe the fundamental value of the asset. Type I agents are informed: in each period t they observe the new fundamental shock u_t and use this to update their beliefs about the asset's fundamental value:

$$\mathbb{E}_{I,t}[D_T] = \mathbb{E}_{I,t-1}[D_T] + u_t \tag{2}$$

Type U agents do not observe any of the fundamental shocks, and instead may learn information from prices. To model the dynamic evolution of the feedback between outcomes and beliefs that is characteristic of the Kindleberger (1978) narrative of bubbles, and informed by experimental evidence by Andreassen and Kraus (1990), we assume that traders learn information from past as opposed to current prices, in the spirit of the positive feedback traders in De Long et al. (1990), Hong and Stein (1999) and Barberis et al. (2018).[6]

Since traders learn information from *past* prices, in each period t they can learn about the previous period fundamental shock, u_{t-1}. Let $\tilde{u}_{t-1}$ be the fundamental shock which uninformed traders learn from P_{t-1}. We can then write their period t beliefs accordingly:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \tilde{u}_{t-1} \tag{3}$$

Throughout the paper we denote by $\tilde{\cdot}$ uninformed agents' beliefs about a variable. In this case, since we assumed that prices are fully revealing, uninformed traders believe they are extracting from P_{t-1} the exact fundamental signal that informed traders observed in $t-1$, so $\tilde{u}_{t-1}$ is uninformed agents' belief about the $t-1$ fundamental shock, u_{t-1}.

Finally, suppose that all traders have mean-variance utility and are only concerned with forecasting the long term fundamental value of the asset, D_T.[7] In Section 2 we show that we can then write the market clearing price as a weighted average of traders' beliefs

[6] This assumption of learning from past as opposed to current prices could be due to agents being boundedly rational, and not having the cognitive capacity to update their beliefs at the same time they submit their trades. We explore the implications of partial equilibrium thinking when agents learn from *current* prices in Bastianello and Fontanier (2021*b*).

[7] We consider agents who are only concerned with forecasting long term fundamentals, as opposed to agents who are interested in timing the market, and who would instead be concerned with forecasting next period prices. We make this assumption to illustrate our notion of PET in the simplest possible framework, and relax this assumption in Section 4.

minus a risk-premium component:

$$P_t = a\mathbb{E}_{I,t}[D_T] + b\mathbb{E}_{U,t}[D_T] - c \tag{4}$$

where $a, b \in [0, 1]$ are weights which capture the influence on prices of informed and uninformed agents' beliefs respectively, and c is the risk-premium component that compensates risk-averse agents for bearing risk. In Section 2 we micro-found this price function and show that the coefficients a, b and c are endogenous objects which are pinned down in equilibrium. Specifically, a and b are weights that depend on the composition of agents in the market and on their relative confidence. For example, the greater the fraction of informed (uninformed) agents in the market, and the more confident they are about their posterior beliefs, the more strongly are their beliefs incorporated into prices, which results in a higher a (b). In addition, the risk-premium component c also depends on the supply of the risky asset and on agents' level of risk aversion: the less scarce the asset is, and the more risk averse agents are, the higher is the premium that agents require to hold the asset. For now take a, b and c to be constants.

Given the price function in (4) and agents' beliefs in (2) and (3), any price change in period t reflects both the instantaneous response of informed agents who receive new information, and the lagged response of uninformed agents who learn information from past prices:[8]

$$\Delta P_t = \underbrace{a u_t}_{\text{instantaneous response}} + \underbrace{b\tilde{u}_{t-1}}_{\text{lagged response}} \tag{8}$$

Understanding what information $\tilde{u}_{t-1}$ uninformed agents learn from past prices lies at the heart of our feedback-loop theory of bubbles. This, in turn, requires us to understand what uninformed agents think is generating the price changes they observe. We now turn to comparing rational agents' inference to that of agents who think in partial equilibrium.

[8] Taking first differences of (2), (3) and (4) we find:

$$\Delta\mathbb{E}_{I,t}[D_T] = u_t \tag{5}$$
$$\Delta\mathbb{E}_{U,t}[D_T] = \tilde{u}_{t-1} \tag{6}$$
$$\Delta P_t = a\Delta\mathbb{E}_{I,t}[D_T] + b\Delta\mathbb{E}_{U,t}[D_T]. \tag{7}$$

Substituting (5) and (6) into (7), we obtain equation (8).

1.2 Rational Expectations

Since traders can only learn information from past prices, in period t they learn information from ΔP_{t-1}. Lagging (8) by one period, ΔP_{t-1} is simply given by:

$$\Delta P_{t-1} = \underbrace{a u_{t-1}}_{\text{instantaneous response}} + \underbrace{b \tilde{u}_{t-2}}_{\text{lagged response}} \tag{9}$$

If uninformed agents hold rational expectations, they perfectly understand what generates this price change, and think that this is due to informed agents updating their beliefs by $\tilde{u}_{t-1}$ (their conjecture of u_{t-1}) and to uninformed agents updating their beliefs by $\tilde{u}_{t-2}$. They then invert the following mapping to learn $\tilde{u}_{t-1}$ from ΔP_{t-1}:

$$\Delta P_{t-1} = \underbrace{a \tilde{u}_{t-1}}_{\text{instantaneous response}} + \underbrace{b \tilde{u}_{t-2}}_{\text{lagged response}} \quad \Longrightarrow \quad \tilde{u}_{t-1} = \left(\frac{1}{a}\right) \Delta P_{t-1} - \left(\frac{b}{a}\right) \tilde{u}_{t-2} \tag{10}$$

Comparing this to the true price function in (9), we see that if agents are rational they are indeed able to recover the right information from prices:

$$\tilde{u}_{t-1} = u_{t-1} \tag{11}$$

When this is the case, price changes follow an MA(1) and any shock takes two periods to propagate through the economy, as intuited in the example in the top panel of Figure 1 in the introduction.[9] However, for uninformed agents to learn the right information from prices, they must perfectly understand what generates the price that they observe at each point in time, which in turn requires them to perfectly understand other agents' actions and beliefs. In what follows, we relax this assumption, and traders who think in partial equilibrium misunderstand what generates the price changes they observe because they fail to realize the general equilibrium consequences of their actions.

[9] If these rational traders were trying to time the market instead of being fundamental traders, they would anticipate the second price change, recognize that this represents an arbitrage opportunity for them, and they would drive the price to its new steady state immediately, in the first period. As will become clear, this assumption is not key in delivering our notion of partial equilibrium thinking.

1.3 Partial Equilibrium Thinking

Agents who think in partial equilibrium fail to realize that all other uninformed agents are thinking and behaving just like them, and are also learning information from past prices (Bastianello and Fontanier (2021*b*)). When thinking about what generates the price change they observe, they then omit the second source of price variation in (9) and attribute any price change they observe to new information alone:

$$\Delta P_{t-1} = \underbrace{a\tilde{u}_{t-1}}_{\text{instantaneous response}} \cancel{\underbrace{+ \quad b\tilde{u}_{t-2}}_{\text{lagged response}}} \implies \tilde{u}_{t-1} = \left(\frac{1}{a}\right)\Delta P_{t-1} \tag{12}$$

PET then provides a micro-foundation for a very simple type of price extrapolation, where uninformed agents become more optimistic (pessimistic) whenever they see a price rise (fall), regardless of the true source of this price change:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \left(\frac{1}{a}\right)\Delta P_{t-1} \tag{13}$$

Moreover, the extrapolation parameter is given by $1/a$: PET agents understand that when the influence on prices of informed agents' beliefs (a) is lower, a given piece of fundamental news leads to a smaller price change, and they must therefore extrapolate price changes more strongly to recover that information.[10]

Since PET agents use a misspecified mapping to infer information from prices, they recover a biased signal. Specifically, substituting the true price function in (8) into the mapping in (12), we find that PET agents recover the following biased information from prices:

$$\tilde{u}_{t-1} = u_{t-1} \underbrace{+ \left(\frac{b}{a}\right)\tilde{u}_{t-2}}_{\text{bias}} \tag{14}$$

where the bias in the information PET agents extract from prices is increasing in the influence on prices of uninformed agents' beliefs (b), and in the extrapolation parameter ($1/a$), as these components lead PET agents to neglect a bigger source of price variation.

[10] This intuition is clearest when prices are fully revealing, and is robust to having a small amount of noise and prices being partially revealing, as discussed in Appendix C.

The AR(1) nature of (14) also makes clear that uninformed agents mistakenly infer a sequence of shocks from a one-off shock, just as we saw in the example in Figure 1 in the introduction. Following a one-off shock, PET agents fail to realize that the second price rise is due to the buying pressure of all other uninformed agents, and instead attribute it to further good news, which in turn fuels even higher prices and more optimistic beliefs, in a self-sustaining feedback loop.[11]

Turning to the properties of equilibrium prices, we can substitute the information uninformed agents extract from prices in (12) into the true price function in (8), to find that price changes also follow an AR(1):

$$\Delta P_t = au_t + \left(\frac{b}{a}\right) \Delta P_{t-1} \tag{15}$$

In this case, t periods after a one-off shock u_0 the price level is given by:

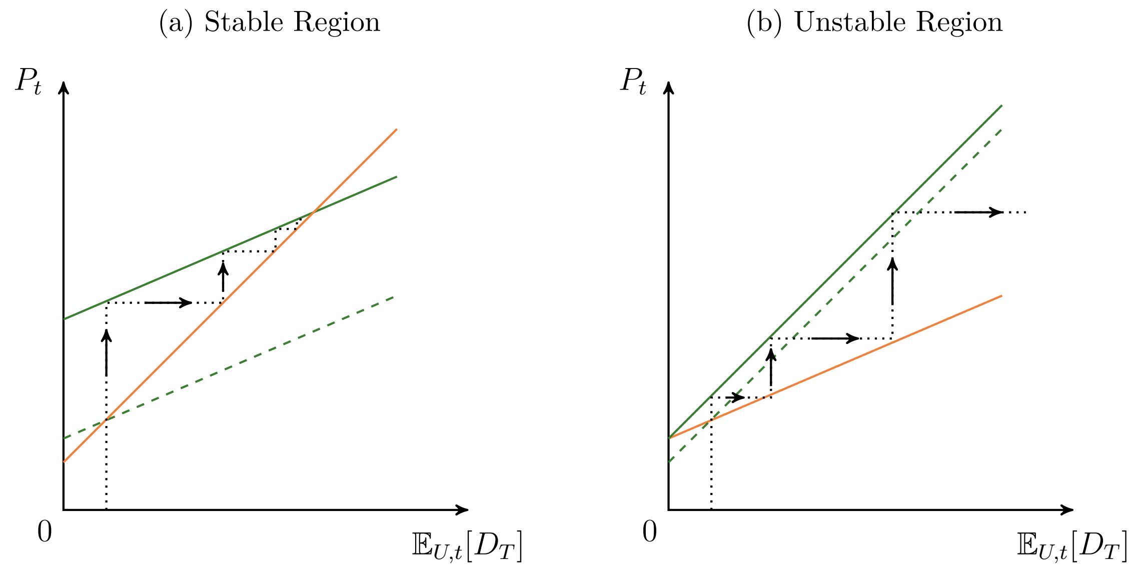
$$P_t = \bar{P} + \sum_{j=0}^{t} \Delta P_j = \bar{P} + \sum_{j=1}^{t} \left(\frac{b}{a}\right)^j (au_0) \tag{16}$$

These expressions illustrate two key points. First, $\frac{b}{a}$ governs the strength of the feedback between outcomes and beliefs, with a higher influence on prices of uninformed agents' biased beliefs (b), and a stronger extrapolation parameter ($1/a$), both fuelling the feedback between outcomes and beliefs.

Second, when $\frac{b}{a} < 1$, the left panel of Figure 2 shows that following a one-off shock, the influence of the feedback on equilibrium outcomes dies out as it gets compounded: consecutive changes in prices and beliefs become smaller over time, and the geometric series in (16) is bounded, so that prices and beliefs converge to a new steady state. On the other hand, the right panel of Figure 2 shows that when $\frac{b}{a} > 1$ the influence of the feedback effect is explosive: consecutive changes in prices and beliefs get larger and larger, and the geometric series in (16) is explosive, so that prices and beliefs accelerate in a convex way and become extreme and decoupled from fundamentals.

[11] Unlike the rational case, even if informed agents were trying to time the market they wouldn't be able to bring the price to its new equilibrium level within a single period because PET agents would extrapolate this price change, regardless of its new level.

Figure 2: Stable and Unstable Regions. Evolution of prices and beliefs following a one-off shock to fundamentals when the economy is in a stable region (left panel), and when the economy is in an unstable region (right panel). The green lines on this graph plot $P_t = aD_t + b\mathbb{E}_{U,t}[D_T] - c$ for $D_t = D_0$ (dashed line) and for $D_t = D_0 + u_1$ (solid line), respectively. These mappings should be read from the horizontal to the vertical axis: fixing the beliefs of informed agents, these mappings return the market clearing price P_t which arises if all uninformed agents trade on $\mathbb{E}_{U,t}[D_T]$. The slope of these mappings in $(P_t, \mathbb{E}_{U,t}[D_T])-$space is b. The orange line plots $\mathbb{E}_{U,t}[D_T] = \frac{1}{a}P_t - \frac{1}{a}P_0 + D_0$, which we obtain by simply solving (13) recursively. This mapping should be read from the vertical to the horizontal axis: given an observable price, this mapping returns uninformed agents' beliefs next period. The slope of this mapping in $(P_t, \mathbb{E}_{U,t}[D_T])-$space is $a = 1/\theta$. In the left panel $\frac{b}{a} < 1$ (the orange mapping is steeper than the green one), and prices and beliefs converge to a new steady state following a shock. In the right panel $\frac{b}{a} > 1$, and prices and beliefs accelerate and become extreme and decoupled from fundamentals.



We summarize these results in the following proposition.

Proposition 1 (Stable and Unstable Regions.). *When the strength of the feedback effect is constant, outcomes and beliefs either converge to a state-dependent equilibrium (if $\frac{b}{a} < 1$), or they accelerate and become extreme and decoupled from fundamentals (if $\frac{b}{a} > 1$). We refer to regions with $\frac{b}{a} < 1$ as stable regions, and regions with $\frac{b}{a} > 1$ as unstable regions.*

Proof. All proofs are in Appendix A. □

While it is implausible to think that the economy always responds to shocks in an unstable way, as we don't usually observe unbounded prices and beliefs, the convexity generated by unstable regions is a noted feature of bubbles and crashes (Greenwood, Shleifer and You, 2018). However, as long as $\frac{b}{a}$ is constant, the economy is either always in a stable region, where prices and beliefs monotonically converge to the new steady state in response to a shock, or it is always in an unstable region, where any shock leads outcomes and beliefs to accelerate away from fundamentals in an unbounded way. While the acceleration characteristic of the unstable regions of this theory may seem well-suited to model the formation of bubbles, it leaves no room for endogenous reversals and crashes.

In the rest of this paper we micro-found this model, and show that the strength of the feedback between outcomes and beliefs ($\frac{b}{a}$) depends on the informational edge of informed agents, which in turn is determined by the composition of agents in the market, and by the relative confidence of informed and uninformed agents.

We show that in normal times informed agents' edge is constant over time and the economy is always in a stable region. For example, normal times shocks may come in the form of earnings announcements. Informed agents understand the implications of these shocks for long term outcomes, and uninformed agents can learn about them more slowly by observing the markets' reaction to such announcements. Informed agents are then always one step ahead of uninformed agents and their edge is constant.

On the other hand, the types of displacements described by Kindleberger generate time-variation in informed agents' edge, and temporarily shift the economy into an unstable region. Specifically, displacements initially wipe out informed agents' edge as even the informed are not able to fully grasp the long term implications of such shocks. As informed agents lose their edge, they trade less aggressively, increasing the influence on prices of uninformed traders' beliefs. PET traders then neglect a greater source of price variation, leading to a greater bias. This strengthens the feedback between outcomes and beliefs and can shift the economy into an unstable region, leading prices and beliefs to become extreme and decoupled from fundamentals. As informed agents gradually learn more about the displacement over time, they regain their edge, leading to a weakening of the feedback effect. Eventually, as the feedback effect runs out of steam the economy returns to a stable region, the bubble bursts and prices and beliefs converge back towards fundamentals. By bringing the explosive properties of unstable regions into play before the convergent properties of stable regions take over again, displacements lead to the formation of bubbles and endogenous crashes.

In the rest of the paper, we formalize these intuitions.

2 Normal Times

In this section we micro-found the model considered in Section 1, and study the properties of partial equilibrium thinking in normal times, when the informational edge of informed agents is constant.

2.1 Setup

Agents solve a portfolio choice problem between a risk-free and a risky asset. The risk-free asset is in zero net supply and we normalize its price and its risk free rate to one. The risky asset is in fixed net supply Z and pays a liquidating dividend when it dies at an uncertain terminal date. In each period, with probability β the asset remains alive and produces $u_t \overset{iid}{\sim} N(0, \sigma_u^2)$ worth of terminal dividends, and with probability $(1 - \beta)$ the asset dies, and all accumulated dividends are paid out. As a result, if the asset dies in period $t + h$, its terminal dividend still evolves as a random walk:

$$D_{t+h} = \bar{D} + \sum_{j=0}^{t+h} u_j \tag{17}$$

where $\bar{D}$ is the prior belief of the asset's terminal dividend, and this is common knowledge. From the point of view of period t, the asset dies in period $t+h$ with probability $(1-\beta)\beta^h$. Taking expectations over all possible terminal dates, the present value of the terminal dividend in period t, conditional on realized future shocks $\{u_{t+h}\}_{h=1}^{\infty}$ can be written as:

$$D_T = D_t + \sum_{h=1}^{\infty} \beta^h u_{t+h} \tag{18}$$

which has the appealing property that β effectively acts as a discount rate such that dividends paid further into the future receive a lower weight. Modelling the present value of the terminal dividend in this way, and modifying (1) with an uncertain terminal date serves two purposes: first, it avoids horizon effects as we approach the terminal date, and second, it bounds the variance perceived by agents even if the terminal date can be arbitrarily far into the future.

Our economy is populated by a continuum of measure one of fundamental traders, who have CARA utility over terminal wealth and trade as if they were going to hold the asset until its death, even though they rebalance their portfolio every period.[12] In each period t all agents then solve the following problem:

$$\max_{X_{i,t}} \quad \left\{ X_{i,t} \left(\mathbb{E}_{i,t}[D_T] - P_t \right) - \frac{1}{2} \mathcal{A} X_{i,t}^2 \mathbb{V}_{i,t}[D_T] \right\} \tag{19}$$

where $X_{i,t}$ is the dollar amount that agent i invests in the risky asset in period t, $\mathcal{A}$ is the coefficient of absolute risk aversion, and $\mathbb{E}_{i,t}[D_T]$ and $\mathbb{V}_{i,t}[D_T]$ refer to agent i's posterior beliefs about the fundamental value of the asset conditional on their information set in period t. The corresponding first order condition yields the following standard demand function for the risky asset:

$$X_{i,t} = \frac{\mathbb{E}_{i,t}[D_T] - P_t}{\mathcal{A}\mathbb{V}_{i,t}[D_T]} \tag{20}$$

which is increasing in agent i's expected payoff, and decreasing in the risk they associated with holding the asset.

Turning to the information structure, we assume that a fraction ϕ of agents are informed, and in each period t they observe the current fundamental shock u_t, so their full information set is $\{u_j\}_{j=1}^t$. The remaining fraction $(1 - \phi)$ of agents are uninformed and do not observe any of the fundamental shocks that determine the fundamental value of the asset, but since informed agents trade on their information advantage, uninformed agents can learn information from *past* prices, as discussed in Section 1.

To solve the model, we proceed in three steps, which closely mirror our discussion in Section 1. First, we solve for the true price function which generates the outcomes that agents observe. Second, we turn to PET agents' beliefs of what generates the prices they observe, which allows us to pin down the mapping that PET agents use to learn information from prices. Finally, we solve the equilibrium recursively, and study the properties of equilibrium outcomes.

[12] The fundamental traders in this section are time-inconsistent in that they trade as if they were going to hold their position forever, even though they rebalance every period. This is a simplifying assumption, which allows us to illustrate our notion of partial equilibrium thinking in the simplest possible framework. In Section 4 we relax this assumption and model traders who time the market, and have CARA utility over next period wealth.

2.2 True Price Function in Normal Times

To solve for the true market clearing price function, we need to specify agents' posterior beliefs, compute agents' asset demand functions, and impose market clearing. Starting from agents' beliefs, we know that in period t all informed agents trade on the information they receive, and update their beliefs accordingly:

$$\mathbb{E}_{I,t}[D_T] = \mathbb{E}_{I,t}\left[D_{t-1} + u_t + \sum_{h=1}^{\infty} \beta^h u_{t+h}\right] = \mathbb{E}_{I,t-1}[D_T] + u_t \tag{21}$$

$$\mathbb{V}_{I,t}[D_T] = \mathbb{V}_{I,t}\left[\sum_{h=1}^{\infty} \beta^h u_{t+h}\right] = \left(\frac{\beta^2}{1-\beta^2}\right)\sigma_u^2 \equiv \mathbb{V}_I \tag{22}$$

where the equivalence in equation (22) highlights that informed agents' uncertainty is constant over time. Moreover, all uninformed agents learn information from past prices, and their posterior beliefs are given by:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t}\left[D_{t-2} + u_{t-1} + u_t + \sum_{h=1}^{\infty} \beta^h u_{t+h}\right] = \mathbb{E}_{U,t-1}[D_T] + \tilde{u}_{t-1} \tag{23}$$

$$\mathbb{V}_{U,t}[D_T] = \mathbb{V}_{I,t}\left[u_t + \sum_{h=1}^{\infty} \beta^h u_{t+h}\right] = \left(\frac{1}{1-\beta^2}\right)\sigma_u^2 \equiv \mathbb{V}_U \tag{24}$$

where the last equality in (24) shows that the uncertainty faced by uninformed agents is also constant over time. Moreover, comparing (24) to (22) we see that informed agents are more confident than uninformed agents as they always see one-period ahead of them. We define ζ to be the aggregate informational edge of informed agents relative to uninformed agents as follows:

$$\zeta \equiv \frac{\phi}{(1-\phi)}\frac{\tau_I}{\tau_U} \tag{25}$$

where $\tau_i = (\mathbb{V}_i)^{-1}$ is the confidence of agent $i \in \{I, U\}$.

Given these posterior beliefs, we can compute agents' asset demand functions and impose market clearing by simply equating the aggregate demand for the risky asset to the fixed supply Z:

$$\underbrace{\phi\left(\frac{\mathbb{E}_{I,t-1}[D_T]+u_t-P_t}{\mathcal{A}\mathbb{V}_I}\right)}_{X_{I,t}}+(1-\phi)\underbrace{\left(\frac{\mathbb{E}_{U,t-1}[D_T]+\tilde{u}_{t-1}-P_t}{\mathcal{A}\mathbb{V}_U}\right)}_{X_{U,t}}=Z \tag{26}$$

The true market clearing price function is then given by:

$$P_t = a\left(\mathbb{E}_{I,t-1}[D_T]+u_t\right)+b\left(\mathbb{E}_{U,t-1}[D_T]+\tilde{u}_{t-1}\right)-c \tag{27}$$

where:

$$a \equiv \frac{\phi\tau_I}{\phi\tau_I+(1-\phi)\tau_U}=\frac{\zeta}{1+\zeta} \tag{28}$$

$$b \equiv \frac{(1-\phi)\tau_U}{\phi\tau_I+(1-\phi)\tau_U}=\frac{1}{1+\zeta} \tag{29}$$

$$c \equiv \frac{\mathcal{A}Z}{\phi\tau_I+(1-\phi)\tau_U} \tag{30}$$

This micro-founds our expression in (4), and shows that prices reflect a weighted average of agents' beliefs minus a risk-premium component which compensates agents for bearing risk. The weight on informed agents' beliefs is increasing in their informational edge, and the opposite comparative static holds for the weight on uninformed agents' beliefs.

Re-writing (27) in changes, we find that:

$$\Delta P_t = au_t + b\tilde{u}_{t-1} \tag{31}$$

which micro-founds (8) in the reduced-form model, and shows that price changes reflect both the instantaneous response to shocks of informed agents, and the lagged response of uninformed agents who learn information from past prices.

2.3 Partial Equilibrium Thinking

To specify what information uninformed agents extract from prices we need to understand what uninformed agents think is generating the prices that they observe. As discussed in

Section 1, the assumption of common knowledge or rationality embedded in the rational expectations equilibrium ensures that all agents perfectly understand the equilibrium forces that generate price changes, and are therefore able to extract the right information from prices. Instead, when agents think in partial equilibrium, they misunderstand what generates the price that they observe because they fail to realize the general equilibrium consequences of their actions. The way that PET manifests itself in this setup is that all agents learn information from prices, but they fail to realize that other agents do too. In other words, PET agents think that they are the only ones inferring information from prices, and that all other agents trade on their unconditional priors.

Formally, PET agents think that in period $t-1$ informed agents update their beliefs with the new fundamental information they receive, $\tilde{u}_{t-1}$:[13]

$$\tilde{\mathbb{E}}_{I,t-1}[D_T] = \tilde{\mathbb{E}}_{I,t-1}\left[D_{t-2} + u_{t-1} + \sum_{h=1}^{\infty} \beta^h u_{t-1+h}\right] = \tilde{D}_{t-2} + \tilde{u}_{t-1} \tag{32}$$

$$\tilde{\mathbb{V}}_{I,t-1}[D_T] = \tilde{\mathbb{V}}_{I,t-1}\left[\sum_{h=1}^{\infty} \beta^h u_{t-1+h}\right] = \left(\frac{\beta^2}{1-\beta^2}\right)\sigma_u^2 \equiv \tilde{\mathbb{V}}_I \tag{33}$$

On the other hand, they think that all other uninformed agents do not learn information from prices, and instead trade on the same unconditional prior beliefs they held in period $t = 0$:

$$\tilde{\mathbb{E}}_{U,t-1}[D_T] = \tilde{\mathbb{E}}_{U,0}\left[\bar{D} + u_0 + \sum_{h=1}^{\infty} \beta^h u_h\right] = \bar{D} \tag{34}$$

$$\tilde{\mathbb{V}}_{U,t-1}[D_T] = \tilde{\mathbb{V}}_{U,0}\left[u_0 + \sum_{h=1}^{\infty} \beta^h u_h\right] = \left(\frac{1}{1-\beta^2}\right)\sigma_u^2 \equiv \tilde{\mathbb{V}}_U \tag{35}$$

where the equivalences in (33) and (35) highlight that in normal times, PET agents understand that all agents face constant uncertainty over time. Moreover, since $\tilde{\mathbb{V}}_I = \mathbb{V}_I < \tilde{\mathbb{V}}_U = V_U$, we see that PET agents are not misspecified about other agents' second moment beliefs, and they understand that informed agents have an informational edge.

Importantly, all agents are atomistic and do not consider the effect of their own asset

[13]The use of $t-1$ subscripts instead of t is to highlight that uninformed agents learn information from past prices, so that in period t they must understand what generated the price in period $t-1$, as this is the price they are extracting new information from.

demand on prices. PET agents then think that the equilibrium price in period $t-1$ is generated by the following market clearing condition:

$$\phi \underbrace{\left(\frac{\mathbb{E}_{U,t-1}[D_T] + \tilde{u}_{t-1} - P_t}{\mathcal{A}\tilde{\mathbb{V}}_I} \right)}_{\tilde{X}_{I,t}} + (1-\phi) \underbrace{\left(\frac{\bar{D} - P_t}{\mathcal{A}\tilde{\mathbb{V}}_U} \right)}_{\tilde{X}_{U,t}} = Z \tag{36}$$

which leads to the following price function:

$$P_{t-1} = \tilde{a} \left(\mathbb{E}_{U,t-1}[D_T] + \tilde{u}_{t-1} \right) + \tilde{b}\bar{D} - \tilde{c} \tag{37}$$

where:

$$\tilde{a} \equiv \frac{\phi \tilde{\tau}_I}{\phi \tilde{\tau}_I + (1-\phi)\tilde{\tau}_U} = \frac{\tilde{\zeta}}{1+\tilde{\zeta}} \tag{38}$$

$$\tilde{b} \equiv \frac{(1-\phi)\tilde{\tau}_U}{\phi \tilde{\tau}_I + (1-\phi)\tilde{\tau}_U} = \frac{1}{1+\tilde{\zeta}} \tag{39}$$

$$\tilde{c} \equiv \frac{\mathcal{A}Z}{\phi \tilde{\tau}_I + (1-\phi)\tilde{\tau}_U} \tag{40}$$

and since the only source of price variation perceived by PET agents is given by changes in informed agents' beliefs, we can rewrite this as:

$$\Delta P_{t-1} = \tilde{a}\tilde{u}_{t-1} \tag{41}$$

This expression micro-founds the reduced-form mapping in (12), and shows that when agents think in partial equilibrium they attribute any price change they observe to new information alone. This also shows PET agents' understanding that new information is incorporated more strongly into prices when informed agents' informational edge is higher, so that a given price change reflects a less extreme piece of news when this is the case. PET agents then invert the mapping in (41) to extract $\tilde{u}_{t-1}$ from prices:

$$\tilde{u}_{t-1} = \left(\frac{1}{\tilde{a}} \right) \Delta P_{t-1} \tag{42}$$

Substituting this into (23) leads to the following posterior beliefs:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \theta \Delta P_{t-1} \tag{43}$$

where:

$$\theta \equiv \frac{1}{\tilde{a}} = \left(1 + \frac{1}{\tilde{\zeta}}\right) \tag{44}$$

These expressions make clear that PET provides a micro-foundation for the type of price extrapolation considered in (13), where the extrapolation parameter is decreasing in uninformed agents' perception of informed agents' edge. Since the informational edge is itself increasing in the fraction of informed agents in the market, and in the confidence of informed agents relative to uninformed agents, the strength of the feedback effect is also decreasing in these quantities. We summarize this in the following proposition.

Proposition 2 (Micro-foundation of Price Extrapolation)**.** *The strength with which PET agents extrapolate past price changes is decreasing in uninformed agents' perception of informed agents' informational edge ($\tilde{\zeta}$). Specifically, PET agents extrapolate more strongly when there are fewer informed agents in the market (ϕ), and when their perception of informed agents' relative confidence is lower ($\tilde{\tau}_I/\tilde{\tau}_U$).*

To understand why PET agents extrapolate price changes more strongly when informed agents have a lower edge, notice that $\tilde{\zeta}$ captures how strongly a given piece of information is incorporated into prices. When informed agents have a lower edge, they trade less aggressively on a given piece of news, leading to a smaller price change. PET agents then recognize that they should extrapolate prices more strongly to recover a given piece of information from a smaller price change.[14]

Finally, it is worth noticing that the rational mapping takes the following form, as

[14]Informed agents' edge is not related to the amount of information that uninformed agents can learn from prices, $u_{t-1} \sim N(0, \sigma_u^2)$, as $\tilde{\zeta} = \left(\frac{\phi}{1-\phi}\right) \frac{\tilde{\tau}_I}{\tilde{\tau}_U} = \left(\frac{\phi}{1-\phi}\right) \frac{1}{\beta^2}$ is independent of σ_u^2. Moreover, since prices are fully revealing, uninformed agents are able to directly extract $\tilde{u}_{t-1}$ (as opposed to a noisy signal of $\tilde{u}_{t-1}$), so the informativeness of the information PET agents extract from prices is also independent of informed agents' edge. Appendix C shows how the extrapolation parameter changes when we introduce noise traders, so that prices are only partially revealing.

shown in (10) in Section 1:

$$\tilde{u}_{t-1} = \underbrace{\frac{1}{a}\Delta P_{t-1}}_{\text{extrapolation}} - \underbrace{\frac{b}{a}\tilde{u}_{t-2}}_{\text{lagged response}} \tag{45}$$

and since PET agents are not misspecified about other agents' second moment beliefs, $a = \tilde{a}$. We then see that it is rational to extrapolate from price changes if uninformed agents are constrained to learning information from past prices, and it is also rational for this extrapolation parameter to be decreasing in informed agents' edge. Comparing the rational mapping in (45) to the PET mapping in (42) shows that the bias in PET agents' beliefs isn't coming from how strongly they extrapolate past prices, but from omitting the correction term which accounts for the price variation due to the lagged response of all other uninformed agents. This bias is then decreasing in informed agents' edge, as a lower edge increases the influence on prices of uninformed agents' beliefs, leading PET agents to omit a greater source of price variation.

2.4 Properties of Equilibrium Outcomes

Combining the expressions in (31) and (42), we find that changes in prices and in beliefs evolve as an AR(1), as we saw in Section 1:

$$\tilde{u}_{t-1} = u_{t-1} + \left(\frac{b}{\tilde{a}}\right)\tilde{u}_{t-2} \tag{46}$$

$$\Delta P_t = a u_t + \left(\frac{b}{\tilde{a}}\right)\Delta P_{t-1} \tag{47}$$

where the strength of the feedback effect now takes the following form:

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1 + \frac{1}{\tilde{\zeta}}\right) \tag{48}$$

This expression makes clear that the strength of the feedback between outcomes and beliefs is decreasing both in the true informational edge (ζ), and in uninformed agents' perception of it ($\tilde{\zeta}$). Intuitively, in Section 2.2 we showed that when uninformed agents' perception of the informational edge is low, they extrapolate past price changes more strongly. Moreover, when the true informational edge of informed agents is low, the influence on prices of uninformed traders' biased beliefs is higher. Both these forces contribute to fuelling the feedback between outcomes and beliefs. We summarize these results in the following proposition.

Proposition 3 (Strength of the Feedback Effect)**.** *When agents think in partial equilibrium, the strength of the feedback between outcomes and beliefs is decreasing both in the true informational edge (ζ), and in uninformed agents' perception of it ($\tilde{\zeta}$). The strength of the feedback effect is stronger when there are fewer informed agents in the market (ϕ), and when the true and perceived confidence of informed agents relative to uninformed agents is low ($\frac{\tau_I}{\tau_U}$, $\frac{\tilde{\tau}_I}{\tilde{\tau}_U}$).*

Equation (46) shows that in response to a one-off shock PET delivers over-reaction, and that the deviation from rationality is increasing in the strength of the feedback effect and therefore decreasing in the true and perceived informational edges. Specifically, when the true and perceived informational edges are lower, the lagged response to information which PET agents neglect is greater, thus leading to a greater bias. This testable empirical prediction holds both in the cross-section, and over time.

Proposition 4 (Deviations from Rationality)**.** *When agents think in partial equilibrium, deviations from rationality in both prices and beliefs are decreasing in the true and perceived informational edges (ζ, $\tilde{\zeta}$). Specifically, environments with a smaller fraction of informed agents (ϕ), and with a lower true and perceived confidence of informed agents relative to uninformed agents ($^{\tau_I}/_{\tau_U}$, $^{\tilde{\tau}_I}/_{\tilde{\tau}_U}$) exhibit greater departures from rationality.*

Turning to the conditions for stability, since in normal times $\tau_i = \tilde{\tau}_i$ for $i \in \{I, U\}$, it follows that $\tilde{\zeta} = \zeta$, and the strength of the feedback effect reduces to:

$$\frac{b}{\tilde{a}} = \frac{1}{\zeta} \tag{49}$$

so that for the response of the economy to normal times shocks not to be explosive it must be that the aggregate confidence of informed agents is greater than the aggregate confidence of uninformed agents.

$$\frac{b}{\tilde{a}} < 1 \iff \zeta > 1 \iff \phi\tau_I > (1-\phi)\tau_U \tag{50}$$

Corollary 1 (Stability in Normal Times)**.** *When agents think in partial equilibrium, stability in normal times requires the aggregate confidence of informed agents to be greater than the aggregate confidence of uninformed agents.*

Figure 3 compares the path of equilibrium outcomes when the economy is in a stable region (left panel) and when it is in an unstable region (right panel). As intuited in Section 1, as long as the feedback between outcomes and beliefs is constant, the economy either responds to shocks by monotonically converging to a new state-dependent steady state, or it accelerates away from fundamentals, leading prices and beliefs to become extreme.[15]

Since empirically shocks are not explosive in normal times, the economy is in a stable region. Figure 3 shows that when this is the case partial equilibrium thinking delivers momentum in response to permanent shocks.[16] Moreover, while the PET impulse response function exhibits over-reaction relative to the rational expectations equilibrium at each point in time, the bias in both prices and beliefs increases over time following a one-off shock. In other words, in normal times PET achieves momentum via delayed over-reaction, and not via under-reaction relative to rational outcomes. However, if we were to run a standard Coibion and Gorodnichenko (2015) regression of forecast errors on forecast revisions, we would find a positive coefficient as positive forecast errors are associated with positive forecast revisions. While the literature often attributes such a positive coefficient to evidence of under-reaction, we caution against such an interpretation, as argued more forcefully in Bastianello and Fontanier (2021*a*).

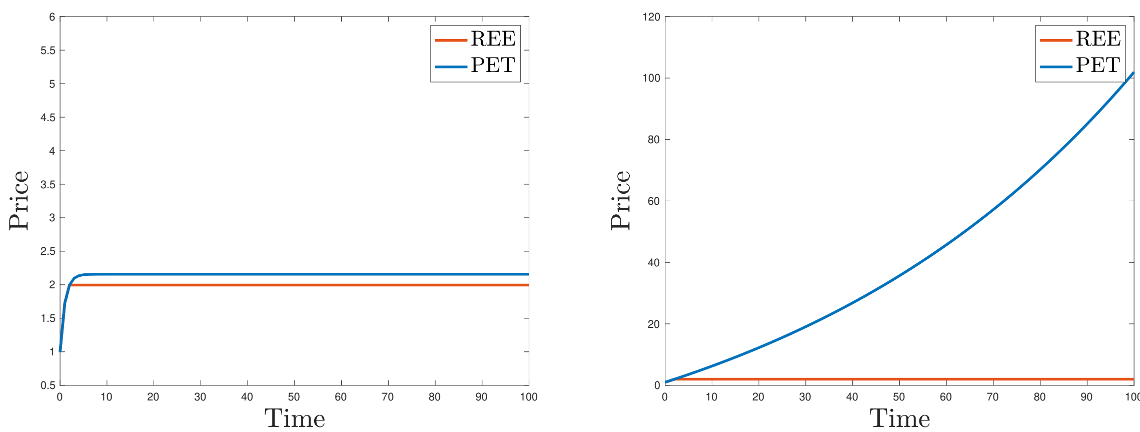
In the next section, we show how displacements can generate time-variation in the feedback effect, and shift the economy across stable and unstable regions, leading to the

[15]Notice that PET outcomes do not converge to the rational expectations equilibrium as $t \to \infty$. Conditional on not observing the liquidating dividend, PET agents never unlearn their misinferred information, as in Gagnon-Bartsch and Rabin (2016). In this respect, PET is attentionally stable in the sense of Gagnon-Bartsch et al. (2021).

[16]In Appendix D we consider alternative setups which also allow us to consider the response of the economy to *temporary* shocks, and we find that in these cases partial equilibrium thinking delivers momentum and reversals.

Figure 3: Impulse response functions following a normal times shock. This Figure compares the path of equilibrium prices following a one-off fundamental shock $u_1 > 0$ under rational expectations (REE) and under partial equilibrium thinking (PET). Panel (a) plots the impulse response function when the economy is in a stable region, with $b/\tilde{a} < 1$, and shows that prices gradually converge to a new steady state level. Panel (b) plots the impulse response function when the economy is in an unstable region, with $b/\tilde{a} > 1$, and shows that prices accelerate away from fundamentals in a convex way, and are unbounded.

(a) Stable Region (b) Unstable Region



formation of bubbles and endogenous crashes.

3 Displacements

"Displacement is some outside event that changes horizons, expectations, profit opportunities, behavior – some sudden advice many times unexpected. Each day's events produce some changes in outlook, but few significant enough to qualify as displacements" (Kindleberger (1978)). The nature of the displacement varies from one bubble episode to the next. Examples include the widespread adoption of a ground-breaking discovery - railroads in the 1840s, radio and automobiles in the 1920s, internet in the 1990s -, financial liberalization in Japan in the 1980s, or financial innovations such as securitization prior to the 2008 financial crisis.

Whatever the source of the displacement, the novelty associated with these shocks means that their full implications for long term outcomes can only be understood gradually over time, as more information becomes available (Pástor and Veronesi (2006), Pástor and Veronesi (2009)). When the internet was first made available to the public in 1993, investors were aware of this new technology, but at the time nobody knew the full potential of this invention. The development of blockchains as decentralized ledgers has paved the way for cryptocurrencies. However, we are yet to learn how wide-spread their adoption will be in the future, and assets that are associated with them have indeed been prone to bubbly behavior.

This seems to be in stark contrast to normal times shocks. When sophisticated investors see a new earnings announcement, they are better able to understand the implications of information such as same store sales for long term outcomes. The uninformed agents can learn about this more slowly by seeing how the market reacts to such announcements. Since informed agents are always one step ahead of uninformed agents, their informational edge is constant. On the other hand, following a displacement, the informational edge of informed agents is wiped out, as not even the most informed agents know what such shocks really mean for long term fundamentals. As the new technology becomes better established, sophisticated investors regain their informational edge as they are better placed to learn information about it, for example through access to management of the companies developing the technology.

In this section we show how the time-variation in informed agents' edge leads to a time-varying extrapolation parameter, and a time-varying strength of the feedback between prices and beliefs. This can shift the economy between stable and unstable regions. When the displacement first materializes, informed agents' edge is wiped out, thus increasing the influence on prices of uninformed agents' beliefs and the strength with which they extrapolate. Both of these forces fuel the feedback between prices and beliefs. If the uncertainty associated with the displacement is high enough, the economy can enter the unstable region, leading prices and beliefs to accelerate away from fundamentals. As informed agents learn about the new technology and regain their edge, the feedback effect weakens, and the economy re-enters the stable region. This leads the bubble to burst and prices and beliefs to return back towards fundamentals.

We conclude this section by discussing how the speed of information arrival shapes the duration and amplitude of bubbles, as well as alternative ways of modeling a displacement.

3.1 Displacement Shocks

We model displacements as an uncertain positive shock to long-term outcomes that agents can learn about only gradually over time. Starting from a normal-times steady state where uninformed agents' beliefs are consistent with the price they observe, in period $t = 0$ both informed and uninformed traders learn that there is "something new under the sun", but do not know the exact implications of such shock for long-term outcomes. Specifically, in period $t = 0$, all agents learn that the terminal dividend changes by an uncertain amount $\omega \sim N(\mu_0, \tau_0^{-1})$, where $\mu_0 > 0$:

$$D_T = \bar{D} + \sum_{j=0}^{\infty} \beta^j u_j + \omega \tag{51}$$

Initially, all agents share the same unconditional prior over ω. Starting in period $t = 1$, each period informed agents observe a common signal that is informative about the displacement, $s_t = \omega + \epsilon_t$ with $\epsilon_t \sim^{iid} N(0, \tau_s^{-1})$. Uninformed agents do not observe these signals but can learn information from past prices.

We solve the model using the same three steps we used in normal times: first, we specify what truly generates price changes agents observe. Second, we specify what uninformed agents think is generating these price changes, and find the mapping PET agents use to extract information from prices. Third, we solve the model recursively, and discuss the properties of equilibrium outcomes.

3.2 True Price Function following a Displacement

Following a displacement, informed agents observe new signals u_t and s_t in each period, and they revise their beliefs accordingly, via standard Bayesian updating:

$$\mathbb{E}_{I,t}[D_T] = E_{I,t}\left[\bar{D} + \sum_{j=1}^{t} u_{t-j} + u_t + \sum_{h=1}^{\infty} \beta^h u_{t+h} + \omega\right] = \mathbb{E}_{I,t-1}[D_T] + u_t + w_t \tag{52}$$

$$\mathbb{V}_{I,t}[D_T] = \mathbb{V}_{I,t}\left[\sum_{h=1}^{\infty} \beta^h u_{t+h} + \omega\right] = \mathbb{V}_I + (t\tau_s + \tau_0)^{-1} \tag{53}$$

where $w_t \equiv \mathbb{E}_{I,t}[\omega] - \mathbb{E}_{I,t-1}[\omega] = \frac{\tau_s}{t\tau_s+\tau_0}\left(s_t - \mathbb{E}_{I,t-1}[\omega]\right)$ is informed agents' revision of their beliefs about the displacement ω in light of the new signal s_t. Equation (53) shows that when the displacement is announced, informed agents face greater uncertainty, but their confidence gradually rises back towards its steady state level as they learn about the displacement over time.

On the other hand, in each period t, uninformed agents are interested in learning $\tilde{u}_{t-1} + \tilde{w}_{t-1}$ from the price change they observe in period $t-1$, and their posterior beliefs are given by:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \tilde{u}_t + \tilde{w}_t \tag{54}$$

$$\mathbb{V}_{U,t}[D_T] = \mathbb{V}_{U,t}\left[u_t + \sum_{h=1}^{\infty}\beta^h u_{t+h} + \omega\right] = \mathbb{V}_U + ((t-1)\tau_s + \tau_0)^{-1} \tag{55}$$

where (53) shows that uninformed agents also face greater uncertainty when the displacement is announced, but their confidence also rises back towards its steady state level as they learn about ω from past prices over time. Specifically, after t periods, PET agents have learnt about the displacement from $(t-1)$ price changes.

Combining the information in (53) and (55), informed agents' edge is initially diluted by the increase in aggregate uncertainty, but then gradually rises back to its steady state level:

$$\zeta_t = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + ((t-1)\tau_s + \tau_0)^{-1}}{\mathbb{V}_I + (t\tau_s + \tau_0)^{-1}}\right) \tag{56}$$

Given these beliefs, the true market clearing price function which generates the price agents observe is given by:

$$P_t = a_t\left(\mathbb{E}_{I,t-1}[D_T] + u_t + w_t\right) + b_t\left(\mathbb{E}_{U,t-1}[D_T] + \tilde{u}_{t-1} + \tilde{w}_{t-1}\right) - c_t \tag{57}$$

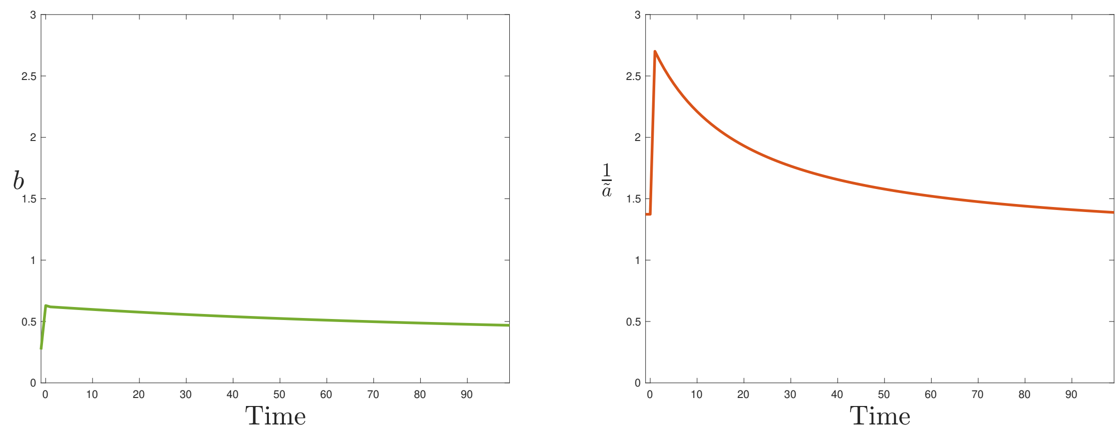
where:

$$a_t \equiv \frac{\zeta_t}{1+\zeta_t} \qquad b_t \equiv \frac{1}{1+\zeta_t} \tag{58}$$

and $c_t \equiv \frac{\mathcal{A}Z}{\phi\tau_{I,t}+(1-\phi)\tau_{U,t}}$, so that (58) shows that the time-variation in informed agents' edge generates changes in the relative influence on prices of informed and uninformed agents' beliefs. The left panel of Figure 4 shows that the influence on prices of uninformed agents' biased beliefs (b_t) initially rises and then gradually falls, as informed agents' edge is originally wiped out by the increase in aggregate uncertainty, and then slowly increases back to its steady state level as uncertainty about the displacement is resolved and informed agents regain their edge.

Figure 4: Time variation in b_t and $1/\tilde{a}_{t-1}$ following a displacement. Panel (a) plots how the influence on prices of uninformed agents' beliefs (b_t) varies over time following a displacement: b_t initially rises and then gradually declines. Panel (b) plots how the strength with which PET agents extrapolate past prices ($1/\tilde{a}_t$) varies over time following a displacement: when the displacement is announced, PET agents initially extrapolate past prices more aggressively, and then the degree with which they extrapolate declines over time. Comparing panels (a) and (b) shows that the extrapolation parameter declines at a faster rate than the influence on prices of uninformed agents' beliefs.

(a) Influence on prices of U agents' beliefs, b_t (b) PET degree of extrapolation, $1/\tilde{a}_{t-1}$



We can re-write the true price function in changes:

$$\Delta P_t = a_t \left(u_t + w_t\right) + b_t \left(\tilde{u}_{t-1} + \tilde{w}_{t-1}\right) + \left(P_{t|t-1} - P_{t-1}\right) \tag{59}$$

where we see that following a displacement, there is an additional source of price variation relative to normal times, as $(P_{t|t-1} - P_{t-1})$ captures the change in prices due to changes in agents' levels of confidence, fixing their mean beliefs:

$$(P_{t|t-1} - P_{t-1}) = \underbrace{(a_t - a_{t-1})\mathbb{E}_{I,t-1}[D_T] + (b_t - b_{t-1})\mathbb{E}_{U,t-1}[D_T]}_{\text{change in average belief}<0} \quad \underbrace{-(c_t - c_{t-1})}_{\text{change in risk-premium}>0} \tag{60}$$

The first term in this expression reflects changes in prices due to changes in average beliefs. To understand why this term is negative, notice that as the informational edge rises over time, informed agents receive more weight. Since informed agents are less optimistic than uninformed PET agents, and they receive a greater weight in prices over time, the average belief becomes less optimistic, pushing towards lower prices. On the other hand, the second term shows that as agents become more confident over time, the risk-premium decreases, and this contributes to higher prices. The overall sign of this expression depends on the relative strength of these two forces.

3.3 Micro-Founding Time-varying Extrapolation

Just as we did in Section 2, to understand what information uninformed agents extract from past prices, we start by specifying what uninformed agents think is generating the price they observe. This, in turn, requires us to work out PET agents' beliefs about other agents' actions and beliefs. Following a displacement, PET agents think that in period $t-1$ informed agents trade on all signals they have received up until period $t-1$, $\{\tilde{u}_j\}_{j=0}^{t-1}$ and $\{\tilde{s}_j\}_{j=1}^{t-1}$:

$$\tilde{\mathbb{E}}_{I,t-1}[D_T] = \tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1} + \tilde{w}_{t-1} \tag{61}$$

$$\tilde{\mathbb{V}}_{I,t-1}[D_T] = \mathbb{V}_I + ((t-1)\tau_s + \tau_0)^{-1} \tag{62}$$

where (62) reflects that after $(t-1)$ periods informed agents have observed $(t-1)$ price changes which incorporate $(t-1)$ signals about the displacement. Notice that $\tilde{\mathbb{V}}_{I,t-1}[D_T]$ it time-varying as uninformed agents recognize that informed agents' confidence decreases when the displacement is announced, and then increases over time as they learn more about it.

Moreover, PET agents think that all other uninformed agents do not learn information from prices, and instead trade on fixed prior beliefs:

$$\tilde{\mathbb{E}}_{U,t-1} = \bar{D} + \mu_0 \tag{63}$$

$$\tilde{\mathbb{V}}_{U,t-1} = \mathbb{V}_U + (\tau_0)^{-1} \tag{64}$$

where (64) shows that following a displacement PET agents believe that other uninformed agents face greater and constant uncertainty as they do not learn new information after the displacement is announced.

Combining the information in (62) and (64), PET agents' perception of informed agents' edge ($\tilde{\zeta}_{t-1}$) is initially diluted by the rise in aggregate uncertainty due to the displacement, and then gradually rises over time as informed agents learn more about it:

$$\tilde{\zeta}_{t-1} = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I + ((t-1)\tau_s + \tau_0)^{-1}}\right) \tag{65}$$

Notice that the initial fall in informed agents' edge is increasing in the amount of uncertainty generated by the displacement, $(\tau_0)^{-1}$, and that $\tilde{\zeta}_t$ rises at a faster rate than ζ_t. Intuitively, since PET agents think that uninformed agents are not learning more information over time, they think that informed agents regain their edge over uninformed agents at a faster rate than they do in reality: in period t PET agents think informed agents know t more signals than uninformed agents, when in reality all uninformed agents learn from past prices and informed agents are only one period ahead of uninformed agents.

Given these beliefs, the price function which PET agents think is generating the price they observe is given by:

$$P_{t-1} = \tilde{a}_{t-1}\left(\tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1} + \tilde{w}_{t-1}\right) + \tilde{b}_{t-1}\left(\bar{D} + \mu_0\right) - \tilde{c}_{t-1} \tag{66}$$

where $\tilde{a}_{t-1} \equiv \frac{\tilde{\zeta}_{t-1}}{1+\tilde{\zeta}_{t-1}}$, $\tilde{b}_{t-1} \equiv \frac{1}{1+\tilde{\zeta}_{t-1}} = 1 - \tilde{a}_{t-1}$, $\tilde{c}_{t-1} \equiv \frac{\mathcal{A}Z}{\phi\tilde{\tau}_{I,t-1}+(1-\phi)\tilde{\tau}_{U'}}$, so uninformed agents think that the influence on prices of informed (uninformed) agents' beliefs initially falls (rises) as informed agents' informational edge is diluted, and then gradually rises (falls) as informed agents learn over time.

According to PET agents, price changes now reflect two components:

$$\Delta P_{t-1} = \tilde{a}_{t-1}\left(\tilde{u}_{t-1} + \tilde{w}_{t-1}\right) + \left(\tilde{P}_{t-1|t-2} - P_{t-2}\right) \tag{67}$$

where $\left(\tilde{P}_{t-1|t-2} - P_{t-2}\right)$ captures an additional source of variation relative to their normal times mapping in (41), and this reflects uninformed agents' perception of price changes due to changes in confidence:

$$\left(\tilde{P}_{t-1|t-2} - P_{t-2}\right) = \underbrace{(\tilde{a}_{t-1} - \tilde{a}_{t-2})\tilde{E}_{I,t-2}[D_T] + (\tilde{b}_{t-1} - \tilde{b}_{t-2})(\bar{D} + \mu_0)}_{\text{change in average belief } >0} \underbrace{-(\tilde{c}_{t-1} - \tilde{c}_{t-2})}_{\text{change in risk-premium } >0} > 0 \tag{68}$$

Following good news, this term is unambiguously positive. Intuitively, PET agents think that informed agents are more optimistic than other uninformed agents. As the perceived informational edge rises over time, and optimistic informed agents receive more weight, PET agents think that the average belief in the market is becoming more optimistic. Moreover, as agents become more confident over time, the risk-premium component decreases, which also contributes to higher prices due to time-varying confidence levels.

PET agents then invert this mapping, and attribute the unexpected part of the price change they observe to new information $(\tilde{u}_{t-1} + \tilde{w}_{t-1})$, leading to the following posterior beliefs:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \theta_t \left(P_{t-1} - \tilde{P}_{t-1|t-2}\right) \tag{69}$$

where:

$$\theta_t \equiv \frac{1}{\tilde{a}_{t-1}} = 1 + \frac{1}{\tilde{\zeta}_{t-1}} \tag{70}$$

Following a displacement, PET agents extrapolate unexpected price changes with a time-varying extrapolation parameter, as is also shown in the right panel of Figure 4. Intuitively, PET agents adjust their mapping to reflect that following the increase in uncertainty associated with the displacement, prices are initially less sensitive to new information as informed agents' edge is diluted, and then gradually become more sensitive to information as informed agents regain their edge.

Proposition 5 (Time-varying Extrapolation)**.** *Following a displacement in period $t = 0$, the degree of extrapolation with which PET agents extrapolate unexpected price changes becomes time-varying. The extrapolation coefficient rises in period $t = 1$, and then gradually declines as uncertainty resolves over time. The initial rise in the extrapolation parameter is increasing in the uncertainty introduced by the shock (τ_0^{-1}) and decreasing in the* *fraction of informed agents in the market* (ϕ).

As well as being consistent with empirical evidence that documents a time-varying extrapolation parameter (Cassella and Gulen (2018)), micro-founding the extrapolation parameter in this way allows us to understand the assumptions implicit in models of constant price extrapolation. A constant extrapolation parameter requires uninformed agents to think that a given piece of information is always incorporated into prices with the same strength. In our model, this requires uninformed agents to think that informed agents' edge is constant over time, which in turn requires them to think that the composition of agents in the market and agents' relative confidence are also constant over time. This assumption seems to be a good characterization of investors' beliefs in normal times, in response to regular earnings announcements.

However, following a Kindleberger type displacement, these assumptions become counterfactual, as these shocks generate large changes to how information is incorporated into equilibrium prices. In this case, as uninformed agents think about what generates the prices they are learning from, they adjust the mapping they use to infer information from prices, thus leading to time-varying extrapolation.

3.4 Displacement, Bubbles and Crashes

By combining the results from Sections 3.2 and 3.3, we find that following a displacement PET agents' beliefs evolve as follows:

$$(\tilde{u}_{t-1} + \tilde{w}_{t-1}) = \left(\frac{a_{t-1}}{\tilde{a}_{t-1}}\right)(u_{t-1} + w_{t-1}) + \left(\frac{b_{t-1}}{\tilde{a}_{t-1}}\right)(\tilde{u}_{t-2} + \tilde{w}_{t-2}) - \frac{1}{\tilde{a}_{t-1}}\left(\tilde{P}_{t-1|t-2} - P_{t-1|t-2}\right) \tag{71}$$

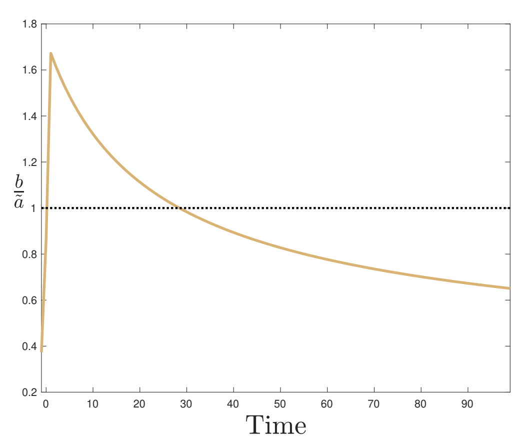
This expression is reminiscent of the AR(1) process in (46), with two key differences. First, the strength of the feedback between prices and beliefs ($\frac{b_{t-1}}{\tilde{a}_{t-1}}$) is now time-varying, allowing the economy to shift between stable and unstable regions. Second, this process now also has an additional correction term, which captures the bias in PET agents' forecasts of price changes due to changes in confidence levels. This additional term provides a pull back force that leads uninformed agents' beliefs to be disappointed at the peak of the bubble, leading to reversals and a crash. We now discuss both of these differences in detail.

Substituting (58) and (70) into the pseudo-AR(1) coefficient in (71), we find that the strength of the feedback effect now takes the following form:

$$\frac{b_{t-1}}{\tilde{a}_{t-1}} = \left(\frac{1}{1+\zeta_{t-1}}\right)\left(1+\frac{1}{\tilde{\zeta}_{t-1}}\right) \tag{72}$$

Figure 5 shows that when the displacement materializes in period $t = 0$, the strength of the feedback effect initially increases as the economy is flooded with uncertainty, and both the true and the perceived informational edges are diluted. However, as agents start learning about the displacement, the strength of the feedback effect gradually declines. Starting from a stable region in normal times, if the increase in uncertainty generated by the displacement is large enough, the economy enters an unstable region (${}^{b_t}/_{\tilde{a}_t} > 1$), allowing prices and beliefs to accelerate away from fundamentals. In the long run the economy always returns into a stable region, as $\lim_{t\to\infty} {}^{b_t}/_{\tilde{a}_t} < {}^{b}/_{\tilde{a}} < 1$ since $\lim_{t\to\infty}(b_t - b) = 0$ and $\lim_{t\to\infty}(\tilde{a}_t - \tilde{a}) > 0$, with prices and beliefs converging to a new steady state.

Figure 5: Time variation in the strength of the feedback effect following a displacement. This figure shows how the strength of the feedback between outcomes and beliefs varies over time following a displacement. The dotted line at ${}^{b}/_{\tilde{a}} = 1$ separates the stable region (${}^{b}/_{\tilde{a}} < 1$) from the unstable region (${}^{b}/_{\tilde{a}} > 1$). Starting from a normal times steady state where the strength of the feedback effect is less than one, a displacement is announced in period $t = 0$, and this leads the strength of the feedback effect to initially rise and then gradually decline over time. The initial increase in ${}^{b}/_{\tilde{a}}$ is increasing in the uncertainty associated with the displacement $(\tau_0)^{-1}$, and this figure depicts a scenario where $(\tau_0)^{-1}$ is large enough to initially shift the economy to an unstable region. Eventually, as informed agents learn more about the displacement, the strength of the feedback effect weakens and the economy returns to a stable region.



It is the last term in (71) that allows for reversals, as we need uninformed agents to infer negative information from prices ($\tilde{u}_{t-1} + \tilde{w}_{t-1} < 0$) for their beliefs to revert back towards fundamentals and the bubble to burst. We show that this can only happen once the economy returns to a stable region. Substituting (60) and (68) into (71), we find that beliefs evolve as follows:

$$\tilde{u}_{t-1}+\tilde{w}_{t-1} = \left(\frac{a_{t-1}}{\tilde{a}_{t-1}}\right)(\mathbb{E}_{I,t-1}[D_T] - \mathbb{E}_0[D_T]) - \left(1 - \frac{b_{t-1}}{\tilde{a}_{t-1}}\right)(\mathbb{E}_{U,t-1}[D_T] - \mathbb{E}_0[D_T]) + \frac{1}{\tilde{a}_{t-1}}(\tilde{c}_{t-1} - c_{t-1}) \tag{73}$$

where $\mathbb{E}_0[D_T] = \bar{D} + \mu_0$ is agents' unconditional prior belief when the displacement is announced. For the bubble to burst, we need $\tilde{u}_{t-1} + \tilde{w}_{t-1}$ to eventually turn negative. If we consider a one-off positive shock, such that $\mathbb{E}_{I,t-1}[D_T] = \mathbb{E}_{I,1}[D_T] > \mathbb{E}_0[D_T]$ for all $t \geq 1$, (73) makes clear that as long as the economy is in a unstable region and $\frac{b_{t-1}}{\tilde{a}_{t-1}} > 1$, PET agents continue to extract positive information from prices, and therefore become increasingly optimistic. In other words, when the economy is in an unstable region, the lagged response of uninformed agents always raises prices by more than what uninformed agents would expect from changes in confidence alone. On the other hand, this is no longer the case once the economy returns to a stable region and the feedback between outcomes and beliefs runs out of steam. At the peak of the bubble uninformed agents' beliefs vastly exceed fundamentals, and the term in $(\mathbb{E}_{U,t-1}[D_T] - \mathbb{E}_0[D_T])$ dominates in determining the sign of the news that uninformed agents extract from past prices in (73). Once the economy returns into a stable region and $\frac{b_{t-1}}{\tilde{a}_{t-1}} < 1$, PET agents expect higher price rises than the ones they observe. As their beliefs are disappointed, they become more pessimistic ($\tilde{u}_{t-1} + \tilde{w}_{t-1} < 0$) and the bubble bursts.

Moreover, the duration of the bubble is longer and its amplitude is greater when the informativeness of the signals that informed agents receive is low and uncertainty takes longer to resolve over time (τ_s is low).

Proposition 6 (Displacements, Bubbles and Crashes.)**.** *Following a displacement-type of shock, the strength of the feedback between prices and beliefs increases on impact, but as uncertainty resolves over time it eventually converges back to a level which is lower than in the original steady state. If the rise in uncertainty produced by the displacement is large* *enough, the economy enters an unstable region, allowing prices and beliefs to accelerate away from fundamentals and leading to the formation of bubbles. As agents learn more about the displacement, the strength of the feedback effect weakens, the economy re-enters a stable region, and the bubble bursts.*

Figure 6 shows the path of equilibrium outcomes following a displacement shock. Initially, as the economy enters the unstable region, prices and beliefs accelerate away from fundamentals in a convex way, and reach levels several multiples of the fundamental value of the asset (Greenwood et al. (2019)). This stage of the bubble is also associated with high trading volume (Barberis (2018), Hong and Stein (2007), DeFusco et al. (2020)). As the strength of the feedback effect weakens, and the economy re-enters the stable region, PET agents' expectations are disappointed, leading the bubble to burst, and prices and beliefs to converge back towards fundamentals. Partial equilibrium thinking naturally delivers these key characteristics of bubbles by exploiting the properties of unstable regions.

Finally, the exact shape of the bubble depends on the speed with which information about the displacement becomes available over time. For example, Figure 7 shows how the model can deliver a slower boom and a faster crash if information about the displacement is revealed slowly at first, and at a faster rate once the bubble bursts.

3.5 Frequency of Information Arrival

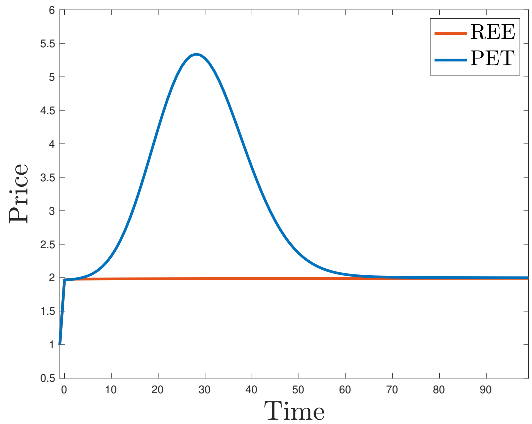
By assuming that informed agents receive new information in each period following a displacement, we are implicitly assuming that uninformed agents understand the frequency with which informed agents receive new information. However, if we change the frequency of information arrival, the true confidence of informed agents becomes decoupled from uninformed agents' perception of it.

In our model, following a displacement, uninformed agents observe a price change in each period, and they attribute each price change to new information. Regardless of the frequency of information arrival, having observed t price changes after t periods, uninformed agents' perception of informed agents' confidence is given by:

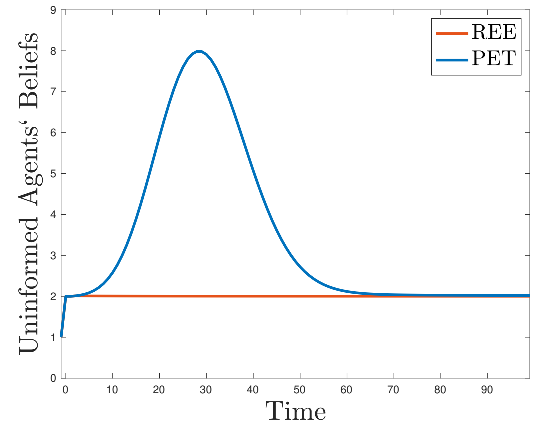
$$\tilde{\tau}_{I,t} = \left(\mathbb{V}_{I,0} + (t\tau_s + \tau_0)^{-1}\right)^{-1} \tag{74}$$

Figure 6: Bubbles and crashes following a displacement. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ with $\epsilon_t \sim N(0, \tau_s^{-1})$ each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0 \ \forall t > 1$. This figure compares the path of equilibrium prices, uninformed agents' beliefs, trading volume and agents' positions in the risky asset following a displacement which temporarily shifts the economy into an unstable region, under rational expectations and under partial equilibrium thinking. As the economy shifts into an unstable region when the displacement is announced, prices and beliefs accelerate away from fundamentals. This phase of the bubble is also associated with high trading volume, and PET agents being long the asset. Eventually, as the strength of the feedback effect weakens, the economy returns to a stable region and uninformed agents' beliefs are disappointed, leading to a crash.

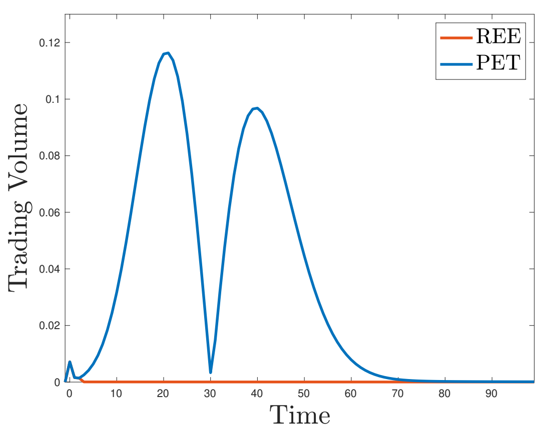
(a) Price



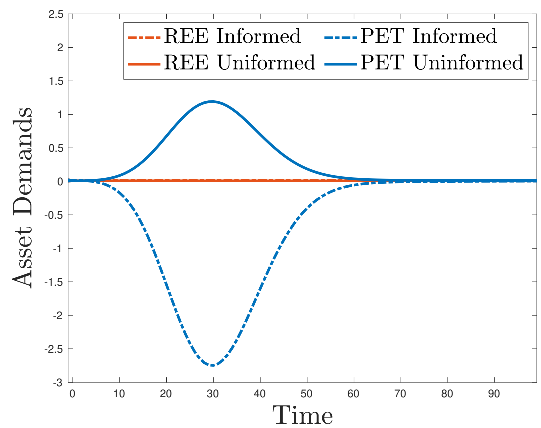
(b) Uninformed agents' beliefs



(c) Trading volume



(d) Asset demands



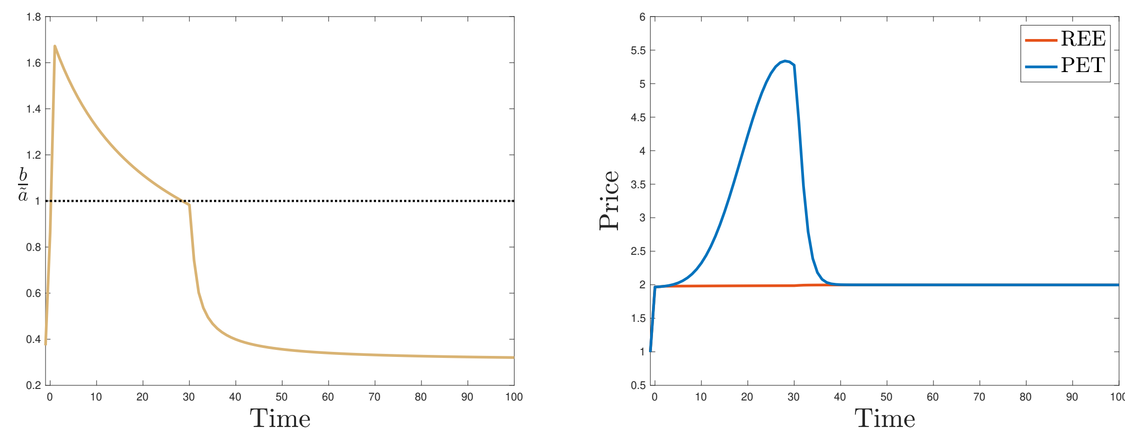
If informed agents receive news in each period, then $\tilde{\tau}_{I,t} = \tau_{I,t}$. Suppose instead that after t period, informed agents have received only $n_t < t$ signals. Their true confidence is now given by:

$$\tau_{I,t} = \left(\mathbb{V}_{I,0} + (n_t \tau_s + \tau_0)^{-1}\right)^{-1} < \tilde{\tau}_{I,t} \tag{75}$$

Figure 7: Asymmetric bubbles and crashes. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ with $\epsilon \sim N(0, \tau_{s,t}^{-1})$ each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0\ \forall t > 1$. Moreover, $\tau_{s,t} = \tau_s$ for $t \leq 30$ and $\tau_{s,t} = \tau_s' > \tau_s$ for $t > 30$, which reflects that information is revealed at a faster rate once the bubble bursts. The left panel illustrates the evolution of the strength of the feedback effect. The right panel illustrates the evolution of equilibrium prices, which now exhibit a slower boom and a faster crash.

(a) Strength of the feedback effect

(b) Price



With this information structure, informed agents need to receive only a finite number of signals for the bubble to burst. Let n_∞ be the total number of signals informed agents receive about the displacement over the whole lifetime of the asset. Long run stability then requires:

$$n_\infty > \bar{n} \tag{76}$$

where $\bar{n} = \frac{1}{\tau_s}\left(\frac{1}{\mathbb{V}_{I,0}(\tilde{\zeta}_\infty \zeta_0 - 1)} - \tau_0\right)$. This implies that bubbles may burst even if the true confidence of informed agents is lower than the true confidence of uninformed agents. This is not the case with models of constant price extrapolation, which instead rely on changes in the true relative confidence of informed and uninformed agents in order to generate bubbles and crashes.

To illustrate this point, Figure 8 shows the response of the economy if informed agents receive a single signal in period $t = 1$, and then receive no further information about the displacement thereafter, so that $n_\infty = 1$. When this is the case, the confidence of uninformed agents rises relative to the confidence of uninformed agents, as shown in the top left panel of Figure 8. However, even though the influence on prices of uninformed agents' biased beliefs rises over time, the economy can still return to a stable region because the strength with which PET agents extrapolate past prices falls over time. Intuitively, PET agents still attribute any price change they observe to additional news about the displacement, and thus think that informed agents' edge is rising over time. Comparing the path of equilibrium prices in the bottom right panel of Figure 8 to the one in Figure 6 we see that when informed agents receive a single shock, the bubble is much more accentuated and takes much longer to die out as the market spends more time in the unstable region. However, the key take-away is that a time-varying extrapolation coefficient allows for bubbles and endogenous crashes that are not driven by changes in agents' relative confidence levels, which would instead be necessary with constant price-extrapolation.

3.6 Other Types of Displacements

A key lesson from our analysis so far is that shocks that generate bubbles and crashes must have two properties: they must shift the economy to an unstable region, and such a shift must be temporary. So far, we have considered one possible way to achieve this via a positive shock that creates uncertainty, which gradually resolves over time. However, the sources of variation in $\frac{b_t}{\tilde{a}_t}$ discussed in Proposition 3 are informative about other types of shocks which may contribute to the formation of bubbles and crashes.

Specifically, we can write the strength of the feedback effect as follows:

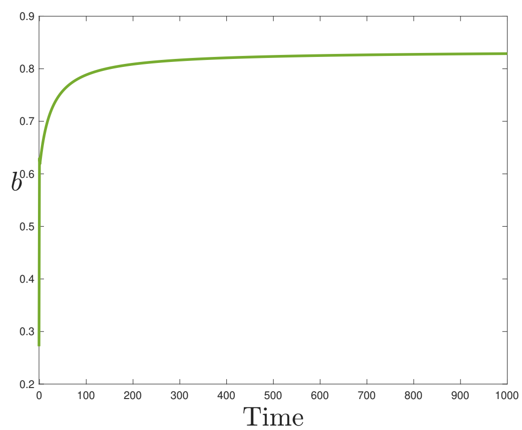
$$\frac{b_t}{a_t} = \left(\frac{1}{1+\zeta_t}\right)\left(1+\frac{1}{\tilde{\zeta}_t}\right) < 1 \iff \left(\frac{\phi_t}{1-\phi_t}\frac{\tau_{I,t}}{\tau_{U,t}}\right)\left(\frac{\tilde{\phi}_t}{1-\tilde{\phi}_t}\frac{\tilde{\tau}_{I,t}}{\tilde{\tau}_{U,t}}\right) > 1 \tag{77}$$

which generalizes our earlier expressions by allowing the fraction of informed agents in the market to be time-varying, and by allowing uninformed agents to be misspecified about this quantity ($\tilde{\phi}_t \neq \phi_t$). There are four components of the information structure that can then lead to time-variation in the strength of the feedback effect: the true and the perceived confidence of informed agents relative to uninformed agents, and the true and the perceived composition of agents in the market. Temporary shocks to these quantities can also contribute to the time-varying strength of the feedback effect.

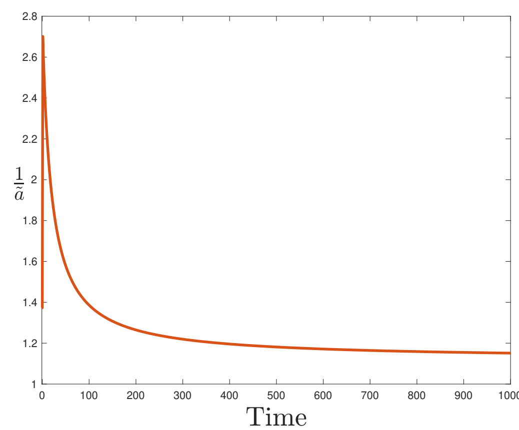
For example, displacements may lead to large changes in the composition of agents

Figure 8: Response of the economy when informed agents receive a single signal in period $t = 1$, and no further information thereafter. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$, and then informed agents receive a *single* signal $s_1 = \omega + \epsilon_1$ with $\epsilon_1 > 0$ and no more signals thereafter. Panels (a) and (b) show how the components of the feedback effect vary over time given this information structure, and Panels (c) and (d) show the evolution of the strength of the feedback effect and of equilibrium prices. Even though b rises over time, the degree of extrapolation still falls after its initial rise, thus allowing the strength of the feedback effect to return to a stable region ($b/\tilde{a} < 1$). Panel (d) shows that the bubble is much more accentuated than the one in Figure 6, as the economy spends longer in the unstable region.

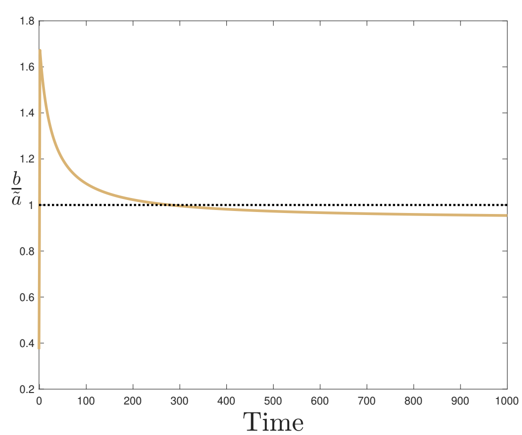
(a) Influence on prices of U agents' beliefs



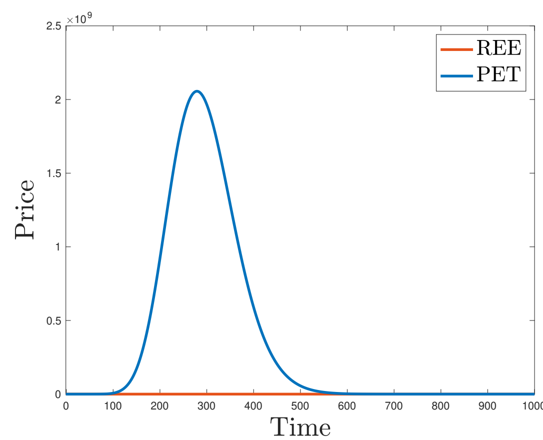
(b) PET degree of extrapolation



(c) Strength of the feedback effect



(d) Price



in the market, either because of the increased attention generated by media coverage, or because of the nature of the displacement itself, as with the introduction of securitization, which led to an expansion in credit during the most recent financial crisis. Changes in investor horizons, in the form of an increased speculative drive, may also generate changes in relative confidence levels and contribute to a stronger feedback effect during bubble episodes.

4 Speculative Motives

A noted feature of bubbles neglected so far is the role of destabilizing speculation. When explaining the stage of 'euphoria' characteristic of bubbles, Kindleberger (1978) describes how "[i]nvestors buy goods and securities to profit from the capital gains associated with the anticipated increases in the prices of these goods and securities".

To model speculative motives, we change agents' objective function to have CARA utility over next period wealth. In this case, agents forecast next period payoffs: with probability ρ the asset is alive next period, and is worth P_{t+1}, and with probability $(1-\rho)$ the asset dies, and pays out a terminal dividend D_t:

$$\Pi_{t+1} = \rho P_{t+1} + (1-\rho) D_t \tag{78}$$

Agents now trade according to the following asset demand function, given their beliefs:

$$X_{i,t} = \frac{\mathbb{E}[\Pi_{t+1}|\mathcal{I}_{i,t}] - P_t}{\mathcal{A}\mathbb{V}[\Pi_{t+1}|\mathcal{I}_{i,t}]} \tag{79}$$

In Appendix B we solve the model with speculative motives using the same three steps as in Section 3, and show that the true price function is linear in agents' beliefs, and that partial equilibrium thinking still provides a micro-foundation for price-based extrapolation:

$$P_t = a_t \mathbb{E}_{I,t}[\Pi_{t+1}] + b_t \mathbb{E}_{I,t}[\Pi_{t+1}] - c_t \tag{80}$$

$$\mathbb{E}_{U,t}[\Pi_{t+1}] = \mathbb{E}_{U,t-1}[\Pi_{t+1}] + \theta_t \left(P_{t-1} - \tilde{P}_{t-1|t-2}\right) \tag{81}$$

where a_t, b_t, c_t and θ_t are once again time-varying and deterministic. While these coefficients still depend on the properties of the environment, their functional form depends on agents' higher order beliefs. Specifically, since agents are forecasting future *endogenous* outcomes, they need to forecast other agents' future beliefs. While partial equilibrium thinking helps to pin down uninformed agents' higher order beliefs (they simply assume that all agents trade on their own private information and that this is common knowledge), it allows for more flexibility about informed agents' higher order beliefs.

In this section, we consider two cases. First, we let informed agents understand uninformed agents' biased beliefs, which in turn means that they understand that mispricing is predictable. Second, we consider the case where informed agents mistakenly believe that all other agents are rational and extract the right information from prices. We refer to the first type of speculators as being "PET-aware", and to the second type as being "PET-unaware". This lines up with the distinction in practical asset management between investors who concentrate on the gap between market prices and their estimates of fundamentals, and those who also think about the behavioral biases in the market.

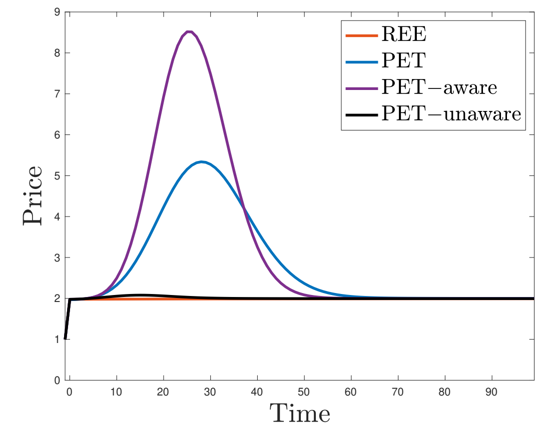
Figure 9 contrasts the dynamics of equilibrium outcomes following a displacement with and without speculative motives. When informed agents understand other agents' biases, they engage in destabilizing speculation and amplify the bubble. Intuitively, when informed agents realize that mispricing is predictable, they understand that higher prices today translate into more optimistic beliefs by uninformed agents and higher prices tomorrow. This increases informed agents' expected capital gains and induces them to demand more of the asset today, inflating prices further (De Long et al. (1990)).

To take advantage of predictable mispricing, "PET-aware" speculators require a high level of understanding of other agents' actions and beliefs. Alternatively, we can consider the case where informed agents mistakenly believe that they live in a rational world and think that uninformed agents are able to recover the right information from past prices. In this case, informed agents believe that any current mispricing will be corrected next period. This leads them to trade more aggressively on their own information, thus keeping prices closer to fundamentals, and effectively arbitraging the bubble away.

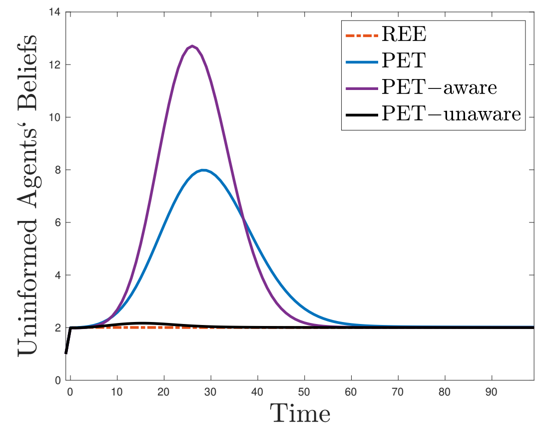
This analysis highlights the importance of higher order beliefs in the formation of bubbles: only if investors think that mispricing is likely to persist do they engage in destabilizing speculation. If instead they think mispricing is temporary, they engage in fundamental speculation and arbitrage it away.

Figure 9: Bubbles and crashes with “PET-aware” and “PET-unaware” speculators. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ in each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0\ \forall t > 1$. This figure compares the path of equilibrium prices, uninformed agents' beliefs, trading volume and agents' positions in the risky asset under rational expectations, partial equilibrium thinking, “PET-aware” speculation, and “PET-unaware” speculation. “PET-aware” speculation amplifies the bubble relative to the case with no speculative motives, while “PET-unaware” speculation arbitrages the bubble away.

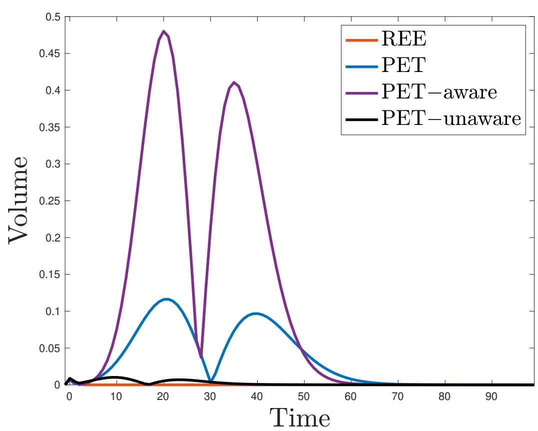
(a) Price



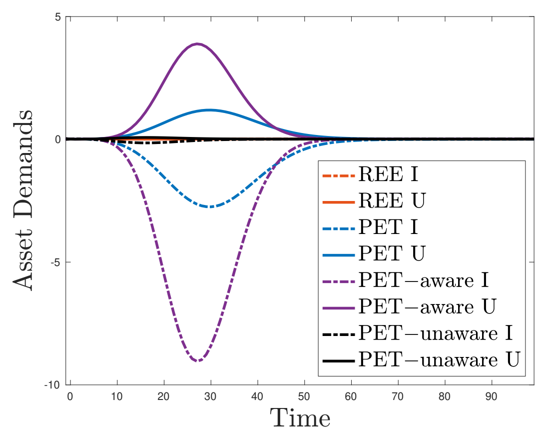
(b) Uninformed agents' beliefs



(c) Trading volume



(d) Asset demands



5 Conclusion

In this paper we provide a micro-foundation for the degree of price extrapolation with a theory of “Partial Equilibrium Thinking” (PET), in which uninformed agents mistakenly attribute any price change they observe to new information alone, when in reality part of the price change is due to other agents' buying/selling pressure. We show that when agents think in partial equilibrium the degree of extrapolation varies with the information structure, and is decreasing in informed agents' informational edge.

This micro-foundation provides a unifying theory of both weak departures from rationality in normal times, and extreme bubbles and crashes following a displacement. These are simply different manifestations of the same two-way feedback between prices and beliefs. In normal times, informed agents' edge is constant, and PET delivers constant price extrapolation. By contrast, following a displacement, informed agents' edge is temporarily wiped out, and PET agents' degree of extrapolation is stronger at first, but then gradually dies down, leading to bubbles and endogenous crashes.

While this paper provides a first step in micro-founding the degree of price extrapolation, our analysis leaves several open avenues for future work. First, a quantitative assessment of our theory would shed light on the extent of amplification that time-varying extrapolation can provide in explaining departures from rationality, and would clarify the importance of this channel. Second, by looking at the variation in the degree of price extrapolation and in individual level forecasts, our model offers two predictions that distinguish it from models of constant price extrapolation, and of fundamental extrapolation: i) unlike models of constant price extrapolation, when agents think in partial equilibrium the degree of price extrapolation is stronger when there are fewer informed agents in the market, and when informed agents' edge is greater; ii) unlike models of fundamental extrapolation, when agents think in partial equilibrium the bias in individual level forecasts depends on the composition of agents in the market, as this affects the extent of misspecification. These predictions can be tested both in the cross-section and over time. As the literature moves to incorporating non rational expectations into macro and finance models, and to studying their quantitative and policy implications, distinguishing between different sources of irrationality is increasingly important, and evidence that sheds light on these issues is a fruitful avenue for future research.

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A Proofs and Derivations

A.1 Proof of Proposition 1: Stable and Unstable Regions

Equation (16) shows that following a one-off shock $u_0 \neq 0$ in period 0, the price level in period t is given by:

$$P_t = \bar{P} + \sum_{j=0}^{t} \left(\frac{b}{a}\right)^j (au_0) \tag{A.1}$$

Moreover, beliefs in period t are given by:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,0}[D_T] + \sum_{j=0}^{t-1} \tilde{u}_j = \mathbb{E}_{U,0}[D_T] + \sum_{j=0}^{t-1} \left(\frac{b}{a}\right)^j u_0 \tag{A.2}$$

where the last equality uses the fact that $\tilde{u}_j = \left(\frac{b}{a}\right) \tilde{u}_{j-1} + u_j$ from (14), and solves recursively.

Since $b > 0$ and $a > 0$, taking the limit of these expressions as t goes to infinity, we notice that if $b/a < 1$ the geometric series in (A.1) and (A.2) are bounded and prices and beliefs converge to:

$$\lim_{t\to\infty} P_t = \bar{P} + \left(\frac{au_0}{1 - \frac{b}{a}}\right) \tag{A.3}$$

$$\lim_{t\to\infty} \mathbb{E}_{U,t}[D_T] = \bar{\mathbb{E}}_{U,0}[D_T] + \left(\frac{u_0}{1 - \frac{b}{a}}\right) \tag{A.4}$$

On the other hand, if $b/a > 1$ the geometric series in (A.1) and (A.2) are prices and beliefs become extreme:

$$\lim_{t\to\infty} P_t = \infty \tag{A.5}$$

$$\lim_{t\to\infty} \mathbb{E}_{U,t}[D_T] = \infty \tag{A.6}$$

Moreover, (15) and (14) show that after a one-off shock u_0, initially $\Delta P_0 = au_0$ and $\tilde{u}_0 = u_0$, and after t periods changes in prices and in beliefs are given by:

$$\Delta P_t = \left(\frac{b}{a}\right) \Delta P_{t-1} \tag{A.7}$$

$$\tilde{u}_{t-1} = \left(\frac{b}{a}\right) \tilde{u}_{t-2} \tag{A.8}$$

so that if $^b/_a < 1$ prices and beliefs converge in a concave way as $\Delta P_t < \Delta P_{t-1}$ and $\tilde{u}_{t-1} < \tilde{u}_{t-2}$, and if instead $^b/_a > 1$, prices and beliefs accelerate away from fundamentals with convex paths as $\Delta P_t > \Delta P_{t-1}$ and $\tilde{u}_{t-1} > \tilde{u}_{t-2}$. □

A.2 Proposition 2: Micro-foundation of Price Extrapolation

Equation (44) shows that when traders think in partial equilibrium, the strength with which they extrapolate past price changes is given by:

$$\theta \equiv 1 + \frac{1}{\tilde{\zeta}} \tag{A.9}$$

Combining this with the definition of $\tilde{\zeta} \equiv \left(\frac{\phi}{1-\phi}\right)\frac{\tilde{\tau}_I}{\tilde{\tau}_U}$, which is uninformed agents' perception of the true informational edge in (25), we find that:

$$\theta \equiv 1 + \frac{1}{\tilde{\zeta}} = 1 + \left(\frac{1}{\phi} - 1\right)\frac{1}{\frac{\tilde{\tau}_I}{\tilde{\tau}_U}} \tag{A.10}$$

The first equality shows that θ is decreasing in uninformed agents' perception of the informational edge $\tilde{\zeta}$. The second equality shows that θ is decreasing in the fraction of informed agents in the market, ϕ, and in uninformed agents' perception of the relative confidence of informed and uninformed agents ($^{\tilde{\tau}_I}/_{\tilde{\tau}_U}$). □

A.3 Proof of Proposition 3: Strength of the Feedback Effect

Equation (48) shows that the strength of the feedback effect is given by:

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1 + \frac{1}{\tilde{\zeta}}\right) \tag{A.11}$$

Combining this with the definitions of the informed agents' edge ζ in (25), and of uninformed agents' perception of it, $\tilde{\zeta}$, we find that:

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1+\frac{1}{\tilde{\zeta}}\right) = \left(\frac{1}{1+\frac{1}{\frac{1}{\phi}-1}\frac{\tau_I}{\tau_U}}\right)\left(1+\left(\frac{1}{\phi}-1\right)\frac{1}{\frac{\tilde{\tau}_I}{\tilde{\tau}_U}}\right) \tag{A.12}$$

The first equality shows that the strength of the feedback effect is decreasing in both the true informational edge, ζ, and in uninformed agents' perception of it, $\tilde{\zeta}$. The second equality shows that the strength of the feedback effect is decreasing in the fraction of informed agents in the market, ϕ (by inspection, both $\frac{1}{1+\zeta}$ and $\frac{1}{\tilde{\zeta}}$ are decreasing in ϕ) and in the true and perceived confidence of informed agents relative to uninformed agents τ_I/τ_U, $\tilde{\tau}_I/\tilde{\tau}_U$. □

A.4 Proof of Proposition 4: Deviations from Rationality

When traders have rational expectations, they infer the right information from prices at each point in time. Following, a one-off shock in period 0, uninformed traders learn the following information from price:

$$\tilde{u}_0^{REE} = u_0 \neq 0 \tag{A.13}$$

$$\tilde{u}_t^{REE} = u_t = 0 \quad \forall t > 0 \tag{A.14}$$

It follows that under rational expectations, uninformed traders' beliefs are given by:

$$\mathbb{E}_{U,0}[D_T]^{REE} = \bar{D} \tag{A.15}$$

$$\mathbb{E}_{U,t}[D_T]^{REE} = \bar{D} + u_0 \quad \forall t > 0 \tag{A.16}$$

This reflects that rational uninformed traders understand that there is no new information after period 0, and that all other price changes they observe are due to the lagged response of all uninformed traders who are also learning information from prices. Therefore, they no longer update their beliefs following the second price rise, as in the example in Figure

1. The corresponding equilibrium prices are then given by:

$$P_0^{REE} = \bar{P} + \Delta P_0 = \bar{P} + au_0 \tag{A.17}$$

$$P_t^{REE} = \bar{P} + \Delta P_0 + \Delta P_1 + \underbrace{\sum_{j=2}^{t} \Delta P_t}_{=0} = \bar{P} + au_0 + bu_0 \quad \forall t > 0 \tag{A.18}$$

where $\sum_{j=2}^{t} \Delta P_t = 0$ as neither informed nor uninformed agents update their beliefs after period $t = 1$, and in normal times the risk-premium component $\left(\frac{\mathcal{A}Z}{\phi\tau_I + (1-\phi)\tau_U}\right)$ is also constant over time.

On the other hand, from (A.1) and (A.2) together with the fact that in normal times $a = \tilde{a}$, we know that when uninformed traders think in partial equilibrium, equilibrium beliefs and prices are given by:

$$\mathbb{E}_{U,0}[D_T] = \bar{D} \tag{A.19}$$

$$\mathbb{E}_{U,1}[D_T] = \bar{D} + u_0 \tag{A.20}$$

$$\mathbb{E}_{U,t}[D_T] = \bar{D} + u_0 + \sum_{j=1}^{t-1} \left(\frac{b}{\tilde{a}}\right)^j u_0 \tag{A.21}$$

and:

$$P_0 = \bar{P} + au_0 \tag{A.22}$$

$$P_1 = \bar{P} + au_0 + bu_0 \tag{A.23}$$

$$P_t = \bar{P} + au_0 + bu_0 + \sum_{j=2}^{t} \left(\frac{b}{\tilde{a}}\right)^j (au_0) \tag{A.24}$$

Comparing PET to REE outcomes, we see that when traders think in partial equilibrium, deviations from rational outcomes are given by:

$$\mathbb{E}_{U,t}[D_T] - \mathbb{E}_{U,t}^{REE}[D_T] = 0 \qquad \text{for } t = 0, 1 \tag{A.25}$$

$$\mathbb{E}_{U,t}[D_T] - \mathbb{E}_{U,t}^{REE}[D_T] = \sum_{j=1}^{t-1} \left(\frac{b}{\tilde{a}}\right)^j u_0 \quad \forall t > 1 \tag{A.26}$$

and:

$$P_t - P_t^{REE} = 0 \qquad \text{for } t = 0,1 \tag{A.27}$$

$$P_t - P_t^{REE} = \sum_{j=2}^{t} \left(\frac{b}{\tilde{a}}\right)^j (au_0) = \sum_{j=1}^{t-1} \left(\frac{b}{\tilde{a}}\right)^j (bu_0) \quad \forall t > 1 \tag{A.28}$$

where the last equality uses the fact that in normal times $\tilde{a} = a$.

From Proposition 3, we know that $\frac{b}{\tilde{a}}$ is decreasing in ζ, $\tilde{\zeta}$, ϕ, $\frac{\tau_I}{\tau_U}$ and $\frac{\tilde{\tau}_I}{\tilde{\tau}_U}$. Moreover, from (29) we know that b is also decreasing in ζ, which is itself increasing in ϕ and $\frac{\tau_I}{\tau_U}$. Combining these results with (A.26) and (A.28), we obtain the comparative statics in Proposition 4 $\forall t > 1$. In particular, when the equilibrium is stable these comparative statics also hold in the limit as $t \to \infty$, as the economy approaches the new steady state. □

A.5 Proof of Corollary 1: Stability in Normal Times

Equation (48) shows that in normal times the strength of the feedback effect is given by:

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1 + \frac{1}{\tilde{\zeta}}\right) \tag{A.29}$$

Since in normal times $\tau_i = \tilde{\tau}_i$ for $i \in \{I, U\}$, it follows that $\tilde{\zeta} = \zeta$, and the strength of the feedback effect reduces to:

$$\frac{b}{\tilde{a}} = \frac{1}{\zeta} \tag{A.30}$$

Using the definition of ζ in (25):

$$\frac{b}{\tilde{a}} < 1 \iff \zeta > 1 \iff \phi\tau_I > (1-\phi)\tau_U \tag{A.31}$$

so that for the response of the economy to normal times shocks not to be explosive it must be that the aggregate confidence of informed agents is greater than the aggregate confidence of uninformed agents. □

A.6 Proof of Proposition 5: Time-varying Extrapolation

Before the displacement is announced, the degree of extrapolation in normal times is simply given by:

$$\theta = 1 + \frac{1}{\tilde{\zeta}} = 1 + \left(\frac{1}{\phi} - 1\right)\frac{\mathbb{V}_I}{\mathbb{V}_U} \tag{A.32}$$

Following a displacement, (70) shows that the degree of extrapolation with which PET agents extrapolate unexpected price changes is given by:

$$\theta_t = 1 + \frac{1}{\tilde{\zeta}_{t-1}} \tag{A.33}$$

In period $t = 0$, when the displacement is first announced, $\tilde{\zeta}_{-1} = \tilde{\zeta}$, and $\theta_0 = \theta$. This simply reflects the fact that PET agents are learning information from *past* prices, and in period $t = 0$ they are still learning information from normal times prices. Therefore, they extrapolate these price changes with the same degree of extrapolation as in normal times.

For $t > 0$, PET agents realize that the prices they are extrapolating from contain information about the displacement, leading to a change in the degree of extrapolation. Substituting (65) into (A.34):

$$\theta_t = 1 + \left(\frac{1}{\phi} - 1\right)\left(\frac{\mathbb{V}_I + ((t-1)\tau_s + \tau_0)^{-1}}{\mathbb{V}_U + (\tau_0)^{-1}}\right) \tag{A.34}$$

This shows that the degree of extrapolation is indeed time-varying.

Starting with the change in the extrapolation parameter between periods $t = 0$ and $t = 1$, we can subtract (A.32) from (A.34) and set $t = 1$:

$$\theta_1 - \theta_0 = \left(\frac{1}{\phi} - 1\right)\frac{(\tau_0)^{-1}}{\left(\mathbb{V}_U + (\tau_0)^{-1}\right)\mathbb{V}_U}(\mathbb{V}_U - \mathbb{V}_I) > 0 \tag{A.35}$$

where the last inequality follows from the fact that $\phi \in (0,1)$, $\tau_0 > 0$, $\mathbb{V}_U > 0$, and $(\mathbb{V}_U - \mathbb{V}_I) > 0$ since in normal times informed agents have an edge relative to uninformed agents, as shown in (22) and (24). Therefore, the degree of extrapolation rises in period

$t = 1$ after the displacement is announced. Moreover, (A.35) also shows that:

$$\frac{\partial(\theta_1 - \theta_0)}{\partial \phi} < 0 \quad \text{and} \quad \frac{\partial(\theta_1 - \theta_0)}{\partial(\tau_0^{-1})} > 0 \tag{A.36}$$

so that the initial rise in the extrapolation parameter between periods $t = 0$ and $t = 1$ is decreasing in the fraction of informed agents in the market and increasing in the uncertainty introduced by the shock.

For $t > 1$, (A.34) shows that:

$$\frac{\partial \theta_t}{\partial t} < 0 \tag{A.37}$$

as uncertainty resolves over time and $\frac{\partial((t-1)\tau_s + \tau_0)^{-1}}{\partial t} < 0$. Therefore the degree of extrapolation gradually declines over time after its initial rise in period $t = 1$. $\square$

A.7 Proof of Proposition 6: Displacements, Bubbles and Crashes

In normal times, the strength of the feedback effect is given by:

$$\frac{b_t}{\tilde{a}_t} = \left(\frac{1}{1+\zeta}\right)\left(1 + \frac{1}{\tilde{\zeta}}\right) = \frac{1}{\zeta} < 1 \tag{A.38}$$

where the second equality follows from the fact that in normal times:

$$\zeta = \tilde{\zeta} = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U}{\mathbb{V}_I} \tag{A.39}$$

and the last inequality in (A.38) follows from the fact that the economy must be in a stable region in normal times.

Following a displacement, the strength of the feedback effect is given by:

$$\frac{b_t}{\tilde{a}_t} = \left(\frac{1}{1+\zeta_t}\right)\left(1 + \frac{1}{\tilde{\zeta}_t}\right) \tag{A.40}$$

where in $t = 0$:

$$\zeta_0 = \tilde{\zeta}_0 = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I + (\tau_0)^{-1}} \tag{A.41}$$

and in $t > 0$:

$$\zeta_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + ((t-1)\tau_s + \tau_0)^{-1}}{\mathbb{V}_I + (t\tau_s + \tau_0)^{-1}} \tag{A.42}$$

$$\tilde{\zeta}_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I + (t\tau_s + \tau_0)^{-1}} \tag{A.43}$$

Combining (A.40) and (A.41), we find that in period $t = 0$ the strength of the feedback effect is given by:

$$\frac{b_0}{\tilde{a}_0} = \frac{1}{\zeta_0} \tag{A.44}$$

$$= \frac{1}{\zeta} + \left(\frac{1}{\zeta_0} - \frac{1}{\zeta}\right) \tag{A.45}$$

$$= \frac{b}{\tilde{a}} + \left(\frac{1-\phi}{\phi}\right)\left(\frac{\mathbb{V}_U - \mathbb{V}_I}{\mathbb{V}_U}\right)\frac{(\tau_0)^{-1}}{\mathbb{V}_U + (\tau_0)^{-1}} \tag{A.46}$$

where the second equality simply adds and subtracts the strength of the feedback effect in normal times $\frac{b}{\tilde{a}} = \frac{1}{\zeta}$, and the last equality uses the expressions for ζ and ζ_0 in (A.39) and (A.41) above, and rearranges.

Ceteris paribus, for the strength of the feedback effect to enter the unstable region we need the uncertainty associated with the displacement $(\tau_0)^{-1}$ to be high enough:

$$\frac{b_0}{\tilde{a}_0} > 1 \iff (\tau_0)^{-1} > \frac{\left(1 - \frac{b}{\tilde{a}}\right)\left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U}{\mathbb{V}_U - \mathbb{V}_I}\right)\mathbb{V}_U}{1 - \left(1 - \frac{b}{\tilde{a}}\right)\left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U}{\mathbb{V}_U - \mathbb{V}_I}\right)} \tag{A.47}$$

In the long run, as uncertainty about the displacement is resolved, we have that:

$$\zeta_\infty \equiv \lim_{t\to\infty} \zeta_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U}{\mathbb{V}_I} = \zeta \tag{A.48}$$

$$\tilde{\zeta}_\infty \equiv \lim_{t\to\infty} \tilde{\zeta}_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I} > \tilde{\zeta} \tag{A.49}$$

Combining these expressions:

$$\lim_{t\to\infty} \frac{b_t}{\tilde{a}_t} = \left(\frac{1}{1+\zeta_\infty}\right)\left(1 + \frac{1}{\tilde{\zeta}_\infty}\right) < \frac{b}{\tilde{a}} < 1 \tag{A.50}$$

which shows that in the long run the economy always returns to a stable region, with a steady state feedback effect that is weaker than the original normal times feedback effect. In the main text we show that when the strength of the feedback effect evolves in this way, prices and beliefs are initially non-stationary and accelerate away from fundamentals in a convex way. As the feedback effect then weakens towards its new steady state level, it eventually returns into a stable region, leading uninformed agents' beliefs to be disappointed, the bubble to burst, and prices and beliefs to converge back towards fundamentals. □

B Adding Speculative Motives

To model speculative motives, we let agents have Constant Absolute Risk Aversion (CARA) utility over *next* period wealth.

When traders have these preferences, their asset demand function conditional on their beliefs is given by:

$$X_{i,t} = \frac{\mathbb{E}_{i,t}[\Pi_{t+1}] - P_t}{\mathcal{A}\mathbb{V}_{i,t}[\Pi_{t+1}]} \tag{B.1}$$

where the expected next period payoff is given by:

$$\Pi_{t+1} \equiv \beta P_{t+1} + (1-\beta) D_t \tag{B.2}$$

and simply reflects that with probability β the asset is alive next period and worth P_{t+1}, and with probability $(1-\beta)$ the asset dies and pays out its terminal dividend D_t.

Since agents are forecasting prices, which are *endogenous* outcomes, they now need to forecast other agents' future beliefs. Therefore, in solving the model with speculative motives, we need to specify agents' higher order beliefs. While partial equilibrium thinking helps to pin down uninformed agents' higher order beliefs (they simply assume that all agents trade on their private information alone, and that this is common knowledge), it allows for more flexibility about informed agents' higher order beliefs.

We consider two cases. In Section B.1 we let informed agents be "PET-aware", so that they perfectly understand uninformed agents' biased beliefs. In Section B.2, we consider a case where informed agents are "PET-unaware" and mistakenly believe that all other agents are rational, and that uninformed agents extract the right information from prices. This lines up with the distinction in practical asset management between investors who concentrate on the gap between market prices and their estimates of fundamentals, and those who also think about the behavioral biases in the market.

B.1 "PET−aware" Speculation

In solving the model, we proceed in the same three steps we used in the baseline model. First, we solve for the true price function which generates the prices agents observe. Second, we specify the mapping that uninformed agents use to extract information from prices. Third, we solve the model forward, starting from the steady state in normal times. The one key difference to our baseline setup is that since all agents are now forecasting an endogenous outcome, we now need to solve for the first two steps by backwards induction. To do so, we use the new steady state after the uncertainty surrounding the displacement has been resolved as our terminal point.

Step 1: True Market Clearing Price Function. To determine the true market clearing condition which determines the prices agents observe, we know that in period t all informed agent trade on the whole history of signals they have received up until that date ($\{u_j\}_{j=0}^t$, $\{s_j\}_{j=1}^t$) and all uninformed agents trade on the information they have learnt from past prices.

We define $\mathcal{D}_t \equiv \bar{D} + \sum_{j=1}^t u_t$ and $\mathcal{W}_t \equiv \frac{\tau_0}{t\tau_s+\tau_0}\mu_0 + \frac{\tau_s}{t\tau_s+\tau_0}\sum_{j=1}^t \tilde{s}_t$ to be informed agents' period t belief of normal times shocks and of the displacement respectively, and $\tilde{\mathcal{D}}_t$ and $\tilde{\mathcal{W}}_t$ are uninformed agents' beliefs about these quantities.

We can then guess that the true price function takes the following form:

$$P_t = A_t(\mathcal{D}_t + \mathcal{W}_t) + B_t(\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1}) - K_t \tag{B.3}$$

where $\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1}$ is the information that uninformed agents extract from past prices, and A_t, B_t and K_t are time-varying and deterministic coefficients that depend on the properties of the environment.

To verify our guess, notice that if informed agents are aware of uninformed agents' bias, their beliefs about next period payoff are given by:

$$\mathbb{E}_{I,t}[\Pi_{t+1}] = (1-\beta+\beta A_{t+1})(\mathcal{D}_t+\mathcal{W}_t)+\beta B_{t+1}\underbrace{\left(\frac{P_t-\tilde{B}_t(\bar{D}+\mu_0)+\tilde{K}_t}{\tilde{A}_t}\right)}_{\mathbb{E}_{I,t}[\tilde{\mathcal{D}}_t+\tilde{\mathcal{W}}_t]}-\beta K_{t+1} \quad \text{(B.4)}$$

$$\begin{aligned}\mathbb{V}_{I,t}[\Pi_{t+1}] = &(\beta A_{t+1})^2\sigma_u^2+\left(\beta A_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(\tau_s)^{-1}\\ &+\left(1-\beta+\beta A_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(t\tau_s+\tau_0)^{-1}=\mathbb{V}_{I,t}\end{aligned} \quad \text{(B.5)}$$

Turning to uninformed agents' beliefs:

$$\mathbb{E}_{U,t}[\Pi_{t+1}] = (1-\beta+\beta\tilde{A}_{t+1})(\tilde{\mathcal{D}}_{t-1}+\tilde{\mathcal{W}}_{t-1})+\beta\tilde{B}_{t+1}(\bar{D}+\mu_0)-\beta\tilde{K}_{t+1} \quad \text{(B.6)}$$

$$\begin{aligned}\mathbb{V}_{U,t}[\Pi_{t+1}] =&\mathbb{V}_{U,t}\left[\beta\tilde{A}_{t+1}\left(u_{t+1}+u_t+\frac{2\tau_s}{(t+1)\tau_s+\tau_0}\omega+\frac{\tau_s}{(t+1)\tau_s+\tau_0}(\epsilon_{t+1}+\epsilon_t)\right)+(1-\beta)(u_t+\omega)\right]\\ =&(\beta\tilde{A}_{t+1})^2\sigma_u^2+(1-\beta+\beta\tilde{A}_{t+1})^2\sigma_u^2\\ &+\left(1-\beta+\beta\tilde{A}_{t+1}\frac{2\tau_s}{(t+1)\tau_s}\right)^2((t-1)\tau_s+\tau_0)^{-1}\\ &+2\left(\frac{\tau_s\beta\tilde{A}_{t+1}}{(t+1)\tau_s+\tau_0}\right)^2(\tau_s)^{-1}=\mathbb{V}_{U,t}\end{aligned} \quad \text{(B.7)}$$

where $\mathbb{V}_{U,t}$ is deterministic and time-varying.

Given these beliefs, the true market clearing condition which generates the prices agents observe is given by:

$$\phi\left(\frac{\mathbb{E}_{I,t}[\Pi_{t+1}]-P_t}{\mathcal{A}\mathbb{V}_{I,t}[\Pi_{t+1}]}\right)+(1-\phi)\left(\frac{\mathbb{E}_{U,t}[\Pi_{t+1}]-P_t}{\mathcal{A}\mathbb{V}_{U,t}[\Pi_{t+1}]}\right)=Z \quad \text{(B.8)}$$

and the resulting market clearing price function is given by:

$$P_t = \left(\frac{\phi\mathbb{V}_{U,t}}{\phi\mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)\mathbb{E}_{I,t}[\Pi_{t+1}] + \left(\frac{(1-\phi)\mathbb{V}_{I,t}}{\phi\mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)\mathbb{E}_{U,t}[\Pi_{t+1}] - \frac{\mathcal{A}Z\mathbb{V}_{I,t}\mathbb{V}_{U,t}}{\phi\mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}} \quad \text{(B.9)}$$

Since (B.4), (B.5), (B.6) and (B.7) show that $\mathbb{E}_{I,t}[\Pi_{t+1}]$ is linear in $(\mathcal{D}_t + \mathcal{W}_t)$ and $(\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1})$, $\mathbb{E}_{U,t}[\Pi_{t+1}]$ is linear in $(\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1})$, and that $\mathbb{V}[\Pi_{t+1}]$ and $\mathbb{V}[\Pi_{t+1}]$ are deterministic, we see that the true price function does indeed take the form in (B.3). Substituting (B.4), (B.5), (B.6) and (B.7) into (B.9), and matching coefficients, yields:

$$A_t = \left(\frac{\frac{\phi}{V_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right) + \frac{(1-\phi)}{\mathbb{V}_{U,t}}}\right)(1-\beta+\beta A_{t+1}) \quad \text{(B.10)}$$

$$B_t = \left(\frac{\frac{1-\phi}{V_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right) + \frac{(1-\phi)}{\mathbb{V}_{U,t}}}\right)(1-\beta+\beta\tilde{A}_{t+1}) \quad \text{(B.11)}$$

$$K_t = \left(\frac{\frac{\phi}{V_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right) + \frac{(1-\phi)}{\mathbb{V}_{U,t}}}\right)\left(\beta K_{t+1} + \beta\frac{B_{t+1}}{\tilde{A}_t}\left(-\tilde{B}_t(\bar{D}+\mu_0) + \tilde{K}_t\right)\right) + \left(\frac{\frac{1-\phi}{V_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right) + \frac{(1-\phi)}{\mathbb{V}_{U,t}}}\right)\left(-\beta\tilde{B}_{t+1}(\bar{D}+\mu_0) + \beta\tilde{K}_{t+1}\right) + \frac{\mathcal{A}Z}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right) + \frac{(1-\phi)}{\mathbb{V}_{U,t}}} \quad \text{(B.12)}$$

These expressions give recursive equations for the coefficients which determine equilibrium prices at each point in time. To solve for this mapping, we then need to solve the model by backward induction. We can do this by using the new steady state after the uncertainty generated by the displacement is resolved as the end point. Specifically, the new steady state is given by:

$$A' = \left(\frac{\frac{\phi}{\mathbb{V}'_I}}{\frac{\phi}{\mathbb{V}'_I}\left(1 - \beta\frac{B'}{\tilde{A}'}\right) + \frac{1-\phi}{\mathbb{V}'_U}} \right)(1 - \beta + \beta A') \tag{B.13}$$

$$B' = \left(\frac{\frac{1-\phi}{\mathbb{V}'_U}}{\frac{\phi}{\mathbb{V}'_I}\left(1 - \beta\frac{B'}{\tilde{A}'}\right) + \frac{1-\phi}{\mathbb{V}'_U}} \right)(1 - \beta + \beta \tilde{A}') \tag{B.14}$$

$$\begin{aligned} K' = & \left(\frac{\frac{\phi}{\mathbb{V}'_I}}{\frac{\phi}{\mathbb{V}'_I}\left(1 - \beta\frac{B'}{\tilde{A}'}\right) + \frac{1-\phi}{\mathbb{V}'_U}} \right)\left(\beta K' + \beta\frac{B'}{\tilde{A}'}\left(-\tilde{B}'(\bar{D} + \mu_0) + \tilde{K}'\right)\right) \\ & + \left(\frac{\frac{1-\phi}{\mathbb{V}'_U}}{\frac{\phi}{\mathbb{V}'_I}\left(1 - \beta\frac{B'}{\tilde{A}'}\right) + \frac{1-\phi}{\mathbb{V}'_U}} \right)\left(-\beta\tilde{B}'(\bar{D} + \mu_0) + \beta\tilde{K}'\right) \\ & + \frac{\mathcal{A}Z}{\frac{\phi}{\mathbb{V}'_I}\left(1 - \beta\frac{B'}{\tilde{A}'}\right) + \frac{1-\phi}{\mathbb{V}'_U}} \end{aligned} \tag{B.15}$$

where $\tilde{A}'$, $\tilde{B}'$ and $\tilde{K}'$ are the coefficients of the mapping PET agents use to extract information from prices in the new steady state, and which we solve for in (B.28), (B.29) and (B.30) in the next section respectively. Moreover, $\mathbb{V}'_I$ and $\mathbb{V}'_U$ are the variances of informed and uninformed agents in the new steady state when uncertainty is resolved:

$$\mathbb{V}'_I = \lim_{t\to\infty} \mathbb{V}_{I,t} = (\beta A')^2\sigma_u^2 \tag{B.16}$$

$$\mathbb{V}'_U = \lim_{t\to\infty} \mathbb{V}_{U,t} = (\beta \tilde{A}')^2\sigma_u^2 + (1 - \beta + \beta\tilde{A}')^2\sigma_u^2 \tag{B.17}$$

Using this steady state as our end point, we can then solve for the true price function which generates the prices agents observe by backward induction.

Step 2: Mapping to Infer Information from Prices. Just as in the baseline model without speculation, PET agents think that in period t informed agents trade on the information they have received so far, $\{u_j\}_{j=1}^t$, $\{s_j\}_{j=1}^t$, and that uninformed agents only trade on their prior beliefs. Therefore, we can guess their beliefs about the equilibrium price function takes the following form:

$$P_t = \tilde{A}_t(\tilde{\mathcal{D}}_t + \tilde{\mathcal{W}}_t) + \tilde{B}_t(\bar{D} + \mu_0) - \tilde{K}_t \tag{B.18}$$

where $\tilde{A}_t$, $\tilde{B}_t$ and $\tilde{K}_t$ are time-varying and deterministic coefficients.

To verify that this is the price function which would arise in equilibrium if agents traded on their own private information alone, notice that, given this price function, informed agents' beliefs would take the following form:

$$\begin{aligned}\tilde{\mathbb{E}}_{I,t}[\Pi_{t+1}] =& \tilde{\mathbb{E}}_{I,t}[\beta \left(\tilde{A}_{t+1}(\tilde{\mathcal{D}}_{t+1} + \tilde{\mathcal{W}}_{t+1}) + \tilde{B}_{t+1}(\bar{D} + \mu_0) - \tilde{K}_{t+1}\right) + (1-\beta)(\tilde{\mathcal{D}}_t + \tilde{\omega})] \\ =& (1-\beta+\beta\tilde{A}_{t+1})(\tilde{\mathcal{D}}_t + \tilde{\mathcal{W}}_t) + \beta\tilde{B}_{t+1}(\bar{D}+\mu_0) - \beta\tilde{K}_{t+1} \qquad \text{(B.19)}\end{aligned}$$

$$\begin{aligned}\tilde{\mathbb{V}}_{I,t}[\Pi_{t+1}] =& \tilde{\mathbb{V}}_{I,t}\left[\beta\tilde{A}_{t+1}\tilde{u}_{t+1} + \beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s + \tau_0}\right)(\tilde{\omega} + \tilde{\epsilon}_{t+1}) + (1-\beta)\tilde{\omega}\right] \\ =& \left(\beta\tilde{A}_{t+1}\right)^2\sigma_u^2 + \left(\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(\tau_s)^{-1} \\ &+ \left(1-\beta+\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(t\tau_s+\tau_0)^{-1} = \tilde{\mathbb{V}}_{I,t} \qquad \text{(B.20)}\end{aligned}$$

where $\tilde{\mathbb{V}}_{I,t}$ is time-varying and deterministic. Turning to PET agents' beliefs of other uninformed agents' beliefs:

$$\tilde{\mathbb{E}}_{U,t}[\Pi_{t+1}] = (1-\beta+\beta\tilde{A}_{t+1} + \beta\tilde{B}_{t+1})(\bar{D}+\mu_0) - \beta\tilde{K}_{t+1} \tag{B.21}$$

$$\begin{aligned}\tilde{\mathbb{V}}_{U,t}[\Pi_{t+1}] =& \tilde{\mathbb{V}}_{I,t}\left[\beta\tilde{A}_{t+1}(\tilde{u}_{t+1} + \tilde{u}_t) + \beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)(2\tilde{\omega} + \tilde{\epsilon}_t + \tilde{\epsilon}_{t+1}) + (1-\beta)(\tilde{u}_t + \tilde{\omega})\right] \\ =& \left(\beta\tilde{A}_{t+1}\right)^2\sigma_u^2 + \left(1-\beta+\beta\tilde{A}_{t+1}\right)^2\sigma_u^2 + 2\left(\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(\tau_s)^{-1} \\ &+ \left(1-\beta+2\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(\tau_0)^{-1} = \tilde{\mathbb{V}}_{U,t} \qquad \text{(B.22)}\end{aligned}$$

where $\mathbb{V}_{U,t}$ is time-varying and deterministic. Notice that we assume that PET agents think other uninformed agents are only uncertain about future (and not past) shocks.

Given these beliefs, the market clearing condition which PET agents think is generating the price that they observe is given by:

$$\phi \left(\frac{\tilde{\mathbb{E}}_{I,t}[\Pi_{t+1}] - P_t}{\mathcal{A}\tilde{\mathbb{V}}_{I,t}[\Pi_{t+1}]} \right) + (1-\phi) \left(\frac{\tilde{\mathbb{E}}_{U,t}[\Pi_{t+1}] - P_t}{\mathcal{A}\tilde{\mathbb{V}}_{U,t}[\Pi_{t+1}]} \right) = Z \tag{B.23}$$

and the resulting market clearing price function is given by:

$$\begin{aligned} P_t = \left(\frac{\phi \tilde{\mathbb{V}}_{U,t}}{\phi \tilde{\mathbb{V}}_{U,t} + (1-\phi)\tilde{\mathbb{V}}_{I,t}} \right) \tilde{\mathbb{E}}_{I,t}[\Pi_{t+1}] \\ + \left(\frac{(1-\phi)\tilde{\mathbb{V}}_{I,t}}{\phi \tilde{\mathbb{V}}_{U,t} + (1-\phi)\tilde{\mathbb{V}}_{I,t}} \right) \tilde{\mathbb{E}}_{U,t}[\Pi_{t+1}] \\ - \frac{\mathcal{A}Z\tilde{\mathbb{V}}_{I,t}\tilde{\mathbb{V}}_{U,t}}{\phi \tilde{\mathbb{V}}_{U,t} + (1-\phi)\tilde{\mathbb{V}}_{I,t}} \end{aligned} \tag{B.24}$$

Since (B.19), (B.20), (B.21) and (B.22) show that $\tilde{\mathbb{E}}_{I,t}[\Pi_{t+1}]$ is linear in $(\tilde{\mathcal{D}}_t + \tilde{\mathcal{W}}_t)$ and $(\bar{D} + \mu_0)$, that $\tilde{\mathbb{E}}_{U,t}[\Pi_{t+1}]$ is linear in $(\bar{D} + \mu_0)$ and that $\tilde{\mathbb{V}}_{I,t+1}[\Pi_{t+1}]$ and $\tilde{\mathbb{V}}_{U,t+1}[\Pi_{t+1}]$ are deterministic, we see that given PET agents' beliefs about other agents, the price function which generates the prices they observe does indeed take the form in (B.18). Substituting (B.19), (B.20), (B.21) and (B.22) into (B.24), and matching coefficients yields:

$$\tilde{A}_t = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) (1 - \beta + \beta \tilde{A}_{t+1}) \tag{B.25}$$

$$\tilde{B}_t = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) \beta \tilde{B}_{t+1} + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) (1 - \beta + \beta \tilde{A}_{t+1} + \beta \tilde{B}_{t+1}) \tag{B.26}$$

$$\tilde{K}_t = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) \beta \tilde{K}_{t+1} + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) \beta \tilde{K}_{t+1} - \frac{\mathcal{A}Z}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \tag{B.27}$$

These expressions give recursive equations for the coefficients with determine equilibrium prices at each point in time. Therefore, to solve for this mapping, we need to solve the model by backward induction. We can do this by using the new steady state after the uncertainty generated by the displacement is resolved. Specifically, uninformed agents think that the new steady state is given by:

$$\tilde{A}' = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}'_I}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \right) (1 - \beta + \beta \tilde{A}') \tag{B.28}$$

$$\tilde{B}' = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}'_I}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \right) \beta \tilde{B}' + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}'_U}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \right) (1 - \beta + \beta \tilde{A}' + \beta \tilde{B}') \tag{B.29}$$

$$\tilde{K}' = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}'_I}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \right) \beta \tilde{K}' + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}'_U}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \right) \beta \tilde{K} - \frac{\mathcal{A} Z}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \tag{B.30}$$

where $\tilde{A}'$, $\tilde{B}'$ and $\tilde{K}'$ are PET agents' beliefs of the coefficients of the price function in the new steady state after the uncertainty associated with the displacement is resolved, and $\tilde{\mathbb{V}}'_I$ and $\tilde{\mathbb{V}}'_U$ are PET agents' beliefs of the variance of informed and uninformed agents in the new steady state when uncertainty is resolved:

$$\tilde{V}'_I = \lim_{t \to \infty} \tilde{\mathbb{V}}_{I,t} = (\beta \tilde{A})^2 \sigma_u^2 \tag{B.31}$$

$$\tilde{V}'_U = \lim_{t \to \infty} \tilde{\mathbb{V}}_{U,t} = (\beta \tilde{A})^2 \sigma_u^2 + (1 - \beta + \beta \tilde{A})^2 \sigma_u^2 + (1 - \beta)^2 (\tau_0)^{-1} \tag{B.32}$$

Using this steady state as our end point, we can then solve for the mapping uninformed agents use to extract information from prices by backward induction.

Given this mapping, uninformed agents extract the following information from prices:

$$\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1} = \frac{P_{t-1} - \tilde{B}_{t-1}(\bar{D} + \mu_0) + \tilde{K}_{t-1}}{\tilde{A}_{t-1}} \tag{B.33}$$

Or, given their information set in period t, they extract the following *new information* from the unexpected price change they observe in period $t-1$:

$$\tilde{u}_{t-1} + \tilde{w}_{t-1} = \frac{1}{\tilde{A}_{t-1}} (P_{t-1} - \mathbb{E}_{U,t-1}[P_{t-1}]) \tag{B.34}$$

where $\tilde{w}_{t-1} = \tilde{\mathcal{W}}_{t-1} - \tilde{\mathcal{W}}_{t-2}$. This verifies our claim in the text that PET agents extrapolate unexpected price changes even when we allow for speculative motives.

Step 3: Solving the Model Recursively. We solve for the normal times steady state before the displacement is announced by solving the system of equations in (B.28), (B.29), (B.30) and (B.13), (B.14), (B.15), using the following normal times variances:

$$\tilde{\mathbb{V}}_I = (\beta\tilde{A})^2\sigma_u^2 \tag{B.35}$$

$$\tilde{\mathbb{V}}_U = (\beta\tilde{A})^2\sigma_u^2 + (1-\beta+\beta\tilde{A})^2\sigma_u^2 \tag{B.36}$$

$$\mathbb{V}_I = (\beta A)^2\sigma_u^2 \tag{B.37}$$

$$\mathbb{V}_U = (\beta\tilde{A})^2\sigma_u^2 + (1-\beta+\beta\tilde{A})^2\sigma_u^2 \tag{B.38}$$

Starting from the normal times steady state, we can then simulate the equilibrium path of our economy forward for a given set of signals.

B.2 "PET−unaware" Speculation - Mistakenly Rational

If informed agents are not omniscient, and instead mistakenly believe that the world is rational, and that uninformed agents are able to recover the correct information form prices, then their posterior beliefs in (B.4) should be replaced by:

$$\mathbb{E}_{I,t}[\Pi_{t+1}] \;=\; (1-\beta+\beta A_{t+1})(\mathcal{D}_t+\mathcal{W}_t)+\beta B_{t+1}(\mathcal{D}_t+\mathcal{W}_t)-\beta K_{t+1} \tag{B.39}$$

Following the same steps as in Section B.1 above, it follows that the equilibrium price becomes:

$$P_t = A_t(\mathcal{D}_t+\mathcal{W}_t)+B_t(\tilde{\mathcal{D}}_{t-1}+\tilde{\mathcal{W}}_{t-1})-K_t \tag{B.40}$$

where:

$$A_t = \left(\frac{\frac{\phi}{\mathbb{V}_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}}\right)(1-\beta+\beta A_{t+1}+\beta B_{t+1}) \tag{B.41}$$

$$B_t = \left(\frac{\frac{1-\phi}{\mathbb{V}_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}}\right)(1-\beta+\beta\tilde{A}_{t+1}) \tag{B.42}$$

$$K_t = \left(\frac{\frac{\phi}{\mathbb{V}_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}}\right)\beta K_{t+1} + \left(\frac{\frac{1-\phi}{\mathbb{V}_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}}\right)\left(-\beta\tilde{B}_{t+1}(\bar{D}+\mu_0)+\tilde{K}_{t+1}\right) + \frac{\mathcal{A}Z}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}} \tag{B.43}$$

Since the mapping used by PET agents to extract information from prices is unchanged relative to the one in Section B.1, we can use this alternative price function to simulate the path of equilibrium prices and beliefs by following the same steps as in Section B.1.

C Partially Revealing Prices

When prices are fully revealing, the extrapolation parameter used by PET agents is decreasing in informed agents' informational edge. In this section, we study how the extrapolation parameter changes if we allow for noise, so that prices are no longer fully revealing.

C.1 Stochastic Supply and Information Structure

To consider the effect of noise on PET agents' inference problem, we assume that the supply of the risky asset is stochastic, and given by $z_t \overset{iid}{\sim} N(Z, \sigma_z^2)$.

To illustrate the effect of noise in the simplest possible way, we assume that agents learn about the realization of the supply of the risky asset after two periods. In each period t, all agents are uncertain about $z_{t-j} \overset{iid}{\sim} N(Z, \sigma_z^2)$ for $j \leq 1$ and they know the realization of z_{t-h} for $h \geq 2$. Even though one period lagged prices are partially revealing, this assumption makes prices fully revealing at further lags, thus simplifying PET agents' inference problem.

C.2 Inference Problem with Noise

When prices are fully revealing, uninformed agents think they can extract from prices the exact information that informed agents received in the previous period. This is no longer true when prices are partially revealing. When this is the case uninformed agents can only infer a noisy signal of fundamentals from prices.

Specifically, in normal times, uninformed agents think that prices take the following form:

$$P_{t-1} = \tilde{a}\left(\tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1}\right) + \tilde{b}\bar{D} - \tilde{c}z_{t-1} \tag{C.1}$$

where $\tilde{a} = \frac{\phi\tilde{\tau}_I}{\phi\tilde{\tau}_I+(1-\phi)\tilde{\tau}_U}$, $\tilde{b} = \frac{(1-\phi)\tilde{\tau}_U}{\phi\tilde{\tau}_I+(1-\phi)\tilde{\tau}_U}$ and $\tilde{c} = \frac{\mathcal{A}}{\phi\tilde{\tau}_I+(1-\phi)\tilde{\tau}_U}$. Since prices are fully revealing in period $t-2$, but they are partially revealing in period $t-1$, uninformed agents extract the following noisy signal from prices:[17]

$$\frac{P_{t-1} - \tilde{a}\tilde{D}_{t-2} - \tilde{b}\bar{D} + \tilde{c}Z}{\tilde{a}} = \tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}\left(z_{t-1} - Z\right) \tag{C.2}$$

and we can re-write this more simply as:

$$\left(\frac{1}{\tilde{a}}\right)\left(P_{t-1} - \mathbb{E}_{t-1}[P_{t-1}]\right) = \tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}\left(z_{t-1} - Z\right) \tag{C.3}$$

This shows that uninformed agents are now uncertain as to whether the unexpected price change they observe is due to new information, or to changes in the stochastic supply of the risky asset. Either way, PET agents still extrapolate past prices to recover a (noisy) signal from them.

Given the noisy information that uninformed agents extract from prices, their beliefs in period t are given by:

$$\mathbb{E}_{U,t}[D_T] = \tilde{D}_{t-2} + \left(\frac{\sigma_u^2}{\sigma_u^2 + \left(\frac{\tilde{c}}{\tilde{a}}\right)^2 \sigma_z^2}\right)\left(\frac{1}{\tilde{a}}\right)\left(P_{t-1} - \mathbb{E}_{U,t-1}[P_{t-1}]\right) \tag{C.4}$$

$$= \tilde{D}_{t-2} + \frac{\kappa}{\tilde{a}}\left(P_{t-1} - \mathbb{E}_{U,t-1}[P_{t-1}]\right) \tag{C.5}$$

where $\kappa = \left(\frac{\sigma_u^2}{\sigma_u^2+\left(\frac{\tilde{c}}{\tilde{a}}\right)^2\sigma_z^2}\right) \leq 1$ is the weight that PET agents put on the noisy signal they extract from past prices. This shows that the extrapolation parameter θ now depends on

[17] The assumption that prices are fully revealing in period $t-2$ means that uninformed agents think they know the exact value of $\tilde{\mathbb{E}}_{I,t-2}[D_T] = \tilde{D}_{t-2}$, as opposed to being uncertain about it.

two components:

$$\theta \equiv \frac{\kappa}{\tilde{a}} = \underbrace{\left(\frac{\sigma_u^2}{\sigma_u^2 + \left(\frac{1}{\phi\tilde{\tau}_I}\right)^2 \sigma_z^2}\right)}_{\text{weight}} \underbrace{\left(1 + \left(\frac{1-\phi}{\phi}\right)\frac{\tilde{\tau}_U}{\tilde{\tau}_I}\right)}_{\text{inference}} \tag{C.6}$$

where $(\tilde{\tau}_U)^{-1} = \left(\frac{1}{1-\beta^2}\right)\sigma_u^2 = (\tilde{\tau}_I)^{-1} + \sigma_u^2$ and $(\tilde{\tau}_I)^{-1} = \left(\frac{\beta^2}{1-\beta^2}\right)\sigma_u^2$. Starting from the second component in (C.6), $1/\tilde{a}$ is the extrapolation parameter that would prevail if $\sigma_z^2 = 0$ and prices were fully revealing: the more sensitive prices are to shocks, the less strongly do PET agents need to extrapolate unexpected price changes to recover the (in their mind unbiased) noisy signal $\tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}(z_{t-1} - Z)$ from prices. Turning to the first component in (C.6), $\kappa \leq 1$ is the weight that PET agents put on the information they extract from prices when forming their posterior beliefs. Whenever $\sigma_z^2 > 0$, $\kappa < 1$, and PET agents extrapolate prices less strongly than when prices are fully revealing, and this simply reflects the noisy nature of the signal they are able to infer from prices.

To draw comparative statics, we can substitute the expressions for $\tilde{\tau}_I$ and $\tilde{\tau}_U$ into (C.6), and re-write the extrapolation parameter in terms of the primitives of the model:

$$\theta = \frac{\kappa}{\tilde{a}} = \underbrace{\left(\frac{1}{1 + \left(\frac{1}{\phi}\right)^2 \left(\frac{\beta^2}{1-\beta^2}\right)^2 \sigma_u^2 \sigma_z^2}\right)}_{\text{weight}} \underbrace{\left(1 + \left(\frac{1-\phi}{\phi}\right)\beta^2\right)}_{\text{inference}} \tag{C.7}$$

From this expression, we see that the extrapolation parameter is decreasing in all sources of noise (σ_u^2 and σ_z^2), as this reduces the informativeness of the signal uninformed agents extract from prices.

On the other hand, increasing the perceived information advantage ($1/\beta^2$) and the fraction of informed agents in the market (ϕ) both have two competing roles. Increasing $1/\beta^2$ (or ϕ) decreases the fully revealing extrapolation parameter $1/\tilde{a}$ as prices are more sensitive to news, but it also increases the weight κ, as prices are a more informative signal. For small enough noise, the first effect dominates, and the extrapolation parameter is decreasing in the informational edge, and in the fraction of informed agents in the market. On the other hand, if there is too much noise in prices, the second effect dominates and the comparative statics are reversed.

D Normal Times and Displacements in Other Setups

In this section, we consider alternative setups to study how partial equilibrium thinking leads to momentum and reversals following a temporary shock, and to show how the results we uncovered in our main model are robust to altering the setup.

D.1 Temporary Shocks

D.1.1 Setup

Assets. Consider an economy where agents are solving a portfolio choice problem between a risky and a riskless asset. The risk-free asset is in zero net supply, and we normalize its price and risk free rate to one, $P_f = R_f = 1$. The risky asset is in fixed net supply Z, and pays off a stream of dividends v_t each period.

$$v_t = (1-\rho)\bar{v} + \rho v_{t-1} + u_t \tag{D.1}$$

where $\bar{v}$ is the unconditional mean of the fundamental value of the asset, $\rho \in [0,1]$ is the persistence coefficient, and $u_t \sim N(0, \tau_u^{-1})$.

Agents and Preferences. There is a continuum of measure one of agents. All agents live for one period. There are no bequest motives, so agents are myopic. Moreover, we assume that all agents are only concerned with forecasting the fundamental value of the asset, so that at time t they have the following demand function for the risky asset:

$$X_{it} = \frac{\mathbb{E}_{it}[v_{t+1}] - P_t}{AVar_{it}[v_{t+1}]} \tag{D.2}$$

where $\mathbb{E}_{it}[\cdot]$ and $Var_{it}[\cdot]$ characterize agent i's beliefs about next period fundamental payoff given the information set they have at time t. Notice that capital gains don't show up in agents' demand functions. While we could extend this framework to allow for speculative motives, we make this assumption to study the basic mechanism in the simplest possible framework. Moreover, we assume that uninformed agents do not observe the history of v_t and they only observe their own realized payoff once they leave the market in period $t+1$.

Information Structure in Normal Times. All agents know $\bar{v}$, as well as all other parameters of the unconditional distribution of v_t and u_t. Moreover, a fraction ϕ of agents are informed, and they observe the whole history u_j for $j \leq t$ before making their portfolio choice in each period. A fraction $(1-\phi)$ of agents are uninformed, and they do not observe u_t, v_t nor their history. However, they can learn information from past prices.

Equilibrium. In equilibrium, uninformed agents' beliefs must be consistent with past prices they observe, given their model of the world. Moreover, all agents trade according to their demand functions in (D.69) given their beliefs, and markets clear. This gives the following price function, conditional on agents' beliefs:

$$P_t = a_t \mathbb{E}_{I,t}[v_{t+1}] + b_t \mathbb{E}_{U,t}[v_{t+1}] - c_t \tag{D.3}$$

where $a_t \equiv \left(\frac{\phi \mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $b_t \equiv \left(\frac{(1-\phi)\mathbb{V}_{I,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$ and $c_t \equiv \left(\frac{\mathbb{V}_{I,t}\mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right) AZ$ $\mathbb{V}_{i,t} = \mathrm{Var}_{i,t}[v_{t+1}]$ for $i \in \{I, U\}$. Therefore, in order to find the equilibrium price, we need to pin down informed and uninformed agents' beliefs about v_{t+1}.

D.1.2 Normal Times

Informed Agents' Beliefs. Informed agents' beliefs are simply given by:

$$\mathbb{E}_{I,t}[v_{t+1}] = (1-\rho)\bar{v} + \rho v_t \tag{D.4}$$

$$\mathbb{V}_{I,t}[v_{t+1}] = \sigma_u^2 \tag{D.5}$$

Uninformed Agents' Beliefs. To compute uninformed agents' beliefs, we start by determining what information they extract from past prices.

Misspecified Mapping used to Extract Information from Past Prices. To construct this mapping, we need to write down uninformed agents' beliefs of the price function which generates the prices they observe. This, in turn, requires us to specify uninformed agents' beliefs of other agents' beliefs about next period fundamentals. We denote by $\tilde{\cdot}$ uninformed agents' beliefs about a variable. When agents think in partial equilibrium, they think that informed agents hold the following posterior beliefs:

$$\tilde{\mathbb{E}}_{I,t-1}[v_t] = (1-\rho)\bar{v} + \rho\tilde{v}_{t-1} \tag{D.6}$$

$$\tilde{\mathbb{V}}_{I,t-1}[v_t] = \sigma_u^2 \tag{D.7}$$

Moreover, PET agents think that all other uninformed agents do not learn information from prices, and instead trade on the unconditional mean and variance:

$$\tilde{\mathbb{E}}_{U,t-1}[v_t] = \bar{v} \tag{D.8}$$

$$\tilde{\mathbb{V}}_{U,t-1}[v_t] = \frac{\sigma_u^2}{1-\rho^2} \tag{D.9}$$

Substituting these expressions into (D.3), we obtain the price function which uninformed agents think is generating the price that they observe.

$$P_{t-1} = a^{CE}\left((1-\rho)\bar{v} + \rho\tilde{v}_{t-1}\right) + b^{CE}\bar{v} - c^{CE} \tag{D.10}$$

where $a^{CE} \equiv \frac{\phi\tilde{\mathbb{V}}_{U,t}}{\phi\tilde{\mathbb{V}}_{U,t}+(1-\phi)\tilde{\mathbb{V}}_{I,t}} = \frac{\phi\left(\frac{\sigma_u^2}{1-\rho^2}\right)}{\phi\left(\frac{\sigma_u^2}{1-\rho^2}\right)+(1-\phi)\sigma_u^2}$, $b^{CE} \equiv \frac{(1-\phi)\tilde{\mathbb{V}}_{I,t}}{\phi\tilde{\mathbb{V}}_{U,t}+(1-\phi)\tilde{\mathbb{V}}_{I,t}} = \frac{(1-\phi)\sigma_u^2}{\phi\left(\frac{\sigma_u^2}{1-\rho^2}\right)+(1-\phi)\sigma_u^2}$, $c^{CE} \equiv \frac{\tilde{\mathbb{V}}_{I,t-1}\tilde{\mathbb{V}}_{U,t-1}}{\phi\tilde{\mathbb{V}}_{U,t}+(1-\phi)\tilde{\mathbb{V}}_{I,t}}AZ = \frac{\sigma_u^2\left(\frac{\sigma_u^2}{1-\rho^2}\right)}{\phi\left(\frac{\sigma_u^2}{1-\rho^2}\right)+(1-\phi)\sigma_u^2}AZ$.[18] Therefore, uninformed agents invert (D.10) to extract the following information from prices:

$$(1-\rho)\bar{v} + \rho\tilde{v}_{t-1} = \frac{1}{a^{CE}}P_{t-1} - \frac{b^{CE}}{a^{CE}}\bar{v} + \frac{c^{CE}}{a^{CE}} \tag{D.11}$$

[18] Throughout Appendix D, we use "CE" to stand for Cursed Equilibrium (Eyster and Rabin (2005)): cursed agents do not learn information from prices, and instead trade on their private information alone. In this respect, PET agents think that all other agents are cursed. Therefore PET agents invert the cursed equilibrium price function to infer information from prices.

Uninformed Agents' Beliefs. Having determined what information uninformed agents extract from past prices they observe, we can compute their beliefs:

$$\begin{aligned}\mathbb{E}_{U,t}[v_{t+1}] =& (1-\rho)\bar{v} + \rho\left((1-\rho)\bar{v} + \rho\tilde{v}_{t-1}\right) & \text{(D.12)}\\ =& \left(\frac{\rho}{a^{CE}}\right) P_{t-1} + \left(1 - \rho - \frac{\rho b^{CE}}{a^{CE}}\right)\bar{v} + \frac{\rho c^{CE}}{a^{CE}} & \text{(D.13)}\end{aligned}$$

$$\mathbb{V}_{U,t}[v_{t+1}] = (1+\rho^2)\sigma_u^2 \tag{D.14}$$

So uninformed agents' beliefs resemble some form of extrapolation:

$$\mathbb{E}_{U,t}[v_{t+1}] = \theta_1 P_{t-1} + \theta_2 \tag{D.15}$$

where:

$$\theta_1 = \frac{\rho}{a^{CE}} \tag{D.16}$$

Equilibrium. Substituting agents' beliefs in (D.4), (D.5), (D.13), (D.14) into (D.3), we obtain the path of equilibrium prices:

$$P_t = \left(\frac{b\rho}{a^{CE}}\right) P_{t-1} + a(1-\rho)(v_t - \bar{v}) + \bar{P}\left(1 - \frac{b\rho}{a^{CE}}\right) \tag{D.17}$$

where $a \equiv \frac{\phi \mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}} = \frac{\phi(1+\rho^2)\sigma_u^2}{\phi(1+\rho^2)\sigma_u^2 + (1-\phi)\sigma_u^2}$, $b \equiv \frac{(1-\phi)\mathbb{V}_{I,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}} = \frac{(1-\phi)\sigma_u^2}{\phi(1+\rho^2)\sigma_u^2 + (1-\phi)\sigma_u^2}$, $c \equiv \frac{\mathbb{V}_{U,t}\mathbb{V}_{I,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}} AZ = \frac{\sigma_u^2(1+\rho^2)\sigma_u^2}{\phi(1+\rho^2)\sigma_u^2 + (1-\phi)\sigma_u^2}$ and $\bar{P}$ is the unconditional mean of prices when agents think in partial equilibrium, and is such that $\bar{P}\left(1 - \frac{b\rho}{a^{CE}}\right) \equiv \left(a + b\left(1 - \rho - \frac{\rho b^{CE}}{a^{CE}}\right)\right)\bar{v} + b\frac{\rho c^{CE}}{a^{CE}} - c$. Let $\mathbb{L}$ denote the lag operator. Then, using the fact that $(v_t - \bar{v}) = (1-\rho\mathbb{L})^{-1}u_t$, and rearranging, we can re-write the dynamics of equilibrium prices as follows:

$$\left(P_t - \bar{P}\right) = \frac{a(1-\rho)}{(1-\rho\mathbb{L})\left(1 - \frac{b\rho}{a^{CE}}\mathbb{L}\right)} u_t \tag{D.18}$$

This makes clear that the equilibrium price follows an AR(2) process. Moreover, for this process to be stationary, we need the roots of the characteristic equation to lie outside the unit circle:

$$\rho < 1 \qquad\qquad \frac{b}{a^{CE}}\rho < 1 \tag{D.19}$$

Rational Expectations Equilibrium Comparison. We can compare the PET impulse response function to the impulse response function which would arise if agents had rational expectations and were able to extract the correct information from past prices.

In this case, informed agents' beliefs are as in (D.4) and (D.13), while uninformed agents' beliefs are as follows:

$$\mathbb{E}_{U,t}[v_{t+1}] = (1-\rho^2)\bar{v} + \rho^2 v_{t-1} \tag{D.20}$$

and with the same conditional variance as in (D.14). Substituting these beliefs into (D.3), we get the following expression for the path of equilibrium prices:

$$P_t = a((1-\rho)\bar{v} + \rho v_t) + b((1-\rho)\bar{v} + \rho v_{t-1}) - c \tag{D.21}$$

$$= a\rho(v_t - \bar{v}) + b\rho(v_{t-1} - \bar{v}) + (a+b)\bar{v} - c \tag{D.22}$$

We can rewrite this as:

$$(P_t - \bar{P}) = \frac{a\rho\left(1 - \frac{b}{a}\mathbb{L}\right)}{1-\rho\mathbb{L}} u_t \tag{D.23}$$

Therefore, with rational expectations, the equilibrium price follows an ARMA(1,1). Moreover, stationarity of an ARMA process depends entirely on the autoregressive parameters, and not on the moving average parameters. Specifically, whenever the roots of $(1-\rho z) = 0$ lie outside the unit circle, this system is stationary. In other words, whenever $\rho < 1$, the rational expectations equilibrium is stationary, while this was not enough to guarantee stationarity of the price dynamics when agents think in partial equilibrium.

Simulation. We simulate the CE, REE and PET equilibrium. We start all three cases from a steady state with $v_0 = \bar{v}$, such that uninformed agents' beliefs are consistent with the prices they observe.

Steady State. For the REE and CE equilibrium concepts, uninformed agents' beliefs in steady state are simply equal to $\mathbb{E}_{U,0}[v_1] = \bar{v}$. On the other hand, for PET agents' beliefs to be consistent with the steady state price they observe, it must be that the steady state extracted fundamental $\tilde{v}_{ss}$ satisfies both these expressions:

$$P_0^{PET} = a\bar{v} + b\left((1-\rho)\bar{v} + \rho\left((1-\rho)\bar{v} + \rho\tilde{v}_{ss}\right)\right) \tag{D.24}$$

$$P_0^{CE} = a^{CE}\left((1-\rho)\bar{v} + \rho\tilde{v}_{ss}\right) + b^{CE}\bar{v} - c^{CE} \tag{D.25}$$

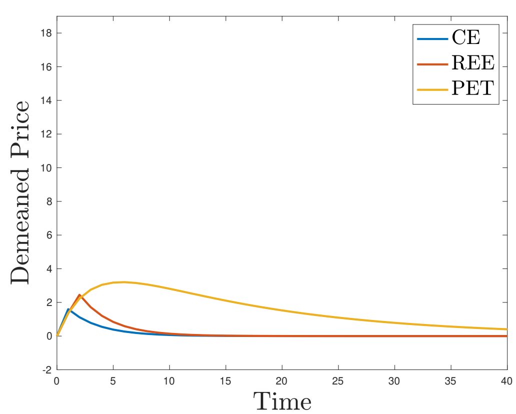
so that:

$$(1-\rho)\bar{v} + \rho\tilde{v}_{ss} = \frac{\left(a + b(1-\rho) - b^{CE}\right)\bar{v} - c + c^{CE}}{a^{CE} - b\rho} \tag{D.26}$$

$$\mathbb{E}_{U,0}^{PET}[v_1] = (1-\rho)\bar{v} + \rho\left(\frac{\left(a + b(1-\rho) - b^{CE}\right)\bar{v} - c + c^{CE}}{a^{CE} - b\rho}\right) \tag{D.27}$$

Impulse Response Function. We then shock the economy in period 1 with $u_1 = 5$ and $u_t = 0$ for $t > 1$, and we compute the impulse response function for each equilibrium concept. We plot the demeaned price path to study the response to shocks while taking into account the difference in steady states.

Figure 10: Normal Times Demeaned Price Path. Impulse response function following a shock to the fundamental value of the asset $u_1 = 5$.



This impulse response function shows PET's ability to generate momentum and reversal to "normal-times" shocks.

D.1.3 Displacement

Information Structure after a Displacement. Kindleberger-style displacements are associated with periods of uncertainty about long term outcomes, and this uncertainty gradually resolves over time. We model a displacement as an unanticipated and uncertain shock to the unconditional mean of the fundamental value of the asset. Specifically, we can write the evolution of the fundamental value of the asset as follows:

$$\begin{cases} v_t = (1-\rho)\bar{v} + \rho v_{t-1} + u_t & \text{if } t \leq 0 \\ v_t = (1-\rho)(\bar{v} + \omega) + \rho v_{t-1} + u_t & \text{if } t > 0 \end{cases} \tag{D.28}$$

When the displacement is "announced" in period $t = 0$, all agents have the same prior unconditional distribution, $\omega \sim (\mu_0, \tau_0^{-1})$. Starting from period $t = 1$ informed agents receive a signal $s_t = \omega + \epsilon_t$, with $\epsilon_t \sim^{iid} N(0, \tau_s^{-1})$ each period, and they also continue to observe u_t. Uninformed agents do not observe these signals, and can still only learn information from past prices.

Starting from the steady state equilibrium, let the shock be announced in period $t = 0$, we can then write the evolution of the fundamental value of the asset as follows:

$$v_t = (1-\rho^t)(\bar{v} + \omega) + \rho^t v_0 + \sum_{j=0}^{t-1} \rho^j u_{t-j} \tag{D.29}$$

We can re-write this as:

$$v_t = (1-\rho^t)(\bar{v} + \omega) + \rho^t v_0 + U_{t-1} + u_t \tag{D.30}$$

where $U_{t-1} = \sum_{j=1}^{t-1} \rho^j u_{t-j}$.

Informed Agents' Beliefs. Informed agents' beliefs are given by:

$$\mathbb{E}_{I,t}[v_{t+1}] = (1-\rho^{t+1}) \left(\bar{v} + \underbrace{\left(\frac{t\tau_s}{t\tau_s + \tau_0} S_t + \frac{\tau_0}{t\tau_s + \tau_0} \mu_0 \right)}_{\mathbb{E}_{I,t}[\omega]} \right) + \rho^{t+1} v_0 + U_t \tag{D.31}$$

$$\mathbb{V}_{I,t}[v_{t+1}] = (1-\rho^{t+1})^2 \underbrace{(t\tau_s+\tau_0)^{-1}}_{\mathbb{V}_{I,t}[\omega]} + \sigma_u^2 \tag{D.32}$$

where $S_t \equiv \sum_{j=1}^{t} s_j$, and since $s_j = \omega + \epsilon_j$, we can re-write this as a stationary AR(1) process with mean ω and AR(1) coefficient $\left(\frac{t-1}{t}\right)$: $(S_t - \omega) = \frac{1}{t\left(1-\left(\frac{t-1}{t}\right)\mathbb{L}\right)}\epsilon_t$.

Uninformed Agents' Beliefs. Turning to uninformed agent's beliefs, we proceed in the same two steps as when solving the model in normal times: first, we determine what unbiased signal uninformed agents extract from prices; second, we determine how they use this information to compute their forecasts about next period fundamentals.

Misspecified Mapping used to Extract Information from Past Prices. Unlike in normal times, uninformed agents now have to gain information about two shocks (u_t and ϵ_t) from prices, and both these shocks are incorporated into prices via informed agents' beliefs. Therefore, uninformed agents extract $\mathbb{E}_{i,t-1}[v_t]$ from P_{t-1}. To do so, they must form beliefs about what generates the prices they observe, which in turn requires them to from beliefs about all other agents' beliefs. Specifically, they correctly understand how informed agents form their beliefs:

$$\tilde{\mathbb{E}}_{I,t-1}[v_t] = (1-\rho^t)\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}S_t + \frac{\tau_0}{(t-1)\tau_s+\tau_0}\mu_0\right) + \rho^t v_0 + U_t \tag{D.33}$$

$$\tilde{\mathbb{V}}_{I,t-1}[v_t] = (1-\rho^t)^2((t-1)\tau_s+\tau_0)^{-1} + \sigma_u^2 \tag{D.34}$$

but they mistakenly think that all other uninformed agents do not infer information from prices:

$$\tilde{\mathbb{E}}_{U,t-1}[v_t] = (1-\rho^t)(\bar{v}+\mu_0) + \rho^t\bar{v} \tag{D.35}$$

$$\tilde{\mathbb{V}}_{U,t-1}[v_t] = (1-\rho^t)^2\tau_0^{-1} + \frac{\sigma_u^2}{1-\rho^2} \tag{D.36}$$

Given these beliefs, they think that market clearing prices are generated by:

$$P_{t-1} = a_{t-1}^{CE}\tilde{\mathbb{E}}_{I,t-1}[v_t] + b_{t-1}^{CE}\tilde{\mathbb{E}}_{U,t-1}[v_t] - c_t^{CE} \tag{D.37}$$

where $a_{t-1}^{CE} \equiv \frac{\phi\tilde{V}_{U,t-1}}{\phi\tilde{\mathbb{V}}_{U,t-1}+(1-\phi)\tilde{V}_{I,t-1}}$, $b_{t-1}^{CE} \equiv \frac{(1-\phi)\tilde{V}_{I,t-1}}{\phi\tilde{\mathbb{V}}_{U,t-1}+(1-\phi)\tilde{V}_{I,t-1}}$, $c_{t-1}^{CE} \equiv \frac{\tilde{V}_{U,t-1}\tilde{V}_{I,t-1}AZ}{\phi\tilde{\mathbb{V}}_{U,t-1}+(1-\phi)\tilde{V}_{I,t-1}}$, and where $\tilde{\mathbb{V}}_{I,t-1}[v_{t+1}]$ and $\tilde{\mathbb{V}}_{U,t-1}[v_{t+1}]$ are given by (D.34) and (D.36) respectively.

Importantly, notice that the mapping that uninformed agents use to extract information from prices is now time-varying (since a_{t-1}^{CE}, b_{t-1}^{CE} and c_{t-1}^{CE} are all time-varying). The time variation in these coefficients stems from the fact that uninformed agents understand that displacements generate changes in uncertainty.

Uninformed agents then invert this mapping to infer information from prices:

$$\tilde{\mathbb{E}}_{I,t-1}[v_t] = \frac{1}{a_{t-1}^{CE}} P_{t-1} - \frac{b_{t-1}^{CE}}{a_{t-1}^{CE}} \tilde{\mathbb{E}}_{U,t-1}[v_t] + \frac{c_t^{CE}}{a_{t-1}^{CE}} \tag{D.38}$$

Uninformed Agents' Beliefs. We are now left to pin down how uninformed agents update their beliefs given the information they extract from prices. For ease of notation, let $\tilde{v}_{t|t-1} \equiv \tilde{\mathbb{E}}_{I,t-1}[v_t]$ from (D.38). We can then write this as:

$$\tilde{v}_{t|t-1} = (1-\rho^t)\left(\bar{v} + \left(\frac{(t-1)\tau_s}{(t-1)\tau_s + \tau_0}\left(\omega + \frac{\sum_{j=1}^{t-1}\epsilon_j}{t-1}\right) + \frac{\tau_0}{(t-1)\tau_s + \tau_0}\mu_0\right)\right) + \rho^t v_0 + U_{t-1} \tag{D.39}$$

Uninformed agents' forecasts are then given by:

$$\mathbb{E}_{U,t}[v_{t+1}] = (1-\rho^{t+1})\left(\bar{v} + \mathbb{E}_{U,t}[\omega|\tilde{v}_{t|t-1}]\right) + \rho^{t+1}\tilde{v}_0 + \rho\mathbb{E}_{U,t}[U_{t-1}|\tilde{v}_{t|t-1}] \tag{D.40}$$

$$\mathbb{V}_{U,t}[v_{t+1}] = (1-\rho^{t+1})^2\mathbb{V}_{U,t}[\omega|\tilde{v}_{t|t-1}] + \rho^2\mathbb{V}_{U,t}[U_{t-1}|\tilde{v}_{t|t-1}] + 2(1-\rho^{t+1})\rho\mathrm{Cov}_{U,t}[\omega, U_{t-1}|\tilde{v}_{t|t-1}] + (1+\rho^2)\sigma_u^2 \tag{D.41}$$

where:

$$\mathbb{E}_{U,t}\begin{bmatrix}\omega \\ U_{t-1}\end{bmatrix} = \begin{bmatrix}\mathbb{E}[\omega] + \frac{\mathrm{Cov}(\omega,\tilde{v}_{t|t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})}\left(\tilde{v}_{t|t-1} - \mathbb{E}[\tilde{v}_{t|t-1}]\right) \\ \mathbb{E}[U_{t-1}] + \frac{\mathrm{Cov}(U_{t-1},\tilde{v}_{t|t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})}\left(\tilde{v}_{t|t-1} - \mathbb{E}[\tilde{v}_{t|t-1}]\right)\end{bmatrix} \tag{D.42}$$

$$\mathrm{Cov}_{U,t}\begin{bmatrix}\omega \\ U_{t-1}\end{bmatrix} = \begin{bmatrix}\mathrm{Var}(\omega) - \frac{\left(\mathrm{Cov}(\omega,\tilde{v}_{t|t-1})\right)^2}{\mathrm{Var}(\tilde{v}_{t|t-1})} & \mathrm{Cov}(w,U_{t-1}) - \frac{\mathrm{Cov}(\omega,\tilde{v}_{t|t-1})\mathrm{Cov}(U_{t-1},\tilde{v}_{t|t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})} \\ \mathrm{Cov}(w,U_{t-1}) - \frac{\mathrm{Cov}(\omega,\tilde{v}_{t|t-1})\mathrm{Cov}(U_{t-1},\tilde{v}_{t|t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})} & \mathrm{V}(U_{t-1}) - \frac{\left(\mathrm{Cov}(U_{t-1},\tilde{v}_{t|t-1})\right)^2}{\mathrm{Var}(\tilde{v}_{t|t-1})}\end{bmatrix} \tag{D.43}$$

and

$$\mathbb{E}\begin{bmatrix}\omega \\ U_{t-1} \\ \tilde{v}_{t|t-1}\end{bmatrix} = \begin{bmatrix}\mu_0 \\ 0 \\ (1-\rho^t)\mu_0 + \rho^t\tilde{v}_0\end{bmatrix} \tag{D.44}$$

$$\mathrm{Cov}\begin{bmatrix}\omega \\ U_{t-1} \\ \tilde{v}_{t|t-1}\end{bmatrix} = \begin{bmatrix} \tau_0^{-1} & 0 & (1-\rho^t)\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}\right)\tau_0^{-1} \\ 0 & \left(\frac{1-\rho^{2(t-1)}}{1-\rho^2}\rho^2\sigma_u^2\right) & \left(\frac{1-\rho^{2(t-1)}}{1-\rho^2}\rho^2\sigma_u^2\right) \\ (1-\rho^t)\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}\right)\tau_0^{-1} & \left(\frac{1-\rho^{2(t-1)}}{1-\rho^2}\rho^2\sigma_u^2\right) & (1-\rho^t)^2\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}\right)^2\left(\tau_0^{-1}+((t-1)\tau_s)^{-1}\right)+\left(\frac{1-\rho^{2(t-1)}}{1-\rho^2}\right)\rho^2\sigma_u^2 \end{bmatrix} \tag{D.45}$$

Therefore, we can write uninformed agents' beliefs as:

$$\mathbb{E}_{U,t}[v_{t+1}] = \theta_{1,t}P_{t-1} + \theta_{2,t} \tag{D.46}$$

where:

$$\theta_{1,t} = \left((1-\rho^{t+1})\frac{\mathrm{Cov}(\omega, \tilde{v}_{t|t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})} + \rho\frac{\mathrm{Cov}(\omega, U_{t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})}\right)\frac{1}{a_{t-1}^{CE}} \tag{D.47}$$

Equilibrium. Given agents' beliefs, equilibrium prices are given by:

$$\begin{aligned} P_t = C_t + \left((1-\rho^{t+1})\frac{\mathrm{Cov}(\omega, \tilde{v}_{t|t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})} + \rho\frac{\mathrm{Cov}(\omega, U_{t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})}\right)\frac{b_t}{a_{t-1}^{CE}}P_{t-1} \\ + a_t\left(\frac{t\tau_s(1-\rho^{t+1})}{t\tau_s+\tau_0}\right)\frac{1}{t\left(1-\left(\frac{t-1}{t}\right)\mathbb{L}\right)}\epsilon_t \end{aligned} \tag{D.48}$$

$$P_t = C_t + b_t\theta_{1,t}P_{t-1} + a_t\left(\frac{t\tau_s(1-\rho^{t+1})}{t\tau_s+\tau_0}\right)\frac{1}{t\left(1-\left(\frac{t-1}{t}\right)\mathbb{L}\right)}\epsilon_t \tag{D.49}$$

where C_t is deterministic. This resembles an AR(2) process, but this time with time-varying roots.

Rational Expectations Equilibrium Comparison. To solve for the rational expectations equilibrium, we compute similar steps as above, with the one difference that uninformed agents are able to recover $v_{t|t-1} = \mathbb{E}_{1,t-1}[v_t]$ from past prices.

Solving for the equilibrium price, we find that:

$$P_t^{REE} = C_t^{REE} + \left((1-\rho^{t+1})\frac{\mathrm{Cov}(\omega, \tilde{v}_{t|t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})} + \rho\frac{\mathrm{Cov}(\omega, U_{t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})}\right)b_t v_{t|t-1} + a_t v_{t+1|t-1} \tag{D.50}$$

$$P_t^{REE} = C_t^{REE} + a_t\left(1 - \left((1-\rho^{t+1})\frac{\mathrm{Cov}(\omega, \tilde{v}_{t|t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})} + \rho\frac{\mathrm{Cov}(\omega, U_{t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})}\right)\frac{b_t}{a_t}\mathbb{L}\right)v_{t+1|t} \tag{D.51}$$

$$\left(P_t^{REE}-\bar{P}\right)=\left(\frac{a_t(1-\rho^{t+1})t\tau_s}{t\tau_s+\tau_0}\right)\frac{\left(1-\left((1-\rho^{t+1})\frac{\mathrm{Cov}(\omega,\tilde{v}_{t|t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})}+\rho\frac{\mathrm{Cov}(\omega,U_{t-1})}{\mathrm{Var}(\tilde{v}_{t|t-1})}\right)\frac{b_t}{a_t}\mathbb{L}\right)}{t\left(1-\left(\frac{t-1}{t}\right)\mathbb{L}\right)}\epsilon_t \tag{D.52}$$

so that the REE equilibrium price resembles an ARMA(1,1) process with time-varying coefficients. Once again, notice that the AR roots are always less than one.

Impulse Response Function. We initiate the economy at the same steady state as in normal times. In period $t=0$, a displacement is announced, and all agents share the same unconditional distribution of the shock to the unconditional mean of the fundamental value of the asset: $\omega \sim N(\mu_0, \tau_0^{-1})$. Finally, starting in period $t=1$, informed agents receive a signal s_t which is informative about the fundamental value of the asset.

Period $t=0$. In period $t=0$ agents learn that starting next period the unconditional mean of the fundamental value of the asset is $\bar{v}+\omega$, where $\omega \sim N(\mu_0, \tau_0^{-1})$. For all equilibrium concepts, informed agents' posterior beliefs are given by:

$$\mathbb{E}_{I,0}[v_1]=(1-\rho)\left(\bar{v}+\mu_0\right)+\rho v_0 \tag{D.53}$$

$$\mathbb{V}_{I,0}[v_1]=(1-\rho)^2(\tau_0)^{-1}+\sigma_u^2 \tag{D.54}$$

Uninformed agents' posterior beliefs differ depending on the equilibrium concept:

$$\mathbb{E}_{U,0}[v_1]=(1-\rho)\left(\bar{v}+\mu_0\right)+\rho\left((1-\rho)\bar{v}+\rho\tilde{v}_{ss0}\right) \tag{D.55}$$

$$\mathbb{E}_{U,0}^{CE}[v_1]=\mathbb{E}_{U,0}^{REE}[v_1]=(1-\rho)\left(\bar{v}+\mu_0\right)+\rho\bar{v} \tag{D.56}$$

$$\mathbb{V}_{U,0}[v_1]=\mathbb{V}_{U,0}^{REE}[v_1]=(1-\rho)^2(\tau_0)^{-1}+(1+\rho^2)\sigma_u^2 \tag{D.57}$$

$$\mathbb{V}_{U,0}^{CE}[v_1]=(1-\rho)^2(\tau_0)^{-1}+\frac{\sigma_u^2}{1-\rho^2} \tag{D.58}$$

where $\tilde{v}_{ss0}$ is the same steady state as in the normal times case, in (D.26). Given these beliefs, we can construct P_0, P_0^{CE}, P_0^{REE} using (D.3), and we can also obtain the mapping that uninformed agents use to extract information from P_0 (this is given by the CE price function).

Period $t = 1$. Informed agents obtain s_1 and their posterior beliefs are given by:

$$\mathbb{E}_{I,1}[v_2] = (1-\rho^2)\left(\bar{v} + \frac{\tau_s}{\tau_s+\tau_0}S_1 + \frac{\tau_0}{\tau_s+\tau_0}\mu_0\right) + \rho^2 v_0 + \rho u_1 \tag{D.59}$$

$$\mathbb{V}_{I,t}[v_2] = (1-\rho^2)^2(\tau_s+\tau_0)^{-1} + \sigma_u^2 \tag{D.60}$$

Uninformed PET agents learn information about u_0 from P_0 by extracting $\tilde{v}_0$ from prices.

$$\mathbb{E}_{U,1}[v_2] = (1-\rho^2)(\bar{v}+\mu_0) + \rho^2\tilde{v}_0 \tag{D.61}$$

$$\mathbb{V}_{U,1}[v_2] = (1-\rho^2)^2(\tau_0)^{-1} + (1+\rho^2)\sigma_u^2 \tag{D.62}$$

where:

$$\tilde{v}_0 = \frac{P_0 - b_0^{CE}\mathbb{E}_{0,U}^{CE}[v_1] + c_0^{CE}}{a_0^{CE}} \tag{D.63}$$

Similarly, uninformed agents' beliefs for the CE and REE equilibrium concpets are given by:

$$\mathbb{E}_{U,1}^{REE}[v_2] = (1-\rho^2)(\bar{v}+\mu_0) + \rho^2 v_0 \tag{D.64}$$

$$\mathbb{V}_{U,1}^{REE}[v_2] = (1-\rho^2)^2(\tau_0)^{-1} + (1+\rho^2)\sigma_u^2 \tag{D.65}$$

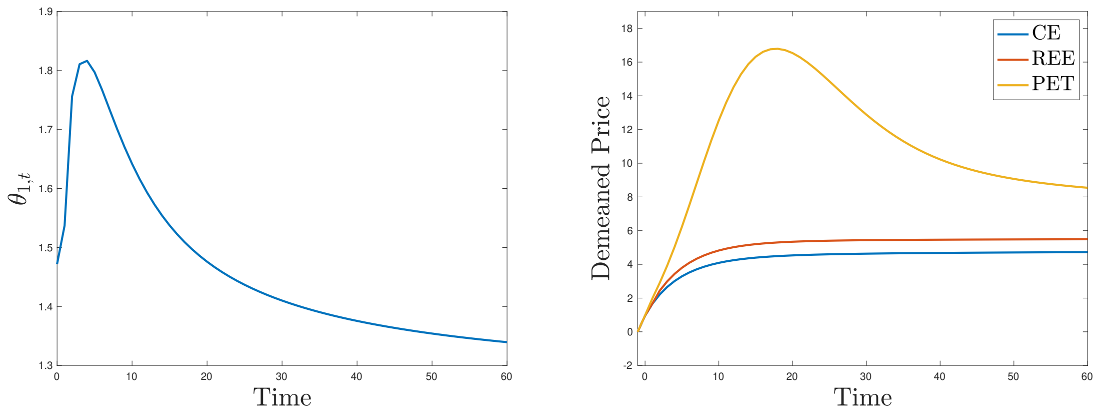
$$\mathbb{E}_{U,1}^{CE}[v_2] = (1-\rho^2)(\bar{v}+\mu_0) + \rho^2\bar{v} \tag{D.66}$$

$$\mathbb{V}_{U,1}^{CE}[v_2] = (1-\rho^2)^2(\tau_0)^{-1} + \frac{\sigma_u^2}{1-\rho^2} \tag{D.67}$$

Given these beliefs, we can solve for the CE, PET and REE equilibrium prices in period $t = 1$.

Period $t > 1$. Starting in period $t = 2$, uninformed agents gain information about both U_t and S_t by learning from past prices, and the economy evolves as described above.

Figure 11: Displacement Demeaned Price Path and Extrapolation Parameter. Impulse response function following a displacement, modeled as an uncertain shock to the unconditional mean of the process.



D.2 Permanent Shocks - Random Walk Fundamentals

The way we have modelled normal times shocks and displacements in Section D.1 draws a distinction between displacements being permanent shock and normal times shocks as being transitory. In what follows, we instead consider the case where fundamentals evolve according to a random walk, so that both normal times and displacement shocks are permanent.

In both cases, displacement shocks differ to normal times shocks because displacements are shocks for which informed agents gain more information about over time (while normal time shocks are effectively revealed next period, so there is no sense in which agents gradually gain more information about these shocks over time, other than by observing their realization).

D.2.1 Setup

Assets. Consider an economy where agents are solving a portfolio choice problem between a risky and a riskless asset. The risk-free asset is in zero net supply, and we normalize its price and risk free rate to one, $P_f = R_f = 1$. The risky asset is in fixed net supply Z, and the fundamental value of the asset evolves according to a random walk:

$$v_t = v_{t-1} + u_t \tag{D.68}$$

where $u_t \sim N(0, \tau_u^{-1})$.

Agents and Preferences. There is a continuum of measure one of agents, and we assume that they are only concerned with forecasting the fundamental value of the asset, so that at time t they have the following demand function for the risky asset:

$$X_{it} = \frac{\mathbb{E}_{it}[v_{t+1}] - P_t}{AVar_{it}[v_{t+1}]} \tag{D.69}$$

where $\mathbb{E}_{it}[\cdot]$ and $Var_{it}[\cdot]$ characterize agent i's beliefs about next period fundamental given the information set they have at time t.

Information Structure in Normal Times. All agents know the unconditional distribution of u_t. Moreover, a fraction ϕ of agents are informed, and they observe the whole history u_j for $j \leq t$ before making their portfolio choice in each period. A fraction $(1-\phi)$ of agents are uninformed, and they do not observe u_t, v_t nor their history. However, they can learn information from past prices.

Equilibrium. In equilibrium, uninformed agents' beliefs must be consistent with past prices they observe, given their model of the world. Moreover, all agents trade according to their demand functions in (D.69) given their beliefs, and markets clear. The market clearing price function, conditional on agents' beliefs, is then given by:

$$P_t = a_t \mathbb{E}_{I,t}[v_{t+1}] + b_t \mathbb{E}_{U,t}[v_{t+1}] - c_t \tag{D.70}$$

where $a_t = \left(\frac{\phi \mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $b_t = \left(\frac{(1-\phi)\mathbb{V}_{I,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $c_t = \left(\frac{\mathbb{V}_{I,t}\mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right) AZ$, and $\mathbb{V}_{i,t} = \mathrm{Var}_{i,t}[v_{t+1}]$ for $i \in \{I, U\}$. Therefore, in order to find the equilibrium price, we need to pin down informed and uninformed agents' beliefs about v_{t+1}.

D.2.2 Normal Times

Agents' Beliefs. Informed agents' beliefs are simply given by:

$$\mathbb{E}_{I,t}[v_{t+1}] = v_{t-1} + u_t \tag{D.71}$$

$$\mathbb{V}_I[v_{t+1}] = \sigma_u^2 \tag{D.72}$$

Since prices in our economy are fully revealing, uninformed agents' beliefs are given by:

$$\mathbb{E}_{U,t}[v_{t+1}] = \tilde{v}_{t-1} \tag{D.73}$$

$$\mathbb{V}_U[v_{t+1}] = 2\sigma_u^2 \tag{D.74}$$

where $\tilde{v}_{t-1}$ is uninformed agents' beliefs of previous period fundamental, which they extract from past prices.

To understand what information uninformed agents extract from prices, we need to pin down what uninformed agents think is generating the price that they observe. Suppose that uninformed agents use a simple heuristic, and think that prices are increasing in fundamentals and decreasing in a constant risk premium component δ as follows:

$$P_{t-1} = \gamma\tilde{v}_{t-1} - \delta \implies \tilde{v}_{t-1} = \frac{1}{\gamma}P_{t-1} + \frac{\delta}{\gamma} \tag{D.75}$$

We can then re-write uninformed agents' beliefs as:

$$\mathbb{E}_{U,t}[v_{t+1}] = \theta P_{t-1} + \theta\delta \tag{D.76}$$

where $\theta = \frac{1}{\gamma}$. By learning from past prices in this way, uninformed agents extrapolate prices to learn about fundamentals, and θ captures the degree of price-based extrapolation.

Partial Equilibrium Thinking. In normal times, PET provides a micro-foundation for θ. This allows to make predictions about the extent of biases in individual level beliefs, depending on the environment. We solve for partial equilibrium using the same steps as in the rest of the paper.

Equilibrium. By substituting these expressions for agents' beliefs in (D.70), we find that equilibrium prices evolve according to:

$$P_t = av_t + b\theta P_{t-1} + b\theta\delta - c \tag{D.77}$$

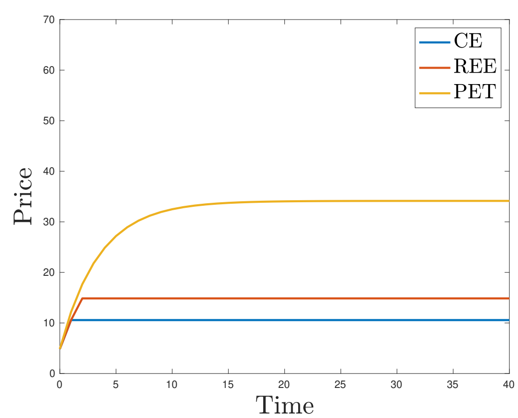
Starting from a steady state where the fundamental value of the asset is constant at v_0, if we study the impulse response function to a shock $u_1 \neq 0$, we have that:

$$P_t = \sum_{j=1}^{t-1} (\beta\theta)^j (av_1 + b\theta\delta - c) + (\beta\theta)^t \tag{D.78}$$

The economy will converge to a new steady state if and only if $\beta\theta < 1$. Otherwise, prices and uninformed agents' beliefs become extreme and decoupled from fundamentals.

Impulse Response Function. We plot the impulse response function in Figure 12. Following a normal times shock, PET leads to momentum as delayed over-reaction.

Figure 12: Path of equilibrium prices in normal times.



D.2.3 Displacement

Displacement Shock and Information Structure. We model a displacement as a one-off shock to fundamentals, ω, whose realization no agent can observe. Instead, agents have a prior distribution of $\omega \sim N(\mu_0, \tau_0^{-1})$. The shock is announced in period $t = 0$, and comes into effect in period $t = 1$.

$$v_t = v_0 + \omega + \sum_{j=1}^{t} u_t \tag{D.79}$$

Starting in period $t = 1$, all informed agents receive a common signal $s_t = \omega + \epsilon_t$ where $\epsilon_t \sim N(0.\tau_s^{-1})$. Uninformed agents do not see these signals, but can still learn information from past prices.

Agents' Beliefs. In period $t = 0$, when the displacement is announced, agents' beliefs are as follows:

$$\mathbb{E}_{I,0}[v_1] = v_{-1} + \mu_0 + u_0 \tag{D.80}$$

$$\mathbb{V}_{I,0}[v_1] = \tau_0^{-1} + \sigma_u^2 \tag{D.81}$$

$$\mathbb{E}_{U,0}[v_1] = \tilde{v}_{-1} + \mu_0 \tag{D.82}$$

$$\mathbb{V}_{U,0}[v_1] = \tau_0^{-1} + 2\sigma_u^2 \tag{D.83}$$

Starting in period $t = 1$, agents' beliefs are given by:

$$\mathbb{E}_{I,t}[v_{t+1}] = v_0 + \left(\frac{t\tau_s}{t\tau_s + \tau_0} S_t + \frac{\tau_0}{t\tau_s + \tau_0}\mu_0\right) + \sum_{j=1}^{t} u_j \tag{D.84}$$

$$\mathbb{V}_{I,t}[v_{t+1}] = (t\tau_s + \tau_0)^{-1} + \sigma_u^2 \tag{D.85}$$

$$\mathbb{E}_{U,t}[v_{t+1}] = \tilde{\mathbb{E}}_{I,t-1}[v_t] \tag{D.86}$$

$$\mathbb{V}_{U,t}[v_{t+1}] = \left(\mathbb{V}_{U,t-1}[v_t] + \sigma_u^2\right) - \frac{\left(\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}\right)\tau_0^{-1} + (t-1)\sigma_u^2\right)^2}{\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}\right)^2 \left(\tau_0^{-1} + ((t-1)\tau_s)^{-1}\right) + (t-1)\sigma_u^2} \tag{D.87}$$

Once again, we need to specify what information uninformed agents extract from prices. When agents think in partial equilibrium, we can write their beliefs as follows:

$$\mathbb{E}_{U,t}[v_{t+1}] = \theta_t P_{t-1} + \theta_t \delta_t \tag{D.88}$$

where PET provides a micro-foundation for the time-varying extrapolation coefficients.

We solve for PET using the same steps as in the rest of the paper. The one step that requires us to make additional assumptions regards the uncertainty faced by uninformed agents. Specifically, the unconditional variance of the process for fundamentals is infinity given the process is a random walk. Instead, we assume that cursed agents have the same variance as uninformed PET agents.

Equilibrium. If we turn off all normal time shocks, on average, in equilibrium, prices evolve as follows:

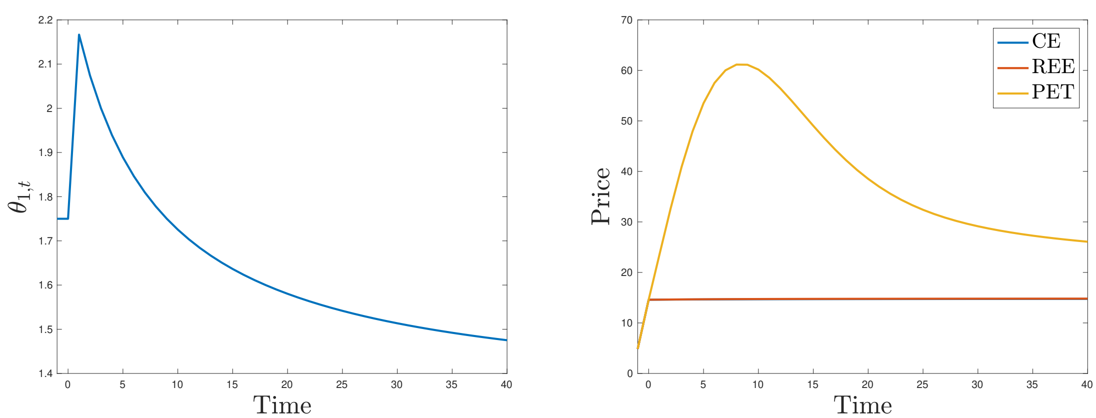
$$P_t = a_t(v_0 + \omega) + b_t\theta_t P_{t-1} + b_t\theta_t\delta_t - c_t \tag{D.89}$$

For simplicity, let $\delta_t = c_t = v_0 = 0$. Then, we can write prices as:

$$P_t = \left(a_t + \sum_{j=1}^{t-1} \prod_{i=1}^{j} \left(\theta_t b_{t+1-i}\right) a_{t-j} \right) \omega + \prod_{j=1}^{t} \left(\theta_t b_{t+1-j}\right) P_0 \tag{D.90}$$

Impulse Response Function. We plot the impulse response function of the displacement shock in Figure 13. Following a displacement, the degree of extrapolation is initially stronger, and then declines over time, leading to bubbles and endogenous crashes.

Figure 13: Time-variation in the extrapolation parameter (left panel) and path of equilibrium prices (right panel) following a displacement.



D.3 Unobservable Growth Rate of Dividends

In this section we consider an alternative setup, where uninformed agents can observe dividends, and they are instead learning about the unobservable growth rate of dividends.

Fundamentals and Shocks. All agents solve a portfolio choice problem between a riskless asset in zero net supply where we normalize the price and risk free rate to 1, and a risky asset in fixed supply Z which pays a stream of dividends D_{t+1} each period. Dividends evolve as follows:

$$D_{t+1} = D_t + g_{t+1} + \xi_{t+1} \tag{D.91}$$

$$g_{t+1} = (1-\rho)\bar{g} + \rho g_t + u_{t+1} \tag{D.92}$$

where $\xi_{t+1} \sim N(0, \sigma_\xi^2)$ and $u_{t+1} \sim N(0, \sigma_u^2)$. Following a displacement, the process for dividend growth is shocked such that:

$$g_{t+1} = (1-\rho)\bar{g} + \rho g_t + \omega + u_{t+1} \tag{D.93}$$

where $\omega \sim N(\mu_0, \tau_0^{-1})$. Therefore this displacement shock is equivalent to shocking the unconditional mean of the growth rate of dividends by $\left(\frac{\omega}{1-\rho}\right)$.

Agents and Preferences. We consider an OLG economy where all agents live for one period, and have the following demand function for the risky asset:

$$X_{it} = \frac{\mathbb{E}_{it}[D_{t+1}] - P_t}{A\mathbb{V}_{it}[D_{t+1}]} \tag{D.94}$$

In this economy, agents are concerned with next period payoff, but we shut down speculative motives to keep things tractable.

Information Structure. In normal times, all agents know $\bar{g}$, ρ, the distribution of ξ_t and u_t, and all agents also observe D_t. Moreover, informed agents observe u_{t+1}. Uninformed agents can learn information from past prices.

Displacements are unanticipated shocks that are announced in period $t = 0$, at which point all agents share the same unconditional distribution for $\omega \sim N(\mu_0, \tau_0^{-1})$. Starting in period $t = 1$, informed agents receive signals $s_t = \omega + \epsilon_t$ where $\epsilon_t \sim (N, \tau_s^{-1})$ each period. Uninformed agents do not observe s_t, but can learn information from past prices.

For tractability, we assume that no agent uses the history of D_t to learn information about g_t. This assumptions allows us to not have to deal with an additional signal that agents receive about g_t, and which they would be combining with the information they either receive or learn from past prices. One way to rationalize this is to think of σ_u^2 as being extremely large, so that ΔD_t provides too noisy a signal of g_{t+1}.

D.3.1 Normal Times

Informed Agents' Beliefs. In normal times informed agents' beliefs are given by:

$$\mathbb{E}_{I,t}[D_{t+1}] = D_t + g_{t+1} \tag{D.95}$$

$$\mathbb{V}_I[D_{t+1}] = \sigma_\xi^2 \tag{D.96}$$

Cursed Agents' Beliefs and Cursed Equilibrium Mapping. [19] Uninformed agents' beliefs depend on the equilibrium concept. We assume that cursed agents form their beliefs based on the unconditional mean of the unobservable growth rate of dividends.

$$\mathbb{E}_{U,t}^{CE}[D_{t+1}] = D_t + \bar{g} \tag{D.97}$$

$$\mathbb{V}_{U,t}^{CE}[D_{t+1}] = \frac{\sigma_u^2}{1-\rho^2} + \sigma_\xi^2 \tag{D.98}$$

The cursed equilibrium price function is therefore given by:

$$P_t^{CE} = D_t + a^{CE} g_{t+1} + b^{CE} \bar{g} - c^{CE} \tag{D.99}$$

[19] Throughout Appendix D, we use "CE" to stand for Cursed Equilibrium (Eyster and Rabin (2005)): cursed agents do not learn information from prices, and instead trade on their private information alone. In this respect, PET agents think that all other agents are cursed. Therefore PET agents invert the cursed equilibrium price function to infer information from prices.

where $a^{CE} = \left(\frac{\phi \mathbb{V}_{U,t}^{CE}}{\phi \mathbb{V}_{U,t}^{CE} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $b^{CE} = \left(\frac{(1-\phi)\mathbb{V}_{I,t}}{\phi \mathbb{V}_{U,t}^{CE} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $c^{CE} = \left(\frac{AZ\mathbb{V}_{I,t}\mathbb{V}_{U,t}^{CE}}{\phi \mathbb{V}_{U,t}^{CE} + (1-\phi)\mathbb{V}_{I,t}}\right)$. The price dividend "ratio" evolves as an AR(1) process which is stationary as long as $\rho < 1$.

$$\left(P_t^{CE} - D_t\right) - \overline{P - D}^{CE} = \frac{u_{t+1}}{1 - \rho \mathbb{L}} \tag{D.100}$$

where $\overline{P - D}^{CE} = \bar{g} - c^{CE}$ is the unconditional mean of $P_t^{CE} - D_t$.

PET Agents' Beliefs and Equilibrium Prices. We assume that PET agents learn information from prices under the mistaken belief that all other agents are cursed. Uninformed agents then extract $\tilde{g}_t$ from prices as follows:

$$\tilde{g}_t = \frac{P_{t-1}^{CE} - D_{t-1} - b^{CE}\bar{g} + c^{CE}}{a^{CE}} \tag{D.101}$$

This leads to PET agents holding the following posterior beliefs:

$$\mathbb{E}_{U,t}[D_{t+1}] = D_t + (1-\rho)\bar{g} + \rho \tilde{g}_t \tag{D.102}$$

$$\mathbb{V}_{U,t}[D_{t+1}] = \sigma_u^2 + \sigma_\xi^2 \tag{D.103}$$

The PET equilibrium price is then given by:

$$P_t = D_t + a g_{t+1} + b\left((1-\rho)\bar{g} + \rho \tilde{g}_t\right) - c \tag{D.104}$$

where $a = \left(\frac{\phi \mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $b = \left(\frac{(1-\phi)\mathbb{V}_{I,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $c = \left(\frac{AZ\mathbb{V}_{I,t}\mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$. Rearranging this expression, and using the results above, we can rewrite the price dividend ratio as:

$$(P_t - D_t) - \overline{P - D} = a \frac{u_{t+1}}{(1 - \rho \mathbb{L})\left(1 - \frac{\rho b}{a^{CE}}\mathbb{L}\right)} \tag{D.105}$$

where $\overline{P - D} = (\bar{g} - c) - \frac{b\rho}{a^{CE}}(\bar{g} - c^{CE})$. This expression in an AR(2). For the price dividend ratio to be stationary in normal times, we need both roots of the autoregressive coefficients to lie outside the unit circle: $\rho < 1$ and $\frac{\rho b}{a^{CE}} < 1$.

REE Agents' Beliefs and Equilibrium Prices. Finally, rational uninformed agents also learn from past prices, but are able to extract the right information from them.

$$\mathbb{E}^{REE}_{U,t}[D_{t+1}] = D_t + (1-\rho)\bar{g} + \rho g_t \tag{D.106}$$

$$\mathbb{V}^{REE}_{U,t}[D_{t+1}] = \mathbb{V}_{U,t}[D_{t+1}] \tag{D.107}$$

The REE equilibrium is then given by:

$$P^{REE}_t = D_t + a g_{t+1} + b((1-\rho)\bar{g} + \rho g_t) - c \tag{D.108}$$

where a, b and c are the same coefficients as in PET, as agents have the same conditional variance in PET and REE.

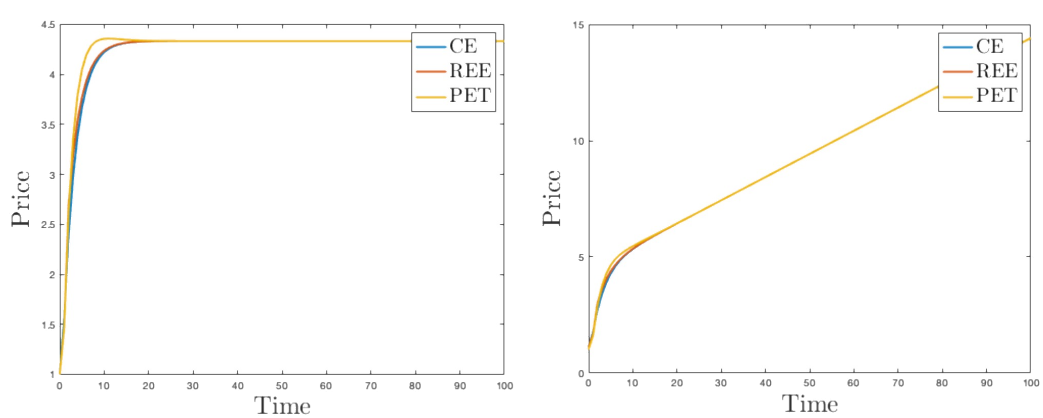
Rearranging and using the results above, we see that in normal times REE prices evolve according to an ARMA(1,1), which is stationary as long as $\rho < 1$:

$$(P^{REE}_t - D_t) - \overline{P-D}^{REE} = a\frac{\left(1 + \frac{b\rho}{a}\mathbb{L}\right)}{(1-\rho\mathbb{L})}u_{t+1} \tag{D.109}$$

where $\overline{P-D}^{REE} = \bar{g} - c$.

Simulation. Figure 14 simulates the path of equilibrium prices when $\bar{g} = 0$ and $\bar{g} > 0$. Regardless of $\bar{g}$, PET leads to mild momentum and reversals in normal times.

Figure 14: Path of equilibrium prices in normal times with an unobservable growth rate of dividend. In the left panel $\bar{g} = 0$, while in the right panel $\bar{g} > 0$.



D.3.2 Displacement

Shock. Starting from the normal times steady state, suppose a displacement shifts the unconditional mean of the growth rate of dividends from $\bar{g}$ to $\bar{g} + \frac{\omega}{1-\rho}$.

$$D_{t+1} = D_t + g_{t+1} + \xi_{t+1} \tag{D.110}$$

$$g_{t+1} = (1-\rho)\bar{g} + \rho g_t + \omega + u_{t+1} \tag{D.111}$$

In period $t = 0$, all agents learn about the existence of this shock, and have the same unconditional prior over it $\omega \sim N(\mu_0, \tau_0^{-1})$. Starting in period $t = 1$, informed agents receive signals $s_t = \omega + \epsilon_t$ each period, where $\epsilon_t \sim N(0, \tau_s^{-1})$. Uninformed agent do not observe this signal, and instead continue to learn information from past prices.

Period $t > 1$. To solve the model for period $t > 1$ it is convenient to rewrite the process for dividends conditional on the information set in period $t = 0$:

$$D_{t+1} = D_t + (1-\rho^{t+1})\left(\bar{g} + \frac{\omega}{1-\rho}\right) + \rho^{t+1} g_0 + U_{t+1} + \xi_{t+1} \tag{D.112}$$

where $U_{t+1} = \sum_{j=0}^{t} \rho^j u_{t+1-j} = \rho^t u_1 + \sum_{j=1}^{t-1} \rho^j u_{t+1-j} + u_{t+1} = \rho U_t + u_{t+1}$.

Informed Agents. In period $t > 2$, informed agents' beliefs are given by:

$$\mathbb{E}_{I,t}[D_{t+1}] = D_t + \underbrace{(1-\rho^{t+1})\left(\bar{g} + \frac{\frac{t\tau_s}{t\tau_s+\tau_0} S_t + \frac{\tau_0}{t\tau_s+\tau_0}\mu_0}{1-\rho}\right) + \rho^{t+1} g_0 + U_{t+1}}_{g_{t+1|t}} \tag{D.113}$$

$$\mathbb{V}_{I,t}[D_{t+1}] = \left(\frac{1-\rho^{t+1}}{1-\rho}\right)^2 (t\tau_s + \tau_0)^{-1} + \sigma_\xi^2 \tag{D.114}$$

CE Agents' Beliefs and Equilibrium. Uninformed cursed agents' beliefs are given by:

$$\mathbb{E}_{U,t}^{CE}[D_{t+1}] = D_t + (1-\rho^{t+1})\left(\bar{g} + \frac{\mu_0}{1-\rho}\right) + \rho^{t+1}\bar{g} \tag{D.115}$$

$$\mathbb{V}_{U,t}^{CE}[D_{t+1}] = \left(\frac{1-\rho^{t+1}}{1-\rho}\right)^2 (\tau_0)^{-1} + \frac{\sigma_u^2}{1-\rho^2} + \sigma_\xi^2 \tag{D.116}$$

The cursed equilibrium price is then given by:

$$P_t^{CE} = D_t + a_t^{CE} g_{t+1|t} + b_t^{CE}\bar{g} - c_t^{CE} \tag{D.117}$$

PET Agents' Beliefs and Equilibrium. PET agents' beliefs are given by:

$$\mathbb{E}_{U,t}[D_{t+1}] = D_t + (1-\rho^{t+1})\left(\bar{g} + \frac{\mathbb{E}_{U,t}[\omega|\tilde{g}_{t|t-1}]}{1-\rho}\right) + \rho^{t+1}\tilde{g}_0 + \rho^t\tilde{u}_1 + \rho\mathbb{E}_{U,t}[U_t|\tilde{g}_{t|t-1}] \tag{D.118}$$

$$\mathbb{V}_{U,t}[D_{t+1}] = \left(\frac{1-\rho^{t+1}}{1-\rho}\right)^2 \mathbb{V}_{U,t}[\omega|\tilde{g}_{t|t-1}] + \rho^2\mathbb{V}_{U,t}[U_t|\tilde{g}_{t+1|t}] + 2\left(\frac{1-\rho^{t+1}}{1-\rho}\right)\rho\mathrm{Cov}_{U,t}(\omega, U_t|\tilde{g}_{t|t-1}) + \sigma_u^2 + \sigma_\xi^2 \tag{D.119}$$

Simulation. Given these beliefs, Figure 15 simulates the path of equilibrium prices when $\bar{g} = 0$ and $\bar{g} > 0$. Parameters are the same as in normal times. Even with this setup, PET delivers bubbles and crashes, and results are robust to the initial level of $\bar{g}$.

Figure 15: Path of equilibrium prices following a displacement which shocks the growth rate of dividends from $\bar{g}$ to $\bar{g} + \frac{\omega}{1-\rho}$. In the left panel $\bar{g} = 0$, while in the right panel $\bar{g} > 0$.