FLEXOELECTRIC ACTUATION AND VIBRATION CONTROL OF RING SHELLS

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ABSTRACT

The converse flexoelectric effect that the gradient of polarization (or electric field) induces internal stress (or strain) can be utilized to control the vibration of flexible structures. This study focuses on the microscopic actuation behavior and effectiveness of a flexoelectric actuator patch on an elastic ring. An atomic force microscope (AFM) probe is placed on the upper surface of the patch to implement the inhomogeneous electric field inducing stresses inside the actuation patch. The flexoelectric membrane force and bending moment, in turn, actuate the ring vibration and its actuation effect is studied. Actuator’s influence in the transverse and circumferential directions is respectively evaluated. For the transverse direction, the gradient of the electric field decays quickly along the ring thickness, resulting in a nonuniform transverse distribution of the induced stress and such distribution is not influenced by the patch thickness. The flexoelectric induced circumferential membrane force and bending moment resembles the Dirac delta function at the AFM contact point. The influence of the actuator can be regarded as a drastic bending on the ring. To evaluate the actuation effect, dynamic response of controllable displacements of the elastic ring under flexoelectric actuation is analyzed by adjusting the geometric parameters, such as the thickness of flexoelectric patch, AFM probe radius, ring thickness and ring radius. This study represents a thorough understanding of the flexoelectric actuation behavior and serves as a foundation of the flexoelectricity based vibration control.

INTRODUCTION

Precision actuation and vibration control has been a constant quest over the years. The phenomenon that the polarization (or electric field) gradient induces internal stress (or strain) is called the converse flexoelectric effect [1-3]. In the last decade, both theoretical studies [4-6] and experimental measurements of the flexoelectric coefficients [7,8] have drawn much attentions and the work has been carried out on different materials [9-11]. The signal analysis and dynamic responses of the flexoelectric materials have been reported recently and their applications on rings and cylindrical shells were studied [12-15]. Static flexoelectric actuation effects of a cantilever beam using the AFM probe were also evaluated [16].

Ring shells are widely used in engineering applications, such as stiffeners, gears, structural sensors, motors, etc. Dynamic behaviors, e.g., natural frequencies and mode shapes, of free floating rings have been thoroughly studied [17]. Piezoelectric sensors and actuators and their modal sensing and actuation effects were analyzed [18]. However, dynamic flexoelectric actuation characteristics and vibration control of shell structures have not been reported previously. This study is to investigate flexoelectric microscopic actuation and control behaviors of ring structures. An inhomogeneous electric field generated by AFM probe is used to actuate the flexoelectric patch laminated on the ring structure. With the flexoelectric membrane force and bending moment, steady-state dynamic response or controllable displacement of the ring is determined. Distribution of the electric field gradient over the transverse direction of different patch thickness is analyzed, followed by actuation effectiveness of the actuator patch thickness, AFM probe radius, ring thickness and ring radius.

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DYNAMICS OF THE RING

Figure 1 illustrates an elastic ring shell laminated with a flexoelectric actuator patch, where \( \alpha_3 \) and \( \psi \) coordinates respectively define the transverse and the circumferential directions; \( R \) is the neutral surface radius of the ring; \( h \) is the ring thickness and \( b \) is the ring width.

Figure 1. Schematic diagram of the elastic ring

The flexoelectric patch perfectly laminated on the ring surface is used as an actuator and its stiffness/mass effect to the ring dynamics is neglected since it is much thinner than the ring thickness. The membrane control force and bending control moment induced by the actuator (i.e., flexoelectric patch) is written as \( N_{\psi \psi}^a \) and \( M_{\psi \psi}^a \) respectively, where the superscript “a” denotes the actuator induced component. The dynamic equation with the mechanical force and actuation force of the ring can be simplified from the dynamic equations of the double-curbvature shell [18].

\[
\rho h \frac{\partial^2 u_\psi}{\partial t^2} + \frac{1}{R} \frac{\partial N_{\psi \psi}^m}{\partial \psi} - \frac{1}{R^2} \frac{\partial M_{\psi \psi}^m}{\partial \psi} = \frac{1}{R} \frac{\partial N_{\psi \psi}^a}{\partial \psi} - \frac{1}{R^2} \frac{\partial M_{\psi \psi}^a}{\partial \psi},
\]

\[
\rho h \frac{\partial^2 u_\psi}{\partial t^2} = \frac{1}{R} \frac{\partial N_{\psi \psi}^m}{\partial \psi} - \frac{1}{R^2} \frac{\partial M_{\psi \psi}^m}{\partial \psi} - \frac{1}{R} \frac{\partial N_{\psi \psi}^a}{\partial \psi} - \frac{1}{R^2} \frac{\partial M_{\psi \psi}^a}{\partial \psi} + F_3,
\]

where \( \rho \) is the mass-density of the ring; \( u_\psi \) is the transverse displacement; \( u_\psi \) is the circumferential displacement which is assumed to be small comparing with \( u_\psi \); \( F_3 \) is the distributed mechanical force in the transverse direction; \( N_{\psi \psi}^m \) and \( M_{\psi \psi}^m \) are the mechanically induced membrane force and bending moment per unit width, where the superscript “m” denotes the mechanical component. Combining the actuator induced components respectively in the circumferential and transverse direction in ring dynamic equations, Love’s control operators can be expressed as [18]

\[
L_3' = \frac{N_{\psi \psi}^a}{R} - \frac{1}{R^2} \frac{\partial^2 M_{\psi \psi}^a}{\partial \psi^2},
\]

where \( L_3 \) and \( L_{\psi \psi} \) are Love’s actuation or control operators, which denote the flexoelectric actuator contributions to the dynamic equations. Note that only the transverse vibration is considered here. According to the modal expansion method, ring’s transverse displacement \( u_\psi \) can be expressed by adding all participating natural modes multiplied by their respective modal participation factors [17]

\[
u_k(t) = \sum_{i=1}^{\infty} \eta_\psi(t) \psi_k(\psi, t),
\]

where \( k \) is the mode number in the transverse direction; \( \eta_\psi \) is the modal participation factor; \( U_{3k} \) is the transverse mode shape function for the \( k \)-th mode. \( \psi \) denotes the temporal contribution of each mode and \( U_{3k} \) denotes the spatial distribution. It is observed that \( k=0 \) is a breathing mode; \( k=1 \) is a rigid body mode, and a set of transverse and circumferential modes occur when \( k \geq 2 \) [17]. When the ring is freely floating in space, the mode shape function of the circumferential direction and the transverse direction can be expressed as [17]

\[
U_{\psi k} = A_k \sin(k \psi - \varphi), \quad U_{3k} = B_k \cos(k \psi - \varphi),
\]

where \( U_{\psi k} \) is the circumferential mode shape function; \( A_k \) and \( B_k \) are the constants of the mode shape functions; \( \varphi \) is an arbitrary phase angle that must be included since the ring does not show a preference for the orientation of its modes. Modal vibration amplitudes of the transverse component modes and the circumferential component modes are coupled at lower \( k \) mode numbers [18]. By substituting the modal expansion expression of transverse displacement, i.e., Eq.(5) into dynamic equations of ring, taking the advantage of the orthogonality of the mode shape functions and introducing the modal damping ratio \( \zeta_k \) yields the independent modal equation [17]

\[
\hat{\eta}_k + 2 \zeta_k \omega_k \hat{\eta}_k + \omega_k^2 \eta_k = \hat{F}_k,
\]

where \( \omega_k \) is the \( k \)-th natural frequency; \( \zeta_k \) represents the modal damping ratio of the ring, which depends on the equivalent damping constant \( c \) and the natural frequency \( \omega_k \), i.e., \( \zeta_k = c/(2 \rho \omega_k) \); \( \hat{F}_k \) is the modal force for the \( k \)-th mode. The modal participation factor equation, i.e., Eq.(8) is a non-homogeneous second-order ordinary differential equation of the \( k \)-th modal participation factor \( \eta_k \). When a ring model is considered, for each value of \( k \geq 2 \), there are two component natural frequencies, i.e., \( \omega_{k1} \) and \( \omega_{k2} \). The component frequency \( \omega_{k1} \) denotes the transverse vibration, and \( \omega_{k2} \) denotes the circumferential vibration. For the oscillation at \( \omega_{k1} \), amplitude
ratio of transverse modes $B_k$ is proportional to circumferential mode’s amplitude $A_k$ as

$$\frac{B_k}{A_k} \approx -k.$$  

(9)

For typical rings $\omega_k \gg \omega_0$, thus the circumferential vibration is not considered and only the transverse $\omega_k$ remains as $\omega_k$ used later [18]. Apparently, the solution is influenced by the modal force, which can be divided into two components: the modal force induced by the mechanical force and the modal force induced by the flexoelectric actuator, i.e.,

$$\hat{F}_k = \hat{F}_k^m + \hat{F}_k^a,$$  

(10)

where $\hat{F}_k^m$ is the modal force induced by the mechanical force; $\hat{F}_k^a$ is the modal force induced by the actuator. Let the external mechanical force be expressed as $F_3$, Fig.1. $\hat{F}_k^m$ and $\hat{F}_k^a$ can be respectively expressed as [18]

$$\hat{F}_k^m = \frac{1}{\rho h N_k} \int_0^{2\pi} F_3 U_{3k} Rd\psi,$$  

(11)

$$\hat{F}_k^a = \frac{1}{\rho h N_k} \int_0^{2\pi} \left(L_3^a U_{\psi k} + L_3^a U_{3k}\right) Rd\psi,$$  

(12)

where

$$N_k = \int_0^{2\pi} \left(U_{\psi k}^2 + U_{3k}^2\right) Rd\psi.$$  

(13)

Note that the patch is lamented from $\psi_i$ to $\psi_f$. When the mechanical force $F_3$ and the actuation voltage $\phi^*$ are harmonic with the mechanical force frequency $\omega$, i.e., $F_3 = F_3^* e^{j\omega t}$, $\phi^*(t) = \phi^* e^{j\omega t}$, the modal response will also be harmonic and $\hat{F}_k \approx \hat{F}_k^* e^{j\omega t}$, where $\hat{F}_k^*$ indicates the magnitude of the $k$-th modal force. The modal response (modal participation factor $\eta_k$) can be solved from the modal participation factor equation, i.e., Eq.(8) as

$$\eta_k(t) = \frac{\hat{F}_k^*}{(\omega_k^2 - \omega^2)^2 + 4\omega_k^2} e^{j\omega t}$$

$$= \frac{\hat{F}_k^*}{\omega_k^2 \left(1 - \frac{\omega_k^2}{\omega^2}\right) + 4\omega_k^2} e^{j(\omega t - \phi^*)},$$  

(14)

where $\phi^*$ is the phase angle expressed as

$$\phi^* = \arctan \left(\frac{\omega_k}{\omega_k^2 - \omega^2} \right).$$  

(15)

With the modal participation factor (Eq.(14)) and the mode expansion expression (Eq.(5)), the transverse deflection induced by the mechanical force and flexoelectric actuator can be obtained as

$$u_3(\psi,t) = \sum_{k=1}^{\infty} \eta_k(t) U_{3k}(\psi,t)$$

$$= \sum_{k=1}^{\infty} \left(\frac{F_3^*}{\omega_k^2 \left(1 - \frac{\omega_k^2}{\omega^2}\right) + 4\omega_k^2} e^{j(\omega t - \phi^*)}\right) U_{3k}(\psi,t)$$

$$= \sum_{k=1}^{\infty} \left(\frac{F_3^*}{\omega_k^2 \left(1 - \frac{\omega_k^2}{\omega^2}\right) + 4\omega_k^2} e^{j(\omega t - \phi^*)}\right) U_{3k}(\psi,t)$$

$$= \sum_{k=1}^{\infty} \frac{N_k^a}{1 - \frac{\omega_k^2}{\omega^2}} e^{j(\omega t - \phi^*)} U_{3k}(\psi,t)$$

(16)

The transverse vibration of the ring is affected by the flexoelectric actuation membrane force and the bending control moment. For the flexoelectric material, the actuation behavior depends on the specific form of the inhomogeneous electric field. The derivation of the actuation force inside the flexoelectric patch from the electric field of the AFM probe is introduced next.

**FLEXOELECTRIC ACTUATION**

An AFM probe is used to actuate the flexoelectric patch. The ring model of the flexoelectric actuator is shown in Fig.2 where $r$ is the radius of the AFM probe and $h^d$ is the thickness of the flexoelectric patch. According to the converse flexoelectric effect, an inhomogeneous electric field is needed to drive the flexoelectric patch. This electric field is induced by an AFM probe locating at $\psi = \psi^*$. The AFM probe tip radius $r$ is far less than the flexoelectric patch thickness and the patch thickness is far less than the ring thickness, i.e., $r \ll h^d \ll h$. 

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To generate the inhomogeneous electric field, an actuation voltage $\phi^e$ is applied between the AFM probe and the electrode under the flexoelectric patch. The electric potential inside the flexoelectric patch can be expressed based on Abplanalp’s approximate [20]. By transforming the rectangular coordinates to the polar coordinates, the potential near the AFM probe can be expressed as

$$\phi = \frac{\phi^e r}{\sqrt{\left(\alpha_3 + R\right) \sin^2 \left(\psi - \psi_o\right) + \left[r + \frac{h}{2} + h^e - \left(\alpha_3 + R\right) \cos \left(\psi - \psi_o\right) + R^2\right]}}. \quad (17)$$

where $\psi_o$ is the AFM probe location. In this study, the net electric field in the $\psi$ direction is almost zero [16], thus only the transverse electric field is considered here. The transverse electric field can be obtained by differentiating the potential expression in the opposite transverse direction.

$$E_3 = \frac{\partial \phi}{\partial \alpha_3} = \phi^e r \left\{ \left(\alpha_3 + R\right) \sin \left(\psi - \psi_o\right) + \left[r + \frac{h}{2} + h^e - \left(\alpha_3 + R\right) \cos \left(\psi - \psi_o\right) + R^2\right] \right\}^{\frac{3}{2}}. \quad (18)$$

where $E_3$ is the electric field in the transverse direction. Note that the transverse electric field (Eq.(18)) only indicates the electric field near the AFM probe. However, for the points away from the AFM probe, both the actually electric field and the value of Eq.(18) are almost zero [20]. Thus it is accurate enough to assume that Eq.(18) can be used to describe the electric field for the whole plane. The circumferential stress induced by the electric field can be derived from the general converse flexoelectric equations [16] and expressed as

$$T_{\psi\psi} = \pi_{12} \frac{\partial E_3}{\partial \alpha_3}$$

$$= \frac{\pi_{12} \phi^e r}{\sqrt{\left(\alpha_3 + R\right) \sin^2 \left(\psi - \psi_o\right) + \left[r + \frac{h}{2} + h^e - \left(\alpha_3 + R\right) \cos \left(\psi - \psi_o\right) + R^2\right]}}$$

$$\times \left\{ \left(\alpha_3 + R\right) \sin^2 \left(\psi - \psi_o\right) + \left[r + \frac{h}{2} + h^e - \left(\alpha_3 + R\right) \cos \left(\psi - \psi_o\right) + R^2\right] \right\}^{\frac{3}{2}}. \quad (19)$$

where $T_{\psi\psi}$ denotes the circumferential stress in the flexoelectric patch. The membrane force induced by the flexoelectric actuator is the overall effect of the stress expressed in Eq.(19). The flexoelectric membrane control force induced by the electric field can be obtained by integrating the stress along the actuator thickness.

$$N_{\psi\psi} = \frac{1}{2} \int T_{\psi\psi} \, d\alpha_3$$

$$= \pi_{12} \phi^e r$$

$$\times \left\{ \left(\alpha_3 + R\right) \sin^2 \left(\psi - \psi_o\right) + \left[r + \frac{h}{2} + h^e - \left(\alpha_3 + R\right) \cos \left(\psi - \psi_o\right) + R^2\right] \right\}^{\frac{3}{2}}. \quad (20)$$

The flexoelectric control moment can be calculated by integrating the product of the actuation stress and the corresponding moment arm (the distance between the local point and ring’s neutral layer). Considering that the patch thickness is far less than the ring thickness, the bending moment can be approximately obtained by multiplying the normal force by the distance between ring’s neutral layer and the patch as

$$M_{\psi\psi} = \int N_{\psi\psi} \, d\alpha_3$$

$$= \pi_{12} \phi^e r$$

$$\times \left\{ \left(\alpha_3 + R\right) \sin^2 \left(\psi - \psi_o\right) + \left[r + \frac{h}{2} + h^e - \left(\alpha_3 + R\right) \cos \left(\psi - \psi_o\right) + R^2\right] \right\}^{\frac{3}{2}}. \quad (21)$$
\[ M_{yy}^{\alpha} = \frac{h + h'}{2} N_{\alpha\alpha}^{\mu} = \frac{h + h'}{2} \int_{\alpha}^{\beta} T_{yy} d\alpha, \]

\[ = \frac{(h + h')}{2} \pi \phi r \]

\[ \times \left\{ \left[ R + \frac{h}{2} \right] \sin \left( \phi - \phi_0 \right) - \left[ \frac{r}{2} + \frac{h}{2} \right] \left[ \cos \left( \phi - \phi_0 \right) \right] \cos \left( \phi - \phi_0 \right) \right\}^{\frac{1}{2}} \]

\[ - \left\{ \left[ R - \frac{h}{2} \right] \sin \left( \phi - \phi_0 \right) + \left[ \frac{r}{2} + \frac{h}{2} \right] \left[ \cos \left( \phi - \phi_0 \right) \right] \cos \left( \phi - \phi_0 \right) \right\}^{\frac{1}{2}} \]

(21)

With the explicit expression of the membrane force and bending moment induced by the flexoelectric actuator, the flexoelectric actuation response of the ring can be obtained by the expression of transverse displacement, i.e., Eq.(16). Based on the formulations of the flexoelectric actuated vibration, microscopic actuation behaviors and specific modal control characteristics are analyzed and evaluated in case studies.

CASE STUDIES

With the geometric and material parameters listed in Table.1, microscopic actuation behavior of the flexoelectric patch and the modal control effects of the ring shell are evaluated. Since the converse flexoelectric effect depends on the electric field gradient, this gradient is analyzed first, followed by a detailed evaluation of the flexoelectric membrane control force and control moments. Steady-state maximal controllable displacements for various ring modes are analyzed with respect to actuator thickness, AFM probe radius, ring thickness and radius when the actuation voltage \( \phi' \) is set at 100 volts.

Table 1. Parameters and properties of the ring model.

<table>
<thead>
<tr>
<th>Properties</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring radius, ( R ), (m)</td>
<td>0.05</td>
</tr>
<tr>
<td>Ring width, ( b ), (m)</td>
<td>0.010</td>
</tr>
<tr>
<td>Ring thickness, ( h ), (m)</td>
<td>0.001</td>
</tr>
<tr>
<td>Young’s modulus of ring, ( Y )</td>
<td>1.556x10^9</td>
</tr>
<tr>
<td>Flexoelectric patch thickness, ( h^\alpha ) (\mu m)</td>
<td>50</td>
</tr>
<tr>
<td>AFM probe tip radius, ( r ), (nm)</td>
<td>50</td>
</tr>
<tr>
<td>AFM probe tip location, ( \psi_0 ), (rad)</td>
<td>0</td>
</tr>
<tr>
<td>Ring mass density, ( \rho ), (kg/m^3)</td>
<td>1100</td>
</tr>
<tr>
<td>Poisson’s ratio, ( \mu )</td>
<td>0.3</td>
</tr>
<tr>
<td>Flexoelectric constant, ( \pi_{12} ) (\mu C/m)</td>
<td>100</td>
</tr>
</tbody>
</table>

Actuator voltage, \( \phi' \) (V) | 100
Starting position of patch, \( \psi_1 \), (rad) | -\pi/40
Ending position of patch, \( \psi_2 \), (rad) | \pi/40

(1) Distribution of the Transverse Electric Field

With the parameters in Table.1, the transverse distribution of the electric field gradient defined in Eqs.(17,18) is analyzed first. Note that by flexoelectric stress equation, i.e., Eq.(19), the induced stress in the flexoelectric actuator is proportional to the gradient of electric field in the \( \alpha \) direction. Figure 3 shows the electric field gradient variation in the transverse direction right under the contact point with the AFM probe. Note that the vertical coordinate is set to start on the top surface of the flexoelectric patch and points downward to the bottom surface, i.e., 50\( \mu m \) in this case.

Figure 3 reveals that the electric field gradient is extremely high near the AFM probe (or thickness equals to zero). The gradient in the lower part of the patch is relatively small comparing with that of the upper surface. More precisely, the magnitude of the electric gradient at the half thickness of the patch (thickness equal to 25\( \mu m \)) is only 7.92x10^-7\% of the top surface (thickness equal to zero). Eq.(19) shows that the induced stress \( T_{yy} \) is proportional to the electric field gradient, thus the stress in the upper part of the flexoelectric patch is much larger than the lower part, which leads to the upper part of the patch an dominant component of the induced membrane control force \( N_{yy}^\alpha \) which is calculated by integrating the induced stress over the thickness of the patch. Further analysis indicates that the gradient distribution of various actuator thicknesses is similar. The top portions of various actuator thicknesses near the AFM probe are the same and the distribution follows the trend extending down as the actuator becomes thicker. Recall that the membrane force induced by upper part of the patch dominates the overall flexoelectric membrane control force. Hence the actuation effect increases little as the patch thickens and this behavior will be validated later.

(2) Microscopic Actuation Behavior of The Flexoelectric Actuator Patch
Microscopic behavior of the flexoelectric patch on ring actuation is analyzed here. Due to the sharpness of the AFM probe induced electric field, the circumferential distributions of the membrane force is a very sharp spike. Figure 4 shows the circumferential distribution (i.e., from $-\pi$ to $+\pi$) of the induced flexoelectric membrane force and control moment respectively marked on different scales. Note that the AFM probe is placed at $\psi_0=0$.

![Figure 4. Circumferential distribution of the actuation](image)

As shown in Fig.5, in the area on the left of the $\psi=0$ line, the circumferential loading points rightward and the right points leftward. In another word, the circumferential actuation points to where the AFM probe placed. Figure 5 also shows that the membrane force component dominates the total induced circumferential actuation. Such result is reasonable because the membrane force mainly influences the circumferential vibrations, while the bending moment mainly influences the transverse ones. The induced microscopic circumferential actuation behavior is shown in Fig.6. Note that the dimenson considered here is very small, thus the ring is nearly straight as a beam in Fig.6. The appearance of the AFM probe’s electric field results in a circumferential control actuation draging the material to where the probe locates.

![Figure 5. The distribution of the circumferential loading induced by actuator](image)

![Figure 6. Distribution of the induced circumferential loading](image)
The schematic diagram of induced microscopic transverse actuation behavior is shown in Fig.8. Again, Fig.8 is drawn at a very small region which is about 0.5μm. The bending control moment plays a dominant role in the transverse actuation, because the transverse deformation can be considered as a drastic bending resembling a Dirac delta function at the AFM probe’s contact point, shown in Fig.8. Note that this sharp bending has been described as the “buckling” characteristic in an earlier research on static deformation control of the cantilever beam based on AFM probe [16].

To analyze the characteristic of the flexoelectric patch under the control of the AFM probe, \( L_\alpha \) and \( L_\beta \) should be considered as a whole. The schematic diagram of the total distributed flexoelectric actuation on the ring is shown in Fig.9.

(3) Flexoelectric Patch Thickness

Recall that the microscopic behavior of the flexoelectric actuator shows that the actuation effect only exists in a very small region, i.e., near the AFM contact point. Thus, the patch size has little influence on the overall actuation effect. A relatively small patch, covers from \(-\pi/40\) to \(\pi/40\), is chosen for displacement control of rings and other material and geometric properties are summarized in Table 1.

The influence of flexoelectric patch thickness on control of ring oscillations is analyzed here. The flexoelectric induced magnitudes (or the maximal controllable displacements) driven at 100V at the steady are used to compensate ring’s vibration amplitudes, i.e., the larger the actuator induced magnitudes, the better the vibration control effects on rings. The maximal flexoelectric induced displacements of 2-6 modes are evaluated with respect to patch thickness of 25μm, 50μm, 75μm and 100μm in Fig.10. Note that steady-state controllable magnitudes of 2-6 modes are calculated when the actuation frequency is set to its natural frequency respectively.

When the ring actuated by the flexoelectric actuator driven by an AFM probe, a transverse deformation is induced as illustrated above. As the deformation occurs, the material is dragged to where the AFM probe locates. While the ring would resist such drag and results in a circumferential actuation. Thus, the overall actuation effect of the flexoelectric patch on the ring can be regarded as a drastic sharp bending effect within a very small region. Steady-state dynamic responses of the ring actuated by a flexoelectric actuator with various design parameters are analyzed next.

![Figure 7. The distribution of the transverse loading induced by actuator](image)

![Figure 8. Distribution of the induced transverse loading](image)

![Figure 9. Circumferential distribution of the total induced flexoelectric actuation near the AFM probe](image)

![Figure 10. Maximal controllable displacement (k=2-6 ring modes) with flexoelectric patch thickness](image)
to the probe. Thus the induced membrane force increases little as the patch thickness grows, which in turn leads to the limited increases of the maximal controllable displacement.

(4) AFM Probe Radius

Earlier analysis indicates that the AFM probe radius is a key factor influencing electric field gradient, membrane control force and control moments, consequently vibration control effectiveness. The influence of the AFM probe radius on the actuation effect is evaluated here. The maximal controllable displacement of $k=2$-$6$ ring modes are evaluated with respect to probe radius of 25 nm, 50 nm, 75nm and 100nm in Fig.11.

As shown in Fig.11, the maximal displacement decreases as the radius of AFM probe increases from 25nm to 100nm. Although the circumferential domain area of the electric field grows with probe radius, the maximal electric field gradient decreases, which leads to the decrease of the maximal membrane force in the flexoelectric patch. The latter decrease outweighs the former increases, as a result the actuation effect decreases as the AFM probe radius increases from 25nm to 100nm.

(5) Ring Thickness

After analyzing flexoelectric actuator characteristics (e.g., the AFM probe and the flexoelectric patch), the influence of the structural characteristics of the elastic ring is evaluated here. The maximal controllable displacements of $k=2$-$6$ modes are calculated with respect to ring thickness of 0.5 mm, 1.0mm, 1.5mm and 2.0 mm in Fig.12.

As shown in Fig.12, the maximal ring displacement increases as the ring thickness decreases. Keeping other parameters unchanged, it is obvious that as the ring thickness increases, the ring becomes stiffer resulting in higher natural frequencies. Generally, it is reasonable that to actuate a thin ring is easier than a thick ring and the actuation effect decreases as the ring thickness increases. For a specific ring mode, Fig.12 shows that the relationship between the maximal controllable displacement and ring thickness is quadric. The bending stiffness is proportional to the cubic of ring thickness and the induced bending moment is proportional to the ring thickness. Thus, the actuation effect, which is influenced by both factors, is proportional to the square of ring thickness.

(6) Ring Radius

The influence of ring radius on the actuation effect is analyzed here. Maximal controllable displacements of modes 2-6 with respect to ring radius of 25mm, 50mm, 75mm and 100mm are plotted in Fig.13.

Figure 13 indicates that the maximal displacement for each mode increases with the ring radius. With other parameters fixed, a ring with larger radius is softer than a smaller radius one. Furthermore, as the ring radius increases the natural frequency decreases, the ring become relatively easier to actuate. Thus, the actuation effect increases with the ring radius. The maximal controllable displacement, for each mode respectively, is proportional to the ring radius as plotted in Fig.13. For two rings with different radius, they share the same bending stiffness and actuation force. Thus, it’s reasonable to deduce that the relationship between actuation effect and the ring size is linear.

CONCLUSIONS

This study focused on the converse flexoelectricity based actuation characteristics of elastic rings. The converse flexoelectric effect depends on an inhomogeneous electric filed implemented by an AFM probe on a flexoelectric actuator patch. The distribution of the induced stress in the flexoelectric actuator patch is extremely nonuniform. The top surface carries most of the induced membrane control force. Furthermore, the transverse distribution of the electric gradient does not change
with the actuator thickness. In the circumferential direction, the distribution of actuation forces resembles the Dirac delta function due to the inhomogeneous electric field gradient induced by the AFM probe. The microscopic behaviors of actuator induced circumferential and transverse control actions were evaluated and the induced flexoelectric actuations were applied to drive a ring shell.

With the basic dynamics of elastic rings, the vibration characteristics driven by the flexoelectric actuator patch were studied and the maximal controllable displacements for five ring modes, i.e., \( k=2-6 \), were evaluated with respect to the patch (thickness), the probe (tip radius) and the ring (thickness and radius). The actuation effect is enhanced with a thicker flexoelectric actuator, a smaller AFM probe radius, thinner ring shell or larger ring radius. Accordingly, this study provides a basic understanding of the microscopic flexoelectric actuation behavior and serves as a foundation for the flexoelectricity based vibration control of flexible structures.

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