APT

Motivation

- CAPM was a eq. pricing model that required strong ass:
  1. agents max. over mean-var. utility.
  2. markets clear
  3. Everyone agrees on $E(r_i)$ and σ_i $\forall i$.

  So, it had strong implications (i.e. price all ass. w/ mkt port)
- APT is an attempt to have an asset pricing model w/ **WEAKER ASS**
  1. Assumes a return generating process. (a set of factors price ALL assets; so that all that's left are idiosyncratic shocks.)
  2. No arbitrage.

$$\boxed{I}\quad \tilde{R}_i = a_i + \vec{f}^T \vec{\beta}_i + \tilde{\varepsilon}_i \qquad \forall i$$

$$\boxed{II}\quad \text{where } E(\tilde{\varepsilon}_i) = E(\tilde{\varepsilon}_i \tilde{f}_k) = Cov(\tilde{\varepsilon}_i, \tilde{f}_k) = 0 \quad \forall i, \forall k$$

KEY $\longrightarrow$

$$\boxed{III}\quad \text{and} \quad E(\tilde{\varepsilon}_i \tilde{\varepsilon}_j) = \begin{cases} \sigma_i^2 \le \bar{\sigma}^2 & \text{if } i=j \\ 0 & \text{if } i \ne j \end{cases}$$

Noise is bdd. and idiosyncratic (i.e. $Cov(\varepsilon_i, \varepsilon_j) = 0$)

This allows us to identify factors.

OR in a system...

$$R_{N\times1} = \vec{A}_{N\times1} + B_{N\times K} f_{K\times1} + E_{N\times1}$$

$$\text{w/} \quad \Sigma_{N\times N} = E(EE^T) = \begin{pmatrix} \sigma_1 & & 0 \\ & \sigma_2 & \\ 0 & & \ddots \; \sigma_N \end{pmatrix} \quad \text{w/ bdd } \sigma_i \; \forall i.$$

$$\text{and} \quad E(E) = 0 \text{ and } E(Ef) = 0.$$

The idea is that ~~once we~~ we can explain movements in asset returns via a set of economic factors! Everything that is left is idiosyncratic specific to the firm.

LIKE CAPM, here we also believe that only systematic risk is priced. But whereas CAPM derives what the systematic risk must be from the bottom up, here we take a top-down approach.

What are factors?

- Comes from ~~the~~ factor analysis.
- Usually are returns ~~lag~~.
- Can be excess returns, so that $E(f_{i,t}) = 0$.

We will stick to this case in Monika's class.

ie) $f_{i,t} = \pi_t - E(\pi_t)$.
ex GDP growth
exc. int rate, ...

In this case, we can write APT as

$$\boxed{\begin{aligned} R_i &= E(R_i) + \beta_{i1} f_1 + \dots + \beta_{iK} f_K + \varepsilon_i \\ &\text{where } E(\varepsilon_i) = E(\varepsilon_i f_K) = 0 \ \forall i \ \forall K \\ &\text{and } E(\varepsilon_i, \varepsilon_j) = \begin{cases} \sigma_i^2 \le \bar{\sigma}^2 & \text{if } i=j \\ 0 & \text{o.w.} \end{cases} \end{aligned}}$$

factor sensitivity/loading

idiosyncratic shock

(ε_i uncorr. across companies)

Observe: In CAPM world, we described returns to comove w/ mkt exc. ret.
Here, we have movements due to movements of various factors.

What do factor models tell us about arbitrage?

If we know returns are generated from

$$r_i = \alpha_i + b_i^T f + \varepsilon_i ,$$

then a portfolio w/ the same <u>factor loadings</u> as r_i should have the <u>same</u> <u>expected</u> return.

Or, two well-diversified portfolios (w/ no idiosyncratic risk) w/ the same factor loadings should have the same exp return.

= ~~(Return)~~ ~~TRACKING PORTFOLIOS~~.

Def: Portfolio of assets that have same exp. return as the asset it tracks.

↓

What this portfolio looks like depends on what the "true" return generating factor model looks like.

eg) Suppose CAPM holds:

$$E(r_{GE}) = r_f + \beta_{GE}[E(r_m) - r_f].$$

Then, we can form a portfolio w/ r_f and r_m by choosing w_m s.t.

$$\frac{\mathrm{Cov}(w_m r_m + (1-w_m) r_f , r_m)}{\mathrm{Var}(r_m)} = \beta_{GE}$$

OR: $\cancel{E(r_m) = r_f + 1\cdot(E(r_m) - r_f)}$

$$E(w_m r_m + (1-w_m) r_f) = r_f + \beta_{GE}[E(r_m) - r_f]$$

$$\Rightarrow w_m[E(r_m) - r_f] = \beta_{GE}(E(r_m) - r_f) \Rightarrow \boxed{w_m = \beta_{GE}}$$

$$\Rightarrow \frac{w_m \, Cov(r_m)}{Var(r_m)} = \boxed{w_m = \beta_{GE}}$$

eg2) Similarly, if a multifactor model generates data, then we need to hold a portfolio of assets that have same β's as the asset.

Much better method

So, if we have 3 assets and 2 factors for GE:

$$r_A = E(r_A) + 1\cdot f_1 - 4f_2 + \varepsilon_A$$
$$r_B = E(r_B) + 3\cdot f_1 + 2f_2 + \varepsilon_B$$
$$r_C = E(r_C) + 1.5 f_1 + \varepsilon_C$$

Then, $1w_A + 3w_B + 1.5(1 - w_A - w_B) = \beta_{1,GE}$

$-4w_A + 2w_B = \beta_{2,GE}$

why?

BC: $w_A r_A + w_B r_B + (1-w_A-w_B) r_C$

$= (w_A E(r_A) + w_B E(r_B) + w_C E(r_C))$

$+ (w_A \cdot 1 + w_B \cdot 3 + w_C \cdot 1.5) f_1$

$+ \ldots$

These factor linearly so we can set them up in a system of equations!

OR, can solve using $\frac{Cov(\ ,\ )}{Var(r_m)}$.

But in multifactor world a little harder.

GENERALLY: Need K+1 securities to track K factors.

FACTOR TRACKING PORTFOLIO (Pure factor Portfolio).
↳ Loads 1 on one factor.
0 on all other factors.

eg) Let's take the 3 asset example.
A portfolio that tracks the 1^{st} factor will be one that solves....

$$W_A + 3W_B + 1.5(1 - W_A - W_B) = 1 \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$-4_A + 2W_B \qquad = 0$$

if track 2nd factor

Risk Premium of factor

λ_K = exp ret of factor tracking portfolio$_K$ − r_f

$$\boxed{\lambda_K = E(r_{f_K}) - r_f}$$

Note: The implied r_f is the return on a portfolio that has 0 loading on all factors.

Note2: If actual r.f. ≠ r.f. tracking portf. ret.
⇒ arbitrage!

Note3: λ_K can be + or −.

Note4: magnitude of λ_K depends on agg. supply of the factor in the economy and taste of investors. ??

(w/ $\beta_1 = 2$, $\beta_2 = 1$)

Notes: We can track original asset by

- Buy 2 factor portfolio 1
- " 1 " " 2
- Sell 2 r.f. portfolios (so weights sum to 1).

How does arbitrage come in? We can track an asset by holding a combination of factor tracking portfolios

Then, if $E(r_{TP}) \neq E(r_i) \Rightarrow$ arbitrage!

If we assume no arbitrage, then it must be that exp. ret. on asset = exp return of the (asset) tracking portfolio!

APT EQN

$$E(r_i) = \underbrace{r_f + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \ldots + \beta_{ik}\lambda_k}_{\text{exp. ret. on asset tracking portfolio}}$$

Market Efficiency

"Prices reflect "information" about fundamentals".

- Weak form
  - ↳ Prices reflect inf. in past data.
    - ⇒ Technical analysis will not produce π's.
  - ↳ Momentum strategies challenge this (Buy ~~past~~ past winners ~~buy~~ sell past losers).

- Semi-strong form.
  - ↳ Prices reflect all past ~~prices~~ data and publicly available inf. now.
    - ⇒ Fundamental analysis won't be π'ble.
  - ↳ Value-based strategies (B/M and E/P) challenge this.

- Strong form
  - ↳ Prices reflect all information (Public & Private).
    - ⇒ Insider trading does not π'ble.
  - ↳ We know insider trading π'ble.
    We observe creep ups b4 announcements.

Market Eff Paradox: If prices are efficient, nobody ~~does~~ should do research → ~~If~~ How do we compound prices in 1st place?