

# Why Portfolio Risk is Lower than Average Risk of Individual Assets

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**Proposition 1** *Let a portfolio consist of 2 risky assets  $S$  and  $B$ , and let  $w \neq 0$  be the portfolio weight on asset  $S$ . Then,  $\rho_{SB} < 1 \Rightarrow \sigma(r_w) < |w\sigma(r_s) + (1-w)\sigma(r_B)|$ . That is, portfolio's risk is lower than the weighted average risk of each security.*

**Proof.** As we saw in Lecture Note 6, portfolio's standard deviation can be written as...

$$\begin{aligned}\sigma(r_w) &= \sqrt{w^2\sigma^2(r_S) + (1-w)^2\sigma^2(r_B) + 2w(1-w)\sigma^2(r_S)\sigma^2(r_B)\rho_{SB}} \\ &= \sqrt{w^2\sigma^2(r_S) + (1-w)^2\sigma^2(r_B) + 2w(1-w)\sigma^2(r_S)\sigma^2(r_B)(1 + (\rho_{SB} - 1))} \\ &= \sqrt{[w\sigma(r_S) + (1-w)\sigma(r_B)]^2 - 2w(1-w)\sigma^2(r_S)\sigma^2(r_B)(1 - \rho_{SB})} \\ &< \sqrt{[w\sigma(r_S) + (1-w)\sigma(r_B)]^2} \text{ since } \rho_{SB} < 1 \\ &= |w\sigma(r_s) + (1-w)\sigma(r_B)|\end{aligned}$$

QED ■