Online appendix

April 30, 2013

A. General equilibrium model of heterogeneous bank shocks

The aggregation exercise in the main text assumes that the credit frictions have no effect on employment at firms borrowing from healthy lenders. The cross-sectional econometric approach requires this assumption, since it implicitly measures the employment shortfall at borrowers of less healthy syndicates relative to employment at borrowers of healthier syndicates. In general equilibrium, however, two opposing channels lead unconstrained firms also to adjust their labor input. First, some demand shifts from constrained to unconstrained firms. The shift may occur because relative prices at unconstrained firms fall, or because constrained firms ration their output. The magnitude of the labor reallocation depends on the substitutability of both the goods produced at the different firms and the labor used in production. Second, the financial shock generates a reduction in aggregate expenditure. The fall in aggregate expenditure reduces labor demand at unconstrained firms.

This appendix presents a stylized general equilibrium model that fully characterizes the relationship between the empirically-estimated relative employment outcomes and the aggregate effects that obtain in general equilibrium. The model contains three sectors: a household that consumes and supplies labor; monopolistically competitive firms that use labor to produce a differentiated good; and financial sector firms that supply credit lines to goods producers. Banking relationships enter the model through the assumption that each financial sector firm operates on an “island” with a subset of the goods producers, such that producers can only obtain financing from the financial sector firm on their island. A financial crisis causes an increase in the interest rate charged on the credit lines.

The model delivers simple closed-form expressions that illustrate the general equilibrium channels. An illustrative calibration using parameters consistent with macroeconomic fluctuations suggests that the general equilibrium effects may either magnify the effects from the partial equilibrium exercise or have at most a modest attenuating effect.
A.1. Household

A household consists of a continuum of individuals who supply labor to firms and pool consumption risk. The household’s objective function takes the form:

$$U = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(C_\tau, L_\tau),$$

and its flow budget constraint is

$$P_tC_t + B_t = w_tL_t + (1 + i_{t-1})B_{t-1} + T_t.$$  \hfill (2)

The consumption bundle $C_t$ is a Dixit-Stiglitz constant elasticity of substitution (CES) aggregation of differentiated varieties:

$$C_t = \left[ \int_{j,s,t} 1 \int_{j,s,t} 1 \xi_{j,s,t}^{\frac{1}{\sigma}} c_{j,s,t}^{\frac{\sigma-1}{\sigma}} djds \right]^{\frac{1}{\sigma-1}},$$

where $c_{j,s,t}$ is the consumption of variety $j$ produced by a firm operating on island $s$; $\sigma$ is the elasticity of substitution across varieties; and $\xi_{j,s,t}$ is a variety-specific taste-shock. Variety $j$ on island $s$ is assumed to also be differentiated from all varieties produced on island $s'$. The bundle price $P_t$ is the cost of purchasing one unit of $C_t$:

$$P_t = \left[ \int_{j,s,t} 1 \int_{j,s,t} 1 \xi_{j,s,t}^{\frac{1}{\sigma}} p_{j,s,t}^{1-\sigma} djds \right]^{\frac{1}{1-\sigma}},$$

where $p_{j,s,t}$ is the price of variety $j$ produced on island $s$.

$L_t$ is a CES aggregation of the labor supplied to different firms, with elasticity of substitution $\nu$:

$$L_t = \left[ \int_{j,s,t} 1 \int_{j,s,t} 1 L_{j,s,t}^{\frac{\nu+1}{\nu}} djds \right]^{\frac{\nu}{\nu+1}},$$

where $L_{j,s,t}$ is the labor supplied to firm $j$ on island $s$. $w_t$ is the composite wage defined as the compensation earned by the household from optimally allocating $L_t = 1$ unit of labor taking the firm wage distribution as given, and satisfies

$$w_t = \left[ \int_{j,s,t} 1 \int_{j,s,t} 1 w_{j,s,t}^{1+\nu} djds \right]^{\frac{1}{1+\nu}},$$

where $w_{j,s,t}$ is the wage paid by firm $j$ on island $s$.

As $\nu \to \infty$, labor becomes a homogenous input and, from the first-order condition \(7\)
below, the real wage must equate across firms. As $\nu \to 0$, it becomes very costly to reallocate labor input and large real wage differentials may obtain. Thus $\nu$ determines how easily labor can shift among firms.\footnote{With utility separable over consumption and labor and a constant Frisch elasticity $\eta = \nu$, this setup collapses to the firm-specific labor input model. In general, it allows the elasticity of labor supply to an individual firm to differ from the intertemporal labor supply decision.} This makes it similar to other frictions such as search costs or labor adjustment costs that would also delay the reallocation of labor from constrained to unconstrained firms.

Finally, $B_t$ denotes purchases of a riskless bond at time $t$, $i_t$ is the nominal interest rate, $\beta < 1$ is the discount factor, and $T_t$ contains dividend payouts from the goods sector and the financial sector and rebates from the financial sector as described further below.

The household’s first order conditions are:

\[
- \left[ \frac{L_{j,s,t}}{L_t} \right]^{\frac{1}{\nu}} \frac{u_{Lt}}{u_{Ct}} = \frac{w_{j,s,t}}{P_t} \forall j, s, \quad (7)
\]

\[
u_{ct} = \mathbb{E}_t \left[ \beta (1 + i_t) \frac{P_t}{P_{t+1}} u_{ct+1} \right], \quad (8)
\]

\[
c_{j,s,t} = \xi_{j,s,t} \left( \frac{p_{j,s,t}}{P_t} \right)^{-\sigma} C_t \forall j, s. \quad (9)
\]

### A.2. Goods producers

The production-side of the economy consists of firms that operate on a unit square, with $s \subseteq [0, 1]$ indexing the first dimension (islands) and $j \subseteq [0, 1]$ the second dimension (firms on an island). Firm $j$ operating on island $s$ produces output using the production technology

\[
y_{j,s,t} = a_{j,s,t} l_{j,s,t}^{1-\gamma} \quad (10)
\]

and subject to the demand curve (9). The firm distributes all profits to the household each period.\footnote{I leave unmodeled the agency problem that gives rise to this dividend distribution policy. For example, it would arise if firms’ managers could abscond with any undistributed profits at the end of the period.}
Firms maximize expected profits, given by

\[ \Pi_{j,s,t} = [1 - \delta] [p_{j,s,t} y_{j,s,t} - (1 + r_{s,t}) w_{j,s,t} l_{j,s,t}] + \delta [0]. \]  

(11)

The profit-maximizing price equals a markup over marginal cost:

\[ p_{j,s,t} = M (1 + r_{s,t}) \frac{w_{j,s,t}}{(1 - \gamma) a_{j,s,t} l_{j,s,t}^{-\gamma}}, \]

(12)

where \( M \equiv \frac{\sigma}{\sigma - 1} \) is the markup.\(^3\) Together, (9)-(12) and the market-clearing condition \( c_{j,s,t} = y_{j,s,t} \) yield the following equation for firm labor demand:

\[ l_{j,s,t} = \left[ \frac{\xi_{j,s,t} a_{j,s,t} (M (1 + r_{s,t}) \frac{w_{j,s,t}}{(1 - \gamma) a_{j,s,t}})^{-\sigma}}{P_t C_t} \right] \frac{1}{1+\gamma(\sigma-1)}. \]  

(13)

The joint distribution of \((\xi_{j,s,t}, a_{j,s,t})\) is the same on every island and time-invariant. Thus the only difference across islands comes from the financial sector, and not from technology or taste. The assumption of time-invariance could be relaxed without changing any of the important results.

### A.3. Financial sector

Each of a continuum of financial firms operates on a single island. Financial firms provide credit lines to the goods producers on their island, using the payments from surviving firms to cover the losses they incur from making wage payments to the workers of exiting firms. In addition, each financial firm diverts a fraction \( \zeta_{s,t} \) of the payments earned from surviving firms to the household sector. \( \zeta_{s,t} \) introduces variation in bank health into the model without requiring an explicit treatment of nonperforming assets. In an extended model, it might reflect writedowns on mortgages that are liabilities of the household sector. The distribution of \( \zeta_{s,t} \) across islands is exogenous, reflecting the assumption that the distress in the financial sector during 2007-09 originated in areas other than corporate lending.

\(^3\)Note that each firm acts as a price-taker in the labor market despite using a differentiated labor input. Following Woodford (2003), one may justify this assumption by having a continuum of “industries” on each island, where each industry consists of a continuum of firms receiving identical technology and taste shocks and hiring from the same pool of workers. All firms in an industry then hire the same amount of labor and set the same wage. However, individual firms do not have monopsony power in wage-setting.

\(^4\)From equation (12), the credit line requirement enters the model isomorphic to a payroll-in-advance constraint that would require the firm to obtain intraperiod working capital (see e.g. Chari, Christiano and Eichenbaum 1995). An important theoretical difference between the working capital and credit line models is that the former requires a supply of intraperiod liquidity in the model whereas the latter does not. Most syndicated loans take the form of credit lines.
A financial firm that diverts $\zeta_{s,t}$ to the household sector lends to the nonfinancial business sector at rate $r_{s,t}$. Without loss of generality, let $r_s < r_{s'}$ if $s > s'$, so that a higher index denotes a lower interest rate. New financial firms can enter an island but must pay the same earnings penalty $\zeta_{s,t}$ as the incumbent firm. In equilibrium no new firms enter but incumbent firms earn zero profits, giving the condition

$$r_{s,t} = \frac{\delta}{1-\delta} + \zeta_{s,t}. \quad (14)$$

A.4. Equilibrium

An equilibrium consists of quantities ($\{c_{j,s,t}\}, \{l_{j,s,t}\}$) and prices ($\{p_{j,s,t}\}, \{w_{j,s,t}\}$) subject to the household’s first-order conditions (7)-(9), the firms’ optimal pricing decision (12) and labor demand (13), the financial sector firms’ lending rate condition (14), the exogenous driving forces ($\{a_{j,s,t}\}, \{\xi_{j,s,t}\}, \{\zeta_{s,t}\}$), and the market-clearing conditions

$$l_{j,s,t} = L_{j,s,t} \forall j, s, \quad (15)$$
$$y_{j,s,t} = c_{j,s,t} \forall j, s, \quad (16)$$
$$B_t = 0. \quad (17)$$

A.5. Relation to empirical work

Let $\hat{x} = \frac{dx}{\bar{x}}$, where $\bar{x}$ is the value in a zero-inflation non-stochastic steady-state. Substituting for the firm real consumption wage $\frac{w_{j,s,t}}{\hat{P}_t}$ in (13) using the intraperiod labor-consumption tradeoff (7), imposing market clearing in the labor market, and using the aggregate relationship $\hat{C}_t = [1 - \gamma] \hat{L}_t$ as well as the economy-wide counterpart to (7) yields the equilibrium employment function

$$\hat{l}_{j,s,t} = \kappa \left[ 1 + \frac{\sigma}{(1-\gamma)\nu} \right] \hat{C}_t - \kappa\sigma \left[ \hat{w}_t - \hat{P}_t \right] - \kappa\sigma \hat{r}_{s,t} + \kappa \left[ \hat{\xi}_{j,s,t} + (\sigma - 1) \hat{a}_{j,s,t} \right], \quad (18)$$

where $\kappa \equiv \frac{\nu}{(1-\gamma)\nu + \sigma(1+\gamma)} \subseteq [0, 1].$

In steady-state, the fraction $\zeta_{s,t}$ diverted to households equals zero at all financial firms, and $r_{s,t} = \bar{r} = \frac{\delta}{1-\delta}$. A “financial crisis” occurs at time $t_0$, resulting in $\zeta_{s,t} \geq 0 \forall s$. Let $m_{s,t} = \chi\hat{r}_{s,t}$ denote an observed variable, with $\chi$ an unknown scalar.

Grouping terms in (18) yields an estimation equation corresponding to the regression
specification in the main text:

\[ \hat{l}_{j,s,t} = \beta_0 + \beta_1 m_{s,t} + \epsilon_{j,s,t}, \]  

(19)

where:

\[ \beta_0 = \alpha_1 \hat{C}_t - \alpha_2 \left[ \hat{w}_t - \hat{P}_t \right] \] depends on aggregate output and the real wage, with \( \alpha_1 = \kappa \left[ 1 + \frac{\sigma}{\rho(1-\gamma)} \right] > 0 \) and \( \alpha_2 = \kappa \sigma > 0 \) the semi-elasticities;

\[ \beta_1 = -\frac{\kappa \sigma}{\chi} \];

\[ \epsilon_{j,s,t} = \kappa \left[ \hat{\xi}_{j,s,t} + (\sigma - 1) \hat{a}_{j,s,t} \right] \] is a composite of the firm-level idiosyncratic shocks and has mean zero by assumption.

Consider first the partial equilibrium exercise of cumulating the employment shortfall relative to firms that borrow from the bank on island 1. Using the approximation that the level employment deviation \( l_{j,s,t} - \bar{l} \approx \hat{l}_{j,s,t} * \bar{l} \), where \( \bar{l} \) is steady-state employment,

\[ \text{Shortfall}^{PE} = \bar{l} \int_0^1 \hat{l}_{j,1,t} dj - \bar{l} \int_0^1 \int_0^1 \hat{l}_{j,s,t} djs 
= \bar{l} \int_0^1 \beta_1 (m_{1,t} - m_{s,t}) ds > 0. \]  

(20)

This is the model counterpart to the aggregation exercise implemented in the main text. Note that while employment falls as a result of the financial crisis, \( \text{Shortfall}^{PE} \) is defined to be positive.

Next consider the general equilibrium exercise of cumulating the employment shortfall relative to firms that borrow from the bank on island 1. Using the approximation that the level employment deviation \( l_{j,s,t} - \bar{l} \approx \hat{l}_{j,s,t} * \bar{l} \), where \( \bar{l} \) is steady-state employment,

\[ \text{Shortfall}^{GE} = -\bar{l} \int_0^1 \int_0^1 \hat{l}_{j,s,t} djs 
= \text{Shortfall}^{PE} - \bar{l} \beta_0 
= \text{Shortfall}^{PE} - \bar{l} \left( \hat{l}_{1,t} \right), \]  

(21)

where \( \hat{l}_{1,t} \) is the average employment deviation at firms on island 1. Equation (21) states that the difference between the partial equilibrium exercise and the general equilibrium deviation of aggregate employment from an economy where all firms can borrow at \( \bar{r} \) is given by the response of employment at unconstrained firms.

Finally, define the percent difference between the partial equilibrium and the general
equilibrium employment losses as

\[
\text{Difference} \equiv -\frac{\text{Shortfall}^{\text{GE}} - \text{Shortfall}^{\text{PE}}}{\text{Shortfall}^{\text{PE}}}
\]

\[
= -\frac{\beta_0}{\beta_1 \tilde{m}_t},
\]

where \( \tilde{m}_t = \int_0^1 m_{s,t} ds \) denotes the average level of bank health. As constructed, a negative value of \( \text{Difference} \) means that the decline in employment in general equilibrium exceeds the partial equilibrium decline.

The sign of \( \text{Difference} \) depends on the average deviation of employment at unconstrained firms, \( \beta_0 \), since \( \beta_1 \tilde{m}_t = -\kappa \sigma \tilde{r} < 0 \). The first term of \( \beta_0 \), \( \alpha_1 \hat{C}_t < 0 \), constitutes the aggregate demand channel, whereby the decline in aggregate consumption in response to the financial shock lowers demand at unconstrained firms. The second term, \( \alpha_2 \left[ \hat{w}_t - \hat{P}_t \right] \), reflects the movement of workers from constrained to unconstrained firms. Writing \( \left[ \hat{w}_t - \hat{P}_t \right] = \left[ \left( \hat{p}_{1,t} - \hat{P}_t \right) + \left( \hat{w}_t - \hat{p}_{1,t} \right) \right] \), the reallocation of workers has two components. The first, \( \hat{p}_{1,t} - \hat{P}_t < 0 \), is the change in the (average) relative price at unconstrained firms. The fall in the relative price shifts product demand to unconstrained firms. The second term, \( \hat{w}_t - \hat{p}_{1,t} < 0 \), is the change in the economy-wide average wage deflated by the product price at unconstrained firms. The fall in the cost of labor relative to the price of their output induces unconstrained firms to move down their labor demand curves. The elasticity \( \alpha_2 \) increases in the substitutibility of the goods produced by the constrained and unconstrained firms \( (\sigma) \), but decreases in the degree of frictions to workers’ switching firms \( (\frac{1}{\nu}) \).

5 It is worth emphasizing that the regression equation (19) and the correction (22) apply to more general settings. Appendixes C and D offer two examples, deriving the corresponding expressions in models allowing for sticky prices and where some firms completely lose access to credit markets, respectively. The general equilibrium logic does not depend on the particulars of the modeling assumptions.

A.6. Solution

The parsimony of the model setup allows for a closed-form solution of \( \beta_0 \) in terms of the model’s primitives. Letting \( \tilde{r}_t \equiv \int_0^1 \tilde{r}_{s,t} ds \) denote the average percent increase in the cost of

\[\frac{\partial \kappa}{\partial \sigma} = (1 - \gamma) \kappa^2 > 0, \quad \text{and} \quad \frac{\partial \kappa}{\partial \nu} = -\sigma^2 \kappa^2 < 0.\]
a credit line, Appendix B proves

$$\beta_0 = \kappa \left[ \sigma - \Upsilon^{-1} \right] \tilde{r}_t,$$

(23)

where \( \Upsilon \) denotes the real rigidity in the model, defined formally as in Ball and Romer (1990) as the elasticity of a firm’s optimal relative price with respect to changes in aggregate expenditure. The appendix provides an expression for \( \Upsilon \) in terms of primitive parameters of the model. The sign of \( \beta_0 \) thus depends on the relative magnitude of the elasticity of substitution across goods and the model’s real rigidity.

Labor demand at unconstrained firms depends on the economy’s real rigidity for the same reason that real rigidity increases the response of output to monetary shocks in sticky price models. With large real rigidity, that is, \( \Upsilon \) small, firms resist changes in their relative prices in response to changes in aggregate conditions. In a sticky price monetary model, this resistance leads to smaller individual price changes, lower inflation, and higher output following a monetary shock. In the present environment, greater real rigidity implies that the optimal relative price at unconstrained firms falls by less in response to a given decline in aggregate demand. This has two effects. First, product demand and employment shift to unconstrained firms only to the extent that they actually lower their relative prices. Second, in equilibrium prices at unconstrained firms must fall relative to prices at constrained firms experiencing higher marginal cost. It follows that with large real rigidity, output must contract even more to induce the unconstrained firms to lower their relative prices, in which case the general equilibrium employment decline exceeds the partial equilibrium decline.

A.7. Calibration

I consider calibrations of \( \beta_0 \) for the case of preferences separable over consumption and labor input (SEP) and allowing for complementarity of the form suggested by Greenwood, Hercowitz and Huffman (1988) (GHH):

$$u(C, L) = \begin{cases} \frac{[C - \phi (L^{1+\frac{1}{\rho}})]^{1-\frac{1}{\rho}}}{C^{1-\frac{1}{\rho}} - L^{1+\frac{1}{\rho}}} & \text{GHH} \\ \frac{C^{1-\frac{1}{\rho}}}{L^{1+\frac{1}{\rho}}} & \text{SEP}. \end{cases}$$

One can show that with these sets of preferences,
\[ \beta_0^{GHH} = \left[ \kappa \sigma - \left( \gamma + \frac{1}{\varepsilon} \right)^{-1} \right] \tilde{r}_t, \quad (24) \]

\[ \beta_0^{SEP} = \left[ \kappa \sigma - \left( \gamma + [1 - \gamma] \frac{1}{\rho} + \frac{1}{\varepsilon} \right)^{-1} \right] \tilde{r}_t. \quad (25) \]

The choice of parameters reflects a compromise between elasticities that match microeconomic studies and those found necessary to generate plausible macroeconomic fluctuations. In all rows I set the elasticity of substitution across goods \( \sigma \) to 6.5, the elasticity of output to labor \( 1 - \gamma \) to 2/3, and the Frisch elasticity of labor supply \( \varepsilon \) to 2. The values of \( \sigma \) and \( \gamma \) are relatively uncontroversial. Hall (2009) argues that a Frisch elasticity of 2 best captures the slope of the aggregate labor supply curve in a model without explicit treatment of the extensive margin, and is consistent with an elasticity along the intensive margin of hours per person of about 0.7. For the case of separable preferences, I report results for an intertemporal elasticity of substitution \( \rho \) of 1 and 3, with \( \rho = 3 \) the preferred parameterization. Rotemberg and Woodford (1997) and Woodford (2003) argue forcefully that in calibrating models where all output takes the form of nondurable consumption goods, one should nonetheless use an intertemporal elasticity of substitution higher than that typically estimated for nondurable consumption to compensate for the absence of the more interest-elastic categories of consumer durables and investment. The value of 3 is still only half of what Rotemberg and Woodford estimate to match U.S. business cycle fluctuations, and not far above recent estimates by Gruber (2006) and Nakamura et al. (2011).

The only parameter not commonly found in macroeconomic models is \( \nu \). From a microeconomic perspective, \( \nu \) is the elasticity of labor supply to an individual firm. Ashenfelter, Farber and Ransom (2010) and Manning (2011) provide recent surveys of studies that estimate this elasticity. Many of the the studies find a very low elasticity, between 0.1 and 2 in the short-run and between 2 and 4 in the long-run. Webber (2011) conducts an analysis of the universe of U.S. firms and finds an average elasticity of 1.08, with 90 percent of firms exhibiting an elasticity below 1.75. From a macroeconomic perspective, \( \nu \) governs the degree of strategic complementarity in price-setting across firms through its influence on \( \kappa \). Woodford (2003) advocates for a value of \( \Upsilon \) of between 0.1 and 0.15, the latter which implies a value of \( \nu \) between 1 and 2.5. Accordingly, I report results for \( \nu = 1, 2, 3 \).\(^6\) For comparison, Manning (2011) raises a number of concerns with the empirical methodologies that may bias the microeconomic estimates down. Nonetheless and as discussed further below, the model abstracts from a number of features that would increase real rigidity, suggesting that estimates at the low end of the microeconomic literature may still be appropriate for macroeconomic purposes.

\(^6\)Manning (2011) raises a number of concerns with the empirical methodologies that may bias the microeconomic estimates down. Nonetheless and as discussed further below, the model abstracts from a number of features that would increase real rigidity, suggesting that estimates at the low end of the microeconomic literature may still be appropriate for macroeconomic purposes.
I also report the case of $\nu = 1000$, corresponding to the frictionless labor market.

### Table 1: Calibrated percent difference between partial equilibrium and general equilibrium effects

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\nu$</th>
<th>GHH</th>
<th>SEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>-0.72</td>
<td>0.04</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-0.12</td>
<td>0.38</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.08</td>
<td>0.49</td>
</tr>
<tr>
<td>1</td>
<td>1000</td>
<td>0.48</td>
<td>0.71</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>-0.72</td>
<td>-0.36</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>-0.12</td>
<td>0.11</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.08</td>
<td>0.27</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td>0.48</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Notes: Each cell of the table reports the percent difference between the employment change in partial equilibrium and general equilibrium, given by the formula $-\frac{\beta_0}{\beta_1 m_t}$, for the values of the cross-sectional labor supply elasticity $\nu$ and intertemporal elasticity of substitution $\rho$ reported in the first two columns. A negative entry indicates that the general equilibrium employment decline exceeds the partial equilibrium decline. The column labeled GHH reports values using the Greenwood, Hercowitz Huffman (1988) preference structure. The column labeled SEP reports values for preferences separable over consumption and labor input. In all cells, $\sigma = 6.5$ is the elasticity of substitution across goods, $\gamma = 1/3$ is one minus the elasticity of output with respect to labor, and $\varepsilon = 2$ is the Frisch elasticity of labor supply.

The table indicates that for the preferred set of parameter values, with $\rho = 3$ and $\nu \in \{1, 2, 3\}$, the model’s general equilibrium channels either amplify the partial equilibrium employment decline or have a modest attenuating effect. Consumption-hours complementarity magnifies the fall in aggregate demand, so that for a given set of parameter values GHH preferences always produce lower employment at unconstrained firms than separable preferences (recall that a negative entry in the table indicates that the general equilibrium employment decline exceeds the partial equilibrium decline). With $\nu = 1$, the partial equilibrium decline underestimates the general equilibrium decline under both preference specifications. With $\nu = 2$, corresponding to the rough midpoint of microeconomic estimates, the general equilibrium decline is within 12 percent of the partial equilibrium decline. Finally, a value of $\nu = 3$ implies a small overestimate of the aggregate effects in partial equilibrium, with the adjustment between 8 and 27 percent depending on the preference structure.

The calibration with separable preferences has some sensitivity to the intertemporal elasticity $\rho$. Many macroeconomic papers set $\rho = 1$, consistent with a balanced growth path. In that case, and with separable preferences and $\nu$ at the upper limit of the preferred range, the partial equilibrium decline exceeds the general equilibrium decline by 49 percent. However, with $\rho = 1$ and no other frictions, the model requires extremely large shocks (from $^7$The calibration with GHH preferences is invariant to $\rho$ because of the absence of wealth effects in the labor supply decision.)
any source) to cause a large movement in output.\(^8\) The smaller general equilibrium decline then occurs because the financial shock generates an implausibly small fall in aggregate output as households smooth consumption. It is in this sense that Rotemberg and Woodford (1997) argue that a model without investment or durable goods needs a higher value of \(\rho\) to generate realistic fluctuations in economic activity (see also Barsky, House and Kimball (2007)). Furthermore, it may be appropriate to assume less consumption smoothing in an environment where credit frictions have increased. Finally, a smaller \(\rho\) does not necessarily imply a large difference between partial and general equilibrium; with \(\rho = \nu = 1\), the difference is 4 percent.

On the other hand, adding a number of features to the model specification would further push the general equilibrium channels toward amplifying the direct effects. For example, variable elasticity preferences that contain positive comovement between a firm’s market share and its optimal markup would increase real rigidity and lower employment at unconstrained firms (Kimball 1995).\(^9\) Likewise, accounting for the input-output structure of the economy would lower employment demand if labor and intermediate inputs are not substitutes in production (Basu 1995). Intuitively, if unconstrained firms use the output of constrained firms in their production process, then their marginal cost also rises as a result of the financial shock. Real wage rigidity reduces the incentive for unconstrained firms to poach labor from constrained firms, which in the extreme case results in a decline in employment at unconstrained firms for any calibration or preference specification.\(^10\) Finally, the relative price component of the reallocation channel would diminish to the extent that constrained firms dis-hoard labor rather than reduce output.

In sum, the partial equilibrium effects in the main text do not appear to substantially overstate the importance of the credit supply channel in general equilibrium.

\(^8\)With \(\rho = 1\) and separable preferences, output falls by 0.44 percent in response to a uniform 1 percent increase in the cost of a credit line. For comparison, with \(\rho = 1\) a 1 percent fall in aggregate technology corresponds to a fall in output of 0.38 percent.

\(^9\)Formally, the model with variable elasticity of demand behaves exactly as that described above, except that \(\kappa\) generalizes to \(\kappa = \frac{\nu}{\sigma(1-\gamma)(1+\sigma\epsilon_{M,c})+\sigma(1+\gamma\nu)}\), where \(\sigma\) now denotes the elasticity of demand in the symmetric steady-state and \(\epsilon_{M,c} \geq 0\) denotes the elasticity of the markup with respect to market share at the symmetric steady-state.

\(^10\)With a fixed real wage and labor demand-determined, the reallocation channel shuts down completely leaving only the aggregate demand channel to act in general equilibrium. The formal result follows from replacing the labor-consumption tradeoff (7) with the assumption \(\frac{w_t}{P_t} = \omega_0\) in Appendix C and taking the limit of the solution as \(\alpha \to 0\).
B. Solution of the model

Log-linearizing equations (7) and (8) gives:

\[ \hat{w}_{j,s,t} - \hat{P}_t = \left[ \frac{1}{\eta} - \frac{1}{\nu} - \omega_{CL} \right] \hat{L}_t + \left[ \frac{1}{\nu} \right] \hat{L}_{j,s,t} + \left[ \frac{1}{\rho} - \omega_{LC} \right] \hat{C}_t, \]  

(26)

\[ -\frac{1}{\rho} \hat{C}_t + \omega_{CL} \hat{L}_t = E_t \left[ i_t - \pi_{t+1} - \frac{1}{\rho} \hat{C}_{t+1} + \omega_{CL} \hat{L}_{t+1} \right], \]  

(27)

where:

\[ \rho = -\left( C_{uL/LC}^{uC} \right)^{-1}; \quad \eta = \left( L_{uL/LL}^{uL} \right)^{-1}; \quad \omega_{LC} = -C_{uLC}^{uC} L_{uL}^{uL}; \quad \omega_{CL} = L_{uC}^{uL} L_{uL}^{uC}; \quad \text{and} \quad \pi_t = \hat{P}_t - \hat{P}_{t-1}. \]

In addition, an aggregate version of (26) obtains:

\[ \hat{w}_t - \hat{P}_t = \left[ \frac{1}{\eta} - \omega_{CL} \right] \hat{L}_t + \left[ \frac{1}{\rho} - \omega_{LC} \right] \hat{C}_t. \]  

(28)

Substituting (28) into the firm labor equilibrium condition production (18), using the production function (10), integrating, and solving gives an expression for the quantity of labor in terms of model primitives and the exogenous shocks:

\[ \hat{L}_t = -\left[ \gamma + (1 - \gamma) \left( \frac{1}{\rho} - \omega_{LC} \right) + \left( \frac{1}{\eta} - \omega_{CL} \right) \right]^{-1} \hat{r}_t. \]  

(29)

From equation (29), the aggregate labor response does not depend on the mobility of labor across firms, governed by \( \nu \). This independence is an artifact of using first-order methods to solve the model. At the first order, only the average marginal cost increase matters to aggregate variables, and in particular the dispersion of the cost shock does not affect aggregate quantities. Higher order terms of \( \hat{L}_t \) include higher moments of the distribution of \( \hat{r}_{s,t} \) and do depend on \( \nu \).

The fact that \( \Upsilon \) measures the model’s real rigidity follows from log-linearizing (12) to write the firm’s optimal relative price as

\[ \hat{p}_{j,s,t} - \hat{P}_t = \left[ \hat{w}_{j,s,t} - \hat{P}_t \right] + \gamma \hat{L}_{j,s,t} + \hat{r}_{s,t} - \hat{a}_{j,s,t} \]

\[ = \left[ \gamma + \frac{1}{\nu} \right] \hat{L}_{j,s,t} + \left[ \frac{1}{\eta} - \frac{1}{\nu} - \omega_{CL} \right] \hat{L}_t + \left[ \frac{1}{\rho} - \omega_{LC} \right] \hat{C}_t, \]

where the second line substitutes the labor supply relationship (26) and drops the shock terms \( \hat{r}_{s,t} - \hat{a}_{j,s,t} \) which have no effect on the subsequent calculations. Replacing \( \hat{L}_{j,s,t} \) with
the equilibrium employment level and rearranging terms gives
\[ \hat{p}_{j,s,t} - \hat{P}_t = \kappa \left\{ \gamma + \left[ (1 - \gamma) \left( \frac{1}{\rho} - \omega_{LC} \right) + \left( \frac{1}{\eta} - \omega_{CL} \right) \right] \right\} \hat{C}_t. \]

By definition, \( Y \equiv \kappa \left\{ \gamma + \left[ (1 - \gamma) \left( \frac{1}{\rho} - \omega_{LC} \right) + \left( \frac{1}{\eta} - \omega_{CL} \right) \right] \right\} \) measures the elasticity of the optimal relative price with respect to aggregate demand.

Using labor-consumption tradeoff (28), the aggregate relationship \( \hat{C}_t = [1 - \gamma] \hat{L}_t \), and equation (29) in the definition of \( \beta_0 \), rearranging, and using the definition of \( Y \) gives the expression in equation (23) in the text.

C. Model with sticky prices

In the sticky price model, a firm can only change its price with Calvo probability \((1 - \alpha)\) each period. Firms discount profits using the household’s stochastic discount factor. Assume that the financing shock disappears with probability \( \mu \) each period. The central bank’s reaction function takes the form
\[ i_t = \phi_i \pi_t + \phi_Y \hat{Y}_t. \] (30)

Note that \( Y_t \) and \( i_t \) in (30) measure deviations from the no financial crisis state, rather than from the natural rate of output and interest that would adjust for the cost-push shock.

The New Keynesian Phillips curve and IS curve complete the model:
\[ \pi_t = \lambda \theta E_t \sum_{r=t}^{\infty} \beta^{r-t} \hat{Y}_r + \frac{\lambda}{1 - \beta} \hat{r}_t \] (31)
\[ \left[ 1 - \frac{\rho \omega_{CL}}{1 - \gamma} \right] \hat{Y}_t = \left[ 1 - \frac{\rho \omega_{CL}}{1 - \gamma} \right] E_t \left[ \hat{Y}_{t+1} \right] - \rho E_t \left[ \hat{r}_t - \pi_{t+1} \right], \] (32)

where \( \lambda \equiv \frac{(1 - \alpha)(1 - \alpha \beta)(1 - \gamma)\kappa}{\alpha} > 0 \), and \( \theta \equiv \left[ \frac{1}{\rho} - \omega_{LC} + \frac{\gamma + \frac{1}{\eta} - \omega_{CL}}{1 - \gamma} \right] \) is the elasticity of average real marginal cost (net of the financial shock) with respect to output.

The solution is via the method of undetermined coefficients, with
\[ \hat{Y}_t = \Theta_Y \hat{r}_t \]
\[ \hat{\pi}_t = \Theta_\pi \hat{r}_t \]
\[ \hat{i}_t = \Theta_i \hat{r}_t. \]
Solving,

\[ \Theta_Y = \frac{\rho\lambda[1-\phi\pi]}{1-\beta\mu} \left[ 1 - \frac{\rho\lambda[1-\phi\pi]}{1-\beta\mu} \right] + \rho\phi_Y \] (33)

\[ \Theta_i = \phi\pi \left[ \frac{\lambda\theta\Theta_Y + \lambda}{1-\beta\mu} \right] + \phi\theta\Theta_Y \] (34)

\[ \Theta_\pi = \frac{\lambda\theta\Theta_Y + \lambda}{1-\beta\mu} \] (35)

Labor demand remains governed by equation (13), which in log deviation form implies

\[ \hat{l}_{j,s,t} = [1 - \gamma]^{-1} \Theta_Y \tilde{r}_t - \sigma [1 - \gamma]^{-1} \left[ \hat{p}_{j,s,t} - \hat{P}_t \right] + [1 - \gamma]^{-1} \left[ \hat{\xi}_{j,s,t} - \hat{\alpha}_{j,s,t} \right]. \] (36)

To characterize the regression model, it remains to specify \( \hat{p}_{j,s,t} - \hat{P}_t \). Profit maximization implies that a firm resetting its price in period \( T \geq t_0 \) during which the financial shock continues to bind sets a relative price (ignoring the idiosyncratic shock terms) of

\[ \hat{p}_{j,s,T}^* - \hat{P}_T = \frac{\alpha\lambda\theta\Theta_Y}{1-\alpha} \left[ 1 - \alpha\beta\mu \right] \tilde{r}_T + \frac{\alpha\beta\mu}{1-\alpha\beta\mu} \Theta_\pi \tilde{r}_T + \frac{\alpha\lambda}{1-\alpha} \left[ 1 - \alpha\beta\mu \right] \tilde{r}_{s,T}. \] (37)

It simplifies the subsequent algebra to set \( T = t_0 \), so that a measure \( \alpha \) of firms have prices set before the financial shock hits and the remaining firms all reset their price in the current period anticipating that the financial crisis will persist with probability \( \mu \). In that case, the relative price of a firm not resetting its price, again ignoring the idiosyncratic terms, is

\[ \hat{p}_{j,s,t}^{\text{noreset}} - \hat{P}_t = -\pi_t \]

\[ = -\Theta_\pi \tilde{r}_t. \] (38)

Using (37) and (38) in (36), taking a conditional expectation over \( \tilde{r}_{s,t} \), and simplifying terms yields

\[ E \left[ \hat{l}_{j,s,t} | \tilde{r}_{s,t} \right] = \beta_0^{\text{sticky}} + \beta_1^{\text{sticky}} m_{s,t}, \] (39)

where \( \beta_0^{\text{sticky}} = \frac{1}{1-\gamma} \left[ \Theta_Y + \frac{\alpha\sigma\lambda}{1-\alpha\beta\mu} \right] \tilde{r}_t \) and \( \beta_1^{\text{sticky}} = -\frac{\alpha\sigma\lambda}{[1-\gamma][1-\alpha\beta\mu]} \). By the Gauss-Markov theorem, an OLS regression of \( \hat{l}_{j,s,t} \) on \( m_{s,t} \) provides the best linear unbiased estimator of \( \beta_0^{\text{sticky}} \) and \( \beta_1^{\text{sticky}} \), implying that \( \beta_0^{\text{sticky}} \) and \( \beta_1^{\text{sticky}} \) give expressions for the OLS intercept and slope coefficient.

Equation (22) for the difference between the employment effects in partial and general
equilibrium continues to hold in this model. Accordingly,

\[
\text{Difference}^{\text{sticky}} = 1 - \left[ \frac{\alpha \beta \mu - 1}{\alpha \sigma \lambda} \right] \Theta_Y. \tag{40}
\]

D. Model with some firms excluded from credit markets

This appendix describes a model that relaxes the assumption that all firms borrow during the financial crisis. To capture that outcome, the model departs from the assumptions of firm exit and credit line requirements used in the text. Instead, a fraction \(1 - \zeta_{s,t}\) of firms on island \(s\) have access to credit markets and can set their payroll at an optimal level. Firms that cannot borrow set their employment to \(l_{j}^{c}\), with the superscript indicating “constrained” and the distribution of \(l_{j}^{c}\) exogenously given. At the firm level, the option to borrow takes the form of a Bernoulli random variable independent of other firm-level idiosyncratic shocks and the constrained employment level. It simplifies the notation without detracting from the argument to assume that firms that borrow face an interest rate of zero and that there is a common wage across firms.

Let \(p_{j,s,t}^*\) denote the price of an unconstrained firm:

\[
p_{j,s,t}^* = M \frac{w_t}{a_{j,s,t}}. \tag{41}
\]

Labor demand at unconstrained firms satisfies:

\[
l_{j,s,t}^* = \left[ \frac{\xi_{j,s,t}}{a_{j,s,t}} \left( \frac{M w_t}{(1 - \gamma) a_{j,s,t}} \right)^{-\sigma} C_t \right]^{\frac{1}{1 + \gamma(\sigma - 1)}}. \tag{42}
\]

Let \(\bar{l}_{t-1}\) denote the employment level at all firms in the \(t - 1\) steady-state, and \(\hat{l}_{j,s,t} = \frac{l_{j,s,t}}{\bar{l}_{t-1}} - 1\) the growth rate of employment. Then taking a conditional expectation of the growth rate over the health measure \(\zeta_{s,t}\) yields

\[
E \left[ \hat{l}_{j,s,t} | \zeta_{s,t} \right] = \bar{l}_t^* + \left[ \bar{l}_t^c - \bar{l}_t^* \right] \zeta_{s,t}, \tag{43}
\]

where \(\bar{x}_t\) denotes the time \(t\) unconditional average value of \(\hat{x}_{j,s,t}\). By the Gauss-Markov theorem, (43) implies that a regression of \(\hat{l}_{j,s,t}\) on \(\zeta_{j,s,t}\) will have an intercept \(\beta_0 = \bar{l}_t^*\) and a slope coefficient \(\beta_1 = \left[ \bar{l}_t^c - \bar{l}_t^* \right].\)
The partial equilibrium employment shortfall is:

\[
\text{Shortfall}^{PE} = \tilde{l}_{t-1} \left[ \tilde{l}_{t} - \tilde{l}^*_t \right] \int_0^1 \zeta_{s,t} ds \\
= \beta_1 \tilde{l}_{t-1} \int_0^1 \zeta_{s,t} ds.
\]  

The partial equilibrium employment gap is the difference between average employment at constrained and unconstrained firms, multiplied by the share of firms in the economy that are constrained.

The difference between the partial and general equilibrium effects is

\[
\text{Difference} = - \frac{\beta_0}{\beta_1 \int_0^1 \zeta_{s,t} ds}.
\]  

References


