Geographic Cross-Sectional Fiscal Spending Multipliers: What Have We Learned?
Online Appendix

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A. A Model of Cross-sectional Multipliers

This appendix derives cross-sectional government spending and transfer multipliers in a model economy. The setup and results closely follow Farhi and Werning (2016). My presentation makes a few functional form assumptions at the outset in order to streamline the derivations and provides sufficient algebraic detail to allow a reader to follow along with minimal interruption.

A.1. Setup

Time is continuous. The economy consists of a unit continuum of local areas inside a currency union. All areas have symmetric preferences. The objective of the model is to determine the relative change in output in a single local area when spending in that area rises or it receives a transfer from the rest of the economy. I will denote the single local region as “Home”. In describing the model’s equations, it will prove easiest in some cases to invoke the symmetry assumption and treat all areas other than “Home” as a composite rest-of-the-economy called “Foreign”. Since the Home region is infinitesimal, Foreign also corresponds to the total closed economy.

Residents in Home produce output $Y_{H,t}$ and consume $C_t$, where $C_t$ is an aggregate of consumption of Home output, $C_{H,t}$, and imports, $M_{H,t}$. All residents choose the same bundle of consumption and labor supply $L_t$. Formally, agents have intertemporal preferences:

\[
U_0 = \int_0^\infty e^{-rt} \left( \ln C_t - \frac{1}{1+\phi} L_t^{1+\phi} \right) dt,
\]

(A.1)
where:

**Home consumption:**
\[ C_t = C_{H,t}^{1-\alpha} M_{H,t}^{\alpha}, \quad (A.2) \]

**Imported consumption:**
\[ \ln M_{H,t} = \int_0^1 \ln M_{j,H,t}^{j} dj, \quad (A.3) \]

and \( M_{j,H,t}^{j} \) denotes imports from area \( j \). The parameter \( r \) is the discount rate and in equilibrium also the real interest rate in Foreign. The parameter \( \alpha \) controls the “Home bias”. The assumption of unitary elasticity of substitution in each of equations (A.2) and (A.3) simplifies substantially the algebra which follows without missing on the key economic concepts. Farhi and Werning (2016) provide expressions for non-unitary elasticities.

Total consumption in Foreign is given by:

**Foreign consumption:**
\[ C^*_t = C_{F,t}^{1-\alpha} M_{F,t}^{\alpha}. \quad (A.4) \]

Also define \( X_{j,t} = M_{j,F,t}^{j} \) as exports from \( j \) to \( F \).

Let \( P_{H,t} \) denote the price of a unit of \( Y_{H,t} \) (in terms of the common currency), \( P_{M,t} \) the price of the imported good, \( P_t \) the domestic CPI, and \( P^*_t \) the Foreign CPI, where:

**Imports price index:**
\[ \ln P_{M,t} = \int_0^1 \ln P_{j,t} dj, \quad (A.5) \]

**Domestic CPI:**
\[ P_t = P_{H,t}^{1-\alpha} P_{M,t}^{\alpha}, \quad (A.6) \]

**Foreign CPI:**
\[ P^*_t = P_{F,t}^{1-\alpha} P_{M,t}^{\alpha}. \quad (A.7) \]

By the symmetry assumption, \( P_{M,t} = P^*_t = P_{F,t} \).

Finally, the government can purchase Home output in quantity \( G_{H,t} \) at the same price as
private agents. The Home flow budget constraint is therefore:

$$\text{NFA: } \dot{N}_t = (P_{H,t} (Y_{H,t} - G_{H,t}) - P_t C_t) + i_t N_t, \quad (A.8)$$

where $N_t$ stands for the region’s net foreign assets, $i_t$ is the instantaneous nominal interest rate and is common across areas, and a dot over a variable denotes the derivative with respect to time.

The following first order and market clearing conditions obtain:

<table>
<thead>
<tr>
<th>Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local consumption</td>
<td>$C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t,$ (A.9)</td>
</tr>
<tr>
<td>Imports</td>
<td>$M_{H,t} = \alpha \left( \frac{P^*_t}{P_t} \right)^{-1} C_t,$ (A.10)</td>
</tr>
<tr>
<td>Exports</td>
<td>$X_{H,t} = \alpha \left( \frac{P_{H,t}}{P^<em>_t} \right)^{-1} C^</em>_t,$ (A.11)</td>
</tr>
<tr>
<td>Output market</td>
<td>$Y_{H,t} = C_{H,t} + X_{H,t} + G_{H,t},$ (A.12)</td>
</tr>
<tr>
<td>PPI inflation</td>
<td>$\pi_{H,t} = \frac{\dot{P}<em>{H,t}}{P</em>{H,t}},$ (A.13)</td>
</tr>
<tr>
<td>CPI inflation</td>
<td>$\pi_t = \frac{\dot{P}_t}{P_t},$ (A.14)</td>
</tr>
<tr>
<td>Euler equation</td>
<td>$\frac{\dot{C}_t}{C_t} = (i_t - \pi_t - r),$ (A.15)</td>
</tr>
<tr>
<td>No Ponzi</td>
<td>$N_0 = - \int_0^\infty e^{-\int_0^t i_s ds} (P_{H,t} (Y_{H,t} - G_{H,t}) - P_t C_t) dt, \quad (A.16)$</td>
</tr>
<tr>
<td>Backus-Smith</td>
<td>$P_t C_t = \Theta_t P^<em>_t C^</em>_t.$ (A.17)</td>
</tr>
</tbody>
</table>

Equations (A.9) to (A.11) follow from the first order conditions for within-period expenditure maximization. Equation (A.12) is the market clearing condition for purchases of Home output. Equations (A.13) and (A.14) are definitional. Equation (A.15) is the intertemporal Euler equation. Equation (A.16) requires that the initial net foreign assets (i.e. transfers) exactly
equal the present value of all current account deficits. Equation (A.17) implicitly defines \( \Theta_t \) as the expenditure gap between Home and Foreign.

I define the supply side of the economy directly in linearized form below.

**A.2. Linearized System**

At time \( t = 0 \), paths of government spending and any transfers are revealed; after \( t = 0 \) the economy is deterministic. I solve for a system of equations in the quantity and price of domestic output by linearizing around a steady state with no deviation of government spending and no transfers. Let variables without time subscripts denote the steady state values. For quantity variables \( Z_t \in \{ Y_{H,t}, C_t, C_{H,t}, X_{H,t}, G_{H,t}, N_t \} \), define the lower case variable \( z_t = \ln \left( \frac{Z_t}{Z} \right) \times \frac{Z}{Y_H} \approx \left( \frac{Z_t}{Z} - 1 \right) / Y_H \). Let \( G \) denote the steady state level of government spending, i.e. \( Y_H = C_H + X_H + G_H \) and \( G = G_H / Y_H \). For variables \( Z_t \in \{ P_{H,t}, P_{t}^*, P_t, \Theta_t \} \), define the lower case variable \( z_t = \ln \left( \frac{Z_t}{Z} \right) \). Because the local area is small, variables pertaining to the whole economy remain at their steady state level, i.e. \( c_t^* = p_t^* = 0 \) and \( i_t = r \ \forall t \).

**Backus-Smith wedge.** Take logs and differentiate equation (A.17) and substitute the Home Euler equation (A.15) and the Foreign equivalent:

Take logs of (A.17): \[ p_t + c_t = \theta_t + p_t^* + c_t^* \]

Time differentiate: \[ \pi_t + \frac{\dot{C}_t}{C_t} = \dot{\theta}_t + \pi_t^* + \frac{\dot{C}_t^*}{C_t^*} \]

Substitute (A.15): \[ i_t - r = \dot{\theta}_t + i_t - r \]

The requirement that the Home and Foreign pricing kernels both must price a bond with interest rate \( i_t \) implies \( \dot{\theta}_t = 0 \). Thus, any expenditure wedge \( \theta \) remains constant over time and
expenditure grows at the same rate in all regions. I therefore drop the $t$ subscript on $\theta$.

**Consumption Euler equation.** In the steady state, $C_H = (1 - \alpha) C$ and $X_H = M_H = \alpha C$, so $Y_H = C + G_H$ and $C/Y_H = (1 - G)$. The log-linearized Euler equation is therefore:

(A.15) and $C/Y_H = (1 - G)$:

$$\dot{c}_t = (1 - G) (\pi_t - \pi_t - r).$$  \hfill (A.18)

**Demand.** Substitute equations (A.9), (A.11) and (A.17) into equation (A.12), linearize, take a time derivative and substitute using equation (A.18) to find:

(A.9) and (A.11) into (A.12): $Y_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t + \alpha \left( \frac{P_{H,t}}{P_t^*} \right)^{-1} C_t^* + G_{H,t}$

(A.17) into above: $= (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t + \alpha \Theta^{-1} \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t + G_{H,t},$

Linearize: $y_{H,t} = - (1 - G) [\alpha \theta + (p_{H,t} - p_t)] + c_t + g_{H,t},$  \hfill (A.19)

Time differentiate: $\dot{y}_{H,t} = - (1 - G) (\pi_{H,t} - \pi_t) + \dot{c}_t + \dot{g}_{H,t}$

Substitute (A.18): $= - (1 - G) (\pi_{H,t} - \pi_t) + (1 - G) (\pi_t - \pi_t - r) + \dot{g}_{H,t}$

Simplify: $= (1 - G) (\pi_t - \pi_{H,t} - r) + \dot{g}_{H,t}.$  \hfill (A.20)

**Net foreign assets.** Next derive a relationship between the Backus-Smith wedge $\theta$ and the initial net foreign assets $n_0$ by linearizing equation (A.16) and using $i_t = r$:

Linearize (A.16): $n_0 = - \int_0^{\infty} e^{-rt} [(1 - G) (p_{H,t} - p_t) + y_{H,t} - (g_{H,t} + c_t)] dt$

Substitute (A.19): $= \int_0^{\infty} e^{-rt} (1 - G) \alpha \theta dt$

Evaluate integral: $= (1 - G) \frac{\alpha}{r} \theta.$  \hfill (A.21)
Intuitively, equation (A.21) states that the difference between Home and foreign expenditure, \( \theta \), is proportional to the initial net foreign asset position between the two.

**Supply side.** I omit a full description of the supply side of the model and instead directly assume a Phillips curve in Home output:

\[
\dot{\pi}_{H,t} = r\pi_{H,t} - \left[ \kappa (y_{H,t} - \Gamma g_{H,t}) + \lambda r n_0 \right], \tag{A.22}
\]

where \( \lambda > 0 \) is a parameter which is increasing in the amount of price flexibility, \( \kappa = \lambda \left( \frac{1}{1 - G} + \phi \right) \) where \( \phi \) is the labor Frisch elasticity defined in equation (A.1), and \( \Gamma = \frac{1}{1 + (1 - G)\phi} < 1 \) is the flexible price closed economy government purchases multiplier. One can derive this relationship from Calvo pricing.

**Initial condition.** To obtain an initial condition, note that the price level cannot jump, and so using equation (A.17):

\[
\text{Linearize (A.17):} \quad c_0 = (1 - G) \theta \tag{A.23}
\]

\[
\text{Substitute (A.21):} \quad = \frac{r}{\alpha} n_0. \tag{A.24}
\]

Evaluate equation (A.19) at time \( t = 0 \), again using the fact that \( p_{H,0} = p_0 = 0 \), and substitute equations (A.23) and (A.24) to find:

\[
\text{(A.19) at } t = 0: \quad y_{H,0} = - (1 - G) \alpha \theta + c_0 + g_{H,0}
\]

\[
\text{Substitute (A.23):} \quad = (1 - G) (1 - \alpha) \theta + g_{H,0}
\]

\[
\text{Substitute (A.24):} \quad = \left( \frac{1 - \alpha}{\alpha} \right) r n_0 + g_{H,0}. \tag{A.25}
\]
A.3. Impact Multipliers

The initial condition (A.25) fully characterizes the impact output multipliers for unanticipated transfers \( n_0 \) or government spending \( g_{H,0} \) which occur at time 0.

**Transfers, impact.** The impact transfer multiplier is:

\[
\beta_{\text{transfer,impact}} = \left( \frac{1 - \alpha}{\alpha} \right) r.
\]  

(A.26)

The annuity value of the transfer is \( rn_0 \). This transfer causes a direct, partial equilibrium increase in expenditure on local output of \((1 - \alpha)rn_0\). The additional increase in local income of \((1 - \alpha)rn_0\) causes a "second round" increase in expenditure on local output of \((1 - \alpha)^2rn_0\), and so on. In general equilibrium, therefore, domestic output rises in response to the transfer by \([\sum (1 - \alpha)^n] rn_0 = \frac{(1 - \alpha)}{\alpha}rn_0\). The increase in domestic income equals \(rn_0 + \frac{(1 - \alpha)}{\alpha}rn_0 = \frac{1}{\alpha}rn_0\), exactly the amount required for domestic agents to purchase an additional \(rn_0\) of output produced in other regions and keep the current account balanced. The equivalence between expenditure and output on impact follows because the price level does not jump.

**Government purchases, impact.** The impact government purchases multiplier is:

\[
\beta_{\text{x}, \text{impact, no transfers}} = 1.
\]  

(A.27)

Since prices cannot jump, there is no expenditure-switching effect for unanticipated purchases on impact. Since the model is Ricardian, there is no other change in private demand for Home output. Therefore, the impact output multiplier is 1.
A.4. Dynamic System Summary

It remains to solve the dynamic system to characterize multipliers at other horizons.

Let $z_{H,t} = y_{H,t} - g_{H,t}$ denote total private demand for Home output (in deviation from steady state). Combine equations (A.20) and (A.22) into a system of differential equations using $i_t = r$:

\[
\begin{pmatrix}
\dot{z}_{H,t} \\
\dot{\pi}_{H,t}
\end{pmatrix}
= \begin{pmatrix}
0 & -(1 - \mathcal{G}) \\
-\kappa & r
\end{pmatrix}
\begin{pmatrix}
z_{H,t} \\
\pi_{H,t}
\end{pmatrix}
- E_2 \left[ \kappa (1 - \Gamma) g_{H,t} + \lambda r n_0 \right],
\]

(A.28)

where $E_2 \equiv \begin{pmatrix} 0 & 1 \end{pmatrix}$. Equation (A.25) gives the initial condition of the system.

Equation (A.28) is a linear non-homogenous system of differential equations with two forcing variables, $g_{H,t}$ and $n_0$. The variable $g_{H,t}$ defines the path of government spending. The variable $n_0$ describes the magnitude of transfers. An outside-financed multiplier consists of a simultaneous increase in $g_{H,t}$ and transfer $n_0 = \int_0^\infty e^{-rt} g_{H,t} dt$. However, in a linear system the combined effect equals the sum of the separate effects. I therefore proceed by solving separately for the response of local output to each forcing variable.

As a preliminary step, define $A = \begin{pmatrix} 0 & -(1 - \mathcal{G}) \\
-\kappa & r \end{pmatrix}$ and diagonalize $A = FDF^{-1}$, where:

Eigenvectors of $A$:

\[F = \begin{pmatrix}
-(1 - \mathcal{G}) & -(1 - \mathcal{G}) \\
d_1 & d_2
\end{pmatrix}, \quad \text{(A.29)}\]

Eigenvectors of $A$:

\[D = \begin{pmatrix}
d_1 & 0 \\
0 & d_2
\end{pmatrix}, \quad \text{(A.30)}\]
\[ d_1 \equiv \frac{r - \sqrt{r^2 + 4(1 - \mathcal{G}) \kappa}}{2} < 0, \quad d_2 \equiv \frac{r + \sqrt{r^2 + 4(1 - \mathcal{G}) \kappa}}{2} > 0 \] are the eigenvalues of \( A \), and \( f_1, f_2 \) are the corresponding eigenvectors defined in equation (A.29) as \( F = \begin{pmatrix} f_1 & f_2 \end{pmatrix} \).

A.5. Transfer Multipliers

Consider first the case of a pure transfer, i.e. \( g_{H,t} = 0 \ \forall t \) and \( n_0 \neq 0 \). Then equation (A.28) is a linear non-homogenous system of differential equations with a constant coefficient \( E_2 \lambda r n_0 \). The generic solution to such a system is:

\[
\begin{pmatrix} z_{H,t} \\ \pi_{H,t} \end{pmatrix} = A^{-1} E_2 \lambda r n_0 + c_1 e^{d_1 t} f_1 + c_2 e^{d_2 t} f_2, \tag{A.31}
\]

where \( c_1 \) and \( c_2 \) are scalar constants. Discard the explosive term with the exponent of the positive root \( c_2 e^{d_2 t} f_2 \) and premultiply both sides by \( E_1' \equiv \begin{pmatrix} 1 & 0 \end{pmatrix} \) to obtain an expression for private output:

Premultiply (A.31):

\[
z_{H,t} = E_1' A^{-1} E_2 \lambda r n_0 + c_1 e^{d_1 t} E_1' f_1
\]

\[= -\frac{\lambda}{\kappa} r n_0 - c_1 (1 - \mathcal{G}) e^{d_1 t}, \tag{A.32}\]

where the second equality uses \( E_1' f_1 = -(1 - \mathcal{G}) \) and \( E_1' A^{-1} E_2 = -\frac{1}{(1 - \mathcal{G}) \kappa} \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} r & (1 - \mathcal{G}) \\ \kappa & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\frac{1}{\kappa} \).

Next evaluate equation (A.32) at \( t = 0 \) and equate to the initial condition equation (A.25) to solve for \( c_1 \):

Equate (A.32) at \( t = 0 \) to (A.25):

\[
\left( \frac{1 - \alpha}{\alpha} \right) r n_0 = -\frac{\lambda}{\kappa} r n_0 - c_1 (1 - \mathcal{G}),
\]

\[^1\text{The characteristic equation associated with the eigenvalues of } A \text{ sets the determinant of } A - dI = \begin{pmatrix} -d & (1 - \mathcal{G}) \\ -\kappa & r - d \end{pmatrix} \text{ to } 0, \ 0 = d^2 - rd - (1 - \mathcal{G}) \kappa, \text{ giving the two roots } d_1 \text{ and } d_2.\]
\[ c_1 (1 - G) = - \left( \frac{\lambda}{\kappa} + \frac{1 - \alpha}{\alpha} \right) r n_0. \]  
(A.33)

Substitute equation (A.33) into equation (A.32):

\[
z_{H,t} = \left[ e^{d_{1t}} \frac{1 - \alpha}{\alpha} - (1 - e^{d_{1t}}) \frac{\lambda}{\kappa} \right] r n_0
= \left[ e^{d_{1t}} \frac{1 - \alpha}{\alpha} - (1 - e^{d_{1t}}) \frac{1}{\frac{1}{1-\gamma} + \phi} \right] r n_0,
\]  
(A.34)

where the second equality uses \( \frac{\lambda}{\kappa} = \frac{1}{\frac{1}{1-\gamma} + \phi} \). Thus, the transfer multiplier is:

\[ \beta^{\text{transfer}}_t = \left[ e^{d_{1t}} \frac{1 - \alpha}{\alpha} - (1 - e^{d_{1t}}) \frac{1}{\frac{1}{1-\gamma} + \phi} \right] r. \]  
(A.35)

The annuity value of the transfer is \( r n_0 \). The term in brackets in equation (A.35) incorporates general equilibrium forces. The weight \( e^{d_{1t}} \) is declining over time \( (d_1 < 0) \) with speed determined by the degree of price flexibility (the \( \kappa \) term in the definition of \( d_1 \)). The first term in brackets \( (1 - \alpha)/\alpha \) translates the direct increase in local consumption demand when prices are sticky into local output. The second term captures the neoclassical wealth effect of a transfer on labor supply as prices adjust and is negative. Comparing the impact transfer multiplier in equation (A.26) to equation (A.35), the peak transfer multiplier occurs on impact.

Further intuition for equation (A.35) comes from computing the response of the price of local output. Following the same steps as above, one finds:

\[ \pi_{H,t} = - \left( \frac{1 - \alpha}{\alpha} + \frac{1}{\frac{1}{1-\gamma} + \phi} \right) d_1 e^{d_{1t}} r n_0. \]  
(A.36)

The price level solves the differential equation \( \dot{P}_{H,t} = \pi_{H,t} P_{H,t} \) with initial condition \( P_{H,0} = P_H \).
giving:

\[ p_{H,t} = \ln\left( P_{H,t} / P_H \right) = \int_0^t \pi_{H,h} dh + c_3 \]

\[ = - \left( \frac{1-\alpha}{\alpha} + \frac{1}{1-\varphi} + \phi \right) r n_0 \int_0^t d_1 e^{d_1 h} dh \]

\[ = (1 - e^{d_1 t}) \left( \frac{1-\alpha}{\alpha} + \frac{1}{1-\varphi} + \phi \right) r n_0. \] (A.37)

The second term in parentheses is positive and \( d_1 < 0 \). Thus, the price level rises over time in response to the transfer. Combining equations (A.34) and (A.37), total expenditure is:

\[ z_{H,t} + p_{H,t} = \left[ e^{d_1 t} \frac{1-\alpha}{\alpha} - (1 - e^{d_1 t}) \frac{1}{1-\varphi} + \phi + (1 - e^{d_1 t}) \left( \frac{1-\alpha}{\alpha} + \frac{1}{1-\varphi} + \phi \right) \right] r n_0 \]

\[ = \left( \frac{1-\alpha}{\alpha} \right) r n_0. \] (A.38)

Equation (A.38) defines the nominal expenditure transfer multiplier:

\[ \beta_{t,\text{transfer,nominal}} = \left( \frac{1-\alpha}{\alpha} \right) r. \] (A.39)

According to equation (A.39), total nominal expenditure on local output (in units of the national price level) jumps on impact and remains fixed thereafter. Total nominal income in the local region rises permanently by \( r n_0 + [(1-\alpha)/\alpha] r n_0 = (1/\alpha) r n_0 \), exactly the amount required for domestic agents to purchase an additional \( r n_0 \) of output produced in other regions and keep the current account balanced. As the price level of local output rises, however, a fixed increase in nominal expenditure implies an ever smaller (and eventually negative) effect on real output. This logic also clarifies why the transfer multiplier for real local output is highest on impact.
A.6. Government Spending Multipliers

Consider next the case where transfers \( n_0 = 0 \) but government spending in the local area deviates from the steady-state level. Then equation (A.28) is a linear non-homogenous system of differential equations with a nonconstant coefficient. The generic solution to this system is:

\[
\begin{pmatrix}
z_{H,t} \\
\pi_{H,t}
\end{pmatrix}
= \int_t^\infty e^{-A(h-t)} E_2 \kappa (1 - \Gamma) g_{H,h} dh + c_3 e^{d_1 t} f_1 + c_4 e^{d_2 t} f_2,
\]

(A.40)

where \( c_3 \) and \( c_4 \) are constants. Discard the explosive term with the exponent of the positive root \( c_4 e^{d_2 t} f_2 \) and premultiply both sides by \( E_1' \) to obtain an expression for private output:

Premultiply:

\[
z_{H,t} = E_1' \int_t^\infty e^{-A(h-t)} E_2 \kappa (1 - \Gamma) g_{H,h} dh + c_3 e^{d_1 t} E_1' f_1
\]

\[
= \frac{(1 - \mathcal{G})(1 - \Gamma) \kappa}{d_2 - d_1} \int_t^\infty \left(e^{-d_1(h-t)} - e^{-d_2(h-t)}\right) g_{H,h} dh - c_3 (1 - \mathcal{G}) e^{d_1 t},
\]

(A.41)

where the second equality again uses \( E_1' f_1 = -(1 - \mathcal{G}) \) and exploits the diagonalization of \( A \) to write:

\[
E_1' e^{-Ax} E_2 = E_1' F e^{-Dx} F^{-1} E_2 = \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix} \begin{pmatrix}
(1 - \mathcal{G}) & (1 - \mathcal{G}) \\
d_1 & d_2
\end{pmatrix} \begin{pmatrix}
e^{-d_1 x} & 0 \\
0 & e^{-d_2 x}
\end{pmatrix} \frac{1}{(1 - \mathcal{G})(d_1 - d_2)} \begin{pmatrix}
d_2 & (1 - \mathcal{G}) \\
-d_1 & -(1 - \mathcal{G})
\end{pmatrix} \begin{pmatrix}
0 \\
1
\end{pmatrix}
\]

\[
= - (1 - \mathcal{G}) \begin{pmatrix}
1 & 1 \\
0 & 0
\end{pmatrix} \begin{pmatrix}
e^{-d_1 x} & 0 \\
0 & e^{-d_2 x}
\end{pmatrix} \frac{1}{(1 - \mathcal{G})(d_1 - d_2)} \begin{pmatrix}
d_2 & (1 - \mathcal{G}) \\
-d_1 & -(1 - \mathcal{G})
\end{pmatrix} \begin{pmatrix}
1 \\
-1
\end{pmatrix}
\]

\[
= \frac{1 - \mathcal{G}}{d_2 - d_1} \left(e^{-d_1 x} - e^{-d_2 x}\right).
\]
Next evaluate equation (A.41) at $t = 0$ and equate to the initial condition equation (A.25) evaluated at $n_0 = 0$ to solve for $c_3$:

Equate (A.41) at $t = 0$ to (A.25):

$$0 = \frac{(1 - \mathcal{G})(1 - \Gamma) \kappa}{d_2 - d_1} \int_0^\infty \left( e^{-d_1 h} - e^{-d_2 h} \right) g_{H,h} dh - c_3 \left(1 - \mathcal{G}\right),$$

$$c_3 \left(1 - \mathcal{G}\right) = \frac{(1 - \mathcal{G})(1 - \Gamma) \kappa}{d_2 - d_1} \int_0^\infty \left( e^{-d_1 h} - e^{-d_2 h} \right) g_{H,h} dh.$$  \hfill (A.42)

Substitute equation (A.42) into equation (A.41):

$$z_{H,t} = \frac{(1 - \mathcal{G})(1 - \Gamma) \kappa}{d_2 - d_1} \left[ \int_t^\infty \left( e^{-d_1 (h-t)} - e^{-d_2 (h-t)} \right) g_{H,h} dh - e^{d_t} \int_0^\infty \left( e^{-d_1 h} - e^{-d_2 h} \right) g_{H,h} dh \right]$$

$$= -\frac{(1 - \mathcal{G})(1 - \Gamma) \kappa}{d_2 - d_1} \left[ \int_0^{t} e^{-d_1 (h-t)} \left(1 - e^{(d_1-d_2)h}\right) g_{H,h} dh + \int_t^\infty e^{-d_2 (h-t)} \left(1 - e^{(d_1-d_2)h}\right) g_{H,h} dh \right],$$  \hfill (A.43)

where in the second equality:

$$\int_t^\infty \left( e^{-d_1 (h-t)} - e^{-d_2 (h-t)} \right) g_{H,h} dh - e^{d_t} \int_0^\infty \left( e^{-d_1 h} - e^{-d_2 h} \right) g_{H,h} dh$$

$$= -e^{d_t} \int_0^t \left( e^{-d_1 h} - e^{-d_2 h} \right) g_{H,h} dh - \int_t^\infty \left[ \left( e^{-d_1 (h-t)} - e^{d_t (h-t) - d_2 h} \right) - \left( e^{-d_1 (h-t)} - e^{-d_2 (h-t)} \right) \right] g_{H,h} dh$$

$$= -\int_0^t \left( e^{-d_1 (h-t)} - e^{d_1 t - d_2 h} \right) g_{H,h} dh - \int_t^\infty \left( e^{-d_2 (h-t)} - e^{d_1 (h-t) - d_2 h} \right) g_{H,h} dh$$

$$= -\int_0^t e^{-d_1 (h-t)} \left(1 - e^{(d_1-d_2)h}\right) g_{H,h} dh - \int_t^\infty e^{-d_2 (h-t)} \left(1 - e^{(d_1-d_2)h}\right) g_{H,h} dh.$$

Equation (A.43) characterizes the change in private purchases of local output in period $t$ as the result of government spending which occurred in period $h$, absent transfers:

$$\beta_{t,h}^{\text{xsp, private, no transfers}} = \begin{cases} \frac{(1 - \mathcal{G})(1 - \Gamma) \kappa}{d_2 - d_1} e^{-d_1 (h-t)} \left(1 - e^{(d_1-d_2)h}\right), & h < t, \\ -\frac{(1 - \mathcal{G})(1 - \Gamma) \kappa}{d_2 - d_1} e^{-d_2 (h-t)} \left(1 - e^{(d_1-d_2)t}\right), & h > t. \end{cases}$$  \hfill (A.44)
According to equation (A.44), both past and anticipated future spending affects private purchases of local output. Thus, the equation defines the multiplier associated with spending in period $h$ at lag $t - h$. Crucially, the effect of government spending on private purchases of local output is negative at all horizons other than impact $h = t = 0$, when it is 0. In contrast, the closed economy output multiplier when monetary policy does not respond to government purchases is (weakly) above one (Woodford, 2011; Christiano et al., 2011; Farhi and Werning, 2016). Therefore, the local output multiplier is less than the closed economy output multiplier when monetary policy does not respond to government purchases.² As a corollary, matching empirical cross-sectional multipliers larger than one requires modification of the basic presented framework here. Nakamura and Steinsson (2014) and Farhi and Werning (2016) show alternative ways of accomplishing this task.

²Recently, Farhi and Werning (2016) show that the closed economy constrained monetary policy multiplier can be less than one in an economy with a fraction of hand-to-mouth agents if prices are sufficiently flexible and the Phillips curve sufficiently forward-looking that current consumption declines because of a decline in inflation in anticipation of a future recession when taxes rise. I do not consider that case further here.


B. Further Exploration of Results in Section 4

This appendix presents additional results related to the exploration in the main text of multipliers associated with ARRA spending. To start, table B.1 reproduces table 1 but showing the coefficients and standard errors for the included covariates. As a reminder, these specifications control for the employment change from 2007M12 to 2008M12, the growth rate of GSP from 2007Q4 to 2008Q4, and the 2008M12 employment level, where the employment change and level are normalized by dividing by the 2008M12 adult population. To ease interpretation, the included covariates are normalized to have unit variance.

Figure B.1 shows the time path of the employment multiplier. Each point in the plot represents the coefficient $\beta_h$ from a separate 2sls regression with second stage:

$$(Y_{s,t+h} - Y_{s,t}) = \alpha_h + \beta_h^{zs} F_s + \gamma_h' X_s + \epsilon_{s,h}, \quad \text{(B.1)}$$

and a fixed first stage:

$$F_s = \Pi_0 + \Pi_1' Z_s + \Pi_2' X_s + \nu_s, \quad \text{(B.2)}$$

where $Z_s$ contains the three instruments as in column (4) of table B.1. Thus, figure B.1 traces out an impulse response function. Summing the first 24 coefficients reproduces the point estimate of 1.99 in column (4) of table B.1. The positive coefficients after horizon 24 likely reflect continued outlays under the ARRA and the enactment of additional federal stimulus measures after the ARRA. According to Council of Economic Advisers (2014, p.101), such additional measures accounted for an additional $709 billion in spending, transfers, or tax reductions with the vast majority coming in 2011 and 2012. Some of these measures extended
## Table B.1: ARRA Example, All Coefficients

<table>
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<tr>
<th>Endogenous variable:</th>
<th>Job years per $100K spent</th>
<th>GSP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2) (3) (4) (5)</td>
<td></td>
</tr>
<tr>
<td>Total ARRA spending</td>
<td>2.29 2.22 1.82 2.01 1.53</td>
<td>1.53</td>
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<td></td>
<td>(0.71) (1.22) (0.69) (0.59)</td>
<td>(1.19)</td>
</tr>
<tr>
<td>Dec-08 employment/population 16+</td>
<td>-3.89 -3.89 -3.87 -3.88 0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(2.51) (2.48) (2.52) (2.52)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Employment change, Dec-07 to Dec-08</td>
<td>12.07 12.06 12.02 12.04 0.04</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(2.94) (2.88) (2.93) (2.93)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>GSP change, 2007Q4-2008Q4</td>
<td>2.78 2.74 2.52 2.62 0.04</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(3.35) (2.92) (3.33) (3.32)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Instruments</td>
<td>FMAP DOT DM ALL ALL</td>
<td></td>
</tr>
<tr>
<td>Estimator</td>
<td>2sls 2sls 2sls 2sls 2sls</td>
<td></td>
</tr>
<tr>
<td>First stage coefficient</td>
<td>0.36 1.66 6.76 . .</td>
<td></td>
</tr>
<tr>
<td>First stage F statistic</td>
<td>35.9 9.8 52.0 46.1 129.3</td>
<td></td>
</tr>
<tr>
<td>First stage $R^2$</td>
<td>0.40 0.23 0.55 0.73 0.87</td>
<td></td>
</tr>
<tr>
<td>Hansen J statistic p-value</td>
<td>0.76 0.34 0.34 0.34 0.34</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.49 0.49 0.49 0.49 0.31</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>50 50 50 50 50</td>
<td>50</td>
</tr>
</tbody>
</table>

Notes: This table reproduces the estimates in table 1 in the main text but includes the coefficients and standard errors for the included covariates. The table reports cross-state regressions of the effect of ARRA spending on employment (columns 1-4) or gross state product (column 5) during 2009 and 2010. ARRA spending is instrumented using pre-recession Medicaid spending ($FMAP$), Department of Transportation formula ($DOT$), and other pre-recession formulae ($DM$) as described in the main text. All specifications also control for the employment change from December 2007 to December 2008 normalized by the December 2008 population 16+, gross state product (GSP) growth from the fourth quarter of 2007 to the fourth quarter of 2008, and the December 2008 ratio of employment to the population 16+. In columns (1)-(4) Total ARRA spending and the instruments are normalized by the December 2008 population 16+. In column (5), Total ARRA spending and the instruments are normalized by 2008Q4 GSP. All variables except ARRA spending are normalized to have unit variance. Eicker-White standard errors in parentheses. Following AEA guidelines, symbolic indicators of significance are omitted.

Table B.2 explores robustness to adding other covariates sometimes used in the literature. To keep the presentation manageable, I start from column (4) of table B.1 which reports the elements contained in the instruments used in table B.1. Because the enactment of most of these measures occurred late in 2010 or after, they plausibly do not affect the estimated multipliers through 2010. However, they do affect employment and output after 2010.
Figure B.1: Employment Impulse Response Function

Notes: The figure plots each coefficient $\beta_h$ from the 2sls regression with second stage dependent variable $Y_{s,t+h} - Y_{s,t}$. ARRA spending is instrumented using pre-recession Medicaid spending (FMAP), Department of Transportation formula (DOT), and other pre-recession formulae (DM) as described in the main text. All specifications also control for the employment change from December 2007 to December 2008 normalized by the December 2008 population 16+, gross state product (GSP) growth from the fourth quarter of 2007 to the fourth quarter of 2008, and the December 2008 ratio of employment to the population 16+. The dashed lines indicate 90% confidence bands based on Eicker-White standard errors.

job-years multiplier of the ARRA based on three instruments drawn from Chodorow-Reich et al. (2012), Wilson (2012), and Dupor and Mehkari (2016). Column (1) of table B.2 reproduces the result in column (4). Columns (2)-(5) add a number of commonly included covariates in accumulating fashion. Many papers have recognized the spatial component of the house price boom and bust around the Great Recession and included controls for house price growth (Chodorow-Reich et al., 2012; Wilson, 2012; Conley and Dupor, 2013; Dupor, 2013). Column (2) therefore controls separately for house price growth during the pre-recession period (December 2003 to December 2007) and the first year of the recession (December 2007 to December 2008) using the log change in the FHFA purchase-only house price index. Column (3) follows Wilson (2012) and adds the change in the three-year moving average of personal income per capita between 2005 and 2006, a component of the hold-harmless provision of the ARRA FMAP spending.
Column (4) follows Chodorow-Reich et al. (2012); Wilson (2012); Dupor (2013); Dupor and Mehkari (2016) and adds the manufacturing share of employment in December 2008. Finally, column (5) adds census region fixed effects. None of these specifications has a material effect on the cost-per-job estimate, with the implied cost ranging from $50,000 (column 2) to $65,000 (column 5).

Table B.3 explores how the ARRA transfers affected state and local finances. The regression specification mirrors the baseline employment specification in column (4) of table 1 but replaces the dependent variable with the (per capita) change in state and local expenditure (columns 1-3, defined as the sum of current operations, construction, and other capital outlays) or taxes (columns 4-6). The state and local finances data come from the Census Bureau. The coefficient in column (1) has the interpretation that an additional $1 of ARRA transfers during 2009 or 2010 increases expenditure by a total of $1.22 during FY2009 and FY2010. Columns (2) and (3) show that slightly more of the increased expenditure occurs in FY2010 than FY2009.\(^3\) Columns (3)-(6) repeat the exercise for taxes and find a negligible effect on state and local tax revenue.

\(^3\)The endogenous variable contains all ARRA spending, including transfers to state and local governments, transfers to persons such as unemployment insurance payments, and direct federal purchases. However, the instruments consist entirely of ARRA transfers to state and local governments. Thus, the coefficient of 1.22 gives the increase in state and local spending when ARRA spending in a state is higher by $1 of transfers (Imbens and Angrist, 1994).
Table B.2: ARRA Example, Robustness

<table>
<thead>
<tr>
<th>Dependent variable: job years (thousands)</th>
<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<tbody>
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<td>Right hand side variables:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total ARRA Spending (hundreds of millions)</td>
<td>2.01</td>
<td>1.95</td>
<td>1.97</td>
<td>1.86</td>
<td>1.53</td>
</tr>
<tr>
<td></td>
<td>(0.59)</td>
<td>(0.53)</td>
<td>(0.48)</td>
<td>(0.50)</td>
<td>(0.66)</td>
</tr>
<tr>
<td>Dec-08 employment/population 16+</td>
<td>−3.88</td>
<td>−3.76</td>
<td>−3.48</td>
<td>−3.50</td>
<td>−1.26</td>
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<tr>
<td></td>
<td>(2.52)</td>
<td>(2.42)</td>
<td>(1.85)</td>
<td>(1.71)</td>
<td>(1.77)</td>
</tr>
<tr>
<td>Employment change, Dec-07 to Dec-08</td>
<td>12.04</td>
<td>11.07</td>
<td>13.74</td>
<td>14.12</td>
<td>11.92</td>
</tr>
<tr>
<td></td>
<td>(2.93)</td>
<td>(2.70)</td>
<td>(3.48)</td>
<td>(3.05)</td>
<td>(2.88)</td>
</tr>
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<td>GSP change, 2007Q4-2008Q4</td>
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<td>3.46</td>
<td>4.03</td>
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<td></td>
<td>(3.32)</td>
<td>(3.31)</td>
<td>(2.37)</td>
<td>(2.23)</td>
<td>(2.04)</td>
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<td>Log change in HPI, Dec-07 to Dec-08</td>
<td>2.41</td>
<td>0.33</td>
<td>1.21</td>
<td>−0.30</td>
<td>−0.30</td>
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<tr>
<td></td>
<td>(1.94)</td>
<td>(2.48)</td>
<td>(2.32)</td>
<td>(2.73)</td>
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<tr>
<td>Log change in HPI, Dec-03 to Dec-07</td>
<td>1.43</td>
<td>4.12</td>
<td>3.29</td>
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<td>(1.20)</td>
<td>(1.79)</td>
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<td>Change in 3 year m.a. of PI per cap., 2005 to 2006</td>
<td>−5.28</td>
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<td>(2.18)</td>
<td>(1.99)</td>
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<td>Manufacturing share of employment, Dec-08</td>
<td>−3.64</td>
<td>−3.59</td>
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<td></td>
<td>(1.67)</td>
<td>(1.94)</td>
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<td>Census region 2</td>
<td>−9.27</td>
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<td>(3.66)</td>
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<td>Census region 3</td>
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<td></td>
<td>(4.24)</td>
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<td>Census region 4</td>
<td>−15.41</td>
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<td>First stage F statistic</td>
<td>46.1</td>
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<td>41.6</td>
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<td>0.60</td>
<td>0.63</td>
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Notes: The table reports cross-state regressions of the effect of ARRA spending on employment during 2009 and 2010. ARRA spending is instrumented using pre-recession Medicaid spending (FMAP), Department of Transportation formula (DOT), and other pre-recession formulae (DM) as described in the main text. All variables except ARRA spending and the census region fixed effects are normalized to have unit variance. Eicker-White standard errors in parentheses. Following AEA guidelines, symbolic indicators of significance are omitted.
Table B.3: ARRA Example, State and Local Finances

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<th></th>
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<td>Total ARRA spending</td>
<td>1.22</td>
<td>0.51</td>
<td>0.71</td>
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<td>(0.41)</td>
<td>(0.27)</td>
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<tr>
<td>$R^2$</td>
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<td>0.20</td>
<td>0.15</td>
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</tbody>
</table>

Notes: The table reports cross-state regressions of the effect of ARRA spending on state and local expenditure and taxes during 2009 and 2010. ARRA spending is instrumented using pre-recession Medicaid spending ($FMAP$), Department of Transportation formula ($DOT$), and other pre-recession formulas ($DM$) as described in the main text. All specifications also control for the employment change from December 2007 to December 2008 normalized by the December 2008 population 16+, gross state product (GSP) growth from the fourth quarter of 2007 to the fourth quarter of 2008, and the December 2008 ratio of employment to the population 16+. Eicker-White standard errors in parentheses. Following AEA guidelines, symbolic indicators of significance are omitted.
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